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Frege's Epistemic Criterion of Thought Individuation

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Abstract

Frege believes that the content of declarative sentences divides into a thought and its 'colouring', perhaps combined with assertoric force. He further thinks it is important to separate the thought from its colouring. To do this, a criterion which determines sameness of sense between sentences must be deployed. But Frege provides three criteria for this task, each of which adjudicate on different grounds. In this article, rather than expand on criticisms levelled at two of the criteria offered, the author focuses on the most promising candidate. As it stands, this criterion has problems, but not insuperable ones. He suggests an adjusted criterion that relies on the epistemic notion of triviality. He recommends this criterion as both harmonious with Frege's broader thought and preferable to alternatives offered. The moral is that Frege individuates thoughts by deploying an epistemic concept, and this is the only suitable way for him to do so.

Keywords

Frege – equipollence – thoughts – sense – criteria – epistemology

Frege believes that the content of declarative sentences divides into a thought and its 'colouring', perhaps combined with assertoric force (CP, 356 [1918–1919, 63]; PW, 198 [1906, 214]; PMC, 67 [1906b, 102]).¹ He further thinks it is important, from a logical point of view, to separate the thought from its colouring. To do this, a criterion which determines sameness of sense between sentences

1 Translated works of Frege will be referenced with shorthand. A key to which can be found in the bibliography. I also include a reference to German texts in square brackets.

must be deployed. The trouble is that Frege provides three such criteria, each of which determines sameness of sense on different grounds. Worse, the first criterion is insufficiently developed, the second collapses the sense-reference distinction, and the third seems either circular or psychologistic.

In this article, rather than expand on criticisms levelled at two of the criteria offered, I focus on the most promising candidate: the criterion of immediate recognition. As it stands, this criterion has problems, but not insuperable ones. By drawing on Frege's views on sense individuation, I suggest an adjustment of the criterion that relies on the epistemic notion of triviality:

*Equipollence:*² Two sentences 'A' and 'B' are equipollent *iff* it is trivial that A *iff* B.

I recommend this criterion as both harmonious with Frege's broader thought and preferable to alternatives offered. The moral is that Frege individuates thoughts by deploying an epistemic concept, and this is the only suitable way for him to do so.

This article divides as follows: §1 provides relevant background. §2 briefly examines the criteria based on consequence. §3 formulates and then examines the immediate recognition criterion. §4 introduces and refines my recommended criterion based on triviality. §5 contrasts it with others.

1 Equipollence, Thoughts, and Colouring

Frege believes declarative sentences (henceforth, the qualification 'declarative' will be dropped) have content. That part of content relevant to truth is the thought. While Frege calls "anything else that goes to make up the content of a sentence the *colouring* of the thought" [my emphasis] (PW, 198 [1906, 214]). (Frege sometimes uses other terms for colouring, such as 'trappings', 'illumination', 'scent', or 'shading', but I shall be consistent.) Declarative sentences, when asserted, also contain assertoric force. The assertoric force "goes hand in hand with the communication of the content" to assert its truth (PW, 197 [1906, 213]). Because thoughts are only part of content, given distinct sentences

2 It is commonplace in the literature to refer to these criteria as 'equipollence criteria'. I shall do the same. I use 'equipollence' to refer (innocently) to a relation between sentences that holds *iff* they express the same thought. As Frege says: "equipollent propositions [sentences] have something in common in their content, and this is what I call the thought they express." (PMC, 67 [1906b, 102]). That this is a term Frege deploys for such a relation without prejudice of cause has been questioned (Sander, 2016, 420–421). I, at least, make no such prejudice.

with distinct contents, it is possible that they express the same thought (PW, 143 [1897, 154]).

If two sentences express the same thought, Frege says that they are *equipollent*. In *Thoughts*, he points to the case of a person saying, 'Alfred has still not come'. In this utterance, he distinguishes between the content that could be communicated by saying merely 'Alfred has not come' and the content the word 'still' adds to it, which hints that the arrival of Alfred is expected. The latter content is not truth-relevant, he says, only the former, since even if Alfred's arrival were not expected, that would not make 'Alfred has still not come' false (CP, 357 [1918, 64]). 'Alfred has still not come' has true content iff 'Alfred has not come' does. The thought, since it is solely the content that is truth-relevant, is the content of 'Alfred has not come'. The content of 'still' in 'Alfred has still not come' is not part of this thought but its colouring. So, 'Alfred has still not come' and 'Alfred has not come', though different sentences with different content, express the same thought, and are thus equipollent.

Many and varied examples of equipollence appear throughout Frege's work. They include natural language cases, such as where one sentence translates the other, e.g. 'Olives are delicious' and 'Oliven sind köstlich', or where one sentence is in active form while the other is in passive form, e.g. 'The Greeks defeated the Persians' and 'The Persians were defeated by the Greeks', or where one sentence uses a neutral term while the other has a term with 'certain associations', e.g. 'this dog howled through the night' and 'this cur howled through the night'. He also repeatedly claims that any sentence '*A*' is equipollent to the sentence 'it is true that *A*'. Other examples concern mathematics. Thus '*a* is parallel to *b*' and 'the direction of *a* is the same as the direction of *b*' are equipollent, so are the trio of sentences 'there is at least one square root of 4', 'the concept *square root of 4* is realised' and 'the number 4 has the property that there is something of which it is the square' (CN, 112 [1879, 2–3]; FOA, 74–75 [1884, 74–75]; CP, 164 [1892, 34], 188–189 [1892b, 199–200], 354–357 [1918, 61–64]; PW, 129, 140–141 [1897, 140, 151–153], 194 [1906a, 210–211], 233–234 [1914, 251–252]).

The diverse examples of colouring Frege presents have led to debate regarding its nature (e.g. Horn, 2008, 2013; Sander, 2019). But he seemed uninterested in such questions and spoke of colouring only to set it aside as irrelevant. I shall follow his lead.

In the following sections I examine the explicit criteria Frege offers for determining equipollence. These criteria appeal only to whole contents of sentences and judgements. Some authors, however, believe Frege is committed, explicitly or implicitly, to a notion of equipollence dependent on alike structure of the sentences and/or the thoughts they express, or dependent on equating the senses of subsentential elements (Kemmerling 2011; Künne 2003; Rumfitt 2016; Dummett

1981b). I disagree with this approach but have no space to properly argue my case here. I will say only that Frege gives clear examples of equipollent sentences that differ in terms of their structure (see above) and makes no appeal to structure when formulating his criteria. So, given that I intend to provide a suitable criterion without such an appeal, I suggest an approach that includes considerations of structure be considered only if my proposal is rejected. That said, I now turn to the equipollence criteria Frege offers based on logical consequence.

2 The Consequence Criteria

Frege presents three different criteria for equipollence in his work. The earliest, in *Begriffsschrift* of 1879, says that two sentences are equipollent if judgements concerning their contents, when combined with other judgements, have the same consequences (CN, 112–3 [1879, 2–3]). It is broadly agreed that Frege's notion of consequence at this time was underdeveloped. And since another of his criteria is based on consequence in a more specified way, I do not discuss this one for the sake of space. (If interested, see Beaney 1996, 56–57, 225–226; 2007, 100; Kremer, 2010, 222–223; Potter, 2020, 43; Sullivan, 2004, 684; Tennant, 2003).

The other criterion that deploys the notion of consequence appears in a letter to Husserl, written in 1906, and is based on logical consequence as determined by logical laws (PMC, 70–1 [1906d, 105–106]). Roughly, two sentences are equipollent iff logical laws can demonstrate a contradiction from assuming the sentences have different truth values (some conditions apply).

There are two major problems with this criterion that render it unworkable. First, the requirement that the contradiction be established by logical laws is at odds with one of Frege's motivations. It places the cart before the horse. Frege wants to put the criterion to use in separating the thought from its colouring to help determine logically relevant features of the sentential structure of natural language. He even calls this the “first and most important task” for logicians (PW, 143 [1897, 154]). If so, the criterion must be of use *before* the logical laws have been established.

Second, the criterion seems wholly unfit for the purpose. Jean van Heijenoort argues that it, catastrophically, does far more than Frege would want when the sentences contain only 'logical notions'. As an example, he says it makes ' $2^2 = 4$ ' and ' $2 + 2 = 4$ ' have the same sense. For it can be shown using (Frege's) logical laws alone that they must have the same truth-value. Van Heijenoort calls this an “unwanted conclusion, which directly contradicts what Frege says about sense” (1977, 106).

Van Heijenoort's objection seems correct (although see Beaney, 1996, 299). But the problem is more widespread than he realises. While the criterion offered here overreaches in cases where only logical notions are involved, it underreaches in many cases where non-logical notions are involved. Consider some Fregean examples:

- (a) 'This dog howled' and 'this cur howled'
- (b) 'The Greeks defeated the Persians' and 'the Persians were defeated by the Greeks'

The first example pair is of the form '*Fa*' and '*Fb*'. Using logical laws alone, a contradiction can be derived from assuming these sentences to have differing truth-values iff ' $a = b$ ' follows from logical laws. Hence the criterion declares the sentences equipollent only under such conditions. Now, ' $a = b$ ' follows from logical laws alone in one of two cases: either '*a*' and '*b*' are names for objects that can be shown identical by logical laws (such as numbers) or '*a*' and '*b*' are the same name. And since elsewhere Frege says '*Fa*' and '*Fb*' are equipollent iff '*a*' and '*b*' have the same sense (PW, 255–256 [1919, 276]), this criterion individuates the senses of some names according to their referent while it individuates the senses of other names according to their sign (see also Sullivan, 2004, 684). The second example pair would also be judged not equipollent if the criterion were strictly applied for similar reasons. It seems to be of the form '*Rab*' and '*Sba*'. By using logical laws alone, there is no way to derive a contradiction from assuming them to have different truth-values.

Certainly, if it were independently argued that, say, 'this dog' and 'this cur' had the same sense then the criterion would produce the correct result for the first pair of sentences, since identity of sense establishes identity of reference. But it would seem clear in this case that the heavy lifting had already been done before the criterion was applied. The thing at issue with the sentences is whether or not 'this dog' and 'this cur' have the same sense. Besides, if independent results can be imported prior to the criterion being applied, there seems little to stop an argument for the co-reference of 'the morning star' and 'the evening star' being used to erroneously justify, via the consequence criterion, the equipollence of 'The morning star is a body illuminated by the Sun' and 'The evening star is a body illuminated by the Sun' (which Frege insists are not equipollent (CP, 162 [1892b, 32])). So it seems the consequence criterion is at best a distraction regarding issues of sense, and at worst plain wrong.

The above considerations points towards the fundamental problem with consequence criteria. Logical consequence only relates to reference-level and sign-level considerations. Yet Frege introduces the level of sense to account for the epistemic value of judgement, which is a matter independent of reference

and sign. Reference contributes only to the truth (or otherwise) of a sentence. While the connection between proper name and object referred to is “arbitrary”, so distinctions of sign need not equate to a distinction in sense (CP, 157–158 [1892a, 25–26]). Senses of proper names must be *more* fine-grained than the objects to which the names refer, as well as *less* fine-grained than proper names.³ Consider, for example, the names ‘this horse’ and ‘this nag’, which Frege says share a sense, and ‘the morning star’ and ‘the evening star’, which he says have distinct senses (CP, 157–158 [1892a, 26–27], 357 [1918, 63]). Here we have four names, three senses, and two objects.

The problem with the logical consequence criteria, then, has to do with the inability of sign-level and reference-level considerations to determine sense-level individuation. If there is to be a suitable criterion of sense individuation, it must draw on sense-level notions. Frege’s remaining criterion has potential in that regard. It will be our focus henceforth.

Note that the above discussion does not amount to a demonstration that Frege realised a consequence criterion conflicts with his other claims regarding sense. And it is remarkable that this criterion was penned in the same year as the one we will next examine, which draws on very different considerations. What we are to make of this diversity splits opinion. Michael Dummett simply labels the consequence criterion an “aberration” (1981b, 325), while Carlo Penco (2003) argues that the two bases for the criteria spring from distinct and incompatible notions of sense that Frege held concurrently (see also Blanchette, 2012, 29–51). I take no stand on this here. My claim is only that if there is to be a criterion of equipollence consistent with Frege’s considered views on sense, a logical consequence criterion is not it.

3 The Immediate Recognition Criterion

The criterion we now turn to has received the most attention in the literature. It is found in *A Brief Survey of my Logical Doctrines* (PW, 197–8 [1906b, 213–4]):

Two sentences *A* and *B* can stand in such a relation that anyone who recognises the content of *A* as true must also recognise the content of *B* as true and, conversely, that anyone who accepts the content of *B* must straightaway accept that of *A*. (*Equipollence*). It is here being assumed that there is no difficulty in grasping the content of *A* and *B* ... I assume

3 At least, this is so if we ignore the fraught issues that surround proper names like ‘Aristotle’, that Frege says may have multiple senses (CP, 158 fn. 4 [1892b, 27]). This issue does not affect the point I wish to make.

there is nothing in the content of either of the two equipollent sentences *A* and *B* that would have to be immediately accepted as true by anyone who had grasped it properly ... [the thought] is the same in equipollent sentences of the kind given above.⁴

This criterion, often named the ‘immediate recognition criterion’, has been formulated in various ways. My first task will be to clarify the most appropriate formulation. After that I will examine it.

The passage under consideration describes conditions said to be sufficient for sentences to be equipollent. But it is also clear that Frege took them to be necessary (PMC, 80 [n.d., 128]; CP, 162 [1892b, 32]). Indeed, Gareth Evans (1982, 18–19) describes the criterion purely in terms of its necessity for equipollence, and, as several others have pointed out, Frege predominantly applies the criterion to distinguish, rather than identify, thoughts. I therefore treat formulations as offering both necessary and sufficient conditions for equipollence.

I begin with a formulation derived from those put forward (separately) by Susanna Schellenberg (2012) and Mark Textor (2018a). Their formulations are similar, and tantamount to:

*Recognition*₁: Two sentences ‘*A*’ and ‘*B*’ are equipollent *iff* if a person acknowledges the content of ‘*A*’ as true, they must immediately acknowledge the content of ‘*B*’ as true (and *vice versa*).

(Proviso: It is assumed (i) that ‘*A*’ and ‘*B*’ do not have a component part of their content that is logically self-evident, and (ii) that neither ‘*A*’ nor ‘*B*’ have content that is difficult to grasp.)⁵

Proviso (i) is added to ensure sentences that express self-evident truths, and those with parts that do, are not thereby classed as equipollent, e.g. ‘*A* or not-*A*’ and ‘*B* or not-*B*’, or ‘*A*’ and ‘*A* and (*B* or not-*B*)’. This proviso will be discussed no further and will be taken as read.

Now, according to both Textor and Schellenberg there is a problem with proviso (ii). They think Frege introduces it to block an obvious complaint. A person might recognise the content of ‘*A*’ to be true, but not that of ‘*B*’, because they simply do not grasp the content of ‘*B*’. This might suggest sentences that ought to be equipollent are not. By restricting the criterion to sentences that have contents that are easily graspable, this situation should not occur. However,

4 The word ‘thereby’ is omitted in the translated quotation as it does not appear in the German original.

5 Textor has an additional third proviso. It is minor, so for the sake of space I ignore it.

both see this as an unhelpful restriction on the sentences that can be covered by the criterion. They claim the criterion should be able to adjudicate between sentences with complex content as well.

The complaint, if it were accurate, would be far more devastating than Textor and Schellenberg suggest. It would render the criterion utterly useless. For Frege explicitly intends the criterion to take account of everyone: "anyone who recognises..." (PW, 197 [1906b, 213]). If proviso (ii) is to restrict sentences covered by the criterion to those that have content easy to grasp, then, to be effective, it would have to restrict it to sentences with content that can be grasped by *everyone*, including children, the mentally disabled, and even those who do not speak the language one of the sentences is spoken/written in. If, for example, the sentences were

- (A) Sweeties are yummy.
- (B) Confectionary is delectable.

then 'B' would have content that would be classified as too difficult to grasp; for a child might recognise the content of 'A' to be true, without immediately recognising the content of 'B' as true. Without proviso (ii), the criterion would brand the expressed thoughts distinct, when (I assume) they ought not to be. Similarly, the content of 'A' would have to be classed as too difficult to grasp; for a monoglot German might not immediately recognise it as true upon recognising the truth of the content of 'Süßwaren sind köstlich'. If (ii) makes the criterion inapplicable to sentences as simple as 'Sweeties are yummy', the criterion is useless.

I propose, instead, that the formulation is to blame. Frege is not placing a proviso on the sentences covered by the criterion but a restriction on the scope of quantification in the criterion. He restricts the scope to agents that grasp the contents of both sentences, and this prevents the problem. I suggest, therefore, he intends the following:

Recognition₂: Two sentences 'A' and 'B' are equipollent *iff* anyone who grasps the contents expressed by 'A' and 'B' and judges that A must immediately judge that B (and *vice versa*).

Currently, though, the criterion has a blind spot regarding contents that people refrain from judging, perhaps because they are widely known to be false. For example, it is conceivable that anyone able to grasp the content of 'dogs read philosophy' and the content of 'philosophy is read by dogs' would refrain from judging these contents true. But it also seems as though these sentences should be equipollent. And it is no good declaring the case where nobody falls under

the scope of quantification for the pair of sentences to be a case in which equipollence holds. For then ‘dogs read philosophy’ would be declared equipollent to ‘cats read philosophy’.

This problem can be corrected by replacing ‘that’ with ‘whether’ (and suitably tweaking the suppressed proviso to include contradictions):

Recognition₃: Two sentences ‘A’ and ‘B’ are equipollent *iff* anyone who grasps the contents expressed by ‘A’ and ‘B’ and judges whether A must immediately judge the same concerning whether B (and *vice versa*).

Recognition₃ is now a suitable formulation of Frege’s intended criterion. In this formulation, equipollence is not something determined by a single case, but is a broader feature of language, demonstrated by the cognitive acts of proficient speakers.

Now that we have a suitable formulation, it is time to examine its credentials. Michael Beaney (1996, 233) sees a major problem with it:

If the ‘content’ of a proposition is precisely the ‘thought’ expressed ... then, if two propositions do express the same thought, to grasp the ‘content’ of one is *ipso facto* to grasp the ‘content’ of the other, and if it is the ‘content’ that we recognize as true or false, then we automatically recognize the ‘content’ of one as true if we recognize the ‘content’ of the other as true. Not only does this make the criterion useless, but it also violates the constraints on an adequate criterion: it is unacceptable to presuppose on the right hand side of the biconditional precisely that notion that we are attempting to specify.

Textor (2018) replies on Frege’s behalf. He disagrees with Beaney’s assumption that the content of a sentence is the thought expressed. Textor is correct. The content of a sentence is not simply the thought it expresses since it includes colouring. Textor thinks this observation avoids the problem. But there is trouble in the vicinity, as I will show.

A criterion of equipollence is intended, in part, to be used to separate the thought from its colouring. The method invoked in *Recognition₃* involves considering everyone that is able to grasp the content of two sentences, and then determine whether or not they, upon recognising one content to be true (or false), immediately recognise the other content to be true (or false).⁶ If they do,

⁶ More correctly, recognise either the content or the contradictory content as true (CP, 385 [1919b, 154]; PW, 185 [1906a, 201]).

then, as the key passage in *A Brief Survey of my Logical Doctrines* continues to say: “whatever distinguishes the content of *A* from that of *B*, does not belong to what is accepted as true; for if it were the case, then it could not be an immediate consequence of anyone’s accepting the content of *B* that he should accept that of *A*” (PW, 197 [1906b, 213]).

At the start of his discussion, Frege presents the *content* as that which is recognised as true: “anyone who recognises the content of *A* as true must ...”. But his conclusion says, “whatever distinguishes the content of *A* from that of *B*, does not belong to what is accepted as true”, i.e. what is recognised as true is the *thought*, for the colouring does not belong to that which is accepted as true. That is one ambiguity. Another is whether there are two acts of judgement or only one. The words ‘straightaway’ and ‘immediately’ used in the passage under consideration suggest two judgements in quick succession. But when writing to Russell, Frege claims that a single act of judgement is a hallmark of equipollence (PMC, 157–158 [1903, 240]):

Now the thoughts contained in those propositions are evidently different; for after having recognised the first as true, we still need a special act to recognise the second as true. If we had the same thought, there would be no need for two acts of recognition but only for a single one.

This is a second ambiguity. The question is whether, if Frege clarifies the object of judgement and the quantity of judgements, the criterion both avoids circularity and remains appropriate for its purpose. This requires a closer look.

From an exegetical perspective, it seems clear that Frege believes thoughts are the objects of judgement (CP, 381 fn. 13 [1919b, 151]). Assuming, then, that what is judged is the thought, it seems the following stages occur in a judgement:

1. A sentence is heard/read.
2. Its content is grasped.
3. The thought is isolated from its colouring.
4. The thought is grasped.
5. The thought is judged.

I will use ‘*A*’ to name stage 1 as it relates to sentence ‘*A*’, and so on; similarly, with sentence ‘*B*’. Beaney’s point, I take it, is that performances of *A*₄ and *B*₄, and *A*₅ and *B*₅ are performances of the same act iff the grasped thoughts are the same. So, the acts are the same iff the sentences are equipollent. If the right side of the biconditional in *Recognition*₃ is read as asking whether when people perform *A*₅ they also perform *B*₅, then this is no different from asking whether the thoughts are the same, and the criterion is useless. To avoid circularity, Frege must understand the relation between the judgements that *A* and

that *B* in terms that are distinct from the question of whether the thought is the same.

There is another problem here too. The criterion is introduced to facilitate A_3 and B_3 . But if everyone is capable of performing A_3 and B_3 when grasping the content of 'A' and 'B', then there is no use for the criterion at all. A person can simply perform A_3 and B_3 and be done with it. No judgement of the expressed thoughts needs to be performed. The judgement to be made is simply whether the thoughts, once isolated from the accompanying colouring, are identical. For these reasons, if the criterion is to be of use, Frege ought to take contents as the objects of judgement rather than thoughts. (Although, as mentioned, that would be in tension with his claims elsewhere.)

Suppose, then, that Frege takes contents to be the objects of judgement. In this case, there are only three stages that occur in a judgement:

1. A sentence is heard/read.
2. Its content is grasped.
3. The content is judged.

Now A_3 and B_3 are distinct acts since the content judged is distinct. And these distinct acts could not, I presume, be performed simultaneously. Let us suppose that A_3 and B_3 are performed in quick succession by everyone able to perform both A_2 and B_2 . In that case, according to *Recognition*₃, the thoughts expressed by 'A' and 'B' must be identical. Otherwise: "it could not be an immediate consequence of anyone's accepting the content of *B* that he should accept that of *A*" (PW, 197 [1906b, 213]).

The conclusion here is probably not tenable. The reason for the quick succession of the judgements could be that the contents, if both grasped, obviously entail each other. The paradigmatic cases are mathematical, e.g. '216 + 216 = 432' and '216 × 2 = 432'. But there are probably other examples. It seems likely that anyone that grasps the contents of both 'water is a solvent' and 'H₂O is a solvent' would infer one content from the other in quick succession. This is because only those able to grasp the contents of both 'H₂O' and 'solvent' fall within the criterion's scope, and it is likely that those people are also sufficiently familiar with the chemical composition of water to know that water is H₂O. But it also the case that these sentences are not equipollent, since it was, at one point, a discovery that water is H₂O, which, by Frege's reasoning, means that 'water' and 'H₂O' have different senses and the sentences express different thoughts (CP, 157–158 [1892b, 25–27]; PW, 255–256 [1919, 276]).

The issue is a broad one. It shows that Frege's equipollence criterion is too reliant on contingent factors concerning what people are capable of. This reliance on psychology in the establishment of logic's objects would surely be unacceptable to him.

But perhaps I have incorrectly interpreted Frege's use of the word 'immediate'. As Eva Picardi (1993, 75) says:

At first blush Frege's approach to the issue of sense-identity may appear almost psychologistic. However, when he alludes to 'immediate recognition' Frege certainly does not mean to refer to what psychologists nowadays call 'reaction times', but to a recognition that requires neither reflection nor inference.

She says nothing further to elaborate the point. But if Picardi is right, the question that should be asked is not whether a performance of B_3 follows from a performance of A_3 in a small amount of time, but whether it follows without reflection or inference. I interpret this to be a matter of the *triviality* of the second judgement, given the first. In this case, perhaps the criterion does not ask whether the judgement that B follows immediately *after* (i.e. temporally) the judgement that A (and *vice versa*), but whether the judgement that B follows immediately *from* (i.e. trivially) the judgement that A (and *vice versa*).

There is something in this, for Frege associates the triviality of an inference with identity of sense in other places (e.g. PW, 208 [1914, 224–225]). I pursue this possibility in the next section.

4 The Criterion of Triviality

In the previous section I suggested that Frege may be aiming at a criterion that determines equipollence by utilising the notion of a trivial judgement. The idea is that, just as the judgement that A trivially follows from the judgement that A , if the judgement that B trivially follows from the judgement that A , they too express the same thought, and the sentences are equipollent.

A reformulation under a new name is in order:

Trivial₁: Two sentences ' A ' and ' B ' are equipollent *iff* anyone who grasps the contents expressed by ' A ' and ' B ' and judges whether A trivially judges the same concerning whether B (and *vice versa*).

The appeal here is to a *trivial act of judgement*. Suppose a person has judged whether A . *Trivial₁* claims that ' A ' and ' B ' are equipollent *iff* judging of the same concerning whether B would be trivial (retaining the given proviso concerning content that is logically self-evident).

Since the notion of a trivial judgement is deployed here as an explainer, a little ought to be said about it up front. A trivial judgement is an act of judgement that lacks epistemic value. No knowledge has been gained by it, i.e. the sum total of knowledge had by the judger is not increased for having made it. The notion of triviality appealed to here is basic and fundamentally epistemic. It is deployed for the purposes of both distinguishing thoughts from colouring and for individuating them.

Taking equipollence to be a matter related to epistemology may seem at odds with some of Frege's examples. Consider the following two pairs of sentences:

- (c) 'The Greeks defeated the Persians' and 'The Persians were defeated by the Greeks'.
- (d) ' $1 + 3 = 2 \times 2$ ' and ' $3 + 1 = 2 \times 2$ '.

The first pair are, according to Frege, equipollent (CN, 112 [1879, 3]). While the second pair are not (CP, 241 [1897b, 369–370]). But whatever epistemic difference there may be appears equally slight in both cases. In fact, the first pair seem *more* distinct from each other since the relations expressed are the *converse* of each other rather than the same (symmetrical) relation.

I suggest, however, that *Trivial*₁ can provide the answer for what otherwise appears to be an inconsistency. For, if *Trivial*₁ is correct, the difference in equipollence between the cases is because inferring the thought expressed by, say, the second of the pair from the thought expressed by the first, is a gain in knowledge only in the mathematical case. And this indeed seems empirically correct. The commutativity of addition is something mathematicians felt the need to *prove* from the basic notion of addition. Linguists feel no such compulsion of proof with active and passive forms of sentences.

Similarly, it seems reasonable to think *Trivial*₁ might label other controversial mathematical examples of equipollence correctly. Consider the (equipollent) sentence pair: '*a* is parallel to *b*' and 'the direction of *a* is the same as the direction of *b*'. This is *not* something mathematicians seek to prove. If a change of expression were required in a geometrical proof, it would likely be justified, if at all, 'by definition'. And a movement between definitionally equivalent expressions in a proof is not, for Frege, a case in which knowledge has been gained (see below).

Admittedly this means the issue of thought identity rests on the rather elusive notion of epistemic equivalence. But given that Frege treats thoughts as objects of judgement and belief, it is in keeping with both his metaphysics of sense and otherwise curious appeals to "special acts of cognition" (CP, 241 [1897b, 369–370]) and distinctions of "cognitive value" (CP, 157–158 [1892b,

25–27]) when arguing for a difference in sense. And the central role of epistemic considerations in such matters is directly supported by the justification he gives for a (comparatively late) example of thought differentiation (PW, 224 [1914, 242]) [my emphasis]:

It is surely not self-evident that $137 + 469 = 606$; on the contrary we only come to see this as a result of first working it out. This sentence says much more than the sentence '606 = 606'; the former *increases our knowledge*, not so the latter. So the thoughts contained in the two sentences must be different too.

Further, the notion of epistemic value attached (or not attached) to judgements is deployed in the separate, but related, case of proper definitions. Here, the triviality of judgements is explicitly tied to sameness of sense with what has gone before. An examination of Frege's claims regarding definitions is insightful (BLA, I 44–45 [1893, 44–45]; see also FOA, 78 [1884, 78]; CN, 167–168 [1879, 55–56]; PW, 207–211 [1914, 224–228]) [original emphasis]:

By means of a *definition* we introduce a new name by determining that it is to have the same sense and the same reference as a name composed of already known signs.

Frege says that a definition is not a judgement. To make this clear he introduces a primitive sign of logical notation used in place of the judgement sign when stating a definition. These definitions take the form of an identity that equates both the sense and reference of the flanking signs. The definition, once introduced, also becomes a possible judgement, and can be restated as one by pre-fixing it with the judgement sign (BLA, I 45 [1893, 44]).

According to Frege, the judgement conveyed by the restatement of a definition lacks epistemic value, i.e. it is trivial.⁷ But not only is this judgement trivial, any inference that follows from it is also trivial. The reason is that the inference involves no change of sense, but only of signs. Definitions are 'epistemically inert', unable to add to our knowledge when stated or deployed. Their purpose is purely psychological. They allow a complex sense to be conveniently expressed by a simple sign, which simplifies proofs. (Frege gives the

⁷ The kind of definition under discussion here (the kind that lacks epistemic value) is what Frege calls a 'proper definition'. It must be distinguished from what he calls 'illustrative examples', 'logical analyses', and, perhaps, the 'fruitful definitions' that appear in his early work (PW, 207–211 [1914, 224–228]; Tappenden, 1995).

integral sign of mathematics as an example.) Any thought that can be proved by deploying a definition could be proved without it. So, logically speaking, definitions are superfluous (PW, 207–209 [1914, 224–226]).

Similar thinking seems to be behind Frege's interest in equipollence. The distinctions between equipollent sentences are either purely at the level of sign or else also involve a distracting distinction of colouring. In either case, they are logically superfluous.

Here is why *Trivial₁*, given this context, might present itself as a good criterion. If 'A' and 'B' express the same thought, then the judgement that B trivially follows from the judgement that A. It is purely an alteration of expression and colouring that adds no knowledge. Inferring that B from that A is no different, in terms of epistemic value, from inferring that A from that A. This, I suggest, is what motivates Frege to present what he does as a criterion.

With some explanation of and motivation for formulating the criterion with *Trivial₁* now established, I can examine it. The criterion, after all, must be something that can be applied to be of use. If it cannot be applied effectively, it cannot be used to separate a thought from its colouring. But whether *Trivial₁* can be applied is unclear. To do so, we would need to know whether an inference from that A to that B (and *vice versa*) would be trivial. This requires some non-circular method of determination, i.e. a method of determination that does not surreptitiously involve equating thoughts. It also requires a method that avoids relying on psychology, else the spectre of psychologism returns.

One method that suggests itself involves an examination of the justification for the inference. Frege is clear that justification is not a psychological affair (FOA, 3–4 [1884, 3–4]), nor need it involve equating thoughts. So it is a method capable of avoiding both pitfalls. To determine what justification would make an inference trivial, we might begin by considering an inference justified by a definition:

1. This female fox is hungry.
2. (By definition) This female fox is this vixen.

3. Therefore: This vixen is hungry.

Suppose we were asked whether the inference is trivial, i.e. whether any knowledge has been gained in concluding that 3. According to Frege, the conclusion *is* trivial. For it is an inference that is justified by a trivial (definitional) judgement of identity, and thus adds no knowledge. Perhaps, then, an inference is trivial iff it is justified by trivial judgements of co-reference. So, if we use *Trivial₁* to determine whether 'A' is equipollent to 'B', it seems we ought to determine whether all those that grasp the contents of 'A' and 'B' infer that

B from that *A* (and *vice versa*) on the basis of a trivial judgement of identity (consciously or otherwise).⁸

Now, in the definitional example above, the substitution effected was subsentential (as is typical with definitions). As we have seen, some of Frege's examples are of the sort that involve (subsentential) names that bear the same sense, e.g. 'this dog' and 'this cur'. But other of Frege's equipollence examples suggest trivial inferences that could not be justified by a trivial judgement identifying the referents of subsentential elements. Rather, the trivial judgement would have to identify the referents of the whole sentences.

Recall a familiar example: 'The Greeks defeated the Persians' and 'The Persians were defeated by the Greeks' (CN, 112 [1879, 3]). The relational concepts of the former defeating the latter and the former being defeated by the latter are the converse of each other. So the relational concept words cannot be merely substituted. If the judgement of the second sentence follows trivially from the judgement of the first sentence, it could only be by trivially recognising that, in general, *x* defeated *y* iff *y* was defeated by *x*. The identity appealed to in this case thus holds between *truth-values*. In the example case, the inference would be as follows:

1. The Greeks defeated the Persians.
2. (Trivially) The Greeks defeated the Persians iff the Persians were defeated by the Greeks.

3. Therefore: The Persians were defeated by the Greeks.

Note also that for any trivial judgement of identity concerning subsentential elements such as 'this female fox = this vixen' there will be an equally trivial judgement of identity at the level of the referents of the sentences concerned: 'this female fox is hungry iff this vixen is hungry'. So we can take the general case to concern an inference that can be justified via a trivial judgement of identity between truth-values.

It now becomes clear that there are unnecessary (and unhelpful) elements of *Trivial₁* as stated. According to *Trivial₁*, to determine whether sentences are equipollent we must determine whether competent speakers would trivially infer the content of one sentence from the content of the other. But since determining this in a non-circular, non-psychologistic way seems to require investigating the *justification* for making the inference, the issue boils down to

⁸ For the sake of simplicity, I use examples in which objects are trivially identified. To expand the discussion to include cases which involve definitionally identified concepts would be possible, but it would introduce complexities that are unnecessary for my argument.

the triviality (or otherwise) of a *single* judgement, namely, the judgement of identity of truth-values that justifies the inference. This means that the judgements of the content of 'A' and 'B' appealed to in *Trivial*₁ are not that which does the heavy lifting. What really determines equipollence in these cases is the nature of the judgement that implicitly justifies the inference. Hence, the requirement that the individual contents be judged can be dropped from the criterion, leaving us with:

*Trivial*₂: Two sentences 'A' and 'B' are equipollent *iff* anyone who grasps the contents expressed by 'A' and 'B' trivially judges that A *iff* B.

The absence of the requirement that the contents of 'A' and 'B' be judged is a virtue of *Trivial*₂. For it now becomes possible to determine equipollence for cases in which it cannot be determined whether those contents are true or false. For example, *Trivial*₂ can be used to determine 'Next year it will snow in Cambridge' is equipollent to 'It will snow in Cambridge next year'. But since the content of these sentences cannot reasonably be judged, it is doubtful whether *Trivial*₁ could be applied to this case.

*Trivial*₂ has a further virtue. It is no longer required that *contents* be the units of judgement to avoid circularity. A trivial judgement that A *iff* B may be a judgement of contents or of thoughts. Neither view is prejudiced in using the criterion, and neither view leads to problems of circularity.

But we are not quite home and dry. As Textor (2018) points out, given that thoughts are expressed entangled with their colouring, it might be difficult for some suitable graspers to make the required trivial judgement of identity at all, trivially or otherwise. For they may be distracted or confused by the colouring, and thus unable to recognise the equivalence. And Frege himself thinks the co-reference of sentences can sometimes be disputed, which leads him to make appeals to the distinction between what is explicitly stated and what is mere implicature and to whether accusations of lying would be appropriate (PW, 140–141 [1897, 151–153]). These considerations suggest the demand for every suitable grasper to perform a particular judgement is too strong.

Relatedly, talk of what suitable graspers judge carries with it a whiff of contingency that is out of place with Frege's metaphysics of thoughts. And even if a person embarked on what seems to be the required empirical investigation, it is unclear how to determine the triviality of a judgement.

My proposal is, once again, to adjust the criterion. The problematic appeal to contingent acts of judgement can be dispensed with if the criterion adjudicates based on the triviality of the thought concerned rather than the triviality of associated judgement acts. For Frege speaks not merely of the epistemic value of acts of judgement, he also speaks of the epistemic (or cognitive) value

of sentences (CP, 157–158 [1892b, 25–27]; PW, 208 [1914, 224–225], 224 [1914, 242]). It seems reasonable to assume, then, that the notion of triviality applies to contents, or thoughts, also. Since we wish to maintain the spirit of Frege criterion while making clear that the triviality of a thought is independent of contingencies surrounding human acts of judgement, I suggest we assume a thought is *itself* trivial iff every judgement of it would be trivial.

(Note that this biconditional is not intended to suggest the triviality of a thought, and thus, ultimately, equipollence, is determined by modal considerations. Rather, in keeping with Frege's metaphysics of thoughts, a thought simply is or is not trivial, and this feature of a thought explains its capacity or incapacity to provide those that judge it with knowledge. I will discuss modality in connection with equipollence in the next section.)

We are now led, finally, to:

Equipollence: Two sentences 'A' and 'B' are equipollent iff it is trivial that A iff B.

Equipollence avoids the need for empirical investigation and can be applied by a single individual. They need only determine whether a thought is trivial, i.e. whether it is capable of contributing knowledge to those that judge it.

Now, it might be thought that *Equipollence* is too strong. For there are equipollent examples Frege gives where the thought to be judged does not seem trivial. For example, it may not seem trivial that this dog howled iff this cur howled. Indeed, this is an example Frege *argues* for. And it seems we often learn something when, by applying the criterion, we successfully separate the thought common to the equipollent sentences from the irrelevant colouring it appears with. If we did not, we would have no use for the criterion.

But this complaint arises from a confusion of levels. *Equipollence* asks whether it is trivial that A iff B, not whether it is trivial that it is trivial that A iff B. Judgements of the triviality of a thought need not themselves be trivial acts. Equipollence is a matter of triviality, but not a trivial matter. And Frege would admit that his examples of it can be disputed.

It might further be asked whether *Equipollence* is guilty of the same circularity that plagued *Recognition*₃. For it might be thought that the question of the triviality of a thought concerning an identity of truth-values is no different from the question of the identity of the relevant thoughts, which is in turn no different from the question of equipollence. But there are two reasons to think this is not the case.

First, the analyses of the judgements are of a different kind. The judgement concerning the triviality of the thought that A iff B concerns whether a (single) thought falls under the unary concept of being trivial, i.e. whether [that A iff B]

is trivial. Whereas the judgement of identity between thoughts concerns two thoughts and a relational concept, i.e. whether $[\text{that } A] = [\text{that } B]$. Of course, having claimed that different structural analyses are no proof of distinctness of thoughts (see §1) this is not enough to show the judgements are substantially different. But, given *Equipollence*, it seems they are. It does not seem trivial that $[\text{that } A \text{ iff } B]$ is trivial iff $[\text{that } A] = [\text{that } B]$. After all, I have just been arguing my case!

Second, the concepts involved are of different kinds. One judgement concerns an *epistemic concept*, that of triviality, while the other concerns a *metaphysical relation*, that of identity. As long as the concept of triviality is more primitive than identity of sense, the former can provide a criterion for the latter, i.e. the metaphysical relation of identity between senses is determined by the extension of the epistemic concept of trivial co-reference. And we have good reason to think Frege *does* take trivial co-reference to be the more primitive. He wrote of the epistemic issues surrounding co-reference (then, ‘identity of content’) as early as *Begriffsschrift* (CN, 124–126 [1879, 13–15]). And it is by appeal to the distinction between identities that contain “actual knowledge” and those that are trivial that he justifies the need to introduce sense in addition to reference (CP, 157–8 [1892b, 25–27]). The epistemic basis for sense individuation thus seems clear. And it is to this that the criterion points.

Now, *Equipollence* is not the criterion Frege puts forward. But it is, I contend, a good way to understand the underlying motivation for him to put forward his third criterion and is a preferable formulation of it. The moral is that Frege attempts to (metaphysically) individuate thoughts *via* an epistemic concept. In the final section, I compare *Equipollence* with some rivals.

5 In Comparison with Rivals

The reader sufficiently convinced by my criterion need not read this section. But, since non-epistemic criteria have been offered by others, I take some time to defend my suggestion by critically examining three alternatives.

First, recall that in §3 it was argued that Frege cannot rely on contingent empirical factors when establishing the identity of thoughts on pain of inconsistency with his stance against psychologism. Upon noting this it might be thought that a criterion could appeal to metaphysical necessity. For example, Jeff Speaks (2014) formulates Frege’s criterion in a way tantamount to:

Modal: Two sentences ‘A’ and ‘B’ are equipollent *iff* necessarily, anyone who grasps the contents expressed by ‘A’ and ‘B’ judges that A iff B.

The idea behind *Modal* seems to be that if it is not possible for a suitable person to judge that the truth-values of the sentences might differ, the thoughts expressed must be identical.

Now, there is textual evidence that might support ascribing something like *Modal* to Frege (PMC, 80 [n.d., 128]):

The sense of the proposition 'Ateb is at least 5000 metres high' is also different from the sense of the proposition 'Aphla is at least 5000 metres high'. Someone who takes the latter to be true need not take the former to be true.

And (CP, 162 [1892b, 32]):

The thought in the sentence 'The morning star is a body illuminated by the Sun' differs from that in the sentence 'The evening star is a body illuminated by the Sun'. Anybody who did not know that the evening star is the morning star might hold the one thought to be true, the other false.

In both passages, the second sentence could be read as a modal claim, and thus motivate *Modal* as a suitably Fregean criterion. But although Frege sometimes helps himself to modal language when discussing equipollence, I think we should be wary of equating it with a metaphysical 'possible worlds' conception for several reasons.

Firstly, Frege believes that the connection between sign and sense can be arbitrary: "nobody can be forbidden to use any arbitrary producible event or object as a sign for something" (CP, 157 [1892b, 25–26]; and see PW, 269 [1924/1925, 288]). For example, it seems he takes it to be possible for 'cur' to be used to express a sense that determines a well-groomed hamster, while 'dog' retains its customary sense. It is possible, after all, to speak a new language. So it seems it is possible for 'A' and 'B' to express different thoughts than the ones they (actually) do. If the names 'the contents expressed by "A" and "B"' and 'that A iff B' on the right-hand side of *Modal* were read *de dicto*, then they might refer to different contents/thoughts in different worlds. So there would be no reason to think the criterion would be satisfied for genuine cases of equipollence. Continuing the example, since it is possible for 'cur' to be used to refer to a well-groomed hamster, it is possible that 'this dog howled all through the night' and 'this cur howled all through the night' would not be judged by suitable graspers as referring to the same truth-value.

This leaves only a *de re* reading as potentially suitable, where the contents/thoughts concerned are rigidly designated. But under a *de re* reading,

given reasonable modal assumptions, *Modal* has a related problem. It incorrectly renders equipollence a necessary relation.

The assumptions are that (i) if *Modal* is true it is necessarily true, and that (ii) if something is necessarily true then it is necessarily necessarily true, i.e. the logic is at least as strong as S4.

Informally: Necessarily, if the right-hand side of *Modal* holds the sentences are equipollent (by (i)). And, since the right-hand side of *Modal* involves a condition of necessity, if it holds, it necessarily holds (by (ii)). So, if sentences are equipollent, they are necessarily equipollent.

Formally:

- p : 'A' and 'B' are equipollent.
- q : Suitable graspers judge that A iff B .
- 1: $p \leftrightarrow \Box q$. (*Modal*)
- 2: $\Box(p \leftrightarrow \Box q)$. (From 1 and (i))
- 3: $\Box p \leftrightarrow \Box \Box q$. (From 2)
- 4: $\Box q \leftrightarrow \Box \Box q$. (From (ii))
- 5: $\Box q \leftrightarrow \Box p$. (From 4 and 3)
- 6: $p \leftrightarrow \Box p$. (From 1 and 5)

Now, p has only a *de re* reading. So, given a *de re* reading *Modal* and reasonable assumptions, if two sentences are equipollent, they are necessarily equipollent. But necessary equipollence is incompatible with the arbitrariness of the connection between sign and sense. Two distinct sentences cannot necessarily express the same thought since it is possible for them to express different things.

Secondly, as Dummett (1981, 132–133) argues, despite it being natural to think of sense being determined by possible reference, this cannot be Frege's notion of sense, for it lacks the distinctly epistemic role Frege says it has.

Thirdly, *Modal* inverts Frege's priority. For he thinks it is sense that determines reference, not (possible) reference that determines sense (PW, 124 [1892–1895, 135]). If we take the people over which the criterion ranges to be possible ideal thinkers (as Speaks does), then it seems *Modal* makes identity of sense dependent on necessary co-reference, which is just that notion of sense that, however natural to us, is not Frege's.

Fourthly, Frege's conception of modality makes it unsuitable as a basis for a Fregean criterion of equipollence. He says that modal claims differ from their assertoric counterparts only in the 'hints' they offer concerning the grounds of judgement. Hence he says, "since this does not affect the conceptual content

of the judgement, the apodictic form of judgement has for us no significance" (CN, 114 [1879, 5]). Similarly, he says assertions regarding what is possible either 'indicate' ignorance concerning implication via laws or are a straightforward existential claim, i.e. 'possibly if Fa then Ga ' means some Fs are Gs (CN, 114 [1879, 5]). It seems, then, that Frege considers modal words like 'necessarily' and 'possibly' to have content that is mere colouring. So modality is not, for Frege, a means by which thoughts could be individuated.

In sum: *Modal* is not a criterion Frege would endorse.

A second alternative non-epistemic criterion is offered by Schellenberg (2012). She thinks Frege should use a normative criterion, based on what a person is 'rationally committed' to judging, i.e. what they ought to judge, given previous judgements.⁹ Her criterion is:

Normative: Two sentences ' A ' and ' B ' are equipollent *iff* anyone who grasps the contents expressed by ' A ' and ' B ' and judges that A is rationally committed to judging that B (and *vice versa*).

Textor (2018) points out that *Normative* is too weak. For a person that judges that $2 + 1 + 3 = 2 + 1 + 3$ is rationally committed to judging that $2 + 1 + 3 = 1 + 1 + 1 + 3$. But, for Frege, these are different thoughts. Schellenberg does have a condition that may perhaps avoid this, for she says the understanding of ' A ' must also be 'constitutively connected' to the understanding of ' B '. But, as Textor says, it is difficult to judge the success of this condition, since she provides no explanation of what the condition requires, and plausible guesses suggest that an understanding of the counterexample sentences are indeed 'constitutively connected'.

Besides Textor's complaint, Schellenberg's justification for *Normative* falls short. She argues it satisfies the two desiderata for any criterion. One desideratum concerns the capacity for the criterion to be applied to sentences with content that is difficult to grasp. I have already argued that this desideratum only fails to apply to Frege's criterion given an incorrect formulation of it (§3). Besides, *Equipollence* also meets this desideratum, so as yet we have no reason to endorse *Normative*. The other desideratum is explained in the following passage (Schellenberg, 2012, 164):

One [desideratum] is that expressions that have the same sense ought to be intersubstitutable *salva veritate* in contexts of belief ascription. So

⁹ Leora Weitzman (1997) seems to agree.

if one holds a certain proposition to be true, then one ought to hold any proposition true that has the very same sense. It is important that this desideratum is formulated as being a matter of what one ought to be able to substitute, rather than what one can substitute. Someone may have inconsistent beliefs or may not be a competent speaker of her language. As a consequence, it may not be possible to substitute *A* with *B* in contexts of beliefs that she endorses, despite *A* and *B* having the same sense.

The consequence drawn in the final sentence here is questionable from a Fregean perspective. It appears to rely on a principle that assumes belief to be coincident with ‘assent’ to sentences, such as the ‘disquotational principle’ outlined by Saul Kripke (2011, 138):

A normal English speaker who is not reticent will be disposed to sincere reflective assent to ‘*p*’ if and only if he believes that *p*.

Let ‘*A*’ and ‘*B*’ be equipollent. Schellenberg seems to assume that if a person is not disposed to assent (sincerely and reflectively) to ‘*B*’ they do not believe that *B*, even if they are disposed to assent to ‘*A*’ and thus believe that *A*. This could be because they are unable to understand ‘*B*’ or because they inconsistently assent to both ‘*A*’ and ‘not *B*’. In that case, so the reasoning goes, ‘*A*’ and ‘*B*’ would not be substitutable in all belief contexts *salva veritate*.

Her point seems to be that, because of this, the desideratum should be couched normatively: equipollent sentences *ought to be* substitutable in belief contexts *salva veritate*. This normative couching of the desideratum paves the way for her normative criterion of equipollence. *Normative* guarantees equipollent sentences ought to be substitutable in belief contexts because it ensures the sentences are equipollent iff the person ought to believe that *B* if they believe that *A* (and *vice versa*).

Now it seems to me that the disquotational principle implicitly appealed to here, even if usually correct, does not apply to the sort of case Schellenberg requires. As Kripke (2011, 138 fn. 23) states in a footnote to the principle:

What if a speaker assents to a sentence, but fails to assent to a synonymous assertion? Say, he assents to “Jones is a doctor,” but not to “Jones is a physician.” Such a speaker either does not understand one of the sentences normally, or he should be able to correct himself “on reflection.” As long as he confusedly assents to ‘Jones is a doctor’ but not to ‘Jones is a physician,’ we cannot straightforwardly apply disquotational principles.

So, there are reasons to doubt the principle applies to Schellenberg's case of a person who has inconsistent beliefs or may not be a competent speaker of her language. And thus reasons to doubt equipollent sentences cannot be substituted *salva veritate* in such contexts. In fact, we have good (Fregean) reasons to think equipollent sentences *must be* substitutable *salva veritate* even in such contexts.

Consider the following pair of sentences that are, I assume, equipollent:

- (e) My mother's father is a builder.
- (f) My maternal grandfather is a builder.

And suppose we put both sentences in the context of a belief ascription to Jemima:

- (g) Jemima believes that my mother's father is a builder.
- (h) Jemima believes that my maternal grandfather is a builder.

Now suppose Jemima is insufficiently competent in English to understand sentence f but understands sentence e. It seems Schellenberg thinks that this would make sentence h false even if g is true. But the referent of 'that my mother's father is a builder' and 'that my maternal grandfather is a builder' is, by hypothesis, the same thought. Since that which is relevant for truth determination is purely at the level of reference, it seems g and h must share a truth-value.¹⁰

This result bears out intuitively. If, after Jemima asserts to me 'your mother's father is a builder', I were to report this assertion to another person by saying 'Jemima believes that my maternal grandfather is a builder', I would not be incorrect, for that is indeed what she believes. We do not risk error when correctly rephrasing or translating a belief report. Otherwise, every belief report I have made concerning Frege in this article would be false if his competence in English were insufficient.

Similarly, although Jemima can have inconsistent beliefs, she cannot both believe and disbelieve the same thought. If upon hearing sentence e she says to me 'I believe that', but upon hearing sentence f she says 'I do not believe

¹⁰ The subsidiary question of whether these sentences also express the same thought is rather difficult. It is a question that exposes a problem with Frege's claim that thoughts can be referred to. Gideon Makin (2000) discusses this issue at length, which he terms the 'no backwards road' problem, borrowing the phrase from Russell (1905).

that', that would show she misunderstood one or other sentence, i.e. she failed to grasp the thought expressed by one or other sentence. If this were not assumed, I could make no sense of her utterances.

Equipollent sentences can thus be substituted *salva veritate* in belief ascriptions just as they can in all indirect contexts. With no appeal required to what 'ought' to be substitutable in belief ascriptions, there is no need to introduce appeals to normativity concerning judgements to individuate senses.

This final point leads nicely on to the third alternative criterion I wish to discuss, based on substitutability *salva veritate*. Penco (2003) offers the following:

Substitution: Two sentences 'A' and 'B' are equipollent *iff* 'B' may be substituted for 'A' in indirect contexts *salva veritate* (and *vice versa*).

Frege is committed to *Substitution*. Since the referent of sentences in indirect contexts is the thought they express in direct contexts, Leibniz's law means equipollent sentences must be intersubstitutable *salva veritate* in indirect contexts. But a biconditional relationship is not enough to constitute a suitable criterion. Frege needs a criterion that can be put to use in determining equipollence (see §2). *Substitution* could only be put to use if the right-hand side could be established prior to the left-hand side. But it could not. It would be impractical for a person to determine the results of every possible substitution. Instead, it would have to be determined whether the sentences co-refer in indirect contexts. But that is the same as determining whether the sentences express the same thought in direct contexts, i.e. determining whether they are equipollent.

In this section I have discussed and dismissed rival criteria that do not appeal to triviality that I have found in the literature. I believe the failings of the cases considered provide good reasons for thinking a suitable criterion of equipollence requires, for Frege, a judgement of trivial identity. And thus reinforces the claim that Frege individuates thoughts by deploying an epistemic criterion.

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