TECHNICAL RESEARCH REPORT

A Carrier Frequency Estimation Method of MPSK Signals and Its Systolic VLSI Implementation

by Yimin Jiang, Wen-Chun Ting, Farhad B. Verahrami, Robert L. Richmond, and John S. Baras

CENTER FOR SATELLITE AND HYBRID COMMUNICATION NETWORKS

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Yimin Jiang *, Wen-Chun Ting *, Farhad B. Verahrami *,

Robert L. Richmond *, John S. Baras +

* Hughes Network Systems, Inc., 11717 Exploration Lane

Germantown, Maryland 20876, USA, Tel: +1-301-601-6494, Fax: +1-301-428-7177

Email: yjiang@hns.com

+ Institute for Systems Research, University of Maryland,

College Park, MD 20742, USA

Abstract

In this paper we present an autocorrelation-based method for estimating the carrier frequency offset of an MPSK signal with random data modulation. Although autocorrelation based techniques imply heavy usage of hardware resources, this technique is scalable and lends itself well to systolic VLSI implementations. The performance of the open-loop estimator presented is close to the Cramer-Rao lower bound (CRLB) for the frequency estimation from a block of random PSK symbols at low signal-to-noise (SNR) ratios. The estimator can be used in frequency acquisition of burst and continuous modems operating under low SNR and large frequency offset conditions.

I.   Introduction

Carrier frequency estimation is a crucial step in the demodulation process of MPSK modems. Fast frequency estimation is necessary for burst satellite modems operating in the presence of large frequency offsets. The additional burden of low signal-to-noise (SNR) ratio can make the task of frequency estimation quite difficult. Several open-loop frequency estimation methods with a large estimation ranges and capable of operating on a relatively small number of random data modulated MPSK symbols are considered in this paper. A new hybrid algorithm is then constructed based on the well-known auto-correlation function, Kay [5], and Chuang's [6] algorithms. Continuous mode modems can also benefit from the faster acquisition times attained by the usage of the estimator.

A frequency estimator, based on power spectral density estimation, was first proposed by Fitz [2] for an unmodulated carrier. For an MPSK signal, the non-linear method in [1] can be used to remove data modulation. A variant of this algorithm was proposed by Luise [3]. The performance of these methods, at low SNR, is close to Cramer-Rao low bound (CRLB) [7] for a carrier with unknown frequency and phase. The maximum frequency error that can be estimated by the Fitz algorithm is $R_s/(2ML)$, where L is maximum auto-correlation lag and M is the number of phase states in MPSK. Under the assumption that the carrier phase has a constant slope equal to the angular frequency offset, Tretter [4] proposed a frequency estimator by performing linear regression or a line fit operation on the received signal phase values. This algorithm can estimate a maximum frequency of $R_s/(2M)$ and performs well at high SNR values. Phase change over time is proportional to the frequency offset. Kay [5] used the same idea and proposed a frequency estimator by weighting the sum of phase differences over consecutive symbols, which is equivalent to Tretter's algorithm. Chuang [6] also presented algorithms based on differential symbol estimates.

In section II we revisit the algorithms of Kay [5] and Chuang [6] and present our frequency estimator. In section III an efficient VLSI implementation for the frequency estimator is presented. In the last section the performance of the estimator is shown through simulations and compared with the CRLB. Compared with other estimators, our estimator has larger estimation range at low SNR, and better variance performance.

II. Frequency Estimator

In order to simplify our presentation, the following assumptions are made for the development of the algorithm:

1. The symbol timing is known,

2. Discrete time sample are taken from the output of a pulse shape matched filter, one sample per symbol, and

3. The pulse shape satisfies the Nyquist Criterion for zero inter-symbol interference.

Let us assume that we have a block of N symbols. The $i\,th$ complex sample derived from matched filter can be expressed as:

$$r_i = d_i \exp[\,j(2\pi\Delta f i T_s + \phi_0)] + n_{i,}\,|d_i| = 1, i = 0,1,...,N-1 \tag{1}$$

where d_i represents the $i\,th$ complex symbol modulating the MPSK carrier, Δf is the frequency offset, T_s is the symbol interval, ϕ_0 is the carrier phase, and n_i represents complex additive Gaussian noise. The channel noise has a two-sided power spectral density $N_o/2$. The variance of the two quadrature components of n_i are $N_o/(2mE_b)$, where E_b is the energy per information bit and $m = \log_2 M$.

At first, the method in [1] can be used to remove modulation from complex sample r_i, obtaining:

$$\Lambda_i = I_i + jQ_i = F(|r_i|)\exp[jM\arg(r_i)],$$
$$F(|r_i|) = |r_i|^k, k \leq M, \text{even} \tag{2}$$

where $k = 0$ is preferred [1]. According to the work done by Tretter [4], the noise term n_i in r_i can be represented as phase noise at high SNRs:

$$r_i = A\exp[j(2\pi\Delta fiT_s + \theta_i + \phi_0 + V_{Qi})] \tag{3}$$

where $A = 1$, θ_i is data modulation, which is multiple of $2\pi/M$, and V_{Qi} is equivalent phase noise. Therefore, the phase ϕ_i of Λ_i $(k = 0)$ can be modeled as:

$$\phi_i \equiv \arg(\Lambda_i) = M(2\pi\Delta fiT_s + \theta_i + \phi_0 + V_{Q_i}) \tag{4}$$

If we differentiate ϕ_i, we obtain:

$$\delta_i \equiv \phi_{i+1} - \phi_i = M(2\pi\Delta fT_s + \theta_{i+1} - \theta_i + V_{Q(i+1)} - V_{Qi}) \tag{5}$$

δ_i is passed through an exponential function $\exp[j(*)]$, and after applying Viterbi's feed-forward phase estimator [1], we arrive at Chuang's frequency estimator:

$$\Delta\hat{f} = \frac{1}{2\pi MT_s}\tan^{-1}\left(\frac{\sum_{i=0}^{N-2}\sin\delta_i}{\sum_{i=0}^{N-2}\cos\delta_i}\right) \tag{6}$$

If $\theta_i = 0$, $\{r_i\}$ is a continuos exponential wave. Let $\phi_i \equiv \arg(r_i), \delta_i \equiv \phi_{i+1} - \phi_i$; we can get to Kay's frequency estimator for sinusoid wave as follows:

$$2\pi\Delta\hat{f}T_s = \sum_{i=0}^{N-2}w_i\delta_i$$
$$w_i = \frac{1.5N}{N^2 - 1}\left\{1 - \left[\frac{i - (N/2 - 1)}{N/2}\right]^2\right\} \tag{7}$$

We develop our frequency estimator based on Chuang and Kay's algorithms. Define ϕ_i as equation (4). We define $R(m)$ as follows:

$$R(m) = \frac{1}{N-m} \sum_{i=m}^{N-1} \exp[j(\phi_i - \phi_{i-m})], m = 1,...,L \ (L < N-1) \tag{8}$$

At high SNR, $R(m)$ represents the average phase change over m symbols with the first term averaged over N-1 terms and the last one over only one value. This can be modeled as:

$$R(m) \approx \exp(j2\pi M\Delta fmT_s) + \text{noise} \tag{9}$$

The sequence $\{R(m)\}$ can be treated as a continuous wave (with frequency $M\Delta f$) which is passed through a noise removal process. Let us define the following process:

$$\theta(m) = \arg[R(m)], m = 1,...,L$$
$$\Delta(m) = \begin{cases} \theta(m+1) - \theta(m), m = 1,...,L-1 \\ \theta(1), m = 0 \end{cases} \tag{10}$$

We then borrow from Kay's frequency estimator; that is weighted sum of phase differences. Because $R(m_1)$ is calculated based on more data than $R(m_2), m_1 < m_2$, after some arithmetic we derived the following carrier frequency offset estimator:

$$\Delta \hat{f} = \frac{1}{2\pi MT_s} \sum_{m=0}^{L-1} w_m^* \Delta(m)$$
$$w_m^* = \frac{3[(2L+1)^2 - (2m+1)^2]}{[(2L+1)^2 - 1](2L+1)}, m = 0,...,L-1 \tag{11}$$

The maximum frequency offset that the algorithm can digest is $1/(2MT_s)$, e.g., for QPSK, $|\Delta f| < 1/(8T_s)$. The weight function w_m^* is shown as Figure 1. It is easy to see that the weights $\{w_m^*\}$ decrease as m increases. That is because as m gets larger and larger, the number of terms required to compute $R(m)$ reduces and makes $\Delta(m)$ less and less accurate.

III.   Implementation

The calculation of $\{R(m)\}$ is a hardware intensive task which requires $(2N-L-1)L/2$ complex multiplication (processing the phase difference through exponential function is treated

as a complex multiplication) and $(2N - L - 3)L/2$ additions. Actually it is equivalent to calculate the auto-correlation of the input symbol with data modulation removal in [1]. In order to make full use of each input data and exploit concurrency, we propose a systolic VLSI implementation (refer to [9] for systolic structure) as shown in Figure 2.
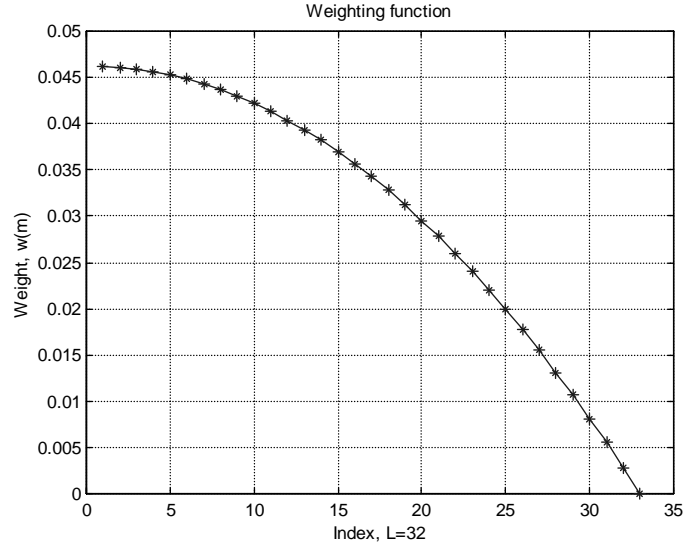

Figure 1 Weighting Function $\{w_m^*\}$

There are two possible schemes for calculating $\{R(m)\}$ as shown in Figure 2.

Scheme 1: equation (4) (implemented via a lookup table (LUT)) is used to remove data modulation, and equation (8) is used to calculate $R(m)$. Lookup tables are then used to calculate the term $\exp[j(\phi_i - \phi_{i-m})]$. If a higher speed clock is available, these LUTs can be shared on time division basis.

Scheme 2: M -th power method is used to remove data modulation, and complex multiplication and accumulations are used to get $R(m)$, i.e.

$$\phi_i \equiv \frac{r_i^M}{|r_i|^M} \tag{12}$$

$$R(m) = \frac{1}{N-m} \sum_{i=m}^{N-1} \phi_i \phi_{i-m}^*, \quad \mathrm{m} = 1,..., L \, (L < N - 1)$$

After this new definition of ϕ_i, $R(m)$ is given by:

$$(13)$$

It is easy to check that Scheme 1 and Scheme 2 are equivalent. If a higher speed clock is available, the complex multipliers can be also shared on time division basis. In both schemes, $\{R(m)\}$ will be available on the clock cycle following the one latching the N th data symbol into the estimator. Frequency offset can then be calculated via equation (11).

$$\Delta \hat{f} = \frac{1}{2\pi M T_s} \sum_{m=0}^{L-1} w_m^* \Delta(m)$$

$$w_m^* = \frac{3[(2L+1)^2 - (2m+1)^2]}{[(2L+1)^2 - 1](2L+1)}, \quad m = 0,...,L-1$$

Scheme 1:
Data Removal: $y = M \arg(x)$
Scheme 2:
Data Removal: $y = \dfrac{x^M}{|x|^M}$

Scheme 1:
$x_{out} = x_{in1}, \quad R(n) = \exp[j(x_{in2} - x_{in1})] + R(n-1), \quad y = R$
Scheme 2:
$x_{out} = x_{in1}, \quad R(n) = x_{in2} x_{in1}^* + R(n-1), \quad y = R,$
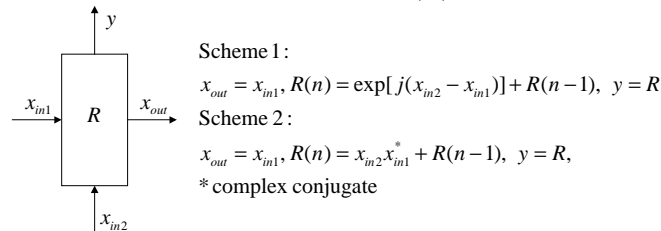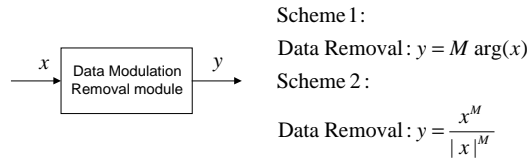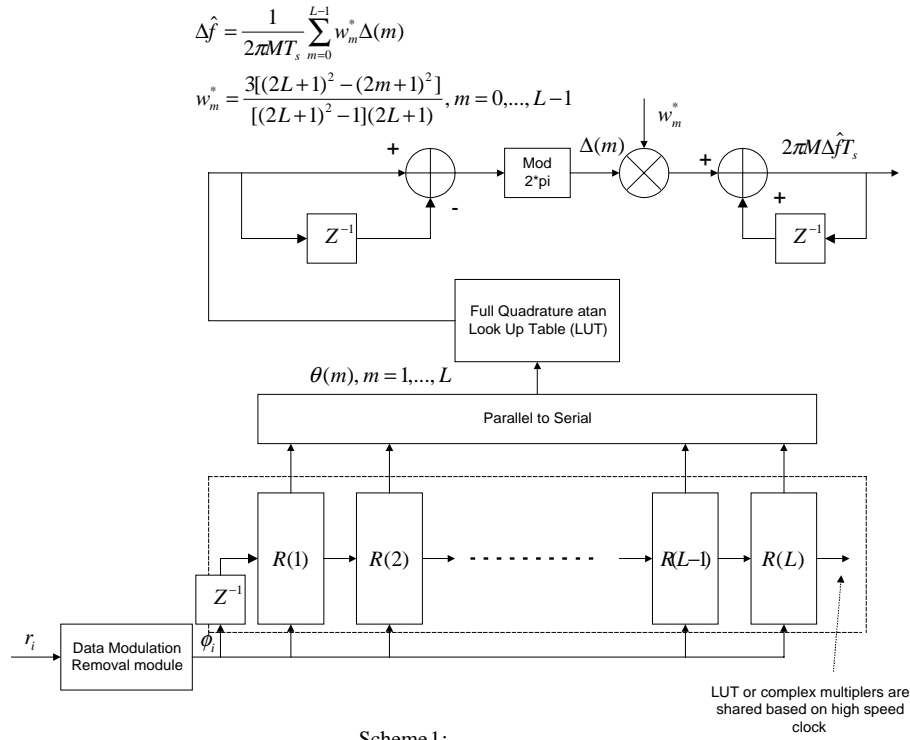* complex conjugate

Figure 2 Systolic VLSI Structure of MPSK Frequency Offset Estimator

One advantage of this structure is that it is scalable. If we want to increase the value of L to get

better performance, we just need to add one more element at the right hand side shown in Figure 2

IV.   Performance and Simulation Results

At high SNR, some approximation methods can be used to analyze the variance of $\Delta \hat{f}$. This is

similar to works presented in [1], [4], and [5]. Unfortunately at low SNR, the analytical

performance approximation is untractable. Hence we resort to computer simulation results. One

of HNS's Universal Modem ASICs which utilizes this technique is under development and

hardware test results will be reported in a subsequent paper.

Two performance indexes are of interests, one is the range over which the frequency estimator is

unbiased and its performance degradation is small.   Second is the variance of the estimate $\Delta \hat{f}$

compared with CRLB. Cowley [8] derived a modified CRLB for the frequency estimators based

on a block of random BPSK or QPSK symbols at low SNRs. The bound for QPSK is given by

Cowley as:

$$CRB(\Delta fT_s) = \frac{6}{4\pi^2 N(N^2-1)\frac{2E_b}{N_o}F_Q(\frac{N_o}{2E_b})}$$

$$F_Q(\sigma^2) = \iint \frac{(\sinh^2(\frac{1+x}{\sigma^2})\frac{y^2}{\sigma^2} + \sinh^2(\frac{y}{\sigma^2})\frac{(1+x)^2}{\sigma^2} - 2\sinh(\frac{1+x}{\sigma^2})\sinh(\frac{y}{\sigma^2})(1+x)\frac{y}{\sigma^2})e^{-(x^2+y)^2/2\sigma^2}}{(\cosh(\frac{1+x}{\sigma^2}) + \cosh(\frac{y}{\sigma^2}))^2} dxdy$$

(14)

Figure 3 shows the root mean square (RMS) frequency estimation error with roll-off factor 0.25

raise cosine shaping, $N = 100, L = 32$, which is compared with the modified CRLB (with

the $F_Q(\sigma^2)$ factor) and CRLB (without the $F_Q(\sigma^2)$ factor). From simulations we can see that the

estimation performance is close to CRLB when SNR>6dB. If SNR<6dB, the RMS estimation

error increases dramatically as SNR decreases. Figure 4 shows the frequency estimation range.

The estimation range increases as SNR increase, which is intuitive. At 6dB, the frequency offset

estimation range is about 7% of symbol rate, at 8dB, it increases to 9%.
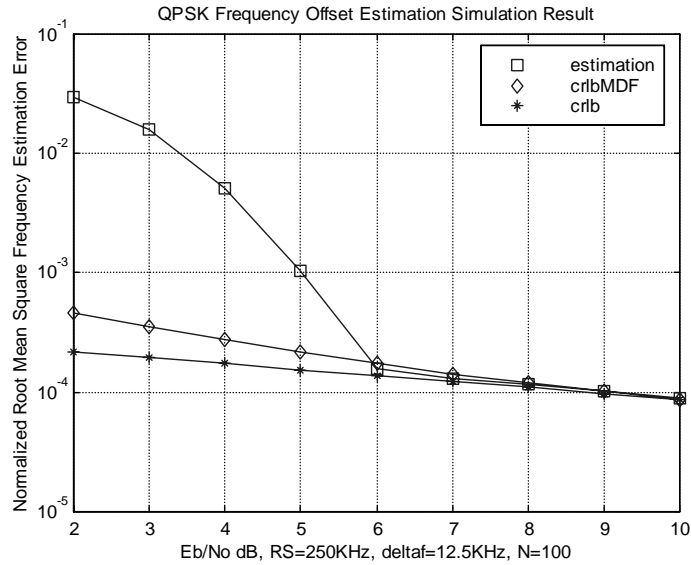

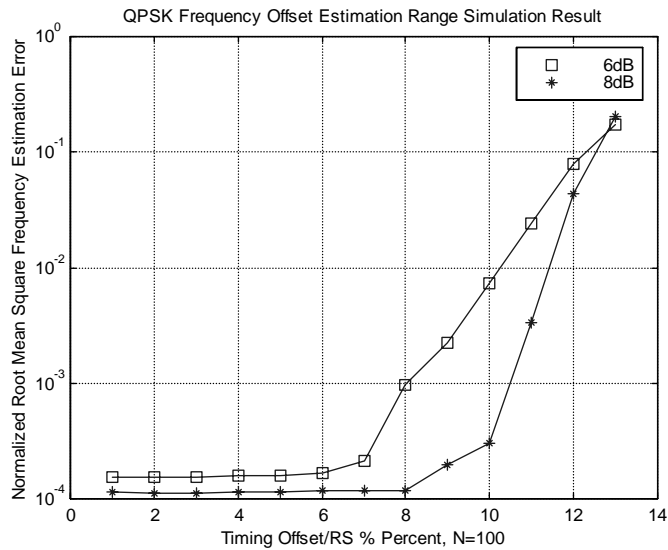**Figure 3 RMS Frequency Estimation Error Compared with Modified CRLB**

Figure 4 Frequency Estimation Range at Different SNR

V.   Conclusion

In this paper we presented a carrier frequency estimation method of MPSK signal with random data modulation and its systolic VLSI implementation. The performance of the estimator is close to CRLB when SNR is no less than 6dB. The estimation range is $\pm 7\% R_s$ at 6dB. For data-aided carrier frequency recovery, correlation can be used to remove the data modulation, the performance of the estimator is much better than PSK signal with random data modulation. We will discuss this feature in further papers.

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