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Optimization of a Three DOF Translational Platform for Well-Conditioned Workspace

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Abstract

Two optimization studies on the design of a three degree of freedom translational parallel platform are conducted and the results are compared. The objective function of the first study maximizes total volume of the manipulator workspace without regard to the quality of the workspace. The second study optimizes the total volume of well conditioned workspace by maximizing a global condition index. The global condition index is a function of the condition number of the Jacobian matrix, providing a means of measuring the amplification error between the actuators and the end effector. Both objective functions involve an integration over the workspace of the manipulator. This integral is approximated using the Monte Carlo method.

1 Introduction

The promise of parallel manipulators continues to motivate research and development as evidenced by machining centers with parallel kinematic structures recently developed by Giddings & Lewis and Ingersoll Milling Machine Co. [1]. As a result of the parallel nature of these manipulators, it is possible for the mechanism to be designed so that the moving structure of the manipulator does not bear the load of the actuators that drive it. This enables large powerful actuators to drive relatively small structures, facilitating the design of manipulators that are faster, stiffer, and stronger than their serial counterparts. These gains are usually realized at the cost of workspace. Consequently, it's incumbent upon the engineer to design the manipulator with the optimization of workspace in mind. This paper addresses that optimization by conducting two optimization studies for a three degree of freedom parallel manipulator recently introduced by Tsai [2]. This manipulator, which constrains the moving platform to only translational motion, maintains the advantages of a parallel manipulator while being less complex than the more common 6 degree of freedom parallel manipulators. The manipulator kinematics were described by Tsai and Stamper [3], showing that there are 4 solutions for the inverse kinematics and 16 solutions for the forward kinematics.

Optimization of manipulator workspace volume is dependent upon a means of determining the workspace of a parallel manipulator for a given set of design variables. Oblak and Kohi [4] described manipulator workspace from the standpoint of a Jacobian analysis and D-surfaces, where one or more of the joints achieve a limit position. A geometric based algorithm to generate a graphical representation of the workspace for a six degree of freedom parallel platform in a given orientation was generated by Gosselin et al. [5] and Gosselin [6]. The workspace of the DELTA4 robot, a 3 degree of freedom platform that is similar to the manipulator being examined in this paper, was described by Clavel [7]. Clavel provided a description of the DELTA4 workspace by defining the workspace as an intersection of geometric primitives that are defined by various robot design parameters. A similar approach was applied to the DELTA4 by Sternheim [8], where the workspace described by the intersection of the geometric primitives was visualized using solid modeling software. Rastegar and Perel [9] and Alciatore and Ng [10] applied the Monte Carlo method to determine workspace boundaries. In this paper, for the purposes of workspace optimization, a numerical value for the workspace volume is computed using the Monte Carlo method.

A parallel manipulator designed for maximum workspace volume may not however be the optimal design for practical applications. It's possible that a parallel manipulator that is optimized for total workspace will result in a manipulator with undesirable kinematic characteristics such as poor dexterity or manipulability. One measure of these characteristics used by Salisbury and Craig [11] and Angeles and Lopez-Cajun [12] is based upon the condition number of the manipulator Jacobian matrix, where the Jacobian matrix maps the actuated joint velocities to the velocity of the moving platform in cartesian space. A summary of other measures of dexterity is presented by Klein and Blaho [13].

The condition of the manipulator in a local sense was considered by Gosselin and Angeles when they examined the optimization of a spherical three degree of freedom parallel manipulator [14] and a planar three degree of freedom parallel manipulator [15]. The criteria they used to evaluate the manipulator designs were symmetry, global workspace, and the condi-

Figure 1: Schematic of the three-DOF manipulator.

tion of the Jacobian of the manipulator at a home position. Global performance indices, that consider the dexterity of the manipulator over the entire workspace, were developed by Park and Brockett [16] and Gosselin and Angeles [17]. The global performance index developed by Gosselin and Angeles is based upon the integration of the reciprocal of the condition number over the entire workspace. This paper takes a similar approach.

2 Description of the Manipulator

A schematic of the manipulator being considered is shown in Fig. 1, where the stationary platform is labeled 0 and the moving platform is labeled 16. Three identical limbs connect the moving platform to the stationary platform. Each limb consists of an upper arm and a lower arm. The lower arms are labeled 1, 2, and 3. Each upper arm is a planar four-bar parallelogram: links 4, 7, 10, and 13 for the first limb; 5, 8, 11, and 14 for the second limb; and 6, 9, 12, and 15 for the third limb. For each limb, the upper and lower arms, and the two platforms are connected by three parallel revolute joints at axes A, B, and E as shown in Fig. 1. The axes of these revolute joints are perpendicular to the axes of the four-bar parallelogram for each limb. There is also a small offset between the axes of B and C, and between the axes of D and E.

A reference frame (XYZ) is attached to the fixed base at point O, located at the center of the fixed platform. For each leg, another coordinate system (UVW) is attached to the fixed base at A_i, such that $\bar{u}_i$ is perpendicular to the axis of rotation of the joint at A_i and at an angle ϕ_i from the x axis as shown in Fig. 1. The angle ϕ_i for the i^{th} leg defines the relative angular position of the legs.

The i^{th} leg of the manipulator is shown in Fig. 2. The vector $\bar{p}$ is the position vector of point P in the (XYZ) coordinate frame, where P is attached at the center of the moving platform. The angle θ_{1i} describes the actuated joint angle and is measured from $\bar{u}_i$ to $\overline{A_iB_i}$. The angle θ_{2i} is defined from the $\bar{u}_i$ direction to $\overline{B_iC_i}$. The angle θ_{3i} is defined by the angle from the $\bar{v}_i$ direction to $\overline{C_iD_i}$. The link lengths as shown in Fig. 2 are assumed to be the same for all limbs.

Considering the manipulator mobility, any single limb constrains rotation of the moving platform about the z and u axes. Hence, the combination of any two limbs constrains rotation about the x, y, and z axes. This leaves the mechanism with three translational degrees of freedom and constrains the moving platform to remain in the same orientation at all times.

3 Manipulator Inverse Kinematics

The inverse kinematics solution will be used during the determination of the workspace volume. The objective of the inverse kinematics problem is to determine the joint angles for each leg given the location of point P in the XYZ coordinate frame. The solutions for θ_{1i} and θ_{3i} for leg i are given by (see Tsai and Stamper [3] for details):

$$\theta_{3i} = \pm \arccos\left(\frac{p_{yi}\cos(\phi_i) - p_{xi}\sin(\phi_i)}{b}\right), \tag{1}$$

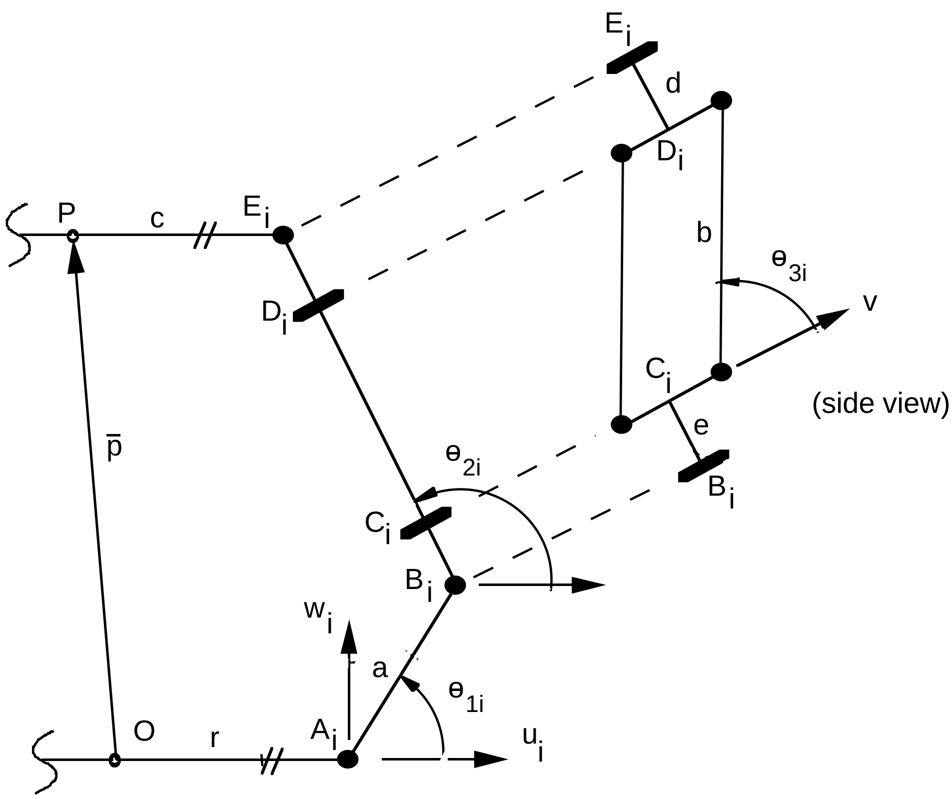

Figure 2: Joint angles and link lengths for leg i.

$$l_{2i}t_{1i}^2 + l_{1i}t_{1i} + l_{0i} = 0, \tag{2}$$

where:

$$\begin{aligned}
l_{0i} = {} & p_{zi}^2 + [p_{xi}\cos(\phi_i) + p_{yi}\sin(\phi_i)]^2 + a^2 \\
& -2a\,[p_{xi}\cos(\phi_i) + p_{yi}\sin(\phi_i)] + (c-r)^2 \\
& +(c-r)\,[2p_{xi}\cos(\phi_i) + 2p_{yi}\sin(\phi_i) - 2a] \\
& -2(d+e)b\sin(\theta_{3i}) + b^2\sin(\theta_{3i})^2 - (d+e)^2, \\
l_{1i} = {} & -4ap_{zi}, \\
l_{2i} = {} & p_{zi}^2 + [p_{xi}\cos(\phi_i) + p_{yi}\sin(\phi_i)]^2 + a^2 \\
& +2a\,[p_{xi}\cos(\phi_i) + p_{yi}\sin(\phi_i)] + (c-r)^2 \\
& +(c-r)\,[2p_{xi}\cos(\phi_i) + 2p_{yi}\sin(\phi_i) + 2a] \\
& -2(d+e)b\sin(\theta_{3i}) + b^2\sin(\theta_{3i})^2 - (d+e)^2, \\
t_{1i} = {} & \tan\left(\frac{\theta_{1i}}{2}\right).
\end{aligned}$$

4 Manipulator Workspace

The manipulator workspace volume, W, is numerically approximated using the Monte Carlo method, as outlined by the following:

Step 1: A hemisphere with a radius equal to that of the total leg length of the manipulator, $a + b + d + e$, is defined that encases the entire possible workspace of the manipulator.

Step 2: A large number of points, n_{total}, are randomly selected within the hemisphere.

Step 3: Each point is tested to determine if it falls within the manipulator workspace. This is accomplished by solving the inverse kinematics problem for each leg as described by equations (1) and (2). If all the joints angles are real, then the point is within the workspace.

Step 4: The number of points that fall within the workspace, n_{in}, is tallied.

Step 5: The workspace volume is estimated by multiplying the volume of the hemisphere by the ratio of points that fall within the workspace to the total number of points selected:

$$W \approx \pi(a+b+d+e)^3 \frac{2n_{in}}{3n_{total}}. \tag{3}$$

This procedure produces a numeric value that is used for the optimization of the total workspace of the manipulator.

5 Optimization Study of Workspace Volume

The objective of the total workspace optimization is to determine the values of the manipulator design variables that result in the largest total manipulator workspace. The design variables considered are:

- the leg link lengths, a, b, and $(d+e)$;
- the relative size of the platforms, $(c-r)$;
- the relative angular position of the legs ϕ_2 and ϕ_3, where leg 1 is assumed to align with the X axis so that $\phi_1 = 0$.

Note that the sum of the two offsets, $d+e$, is treated as a single design variable as shown in equations (1) and (2). Similarly the relative size of the platforms, $(c-r)$, is considered as a single design variable.

In order to bound the solution and to ensure a practical realization, the objective function is subject to the following constraints:

- the total leg length is not to exceed one, $a+b+d+e \leq 1$;
- each leg must have an angular separation of at least 5° from each of the other legs;
- all link lengths must be positive.

Given this problem formulation, the optimization is computed using the Matlab optimization toolbox and produced the following results: $a = .4,\ b = .6,\ d = e = 0,\ (c-r) = 0,\ \phi_2 = 5°$, and $\phi_3 = 355°$. Both the angular leg separation constraint and the total leg length constraints are active. A plot of the manipulator workspace with these design variables is shown in Fig. 3. Note that the optimization routine drove a value at the edge of the allowable design variable space. That is, the angles ϕ were driven to be $5°$ from each other. This is because the volume of the manipulator workspace is the intersection of three torii, which reaches a maximum when they align.

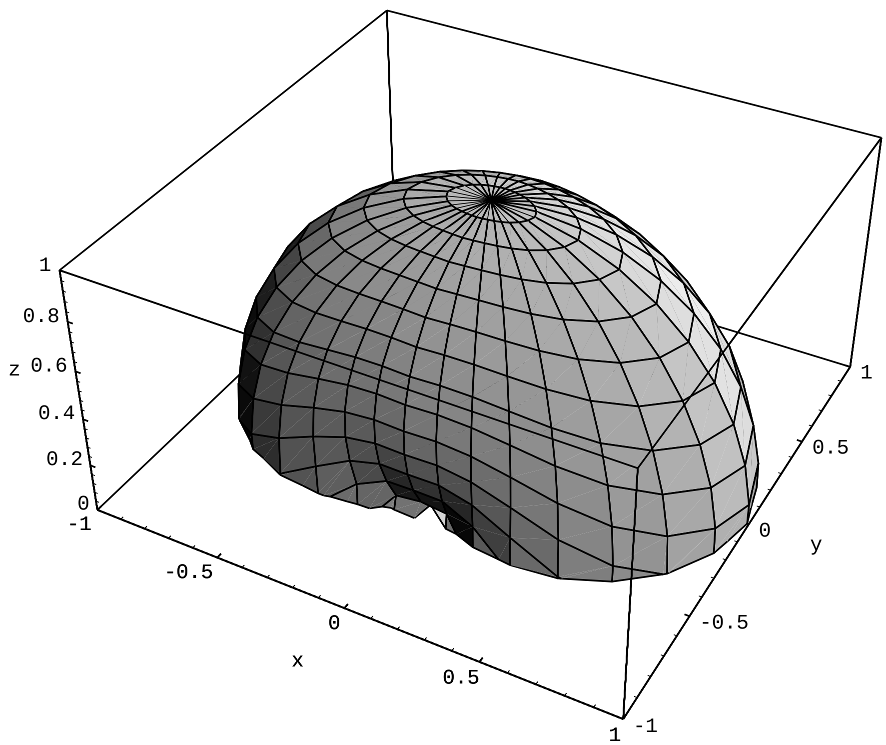

Figure 3: Workspace of manipulator designed for maximum workspace.

6 Manipulator Global Condition Index

A global condition index, η, that considers the condition number of the Jacobian over the entire workspace is defined for the manipulator as:

$$\eta = \int_W \frac{1}{\lambda} \, dW, \tag{4}$$

where λ is the condition number of the Jacobian at a given position in the workspace and W is the manipulator workspace.

The condition number of the Jacobian matrix, J, is defined as:

$$\lambda = \| J \| \| J^{-1} \|, \tag{5}$$

where $\| \cdot \|$ is the 2 norm of the matrix.

For a parallel manipulator, the Jacobian matrix maps the velocity of the moving platform in cartesian space to the actuated joint velocities. In this case, the Jacobian was calculated by differiniating the loop closure equation of each leg as follows:

$$\begin{aligned} - \dot{\theta}_{1i}\overline{v} \times \overline{AB_i} &= \overline{V_p} - \dot{\theta}_{3i}\left[\sin(\theta_{2i})\overline{u} - \cos(\theta_{2i})\overline{w}\right] \times \overline{DC_i} \\ &-\dot{\theta}_{2i}\overline{v} \times \left(\overline{ED_i} + \overline{CB_i} + \overline{DC_i}\right) \end{aligned} \tag{6}$$

Equation (6) is then solved for for $\dot{\theta}_{1i}$, $\dot{\theta}_{2i}$, and $\dot{\theta}_{3i}$ which leads to the Jacobian matrix for the manipulator as:

$$\begin{bmatrix} \dot{\theta}_{11} \\ \dot{\theta}_{12} \\ \dot{\theta}_{13} \end{bmatrix} = \begin{bmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{bmatrix} \begin{bmatrix} v_{p,x} \\ v_{p,y} \\ v_{p,z} \end{bmatrix} \tag{7}$$

where:

$$\begin{aligned} j_{i1} &= \frac{\cos(\theta_{3i})\sin(\theta_{0i}) - \cos(\theta_{2i})\cos(\theta_{0i})\sin(\theta_{3i})}{a\sin(\theta_{3i})\sin(\theta_{1i} - \theta_{2i})}, \\ j_{i2} &= \frac{-\cos(\theta_{3i})\cos(\theta_{0i}) - \cos(\theta_{2i})\sin(\theta_{0i})\sin(\theta_{3i})}{a\sin(\theta_{3i})\sin(\theta_{1i} - \theta_{2i})}, \\ j_{i3} &= \frac{-\sin(\theta_{2i})}{a\sin(\theta_{1i} - \theta_{2i})}, \end{aligned}$$

for $i = 1, 2,$ and 3, where the joint angles are determined using the inverse kinematic solution.

The complexity of generating a closed-form solution to equation (4), compels the use of a numerical solution technique. For this paper, a Monte Carlo method is employed and is outlined as follows:

Steps 1-3: Same as steps 1-3 for workspace estimation.

Step 4: The condition index sum, S, which is the sum of the reciprocal of the condition number of each point that falls within the workspace, is calculated by $S = \sum_i \frac{1}{\lambda_i}$, for the i points that fall within the workspace.

Step 5: The global condition index, η, is determined by multiplying the volume of the hemisphere and the condition index sum, and then dividing by the total number of points selected, i.e.:

$$\eta = \frac{2\pi(a+b+d+e)^3 S}{3n_{total}} \tag{8}$$

7 Optimization Study of Global Condition Index

The objective of the well-conditioned workspace optimization is to determine the values of the manipulator design variables that result in the best global condition index. The same set of design variables that were used during the total workspace optimization is also used for the well conditioned workspace optimization. The objective function is also subject to the same constraints as were used during the total workspace optimization. The well-conditioned workspace optimization is computed using the Matlab optimization toolbox and produced the following results: $a = .44$, $b = .56$, $d = e = 0$, $(c - r) = 0$, $\phi_2 = 120°$, and $\phi_3 = 240°$ when 200,000 points were used for the Monte Carlo method. The only active constraint is the total leg length constraint. A plot of the manipulator workspace with these design variables is shown in Fig. 4.

The impact of the design on the condition of the workspace is significant, and can be seen in Figs. 5 and 6, where the condition number is plotted for a plane of the workspace at z=.5 for both the manipulator optimized for total workspace and the manipulator optimized for well conditioned workspace. Figure 5 shows that the workspace is ill conditioned for the manipulator that is optimized for total workspace volume with a maximum condition index, $\frac{1}{\lambda}$ of ~ 0.002 in the z=.5 plane. This results in a poor manipulator since positioning errors at the actuator are significantly magnified at the end effector. Whereas Fig. 6 shows a much better conditioned workspace with a maximum condition condition index of ~ 0.6 in the z=.5 plane, suggesting that this is the better design when considering the positioning performance of the manipulator. It's also interesting to note that when the position of the legs about the platform is constrained to be symmetrical, so that $\phi_2 = 120°$ and $\phi_3 = 240°$,

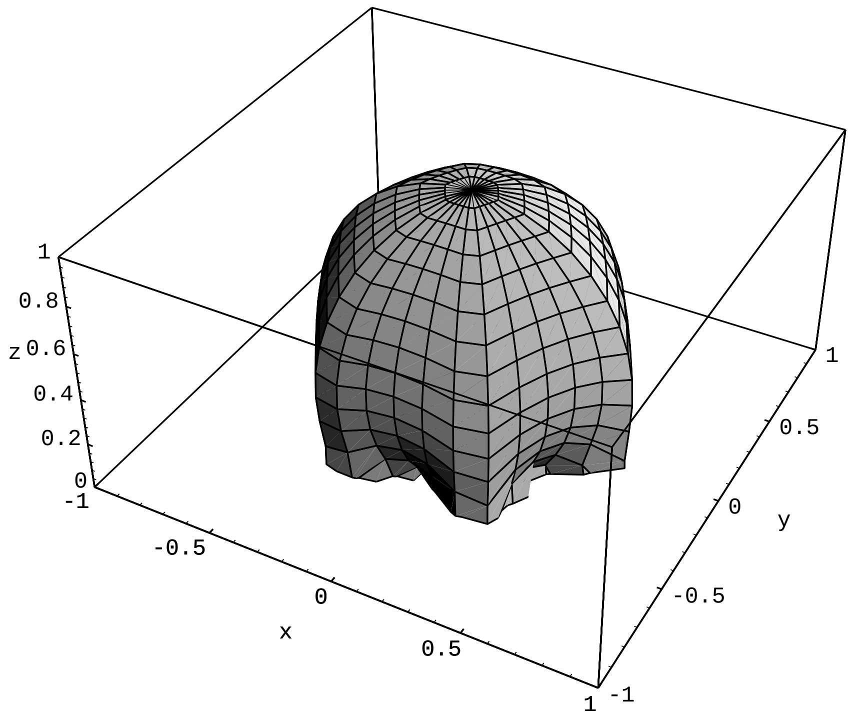

Figure 4: Workspace of manipulator designed for maximum global condition index.

the total workspace volume optimization results differ significantly from the results obtained from the global condition index optimization. The link lengths produced are $a = .32$, $b = .68$, $(d + e) = 0$, and $(c - r) = 0$ when the manipulator is optimized for total workspace with legs that are constrained to be symmetrically located about the platform.

8 Conclusion

The design of a three degree of freedom translational parallel manipulator is optimized for both total workspace and global conditioning index. The optimization of the total workspace shows that the lower leg of the manipulator should comprise 40% of the total leg length and the upper arm parallelogram should should comprise the remaining 60% of the total leg length, while the links that provide the offset between joints B and C and also between D and E should have a length of 0. Furthermore, the legs should also be at the smallest

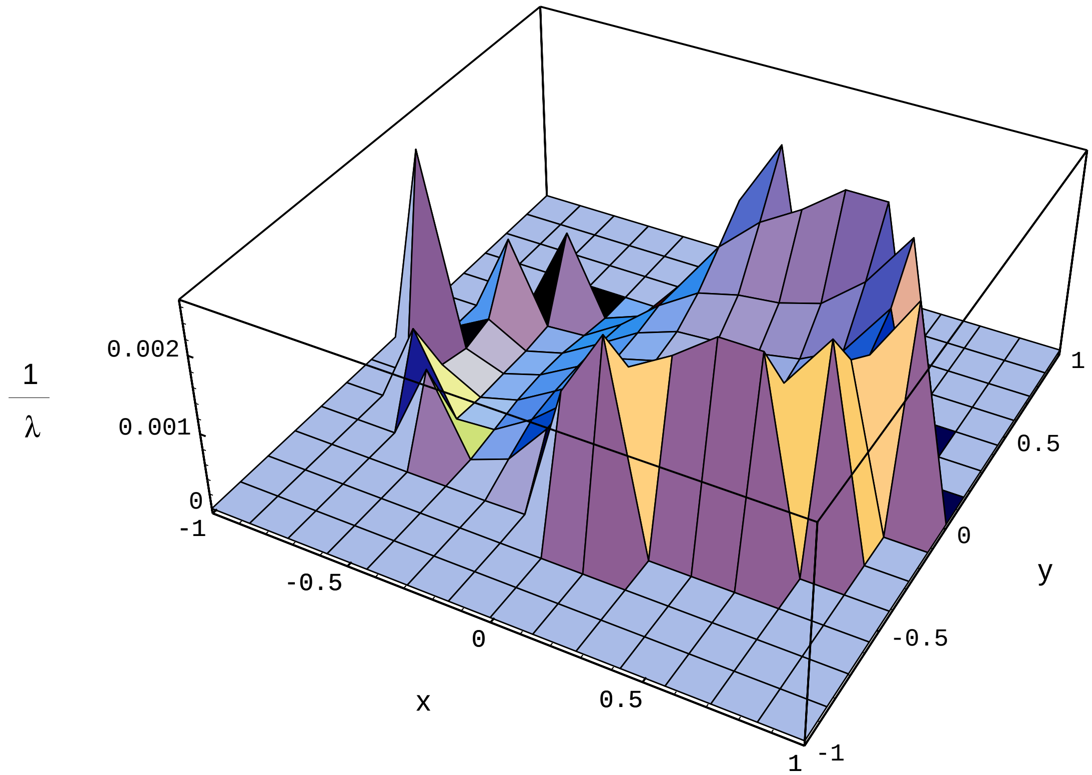

Figure 5: Reciprocal of the condition number at the $z = .5$ plane for the total workspace optimized manipulator.

angular offset possible. The total workspace of the manipulator is determined by a Monte Carlo technique.

The optimization of the manipulator design for the global condition index results in a manipulator where the lower leg comprises 44% of the the total leg length and the upper arm comprises 56% of the total leg length, while each leg has an angular separation of 120°. The global condition index is a function that considers the condition number of the manipulator Jacobian over the entire workspace. A numerical value for the global condition index is determined with the application of a Monte Carlo technique.

These results show that a parallel manipulator of this type that is designed to optimized total workspace volume is significantly different from one that is optimized for a well conditioned workspace. Moreover, these results show that a manipulator of this type that is designed to maximize total workspace volume will result in an ill conditioned workspace.

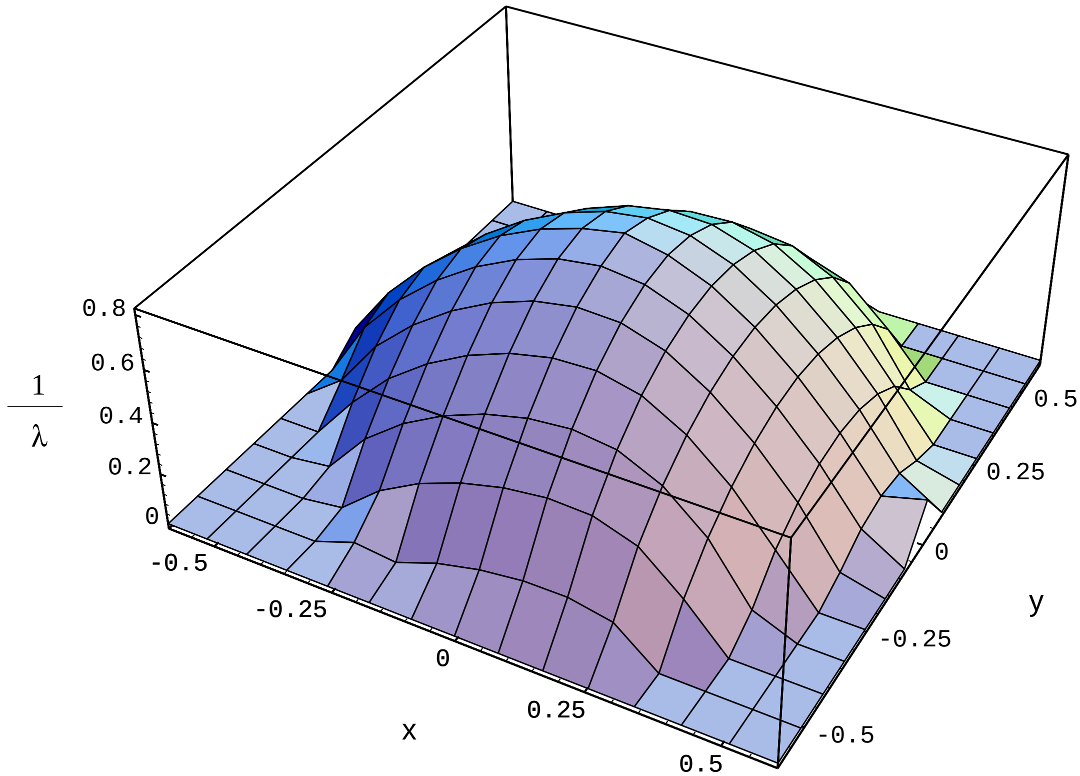

Figure 6: Reciprocal of the condition number at the $z = .5$ plane for the global condition index optimized manipulator.

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