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THE TWO-TEMPERATURE EFFECT ON A SEMICONDUCTING THERMOELASTIC SOLID CYLINDER BASED ON THE MODIFIED MOORE – GIBSON – THOMPSON HEAT TRANSFER

I. Kaur [1]✉, K. Singh [2]

[1] Government College for Girls (Palwal), Kurukshetra, India;

[2] Kurukshetra University, Kurukshetra, India

✉ bawahanda@gmail.com

Abstract. This study introduces a new modified Moore – Gibson – Thompson Photo-Thermal (MGTPT) theory with two temperatures for semiconductor material. The photo-thermoelastic effects have been been investigated in an infinitely constrained semiconducting solid cylinder subjected to variable heat flux in the form of an exponential laser pulse. The Laplace transforms were used for the solution of the mathematical model in the transformed domain. The numerical inversion was applied to obtain the displacement components, the conductive temperature, the carrier density, and the thermal stresses in the physical domain. The impact of different theories of thermoelasticity with two temperatures on the displacement, temperature, thermal stresses and carrier density were represented graphically and discussed using Matlab software.

Keywords: semiconducting media, two-temperature effect, modified Moore – Gibson – Thompson heat transfer

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ДВУХТЕМПЕРАТУРНЫЙ ПОДХОД К АНАЛИЗУ СВОЙСТВ ПОЛУПРОВОДЯЩЕГО ТЕРМОУПРУГОГО СПЛОШНОГО ЦИЛИНДРА, ОСНОВАННЫЙ НА МОДИФИЦИРОВАННОЙ ТЕОРИИ ТЕПЛОПЕРЕДАЧИ МУРА – ГИБСОНА – ТОМПСОНА

И. Каур [1]✉, К. Сингх [2]

[1] Государственный колледж для девушек (Палвал), г. Курукшетра, Индия;

[2] Университет Курукшетры, г. Курукшетра, Индия

✉ bawahanda@gmail.com

Аннотация. В работе вводится новая модифицированная двухтемпературная фототермическая теория Мура – Гибсона – Томпсона (MGTPT) для полупроводникового материала. Исследованы фототермоупругие эффекты в полупроводниковом твердом круговом цилиндре бесконечной длины, который подвергается переменному тепловому потоку, образованному экспоненциальным лазерным импульсом. Для решения математической модели в преобразованной области используются преобразования Лапласа. Для определения компонент смещения, температуры проводимости, плотности носителей и тепловых напряжений в физической области применяется численная инверсия. Влияние различных двухтемпературных моделей с термоупругостью на смещение, температуру, тепловые напряжения и плотность носителей представлено графически с помощью программного обеспечения Matlab, и полученные результаты обсуждаются.

Ключевые слова: полупроводниковая среда, двухтемпературная модель, модифицированная модель теплопередачи Мура – Гибсона – Томпсона

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Introduction

In recent years, a lot of attention has been paid to semiconductor materials due to their physical properties. Unlike copper or aluminum, semiconductor materials are not very good conductors at room temperature. At room temperature, they are also not good insulators too, for example, such as glass. The resistance of semiconductor materials gradually decreases with increasing temperature. As a consequence, they provide great electrical conductivity. Because of this property, semiconductor materials become important in modern very large-scale integration logic (VLSI)/electronics/Electrical industries. Under high temperatures, semiconductor materials deform and their physical properties change internally. Two major changes occur as a result of thermal effects: one is thermoelastic deformation and another is electronic deformation. The thermal effect occurs when semiconductor media are exposed to focused laser beams or sunlight beams and, owing to this effect, they will vibrate. These materials have many uses in renewable energy, especially in the solar cells industry which depends heavily on semiconductor materials.

J. M. Duhamel [1] presented the theory of classical uncoupled thermoelasticity. This theory has two limitations. To begin with, the elastic state of material is not related to temperature. Moreover, the parabolic heat equation predicts that the temperature travels at an infinite speed, which is again contradictory to physical experiments. M. A. Biot [2] set up the hypothesis of coupled thermoelasticity to conquer these impediments. According to his theory, the heat conduction equations and the equations of elasticity are related. However, the shortcoming of this theory is that it only predicts heat waves with an unlimited speed of propagation. C. Cattaneo [3] and P. Vernotte [4, 5] suggested a wider form of the Fourier law for the homogeneous and isotropic medium by introducing thermal relaxation time τ_0 to the heat flux vector $\mathbf{q}$ to establish a steady state at a point when a temperature gradient ∇T is abruptly imposed on it, as,

$$(1 + \tau_0\, \partial/\partial t)\mathbf{q} = -K_{ij} \nabla T, \tag{1}$$

where K_{ij} is the coefficient of thermal conductivity, t is the time.

Then the generalized theory of thermoelasticity with one relaxation time was put forward by H. W. Lord and V. Shulman [6] for the particular case of an isotropic body. According to this theory, the heat equation being hyperbolic has a finite speed of propagation for temperature. After that a more precise version of thermoelasticity was presented by the two-temperature theory of thermoelasticity introduced by P. J. Chen and M. E. Gurtin [7]. This model includes the conductive temperature φ (the outcome of thermodynamic processes) and the thermodynamic temperature T (the result of mechanical processes). A. E. Green and K. A. Lindsay [8] showed that the linear heat conduction tensor is symmetric. R. S. Dhaliwal and H. H. Sherief [9] gave the equations of generalized thermoelasticity for an anisotropic medium. However, A. E. Green and P. M. Naghdi [10 – 12] further contributed to the thermoelastic theories with and without energy dissipation and further developed the Fourier law as

$$q = -K_{ij}\ \nabla T - K_{ij}^*\ \nabla\theta,\ \dot{\theta} = T,\ T = \varphi - a_{ij}\ \varphi_{ij}, \tag{2}$$

where K_{ij}^* is the materialistic constant; a_{ij} is the two-temperature parameter.

Now the law can be formulated as follows:

Based on entropy equality, they proposed three new thermoelastic theories. Their theories are known as the thermoelasticity theory of type I, the thermoelasticity theory of type II (i.e., thermoelasticity without energy dissipation), and the thermoelasticity theory of type III (i.e., thermoelasticity with energy dissipation). On linearization, type I becomes the classical heat equation whereas on linearization of type-II, as well as type-III theories, give the finite speed of thermal wave propagation.

Recently, there has been a myriad of academic articles that study and interpret the Moore – Gibson – Thompson (MGT) equation. I. Lasiecka and X. Wang [13] founded their theory on a 3rd-order differential equation, important to various fluid dynamics. Using the MGT equation with 2T, R. Quintanilla [14, 15] devised a novel heat conduction model. MGT equation of the modified Fourier law is, as follows:

$$(1 + \tau_0\ \partial/\partial t)\mathbf{q} = -K_{ij}\ \nabla T - K_{ij}^*\ \nabla\theta,\ \text{where}\ \dot{\theta} = T. \tag{3}$$

J. R. Fernandez and R. Quintanilla [16] discussed linear thermoelastic deformations of dielectrics. N. Bazarra et al. [17] studied a thermoelastic problem using MGT thermoelastic equation. Suppose that the semiconductor elastic media is exposed to external laser beams and which create carrier-free charge density due to excited free electrons with semiconductor gap energy E_g. In response to absorbed optical energy, there is a change in the electronic deformation and the elastic vibration. In this case, heat conductivity equations will be affected by thermal-elastic-plasma waves. The revised Fourier law for semiconductor materials with plasma impact in a generalized form can be written as

$$(1 + \tau_0\ \partial/\partial t)\mathbf{q} = -K_{ij}\nabla T - K_{ij}^*\nabla\theta - \int (E_g N/\tau)dx,\ \text{where}\ \ \dot{\theta} = T; \tag{4}$$

here N is the carrier density.

The photoexcitation effect is represented by the final term in Eq. (4). When the above equation is differentiated with respect to x, the result is

$$(1 + \tau_0\ \partial/\partial t)\nabla\cdot\mathbf{q} = -\ \nabla\cdot(K_{ij}\nabla T - K_{ij}^*\nabla\theta) - E_g N/\tau,\ \text{where}\ \dot{\theta} = T. \tag{5}$$

Some other researchers also worked on similar research on Hall current effect and semiconductor medium such as M. Marin [18], P. Lata et al. [19], A. M. S. Mahdy et al. [20], I. Kaur and K. Singh [21, 22], M. Marin et al. [23, 24], I. Kaur et al. [25, 26], M. M. Bhatti et al. [27, 28], M. Conti et al. [29], M. I. A. Othman and M. Marin [30], J. A. Conejero et al. [31], M. Marin et al. [32], E. M. Craciun et al. [33], M. E. Nasr et al. [34], A. E. Abouelregal et al. [35]. However, from the literature review, it has been observed that no work has been carried out in the transient study of semiconductor cylinders exposed to ultrashort pulsed laser heating and photogenerated plasma with two temperatures. The importance of this issue is the main motivation for its in-depth review in this study.

This study aimed to explore the photo-thermoelastic interactions in an infinite semiconducting solid cylinder acted upon by the variable heat flux in the form of an exponential laser pulse along the boundary surface.

The governing equations are expressed using a new generalized photo-thermoelastic model as MGTPT heat transfer for semiconducting medium with two temperatures. The Laplace transforms are used for the solution of the mathematical model in the transformed domain. And the numerical inversion is applied to obtain the displacement components, the conductive temperature, the carrier density and the thermal stresses in the physical domain. The impact of different theories of thermoelasticity with two temperatures on the displacement, temperature, thermal stresses, and the carrier density are represented graphically using Matlab software.

Basic equations

Following the work of A. E. Abouelregal and D. Atta [36], we present below the constitutive relations, the equation of motion, the plasma diffusion equation governing the plasma transportation process in semiconductor nanostructure medium, the MGTPT heat conduction equation with thermal-plasma-elastic interaction.

The constitutive relations:

$$\sigma_{ij} = \left(\lambda u_{kk} - \beta T - \delta_n N\right)\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right),$$
$$\beta = \left(3\lambda - 2\mu\right)\alpha_T \delta_n = \left(3\lambda + 2\mu\right) d_n, T = \varphi - a_{ij}\varphi_{ij}, \tag{6}$$

where σ_{ij}, N/m^2, are the stress tensor components; λ, μ, Pa, are the Lame's elastic constants; $u_{i,j}$, $u_{j,i}$, m, are the displacement tensor components; d_n is the coefficient of electronic deformation; δ_{ij} is the Kronecker delta; α_T, K^{-1}, is the linear thermal expansion coefficient.

The equation of motion:

$$\sigma_{ij,j} + F_i = \rho \ddot{u}_i, \tag{7}$$

where F_i, N, is the body force; ρ, kg/m^3, is the medium density; $\ddot{u}_i$, m/s^2, are components of the body acceleration.

The plasma diffusion equation:

$$\frac{\partial N}{\partial t} = D_E \nabla^2 N - \frac{N}{\tau} + \kappa\left(1 - a\nabla^2\right)\varphi, \tag{8}$$

where D_E is the carrier diffusion coefficient; τ, s, is the photo-generated carrier lifetime; a is the measure of thermoelastic diffusion effect; κ is the coupling parameter for thermal activation, $\kappa = (T/\tau)\, \partial N_0/\partial T$ (N_0 is the carrier concentration at equilibrium position).

The modified Moore – Gibson –Thompson heat conduction equation with two temperature quantities:

$$\left(K_{ij}\varphi_j\right)_{,i} + \left(K^*_{ij}\dot{\varphi}_{,j}\right)_{,i} + \frac{E_g \dot{N}}{\tau} = \left(1 + \tau_0 \frac{\partial}{\partial t}\right)\left[\rho C_E \left(1 - a\nabla^2\right)\ddot{\varphi} + \beta_{ij} T_0 e_{ij} - \rho \dot{Q}\right], \tag{9}$$

where C_E is the specific heat at constant strain; e_{ij} are the strain tensor components; Q is the source of heat; $K_{ij} = K_i \delta_{ij}$, $K^*_{ij} = K^*_i \delta_{ij}$, i is not summed.

The subscripts marked by commas mean the partial derivatives with respect to the spatial coordinates, whereas one or two dots on top of the notation mean the first or second derivative with respect to the time variable t.

Formulation and solution of the problem

A one-dimensional (1D) symmetrical, thermally homogenous, semiconductor solid cylinder of radius r_0 was considered (Fig. 1). An external laser pulse heating system was used to irradiate the external surface of the solid cylinder. A cylindrical coordinate system (r, θ, z) with the z-axis arranged along the cylinder axis was taken. Initially, it was believed the cylinder to maintain constant and uniform temperature T_0.

Furthermore, all examined fields were assumed to be finite within the medium for the regularity condition. Due to symmetry, and for the 1D problem, all the functions considered depended on the radial distance r and the time t.

For the 1D problem, displacement components of **u** and the displacement-strain relations are given by

$$\mathbf{u} = (u,0,0)\,(r, t),\ e_{rr} = u/r,\ e_{\theta\theta} = \partial u/\partial r,\ e_{r\theta} = e_{rz} = e_{\theta z} = e_{zz} = 0, \tag{10}$$

where e_{ij} are the strain tensor components.

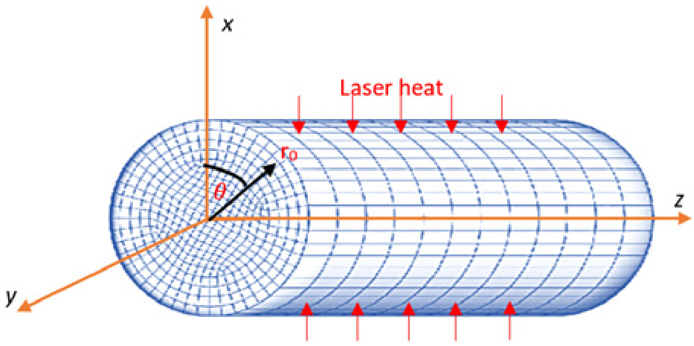

Fig. 1. The illustration to the problem formulation:
a semiconductor solid cylinder of radius r_0 and an external laser heat

The stress-strain-temperature-carrier relations (6) using Eqs. (10) will be the form

$$\sigma_{rr} = 2\mu\frac{\partial u}{\partial r} + \lambda e - \left[\beta\left(1 - a\nabla^2\right)\varphi + \delta_n N\right], \tag{11}$$

$$\sigma_{\theta\theta} = 2\mu\frac{u}{r} + \lambda e - \left[\beta\left(1 - a\nabla^2\right)\varphi + \delta_n N\right], \tag{12}$$

$$\sigma_{zz} = \lambda e - \left[\beta\left(1 - a\nabla^2\right)\varphi + \delta_n N\right], \tag{13}$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$$

where e_{kk} is the cubical dilatation, $e_{kk} = e = (1/r)\,\partial(ru)\,/\,\partial r$.

Hence, the dynamic motion equation becomes

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\left(\sigma_{rr} - \sigma_{\theta\theta}\right) = \rho\left(\frac{\partial u^2}{\partial t^2} - \Omega^2 u\right). \tag{14}$$

Using Eqs. (11) – (13), in Eqs. (14) and (8), (9), the governing equations for the considered semiconducting medium are:

$$(\lambda + 2\mu)\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial(ru)}{\partial r}\right] - \beta\frac{\partial}{\partial r}\left[\left(1 - a\nabla^2\right)\varphi\right] - \delta_n\frac{\partial N}{\partial r} = \rho\left(\frac{\partial^2 u}{\partial t^2}\right), \tag{15}$$

$$\frac{\partial N}{\partial t} = D_E\left(\nabla^2 N\right) - \frac{N}{\tau} + \kappa\left(1 - a\nabla^2\right)\varphi, \tag{16}$$

$$K\frac{\partial}{\partial t}\nabla^2\varphi + K^*\nabla^2\varphi + \frac{E_g\dot{N}}{\tau} = \left(1 + \tau_0\frac{\partial}{\partial t}\right)\left\{\rho C_E\frac{\partial^2}{\partial t^2}\left[\left(1 - a\nabla^2\right)\varphi\right] + \beta T_0\frac{\partial^2 e}{\partial t^2}\right\}, \tag{17}$$

Pre-operating both sides of Eq. (15) by $(1/r + \partial/\partial r)$, we get:

$$\left(\lambda + 2\mu\right)\nabla^2 e - \beta\nabla^2\left(1 - a\nabla^2\right)\varphi - \delta_n\nabla^2 N = \left(\frac{\partial^2 e}{\partial t^2}\right). \tag{18}$$

In order to obtain the above equations in dimensionless form, the dimensionless quantities are given by the following:

$$\begin{gathered}\left(r', u'\right) = \nu_0\eta\left(r, u\right),\ \left(T', N', \sigma'_{ij}, \varphi'\right) = \frac{1}{\rho\nu_0^2}\left(\beta T, \delta_n N, \sigma_{ij}, \varphi\right),\\ \left(\tau'_0, \tau', t'\right) = \nu_0^2\eta\left(\tau'_0, \tau', t'\right),\ \eta = \frac{\rho C_E}{K},\ \rho\nu_0^2 = \lambda + 2\mu,\ \gamma = \sqrt{\frac{\mu}{\lambda + 2\mu}}.\end{gathered} \tag{19}$$

Here, the magnetic parameter M (also known as the Hartmann number) measures the strength of the magnetic field. Using Eq. (19) in Eqs. (16) – (18) and after suppressing the primes, yields:

$$\nabla^2 e-\nabla^2\left(1-a\nabla^2\right)\varphi-\nabla^2 N=\left(\frac{\partial^2 e}{\partial t^2}\right), \tag{20}$$

$$\frac{\partial N}{\partial t}=\delta_1\left(\nabla^2 N\right)-\delta_2 N+\delta_3\left(1-a\nabla^2\right)\varphi, \tag{21}$$

$$\frac{\partial}{\partial t}\nabla^2\varphi+\delta_4\nabla^2\varphi+\delta_5 N=\left(1+\tau_0\frac{\partial}{\partial t}\right)\left[\frac{\partial^2}{\partial t^2}\left(1-a\nabla^2\right)\varphi+\delta_6\frac{\partial^2 e}{\partial t^2}\right], \tag{22}$$

where δ_n are the electronic deformation coefficients that follow the expressions

$$\begin{gathered}\delta_1=D_E\eta,\ \delta_2=\frac{1}{\tau},\ \delta_3=\frac{\kappa\delta_n}{\beta},\ \delta_4=\frac{K^*}{(\lambda+2\mu)C_E},\\ \delta_5=\frac{E_g}{\delta_n C_E(\lambda+2\mu)\eta\tau},\ \delta_6=\frac{\beta^2 T_0}{\rho C_E(\lambda+2\mu)}.\end{gathered} \tag{23}$$

Making use of dimensionless quantities defined by Eq. (19) in Eqs. (10) – (12) and after suppressing the primes, yields:

$$\sigma_{rr}=2\gamma^2\frac{\partial u}{\partial r}+\left(1-2\gamma^2\right)e-\left[\left(1-a\nabla^2\right)\varphi+N\right], \tag{24}$$

$$\sigma_{\theta\theta}=2\gamma^2\frac{u}{r}+\left(1-2\gamma^2\right)e-\left[\left(1-a\nabla^2\right)\varphi+N\right], \tag{25}$$

$$\sigma_{zz}=\left(1-2\gamma^2\right)e-\left[\left(1-a\nabla^2\right)\varphi+N\right]. \tag{26}$$

The initial conditions of the problem are taken as

$$u(r,0)=0=\frac{\partial u}{\partial r}(r,0), \tag{27}$$

$$\varphi(r,0)=0=\frac{\partial\varphi}{\partial r}(r,0), \tag{28}$$

$$N(r,0)=0=\frac{\partial N}{\partial r}(r,0). \tag{29}$$

The Laplace transform of a function f with respect to the time variable t, with s as a Laplace transform variable, is defined as

$$\mathcal{L}\left(f(t)\right)=\overline{f}(s)=\int_0^\infty f(t)e^{-st}dt. \tag{30}$$

Using Laplace transforms on Eq. (30) to Eqs. (20) – (22) we obtain:

$$\left(\nabla^2-s^2\right)\overline{e}-\nabla^2\left(1-a\nabla^2\right)\overline{\varphi}-\nabla^2\overline{N}=0, \tag{31}$$

$$\left[\delta_1\nabla^2-\left(\delta_2+s\right)\right]\overline{N}+\delta_3\left(1-a\nabla^2\right)\overline{\varphi}=0, \tag{32}$$

$$\left(1+\tau_0 s\right)\delta_6 s^2\overline{e}+\left[-\left(s+\delta_4\right)\nabla^2+\left(1+\tau_0 s\right)s^2\left(1-a\nabla^2\right)\right]\overline{\varphi}-\delta_5\overline{N}=0. \tag{33}$$

Using Laplace transforms on Eq. (30) to Eqs. (24) – (26), we obtain:

$$\overline{\sigma}_{rr}=2\gamma^2\frac{\partial\overline{u}}{\partial r}+\left(1-2\gamma^2\right)\overline{e}-\left[\left(1-a\nabla^2\right)\overline{\varphi}+\overline{N}\right], \tag{34}$$

$$\bar{\sigma}_{\theta\theta} = 2\gamma^2 \frac{\bar{u}}{r} + \left(1 - 2\gamma^2\right)\bar{e} - \left[\left(1 - a\nabla^2\right)\bar{\varphi} + \bar{N}\right], \tag{35}$$

$$\bar{\sigma}_{zz} = \left(1 - 2\gamma^2\right)\bar{e} - \left[\left(1 - a\nabla^2\right)\bar{\varphi} + \bar{N}\right]. \tag{36}$$

When Eqs. (31) to (33) are decoupled, we get

$$\left(\nabla^6 - B\nabla^4 + C\nabla^2 - D\right)\left(\bar{e}, \bar{\varphi}, \bar{N}\right) = 0, \tag{37}$$

where

$$A = -\delta_1\delta_{11},$$

$$B = -\left[a\delta_3\delta_5 s + \left(-A\delta_7 - \delta_1\delta_{10} - \delta_1\delta_9 + \delta_8\delta_{11} - a\delta_8\delta_9\right)\right] / A,$$

$$C = \left(-\delta_3\delta_5 s + a\delta_3\delta_5\delta_7 + \delta_3\delta_9 - \delta_1\delta_7\delta_{10} + \delta_8\delta_7\delta_{11} + \delta_8\delta_{10} + \delta_8\delta_9\right) / A,$$

$$D = \left(\delta_3\delta_7\delta_5 s - \delta_8\delta_7\delta_{10}\right) / A,$$

$$\delta_7 = \left(-s^2\right),\ \delta_8 = \delta_2 + s,\ \delta_9 = \left(1 + \tau_0 s\right)\delta_6 s^2,$$

$$\delta_{10} = \left(1 + \tau_0 s\right)s^2,\ \delta_{11} = -\left(s + \delta_4\right) - \left(1 + \tau_0 s\right)s^2 a.$$

Presenting λ_i (i = 1, 2, 3) in Eqs. (39) we obtain:

$$\left(\nabla^2 - \lambda_1^2\right)\left(\nabla^2 - \lambda_2^2\right)\left(\nabla^2 - \lambda_3^2\right)\left(\bar{e}, \bar{\varphi}, \bar{N}\right) = 0, \tag{38}$$

where λ_i^2 (i = 1, 2, 3) are the roots of the equation

$$\left(\lambda^6 - B\lambda^4 + C\lambda^2 - D\right) = 0,$$

that are given by

$$\lambda_1^2 = \frac{1}{3}\left(2\omega \sin\xi + B\right), \tag{39}$$

$$\lambda_2^2 = \frac{1}{3}\left(-\omega \sin\xi - \sqrt{3}\omega\cos\xi + B\right), \tag{40}$$

$$\lambda_3^2 = \frac{1}{3}\left(-\omega \sin\xi + \sqrt{3}\omega\cos\xi + B\right) \tag{41}$$

with

$$\omega = \sqrt{B^2 - 3C},\ \xi = \frac{1}{3}\left[\sin\left(-\frac{2B^3 - 9BC + 27D}{2\omega}\right)\right]^{-1}.$$

The general solution of Eq. (38) can be written in the form

$$(\bar{e}, \bar{\varphi}, \bar{N}) = \sum_{i=1}^{3}\left(1, \zeta_i, \eta_i\right) g_i I_0\left(\lambda_i r\right), \tag{42}$$

where $I_n(\ )$ indicates the second type of modified Bessel functions of order n.

We get the following relations by inserting Eq. (42) into Eqs. (31) – (33):

$$\zeta_i = \frac{-\left(\lambda_i^2 + \delta_7\right)\left(\delta_9\lambda_i^2 - \delta_5\right)}{\delta_3\delta_5 + \left(\delta_{11}\lambda_i^2 + \delta_{10}\right)\left(\delta_1\lambda_i^2 - \delta_8\right)}, \tag{43}$$

$$\eta_i = \frac{-\left(\lambda_i^2 + \delta_7\right)\delta_3}{\delta_3\delta_5 + \left(\delta_{11}\lambda_i^2 + \delta_{10}\right)\left(\delta_1\lambda_i^2 - \delta_8\right)}. \tag{44}$$

The displacement **u** may be represented in the Laplace transform domain, as follows:

$$\bar{u} = \sum_{i=1}^{3} \frac{1}{\lambda_i} g_i I_1\left(\lambda_i r\right). \tag{45}$$

We obtained Eq. (45) with the help of the Bessel function relation

$$\int x I_0\left(x\right) dx = x I_1\left(x\right). \tag{46}$$

Differentiating Eq. (45) in terms of r gives

$$\frac{\partial \bar{u}}{\partial r} = \sum_{i=1} g_i \left[I_0\left(\lambda_i r\right) - \frac{1}{\lambda_i r} I_1\left(\lambda_i r\right) \right]. \tag{47}$$

Thus, the final thermal stress solutions are generated in closed form as follows:

$$\bar{\sigma}_{rr} = \sum_{i=1}^{3} g_i \left[l_i I_0\left(\lambda_i r\right) - \frac{2\gamma^2}{\lambda_i r} I_1\left(\lambda_i r\right) \right], \tag{48}$$

$$\bar{\sigma}_{\theta\theta} = \sum_{i=1}^{3} g_i \left[\frac{2\gamma^2}{\lambda_i r} I_1\left(\lambda_i r\right) + l_i I_0\left(\lambda_i r\right) \right], \tag{49}$$

$$\bar{\sigma}_{zz} = \sum_{i=1} g_i l_i I_0\left(\lambda_i r\right), \tag{50}$$

$$l_i = 1 - 2\gamma^2 - \left[\zeta_i \left(1 - a\lambda_i^2\right) + \eta_i \right].$$

Boundary conditions

We presume that the cylinder's outside surface is compelled. Therefore, the mechanical boundary condition can be expressed as

$$\mathrm{u}(r, t) = 0 \text{ at } r = r_0. \tag{51}$$

Also, the boundary condition for variable heat flux (exponentially laser pulsed heat) is applied to the boundary surface:

$$q_p = q_0 \frac{t^2}{16 t_p^2} e^{-\frac{t}{t_p}} \text{ at } r = r_0. \tag{52}$$

Using dimensionless variables (32) on Eq. (3) yields the equation

$$\left(1 + \tau_0 \frac{\partial}{\partial t}\right) \dot{q}_p = -\left(\frac{\partial}{\partial t} + \delta_4\right) \frac{\partial T}{\partial r}, \tag{53}$$

Eqs. (52) and (53) give the following boundary condition:

$$\frac{q_0}{16 t_p^2} \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial}{\partial t} \left(t^2 e^{-\frac{t}{t_p}}\right) = -\left(\frac{\partial}{\partial t} + \delta_4\right) \frac{\partial T}{\partial r} \text{ at } r = r_0. \tag{54}$$

The carriers can reach the sample surface during the diffusion phase, with a finite probability of recombination.

The boundary condition for the carrier density:

$$D_E \frac{\partial N}{\partial r} = s_v N \text{ at } r = r_0, \tag{55}$$

where D_E is the carrier diffusion coefficient, s_v is the surface recombination velocity.

The boundary conditions (51), (54), and (55) have the following forms after performing the Laplace transform:

$$\bar{u}(r_0, s) = 0, \tag{56}$$

$$\left.\frac{\partial\left(1 - a\nabla^2\right)\bar{\varphi}}{\partial r}\right|_{r=r_0} = -\frac{q_0\left(1+\tau_0 s\right)s}{8\left(1+st_p\right)^3\left(s+\delta_4\right)} = -\bar{G}(s), \tag{57}$$

$$D_E \left.\frac{\partial \bar{N}}{\partial r}\right|_{r=r_0} = s_v \bar{N}\left(r_0, s\right). \tag{58}$$

Eqs. (42) and (45) are substituted into Eqs. (56) – (58), giving

$$\sum_{i=1}^{3} g_i \frac{1}{\lambda_i} I_1\left(\lambda_i r_0\right) = 0, \tag{59}$$

$$\sum_{i=1}^{3} g_i I_1\left(\lambda_i r_0\right)\zeta_i \lambda_i\left(1 - a\lambda_i^2\right) = -\frac{q_0\left(1+\tau_0 s\right)st_p}{8\left(1+st_p\right)^3\left(s+\delta_4\right)} = -\bar{G}(s), \tag{60}$$

$$\sum_{i=1}^{3} \eta_i g_i\left[D_E \lambda_i I_1\left(\lambda_i r_0\right) - s_v I_0\left(\lambda_i r_0\right)\right] = 0. \tag{61}$$

The values of g_i (i = 1, 2, 3) can be obtained by solving Eqs. (59) – (61) by the Cramer's rule:

$$g_i(s)\frac{\Delta_i}{\Delta},$$

$$\Delta = G_1\left[G_5 G_9 - G_8 G_6\right] - G_2\left[G_4 G_9 - G_6 G_7\right] + G_3\left[G_4 G_8 - G_5 G_7\right],$$

$$\Delta_1 = \bar{G}(s)\left[G_2 G_9 - G_8 G_3\right],$$

$$\Delta_2 = -\bar{G}(s)\left[G_1 G_9 - G_7 G_3\right],$$

$$\Delta_3 = \bar{G}(s)\left[G_1 G_8 - G_2 G_7\right],$$

$$G_i = \frac{1}{\lambda_i}\varphi_i,$$

$$G_{i+3} = \varphi_i \zeta_i \lambda_i\left(1 - a\lambda_i^2\right),$$

$$G_{i+6} = \eta_i\left[D_E \lambda_i \varphi_i - s_v \psi_i\right],$$

$$I_1\left(\lambda_i r_0\right) = \varphi_i,\ I_0\left(\lambda_i r_0\right) = \psi_i,\ i = 1,\ 2,\ 3,$$

and putting the values of $g_i(s)$ into Eqs. (42), (45), (48) – (50), the various components of displacement, temperature distribution, carrier density, and stresses are

$$\bar{u} = \frac{\bar{G}(s)}{\Delta}\left\{\left[G_2 G_9 - G_8 G_3\right]\frac{\theta_1}{\lambda_1} - \left[G_1 G_9 - G_7 G_3\right]\frac{\theta_2}{\lambda_2} + \left[G_1 G_8 - G_2 G_7\right]\frac{\theta_3}{\lambda_3}\right\}, \tag{62}$$

$$\bar{T} = \frac{\bar{G}(s)}{\Delta}\left\{\left[G_2G_9 - G_8G_3\right]\zeta_1\vartheta_1 - \left[G_1G_9 - G_7G_3\right]\zeta_2\vartheta_2 + \left[G_1G_8 - G_2G_7\right]\zeta_3\vartheta_3\right\}, \tag{63}$$

$$\bar{N} = \frac{\bar{G}(s)}{\Delta}\left\{\left[G_2G_9 - G_8G_3\right]\eta_1\vartheta_1 - \left[G_1G_9 - G_7G_3\right]\eta_2\vartheta_2 + \left[G_1G_8 - G_2G_7\right]\eta_3\vartheta_3\right\}, \tag{64}$$

$$\begin{aligned}\bar{\sigma}_{rr} = \frac{\bar{G}(s)}{\Delta}\Big\{&\left[G_2G_9 - G_8G_3\right]\left(l_1\vartheta_1 - \mu_1\right) - \left[G_1G_9 - G_7G_3\right]\left(l_2\vartheta_2 - \mu_2\right) + \\ &+\left[G_1G_8 - G_2G_7\right]\left(l_3\vartheta_3 - \mu_3\right)\Big\},\end{aligned} \tag{65}$$

$$\begin{aligned}\bar{\sigma}_{\theta\theta} = \frac{\bar{G}(s)}{\Delta}\Big\{&\left[G_2G_9 - G_8G_3\right]\left(\mu_1 + l_1\vartheta_1\right) - \left[G_1G_9 - G_7G_3\right]\left(\mu_2 + l_2\vartheta_2\right) + \\ &+\left[G_1G_8 - G_2G_7\right]\left(\mu_3 + l_3\vartheta_3\right)\Big\},\end{aligned} \tag{66}$$

$$\bar{\sigma}_{zz} = \frac{\bar{G}(s)}{\Delta}\left\{\left[G_2G_9 - G_8G_3\right]l_1\vartheta_1 - \left[G_1G_9 - G_7G_3\right]l_2\vartheta_2 + \left[G_1G_8 - G_2G_7\right]l_3\vartheta_3\right\}, \tag{67}$$

where

$$\vartheta_i = I_0\left(\lambda_i r\right),\ \theta_i = I_1\left(\lambda_i r\right),\ \mu_i = \frac{2\gamma^2}{\lambda_i r} I_1\left(\lambda_i r\right),\ i = 1,\ 2,\ 3.$$

Inversion of the transforms

In order to obtain the result of the problem in the physical domain, transforms in Eqs. (62) – (67) are inverted using

$$f\left(x,t\right) = \frac{1}{2\pi i}\int_{e^{-i\infty}}^{e^{+i\infty}} \tilde{f}\left(x,s\right)e^{-st}ds. \tag{68}$$

Finally, let us evaluate the integral in Eq. (68) using the Romberg's integration (W. H. Press et al. [37]) with an adaptive step size.

Particular cases:

i. If $K^* \neq 0$, $K \neq 0$ and $\tau_0 \neq 0$, $a \neq 0$ in Eqs. (62) – (67), the results for the MGTPT + $2TT$ can be obtained ($2TT$ are the two temperature quantities).

ii. If $K^* \neq 0$, $K \neq 0$ and $\tau_0 = 0$, $a \neq 0$ in Eqs. (62) – (67), the results for the Photothermal Green and Naghdi III model (PGN-III) [12] with $2TT$ can be obtained.

iii. If $K \neq 0$ and $\tau_0 = 0$, $a \neq 0$ in Eqs. (62) – (67), the results for the Photothermal Green and Naghdi II model (PGN-II) [11] with $2TT$ can be obtained.

iv. If $K^* = 0$ and $\tau_0 = 0$, $a \neq 0$ in Eqs. (62) – (67), the results for the Coupled Photothermoelasticity theory (CPTE) with $2TT$ are obtained.

v. If $K^* = 0$ and $a \neq 0$ in Eqs. (62) – (67), the results for the generalized Lord and Shulman Photothermoelasticity model (PLS) [6] with $2TT$ are obtained.

vi. If $a = 0$ in Eqs. (62) – (67), the results for the MGTPT model are obtained.

Numerical results and discussion

In order to demonstrate the theoretical results and to show the effect of Hall current, rotation, and the effect of the modified photothermal heat equation (MGTPT) graphically using Matlab software, the isotropic silicon material (Si) with its physical properties was taken. The used data are listed in Table.

Fig. 2, *a* depicts the variation in the radial displacement **u** for MGTPT and PGN-III theories with two temperatures. Notice that the radial displacement is minimum in the absence of two temperature quantities in the MGTPT theory. However, as the value of the two-temperature parameters in the MGTPT theory increases, the radial displacement increases as well. Moreover, PGN-III theories with and without two temperatures show a higher variation in the radial displacement. Furthermore, as the radial distance increases, the radial displacement decreases.

Table

Parameter values of silicon material [36] used in our study

Parameter	Notation	Unit	Value
Lame's elastic constants	λ	GPa	36.4
	μ		54.6
Thermal elastic coupling	β	MPa/(deg)	7.04
Electronic deformation coefficient	δ_n	m^3	$-9\cdot10^{-31}$
Medium density	ρ	kg/m^3	$2.33\cdot10^3$
Specific heat at constant strain	C_E	J/(kg·K)	695
Coefficient of thermal conductivity	K	W/(m·K)	150
Materialistic constant	K^*	kW·s	1.54
Carrier diffusion coefficient	D_E	m^2/s	$2.5\cdot10^{-3}$
Magnetic constant	μ_0	H/m	$1.257\cdot10^{-6}$
Reference temperature (s. t. $\lvert T/T_0\rvert <<1$)	T_0	K	300
Photo-generated carrier lifetime	τ	μs	50
Carrier concentration at equilibrium position	N_0	m^{-3}	10^{20}
Electric constant	ε_0	F/m	$8.854\cdot10^{-12}$
Semiconductor gap energy	E_g	eV	1.11
Linear thermal expansion coefficient	α_T	K^{-1}	$3\cdot10^{-6}$
Surface recombination velocity	s_v	m/s	2

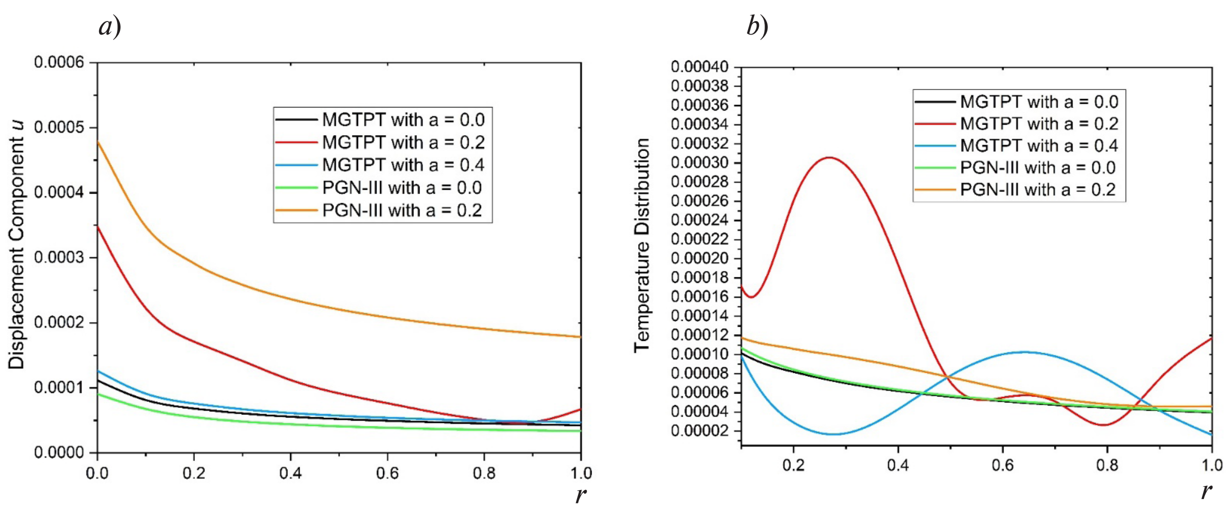

Fig. 2. The radial displacement (*a*) and temperature (*b*) variations for various models with two temperature quantities (*a* is the two-temperature parameter)

Fig. 2, *b* illustrates the variation in the temperature distribution for MGTPT and PGN-III theories with two temperatures. Note that temperature distribution is higher in the inner core of the cylinder as compared to the outer one of the cylinder. In addition, the presence of two temperatures causes a higher temperature distribution.

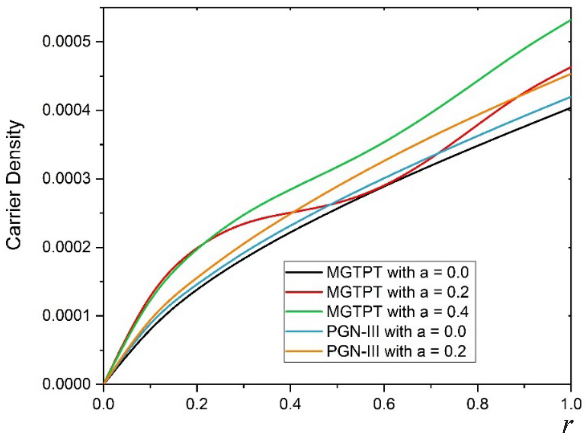

Fig. 3. The variation in the carrier density for various models with two temperature quantities

Fig. 3 shows the variation in the carrier density for MGTPT and PGN-III theories with two temperatures. It should be noted that PGN-III depicts the minimum variation in carrier density than MGTPT theory in the absence of two temperature quantities. At the same time, PGN-III theory results in maximum variation in carrier density. Notice that variation in the carrier density sharply decreases as the radial distance r increases.

Fig. 4 shows the variation in the stress components for MGTPT and PGN-III theories with two temperatures. It has been noticed that in the absence of two temperature quantities in the MGTPT theory, the variation in the stress components are minimum as compared to the PGN-III theory. However, as the value of the

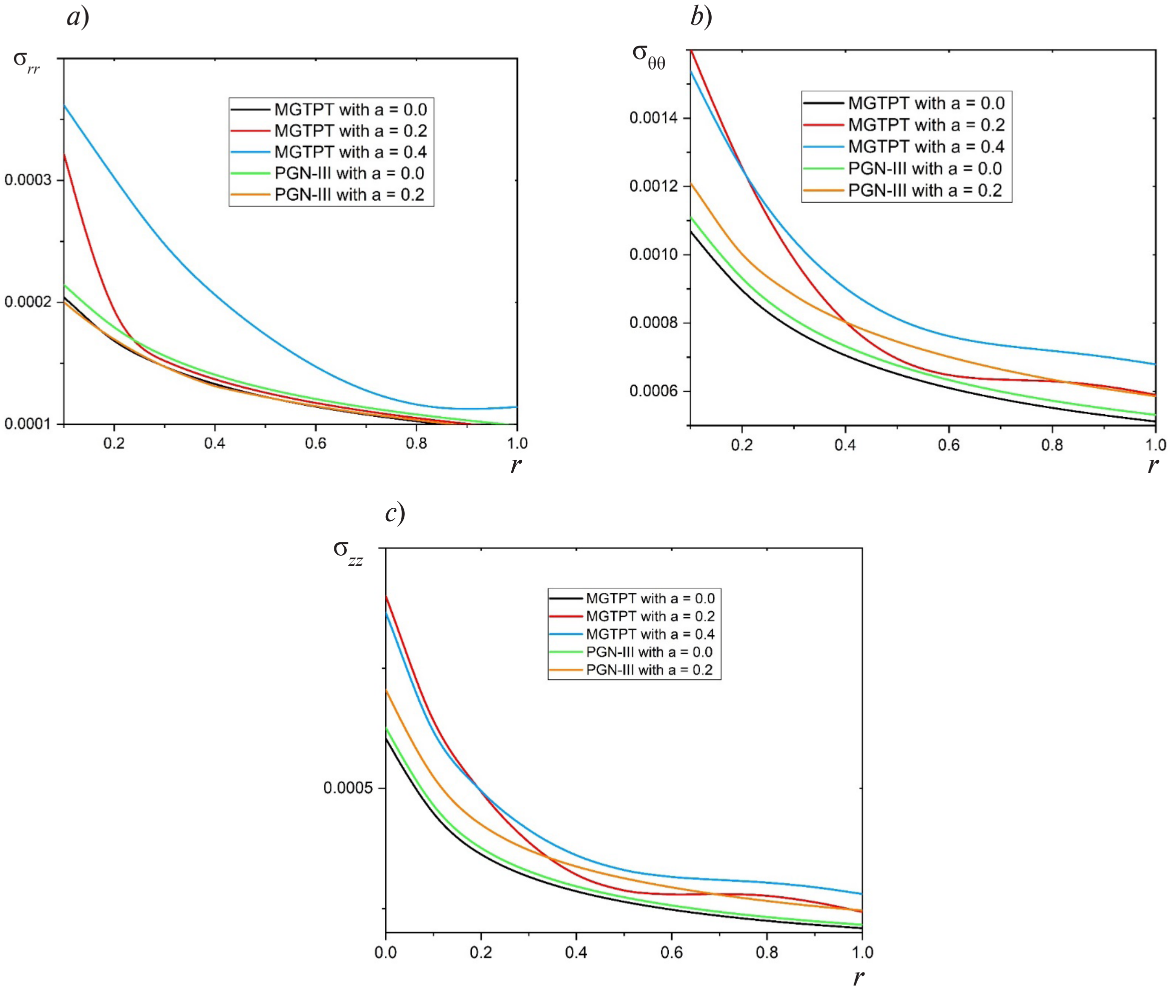

Fig. 4. The deviations in radial (*a*), hoop (*b*) and vertical (*c*) stresses for various models with two temperatures

two- temperature parameter increases, there is a sharp increase in the stress components. Note that the variation in the stress components decreases drastically as the radial distance r increases.

Conclusions

This study lays out several photo-thermoelasticity models that are generalized in the Moore – Gibson – Thompson photo-thermal (MGTPT) model with two temperatures. In addition, this study included the photo-thermoelastic Green – Naghdi type III model. The generalized MGTPT model is used to solve some of the physical consequences of some earlier models. In this study, the infinite semiconducting solid cylinder subjected to the exponential laser pulse on its boundary surface has been studied. The governing equations are expressed with the help of the MGTPT with two temperatures.

Effects of different thermoelastic theories with two temperatures on the components of displacement, temperature field, carrier density, and thermal stresses are represented graphically. As seen in the graphs, all of the domains examined are significantly impacted by the two temperatures. The thermal effect occurs when semiconductor media are exposed to focused laser beams or sunlight ones and, because of the thermal effect, they will vibrate. These materials have many uses in renewable energy, especially in the solar cells industry which depends heavily on semiconductor materials.

Photothermal methods are not only simple and sensitive but they can also be used to gain some insight into the process of de-excitation in materials with optical absorption. The ideas presented in this paper will come in handy for physicists, material designers, thermal engineers, and geophysicists. A wide range of photo-thermoelasticity and thermodynamic problems can be solved using the technique used in the above study.

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THE AUTHORS

KAUR Iqbal
Government College for Girls, Department of Mathematics
Palwal, Kurukshetra, Haryana, India
bawahanda@gmail.com
ORCID: 0000-0002-2210-7701

SINGH Kulvinder
Kurukshetra University, University Institute of Engineering & Technology, Department of Computer Science & Engineering
Kurukshetra, Haryana, India-136119
ksingh2015@kuk.ac.in
ORCID: 0000-0002-2717-0419

СВЕДЕНИЯ ОБ АВТОРАХ

КАУР Икбал — *Ph.D., адъюнкт-профессор математики кафедры математики Государственного колледжа для девушек (городок Палвал), г. Курукшетра, штат Харьяна, Индия.*
Palwal, Kurukshetra, Haryana, India
bawahanda@gmail.com
ORCID: 0000-0002-2210-7701

СИНГХ Кулвиндер — *Ph.D., доцент кафедры информатики и инженерии Инженерно-технологического института Университета Курукшетры, г. Курукшетра, штат Харьяна, Индия.*
Kurukshetra, Haryana, 136119, India
ksingh2015@kuk.ac.in
ORCID: 0000-0002-2717-0419

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