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Global dynamics of a dengue fever model incorporating transmission seasonality*

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Abstract. The changes of seasons cause that the transmission of dengue fever is characterized by periodicity. We develop a dengue fever transmission model incorporating seasonal periodicity and spatial heterogeneity. Based on the well-posedness of solution for this model, we propose its basic reproduction number R_0, and we discuss the properties of this number including its limiting form when the diffusion coefficients change. Moreover, the dynamical behavior of this model infers that if $R_0 \leqslant 1$, then the disease-free periodic solution is globally asymptotically stable, and if $R_0 > 1$, then the model possesses a positive periodic solution, which is globally asymptotically stable. These theoretical findings are further illustrated by the final numerical simulations. Additionally, we add that the similar problem has been investigated by M. Zhu and Y. Xu [A time-periodic dengue fever model in a heterogeneous environment, *Math. Comput. Simul.*, 155:115–129, 2019] in which some dynamical results have been studied only on the cases $R_0 < 1$ and $R_0 > 1$. Our results not only include the scenario on the case $R_0 = 1$, but also involve the more succinct conditions on the cases $R_0 < 1$ and $R_0 > 1$.

Keywords: dengue fever model, diffusion-reaction system, periodicity, global stability.

1 Introduction and model description

With the deterioration of global climate, the frequency of human activities, the aggravation of environment pollution and the emergence of virus resistance, not only new infectious diseases such as AIDS, SARS, Ebola and COVID-19 have emerged, but also the

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original contagions such as malaria and dengue fever have appeared repeatedly. Dengue fever (DF) is one of the most destructive mosquito-borne diseases globally. In 2014, the World Health Organization (WHO) called it the fastest spreading insect-borne infectious disease around the world [34]. Dengue is a kind of cute contagion in which mosquitoes, especially, *Aedes albopictus* and *Aedes aegypti* form an intermediate step, as well as dengue virus (DENV) is transmitted [37]. After being infected by DENV, mosquitoes cannot only carry and transmit the virus for life, but also transmit the virus to their offspring through their own eggs, this makes dengue fever erupt over and over again. The World Health Report published by WHO has shown that the epidemic areas contaminated by dengue expanded from original nine countries before 1970 to more than 100 countries at present [35, 37]. America, Africa, Southeast Asia, the Western Pacific and the Eastern Mediterranean are all high-risk areas. Therefore, human health is still seriously threatened by infectious diseases including dengue fever, and the task of fighting against contagion is even more arduous.

Mathematical modeling of infections diseases, as we know, has progressed dramatically over the past 3 decades and continues to flourish at the relationship of epidemiology, infectious diseases and mathematics research [12]. Take again DF example, some mathematical epidemiology studies for dengue transmission were proposed. As far as we know, Fischer and Halstead [10] first developed the mathematical model of DF in 1970. Next, a competitive exclusion principle in a two-strain dengue model was explored by Feng and Velasco-Hernández [9]. Moreover, a classical SIR-SI dengue model was also proposed by Esteva and Vargas in [8] in which the global stability of the endemic equilibrium was investigated through the stability of periodic orbits. Except for the above ordinary differential dengue models and others [6, 23, 27], there appear many models described by partial differential equations. Wang and Zhao [21] considered a nonlocal reaction-diffusion model of dengue transmission with time-delay and proved that for spatially heterogeneous infections, if the basic reproduction number is computed through the spatially averaged parameters, then the infection risk may be underestimated. In [18], the authors explored a mathematical model, incorporating the spatially heterogeneous temperature, and revealed that temperature heterogeneity could place a crucial impact on transmission of dengue epidemics. The authors of [30] incorporated the media coverage parameter and free boundary on the dengue transmission model, and investigated the impact of media coverage and the virus expanding capability on the spreading of dengue.

Inspired by the modeling idea in [8], we first evolve a dengue fever model, which characterizes the transmission mechanism of DF. Let N_h be the human population size, and let N_v be the mosquito one in which N_h is assumed as a constant. According to [8], the quantity change of mosquito population is described by the differential equation

$$N_v' = A - \mu_v N_v,$$

where A represents a constant recruitment, and μ_v is the per capita mortality rate of mosquitoes. Obviously, the solutions N_v of this equation approach the equilibrium A/μ_v as $t \to \infty$.

Since mosquitoes never recover once infected, we divide the mosquito population into two subpopulations, that is, susceptible (S_v) and infectious (I_v). On the other hand, the research in [33] shows that dengue infection caused by one serotype will be permanently immune to this virus, but to the other three viruses, it is only of short-time immunity, even of no immunity. Therefore, recovery persons can very easily become the susceptible again, and subsequent infection caused by the different serotype increases the risk of severe syndrome. Based on this fact, human population is also divided into two subclasses, that is, susceptible (S_h) and infectious (I_h). According to some related introduction in [8], we know that the contagion rates of per susceptible mosquito and susceptible person are embodied by

$$\beta_v b \frac{N_h}{N_h+m}\frac{I_h}{N_h} = \frac{\beta_v b}{N_h+m} I_h \quad \text{and} \quad \beta_h b \frac{N_v}{N_h}\frac{N_h}{N_h+m}\frac{I_v}{N_v} = \frac{\beta_h b}{N_h+m} I_v,$$

respectively, where β_v represents the infection probability from human to mosquito, β_h is the infection probability from mosquito to human, b is the average number of bites per mosquito per day, in simple terms, the biting of mosquitoes, m means the number of alternative hosts available as blood sources. Finally, motivated by the SIS compartment model, we suppose that get treated or infected humans recover at a rate γ_h, and death rate of human is μ_H, then preliminarily construct an ordinary differential dengue model as follows:

$$\begin{aligned} S_h'(t) &= \mu_h N_h - \frac{\beta_h b}{N_h+m} S_h I_v + \gamma_h I_h - \mu_H S_h, \\ I_h'(t) &= \frac{\beta_h b}{N_h+m} S_h I_v - \gamma_h I_h - \mu_H I_h, \\ S_v'(t) &= A - \frac{\beta_v b}{N_h+m} S_v I_h - \mu_v S_v, \\ I_v'(t) &= \frac{\beta_v b}{N_h+m} S_v I_h - \mu_v I_v, \\ S_h(0) &> 0, \qquad I_h(0) \geqslant 0, \qquad S_v(0) > 0, \qquad I_v(0) \geqslant 0. \end{aligned} \tag{1}$$

In epidemic models, the basic reproduction number encapsulates the contagiousness of an infectious agent circulating in a host population and reflects the contagion risk of infectious disease. Utilizing the method of next generation matrix [20], we can directly calculate out the basic reproduction number of system (1)

$$\mathfrak{R}_0 = \sqrt{\frac{\frac{\beta_h b}{N_h+m}\frac{\mu_h N_h}{\mu_H} \cdot \frac{\beta_v b}{N_h+m}\frac{A}{\mu_v}}{(\gamma_h+\mu_H)\mu_v}}. \tag{2}$$

Admittedly, ordinary differential system (1) describes the transmission process of DF, but it does not embody the time periodicity and spatial heterogeneity of transmission for infectious diseases. In [29], the authors stress that variations in climate and weather can influence the Aedes mosquitoes and dengue virus through multiple mechanism. For example, precipitation provides habits for the aquatic stages of the Aedes mosquito life cycle and strongly affects mosquito distribution. The effects of precipitation and evaporation on available water sources can regulate the size, population and behavior of Aedes. On the other hand, DF cases also present significant characteristics of spatial aggregation. According to the China Notifiable Disease Surveillance System, the authors in [29] point out that 49,290 local DF cases occurred in China between 2010 and 2014, with those in the Pearl River Delta (PRD) and the Border of Yunnan and Myanmar (BYM) accounting for 97.06%. Especially, the authors further indicate that the DF epidemic of the PRD was highly aggregated, while that of the BYM was relatively decentralized. These information claims that geographic heterogeneity is a significant factor in the spreading of DF. Motivated by [13, 19, 24], we introduce the spatial heterogeneity and temporal periodicity into the dengue model and extend model (1) as follows:

$$\begin{aligned}
\frac{\partial S_h}{\partial t} &= \nabla\cdot\big(d_h(x,t)\nabla S_h\big) + \mu_h\big(x,N_h(x,t)\big)N_h(x,t) - \frac{\beta_h(x,t)b(x,t)}{N_h(x,t)+m(x,t)}S_hI_v \\
&\quad + \gamma_h(x,t)I_h - \mu_H(x,t)S_h, \quad (x,t)\in\Omega\times(0,\infty), \\
\frac{\partial I_h}{\partial t} &= \nabla\cdot\big(d_h(x,t)\nabla I_h\big) + \frac{\beta_h(x,t)b(x,t)}{N_h(x,t)+m(x,t)}S_hI_v - \gamma_h(x,t)I_h - \mu_H(x,t)I_h, \\
&\quad (x,t)\in\Omega\times(0,\infty), \\
\frac{\partial S_v}{\partial t} &= \nabla\cdot\big(d_v(x,t)\nabla S_v\big) + A(x,t) - \frac{\beta_v(x,t)b(x,t)}{N_h(x,t)+m(x,t)}S_vI_h - \mu_v(x,t)S_v, \\
&\quad (x,t)\in\Omega\times(0,\infty), \\
\frac{\partial I_v}{\partial t} &= \nabla\cdot\big(d_v(x,t)\nabla I_v\big) + \frac{\beta_v(x,t)b(x,t)}{N_h(x,t)+m(x,t)}S_vI_h - \mu_v(x,t)I_v, \\
&\quad (x,t)\in\Omega\times(0,\infty), \\
\frac{\partial S_h}{\partial \nu} &= \frac{\partial I_h}{\partial \nu} = \frac{\partial S_v}{\partial \nu} = \frac{\partial I_v}{\partial \nu} = 0, \quad (x,t)\in\partial\Omega\times(0,\infty).
\end{aligned} \tag{3}$$

Here Ω is the spatial habitat with smooth boundary $\partial\Omega$, and ν is the outward unit normal vector on $\partial\Omega$. Motivated by the idea in [4], we assume that μ_h, representing the birth rate of human, is a nonnegative function. Two prototypical birth rate functions in the biological literature are $\mu_h(x,u) = b_h\mathrm{e}^{-u/K(x)}$ and

$$\mu_h(x,u) = \begin{cases} b_h[1-\frac{u}{K(x)}], & 0\leqslant u\leqslant K(x),\ x\in\bar{\Omega}, \\ 0, & u > K(x),\ x\in\bar{\Omega}, \end{cases}$$

where $b_h > 0$ is the maximal individual birth rate of human, as well as $K(x)$ stands for the local carrying capacity and is supposed to be a positive function of location x.

Concurrently we raise the following basic assumption.

(A1) Functions $\beta_h(x,t), b(x,t), \beta_v(t,x), m(x,t) \not\equiv 0$, $\gamma_h(x,t)$, $\mu_H(x,t)$ and $\mu_v(x,t)$ are all positive nontrivial, and all of which are Hölder-continuous on $\bar{\Omega}\times\mathbb{R}$ and T-periodic in t; diffusion coefficients $d_h(x,t)$ and $d_v(x,t)$ are positive and Hölder-continuous on $\bar{\Omega}\times\mathbb{R}$ and T-periodic in t.

Since $N_h = S_h + I_h$, it follows from the first two equations in (3) that

$$\begin{aligned}\frac{\partial N_h(x,t)}{\partial t} &= \nabla \cdot \bigl(d_h(x,t) \cdot \nabla N_h(x,t)\bigr) + \mu_h\bigl(x, N_h(x,t)\bigr)N_h(x,t)\\ &\quad - \mu_H(x,t)N_h(x,t), \quad x \in \Omega,\ t > 0,\\ \frac{\partial N_h(x,t)}{\partial \nu} &= 0, \quad x \in \partial\Omega,\ t > 0.\end{aligned} \tag{4}$$

According to Theorems 3.1.5 and 3.1.6 in [28], system (4) admits a unique positive T-periodic steady state $H^*(x,t)$ in $C(\bar{\Omega}, \mathbb{R}_+) \setminus \{0\}$, which is globally attractive under appropriate assumptions, that is,

$$\lim_{t\to\infty} N_h(x,t) = H^*(x,t)$$

uniformly in $x \in \bar{\Omega}$ for $t > 0$.

Furthermore, we set $V = S_v + I_v$ and deduce by (3) that

$$\begin{aligned}\frac{\partial V(x,t)}{\partial t} &= \nabla \cdot \bigl(d_v(x,t) \cdot \nabla V(x,t)\bigr) + A(x,t)\\ &\quad - \mu_v(x,t)V(x,t), \quad x \in \Omega,\ t > 0,\\ \frac{\partial V(x,t)}{\partial \nu} &= 0, \quad x \in \partial\Omega,\ t > 0.\end{aligned} \tag{5}$$

Through [25, Lemma 2.1], system (5) also admit the unique positive T-periodic steady state $V^*(x,t)$, which is globally asymptotically stable in $C(\bar{\Omega}, \mathbb{R}_+) \setminus \{0\}$, so one has

$$\lim_{t\to\infty} V(x,t) = V^*(x,t)$$

uniformly in $x \in \bar{\Omega}$ for $t > 0$. Putting $(u_1, u_2) := (I_h, I_v)$, one derives that the limiting system for (3) takes the form

$$\begin{aligned}\frac{\partial u_1}{\partial t} &= \nabla \cdot \bigl(d_h(x,t) \cdot \nabla u_1\bigr) + \beta_1(x,t)\bigl(H^*(x,t) - u_1\bigr)u_2\\ &\quad - \gamma_h(x,t)u_1 - \mu_H(x,t)u_1, \quad x \in \Omega,\ t > 0,\\ \frac{\partial u_2}{\partial t} &= \nabla \cdot \bigl(d_v(x,t) \cdot \nabla u_2\bigr) + \beta_2(x,t)\bigl(V^*(x,t) - u_2\bigr)u_1\\ &\quad - \mu_v(x,t)u_2, \quad x \in \Omega,\ t > 0,\\ \frac{\partial u_1}{\partial \nu} &= \frac{\partial u_2}{\partial \nu} = 0, \quad x \in \partial\Omega,\ t > 0,\end{aligned} \tag{6}$$

with the initial function

$$(u_1, u_2)(0, x) = \bigl(\phi_1(x), \phi_2(x)\bigr), \quad x \in \bar{\Omega}.$$

Here we denote

$$\beta_1(x,t) := \frac{\beta_h(x,t)b(x,t)}{H^*(x,t) + m(x,t)}, \qquad \beta_2(x,t) := \frac{\beta_v(x,t)b(x,t)}{H^*(x,t) + m(x,t)} \tag{7}$$

for the convenience of writing.

In this paper, we will be interested primarily in the effects of the time periodicity and space heterogeneity in the distribution of human and mosquito populations on the dengue spreading dynamics. The forthcoming content of the paper is described briefly below. Section 2 is concerned with the well-posedness of system (6). The basic reproduction number R_0 of model (6) and its relevant properties are exhibited in Section 3. In Section 4, we establish the threshold-type conclusion on the global stability with respect to R_0. Finally, some numerical simulations and brief discussion further reveal the influence of temporal periodicity on the dengue virus in the heterogeneous environment.

2 The well-posedness for system (6)

Facing system (6), we endeavor to solve the existence, uniqueness and boundedness of its solutions in this section. First, for any $t \geqslant 0$, let $\mathbb{X} := C(\bar{\Omega}, \mathbb{R})$ and $\mathbb{W} := C(\bar{\Omega}, \mathbb{R}^2)$ be Banach spaces with the supremum norms $\|\cdot\|_{\mathbb{X}}$ and $\|\cdot\|_{\mathbb{W}}$, respectively. Further, define $\mathbb{X}^+ := C(\bar{\Omega}, \mathbb{R}_+)$, $\mathbb{W}^+ := C(\bar{\Omega}, \mathbb{R}_+^2)$ and

$$\begin{aligned} W(t) :=& \bigl\{(\varphi_1, \varphi_2) \in \mathbb{W}^+ \colon\ 0 \leqslant \varphi_1(x) \leqslant H^*(x,t), \\ & 0 \leqslant \varphi_2(x) \leqslant V^*(x,t)\ \forall x \in \bar{\Omega}\bigr\} \end{aligned} \tag{8}$$

for each $t \geqslant 0$. We have the following statement.

Theorem 1. *Let* (A1) *holds. For any* $\phi = (\phi_1, \phi_2) \in W(0)$*, system* (6) *possesses a unique solution* $u(\cdot,t,\phi) = (u_1(\cdot,t,\phi), u_2(\cdot,t,\phi)) \in W(t)$ *with* $u(\cdot,0,\phi) = \phi$ *for* $t \geqslant 0$*, and this solution is uniformly bounded and ultimately bounded.*

Proof. Set $E_1(t,s), E_2(t,s) : \mathbb{X} \to \mathbb{X}$, $t \geqslant s$, to be the linear evolution operator caused by

$$\frac{\partial v_1}{\partial t} = \nabla \cdot \bigl(d_h(x,t)\nabla v_1\bigr) - \gamma_h(x,t)v_1 - \mu_H(x,t)v_1, \tag{9}$$

and

$$\frac{\partial v_2}{\partial t} = \nabla \cdot \bigl(d_v(x,t)\nabla v_2\bigr) - \mu_v(x,t)v_2 \tag{10}$$

for $(x,t) \in \Omega \times (0,\infty)$ and associated with the Neumann boundary condition, respectively. Noting that $\gamma_h(x,t)$, $\mu_H(x,t)$ and $\mu_v(x,t)$ are all T-periodic in t, one can conclude that $E_i(t+T, s+T) = E_i(t,s)$ for any $(t,s) \in \mathbb{R}^2$, $i = 1,2$, with $t \geqslant s$, as well as $E_i(t,s)$ is compact and strongly positive.

For any $\eta = (\eta_1, \eta_2) \in \mathbb{W}$ and $t \geqslant 0$, we further define

$$\begin{aligned} F_1(\eta,t) &= F_1(\eta_1,\eta_2,t) := \beta_1(\cdot,t)\bigl(H^*(\cdot,t) - \eta_1(\cdot)\bigr)\eta_2(\cdot), \\ F_2(\eta,t) &= F_2(\eta_1,\eta_2,t) := \beta_2(\cdot,t)\bigl(V^*(\cdot,t) - \eta_2(\cdot)\bigr)\eta_1(\cdot) \end{aligned}$$

and

$$E(t,s) = \operatorname{diag}\bigl(E_1(t,s), E_2(t,s)\bigr), \qquad F = (F_1, F_2) : \mathbb{W} \times [0,\infty) \to \mathbb{W}. \tag{11}$$

Thus, system (6) can be written as

$$u_1(\cdot,t,\phi)=E_1(t,0)\phi_1(\cdot)+\int_0^t E_1(t,s)F_1\bigl(u(s),s\bigr)\,\mathrm{d}s,$$

$$u_2(\cdot,t,\phi)=E_2(t,0)\phi_2(\cdot)+\int_0^t E_2(t,s)F_2\bigl(u(s),s\bigr)\,\mathrm{d}s.$$

In this way, system (6) can be translated into an abstract integral equation

$$u(t,\phi)=E(t,0)\phi+\int_0^t E(t,s)F\bigl(u(s),s\bigr)\,\mathrm{d}s\quad \forall t\geqslant 0,\ \phi\in\mathbb{W}.$$

In what follows, we claim that F is quasimonotone on $W:=\{(\varphi,t)\in\mathbb{W}^+\times[0,\infty),\ \varphi\in W(t)\}$, in other words,

$$\lim_{h\to 0^+}\frac{1}{h}\operatorname{dist}\bigl((\varphi-\psi)+h\bigl[F(\varphi,t)-F(\psi,t)\bigr],\quad \mathbb{W}^+\bigr)=0 \tag{12}$$

holds for all $(\varphi,t),(\psi,t)\in W$ with $\varphi\geqslant\psi$.

Actually, for any given $(\varphi,t),(\psi,t)\in W$, the precondition $\varphi\geqslant\psi$ can lead to

$$\begin{aligned}
&\varphi-\psi+h\bigl[F(\varphi,t)-F(\psi,t)\bigr]\\
&\quad=\begin{pmatrix}\varphi_1(\cdot)-\psi_1(\cdot)+h\bigl[\beta_1(\cdot,t)(H^*(\cdot,t)-\varphi_1(\cdot))\varphi_2(\cdot)-\beta_1(\cdot,t)(H^*(\cdot,t)-\psi_1(\cdot))\psi_2(\cdot)\bigr]\\ \varphi_2(\cdot)-\psi_2(\cdot)+h\bigl[\beta_2(\cdot,t)(V^*(\cdot,t)-\varphi_2(\cdot))\varphi_1(\cdot)-\beta_2(\cdot,t)(V^*(\cdot,t)-\psi_2(\cdot))\psi_1(\cdot)\bigr]\end{pmatrix}\\
&\quad=\begin{pmatrix}(1-h\beta_1(\cdot,t)\psi_2(\cdot))(\varphi_1(\cdot)-\psi_1(\cdot))+h\beta_1(\cdot,t)(H^*(\cdot,t)-\varphi_1(\cdot))(\varphi_2(\cdot)-\psi_2(\cdot))\\ (1-h\beta_2(\cdot,t)\psi_1(\cdot))(\varphi_2(\cdot)-\psi_2(\cdot))+h\beta_2(\cdot,t)(V^*(\cdot,t)-\varphi_2(\cdot))(\varphi_1(\cdot)-\psi_1(\cdot))\end{pmatrix}\\
&\quad\geqslant\begin{pmatrix}(1-h\beta_1^M\psi_2(\cdot))(\varphi_1(\cdot)-\psi_1(\cdot))+h\beta_1^m(H^*(\cdot,t)-\varphi_1(\cdot))(\varphi_2(\cdot)-\psi_2(\cdot))\\ (1-h\beta_2^M\psi_1(\cdot))(\varphi_2(\cdot)-\psi_2(\cdot))+h\beta_2^m(V^*(\cdot,t)-\varphi_2(\cdot))(\varphi_1(\cdot)-\psi_1(\cdot))\end{pmatrix},
\end{aligned}$$

where $\beta_i^m=\min_{x\in\bar\Omega,t\in[0,T]}\beta_i(x,t)$, $\beta_i^M=\max_{x\in\bar\Omega,\,t\in[0,T]}\beta_i(x,t)$, $i=1,2$. The above inequality set shows $(\varphi-\psi)+h[F(\varphi,t)-F(\psi,t)]\in\mathbb{W}^+$ for all sufficiently small $h>0$, hence, (12) is valid.

Meanwhile, we know that $H^*(x,t)$ and $V^*(x,t)$ meet with (4), (5), respectively, which combined with assumption (A1) can result in the following inequalities:

$$\frac{\partial H^*(x,t)}{\partial t}>\nabla\cdot\bigl(d_h(x,t)\cdot\nabla H^*(x,t)\bigr)-\mu_H(x,t)H^*(x,t) \tag{13}$$

and

$$\frac{\partial V^*(x,t)}{\partial t}>\nabla\cdot\bigl(d_v(x,t)\cdot\nabla V^*(x,t)\bigr)-\mu_v(x,t)V^*(x,t). \tag{14}$$

Therefore, we choose $u^+(t)=(H^*(\cdot,t),V^*(\cdot,t))$, $u^-(t)=(0,0)$, and we set $S^+=S^-=S=\emptyset$, $B^+=B^-=F$, which can verify assumptions (C_1)–(C_6) in [16]. So it follows form Corollary 5 in [16] that for any $\phi \in W(0)$, system (6) has a unique solution $u(\cdot,t,\phi)$ with $u(\cdot,0,\phi)=\phi$ such that $u(\cdot,t,\phi)\in W(t)$, where t is in its maximal existence interval $[0,t_\phi)$, and $t_\phi \leqslant \infty$. Additionally, in terms of the analyticity of $E(t,s)$ with $(t,s)\in\mathbb{R}^2$, $t>s$, we know that $u(\cdot,t,\phi)$ with initial value $\phi\in W(0)$ is a classical solution and exists globally on $[0,+\infty)$.

Besides, it is obvious that $u^+(t)=(H^*(\cdot,t),V^*(\cdot,t))$ is an upper solution of (6) through (13) and (14) again, and $W(t)$ is a bounded domain with respect to (φ_1,φ_2) for any $t\geqslant 0$, so one can directly conclude that the solutions of (6) are uniformly bounded and ultimately bounded, which means that the proof is completed. $\square$

3 Basic reproduction number and its properties

The basic reproduction number $\mathfrak{R}_0$ of ODE system (1) has been exhibited in (2) through the next generation matrix method. In this section, the one of PDE systems (3) and (6) will be introduced by the spectral radius of next infection operator [28]. Then we will exhibit a threshold-type result and some relevant properties about this number.

Let $C_T(\mathbb{R},\mathbb{W})$ be the Banach space consisting of continuous and T-periodic functions from $\mathbb{R}$ to $\mathbb{W}$ for which $\|\varphi\|_{C_T(\mathbb{R},\mathbb{W})}=\max_{\zeta\in[0,T]}\|\varphi(\zeta)\|_{\mathbb{W}}$ for any $\varphi\in C_T(\mathbb{R},\mathbb{W})$. Now, we linearize system (6) at the disease-free equilibrium, that is, periodic null-solution $(0,0)$, and acquire

$$\begin{aligned}
\frac{\partial u_1}{\partial t} &= \nabla\cdot\big(d_h(x,t)\nabla u_1\big)+\beta_1(x,t)H^*(x,t)u_2\\
&\quad -\gamma_h(x,t)u_1-\mu_H(x,t)u_1,\quad (x,t)\in\Omega\times(0,\infty),\\
\frac{\partial u_2}{\partial t} &= \nabla\cdot\big(d_v(x,t)\nabla u_2\big)+\beta_2(x,t)V^*(x,t)u_1-\mu_v(x,t)u_2,\\
&\quad (x,t)\in\Omega\times(0,\infty),\\
\frac{\partial u_1}{\partial\nu} &= \frac{\partial u_2}{\partial\nu}=0,\quad (x,t)\in\partial\Omega\times(0,\infty).
\end{aligned}\tag{15}$$

Define the function vector $v(x,t)=(u_1(x,t),u_2(x,t))^{\mathrm{T}}$ and matrixes

$$\begin{aligned}
J(x,t)&=\begin{pmatrix}0 & \beta_1(x,t)H^*(x,t)\\ \beta_2(x,t)V^*(x,t) & 0\end{pmatrix},\\
\Gamma(x,t)&=\begin{pmatrix}\gamma_h(x,t)+\mu_H(x,t) & 0\\ 0 & \mu_v(x,t)\end{pmatrix},
\end{aligned}\tag{16}$$

as well as the operator

$$D(\cdot,t)v=\nabla\cdot\big(d(\cdot,t)\nabla v\big),$$

where $d(\cdot,t)=\mathrm{diag}(d_h(\cdot,t),d_v(\cdot,t))$. Then system (15) can be simplified as

$$\frac{\partial v}{\partial t}=D(x,t)v+J(x,t)v-\Gamma(x,t)v,\quad x\in\bar{\Omega},\ t\geqslant 0.$$

According to the definitions (9), (10) and (11), we know that $E(t,s)$ is the evolution family on $\mathbb{W}$ caused by the linear system

$$\begin{aligned}&\frac{\partial v}{\partial t} = D(x,t)v - \Gamma(x,t)v, \quad (x,t) \in \Omega \times (0,\infty),\\ &\frac{\partial v}{\partial \nu} = 0, \quad (x,t) \in \partial\Omega \times (0,\infty).\end{aligned}$$

Following [14], we further define a linear operator on $C_T(\mathbb{R},\mathbb{W})$ by

$$[\mathcal{L}v](t) := \int\limits_0^\infty E(t,\, t-s)J(\cdot,\, t-s)v(\cdot,\, t-s)\,\mathrm{d}s \quad \forall v \in C_T(\mathbb{R},\mathbb{W}),\ t \in \mathbb{R}.$$

Thus, we obtain the basic reproduction number of systems (6), which is the spectral radius of $\mathcal{L}$,

$$R_0 := \rho(\mathcal{L}). \tag{17}$$

Naturally, this number R_0 is just the basic reproduction number of system (3).

For any given $t \geqslant 0$, set $Q(t)$ to be the solution map of system (15) on $\mathbb{W}$, that is, $Q(t)\phi = v(t,\phi)$, where $v(t,\phi)(x) = v(x,t,\phi)$ is the unique solution of system (15) with $v(x,0,\phi) = \phi(x)$. Then $Q := Q(T)$ is the Poincaré map encountered with problem (15). Let $\rho(Q)$ be the spectral radius of Q, then we have the following result through [14, Theorem 3.7] with $\tau = 0$.

Lemma 1. $\operatorname{sign}(R_0 - 1) = \operatorname{sign}(\rho(Q) - 1)$*, that is, both of which have the same sign.*

Additionally, if we set $\tau = 0$, $X = \mathbb{W}$, and $F(t)(x) = J(x,t)$, then one can verify that (H1), (H2) and (H6) in [14] hold, so that it follows from Proposition 3.9 and Theorem 3.8 in [14] that the forthcoming results hold.

Lemma 2. *The basic reproduction number* $R_0 = \mu_0$*, where* μ_0 *is the unique principal eigenvalue of the periodic parabolic eigenvalue problem*

$$\begin{aligned}&\frac{\partial \phi}{\partial t} = \nabla\cdot\bigl(d_h(x,t)\cdot\phi\bigr) + \frac{1}{\mu}\beta_1(x,t)H^*(x,t)\psi - \gamma_h(x,t)\phi\\ &\qquad - \mu_H(x,t)\phi, \quad (x,t) \in \Omega \times (0,\infty),\\ &\frac{\partial \psi}{\partial t} = \nabla\cdot\bigl(d_v(x,t)\cdot\psi\bigr) + \frac{1}{\mu}\beta_2(x,t)V^*(x,t)\phi - \mu_v(x,t)\psi,\\ &\quad (x,t) \in \Omega \times (0,\infty),\\ &\frac{\partial \phi}{\partial \nu} = \frac{\partial \psi}{\partial \nu} = 0, \quad (x,t) \in \partial\Omega \times (0,\infty),\\ &\phi(x,0) = \phi(x,T), \quad \psi(x,0) = \psi(x,T), \quad x \in \Omega,\end{aligned} \tag{18}$$

and the eigenfunction pair $(\phi,\psi) \in C_T(\mathbb{R},\mathbb{W}) \times C_T(\mathbb{R},\mathbb{W})$*, and* $\phi,\psi > 0$ *in* $\bar{\Omega} \times [0,\infty)$.

In fact, if the relevant coefficients are all constants, then we can give the explicit expression about R_0.

Theorem 2. *Suppose that* $g(x,t) \equiv g$ *when* $g = \beta_1$, β_2, H^*, V^*, γ_h, μ_H, μ_v *are all positive constants in* (6)*, that is, suppose that*

$$\frac{\beta_h(x,t)b(x,t)}{H^*(x,t)+m(x,t)} \equiv \frac{\beta_h b}{H^*+m}, \qquad \frac{\beta_v(x,t)b(x,t)}{H^*(x,t)+m(x,t)} \equiv \frac{\beta_v b}{H^*+m}$$

hold in (6) *and* (7)*. Then* R_0 *is expressed by*

$$R_0 = \sqrt{\frac{\beta_1 H^* \cdot \beta_2 V^*}{(\gamma_h+\mu_H)\mu_v}}.$$

Proof. Choose $\phi^*(x,t) \equiv 1$ in $\Omega \times (0,\infty)$, and

$$C = \sqrt{\frac{(\gamma_h+\mu_H)\cdot\beta_2 V^*}{\mu_v \cdot \beta_1 H^*}}, \qquad R = \sqrt{\frac{\beta_1 H^* \cdot \beta_2 V^*}{(\gamma_h+\mu_H)\mu_v}}.$$

We can directly check that $(\phi,\psi;\mu) = (\phi^*, C\phi^*; R)$ is one positive solution of problem (18). So one can obtain $R_0 = \mu_0 = R$ in terms of the uniqueness of the principal eigenpair for (18). □

Denote

$$g^m = \min_{\bar\Omega\times[0,T]} g(x,t), \qquad g^M = \max_{\bar\Omega\times[0,T]} g(x,t).$$

It is straightforward to obtain the following estimate through Theorem 2 and the monotonicity of the eigenvalue on some relevant coefficients [1,3].

Corollary 1. *Under the assumptions of Theorem* 2*, the basic reproduction number* R_0 *for problem* (6) *satisfies*

$$\sqrt{\frac{\beta_1^m H^* \cdot \beta_2^m V^*}{(\gamma_h^M+\mu_H)\mu_v}} \leqslant R_0 \leqslant \sqrt{\frac{\beta_1^M H^* \cdot \beta_2^M V^*}{(\gamma_h^m+\mu_H)\mu_v}}. \tag{19}$$

We now assume that the diffusion coefficients in system (6) satisfy

$$d_h(x,t) = d_1\delta_h(x,t), \qquad d_v(x,t) = d_2\delta_v(x,t),$$

where both d_1 and d_2 are positive constants. Then R_0 defined in (17) can be also written as $R_0(d_1,d_2)$. Subsequently, we discuss the asymptotic behavior of $R_0(d_1,d_2)$ as $\max(d_1,d_2) \to 0$ and $\min(d_1,d_2) \to +\infty$. For this, we first define the space

$$\mathbb{Z} := \bigl\{\mathbf{z} \in C\bigl(\mathbb{R},\mathbb{R}^2\bigr)\colon \mathbf{z}(t) = \mathbf{z}(t+T),\ t \in \mathbb{R}\bigr\}$$

and denote

$$\bar{\bar{h}}(t) = \frac{1}{|\Omega|}\int_\Omega h(x,t)\,\mathrm{d}x.$$

For any $x \in \bar{\Omega}$, let $\{E_{x,0}(t,s)\colon t \geqslant s\}$ be the evolution family on $\mathbb{R}^2$ of

$$\begin{aligned}
\frac{\partial u_1}{\partial t} &= -\gamma_h(x,t)u_1 - \mu_H(x,t)u_1, \quad x \in \bar{\Omega},\ t > 0\\
\frac{\partial u_2}{\partial t} &= -\mu_v(x,t)u_2, \quad x \in \bar{\Omega},\ t > 0,
\end{aligned}$$

that is,

$$E_{x,0}(t,s)\phi(\cdot) = \bigl(\phi_1(\cdot)\mathrm{e}^{-\int_s^t (\gamma_h(\cdot,\tau)+\mu_H(\cdot,\tau))\,\mathrm{d}\tau},\ \phi_2(\cdot)\mathrm{e}^{-\int_s^t \mu_v(\cdot,\tau)\,\mathrm{d}\tau}\bigr), \quad \phi \in \mathbb{W}.$$

Using the definition of $J(x,t)$ in (16) again, we define a positive bounded linear operator $\mathcal{L}_{x,0} : \mathbb{Z} \to \mathbb{Z}$ by

$$[\mathcal{L}_{x,0}\mathbf{z}](t) := \int_0^\infty E_{x,0}(t,\, t-s)J(x,\, t-s)\mathbf{z}(t-s)\,\mathrm{d}s \quad \forall \mathbf{z} \in \mathbb{Z},\ t \in \mathbb{R}.$$

Furthermore, we set $\{E_\infty(t,s)\colon t \geqslant s\}$ to be the evolution family on $\mathbb{R}^2$ of

$$\begin{aligned}
\frac{\partial u_1}{\partial t} &= -\bar{\bar{\gamma}}_h(t)u_1 - \bar{\bar{\mu}}_H(t)u_1, \quad x \in \bar{\Omega},\ t > 0,\\
\frac{\partial u_2}{\partial t} &= -\bar{\bar{\mu}}_v(t)u_2, \quad x \in \bar{\Omega},\ t > 0,
\end{aligned}$$

that is,

$$E_\infty(t,s)\phi(\cdot) = \bigl(\phi_1(\cdot)\mathrm{e}^{-\int_s^t (\bar{\bar{\gamma}}_H(\tau)+\bar{\bar{\mu}}_H(\tau))\,\mathrm{d}\tau},\ \phi_2(\cdot)\mathrm{e}^{-\int_s^t \mu_v(\tau)\,\mathrm{d}\tau}\bigr), \quad \phi \in \mathbb{W},$$

and define

$$\bar{\bar{J}}(t) = \begin{pmatrix} 0 & \bar{\bar{h}}_{\beta H}(t) \\ \bar{\bar{h}}_{\beta V}(t) & 0 \end{pmatrix}, \quad t \in \mathbb{R},$$

where

$$\bar{\bar{h}}_{\beta H}(t) := \frac{1}{|\Omega|}\int_\Omega \beta_1(x,t)H^*(x,t)\,\mathrm{d}x, \qquad \bar{\bar{h}}_{\beta V}(t) := \frac{1}{|\Omega|}\int_\Omega \beta_2(x,t)V^*(x,t)\,\mathrm{d}x.$$

Now, we introduce another bounded linear positive operator $\mathcal{L}_\infty : \mathbb{Y} \to \mathbb{Y}$ by

$$[\mathcal{L}_\infty\mathbf{z}](t) := \int_0^\infty E_\infty(t,\, t-s)\bar{\bar{J}}(t-s)\mathbf{z}(t-s)\,\mathrm{d}s \quad \forall \mathbf{z} \in \mathbb{Z},\ t \in \mathbb{R}.$$

Finally, we define $\mathcal{R}_0(0,x) := \rho(\mathcal{L}_{0,x})$, $x \in \bar{\Omega}$, and $\mathcal{R}_0(\infty) := \rho(\mathcal{L}_\infty)$. According to Theorem 4.1 in [26], one can acquire the trends of threshold value $R_0(d_1,d_2)$.

Lemma 3. *$R_0(d_1,d_2)$ admits the following properties*:

$$\lim_{\max(d_1,d_2)\to 0} R_0(d_1,d_2) = \max_{x\in\bar{\Omega}} \mathcal{R}_0(0,x)$$

and

$$\lim_{\min(d_1,d_2)\to +\infty} R_0(d_1,d_2) = \mathcal{R}_0(\infty).$$

4 The global dynamics of solution to system (6)

According to Theorem 1, we still let $u(x,t,\phi)=(u_1(x,t,\phi_1),u_2(x,t,\phi_2))$ be the unique solution of (6) with $u(x,0,\phi)=(u_1(x,0,\phi_1),u_2(x,0,\phi_2))=(\phi_1,\phi_2)$. For any given $t\geqslant 0$, we define an operator $G(t):W(0)\to W(t)$ by

$$G(t)(\phi)(x)=u(x,t,\phi)\quad \forall\phi\in W(0),\ x\in\bar{\Omega},$$

where $W(t)$ has been defined by (8), so $G(t)$ is a T-periodic semiflow, and

$$G:=G(T):W(0)\to W(T)=W(0)$$

is the Poincaré map associated with system (6). The following theorem presents a characteristic of map $G(t)$.

Lemma 4. *For each $t>0$, the map $G(t):W(0)\to W(t)$ is strongly monotone. In other words, for any ψ and ϕ in $W(0)$ with $\psi>\phi$ (that is, $\psi\geqslant\phi$ but $\psi\not\equiv\phi$), the solution $\bar{u}(\cdot,t,\psi)$ and $u(\cdot,t,\phi)$ of system* (6) *with $\bar{u}(\cdot,0,\psi)=\psi$, $u(\cdot,0,\phi)=\phi$, respectively, meet with $\bar{u}(\cdot,t,\psi)\gg u(\cdot,t,\phi)$ for all $t>0$.*

Proof. From the comparison principle for cooperative parabolic system, obviously, one has $\bar{u}_i(\cdot,t,\psi)\geqslant u_i(\cdot,t,\phi)$ for all $t\geqslant 0$, $i=1,2$. Let $\psi,\phi\in W(0)$ satisfy $\psi>\phi$, and define

$$\begin{aligned}\bar{u}(\cdot,t)&=\bar{u}(\cdot,t,\psi)=\big(\bar{u}_1(\cdot,t,\psi_1),\bar{u}_2(\cdot,t,\psi_2)\big),\\ u(\cdot,t)&=u(\cdot,t,\phi)=\big(u_1(\cdot,t,\phi_1),u_2(\cdot,t,\phi_2)\big).\end{aligned}$$

Without loss of generality, we suppose $\psi_1(\cdot)>\phi_1(\cdot)$. Clearly, it follows from (6) that

$$\begin{aligned}\frac{\partial(\bar{u}_1-u_1)}{\partial t}&=\nabla\cdot\big(d_h(x,t)\nabla(\bar{u}_1-u_1)\big)+\beta_1(x,t)H^*(x,t)(\bar{u}_2-u_2)\\&\quad-\beta_1(x,t)\bar{u}_1\bar{u}_2+\beta_1(x,t)u_1u_2-\big(\gamma_h(x,t)+\mu_H(x,t)\big)(\bar{u}_1-u_1)\\&=\nabla\cdot\big(d_h(x,t)\nabla(\bar{u}_1-u_1)\big)+\beta_1(x,t)\big(H^*(x,t)-\bar{u}_1\big)(\bar{u}_2-u_2)\\&\quad-\beta_1(x,t)u_2(\bar{u}_1-u_1)-\big(\gamma_h(x,t)+\mu_H(x,t)\big)(\bar{u}_1-u_1)\\&\geqslant\nabla\cdot\big(d_h(x,t)\nabla(\bar{u}_1-u_1)\big)-\beta_1(x,t)u_2(\bar{u}_1-u_1)\\&\quad-\big(\gamma_h(x,t)+\mu_H(x,t)\big)(\bar{u}_1-u_1)\end{aligned}$$

for $x\in\bar{\Omega}$, $t>0$, and

$$\begin{aligned}&\frac{\partial(\bar{u}_1-u_1)}{\partial\nu}=0,\quad x\in\partial\Omega,\ t>0,\\&\bar{u}_1(x,0)-u_1(x,0)=\psi_1(x)-\phi_1(x)>0,\quad x\in\bar{\Omega}.\end{aligned}$$

As an application of the maximal principle and parabolic comparison theorem, it turns out that $\bar{u}_1(x,t)>u_1(x,t)$ for $t>0$, $x\in\bar{\Omega}$, that is, $\bar{u}_1(\cdot,t)\gg u_1(\cdot,t)$, $t>0$. With the same manner as $\bar{u}_1$ and u_1, we can also obtain $\bar{u}_2(\cdot,t) \gg u_2(\cdot,t)$ for each $t > 0$, provided with $\psi_2(\cdot) > \phi_2(\cdot)$. Afterwards, it suffices to prove that $\bar{u}_2(\cdot,t) \gg u_2(\cdot,t)$ for $t > 0$ on the case $\psi_1(\cdot) > \phi_1(\cdot), \psi_2(\cdot) \equiv \phi_2(\cdot)$.

First, we claim that $\bar{u}_2(\cdot,t) > u_2(\cdot,t)$ for all $t > 0$. In fact, were the stated result false, there would exist some $t_0 > 0$ such that $\bar{u}_2(t_0,\cdot) = u_2(t_0,\cdot)$. Naturally, the maximum principle guarantees $\bar{u}_2(\cdot,t) = u_2(\cdot,t)$ for all $t \in [0,t_0]$, which leads to

$$\frac{\partial \bar{u}_2(x,t)}{\partial t} = \frac{\partial u_2(x,t)}{\partial t} \quad \forall x \in \bar{\Omega},\ t \in [0,t_0].$$

Recalling the second equation in (6) again, hence, one acquires

$$\beta_2(\cdot,t)\bigl(V^*(\cdot,t) - u_2(\cdot,t)\bigr)\bigl(\bar{u_1}(\cdot,t) - u_1(\cdot,t)\bigr) = 0 \quad \forall t \in [0,t_0].$$

Noting that $\bar{u}_1(\cdot,t) \gg u_1(\cdot,t)$ for all $t > 0$, the above equality leads to $V^*(\cdot,t) = u_2(\cdot,t)$ for all $t \in (0,t_0]$. Therefore, the second equation in (6) implies that

$$\frac{\partial V^*(x,t)}{\partial t} = \nabla \cdot \bigl(d_v(x,t)\nabla V^*(x,t)\bigr) - \mu_v(x,t)V^*(x,t) \quad \forall t \in (0,t_0],$$

which contradicts with (14). So the claim is valid.

In what follows, we denote

$$\begin{aligned} f(x,t,w) = {}& \nabla \cdot \bigl(d_v(x,t)\nabla w\bigr) + \beta_2(x,t)\bigl(V^*(x,t) - w\bigr)u_1(x,t) \\ & - \mu_v(x,t)w \quad x \in \bar{\Omega},\ t > 0, \end{aligned}$$

and then acquire

$$\begin{aligned} \frac{\partial \bar{u}_2}{\partial t} &= \nabla \cdot (d_v(x,t)\nabla \bar{u}_2) + \beta_2(x,t)\bigl(V^*(x,t) - \bar{u}_2\bigr)\bar{u}_1 - \mu_v(x,t)\bar{u}_2 \\ &\geqslant \nabla \cdot \bigl(d_v(x,t)\nabla \bar{u}_2\bigr) + \beta_2(x,t)\bigl(V^*(x,t) - \bar{u}_2\bigr)u_1 - \mu_v(x,t)\bar{u}_2 = f(x,t,\bar{u}_2), \end{aligned}$$

which results in

$$\frac{\partial \bar{u}_2}{\partial t} - f(x,t,\bar{u}_2) \geqslant \frac{\partial u_2}{\partial t} - f(x,t,u_2), \quad x \in \Omega,\ t > 0,$$

with $\partial \bar{u}_2/\partial \nu = \partial u_2/\partial \nu = 0$, $x \in \partial\Omega$, $t > 0$. Meanwhile, the above claim shows that $\bar{u}_2(t_1,\cdot) > u_2(t_1,\cdot)$ for any given $t_1 > 0$. Therefore, it follows that $\bar{u}_2(\cdot,t) \gg u_2(\cdot,t)$ for all $t > t_1$ by using the parabolic maximum principle. Since t_1 is arbitrary, one has $\bar{u}_2(\cdot,t) \gg u_2(\cdot,t)$ for all $t > t_1$, which means that $\bar{u}_2(\cdot,t,\phi) \gg u_2(\cdot,t,\psi)$ holds for all $t > 0$ by the arbitrariness of t_1. Hence, $\bar{u}(\cdot,t,\phi) \gg u(\cdot,t,\psi)$ for all $t > 0$. □

Below we exhibit the main results of this section in detail.

Theorem 3. *The following statements holds*:

(i) *If* $R_0 > 1$, *then system* (6) *admits a unique positive* T*-periodic solution* $(u_1^*(x,t), u_2^*(x,t))$, *which is globally asymptotically stable for system* (6) *in* $E(0)\backslash\{(0,0)\}$.

(ii) *If* $R_0 \leqslant 1$, *then the disease-free periodic solution* $(0,0)$ *is globally asymptotically stable for system* (6) *in* $W(0)$.

Proof. Our arguments are inspired by those of [13, Theorem 1]. For any given $\phi \in W(0)$, $\lambda \in [0,1]$, we suppose $u(x,t,\phi)$ and $u(x,t,\lambda\phi)$ be the solutions of system (6) associated with $u(x,0,\phi) = \phi(x)$ and $u(x,0,\lambda\phi) = \lambda\phi(x), x \in \bar{\Omega}$, respectively.

Define

$$w(x,t) := \lambda u(x,t,\phi) = \bigl(w_1(x,t), w_2(x,t)\bigr),$$

then one has

$$\begin{aligned}\frac{\partial w_1}{\partial t} &= \lambda\frac{\partial u_1}{\partial t}\\ &= \nabla\cdot\bigl(d_h(x,t)\nabla(\lambda u_1)\bigr)+\beta_1(x,t)\bigl(H^*(x,t)-u_1\bigr)\lambda u_2-\bigl(\gamma_h(x,t)+\mu_H(x,t)\bigr)\lambda u_1\\ &\leqslant \nabla\cdot\bigl(d_h(x,t)\nabla(\lambda u_1)\bigr)+\beta_1(x,t)\bigl(H^*(x,t)-\lambda u_1\bigr)\lambda u_2-\bigl(\gamma_h(x,t)+\mu_H(x,t)\bigr)\lambda u_1\\ &= \nabla\cdot\bigl(d_h(x,t)\nabla w_1\bigr)+\beta_1(x,t)\bigl(H^*(x,t)-w_1\bigr)w_2-\bigl(\gamma_h(x,t)+\mu_H(x,t)\bigr)w_1.\end{aligned}$$

Similarly,

$$\frac{\partial w_2}{\partial t} \leqslant \nabla\cdot\bigl(d_v(x,t)\nabla w_2\bigr) + \beta_2(x,t)\bigl(V^*(x,t) - w_2\bigr)w_1 - \mu_v(x,t)w_2.$$

Therefore, $w(x,t)$ is a lower solution of system (6) with $w(x,0) = \lambda u(x,0,\phi) = \lambda\phi$, which means that $\lambda u(x,t,\phi) \leqslant u(x,t,\lambda\phi)$ for all $t > 0, x \in \bar{\Omega}$. Thus, the solution map $G(t) : W(0) \to W(t)$ is subhomogeneous.

In what follows, we claim that $G(t) : W(0) \to W(t)$ is strictly subhomogeneous for each $t > 0$, that is, for all $\lambda \in (0,1)$, $\phi \in W(0)$ with $\phi \gg 0$, there holds $G(t)(\lambda\phi) > \lambda G(t)\phi$, which is equivalent to $u(\cdot,t,\lambda\phi) > \lambda u(\cdot,t,\phi)$.

Actually, for any $\phi \in W(0)$ with $\phi \not\equiv 0$, $\lambda \in (0,1)$, we set

$$z(x,t) = u(x,t,\lambda\phi) - \lambda u(x,t,\phi).$$

Then $z(x,0) = 0, z(x,t) \geqslant 0$ for $(x,t) \in \bar{\Omega} \times [0,\infty)$. We further indicate $z(x,t) > 0$ for all $(x,t) \in \bar{\Omega} \times (0,\infty)$.

Because one can calculate out

$$\begin{aligned}\frac{\partial z_1}{\partial t} &= \frac{\partial u_1(x,t,\lambda\phi)}{\partial t} - \lambda\frac{\partial u_1(x,t,\phi)}{\partial t}\\ &= \nabla\cdot\bigl(d_h(x,t)\nabla u_1(x,t,\lambda\phi)\bigr) + \beta_1(x,t)\bigl(H^*(x,t) - u_1(x,t,\lambda\phi)\bigr)u_2(x,t,\lambda\phi)\\ &\quad - \bigl(\gamma_h(x,t) + \mu_H(x,t)\bigr)u_1(x,t,\lambda\phi)\\ &\quad - \lambda\bigl[\nabla\cdot\bigl(d_h(x,t)\nabla u_1(x,t,\phi)\bigr) + \beta_1(x,t)\bigl(H^*(x,t) - u_1(x,t,\phi)\bigr)u_2(x,t,\phi)\\ &\quad - \bigl(\gamma_h(x,t) + \mu_H(x,t)\bigr)u_1(x,t,\phi)\bigr]\\ &\quad = \nabla\cdot\bigl(d_h(x,t)\nabla z_1\bigr) + \beta_1(x,t)\bigl(H^*(x,t) - u_1(x,t,\lambda\phi)\bigr)u_2(x,t,\lambda\phi)\\ &\quad - \lambda\beta_1(x,t)\bigl(H^*(x,t) - u_1(x,t,\phi)\bigr)u_2(x,t,\phi) - \bigl(\gamma_h(x,t) + \mu_H(x,t)\bigr)z_1\\ &= \nabla\cdot\bigl(d_h(x,t)\nabla z_1\bigr)-\beta_1(x,t)u_2(x,t,\lambda\phi)z_1+\beta_1(x,t)\bigl(H^*(x,t)-\lambda u_1(x,t,\phi)\bigr)z_2\\ &\quad - \bigl(\gamma_h(x,t) + \mu_H(x,t)\bigr)z_1 + g(x,t)\\ &\geqslant \nabla\cdot\bigl(d_h(x,t)\nabla z_1\bigr) - \beta_1(x,t)u_2(x,t,\lambda\phi)z_1 - \bigl(\gamma_h(x,t)+\mu_H(x,t)\bigr)z_1 + h(x,t),\end{aligned}$$

where $h(x,t) = \lambda\beta_1(x,t)u_2(x,t,\phi)[u_1(x,t,\phi) - \lambda u_1(x,t,\phi)]$, one can acquire that $h(x,t) > 0$ from the fact $u_2(x,t,\phi) > 0$ created by Theorem 1.

Owing to the boundedness of $u_2(x,t,\lambda\phi)$, there exists a positive constant K such that

$$\frac{\partial z_1}{\partial t} \geqslant \nabla \cdot (d_h(x,t)\nabla z_1) - K z_1 + h(x,t).$$

Put $\hat{E}(t,s) : \mathbb{X} \to \mathbb{X}$, $0 \leqslant s \leqslant t$, be the evolution operator of

$$\begin{aligned} &\frac{\partial U}{\partial t} = \nabla \cdot \big(d_h(x,t)\nabla U\big) - K \cdot U, \quad x \in \Omega,\ t > 0, \\ &\frac{\partial U}{\partial \nu} = 0, \quad x \in \partial\Omega,\ t > 0. \end{aligned}$$

Then the solution of problem

$$\begin{aligned} &\frac{\partial U}{\partial t} = \nabla \cdot \big(d_h(x,t)\nabla U\big) - K \cdot U + h(x,t), \quad x \in \Omega,\ t > 0, \\ &\frac{\partial U}{\partial \nu} = 0, \quad x \in \partial\Omega,\ t > 0, \qquad U(x,0) = \varphi \in \mathbb{X}, \quad x \in \bar{\Omega}, \end{aligned} \tag{20}$$

can be expressed by

$$U(x,t,\varphi) = \hat{E}(t,0)(\varphi)(x) + \int_0^t \hat{E}(t,s)h(x,s)\,\mathrm{d}s, \quad x \in \bar{\Omega},\ t \geqslant 0,\ \varphi \in \mathbb{X}. \tag{21}$$

In view of $h(x,t) > 0$ for $(x,t) \in \bar{\Omega} \times (0,\infty)$, it follows that the solution of (20) satisfies $U(x,t,\varphi) > 0$ for any $\varphi \geqslant 0$ with $\varphi \not\equiv 0$ from equation (21) and the properties of $\hat{E}(t,s)$. Hence, we acquire $z_1(x,t) > 0$. Similarly, $z_2(x,t) > 0$. Then $u(\cdot,t,\lambda\phi) > \lambda u(\cdot,t,\phi)$, which means that $G(t)(\lambda\phi) > \lambda G(t)(\phi)$ for all $t > 0$. So the map $G(t)$ is strictly subhomogeneous.

Considering the continuity and differentiability of solutions about the initial value, furthermore, we know that $G := G(T)$ is differentiable at zero and its Fréchet derivate is $DG(0) = Q$. By analyzed above and Theorem 4, one can yield that G is a strictly subhomogeneous and strongly monotone map on $W(0)$. Because $G(t)$ is compact for any $t > 0$, G is asymptotically smooth on $W(0)$. Meanwhile, Q is also compact and strongly positive. According to [28, Thm. 2.3.4] and [22, Lemma2.1], for G, we come to the following threshold-type result:

(i) If $\rho(Q) > 1$, then system (6) possesses a unique positive T-periodic solution $(u_1^*(x,t), u_2^*(x,t))$, which is globally asymptotically stable for system (6) in $W(0) \setminus \{(0,0)\}$.
(ii) If $\rho(Q) \leqslant 1$, then $(0,0)$ is globally asymptotically stable for system (6) in $W(0)$.

Combining with Lemma 1, we directly obtain the desired result in terms of R_0. □

5 Numerical Simulations and discussion

To further understand the seasonal periodicity on the transmission of dengue fever, we carry out some numerical simulations in this section. Specifically, we apply system (6) to Guangzhou, which is one of cities that breaks out dengue fever most easily in China.

In Guangzhou, it is largely *Ae.albopictus* that transmits dengue fever [5]. The literature [17] emphasis that all mosquito traits relevant to transmissionbiting, egg-to-adult survival and development, fecundityrespond strongly to temperature and peak between $23\,^{\circ}\mathrm{C}$ and $34\,^{\circ}\mathrm{C}$ for *Ae.aegypti* and *Ae.albopictus*. This literature further shows that *Ae.albopictus* transmission peaked at $26.4\,^{\circ}\mathrm{C}$ and declined to zero below $16.2\,^{\circ}\mathrm{C}$ and above $31.6\,^{\circ}\mathrm{C}$. Based on these information, we can assume that the biting rate and transmission probability are significantly affected by temperature. In Guangzhou, additionally, the relevant data [32] shows that permanent resident population density is 2521 per square kilometer, and human mortality rate is 6.39% in 2020. We list some functions influenced by temperature and some values irrelevant to temperature in Table 1 by employing the above data and other published data.

In Table 2, meanwhile, we present the monthly mean temperatures for Guangzhou, China, in 2020 [36], which are viewed as some references in simulation process.

Combining the aforementioned information with Tables 1 and 2, we assume that the biting rate $b(C)$, transmission probabilities $\beta_h(C)$ and $\beta_v(C)$ satisfy the following

Table 1. Constant parameters and temperature C -dependent functions (in °C)

Parameters	Description	Value or formula	Ref.
H^*	human population density	2521	[32]
μ_H	death rate of human	$6.39\% \cdot \frac{1}{12}$ Month^{-1}	[32]
m	population density of other alternative hosts	0	[8]
γ_h	recovery rate of human	$\frac{1}{6} \cdot \frac{1}{12}$Month^{-1}	[2]
A	recruitment density of mosquito	$1 \leqslant \frac{A}{N_h} \leqslant 10$	[7]
μ_v	per capita mortality of mosquito	$\frac{1}{14.49} \cdot 30.4$ Month^{-1}	[2]
$\hat{b}(C)$	biting rate of mosquito	$2.02C(C-13.35)(40.08-C)^{1/2}$ $\times 10^{-4} \cdot 30.4$ Month^{-1}	[13, 17]
$\hat{\beta}_H(C)$	transmission probability from infectious mosquitoes to susceptible humans	$8.49C(C-17.05)(35.83-C)^{1/2}$ $\times 10^{-4}$	[13, 17]
$\hat{\beta}_V(C)$	transmission probability from susceptible mosquitoes to infectious humans	$4.91C(C-12.22)(37.46-C)^{1/2}$ $\times 10^{-4}$	[13, 17]
d_h	human diffusion coefficient	$1 \cdot 30.4$km^2/Month	[13, 15]
d_v	mosquito diffusion coefficient	$1.25 \cdot 10^{-2} \cdot 30.4$km^2/Month	[13, 15]

Table 2. Monthly mean temperatures for Guangzhou in 2020 (°C).

Month	Jan.	Feb.	Mar.	Apr.	May	Jun.
Temperature	13.3	14.4	17.9	21.9	25.6	27.2
Month	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Temperature	28.4	28.1	26.9	23.7	19.4	15.2

Table 3. Monthly transmission probabilities β_h, β_v and biting rate b for Guangzhou in 2020.

Month	Jan.	Feb.	Mar.	Apr.	May	Jun.
β_h	0	0	0.0547	0.3366	0.5944	0.6886
β_v	0	0	0.2208	0.4106	0.5792	0.6408
b	0	0	2.3554	4.9027	7.3280	8.3024
Month	**Jul.**	**Aug.**	**Sep.**	**Oct.**	**Nov.**	**Dec.**
β_h	0.7460	0.7330	0.6722	0.4660	0.1569	0
β_v	0.6791	0.6703	0.6301	0.4955	0.2906	0
b	8.9702	8.8095	8.1260	6.0964	3.2776	0

functions, respectively, for Guangzhou in 2020:

$$b(C) = \begin{cases} 0, & 0 < C < 16.2 \text{ or } C > 31.6, \\ \hat{b}(C) \text{ in Table 1}, & 16.2 \leqslant C \leqslant 31.6, \end{cases} \tag{22}$$

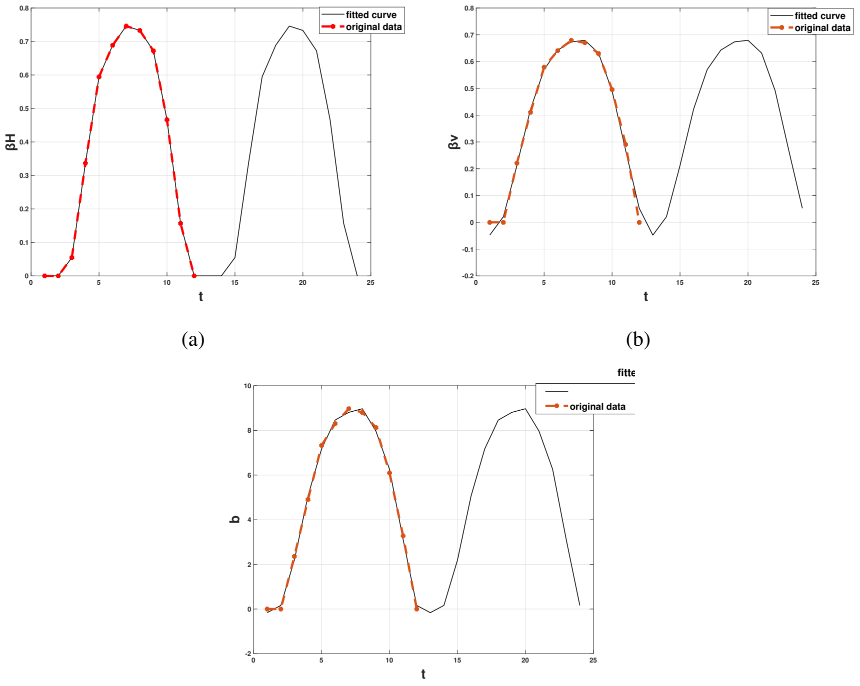
$$\beta_h(C) = \begin{cases} 0, & 0 < C < 16.2 \text{ or } C > 31.6, \\ \hat{\beta_h}(C) \text{ in Table 1}, & 16.2 \leqslant C \leqslant 31.6, \end{cases} \tag{23}$$

and

$$\beta_v(C) = \begin{cases} 0, & 0 < C < 16.2 \text{ or } C > 31.6, \\ \hat{\beta_v}(C) \text{ in Table 1}, & 16.2 \leqslant C \leqslant 31.6, \end{cases} \tag{24}$$

Putting the data into formulaes (22)–(24), respectively, we obtain all values of β_h, β_v and b from January to December, as well as three scatter diagrams associated with month-axis form 0 to 12, both of which are exhibited in Table 3 and Figs. 1–3 (Figs. 1–3 represent the data points in Table 3 and fitted curves of $\beta_h(t)$, $\beta_v(t)$ and $b(t)$, respectively). Furthermore, we adopt the curve fitting tool by using the scatter values in Fig. 1 and fit the following relevant time-periodic functions (set the period $T = 12$), whose approximate image are also exhibited in Fig. 1.

$$\begin{aligned} \beta_h(t) = {} & 0.3737 \\ & - 0.3359\cos\frac{\pi t}{6} - 0.2531\sin\frac{\pi t}{6} - 0.009587\cos\frac{2\pi t}{6} - 0.01082\sin\frac{2\pi t}{6} \\ & - 0.01418\cos\frac{3\pi t}{6} + 0.05151\sin\frac{3\pi t}{6} - 0.01681\cos\frac{4\pi t}{6} + 0.00929\sin\frac{4\pi t}{6} \\ & + 0.005763\cos\frac{5\pi t}{6} - 0.004155\sin\frac{5\pi t}{6}, \\ \beta_v(t) = {} & 0.3848 \\ & - 0.2953\cos\frac{\pi t}{6} - 0.2098\sin\frac{\pi t}{6} - 0.0373\cos\frac{2\pi t}{6} - 0.06157\sin\frac{2\pi t}{6} \\ & - 0.009244\cos\frac{3\pi t}{6} + 0.00312\sin\frac{3\pi t}{6} - 0.01184\cos\frac{4\pi t}{6} + 0.006509\sin\frac{4\pi t}{6} \\ & - 0.001591\cos\frac{5\pi t}{6} + 0.00825\sin\frac{5\pi t}{6}, \end{aligned}$$



(a) (b)

(c)

Figure 1. Data points in Table 3 and fitted curves of $\beta_h(t)$, $\beta_v(t)$ and $b(t)$, respectively.

$$\begin{aligned}b(t) = {} & 4.847 \\ & - 3.898\cos\frac{\pi t}{6} - 2.816\sin\frac{\pi t}{6} - 0.3826\cos\frac{2\pi t}{6} - 0.5521\sin\frac{2\pi t}{6} \\ & - 0.1144\cos\frac{3\pi t}{6} + 0.1418\sin\frac{3\pi t}{6} - 0.1514\cos\frac{4\pi t}{6} + 0.07998\sin\frac{4\pi t}{6} \\ & - 0.139\cos\frac{5\pi t}{6} + 0.0721\sin\frac{5\pi t}{6}.\end{aligned}$$

We now simulate the following system, which is simplified by (6) and (7):

$$\begin{aligned}&\frac{\partial I_h}{\partial t} = d_h\Delta I_h + \frac{\beta_h(t)b(t)}{H^*+m}\bigl(H^*-I_h\bigr)I_v - \gamma_h I_h - \mu_H I_h, \quad x\in\bar{\Omega},\ t>0,\\ &\frac{\partial I_v}{\partial t} = d_v\Delta I_v + \frac{\beta_v(t)b(t)}{H^*+m}\left(\frac{A}{\mu_v} - I_v\right)I_h - \mu_v I_v, \quad x\in\bar{\Omega},\ t>0,\\ &\frac{\partial I_h}{\partial \nu} = \frac{\partial I_v}{\partial \nu} = 0, \quad x\in\partial\Omega,\ t>0,\end{aligned} \tag{25}$$

with the initial function

$$(I_h, I_v)(x,0) = \bigl(\phi_1(x), \phi_2(x)\bigr).$$

In system (25), we set $\varOmega = (0, \pi)$ and

$$\begin{aligned} I_h(x,0) &= \phi_1(x) = 2000 + 20\cos(4x), \\ I_v(x,0) &= \phi_2(x) = 20 + 8\cos(4x). \end{aligned}$$

Meanwhile, a noteworthy fact is that dengue fever usually occurs from April to November in Guangzhou [5]. Therefore, according to the values in Table 3, we assume that

$$\begin{aligned} \beta_h^m &= \beta_h(11) \approx 0.1569, & \beta_h^M &= \beta_h(7) \approx 0.7460, \\ \beta_v^m &= \beta_v(11) \approx 0.2906, & \beta_v^M &= \beta_v(7) \approx 0.6791, \\ b^m &= b(11) \approx 3.2776, & b^M &= b(7) \approx 8.9702. \end{aligned}$$

In what follows, we choose the different recruitment density of mosquito A and then observe the asymptotic behavior of solutions for system (25).

Example 1. Select $A = 0.0008H^*$. Combining with the values in Table 1 and recalling formula (19), we can obtain

$$\begin{aligned} R_0 &\leqslant \sqrt{\frac{\frac{\beta_h^M b^M}{H^*+m} H^* \cdot \frac{\beta_v^M b^M}{N_h+m} \frac{A}{\mu_v}}{(\gamma_h + \mu_H)\mu_v}} = \sqrt{\frac{\frac{0.7460 \cdot 8.9702}{2521} \cdot 2521 \cdot \frac{0.6791 \cdot 8.9702}{2521} \cdot \frac{0.008 \cdot 2521}{(1/14.49) \cdot 30.4}}{(\frac{1}{6} \cdot \frac{1}{12} + 6.39\% \cdot \frac{1}{12}) \cdot \frac{1}{14.49} \cdot 30.4}} \\ &\leqslant \sqrt{\frac{0.00018}{0.00045}} \approx 0.6325 < 1. \end{aligned}$$

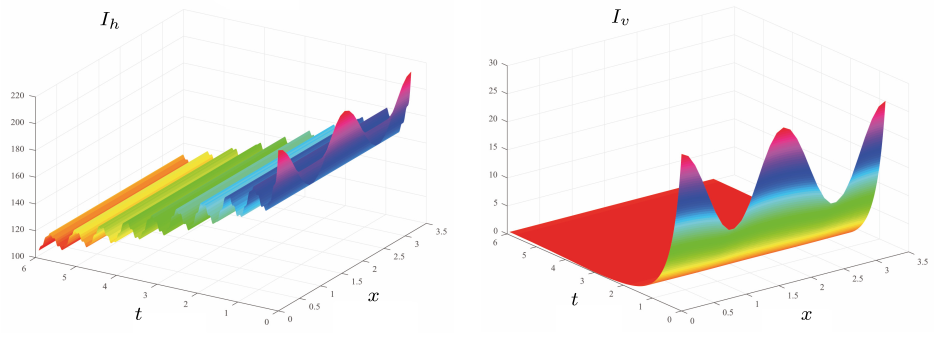
With the help of Theorem 3, the above result infers that the variables I_h and I_v in system (25) will decay to zero little by little as time evolves. Figures 2(a) and 2(b) also exhibit the long-time behavior of I_h and I_v in which both I_h and I_v is vanishing on the case $R_0 \leqslant 1$.

Example 2. Select $A = 2.40H^*$. It follows from formula (19) that

$$\begin{aligned} R_0 &\geqslant \sqrt{\frac{\frac{\beta_h^m b^m}{H^*+m} H^* \cdot \frac{\beta_v^m b^m}{H^*+m} \frac{A}{\mu_v}}{(\gamma_h + \mu_H)\mu_v}} = \sqrt{\frac{\frac{0.1569 \cdot 3.2776}{2521} \cdot 2521 \cdot \frac{0.2906 \cdot 3.2776}{2521} \cdot \frac{2.40 \cdot 2521}{(1/14.49) \cdot 30.4}}{(\frac{1}{6} \cdot \frac{1}{12} + 6.39\% \cdot \frac{1}{12}) \cdot \frac{1}{14.49} \cdot 30.4}} \\ &\geqslant \sqrt{\frac{0.00625}{0.000451}} \approx 3.6858 > 1, \end{aligned}$$

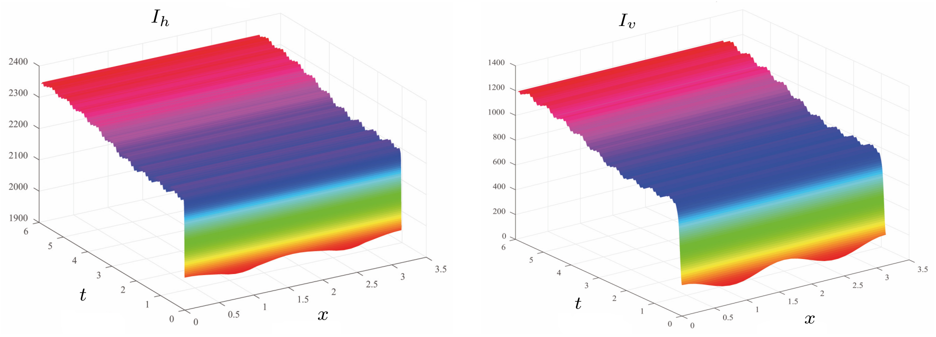
which means that the solution (I_h, I_v) of system (24) finally converges to a positive periodic solution. In Figs. 3(a) and 3(b), we also observe that the infectious individuals (I_h) and I_v gradually approach the periodic steady-state and exhibit periodic fluctuation as time increases when $R_0 > 1$.

The authors in [11] stress that climate change influences the epidemiology of vector-borne disease by influencing the ecology or biology of either the reservoir host or the insect vector. Further, they state that many mosquito-borne diseases are sensitive to climate variables such as temperature and rainfall. It is well known that climate characterizes



(a) (b)

Figure 2. The vanishing of dengue virus on the case $R_0* \leqslant 1$. Graphs (a) and (b) suggest that the solution (I_h, I_v) of system (25) goes to zero as time evolves, which implies that dengue fever gradually disappears.



(a) (b)

Figure 3. The spreading of dengue fever on the case $R_0^* > 1$. Through graphs (a) and (b), we can observe that the solution (I_h, I_v) gradually stabilizes to an positive periodical solution, which means that the virus is in a state of propagation.

seasonality and periodicity upon most occasions. In this paper, we have built and studied a dengue fever model in the heterogeneous environment associated with time periodicity. For the dynamics of this model, we have acquired its basic reproduction number R_0 through the method of next infection operator [28], as well as discussed some limiting forms when the diffusion coefficients increase or decrease infinitely. By means of the theories of monotone dynamical systems, furthermore, we have obtained some global stability results decided by the threshold R_0. Our findings demonstrate that dengue fever will be controlled and gradually vanish if $R_0 \leqslant 1$, while the virus will stabilize to a positive periodic steady state if $R_0 > 1$.

Actually, the similar periodic dengue model has been investigated in [31]. For the preceding model (1.5) appeared in [31], the novelties of our paper lie in the following two aspects. Firstly, model (1.5) in [31] only considered the time periodicity and space heterogeneity of transmission rates β_h and β_v, as well as biting rate b, but the values related with the population quantity, such as N_H, A, m, μ_h, μ_H and μ_v, are all assumed as constants, whose kind of hypothesis offers convenience for the upper-lower solutions method adopted in the analysis of dynamics. In the present paper, we have assumed that these values N_H, A and so on are also periodic in time and spatially heterogeneous, which would be more in line with the transmission mechanism of dengue in reality. In brief, contrasting with model (1.5) in [31], our limiting system (6) possesses the more general forms. Secondly, the dynamical results in [31] involved two aspects only on either the condition $R_0 < 1$ or $R_0 > 1$ associating with other conditions. However, our discussions on system (6) have specified the relevant results on the case either $R_0 \leqslant 1$ or $R_0 > 1$ with no need for other additional conditions, except the fundamental assumptions (A1). Meanwhile, it should be added that we can further use the theory of chain transitive sets (see [28, Chap. 1]) to lift the global stability results on system (6) on model (3). That is, model (3) admits the same dynamical behavior as the ones in Theorem 3. Therefore, our conclusions are more complete than ones in [31].

Through the detailed analysis in the present paper and [31], we are fully aware of the profound influence of seasons changing on the transmission of dengue fever. In order to effectively control the virus, the human should make some preparations, such as mosquito eradication in large-scale and vaccine injection, before high-incidence season of dengue virus approach.

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