Applying the proportional hazard premium calculation principle

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Abstract

We discuss the application of the proportional hazard premium calculation principle in the parametric and non parametric framework.

In the parametric approach, we propose a method to calculate the premium of a compound risk when the severity distribution is subexponential.

In the non parametric approach, the use of the empirical distribution to calculate the premium using the proportional hazard principle leads to a systematic underestimation of the premium. After studying the bias of the premium calculated using this non-parametric approach we use the bootstrap technique with subsampling to reduce it.

Keywords: Proportional hazard premium principle; subexponential distributions; bootstrap; subsampling;

1   Introduction

As it is well known the proportional hazard premium principle (PH premium principle), introduced by Wang (1995), satisfies properties which make of it a very attractive premium from the theoretical point of view, see e.g. Wang (1996) and Andrade e Silva and Centeno (1998). However its use depends on the complete knowledge of the distribution of the aggregate claims amount. In practical terms this means that one must fit a distribution to the data set or use the empirical distribution function. When the aggregate claims amount follows a compound distribution the calculation of the PH premium based on the parametric approach can raise some problems. This will be discussed in the following section.

The non-parametric approach is more appealing, from the practical point of view, but can lead to a significative underestimation of the premium. In subsection 3.1 we calculate the bias of the premium based on the empirical distribution for some

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distributions, namely the exponential, Pareto and uniform. We also study the rate of convergency of the bias in the exponential case. In subsection 3.2 we show how to use the bootstrap technique with subsampling to reduce the bias of the estimator of the premium based on the empirical distribution. We also perform two simulation studies to give some insight to this technique.

2 Applying the PH transform to compound distributions in the parametric model

Let Y, the aggregate claims amount, be a nonnegative random variable, with distribution function $F(y)$. Let $S(y) = 1 - F(y)$ be the survival function. The PH premium principle assigns to the distribution $F(y)$ the premium

$$\pi_\rho = \int_0^\infty (S(y))^{1/\rho} dy, \tag{1}$$

where ρ, with $\rho \geq 1$, is the *risk aversion index*.

When using the collective risk model, Y is equal to $X_1 + ... + X_N$, where N is the number of claims occurred in a given period and $\{X_i\}_{i=1,2,...}$ are the individual claims amounts which are assumed to be nonnegative i.i.d. random variables and independent of N. In this case Y has a compound distribution and the company often estimates claim frequency and claim severity separately. Various numerical techniques are available to calculate the compound distribution, being Panjer's recursion formula the most well known. When using the recursion formula we will have to stop somewhere the calculations. When the individual claims amount has distribution G with unlimited support, i.e. $G(x) < 1$, for all $x > 0$, and it is very skewed, the choice of the value where to stop the calculations to obtain a reasonable estimate of the premium is a critical aspect of the parametric model. When X follows a heavy tail distribution $\int_t^\infty (S(y))^{1/\rho} dy$ can be of a significative size even for large values of t. In this case, when the probability generating function of N is analytic in the neighbourhood of 1 (which happens both for the Poisson and the negative binomial case) and G is subexponential then (see Embrechts *et al* (1997), pp. 45-46)

$$1 - F(x) \sim \mathrm{E}[N](1 - G(x)), \ x \longrightarrow \infty. \tag{2}$$

Hence for heavy tail severity distributions we will estimate the premium by

$$\pi_\rho^* = \int_0^t (S^*(y))^{1/\rho} dy + (\mathrm{E}[N])^{1/\rho} \int_t^\infty (1 - G(y))^{1/\rho} dy, \tag{3}$$

where $S^*(y)$ is the approximation to $S(y)$ obtained by Panjer's recursion formula, and t is a suitable high value.

Example 1 *Suppose that an actuary has arrived to the following estimates for a risk: the number of claims greater than a given observation point $d = \$100K$ is Poisson distributed with mean $\lambda_d = 6$ and the size of each claim greater than d is Pareto with*

parameter $\alpha = 1.647$. *Let us consider an excess of loss reinsurance treaty. For a layer* l *xs* m, *with* $m \geq d$ *and* $l \leq +\infty$, *the aggregate claims amount are compound Poisson with expected number of claims* $\lambda = (d/m)^\alpha \lambda_d$ *and severity distribution*

$$G(x) = \begin{cases} 1 - \left(\frac{m}{m+x}\right)^\alpha & \text{if } 0 \leq x < l \\ 1 & \text{if } x \geq l. \end{cases} \qquad (4)$$

Using $1/\rho = 0.9025$, Table 1 shows the PH transform premium, as percentage of the subject earned premium, $SEP = \$10,000K$, associate to the following layers:

1. $\$400K$ xs $\$100K$

2. $\$500K$ xs $\$500K$

3. $\$900K$ xs $\$100K$

4. the excess over $\$1,000K$

This example for the limited layers was considered by Wang (1998). For the calculation of the premium of the unlimited layer we have used (3) with $t = \$10^5K$, for which the difference between the aggregate survival function $S^*(t)$ and $\lambda(1-G(t))$, is 2.3×10^{-7}. The step used in the arithmetization of the severity distribution to perform the recursion was $h = \$1K$.

Table 1: PH- transform premiums

Layer l xs m	Pure Premium as percentage of SEP	π_ρ^* as percentage of SEP
$\$400K$ xs $\$100K$	6.000%	6.384%
$\$500K$ xs $\$500K$	1.183%	1.408%
$\$900K$ xs $\$100K$	7.183%	7.742%
xs $\$1,000K$	2.090%	3.388%

The reason to chose such a high value for t in formula (3) is related to the very small value of α. If instead of $\$10^5K$ we had used $\$10^4K$ for t, the result of applying formula (3) with $\rho = 1$ would be 2.086% of SEP, compared to a theoretical value of 2.090%. Using $t = \$10^5K$ the two terms in the right hand side of (3) are 1.9848% and 0.1055% for $\rho = 1$ and 3.0298% and 0.3580% for $1/\rho = 0.9025$. Note that the relative weight of the second term increases with ρ and is far from negligible in spite of the high values of t.

3 Applying the PH transform to the empirical distribution

3.1 The bias of the premium

Let $F_n(y)$ be the empirical distribution function of Y based on a random sample of size n, $(Y_1, ...Y_n)$, let $S_n(y)$ be the corresponding empirical survival function and

$$\widehat{\pi}_\rho = \int_0^\infty (S_n(y))^{1/\rho} dy, \tag{5}$$

the premium estimator. Although $S_n(y)$ is an unbiased estimator of $S(y)$, $\widehat{\pi}_\rho$ given by (5) is a biased estimator of π_ρ, since

$$E[\widehat{\pi}_\rho] = \int_0^\infty E\left[(S_n(y))^{1/\rho}\right] dy \le \int_0^\infty [E(S_n(y))]^{1/\rho} dy = \pi_\rho$$

by Jensen inequality, with the inequality being strict unless $\rho = 1$.

Let $Y_{k:n}$ be the k-th order statistic in a sample of size n. Then

$$S_n(y) = \begin{cases} 1 & y < Y_{1:n} \\ (n-k)/n, & Y_{k:n} \le y < Y_{k+1:n}, \ k = 1, ..., n-1 \\ 0 & y \ge Y_{n:n}, \end{cases}$$

which implies that

$$\widehat{\pi}_\rho = \sum_{k=0}^{n-1} \left(\frac{n-k}{n}\right)^{1/\rho} (Y_{k+1:n} - Y_{k:n}), \tag{6}$$

where $Y_{0:n} \equiv 0$.

Let $F_{k:n}(y)$ be the distribution function of the k-th order statistic, i.e. $F_{k:n}(y) = \Pr\{Y_{k:n} \le y\}$. Given that

$$F_{k:n}(y) = \sum_{r=k}^n \binom{n}{r} (F(y))^r (1 - F(y))^{n-r},$$

then

$$1 - F_{k:n}(y) = 1 - F_{k+1:n}(y) - \binom{n}{k} (F(y))^k (1 - F(y))^{n-k}$$

and integrating we obtain

$$E[Y_{k+1:n} - Y_{k:n}] = \int_0^\infty \binom{n}{k} (F(y))^k (1 - F(y))^{n-k} dy. \tag{7}$$

Then, using (6) and (7) we get

$$\begin{aligned} E[\widehat{\pi}_\rho] &= \sum_{k=0}^{n-1} \left(\tfrac{n-k}{n}\right)^{1/\rho} \binom{n}{k} \int_0^\infty (F(y))^k (S(y))^{n-k} dy \\ &= \sum_{k=1}^n \left(\tfrac{n-k+1}{n}\right)^{1/\rho} \binom{n}{k-1} \int_0^\infty (F(y))^{k-1} (S(y))^{n-k+1} dy \end{aligned} \tag{8}$$

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which is, when F is absolutely continuous for $x > 0$, equivalent to

$$E[\widehat{\pi}_\rho] = \sum_{k=1}^{n} \left(\frac{n-k+1}{n}\right)^{1/\rho} \binom{n}{k-1} \int_0^1 (1-x)^{k-1} x^{n-k+1} \frac{1}{f[S^{-1}(x)]} dx, \quad (9)$$

where $f(x) = F'(x)$.

Expressions (8) and (9) can be used to calculate the bias of $\widehat{\pi}_\rho$

$$B(\widehat{\pi}_\rho) = E[\widehat{\pi}_\rho] - \pi_\rho.$$

For some distributions (not in the compound case) the bias is easily calculated, as it is the case when Y is exponential, Pareto or uniform, as can be seen in the following examples.

Example 2 *Let Y be exponential distributed, i.e. $S(y) = \exp(-\theta y)$, $y > 0$ ($\theta > 0$). In this case $f[S^{-1}(x)] = \theta x$, and using (9) we get, after some calculations,*

$$E[\widehat{\pi}_\rho] = n^{-1/\rho}\theta^{-1} \sum_{k=1}^{n} k^{1/\rho-1}$$

and

$$B(\widehat{\pi}_\rho) = \left(n^{-1/\rho} \sum_{k=1}^{n} k^{1/\rho-1} - \rho\right) \theta^{-1}. \quad (10)$$

Example 3 *Let Y be Pareto distributed, i.e. $S(y) = \left(\frac{\beta}{\beta+y}\right)^\alpha$, $y > 0$ ($\alpha, \beta > 0$). In this case $f[S^{-1}(x)] = \alpha\beta^{-1}x^{1+1/\alpha}$, and using (9) we get,*

$$E[\widehat{\pi}_\rho] = \beta\alpha^{-1}n^{-1/\rho}n! \sum_{k=1}^{n} k^{1/\rho} \frac{\Gamma(k-1/\alpha)}{k!\Gamma(n+1-1/\alpha)}$$

and

$$B(\widehat{\pi}_\rho) = \beta \left[\frac{n!}{\alpha n^{1/\rho}} \sum_{k=1}^{n} k^{1/\rho} \frac{\Gamma(k-1/\alpha)}{k!\Gamma(n+1-1/\alpha)} - \frac{\rho}{\alpha-\rho}\right]. \quad (11)$$

Example 4 *When Y is uniformly distributed in (0,1) we get*

$$E[\widehat{\pi}_\rho] = \frac{1}{(n+1)n^{1/\rho}} \sum_{k=1}^{n} k^{1/\rho}$$

and

$$B(\widehat{\pi}_\rho) = \frac{1}{(n+1)n^{1/\rho}} \sum_{k=1}^{n} k^{1/\rho} - \frac{\rho}{\rho+1}.$$

For the exponential distribution it is possible to relate the order of the bias with the sample size, as stated in the result that follows. We were not able of developing similar results for other distributions, but we are strongly convinced that the order of the bias is smaller for the uniform and bigger for the Pareto.

Result. *When Y follows an exponential distribution $|B(\widehat{\pi}_\rho)|$ converges to zero at the same rate as $n^{-1/\rho}$.*

Proof. Noticing that

$$\rho = n^{-1/\rho} \sum_{k=1}^{n} \int_0^1 (k-x)^{1/\rho-1} dx,$$

and using (10) we have

$$|B(\widehat{\pi}_\rho)| = \theta^{-1} n^{-1/\rho} \sum_{k=1}^{n} \int_0^1 \left[(k-x)^{1/\rho-1} - k^{1/\rho-1} \right] dx.$$

As $(k-x)^{1/\rho-1}$ is an increasing function of x for $k \geq 1$ and $0 < x < 1$, we have that

$$\sum_{k=1}^{n} \int_0^1 \left[(k-x)^{1/\rho-1} - k^{1/\rho-1} \right] dx \leq \int_0^1 \left[(1-x)^{1/\rho-1} - 1 \right] dx \qquad (12)$$

$$+ \sum_{k=2}^{n} \left[(k-1)^{1/\rho-1} - k^{1/\rho-1} \right] = \rho - n^{1/\rho-1} \leq \rho.$$

On the other hand as $\int_0^1 \left[(k-x)^{1/\rho-1} - k^{1/\rho-1} \right] dx$ is strictly positive we can conclude that left hand side of (12) converges to a positive value A as n goes to infinity. Consequently

$$\lim_{n \longrightarrow \infty} |B(\widehat{\pi}_\rho)| = \theta^{-1} A \lim_{n \longrightarrow \infty} n^{-1/\rho}.$$

3.2 Correcting the bias via bootstrapping

As we have seen the distortion of the empirical distribution leads to an underestimation of the premiums. We use the bootstrap technique, see e.g. Efron and Tibshirani (1993), to correct, at least partially, the bias of that estimator.

The bootstrap technique consists, as it is well known, on the resampling of the original data set. The bootstrap estimator of the bias of $\widehat{\pi}_\rho$, is

$$\widehat{B(\widehat{\pi}_\rho)} = \widehat{\pi}_\rho^* - \widehat{\pi}_\rho$$

with

$$\widehat{\pi}_\rho^* = \frac{1}{M} \sum_{b=1}^{M} \widehat{\pi}_\rho^{*b}$$
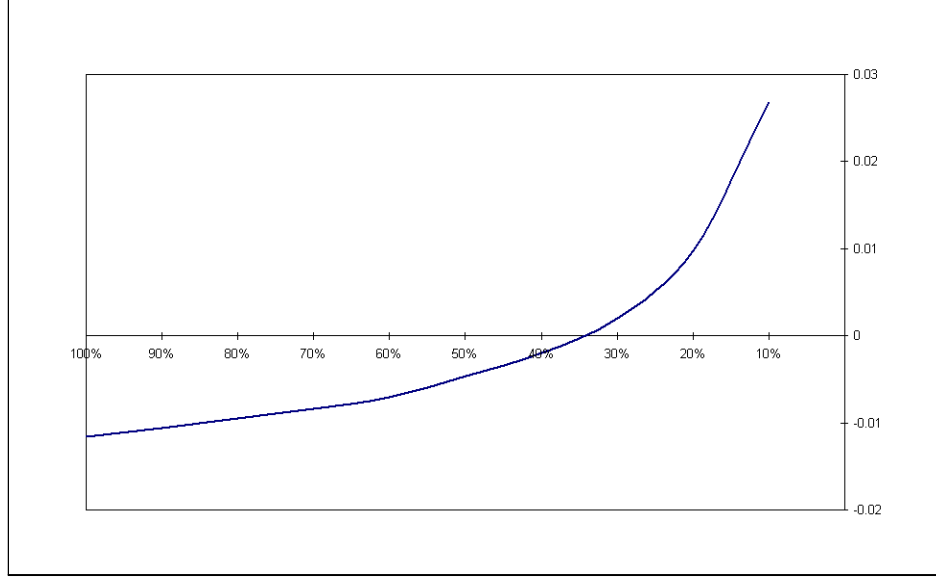
$$0.03$$
$$0.02$$
$$0.01$$
$$0$$

100% 90% 80% 70% 60% 50% 40% 30% 20% 10%

$$-0.01$$
$$-0.02$$

Figure 1: $\overline{B\left(\widehat{\widehat{\pi}}_\rho\right)}$ as function of n'/n

where M is the number of bootstrap samples and $\widehat{\pi}_\rho^{*b}$ is calculated using the b-th bootstrap sample. Hence the bootstrap estimator of the premium π_ρ is

$$\widehat{\widehat{\pi}}_\rho = \widehat{\pi}_\rho - \widehat{B(\widehat{\pi}_\rho)}. \tag{13}$$

Usually the resampling is made with replacement and using bootstrap samples of size n (i.e. the same size as the original sample). However in this case, as $B(\widehat{\pi}_\rho)$ is always negative, we can improve, in principle, the results by using bootstrap samples of size $n' < n$. Figure 1 shows how the bias of the corrected estimator given by (13) varies with the sample size n' (measured as a percentage of n). The figure is representative: it starts, when $n' = n$, with a negative value for the mean of the observed bias of the bootstrap premiums, which we denote $\overline{B\left(\widehat{\widehat{\pi}}_\rho\right)}$, than there is an optimal value of n' where $\overline{B\left(\widehat{\widehat{\pi}}_\rho\right)}$, is almost zero, and becomes positive when n' is very small. The question is how to choose n', independently of the distribution.

To give some insight to the problem we performed two simulation studies.

Simulation 1 This case is based on two families of distributions, the Pareto and the Gamma. In the Pareto case we have considered that the parameter α takes the value 2, 3 or 4, originating what we call Pareto$_2$, Pareto$_3$ and Pareto$_4$ respectively. The other parameter was chosen such that the distribution has mean equal to 1, i.e. we have assumed that the survival function for the Pareto$_\alpha$ is

$$S_\alpha(x) = \left(\frac{\alpha - 1}{x + \alpha - 1}\right)^\alpha, \quad x > 0.$$

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For the gamma distribution we have kept the same mean and assumed that the variance was equal to 1 (exponential) or 2. We have considered that the original sample size is either 100, 500 or 1000. Tables 2 and 3 give the bootstrap results for $\rho = 1.2$ and $\rho = 1.15$ respectively, with $M = 2000$ and using 2000 replicas.

As we have mentioned, the bias of the premium based on the empirical distribution (column labelled $B(\widehat{\pi}_\rho)$, and calculated using expressions (8), (10) and (11), for the gamma, exponential and Pareto distributions) can be of a considerable size, namely for heavy tail distributions when n is small and ρ is high. For instance for the Pareto$_2$, with $n = 100$ and $\rho = 1.2$, the absolute value of the bias is 8.4% of the theoretical premium. The average bias of the premiums based on the 2000 replicas (before the bootstrap is performed), labelled $\overline{B(\widehat{\pi}_\rho)}$, is in those cases still a bit far from $B(\widehat{\pi}_\rho)$, but it was out of our computing facilities to consider more replicas in the simulation study. The following 10 columns show the average bias for the bootstrap premiums. The bootstrap with $n' = n$, only corrects the bias partially, since after that correction we still observe a negative bias in all the situations of our example. As we can see from Tables 2 and 3, the pattern shown in Figure 1 is characteristic of the behaviour of the bias as function of n'/n. The optimal proportion n'/n depends, of course, on the distribution, on n and on ρ. As a rule n'/n should increase with n and decrease with ρ. For instance, for $\rho = 1.2$, a bootstrap with a resampling of $n'/n = 40\%$ performs, in average, better than the bootstrap with full resampling (and certainly better than with no bootstrap), at least for the sample sizes considered in the study.

The last column of the tables, provides the average of the premiums if we have used the maximum likelihood estimators of the parameters. For the Pareto$_2$, the figures are not presented because for a significative number of samples (20 for $\rho = 1.2$ and 11 for $\rho = 1.15$) we got a maximum likelihood estimate $\widehat{\alpha}$ smaller than ρ. In some other situations the estimate was very close to ρ, what would imply an extremely high premium. For the Pareto$_3$ and Pareto$_4$ we didn't get any sample with a maximum likelihood estimate smaller than ρ, but we got values not very far from ρ, which explain the positive values of the bias on those cases. The maximum likelihood performs well (in association with the PH premium calculation principle) if the distribution does not have a very heavy tail, or if the sample size is very big.

As a conclusion, and for values of ρ around 1.15 -1.2 we can say that use of the bootstrap technique with a resampling proportion size of 40% of the original sample size provides good results in general, but when we know that we are dealing with heavy tail loss distributions, we could use a smaller resampling size.

Simulation 2 The goal of the second simulation study is to analyse the behaviour of our procedures when the aggregate claims amount are generated according to a compound Poisson distribution. For the individual claims amount we have considered the three Pareto's and the exponential distributions of the former example. The expected number of claims was set at 1. As in the previous simulation study $\rho = 1.2$ and $\rho = 1.15$, $M = 2000$, we use 2000 replicas and n is equal to 100, 500 or 1000.

Tables 4 and 5 are similar to tables 2 and 3, with the exceptions of $B(\widehat{\pi}_\rho)$ and ML, which are not considered now. Column labelled π_ρ^* was obtained using (3), with t equal to 10000, 926 and 297 for α equal to 2, 3 and 4 respectively, values for which

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the $\Pr\{X > t\} = 10^{-8}$. For the exponential case we only considered the first term of (3), with $t = 9000$.

As expected with $\lambda = 1$ the premiums associated to the compound distribution are greater than for the corresponding Pareto's.

The results are very similar to the former case. We obtained the same behaviour for the bias as a function of n'/n, and again we would recommend a resampling of $n'/n = 40\%$, but if we knew that we were dealing with heavy tail loss distributions, we could use a smaller resampling size.

Acknowledgments

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Table 2: Bootstrap Results, $\rho = 1.2$

n	Distribution	π_ρ	$B(\widehat{\pi}_\rho)$	$\overline{B(\widehat{\pi}_\rho)}$	$\overline{B\left(\widehat{\overline{\pi}}_\rho\right)}$ for n'/n equal to										ML
					100%	90%	80%	70%	60%	50%	40%	30%	20%	10%	
	Pareto$_2$	1.5000	-0.1261	-0.1388	-0.1134	-0.1108	-0.1075	-0.1033	-0.0980	-0.0913	-0.0822	-0.0685	-0.0463	0.0004	—
	Pareto$_3$	1.3333	-0.0458	-0.0508	-0.0361	-0.0345	-0.0325	-0.0300	-0.0268	-0.0227	-0.0168	-0.0080	0.0069	0.0403	0.0492
100	Pareto$_4$	1.2857	-0.0296	-0.0330	-0.0217	-0.0204	-0.0188	-0.0169	-0.0143	-0.0110	-0.0063	0.0008	0.0132	0.0418	0.0161
	Gamma	1.2865	-0.0170	-0.0166	-0.0073	-0.0063	-0.0050	-0.0033	-0.0012	0.0018	0.0060	0.0126	0.0244	0.0529	-0.0012
	Exponential	1.2000	-0.0098	-0.0112	-0.0056	-0.0049	-0.0041	-0.0031	-0.0018	0.0000	0.0025	0.0066	0.0139	0.0324	-0.0013
	Pareto$_2$	1.5000	-0.0743	-0.0838	-0.0687	-0.0670	-0.0650	-0.0625	-0.0593	-0.0552	-0.0497	-0.0416	-0.0279	0.0011	0.0452
	Pareto$_3$	1.3333	-0.0209	-0.0238	-0.0168	-0.0160	-0.0151	-0.0139	-0.0124	-0.0103	-0.0075	-0.0034	0.0040	0.0208	0.0069
500	Pareto$_4$	1.2857	-0.0120	-0.0139	-0.0091	-0.0086	-0.0079	-0.0070	-0.0060	-0.0045	-0.0025	0.0006	0.0060	0.0190	0.0021
	Gamma	1.2865	-0.0051	-0.0059	-0.0029	-0.0025	-0.0021	-0.0015	-0.0008	0.0002	0.0016	0.0039	0.0080	0.0188	-0.0014
	Exponencial	1.2000	-0.0029	-0.0039	-0.0021	-0.0019	-0.0017	-0.0013	-0.0009	-0.0003	0.0005	0.0018	0.0042	0.0105	-0.0010
	Pareto$_2$	1.5000	-0.0590	-0.0602	-0.0482	-0.0468	-0.0452	-0.0432	-0.0406	-0.0372	-0.0327	-0.0259	-0.0150	0.0089	0.0224
	Pareto$_3$	1.3333	-0.0149	-0.0148	-0.0099	-0.0093	-0.0086	-0.0077	-0.0066	-0.0051	-0.0031	0.0000	0.0053	0.0177	0.0041
1000	Pareto$_4$	1.2857	-0.0081	-0.0081	-0.0049	-0.0045	-0.0041	-0.0035	-0.0028	-0.0018	-0.0004	0.0018	0.0055	0.0146	0.0015
	Gamma	1.2865	-0.0030	-0.0039	-0.0021	-0.0019	-0.0016	-0.0013	-0.0009	-0.0003	0.0006	0.0020	0.0046	0.0111	-0.0015
	Exponential	1.2000	-0.0017	-0.0021	-0.0011	-0.0010	-0.0008	-0.0006	-0.0004	0.0000	0.0005	0.0013	0.0027	0.0066	-0.0006

Table 3: Bootstrap Results, $\rho = 1.15$

n	Distribution	π_ρ	$B(\widehat{\pi}_\rho)$	$\overline{B(\widehat{\pi}_\rho)}$	$B\left(\overline{\widehat{\widehat{\pi}}_\rho}\right)$ for n'/n equal to										ML
					100%	90%	80%	70%	60%	50%	40%	30%	20%	10%	
	Pareto$_2$	1.3529	-0.0771	-0.0881	-0.0705	-0.0687	-0.0664	-0.0635	-0.0598	-0.0551	-0.0487	-0.0391	-0.0232	0.0106	—-
	Pareto$_3$	1.2432	-0.0293	-0.0337	-0.0235	-0.0223	-0.0209	-0.0192	-0.0169	-0.0140	-0.0099	-0.0036	0.0071	0.0314	0.0337
100	Pareto$_4$	1.2105	-0.0193	-0.0223	-0.0143	-0.0134	-0.0123	-0.0110	-0.0092	-0.0068	-0.0035	0.0016	0.0105	0.0314	0.0121
	Gamma	1.2137	-0.0115	-0.0112	-0.0046	-0.0038	-0.0029	-0.0018	-0.0002	0.0019	0.0049	0.0097	0.0182	0.0390	-0.0009
	Exponential	1.1500	-0.0067	-0.0080	-0.0039	-0.0035	-0.0029	-0.0022	-0.0012	0.0000	0.0019	0.0048	0.0102	0.0238	-0.0012
	Pareto$_2$	1.3529	-0.0429	-0.0507	-0.0408	-0.0398	-0.0384	-0.0368	-0.0347	-0.0319	-0.0283	-0.0229	-0.0137	0.0062	0.0302
	Pareto$_3$	1.2432	-0.0127	-0.0152	-0.0105	-0.0100	-0.0094	-0.0086	-0.0076	-0.0062	-0.0043	-0.0015	0.0035	0.0151	0.0050
500	Pareto$_4$	1.2105	-0.0074	-0.0091	-0.0059	-0.0055	-0.0051	-0.0045	-0.0038	-0.0028	-0.0015	0.0006	0.0043	0.0134	0.0014
	Gamma	1.2137	-0.0033	-0.0042	-0.0022	-0.0019	-0.0016	-0.0012	-0.0007	-0.0001	0.0009	0.0025	0.0054	0.0130	-0.0013
	Exponential	1.1500	-0.0019	-0.0028	-0.0016	-0.0015	-0.0013	-0.0011	-0.0008	-0.0004	0.0002	0.0011	0.0027	0.0072	-0.0010
	Pareto$_2$	1.3529	-0.0333	-0.0342	-0.0266	-0.0257	-0.0247	-0.0234	-0.0217	-0.0195	-0.0166	-0.0122	-0.0050	0.0110	0.0155
	Pareto$_3$	1.2432	-0.0088	-0.0088	-0.0056	-0.0053	-0.0048	-0.0043	-0.0035	-0.0026	-0.0012	0.0009	0.0044	0.0127	0.0031
1000	Pareto$_4$	1.2105	-0.0049	-0.0050	-0.0029	-0.0027	-0.0024	-0.0020	-0.0015	-0.0009	0.0001	0.0015	0.0040	0.0102	0.0011
	Gamma	1.2137	-0.0019	-0.0030	-0.0017	-0.0016	-0.0014	-0.0012	-0.0009	-0.0005	0.0001	0.0010	0.0028	0.0074	-0.0014
	Exponential	1.1500	-0.0011	-0.0015	-0.0009	-0.0008	-0.0007	-0.0006	-0.0004	-0.0001	0.0002	0.0007	0.0017	0.0044	-0.0006

Table 4: Bootstrap Results for Compound Distributions, $\rho = 1.2$

n	Severity Distribution	π_ρ^*	$\overline{B(\widehat{\pi}_\rho)}$	$\overline{B\left(\widehat{\widehat{\pi}}_\rho\right)}$ for n'/n equal to									
				100%	90%	80%	70%	60%	50%	40%	30%	20%	10%
	Pareto$_2$	1.5498	-0.1493	-0.1236	-0.1207	-0.1172	-0.1129	-0.1076	-0.1006	-0.0910	-0.0763	-0.0527	-0.0021
	Pareto$_3$	1.3939	-0.0589	-0.0433	-0.0414	-0.0393	-0.0366	-0.0332	-0.0286	-0.0222	-0.0122	0.0047	0.0438
100	Pareto$_4$	1.3515	-0.0401	-0.0276	-0.0261	-0.0244	-0.0222	-0.0194	-0.0156	-0.0103	-0.0018	0.0128	0.0477
	Exponential	1.2822	-0.0180	-0.0103	-0.0093	-0.0082	-0.0068	-0.0051	-0.0026	0.0010	0.0068	0.0171	0.0436
	Pareto$_2$	1.5498	-0.0817	-0.0661	-0.0644	-0.0622	-0.0597	-0.0564	-0.0522	-0.0465	-0.0381	-0.0238	0.0065
	Pareto$_3$	1.3939	-0.0243	-0.0171	-0.0163	-0.0153	-0.0140	-0.0125	-0.0103	-0.0074	-0.0029	0.0051	0.0235
500	Pareto$_4$	1.3515	-0.0145	-0.0094	-0.0088	-0.0081	-0.0072	-0.0061	-0.0045	-0.0023	0.0011	0.0072	0.0220
	Exponencial	1.2822	-0.0049	-0.0025	-0.0022	-0.0019	-0.0014	-0.0009	-0.0001	0.0011	0.0029	0.0064	0.0153
	Pareto$_2$	1.5498	-0.0643	-0.0517	-0.0503	-0.0486	-0.0466	-0.0438	-0.0405	-0.0356	-0.0287	-0.0174	0.0070
	Pareto$_3$	1.3939	-0.0180	-0.0128	-0.0122	-0.0115	-0.0106	-0.0094	-0.0079	-0.0057	-0.0025	0.0031	0.0161
1000	Pareto$_4$	1.3515	-0.0106	-0.0071	-0.0067	-0.0062	-0.0056	-0.0048	-0.0037	-0.0022	0.0001	0.0041	0.0140
	Exponential	1.2822	-0.0034	-0.0019	-0.0018	-0.0016	-0.0013	-0.0010	-0.0005	0.0002	0.0012	0.0033	0.0086

Table 5: Bootstrap Results for Compound Distributions, $\rho = 1.15$

n	Severity Distribution	π_ρ^*	$\overline{B(\widehat{\pi}_\rho)}$	$\overline{B\left(\widehat{\widehat{\pi}}_\rho\right)}$ for n'/n equal to									
				100%	90%	80%	70%	60%	50%	40%	30%	20%	10%
	Pareto$_2$	1.3903	-0.0969	-0.0791	-0.0771	-0.0747	-0.0717	-0.0680	-0.0631	-0.0564	-0.0459	-0.0291	0.0074
	Pareto$_3$	1.2887	-0.0405	-0.0296	-0.0283	-0.0268	-0.0249	-0.0225	-0.0193	-0.0148	-0.0076	0.0045	0.0329
100	Pareto$_4$	1.2599	-0.0283	-0.0194	-0.0184	-0.0171	-0.0156	-0.0136	-0.0110	-0.0072	-0.0010	0.0094	0.0349
	Exponential	1.2115	-0.0133	-0.0077	-0.0071	-0.0063	-0.0053	-0.0040	-0.0022	0.0003	0.0045	0.0120	0.0315
	Pareto$_2$	1.3903	-0.0491	-0.0389	-0.0378	-0.0364	-0.0347	-0.0326	-0.0298	-0.0260	-0.0203	-0.0107	0.0101
	Pareto$_3$	1.2887	-0.0156	-0.0108	-0.0103	-0.0097	-0.0088	-0.0078	-0.0063	-0.0043	-0.0013	0.0042	0.0170
500	Pareto$_4$	1.2599	-0.0097	-0.0062	-0.0059	-0.0054	-0.0048	-0.0040	-0.0029	-0.0014	0.0009	0.0052	0.0154
	Exponencial	1.2115	-0.0036	-0.0019	-0.0017	-0.0015	-0.0012	-0.0008	-0.0003	0.0005	0.0018	0.0043	0.0106
	Pareto$_2$	1.3903	-0.0379	-0.0298	-0.0290	-0.0279	-0.0266	-0.0248	-0.0226	-0.0195	-0.0149	-0.0075	0.0089
	Pareto$_3$	1.2887	-0.0116	-0.0082	-0.0078	-0.0074	-0.0067	-0.0060	-0.0050	-0.0036	-0.0014	0.0023	0.0112
1000	Pareto$_4$	1.2599	-0.0071	-0.0048	-0.0046	-0.0042	-0.0038	-0.0033	-0.0026	-0.0016	-0.0001	0.0027	0.0094
	Exponential	1.2115	-0.0025	-0.0015	-0.0014	-0.0012	-0.0010	-0.0008	-0.0005	-0.0001	0.0007	0.0021	0.0058