



How is new information capitalized in asset values? The role of kurtosis

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Abstract

Purpose – The purpose of this paper is to further understanding of how new information impacts the market value of financial assets.

Design/methodology/approach – The paper uses a Bayesian approach to asset valuation, whereby investors use signals conveyed by new information to update their estimate of a structural valuation parameter. The underlying distributions – i.e. the distribution of the information signal and the prior distribution of the valuation parameter – are allowed to exhibit a degree of kurtosis greater than that of the normal distribution.

Findings – The revision in asset value as a function of the realization of the information signal is an S-shaped function (in the local region centred on the zero-surprise level of the signal), if the distribution of the information signal features excess kurtosis; conversely, if the prior of the valuation parameter features excess kurtosis, the revision in asset value is an inverted S-shaped function.

Research limitations/implications – The paper generates clear implications with respect to the shape of the function relating the revision in asset value to the realization of the signal only in the local region centred on the zero-surprise level of the signal.

Practical implications – The paper helps to shed light on the well-known empirical result that the stock price reaction to earnings' announcements is an S-shaped function, centred on the zero-surprise level of reported earnings.

Originality/value – In the financial accounting literature, the paper helps one to understand the role of the distributional assumptions underlying the stock price reaction to earnings' announcements, namely, the role of excess kurtosis both in reported earnings and in the prior of means earnings.

Keywords Stock prices, Case studies, Financial information, Assets valuation

Paper type Research paper

I. Introduction

In many applications in economics and finance researchers seek to estimate the impact of new information on asset values. Using an event study methodology, this is achieved by measuring the change in asset value around the time of occurrence of the information event – i.e. during the so-called event window. When the information event is the realization of a quantitative variable, the standard approach is to regress the change in asset value during the “event window” against the unexpected component of the information variable. The estimated regression coefficient is then interpreted as a “signal response coefficient”, i.e. the impact on asset value “per unit” of the information signal[1].

Underlying this approach is the ancillary hypothesis that the marginal response of asset values to new information is constant. In a context of Bayesian learning,



however, strong distributional assumptions are required for such hypothesis to hold. To understand this, consider a setting in which the value of an asset is a multiple of a market-based estimate of an unknown structural parameter, which is updated as new information is publicly revealed (see, for example, Grossman (1976) or Stapleton and Subrahmanyam (1978)). If the updating process takes place within the normal conjugate family – i.e. if the prior distribution of the unknown structural parameter as well as the distribution of the information signal are normal – then the revision in asset value following the information event is a linear function of the information signal (see, for example, Holthausen and Verrecchia (1988)). Any departure from normality in the distributions surrounding the learning process, however, suffices to invalidate this result.

The standard event-study methodology therefore rests on narrow foundations. Given that such methodology is the primary tool to measure the impact of news on asset prices, it is important to examine it under more general distributional assumptions. In recent years, a number of authors have examined extensions of the standard Gaussian model of Bayesian learning in different settings. Some have explored the role of signal precision and the quality of new information in accounting for the observed variability in the reaction of asset prices' to news across different market environments (Veronesi, 2000; Hautsch and Hess, 2004). Others have considered settings in which either the prior distribution held by investors over state variables is not normal or state variables do not follow Gaussian diffusion processes, in an effort to explain time-shifting volatility of asset returns and time-shifting risk premia (Detemple, 1991; David, 1997; Veronesi, 1999). In the financial accounting field, an extensive body of evidence documenting a non-linear response of stock values to earnings' surprises (see, for example, Freeman and Tse, 1992; Das and Lev, 1994; Lipe *et al.*, 1998) has prompted some to explore settings in which the precision of the signal conveyed by earnings is unknown (Subrahmanyam, 1996).

Our concern in this paper is the shape of the function relating the price response to the magnitude of the signal (i.e. the signal response function of SRF for short). Specifically, we want to know the impact of prior and signal distributions with fat tails on the shape of the SRF. The ideas are developed using an asset valuation model in which the value of a share is a multiple of the current estimate of mean periodic earnings. Share prices respond to the disclosure of an earnings reports as investors use the earnings signal conveyed in the report to update their estimate of mean periodic earnings.

Going into the specifics, we present two different departures from the linear model, each of them introducing fat tail behavior in a particular fashion. The first approach considers a fat tail distribution for the earnings signal while assuming normality for the prior uncertainty about mean earnings. Conversely, the second approach assumes a fat tail for the prior distribution and normality for the earnings signal. In either case, fat tails are generated by mixing normal distributions with the same mean but with different variances[2]. We find that fat-tail behavior imparts a particular pattern of non-linearity to the SRF, in the local region centered around the zero-surprise level of earnings[3]. When only the earnings signal is fat-tailed, the SRF exhibits an S-shape; in contrast, when only the prior distribution is fat-tailed, the SRF has an inverted S-shape[4].

What may cause fat-tailed behavior or excess kurtosis? One obtains a signal with a fat-tailed distribution if the precision of the signal is unknown. In the context of our illustrative setting, we consider a set up in which firms generate transitory earnings sporadically and unexpectedly over time – in addition to permanent earnings – causing investors to bear uncertainty about the presence of transitory earnings in current reported earnings. This is achieved by assuming that earnings are sampled from an unknown mixture of two normal distributions with different variances, yielding an earnings signal with fat tails. Since the presence of transitory earnings in current earnings tends to amplify deviations from expected earnings, big surprises in the earnings' signal yield a smaller impact on the share price – per unit of surprise – than small surprises.

A prior distribution will display fat tails, on the other hand, if there is uncertainty *vis-à-vis* the degree of homogeneity of the population from which the unknown structural parameter is drawn from. In our particular setting, we generate a prior with fat tails by adopting a framework in which there exist two types of firms, each associated with a different degree of uncertainty surrounding mean earnings. When reacting to an earnings announcement, investors bear uncertainty about which firm type they are dealing with, and use the earnings signal disclosed by the firm to help resolve such uncertainty. We capture this set up by assuming that the prior distribution of mean earnings is a mixture of two normal distributions with different variances. Such an assumption yields a prior with fat tails[5].

The paper that is closer to ours is Subramanyan (1996). He shows that when investors are uncertain about the precision of earnings signals, they view large earnings surprises as less accurate signals of firm value. This entails a non-linear SRF with a symmetric S-shape. The model of Subramanyan (1996) can be viewed as a particular case of our setting since it assumes a specific distribution for the precision of the signal whereas we allow for a wide range of distributions. Furthermore, the scope of our analysis is broader since it also encompasses fat-tail behavior for the prior distribution.

II. General set-up

Let V^F denote the fundamental value of the firm's equity. Assume that V^F can be written as:

$$V^F = \lambda E(e) \quad (1)$$

where λ is a constant, e is a random variable representing the firm's periodic economic earnings and E is the expectation operator. Periodic earnings, e , are iid with a probability density function $f(e)$. The market is uncertain about the parameters of the pdf $f(e)$ and uses the earnings information disclosed by management to develop more precise estimates. One parameter of special interest is the mean of the distribution of earnings – which we represent as μ – since the market value of the equity is a multiple of the current estimate of such parameter.

The linear model is obtained by assuming that earnings are sampled from a normal distribution with an unknown mean (but known variance), whose prior is described by a normal distribution (see, for example, Holthausen and Verrecchia, 1988). As it is well known, these assumptions guarantee that the posterior distribution of the mean will

also be normal, so that the updating process of model parameters takes place within the normal conjugate family.

Specifically, assume that $e \sim N(\mu, \sigma_e^2)$ and that the market's prior distribution of μ is a normal with mean μ_0 and variance σ_0^2 (the variance σ_e^2 is known by the market). If we let V_0 represent the market value of the equity before the earnings information is publicly disclosed, then V_0 is equal to:

$$V_0 = E(V^F) = \lambda\mu_0. \quad (2)$$

The manager observes the earnings signal, e , and then discloses it to the market. Denote by $V_1(e)$ the market value of the equity immediately after the earnings signal is publicly disclosed – referred to as the Signal Response Function or SRF for short – and denote by $\mu(e)$ the mean of the posterior distribution of μ . Under the stated assumptions, $V_1(e)$ is the product of λ by the weighted average of the pre-signal expected value of the earnings, μ_0 , and the new earnings information, e , where the weights are determined by the relative precisions of the two information components:

$$\begin{aligned} V_1(e) &= E(V^F|e) = \lambda\mu(e) = \lambda(\mu_0(\sigma_e^2/(\sigma_0^2 + \sigma_e^2)) + e(\sigma_0^2/(\sigma_0^2 + \sigma_e^2))) \\ &= \lambda(\mu_0(1 - \beta) + e\beta) \\ &= V_0 + \lambda\beta(e - \mu_0) \end{aligned} \quad (3)$$

where:

$$\beta = \sigma_0^2/(\sigma_0^2 + \sigma_e^2) \quad (4)$$

is the earnings response coefficient.

We can see that the SRF – i.e. $V_1(e)$ – is linear in e . Although the linear model is simple and appealing, it makes strong assumptions *vis-à-vis* the underlying distributions. In the rest of this paper, we examine how non-linear patterns arise from distributions with fat-tails for the earnings signal and the prior of mean earnings.

III. Non-linear signal response functions

III.1 Distribution of earnings with fat tails

In this section, we consider a set-up in which the signal has an unknown precision, as a way to generate excess kurtosis. Suppose that firms generate two earnings streams: one stream – labeled permanent earnings – is generated by ongoing and recurring business activities; a second stream –labeled transitory earnings – is generated by one-off activities. Permanent earnings are distributed normally with an unknown mean μ and a known variance σ_{eP}^2 ; the prior distribution of μ is normal with mean μ_0 and variance σ_0^2 . Transitory earnings are distributed normally with mean zero and a known variance equal to σ_{eT}^2 . The two processes are uncorrelated. At the end of each reporting period, firms disclose a global earnings figure corresponding to the joint amount of permanent and transitory earnings generated over the period.

Additionally, suppose that firms generate permanent earnings every single period but generate transitory earnings only sporadically. Reported earnings are then equal to either the sum of a draw from the permanent earnings process plus a draw from the transitory earnings process or just a draw from the permanent earnings process. When

reacting to an earnings announcement, investors do not know the generating process behind the earnings figure being reported, i.e. they do not know whether reported earnings consist exclusively of permanent earnings or it includes transitory earnings as well. They use the level of reported earnings plus what they know about the properties of the two processes, to infer the generating process behind current reported earnings. For instances, an extreme earnings surprise supports the belief that the earnings process behind reported earnings encompasses a draw from transitory earnings. Since transitory earnings increase the noise of the earnings signal, such belief leads investors to downplay the information conveyed by extreme reports.

To formalize the intuition, assume that the firm's periodic reported earnings are generated by the process:

$$e = Ie_{HV} + (1 - I)e_{LV} \quad (5)$$

where I is a Bernoulli random variable that takes a value equal to 1 with probability p and 0 with probability $1 - p$, e_{HV} is a normally distributed random variable with mean μ and variance $\sigma_{eH}^2 = \sigma_{eP}^2 + \sigma_{eT}^2$ and e_{LV} is a normally distributed random variable with mean μ and variance $\sigma_{eL}^2 = \sigma_{eP}^2$. The three random variables are mutually independent. The random variable e_{LV} represents the permanent earnings process, whereas variable e_{HV} represents the sum of permanent and transitory earnings. Henceforth, we refer to e_{HV} and e_{LV} as, respectively, the high-variance and the low-variance earnings process. Hence, we have a set up in which the earnings signal conveyed by the firm is a mixture of two normal earnings processes, with identical means but with different variances.

Under the stated assumptions, the pdf of the earnings' signal is equal to:

$$\begin{aligned} f(e) = & p(\sigma_0^2 = \sigma_{eH}^2)^{-1/2} \phi\left(\frac{e - \mu}{(\sigma_0^2 + \sigma_{eH}^2)^{1/2}}\right) + (1 - p) \\ & \times (\sigma_0^2 + \sigma_{eL}^2)^{-1/2} \phi\left(\frac{e - \mu}{(\sigma_0^2 + \sigma_{eL}^2)^{1/2}}\right) \end{aligned} \quad (6)$$

where ϕ stands for the normal standardized pdf. The pdf (6) is a distribution with fat tails *vis-à-vis* the normal distribution; indeed, while the kurtosis of the normal distribution is equal to 3, the kurtosis of the pdf (6) is equal to:

$$\frac{3(p(\sigma_0^2 + \sigma_{eH}^2)^2 + (1 - p)(\sigma_0^2 + \sigma_{eL}^2)^2)}{(p(\sigma_0^2 + \sigma_{eH}^2) + (1 - p)(\sigma_0^2 + \sigma_{eL}^2))^2} > 3. \quad (7)$$

We may now proceed to characterize the SRF. Conditional on process type (i.e. on whether reported earnings have been sampled from the low-variance process or from the high-variance process), the mean of the posterior distribution of μ is equal to the weighted average of the prior mean and the earnings signal, i.e.:

$$V_1(e|I = i) = E(V^F | e, I = i) = \lambda(\mu_0(\sigma_{ei}^2/(\sigma_0^2 + \sigma_{ei}^2)) + e(\sigma_0^2/(\sigma_0^2 + \sigma_{ei}^2))) \quad (8)$$

where I is the Bernoulli random variable which takes value $i = 1$ for the high variance process and value $i = 0$ for the low variance process.

To find the unconditional value of the firm following the earnings disclosure, one simply needs to integrate (8) over process types, using the updated probabilities of process type. If we denote $p(e) = \Pr(I = 1 | e)$ as the probability of earnings having been drawn from the high-variance process given the earnings signal, e , then the unconditional value of the firm following the earnings disclosure is equal to:

$$V_1(e) = E(V^F | e) = \lambda p(e) (\mu_0 (\sigma_{eH}^2 / (\sigma_0^2 + \sigma_{eH}^2)) + e (\sigma_0^2 / (\sigma_0^2 + \sigma_{eH}^2))) + \lambda (1 - p(e)) (\mu_0 (\sigma_{eL}^2 / (\sigma_0^2 + \sigma_{eL}^2)) + e (\sigma_0^2 / (\sigma_0^2 + \sigma_{eL}^2))). \quad (9)$$

Since the *ex-ante* probability of the high-variance process is p , then:

$$p(e) = \frac{p (\sigma_0^2 + \sigma_{eH}^2)^{-1/2} \phi \left(\frac{e - \mu_0}{(\sigma_0^2 + \sigma_{eH}^2)^{1/2}} \right)}{p (\sigma_0^2 + \sigma_{eH}^2)^{-1/2} \phi \left(\frac{e - \mu_0}{(\sigma_0^2 + \sigma_{eH}^2)^{1/2}} \right) + (1 - p) (\sigma_0^2 + \sigma_{eL}^2)^{-1/2} \phi \left(\frac{e - \mu_0}{(\sigma_0^2 + \sigma_{eL}^2)^{1/2}} \right)}. \quad (10)$$

Substituting (10) into (9) yields an SRF exhibiting an S-shape in the local region around the zero earnings -surprise level. This result is generalized in the following proposition.

PI

Suppose that the sampling distribution of the earnings signal observed by investors, e , is a fat-tailed distribution generated by a mixture of N normal distributions with identical means but with different variances. Investors know the variances of the N distributions but do not know their common mean. Moreover, the prior distribution of the mean is normal with known parameters. Then, locally around the zero-surprise level of earnings the SRF is monotonically increasing, symmetric and with an S-shape.

Proof in Appendix 1.

We conclude that a signal with a fat-tail distribution arising from uncertainty about signal precision, yields an SRF with an S-shape. Subramanyan (1996) examines the case where reported earnings are drawn from a normal distribution with an unknown mean and an unknown variance, with priors equal to, respectively, a normal distribution and a gamma distribution. Since our proposition is valid for mixtures of arbitrarily large number of normal distributions – each with its own variance – we can approximate to any desired degree of accuracy the gamma distribution assumed by Subramanyan, or any other distribution for that matter. Thus our results are a generalization of Subramanyan (1996) to any distribution of the prior of the variance of the signal.

Regarding the empirical support for non-linear SRFs, Freeman and Tse (1992), Das and Lev (1994) and Lipe *et al.* (1998) have found that the marginal response of stock price to unexpected earnings depends on the level of the earnings surprise. A widely documented feature of the data is the observed S-shape departure from linearity when one plots unexpected earnings against the corresponding abnormal stock price reaction. According to *PI*, an earnings' signal with excess kurtosis is a possible explanation for such empirical feature.

III.2 Distribution of the prior of mean earnings with fat tails

This section examines the impact of fat-tail behavior in the prior distribution of the unknown structural parameter. As in the previous section, we create fat tails by mixing normal distributions with different variances.

Let reported earnings be sampled from a normal distribution with an unknown mean, μ , and known variance, σ_e^2 as in the linear model. Now suppose that the prior uncertainty about mean earnings, μ , is characterized, with a probability equal to p , by a Normal distribution with mean μ_0 and variance σ_{0H}^2 and, with probability equal to $1 - p$, by a Normal distribution with mean μ_0 and variance σ_{0L}^2 (where $\sigma_{0L}^2 < \sigma_{0H}^2$). The prior of μ can then be written as:

$$g(\mu) = p\sigma_{0H}^{-1}\phi((\mu - \mu_0)/\sigma_{0H}) + (1 - p)\sigma_{0L}^{-1}\phi((\mu - \mu_0)/\sigma_{0L}) \quad (11)$$

where ϕ , as in the previous section, represents the pdf of the standard Normal distribution. The kurtosis of the prior distribution is equal to:

$$\frac{3(p\sigma_{0H}^4 + (1 - p)\sigma_{0L}^4)}{(p\sigma_{0H}^2 + (1 - p)\sigma_{0L}^2)^2} > 3 \quad (12)$$

thus greater than that of the normal distribution[6].

We can motivate the assumption of a prior as a mixture of two normal distributions by arguing that there exist two types of firms, H and L , each sampled from a population with its own distribution of a structural valuation parameter – in this case, mean earnings. Type L would correspond to firms with homogeneous characteristics. Firms within this group would sell similar products, supply similar markets, use similar business models and employ similar technologies. Since there would be little differentiation among firms, the variation in mean earnings across firms would tend to be small (σ_{0L}^2). Type H , in turn, would correspond to firms with heterogeneous characteristics. Firms operating under a large degree of uncertainty surrounding environmental constraints such as customer preferences and technology would fall into this group. Within this group, firms would undergo intense innovation and experimentation *vis-à-vis* their business models, their products, their markets, their technologies and so on. This group should, therefore, exhibit a lot of variation in mean earnings (σ_{0H}^2), as the strategies adopted by its members would tend to generate extreme outcomes.

Over time firms might shift between the two types. That may happen because the competitive environment surrounding the firm changes, because the firm enters new markets (or it abandons existing ones) or yet because the firm changes its business strategy. The key point is that the underlying uncertainty about the structural parameter driving firm value is dynamic. Investors try to keep track of these shifts by constantly revising the probabilities of the firm falling into each type, as they observe, scrutinize and interpret the ongoing activities of the firm.

Let I represent a Bernoulli random variable, assuming value $i = 1$ when earnings are being reported by a firm of type H and $i = 0$ otherwise; the ex-ante probability of a type- H firm is p . Conditional on firm type (i.e. on whether earnings are being reported by a firm of type H or type L), the mean of the posterior distribution of μ is equal to the weighted average of the prior mean and the earnings signal:

$$V_1(e|I = i) = E(V^F|e, I = i) = \lambda(\mu_0(\sigma_e^2/(\sigma_{0i}^2 + \sigma_e^2)) + e(\sigma_{0i}^2/(\sigma_{0i}^2 + \sigma_e^2))) \quad (13)$$

To find the unconditional value of the firm following the earnings disclosure, one needs to integrate (13) over firm types, using the updated probabilities of firm type. Let $p(e) = \Pr(I = 1 | e)$ denote the probability of earnings being reported by a firm of type H given the earnings signal, e . Then, the unconditional value of the firm following the earnings disclosure is equal to:

$$V_1(e) = E(V^F | e) = \lambda p(e) (\mu_0 (\sigma_e^2 / (\sigma_{0H}^2 + \sigma_e^2)) + e (\sigma_{0H}^2 / (\sigma_{0H}^2 + \sigma_e^2))) + \lambda (1 - p(e)) (\mu_0 (\sigma_e^2 / (\sigma_{0H}^2 + \sigma_e^2)) + e (\sigma_{0H}^2 / (\sigma_{0H}^2 + \sigma_e^2))) \quad (14)$$

Since the *ex-ante* probability of a type- H firm is p then:

$$p(e) = \frac{p (\sigma_{0H}^2 + \sigma_e^2)^{-1/2} \phi \left(\frac{e - \mu_0}{(\sigma_{0H}^2 + \sigma_e^2)^{1/2}} \right)}{p (\sigma_{0H}^2 + \sigma_e^2)^{-1/2} \phi \left(\frac{e - \mu_0}{(\sigma_{0H}^2 + \sigma_e^2)^{1/2}} \right) + (1 - p) (\sigma_{0L}^2 + \sigma_e^2)^{-1/2} \phi \left(\frac{e - \mu_0}{(\sigma_{0L}^2 + \sigma_e^2)^{1/2}} \right)}. \quad (15)$$

Substituting (15) into (13) yields an SRF exhibiting an inverted S-shape in the local region around the zero earnings-surprise level. This result is generalized in the following proposition.

P2

Suppose that the sampling distribution of the earnings signal observed by investors, e , is a normal distribution with an unknown mean and a known variance. Moreover, suppose that the prior of mean earnings is a fat-tailed distribution generated by a mixture of N normal distributions with the same mean but with different variances. Investors know the variances and the common mean of the N distributions. Then, locally around the zero-surprise level of earnings the SRF is monotonically increasing, symmetric and with an inverted S-shape.

Proof in Appendix 2.

When the prior of mean earnings is a normal distribution with an unknown variance we obtain a result that is the symmetric of that obtained when the signal is assumed to be normal with an unknown variance. We thus find that fat-tailed behavior in the prior distribution of the structural valuation parameter and fat-tailed behavior in the distribution of the signal impart opposite patterns of non-linearity to the SRF.

In which circumstances is an inverted S-shape expected to hold empirically? As mentioned in the previous section, in financial accounting there is evidence of S-shaped SRFs but none, to our knowledge, of inverted S-shaped SRFs. Is an inverted S-shape just a theoretical curiosity?

Our earlier discussion for the motivation for a prior with fat tails suggests that firms caught in transition between business strategies or business models with distinct risk profiles are those most likely to feature inverted S-shape SRFs. An extreme earnings realization tells investors that such a firm is likely to have adopted the high-risk business model. The adjustment of traditional businesses to the threats and opportunities created by new technologies offers examples of this idea: in the book business, the attempt of traditional bookstore chains to incorporate new technologies to

stand up against on-line booksellers such as Amazon; in telecommunications, the effort made by incumbents to deal with the new competition brought by the internet, mobile telephony and VoIP. In the airline business, the response of traditional carriers to low-cost operators. In all these cases, firms further along in the transition process are more prone to experience extreme economic outcomes. Hence, a small earnings surprise tells investors that the firm is hanging on to its traditional low-risk business model whereas a big surprise is evidence of a shift toward the new high-risk model. In sum, an inverted S-shaped SRF, if it exists at all, should be found among firms in the process of shifting between business strategies with different levels of risk.

IV. Conclusions

This paper seeks to illuminate the role of fat-tail distributions in the process through which information signals are capitalized in asset values. Specifically, it examines how fat-tail distributions associated either with a signal or the prior of a structural valuation parameter influence the response of asset prices to information disclosures. The analysis shows that the price adjustment is driven by the relative kurtosis of the distributions of the signal and the prior, when one of the distributions has fat tails. The price adjustment as a function of the magnitude of the signal displays either an S-shape or an inverted S-shape depending on which distribution features excess kurtosis.

As mentioned in the introduction, the typical approach in event studies is to regress the price adjustment occurring immediately after the release of an information signal on the unexpected component of signal. The slope of the regression is interpreted as the information impulse per unit of the signal (in financial accounting, the slope coefficient is known as the “earnings response coefficient” or ERC for short). Our analysis suggests that an ERC estimated in the traditional fashion will be biased if the underlying distributions feature excess kurtosis. Furthermore, the bias will follow a particular pattern: If investors are uncertain in about the precision of the earnings signal then the ERC underestimates the price impact of small information events whereas it overestimates the price impact of big information events; the converse holds if investors are uncertain about the risk type of the firm disclosing earnings.

Notes

1. In the macro-finance literature, examples are McQueen and Roley (1993) and Hakkio and Pearce (1985). The former study the impact of fundamental macro news on aggregate stock prices and market interest rates whereas the latter examine the impact of the same sort of news on exchange rates. In agricultural economics, Rucker *et al.* (2005) study how the release of new data on monthly housing starts effects lumber future prices and Collins and Irwin (1990) study the reaction of live hog future prices to USDA Hogs and Pigs reports. In environmental economics, Bui and Mayer (2003) analyse how disclosures of toxic emissions under the Toxic Release Inventory regulation impact house prices whereas Konar and Cohen (1997) investigate the impact of such disclosures on the stock prices of polluting firms. Perhaps the field where this approach has been most extensively used, however, is financial accounting. Authors such as Beaver *et al.* (1979), Beaver *et al.* (1980), Kormendi and Lipe (1987), Collins and Khotari (1989), Easton and Zmijewsky (1989) and Lipe (1990) evaluate the information content of earnings reports by regressing the unexpected stock return around the time of the disclosure of the report on the earnings’ surprise. The slope coefficient of this regression is typically referred to as the “earnings response coefficient” (see, for example, Freeman and Tse (1992)).

2. Mixtures of normal distributions has been used extensively in finance as a general and flexible approach to generate random variables with excess kurtosis, such as the returns on financial assets. See, for example, Kon (1984) and Hull and White (1998).
3. Regarding the behavior of SRFs at the tails, Das and Lev (1994) find that non-linearities persist in the data even after fitting various parametric S-shaped functions (e.g. the arctan function and a modified quadratic function), albeit the evidence is weak. At any rate, the authors argue that due to paucity of data at the extremes of the distribution of unexpected earnings, tests aimed at detecting different types of non-linearities in this range of the data have low power.
4. Our analysis emphasizes the role of kurtosis in generating non-linear SRFs. Under the standard normality assumptions underlying the Gaussian model, the kurtosis of the prior distribution is identical to the kurtosis of the distribution of the signal. When we depart from the standard case by assuming that the signal is sampled from a mixture of normal distributions, the kurtosis of the signal becomes higher than that of the prior; the converse is true when the departure from the standard case consists in assuming a mixture of normals for the prior distribution.
5. Financial analysts know that companies are more hard to value in industries with a greater degree of heterogeneity. The degree of dispersion in market multiples such as price-earnings ratios or book-to-market ratios within an industry gives a measure of the degree of heterogeneity in the population of firms from that industry.
6. Recall that the kurtosis of the Normal is equal to 3.

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Further reading

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Appendix 1. Proof of P1

Let us generalize the model to consider N normal distributions for the signal. Given μ and the earnings process given by $Y = i$ ($i = 1, 2, \dots, N$), e follows a normal distribution with mean μ and known variance σ_{ei}^2 . The prior for μ is normal with known mean μ_0 and known variance σ_0^2 . The distribution of Y is given by $\Pr(Y = i) = p_i$ with $\sum_{i=1}^N p_i = 1$. Note that the variable Y generalizes the Bernoulli variable I to accommodate more than two situations.

Let us define $\beta_i = \sigma_0^2 / (\sigma_0^2 + \sigma_{ei}^2)$ and $p_i(e) = \Pr(Y = i | e)$ and let us remember that, given the earnings process, i.e. given $Y = i$, and the observation of the earnings value e , the posterior expected value of μ will be given by:

$$E(\mu | e, Y = i) = \beta_i e + (1 - \beta_i) \mu_0. \quad (A1)$$

Using the law of iterated expectations and expression (A1), we obtain:

$$\begin{aligned} \mu(e) &= E(\mu | e) = E[E(\mu | e, Y) | e] \\ &= \sum_{i=1}^N (\beta_i e + (1 - \beta_i) \mu_0) \Pr(Y = i | e) \\ &= \mu_0 + (e - \mu_0) \sum_{i=1}^N \beta_i p_i(e) \\ &= \mu_0 + (e - \mu_0) r(e) \end{aligned} \quad (A2)$$

where:

$$r(e) = \sum_{i=1}^N \beta_i p_i(e). \quad (A3)$$

Note that we can define a random variable W assuming the value β_i with probability (given e) $p_i(e)$ ($i = 1, 2, \dots, N$) and, in this framework, $r(e) = E(W | e)$.

To obtain the shape of the SRF curve we need to differentiate $\mu(e)$ twice in order to e . The first derivative will be given by:

$$\mu'(e) = r(e) + (e - \mu_0) r'(e) = r(e) + (e - \mu_0) \sum_{i=1}^N \beta_i p_i'(e). \quad (A4)$$

But, noting that:

$$p_i(e) = a_i(e) / \sum_{j=1}^N a_j(e) \text{ with } a_i(e) = p_i \beta_i^{1/2} \exp \left[-\frac{\beta_i (e - \mu_0)^2}{2\sigma_0^2} \right]$$

we obtain, after some calculus, $p_i'(e) = -(e - \mu_0) \sigma_0^{-2} p_i(e) [\beta_i - r(e)]$ and consequently:

$$r'(e) = -(e - \mu_0) \sigma_0^{-2} (s(e) - r(e)^2) \quad (A5)$$

and

$$\mu'(e) = r(e) - (e - \mu_0)^2 \sigma_0^{-2} (s(e) - r(e)^2) \quad (A6)$$

where $s(e) = \sum_{i=1}^N \beta_i^2 p_i(e)$

Note that $s(e) - r^2(e) = \text{var}(W | e)$.

Now, we must calculate the second derivative of $\mu(e)$:

$$\mu''(e) = r'(e) - \frac{2(e - \mu_0)}{\sigma_0^2} [s(e) - r^2(e)] - \frac{(e - \mu_0)^2}{\sigma_0^2} [s'(e) - 2r(e)r'(e)].$$

But we can see that $s'(e) = -(e - \mu_0)\sigma_0^{-2}[t(e) - s(e) r(e)]$, where $t(e) = \sum_{i=1}^N \beta_i^3 p_i(e)$, that is $t(e) = E(W^3|e)$, and we can use (A5).

We can then rewrite $\mu''(e)$ as:

$$\begin{aligned}\mu''(e) &= -\frac{3(e-\mu_0)}{\sigma_0^2} [s(e) - r(e)^2] + \frac{(e-\mu_0)^3}{\sigma_0^4} [t(e) - 3r(e)s(e) + 2r(e)^3] \\ &= -\frac{3(e-\mu_0)}{\sigma_0^2} \text{var}(W|e) + \frac{(e-\mu_0)^3}{\sigma_0^4} E(W - E(W|e)|e)^3.\end{aligned}\quad (\text{A7})$$

As is obvious, $\mu''(\mu_0) = 0$. It is straightforward to see that the third central moment of W always exists and consequently, when e is in a close enough neighborhood of μ_0 , the signal of $\mu''(e)$ will be given by the first part of the expression, that is to say that, when $e > \mu_0$, $\mu''(e)$ is negative as $\text{var}(W|e) > 0$. Of course, when $e < \mu_0$, $\mu''(e)$ is positive.

Using a similar argument we can prove that, in the neighborhood of μ_0 , $\mu'(e)$ is positive, since $r(e)$ is the expected value of a positive valued random variable and consequently is positive.

As $V_1 = \lambda\mu(e)$, (with $\lambda > 0$) we will obtain an S-shape curve when in the neighborhood of μ_0 .

Appendix 2. Proof of P2

The proof of P2 is quite similar to the proof of P1. Given μ , e follows a normal distribution with mean μ and known variance σ_e^2 . Let us now assume that there are N types of firms and, consequently, given $Y = i$ ($i = 1, 2, \dots, N$), the prior for μ is normal with known mean μ_0 and known variance σ_{0i}^2 . The distribution of Y is given by $\Pr(Y = i) = p_i$ with $\sum_{i=1}^N p_i = 1$.

Let us define $\delta_i = \sigma_e^2 / (\sigma_e^2 + \sigma_{0i}^2)$ and $p_i(e) = \Pr(Y = i|e)$ and let us remember that, given the observation of the earnings value e and the type of firm, i.e. given $Y = i$, the posterior expected value of μ will be given by:

$$E(\mu|e, Y = i) = \delta_i \mu_0 + (1 - \delta_i)e. \quad (\text{B1})$$

Using the law of iterated expectations, we obtain:

$$\begin{aligned}\mu(e) &= E(\mu/e) = E[E(\mu|e, Y)|e] \\ &= \sum_{i=1}^N (\delta_i \mu_0 + (1 - \delta_i)e) \Pr(Y = i|e) \\ &= e - (e - \mu_0) \sum_{i=1}^N \delta_i p_i(e) \\ &= e - (e - \mu_0)r(e)\end{aligned}\quad (\text{B2})$$

where

$$r(e) = \sum_{i=1}^N \delta_i p_i(e). \quad (\text{B3})$$

As in Appendix 1, we can define a random variable W assuming the value δ_i with probability (given e) $p_i(e)$ ($i = 1, 2, \dots, N$) and $r(e) = E(W \dots | e)$.

To obtain the shape of the SRF curve we need to differentiate $\mu(e)$ twice in order to e . The first derivative will be given by:

$$\mu'(e) = 1 - r(e) - (e - \mu_0)r'(e) = 1 - r(e) - (e - \mu_0) \sum_{i=1}^N \delta_i p_i'(e). \quad (\text{B4})$$

But, noting that $p_i(e) = \alpha_i(e) / \sum_{j=1}^N \alpha_j(e)$ with $\alpha_i(x) = p_i \delta_i^{1/2} \exp \left[\frac{\delta_i(e - \mu_0)^2}{2\sigma_e^2} \right]$, we obtain, after some calculus, $p_i'(e) = -(e - \mu_0)\sigma_e^{-2} p_i(e) [\delta_i - r(e)]$ and consequently:

$$r'(e) = -(e - \mu_0)\sigma_e^{-2} (s(e) - r(e)^2) \tag{B5}$$

and

$$\mu'(e) = 1 - r(e) - (e - \mu_0)^2 \sigma_e^{-2} (s(e) - r(e)^2) \tag{B6}$$

where $s(e) = \sum_i \delta_i^2 p_i(e)$. Note that $s(e) - r^2(e) = \text{var}(W|e)$.

The second derivative of $\mu(e)$ will be given by:

$$\mu''(e) = -r'(e) + 2(e - \mu_0)\sigma_e^{-2} [s(e) - r(e)^2] + (e - \mu_0)^2 \sigma_e^{-2} [s'(e) - 2r(e)r'(e)]. \tag{B7}$$

But we can see that $s'(e) = -(e - \mu_0)\sigma_e^{-2} [t(e) - s(e)r(e)]$ where $t(e) = \sum_i \delta_i^3 p_i(e)$, that is $t(e) = E(W^3|e)$. We can then rewrite $\mu''(e)$ as:

$$\begin{aligned} \mu''(e) &= \frac{3(e - \mu_0)}{\sigma_e^2} [s(e) - r(e)^2] - \frac{(e - \mu_0)^3}{\sigma_e^2} [t(e) - 3r(e)s(e) + 2r(e)^3] \\ &= \frac{3(e - \mu_0)}{\sigma_e^2} \text{var}(W|e) - \frac{(e - \mu_0)^3}{\sigma_e^2} E(W - E(W|e)|e)^3. \end{aligned} \tag{B8}$$

As is obvious, $\mu''(\mu_0) = 0$. It is straightforward to see that the third central moment of W always exists and consequently, when e is in a close enough neighborhood of μ_0 , the signal of $\mu''(e)$ will be given by the first part of the expression, that is to say that, when $e > \mu_0$, $\mu''(e)$ is positive as $\text{var}(W|e) > 0$. Of course, when $e < \mu_0$, $\mu''(e)$ is negative. As $V_1 = \lambda\mu(e)$, we will obtain an inverse S-shape curve when in the neighborhood of μ_0 , that is for small surprises.

Using a similar argument we can prove that, in the neighborhood of μ_0 , $\mu'(e)$ is positive, since $r(e)$ is the expected value of a random variable assuming values between 0 and 1, and consequently $0 < r(e) < 1$.

As $V_1 = \lambda\mu(e)$, (with $\lambda > 0$) we will obtain an inverted S-shape curve when in the neighborhood of μ_0 .

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