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Structural change test in duration of bull and bear markets

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HIGHLIGHTS

- Equity prices over long periods can be described as bull and bear market cycles.
- A test to analyze structural changes in these cycle durations is proposed.
- There is evidence of structural break in the bull market duration in April, 1942.

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ABSTRACT

We propose a recursive test to analyze structural changes in duration of bull and bear markets. Using the Dow Jones Industrial Average index, we detected a single structural break in the bull market duration in April, 1942.

1. Introduction

Equity prices over long periods can be described as bull and bear (BB) market cycles. These cycles have been the object of enormous attention by investors and noise traders, and more recently by academics (see for example, Pagan and Sossounov, 2003, Lunde and Timmermann, 2004, Maheu et al., 2012, Kole and van Dijk, forthcoming).

The stability of the expansion and contraction of the business cycle has been studied for many years (see for example Stock and Watson, 1993, among many others). However, as far as we know, no formal statistical tests have been devised to analyze possible structural changes in cycle durations. The stability of BB markets is much less studied and also there are no formal tests to analyze structural changes in cycle durations.

The main goal of this paper is to study the BB market cycle stability (i.e. duration) across time, through a structural change test.

2. Statistical inference on duration stability

2.1. Building the bull and bear cycles

Because there are several definitions of BB markets and since the BB regimes are not directly observable, there are several alternative techniques to estimate the cycles. One can distinguish two main approaches. One is based on nonparametric simple mechanic rules. The other is based on a parametric statistical model. Although the parametric-based approach has his own merits, as is well documented in Maheu et al. (2012) (for example in terms of forecasting), we use a rules-based method in this paper, because, as Kole and van Dijk (forthcoming) argue, it purely reflects the tendency of the market, is more transparent and robust to misspecification, and works best for *ex post* identification. Among the rules-based procedures we selected the Lunde and Timmermann (2004) method, because it does not restrict the cycle duration (as the Pagan and Sossounov (2003) procedure does), which helps us guard against interval censoring issues.

In Lunde and Timmermann's algorithm there are two parameters, λ_1 and λ_2, that are crucial, as they control the identification of peaks and troughs. We will return to this topic in Section 3.

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2.2. The structural change test

Let S_t be the bull market indicator variable taking the value 1 if the stock market is in a bull state at time t and 0 otherwise. To analyze the cycle durations we consider the random variable $T_{Bull} =: \min\{t > 0 : S_t = 0 | S_0 = 1\}$, the first passage time to the bear state given that S started at the bull state. Thus $\mathrm{E}(T_{Bull})$ is the expected time (or duration) of the bull market. Likewise, we define $T_{Bear} =: \min\{t > 0 : S_t = 1 | S_0 = 0\}$.

Throughout the paper we assume the following hypothesis. $\mathcal{H}$: $\{S_t\}$ is a stationary first order Markov chain. We have

$$\theta_1 := \mathrm{E}(T_{Bull}) = \frac{1}{1-p_{11}}, \qquad \theta_0 := \mathrm{E}(T_{Bear}) = \frac{1}{1-p_{00}}$$

where $p_{ii} = P(S_t = i | S_{t-1} = i)$, $i = 0, 1$ (see, for example, Taylor and Karlin, 1998). Given a sample path of S, the maximum likelihood estimate of p_{11} is $\hat{p}_{11} = n_{11}/n_1$ where n_{11} counts the number of times that $S_{t-1} = 1$ is followed by $S_t = 1$, and n_1 counts the ones in the sequence (see, for example, Basawa and Rao, 1980). Hence

$$\hat{\theta}_1 = \frac{1}{1-\hat{p}_{11}} = \frac{n_1}{n_1 - n_{11}} \tag{1}$$

and similarly for $\hat{\theta}_0 = n_0/(n_0 - n_{00})$. Notice that (1) is the same expression used by Pagan and Sossounov (2003, page 27) for the average duration of an expansion. We can say something further about the asymptotic behavior of $\hat{\theta}_i$ $(i = 0, 1)$: Under the hypothesis $\mathcal{H}$ we have for $i = 0, 1$

$$\hat{\theta}_i \xrightarrow{p} \theta_i,$$

$$\sqrt{n}\left(\hat{\theta}_i - \theta_i\right) \xrightarrow{d} N\left(0, \frac{p_{ii}}{(1-p_{ii})^3 \pi_i}\right),$$

where $0 < p_{ii} < 1$ and $\pi_i = P(S_t = i)$. To prove this result we notice that, $\hat{p}_{ii} = n_{ii}/n_i \xrightarrow{p} p_{11}$ and $\sqrt{n}\left(\hat{p}_{ii} - p_{ii}\right) \xrightarrow{d} N(0, p_{ii}(1 - p_{ii})/\pi_i)$ where π_i is such that $n_i/n \xrightarrow{p} \pi_i := P(S_t = i)$ (see, for example, Basawa and Rao, 1980). Finally, the results follow by the invariance principle of the maximum likelihood estimation and the delta method.

Our goal is to investigate whether θ_i is constant over time. Let $\theta_{i,t}$ be the duration or expected time of a bull $(i = 1)$ or a bear regime $(i = 0)$ at time t, and $\hat{\theta}_{i,t}$ the corresponding estimate. Let $[x]$ be the integer part of x. We focus on observations $t = [rn]$ for $r \in R$, where R is a pre-specified compact subset of $(0, 1)$. The null hypothesis of constancy then takes the form

$$H_0 : \theta_{i,[rn]} = \theta_i, \quad \forall r \in R$$

with the alternative $H_0 : \theta_{i,[rn]} \neq \theta_i$ for some $r \in R$. Define

$$Q_{i,n}([rn]) = \sqrt{\frac{[rn]-w}{n-w}} \frac{\sqrt{[rn]}}{\hat{\sigma}_i}\left(\hat{\theta}_{i,[rn]} - \hat{\theta}_{i,n}\right),$$

where $i = 0, 1$ and w is the subsample size corresponding to the first estimate of $\theta_{i,w}$ (i.e. the start-up value) and n is the size of the full sample. First we need the following lemma.

Lemma 1. *Let*

$$Z_t = \frac{(1-\theta_1) S_t + \theta_1 S_t S_{t-1}}{\sigma_1 \pi_1 (1-p_{11})}, \qquad \sigma_1^2 = \frac{p_{11}}{(1-p_{11})^3 \pi_1}.$$

Under $\mathcal{H}$ we have (i)

$$X_n(1) = \frac{1}{\sqrt{n}} \sum_{t=1}^{n} \frac{(1-\theta_1) S_t + \theta_1 S_t S_{t-1}}{\sigma_1 \pi_1 (1-p_{11})} = \frac{1}{\sqrt{n}} \sum_{t=1}^{n} Z_t \xrightarrow{d} N(0, 1),$$

and (ii)

$$\frac{[rn]}{\sqrt{n}} \frac{1}{\sigma_1}\left(\hat{\theta}_{1,[rn]} - \theta_i\right) \stackrel{d}{=} X_n(r) = \frac{1}{\sqrt{n}} \sum_{t=1}^{[rn]} Z_t \xrightarrow{d} W(r).$$

All the proofs are in the Appendix. The case $i = 0$ is entirely analogous. Our main result is presented in the following theorem.

Theorem 2. *Under $\mathcal{H}$ and H_0 we have*

$$\sup_{r \in R} Q_{i,n}^2([rn]) \xrightarrow{d} \sup_{r \in R}(W(r) - rW(1)), \quad i = 0, 1. \tag{2}$$

This is a recursive type test derived from the fluctuations test of Ploberger et al. (1989), which allows us to identify a single unknown breakpoint in duration. However, this test may also be used again, in subsamples, to identify other possible breaks. In the empirical application we consider the Candelon and Straetmans (2006) algorithm for detecting multiple regimes in the tail behavior.

The test is implemented as follows: (1) A sequence of duration estimates are obtained by successively using the subsamples $\{1, \ldots, w\}, \{1, \ldots, w+1\}, \ldots, \{1, \ldots, n\}$ (hence r runs from w/n to 1). One then obtains $\left\{\hat{\theta}_{i,w}, \hat{\theta}_{i,w+1}, \ldots, \hat{\theta}_{i,n}\right\}$ and $\left\{Q_{i,n}^2(w), Q_{i,n}^2(w+1), \ldots, Q_{i,n}^2(n)\right\}$. The maximum value of this later sequence is then compared to the corresponding critical value.

Remark 1. The parameter $r \in R$ cannot start at value zero as the estimator of θ_i needs a minimum number of observations to be implemented. In typical recursive tests, the way R is defined obviously affects the critical values. In our test we propose an adjustment similar to the one described in Nicolau and Rodrigues (forthcoming) for tail breaks, that mimics the case $R = (0, 1)$ and consequently allows us to always use the same critical values regardless of how the left endpoint of R is defined. A brief explanation is given below. According to the proof of Theorem 2, in the Appendix, our test can be written as

$$Q_{i,n}([rn]) = \sqrt{\frac{[rn]-w}{n-w}} \frac{\sqrt{n}}{\sqrt{[rn]}} \frac{1}{\sqrt{n}} \sum_{t=1}^{[rn]} Z_t - \sqrt{\frac{[rn]-w}{n-w}} \frac{\sqrt{[rn]}}{\sqrt{n}} \frac{1}{\sqrt{n}}\left(\sum_{t=1}^{n} Z_t\right).$$

Define $r^* = \frac{[rn]-w}{n-w}$. This parameter varies in the set $[0, 1]$ as $r \in [[w/n], 1]$. We have

$$\begin{aligned} Q_{i,n}([rn]) &= \sqrt{\frac{r^*}{r}} \frac{1}{\sqrt{n}} \sum_{t=1}^{[rn]} Z_t - \sqrt{r^* r} \frac{1}{\sqrt{n}}\left(\sum_{t=1}^{n} Z_t\right) \\ &\stackrel{d}{=} \sqrt{\frac{r^*}{r}} W(r) - \sqrt{r^* r} W(1) \\ &= W(r^*) - \sqrt{r^* r} W(1) \\ &\stackrel{d}{=} W(r^*) - r^* W(1) \quad \text{for } r^* \in [0, 1] \end{aligned}$$

as $r \simeq r^*$ for moderate sample size n, and $|r - r^*| \to 0$ as $n \to \infty$.

Critical values for the test are well known: 1.46, 1.78 and 2.54 for 10%, 5% and 1% respectively.

Monte Carlo simulations carried out by the author (available upon request) show that the test presents approximately the correct nominal size, even for a small/moderate sample size, and it is consistent. Moreover, we find that the power of the test depends on the location of the breakpoint and on the particular values θ_1 and θ_0, in addition to the sample size.

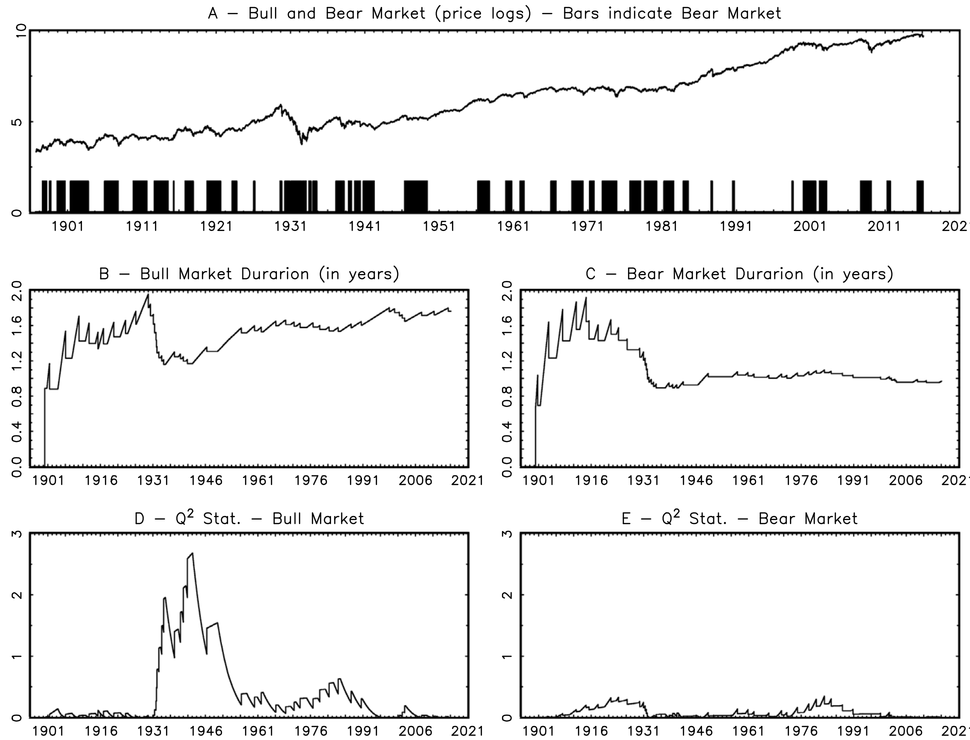

Fig. 1. Bull and bear market duration and Q^2 statistics (critical level at 5%: 1.78).

3. Empirical application

Our dataset consists of daily closing values of the Dow Jones Industrial Average from the 7/10/1896 to 25/2/2016 period (32,559 daily observations), which was obtained from Williamson (2016) (http://www.measuringworth.com/DJA/). Missing observations in 1914 due to the closure of the NYSE at the outbreak of World War I were linearly interpolated (as in Pagan and Sossounov, 2003).

We are interested in whether there was at least one change in the duration of the bull and bear markets. Fig. 1 presents the main results for $(\lambda_1, \lambda_2) = (0.20, 0.15)$. Panel A shows the log-prices over the period and below the bull and bear cycles—for convenience the bear periods are highlighted. The blank spaces represent the bull cycles and, apparently, it seems that the duration tends to increase over time. On the contrary, the duration of the bear periods seems to remain constant. Panel B shows that the duration of the bull cycle was relatively short at the beginning of the period, and then increases until the beginning of the 1929 crisis to fall again at the end of the 20's and early 40's. From there it has been increasing most of the time up to today. Turning now to the statistical hypothesis $H_0 : \theta_{1,[rn]} = \theta_i, \forall r \in R$ we conclude that there is evidence to reject the null at the 1% level ($\sup_{r\in R} Q^2_{1,n}([rn]) = 2.683 > 2.54$—see panel D). Therefore, there is evidence that the bull market duration was not constant across time. The breakpoint, the period at which $Q^2_{i,n}([rn])$ is maximum, was reached on April 29, 1942, during World War II. Interestingly the other null hypothesis $H_0 : \theta_{0,[rn]} = \theta_i, \forall r \in R$ cannot be rejected at any conventional level. In other words, there is no evidence that bear market durations have changed in the period. Our results are robust for other choices of (λ_1, λ_2) mentioned by Lunde and Timmermann (2004), i.e. for (λ_1, λ_2), $(0.20, 0.15)$, $(0.20, 0.10)$, $(0.15, 0.15)$ and $(0.15, 0.10)$.

We applied the Candelon and Straetmans (2006) algorithm to detect other possible breaks beyond the one on April 29, 1942. No further breaks were detected at a 10% significance level. Therefore, statistical evidence points to the existence of a single break in the full sample.

Interestingly, several authors have documented an increase in the length of business cycle expansion from the prewar to the postwar period (see for example Watson, 1992). It seems, therefore, that the durations of the business cycle and the bull market suffered a structural change around the same period. This phenomenon needs further investigation.

Possible explanations for the bull market duration breakpoint in 1942 are the following: (1) the volatility of Dow Jones returns has been smaller in the postwar period, which potentially makes the Dow Jones less cyclical, and therefore prone to higher durations. We have confirmed this fact through a GARCH type model (for daily log-return, in percentage, the marginal variance estimate before the break was 1.49, whereas after the break was 0.963. Further results are available upon request). It is still unclear whether this higher volatility in the prewar period reflects more shocks to the economy, or the changing composition of the Dow Jones across time. (2) The Dow Jones' components have changed from more cyclical sectors such as Railroads and Basic Materials to less cyclical sectors such as Telecommunications and Technology. (3) The Dow Jones started out with 12 components in 1896, rising to 20 in 1916 and only reached the current value (30-stock) in 1928. It is possible that fewer components in the index make it more volatile and hence more cyclical. The causes of a possible duration shift in the bull market needs further attention.

4. Conclusions

This paper focused on structural changes of bull and bear market cycles. We estimated the duration or expected time of each cycle across the sample, and subsequently we propose a recursive test derived from the fluctuations test of Ploberger–Krämer–Kontrus. Our findings point to the existence of a single structural break in the bull market duration in April, 1942, but none in bear market duration.

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Appendix. Proofs

Proof of Lemma 1. (i) This follows from: $\mathrm{E}(Z_t) = 0$, $\mathrm{Var}(Z_t) = 1$, $\mathrm{Cov}(Z_t, Z_{t-k}) = 0$, $k \in \mathbb{Z}$, $\mathrm{E}\left(|Z_t|^{2+k}\right) < \infty$ for any $k > 0$. In fact,

$$\mathrm{E}(Z_t) = \frac{1}{\sigma_1 \pi_1 (1-p_{11})} \left((1-\theta_1)\pi_1 + \theta_1 p_{11}\pi_1\right) = 0.$$

$$\mathrm{Var}(Z_t) = \frac{1}{\sigma_1^2 \pi_1^2 (1-p_{11})^2} \left((1-\theta_1)^2 \pi_1 + 2(1-\theta_1)\theta_1\pi_1 p_{11} + \theta_1^2 \pi_1 p_{11}\right) = 1$$

and $0 < p_{11} < 1$, $\pi_1 > 0$ since the Markov chain is stationary. To show $\mathrm{Cov}(Z_t, Z_{t-k}) = 0$, let $p_{11}^{[k]} := P(S_t = 1 | S_{t-k} = 1)$ and $p_{11}^{[1]} = p_{11}$. The result $\mathrm{Cov}(Z_t, Z_{t-k}) = 0$ follows from

$$\mathrm{E}(S_t S_{t-k}) = p_{11}^{[k]}\pi_1, \qquad \mathrm{E}(S_t S_{t-k} S_{t-k-1}) = p_{11}^{[k]} p_{11}\pi_1$$
$$\mathrm{E}(S_t S_{t-1} S_{t-k}) = p_{11}^{[k-1]} p_{11}\pi_1,$$
$$\mathrm{E}(S_t S_{t-1} S_{t-k} S_{t-k-1}) = p_{11}^{[k-1]} p_{11}^2 \pi_1.$$

Finally $\mathrm{E}\left(|Z_t|^{2+k}\right)$ is bounded for any $k > 0$ given that the $|Z_t| < \infty$ for all t. Hence, the central limit theorem applies (see for example, White, 2001).

(ii) We consider the Functional Central Limit theorem 7.17 of White (2001) for stationary processes. All the conditions of theorem 7.17 are automatically verified, given that Z_t is stationary, uncorrelated, and $\sigma^2 = \lim_{n\to\infty} \mathrm{Var}\left(\frac{1}{\sqrt{n}}\sum_{t=1}^{n} Z_t\right) = 1 < \infty$. Hence, $X_n(r) = \frac{1}{\sqrt{n}}\sum_{t=1}^{[rn]} Z_t \xrightarrow{d} W(r)$. Given that

$$\frac{n_1([rn])}{[rn]} = \frac{\sum_{t=1}^{[rn]} S_t}{[rn]} \xrightarrow{p} \pi_1 \quad \text{as } n \to \infty, \text{ for } 0 < r \le 1$$

$$\frac{n_{11}([rn])}{n_1([rn])} = \frac{\sum_{t=1}^{[rn]} S_t S_{t-1}}{\sum_{t=1}^{[rn]} S_t} \xrightarrow{p} p_{11} \quad \text{as } n \to \infty, \text{ for } 0 < r \le 1$$

(see Basawa and Rao, 1980) and consequently $n_1([rn]) \stackrel{d}{=} \pi_1 [rn]$ and $n_{11}([rn]) \stackrel{d}{=} p_{11} n_1([rn]) \stackrel{d}{=} p_{11}\pi_1 [rn]$, we have

$$\begin{aligned}
\frac{1}{\sigma_1}\left(\hat{\theta}_{1,[rn]} - \theta_1\right) &= \frac{1}{\sigma_1}\left(\frac{n_{11}([rn])}{n_1([rn]) - n_{11}([rn])} - \theta_1\right) \\
&\stackrel{d}{=} \frac{1}{\sigma_1}\left(\frac{n_{11}([rn])}{\pi_1[rn] - p_{11}\pi_1[rn]} - \theta_1\right) \\
&= \frac{1}{[rn]}\left(\frac{(1-\theta_1)n_1([rn]) + \theta_1 n_{11}([rn])}{\sigma_1\pi_1(1-p_{11})}\right) \\
&= \frac{1}{[rn]}\left(\sum_{t=1}^{[rn]} \frac{(1-\theta_1)S_t + \theta_1 S_t S_{t-1}}{\sigma_1\pi_1(1-p_{11})}\right) \\
&= \frac{1}{[rn]}\left(\sum_{t=1}^{[rn]} Z_t\right).
\end{aligned}$$

Hence

$$\begin{aligned}
\sqrt{[rn]}\frac{1}{\sigma_1}\left(\hat{\theta}_{1,[rn]} - \theta_1\right) &= \frac{1}{\sqrt{[rn]}}\left(\sum_{t=1}^{[rn]} Z_t\right) \\
&= \frac{\sqrt{n}}{\sqrt{[rn]}} X_n(r) \xrightarrow{d} \frac{1}{\sqrt{r}} W(r) \stackrel{d}{=} W(1)
\end{aligned}$$

and

$$\begin{aligned}
\frac{[rn]}{\sqrt{n}}\frac{1}{\sigma_1}\left(\hat{\theta}_{1,[rn]} - \theta_1\right) &\stackrel{d}{=} X_n(r) \\
&= \frac{1}{\sqrt{n}}\sum_{t=1}^{[rn]} Z_t \xrightarrow{d} W(r). \quad \blacksquare
\end{aligned}$$

Remark. These results can be immediately extended to the bear case (i.e. θ_0).

Proof of Theorem 2. Given the Lemma 1, we have,

$$\begin{aligned}
Q_{i,n}([rn]) &= \sqrt{\frac{[rn]-w}{n-w}}\frac{\sqrt{[rn]}}{\hat{\sigma}_i}\left(\hat{\theta}_{i,[rn]} - \hat{\theta}_{i,n}\right) \\
&\stackrel{d}{=} \sqrt{\frac{[rn]-w}{n-w}}\frac{\sqrt{[rn]}}{\sigma_i}\left(\hat{\theta}_{i,[rn]} - \hat{\theta}_{i,n}\right) \\
&= \sqrt{\frac{[rn]-w}{n-w}}\frac{\sqrt{[rn]}}{\sqrt{[rn]}}\frac{\sqrt{n}}{\sqrt{n}}\frac{\sqrt{[rn]}}{\sigma_i}\left(\hat{\theta}_{i,[rn]} - \theta_i\right) \\
&\quad - \sqrt{\frac{[rn]-w}{n-w}}\frac{\sqrt{[rn]}}{\sqrt{n}}\frac{\sqrt{n}}{\sigma_i}\left(\hat{\theta}_{i,n} - \theta_i\right) \\
&= \sqrt{\frac{[rn]-w}{n-w}}\frac{\sqrt{n}}{\sqrt{[rn]}}\frac{[rn]}{\sqrt{n}\sigma_i}\left(\hat{\theta}_{i,[rn]} - \theta_i\right) \\
&\quad - \sqrt{\frac{[rn]-w}{n-w}}\frac{\sqrt{[rn]}}{\sqrt{n}}\frac{\sqrt{n}}{\sigma_i}\left(\hat{\theta}_{i,n} - \theta_i\right) \\
&= (1+o(1))\frac{1}{\sqrt{n}}\sum_{t=1}^{[rn]} Z_t - (r+o(1))\frac{1}{\sqrt{n}}\left(\sum_{t=1}^{n} Z_t\right) \\
&\stackrel{d}{=} X_n(r) - rX_n(1) \xrightarrow{d} W(r) - rW(1).
\end{aligned}$$

Finally, the result follow by the continuous mapping theorem. $\blacksquare$

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