Quantification and Second-Order Monadicity
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Frege's (1879, 1892) treatment of quantification is rightly regarded as a major advance in the study of logic and language. But it led Frege and others to some views that we can and should reject, given developments in the study of transformational grammar and compositional semantics. Once these views are put aside, the developments suggest that the common logical form of sentences like (1) and (2)

(1) Every bottle is red (2) Pat kicked every bottle

is '∃O[EVERY(O) & $\mathscr{E}$(O) & $\mathscr{F}$(O)]'; where 'O' ranges over ordered pairs of a special sort, but *not* ordered pairs of sets. If this is correct, some familiar conceptions of logical form must be modified.

1. Overview

Pace Frege, there is no important mismatch between the structure of a sentence like (2) and the structure of the proposition (Thought, potential premise/conclusion) expressed by using the sentence. This point— about grammatical form, logical form, and the need to revise pre-Fregean conceptions of both—will be familiar to specialists. But it has not yet pervaded the wider community that learned, often via Russell (1905, 1918), about the alleged mismatch. Correlatively, many philosophers have held that logical forms are *regimentations* of sentences in a natural language. So given a preference for first-order regimentation, appeal to second-order logical forms can seem misguided. But as Boolos (1998) shows, quantification into monadic predicate positions is motivated and unobjectionable, when it is construed as *plural* quantification (over whatever first-order variables range over) as opposed to quantification over sets.

We can also reject the idea that transitive verbs are associated with functions from pairs of entities to truth values. Instead, we can adopt the following view, motivated by extensions of Davidson's (1967, 1985) event analyses: all verbs are semantically monadic; combining verbs with other expressions corresponds to predicate-conjunction; and grammatical relations that verbs bear to their arguments are semantically associated with participation relations, like *being an Agent of*, that individuals can bear to things like events. Schein (1993) adopts this approach to plural constructions like (3).

(3) The guests brought four bottles

He argues that verbs and quantificational arguments are understood as plural monadic predicates, as in '∃E[Agents(E, the guests) & Past-Bringings(E) & Themes(E, four bottles)]': there were some events such that their Agents were the guests, they were events of bringing, and their Themes were four bottles.

Given these considerations, various parallels between transitive verbs and determiners—words like 'the' and 'every', which can combine with nouns to form restricted quantifiers like 'every bottle'—create pressure to treat determiners as second-order monadic predicates. The proposal, still Fregean at its core, is that these quantificational words are satisfied by ordered pairs that associate entities in the relevant domain with "truth" values, **t** or **f**. Each such pair is of the form <**t**, x> or <**f**, x>. And the idea is that a given determiner is satisfied by some pairs of this form iff *they* meet a certain condition: some value-entity pairs satisfy 'every'/'no'/'most' iff all/none/most of them associate their entities with **t**. One can go on to provide a compositional semantics (with independent virtues) according to which (1) is true iff there are some entity-value pairs that meet the following three conditions: every one of them associates its entity with **t**; their entities are (all and only) the bottles; and each of them associates its entity with **t** iff its entity is red.

2. Some Reconstructed History

If a speaker of a natural language uses a declarative sentence to express a proposition, we can ask: what is the *grammatical* structure of the sentence; what is the *logical* structure of the proposition; and how is the former related to the latter?[1] Prior to Frege, a standard view was that sentences and propositions have the same basic (subject-predicate) form. After Russell, it seemed that sentential structure differs in kind from propositional structure. But as Chomsky and others showed, "surface appearances" can mask both grammatical form and logical form. Sentences may mirror propositions after all. This story has been told before. But it illustrates how the study of language and thought can benefit from mutually influencing contributions by philosophers and linguists. It is also needed background for what follows.

2.1 The Subject-Predicate Hypothesis and Trouble

Focussing initially on examples like (4-6),

(4) Every politician is deceitful   (5) No logician is deceitful

(6) Some logician is a politician

Aristotle initiated a research program that medieval logicians pursued ingeniously. For present purposes, we can ignore most of the details and recall what came to be a leading idea: in each basic proposition, a subject is linked (perhaps via some tense-bearing copula) to a predicate, with the whole being truth-evaluable in a way corresponding to kind of the subject—universal as in (4), existential as in (6), deictic as in 'He is deceitful', etc. (Nonbasic propositions were said to consist of basic propositions and syncategorematic elements, corresponding to words like 'or', 'not', and 'only'.) This provided a framework for explaining logical relations: given the nature of propositional structure and quantificational elements, propositions that share parts in certain ways are inferentially related in certain ways. For example, it must be the case that if the propositions indicated with (4) and (5) are true, the proposition indicated with (6) is false.

More generally, if every Φ is Ψ, and no Ω is Ψ, then it is false that some Ω is Φ. Though one suspects that this particular inference pattern is somehow a reflection of more primitive patterns. This invites attempts to characterize as many inferential relations among propositions as possible in terms of the fewest primitive inferential patterns. Medieval logicians were remarkably successful at expanding the scope of Aristotle's syllogistic logic while reducing the number of primitive patterns.[2] But the program also faced serious difficulties, as its proponents recognized.

The predicative components of propositions were said to be monadic. Variables like 'Φ' and 'Ψ' above, for example, range over monadic predicates. This suggests that (7)

(7) Brutus stabbed Caesar

expresses a proposition whose structure is reflected by 'Brutus was one-who-stabbed-Caesar'; where the

hyphenated expression indicates a predicate Ψ relevantly like 'deceitful'. This is plausible enough, until one adds that Ψ is *not* a complex predicate with a relational component. For (7) and (8) imply (9).

(8) Caesar was a tyrant (9) Brutus stabbed a tyrant

But how can this be explained if 'stabbed Caesar' and 'stabbed a tyrant' do not indicate predicates that share a nonmonadic part corresponding to 'stabbed'? By allowing for relational predicates, one can abstract an inference pattern from any case.[3] But such patterns are not intuitively primitive. And the difficulty generalizes, suggesting that no finite number of principles will cover the full range of cases.

Given relative clauses, endlessly many sentences have a transitive verb and a quantificational direct object, which may contain pronouns bound by the subject. Consider (10-11)

(10) Pat kicked every bottle that no logician saw

(11) [No logician]$_x$ saw every bottle that was kicked by someone who refuted him$_x$

The corresponding propositions exhibit logical relations that cry out for explanation. For example, if no logician saw any red bottle, the truth of (10) ensures the truth of (12),

(12) Pat kicked every red bottle

which also follows from (2). More interestingly, (13-17) illustrate facts known to the medievals.

(13) Every dog saw a cat (14) Every brown dog saw a cat

(15) Every dog saw a brown cat (16) No man who found every dog saw a cat

(17) No man who found every brown dog saw a cat

One can allow for complex predicates, corresponding to phrases like 'brown dog'; where such a predicate is satisfied (to a first approximation) by whatever satisfies its constituents. Then one can say that the propositions indicated by (13-17) share a part corresponding to 'dog'. But this raises the question of how to characterize the logical relations exhibited by propositions that share parts in this way. In grammatical terms, is the direction of entailment from NOUN to ADJECTIVE^NOUN, or *vice versa*?

Given a subject-predicate conception of propositional structure, there seems to be no suitably general answer. While (13) implies (14), (15) implies (13). Given a subject of the form 'every Φ', the direction of entailment depends on whether or not the noun in question is part of the subject. And while (13) implies (14), the valid inference being from 'every dog' to 'every brown dog', (17) implies (16). In the latter case, the valid inference is from 'every brown dog' to 'every dog', even though 'dog' is part of the subject in each case. So the direction of entailment also depends on whether or not 'every' is part of a complex subject headed by a negative expression. Reflection on such examples reveals an array of facts that, in retrospect, we can see as indicating the need for a new conception of propositions.[4]

2.2 The Function-Argument Hypothesis and Exaggeration

Thanks to Frege, it now seems obvious that there was another way to generalize from Aristotle's treatment of simple cases like (4-6). Instead of focussing on the subject-predicate structure of quantificational sentences, one can focus on the fact that such sentences consist of a quantificational word combined with two monadic predicates. Then one can explore the idea that each quantificational element of a proposition combines (somehow) with two predicative elements, allowing for complex predicates with relational parts (indicated by transitive verbs and certain other linguistic expressions).

From this perspective, a complex proposition P may have a quantifier-predicate(s) structure that differs from the subject-predicate structure of a sentence used to express P. For example, one can say that the logical forms corresponding to (13-17) are as shown in (13a-17a).

(13a) $\forall x[Dx \rightarrow \exists y(Cy \& Sxy)]$

(14a) $\forall x[Bx \& Dx \rightarrow \exists y(Cy \& Sxy)]$

(15a) $\forall x[Dx \rightarrow \exists y(By \& Cy \& Sxy)]$

(16a) $\neg \exists x[Mx \& \forall y(Dy \rightarrow Fxy) \& \exists z(Cz \& Sxz)]$

(17a) $\neg \exists x[Mx \& \forall y(By \& Dy \rightarrow Fxy) \& \exists z(Cz \& Sxz)]$

This lets one capture the relevant entailments without recourse to *ad hoc* inferential principles. Likewise,

one can say that the logical forms corresponding to (1) and (9) are as shown in (1a) and (9a),

(1) Pat kicked every bottle   (9) Brutus stabbed a tyrant

(1a) ∀x[Bx --> Kpx]   (9a) ∃x[Tx & Sbx]

which do *not* mirror the subject-predicate structures of (1) and (9). No constituent of (1a) corresponds to 'kicked every bottle'; and 'Bx –> Kpx', which does not correspond to a grammatical constituent of (1), stands for a function that maps things of the sort 'x' ranges over to truth values—a function of type <x, **t**>. But presumably, 'every' does not itself indicate a function that maps each function of type <x, **t**> to a truth value, since 'every bottle' is not a truth-evaluable claim. (Compare 'Everything is a bottle'.)

This suggested that sentential structure and propositional structure differ in kind. Russell (1905) famously extended this idea to claims involving definite descriptions, offering a theory according to which the logical form of (18) is as shown in (18a).

(18) Pat kicked every bottle that the logician dropped

(18a) ∀x{Bx & ∃y[Ly & ∀z(Lz --> z = y) & Dyx] --> Kpx}

The virtues of Russell's theory thus made it seem that the form of a proposition P can diverge *radically* from the form of a sentence used to express P. (Wittgenstein [1921] elaborated this thought in presenting a picture of propositional structure and logical necessity.) But this exaggerated the gap between logical and grammatical form, even given the prevailing subject-predicate conception of the latter.

Frege himself often thought of quantificational words, on analogy with transitive verbs, as expressing *relations* of a special sort. In connection with his study of how arithmetic is connected to logic, Frege associated cardinality expressions like 'three' (as in 'Three bottles are red') with relations between extensions of monadic predicates, like the relation one set bears to another iff their intersection has three members. The second-order character of this proposal can be made explicit as follows: 'Three bottles are red' is true iff ∃X∃Y[X = {x: x is red} & Y = {x: x is a bottle} & |X ∩ Y| = 3].[5] On this kind of view, 'some' and 'every' signify intersection and inclusion. One can also accommodate

'most' and other quantificational expressions, like 'nine of ten', whose essentially relational meanings cannot be captured with sentential connectives on the model of (1a); see Rescher (1962), Wiggins (1980). There are various ways to do this. But since natural language employs *restricted* quantifiers, the obvious proposal is that 'most bottles' has the logical form '$\mu x:\Phi x$', where each instance of '$\mu x:\Phi x(\Psi x)$' is true iff the Φs that are Ψs *outnumber* the Φs that are not Ψs. In set-theoretic terms, the end, the idea is that 'most' expresses the relation that $\{x: \Phi x\}$ bears to $\{x: \Psi x\}$ iff $|\{x: \Phi x \ \& \ \Psi x\}| > |\{x: \Phi x \ \& \ \neg\Psi x\}|$.

For present purposes, the main point is that we can replace (1a) and (9a) with (1b) and (9b).

(1a) $\forall x[Bx \dashrightarrow Kpx]$ (9a) $\exists x[Tx \ \& \ Sbx]$

(1b) $\forall x:Bx(Kpx)$ (9b) $\exists x:Tx(Sbx)$

One can add, if one wants to, that a certain aspect of the formal parallel between '$\forall x$' and 'K' in (1b)— viz., that each combines with *two* expressions (of the right type) to form a sentence—is inessential for capturing the logical implications of (1). But since the connective in (1a) is also inessential, we should not conclude that this constituent of (1a) reflects a constituent of the propositions expressed with (1).[6] Montague (1974) and Neale (1990, 1993) make this point, in different ways, by showing how to recode Russell's theory. For example, we can replace (18a) with (18b), (18c) or (18d);

(18a) $\forall x\{Bx \ \& \ \exists y[Ly \ \& \ \forall z(Lz \dashrightarrow z = y) \ \& \ Dyx] \dashrightarrow Kpx\}$

(18b) $\forall x:\{Bx \ \& \ [\exists y:Ly \ \& \ \forall z(Lz \dashrightarrow z = y)](Dyx)\}(Kpx)$

(18c) $\forall x:\{Bx \ \& \ [\exists y:Ly \ \& \ |L| = 1](Dyx)\}(Kpx)$

(18d) $\forall x:[Bx \ \& \ \iota y:Ly(Dyx)](Kpx)$

where '$\iota y: \Phi y(\Psi y)$' means that for some y such that y is Φ and exactly one thing is Φ, y is Ψ.

Even (18d) differs structurally from (18), since no constituent of (18d) corresponds to 'kicked every bottle that the logician dropped', just as no constituent of (1b) corresponds to 'kicked every bottle'. But (18d) has constituents corresponding to 'the', 'the logician', 'bottle that the logician dropped' and 'every bottle that the logician dropped'. So while these constituents of (18) do not correspond to

consitutents of (18a), this is an artifact of the notation Russell used to formalize his main insight about definite descriptions: they are quantificational expressions, and in this respect, like 'some king' or 'every bottle'. The arguments in favor of this view do *not* suggest that the proposition expressed with (18) has the function-argument structure indicated in (18a) as opposed to (18d).

Moreover, as Montague (1974) showed, one can characterize a suitably general algorithm that pairs subject-predicate structures of natural language with the corresponding logical structures. In this sense, predicates with quantificational constituents do not *themselves* preclude systematic theories of truth for natural languages. Each sentence like (1) can be mechanically associated with at least one formal sentence like (1b), or an analog of the lambda calculus; where the stipulated interpretation of the formal sentence, specified by a recursive theory of truth for the formal language (see Tarski [194x]), is said to be (an idealized version of) the interpretation of the associated natural sentence. Similarly, (18) can be associated with (18b, c, or d). Any such semantic theory will be "indirect" in that it assigns interpretations to sentences of English by first associating them with (nonisomorphic) formal sentences, instead of assigning interpretations directly and compositionally to English expressions. One might wonder if such indirection can be both essential and explanatory, given that interpretations of the formal sentences are stipulated by using a natural language. But in any case, recoding Frege and Russell at least makes the (alleged) logic-grammar gap seem less dramatic.

2.3 The Relevance of Transformations and Nonambiguity

Montague (1974, p. 188) held that the 'basic goal of serious syntax and semantics' is 'the construction of a theory of truth—or rather, of the more general notion of truth under an arbitrary interpretation', and that 'developments emanating from the Massachusetts Institute of Technology offer little promise to that end'. But in fact, Chomsky's proposals concerning transformational grammar were directly relevant to syntax, semantics, and closing the gap.

Obvious parallels between declarative sentences like (19) and questions like (20)

(19) The child was lost   (20) Was the child lost

invite the hypothesis that (20) is somehow the result of displacing the auxiliary verb in (19). Examples

like (21-23) confirm this hypothesis.

(21) Was the child who lost kept crying   (22) The child who was lost kept crying

(23) The child who lost was kept crying

For strikingly, (21) is *un*ambiguous. Speakers of English cannot hear it as the question corresponding to

(22). Rather, (21) can only be heard as the unexpected question corresponding to (23). This suggests that

there is a *constraint* on the relevant transformation (and hence a transformation): to a first appoximation,

only the *main* auxiliary verb can be displaced to form a question; an auxiliary verb embedded in a

relative clause, as in (22), cannot be so displaced.[7] By offering many examples of this sort, Chomsky

(1957, 1965, 1977, 1981) and others argued that: natural language grammars are indeed transformational;

"surface structure" (SS) often differs from "deep structure" (DS); and facts about what strings of words

cannot mean are often explained by constraints on (DS, SS, and) transformations.

With this in mind, consider the much discussed contrast between (24) and (25).

(24) John is eager to please   (25) John is easy to please

While (24) means roughly that John is eager that he please relevant parties, (25) means roughly that it is

easy for relevant parties to please John. These expressions are unambiguous: 'John' is associated with

only the subject of 'please' in (24), and only the direct object of 'please' in (25). But we can specify an

algorithm that associates (24) and (25) with meanings they do not have. So *explaining* the negative facts

requires more than just a method for associating word-strings with the meanings they have (and no

others). We want to know *why* certain strings fail to have certain meanings. And evidently, constraints on

natural language—facts about how one expression can be *grammatically* related to another (potentially

distant) expression—impose limits on how strings of words can be associated with meanings.

Accounting for nonambiguity in this Chomsky-esque fashion requires two related assumptions: at least for a wide range of cases, ambiguity is homophony; and form constrains meaning in substantive ways. If a spoken language associates a given sound with more than one meaning, then two or more expressions of the language share that sound. Just as the sound of 'bear/bare' corresponds to more than one word, a string of words can correspond to more than one sentence, as in (26-28).

(26) Visiting relatives can be unpleasant

(27) You cannot stop a philosopher with an argument

(28) The prince attacked the duke from Essex

By contrast, (25) *cannot* support a certain coherent meaning composed (coherently) from the meanings of its words. This suggests that ambiguity is limited by *independent* constraints on grammatical forms and how such forms are related to meanings; see Higginbotham (1985). Moreover, even an ambiguous string fails to be ambiguous in many ways. While (28) can be used to say either that the duke or the attack was from Essex, it does not have a third reading according to which the prince is from Essex.

This is partly because given the word order in (28), the prepositional phrase could modify 'duke' or 'attacked the duke', but not 'prince'. The complex verb phrase is structured in one of *two* ways: [attacked [the duke [from Essex]]]; or [[attacked [the duke]][from Essex]]. And crucially, the latter is not understood as a predicate satisfied by x iff x attacked the duke, and x is from Essex. So even *given* a grammatical structure, there are negative facts to account for. I return to the details, which cast doubt on the idea that the simple verb phrase [attacked [the duke]] is itself satisfied by x iff x attacked the duke. For now, though, the important point is that accounting for *non*ambiguities seems to require a theory according to which (i) semantic structure tracks grammatical structure, and (ii) semantic composition is governed by principles that rule out certain coherent meanings for a given structure. So indirect semantic theories *a la Montague* are at best incomplete, absent an account of why strings of English words are *not* associated with formal sentences whose meanings are not possible readings of the English sentences.

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Since we could provide an algorithm that associates (28) with (28a),

(28a) ιx:Prince(x){ιy:Duke(y)[Attacked(x, y) & From(x, Essex)]}

merely providing a different algorithm doesn't *explain* why (28a) is not a reading of (28).[8]

In this context, it becomes significant that (29)

(29) It is false that Pat kicked every bottle

fails to have a reading according to which for every bottle x, it is false that Pat kicked x. We can easily

specify an algorithm that associates (29) with both '¬∀x:Bx(Kpx)' and '∀x:Bx¬(Kpx)'. So one wants to

know why speakers of English can associate (29) with the first logical form but not the second. Internal

to Chomsky's program, a natural conjecture is that 'every bottle' is the target of a transformation that is

subject to constraint: 'every bottle' is displaced in a way reflected by '¬∀x:Bx(Kpx)'; but it cannot move

further, into a grammatical position interpreted as having scope over the negation. Similarly, (30) has the

reading indicated with (30a), but not the one indicated with (30b).

(30) Every child who saw every dog is happy

(30a) ∀x:[Cx & ∀y:Dy(Sxy)](Hx)   (30b) ∀y:Dy[∀x:Cx & Sxy(Hx)]

This invites the thought that 'every dog' is displaced, but only so far.

Following May (1985) and others, one can hypothesize that quantificational sentences have

grammatical structures like the following: [[Every bottle]$_x$ [Pat [kicked t$_x$]]], where 't$_x$' indicates a trace

of the displaced expression. This is to posit "covert" transformations—*i.e.*, transformations not reflected

in audible word order. But this fits nicely with idea that determiners express second-order relations, since

[Pat [kicked t$_x$]] can be interpreted as a complex predicate satisfied by x iff Pat kicked x. One can say that

'every' combines with a first (or *internal*) predicative argument like 'bottle', and that because the

resulting phrase is displaced, 'every' combines with a second (or *external*) predicative argument like [Pat

[kicked t$_x$]]. By analogy, in [Pat [kicked it]], 'it' is the internal argument of the verb; 'Pat' is the external

argument, since 'Pat' and 'kicked' do not form a phrase.

Saying that quantifiers are displaced in a way that bears on interpretation but not pronunciation, as depicted in the (now familiar) diagram below, is a way of *preserving* the main Frege-Russell insight.

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        DS
        |
        SS
       / \
     PF   LF
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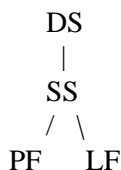
There is indeed a mismatch between the subject-predicate structure of 'Pat kicked every bottle' and the corresponding semantic structure. But instead of saying that grammatical structure fails to reflect logical structure, one can posit a distinction *in natural language* between aspects of sentence structure relevant to pronunciation and aspects relevant to meaning; where such talk of meaning (interpretation) should be no more tendentious than talk of pronunciation (sound). The crucial point is that a language can associate sounds with interpretations in a way that leads to PF-LF divergences, which might be *mis*diagnosed as symptoms of an underlying mismatch between sentence structure and propositional structure.

2.4 Frege as (Unintentionally) Prescient Grammarian

There is considerable evidence for the hypothesized covert transformations. This is not the place for a full review; see Huang (1995), Hornstein (1995). But let me illustrate of the *kind* of data that is relevant.

In English, questions like 'Who did John see' seem to be formed by displacing 'who', with [Who$_x$ did [John see t$_x$]] having a quantificational meaning like the following: which person x is such that John saw x. So a natural thought is that the French translation 'Jean a vu qui' has the same kind of LF, even though the audible word order does not reflect displacement of 'qui' ('who'). In Chinese, wh-elements are also pronounced "*in situ.*" But (31), which has the indicated transliteration,

(31) Zhangsan zhidao Lisi mai-te sheme   [Zhangsan know Lisi bought what]

is ambiguous as between the interrogative (31a) and the complex declarative (31b).

(31a) what$_x$ is such that Zhangsan knows Lisi bought it$_x$

(31b) Zhangsan knows what$_x$ (is such that) Lisi bought (it$_x$)

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This suggests covert wh-movement in Chinese; see Huang (1982, 1995). And compare (32) with (33).

(32) Who said that he has the best smile (33) Who did he say has the best smile

In (32), 'he' has a bound-variable reading: who$_x$ [t$_x$ said that [he$_x$ has the best smile]]; which person x is such that x said that x has the best smile. But (33) cannot be used to ask this question, suggesting that some constraint rules out the relevant structure: who$_x$ did [he$_x$ say [t$_x$ has the best smile]]. Similarly, (34)

(34) When did he say Pat has the best smile

has no reading on which 'Pat' and the pronoun are referentially linked, suggesting that some constraint rules out the relevant structure: when did [he$_x$ say [Pat$_x$ has the best smile]]. Chomsky (1981) argues that the inaudible trace of 'who' in (33) is relevantly like 'Pat' in (34), and that the same constraint applies in both cases. This connects facts about names and pronouns with facts about wh-expressions (which have a quantificational semantic character) via the hypothesis that such expressions are displaced, whether or not such displacement is reflected in audible word order. This in turn leads to theoretical economy with respect to how we describe (speakers' tacit knowledge of) the constraints on natural languages.[9]

Moreover, among the facts concerning (17) is that it fails to have the reading indicated in (17b).

(17) No man who found every brown dog saw a cat

(17a) ¬∃x:{Mx & [∀y:By & Dy](Fxy)}[∃z:Cz(Sxz)]

(17b) ∀y:By & Dy{¬∃x:(Mx & Fxy)[∃z:Cz(Sxz)]}

There are independent reasons for thinking that English abhors this kind of "fronting" from within an embedded relative clause. We already saw a manifestation of this constraint in (21).

(21) Was the child who lost kept crying

And why would speakers associate (17) with (17a) *but not* (17b), unless their grammar imposes a constraint to the effect that 'every brown dog' must raise, though not too far? This also suggests that (17a) is a better representation of logical form than the logically equivalent (17c).

(17c) ¬∃x[Mx & ∀y(By & Dy --> Fxy) & ∃z(Cz & Sxz)]

If we represent the relevant propositional structure with (17c) and the contrasting structure with (17d),

(17d) $\forall y\{By \ \& \ Dy \rightarrow \neg\exists x[Mx \ \& \ Fxy \ \& \ \exists z(Cz \ \& \ Sxz)]\}$

which is logically equivalent to (17b), we need to explain why speakers associate (17) with (17c) but not (17d). And here a Frege-Russell point is relevant: '$\forall y\{By \ \& \ Dy \rightarrow$' is not a constituent of (17d), and hence, not a constituent corresponding to any part of (17) governed by a principle of natural language.

Covert displacement also lets one maintain that ambiguity is homophony, even for (35),

(35) Some logician kicked every bottle

which is associated with two LFs indicated as follows: [(some logician)$_x$ [(every bottle)$_y$ [t$_x$ (kicked t$_y$)]]]; [(every bottle)$_y$ [(some logician)$_x$ [t$_x$ (kicked t$_y$)]]]. This removes one motivation for indirect semantic theories—viz., that sentences of natural language must be *dis*ambiguated before applying Frege-Tarski tools. *Sentences* (as opposed to word-strings, or word-strings associated with surface structures) may not be ambiguous after all, at least not in ways that preclude systematic and compositional theories of meaning. Of course, one must not allow too much ambiguity. And examples like (36-37) suggest

(36) Few dogs saw every cat   (37) Some dogs saw no cats

that not all direct objects can take scope over subjects; see Pietroski and Hornstein (2002) for discussion. So defenders of the "LF-hypothesis," according to which the quantifiers in (36-37) are covertly displaced, need to describe the relevant constraints in a plausible way. But others face the less enviable task of providing an alternative explanation for the full range of facts concerning scope (non)ambiguity.

The LF-hypothesis also lets one avoid unpleasant questions raised by mismatch proposals. How can a speaker associate each of *endlessly many* natural sentences with one or more *structurally different* propositions; and in so far as adult speakers *agree* about this association, how do *children* manage to converge on such agreement? Instead, one asks how children come to associate PFs with LFs in the way that adult speakers do. This makes a substantive nativism about human language acquisition all but inevitable. But as Chomsky has long stressed, if grammars are transformational and subject to specific

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constraints, children are presumably disposed to acquire grammars with a character not predicted by general considerations of learnability or communication. This seems to be the case; see, for example, Crain and Pietroski (2001, 2002) and references there. Moreover, linguists have been at least partially successful in characterizing a substantive "Universal Grammar" that would allow each child to acquire local grammar(s)—i.e., the local adult ways of associating PFs with LFs—given a normal course of experience (that is in many respects impoverished relative to what children come to know about how sounds are, and are not, associated with meanings). By contrast, we have no account of how a typical child could come to associate PFs with syntactic structures and *mismatching* logical forms.

As noted earlier, displacement of quantifiers also becomes theoretically attractive once we stress parallels between determiners and transitive verbs. If a determiner combines with *two* predicates to form a sentence, covert displacement is expected. In [Pat [kicked [every bottle]]], there is no second argument for 'every', whose first argument is 'bottle'. (And if 'every' expresses a relation, the sentence can be evaluated for truth only if both relata are indicated.) But in [[every bottle]$_x$ [Pat [kicked t$_x$]]], 'every' combines with two arguments of type $<x, \mathbf{t}>$, and likewise for [[every bottle]$_x$ [t$_x$ [is red]]].[10]

To summarize: instead of saying propositions have subject-predicate structures, Frege said that they have function-argument structures. But if natural sentences have LFs that mirror Fregean propositional structures, we can return to the idea that grammatical structure matches logical structure, by modifying the traditional conception of both.

3. Complications

Alas, it isn't quite *that* easy to reconcile Fregean logic and natural language, not even for the simple cases considered here. Frege's own interpretation of second-order quantification can encourage a preference for first-order analyses, leading to further mismatches between logical and grammatical form. Boolos (1984, 1998) offers a better interpretation, though it faces what seems to be a serious problem. There is a solution. But I think it requires a conception of propositions/sentences according to which:

determiners are semantically monadic *without* being predicates satisfied by ordered pairs of sets. Luckily, this turns out to be plausible. In this section, I review the considerations that (together) motivate the idea that determiners (and transitive verbs) are semantically monadic Boolos-style second-order predicates. In section four, I defend a specific proposal about what determiners are satisfied by.

3.1 The Trap of First-Order Regimentation

Frege's *Begriffsschrift* was a second-order language. But his intellectual descendants often took logical forms to be sentences of first-order predicate calculus, even after Montague (1974) and the developments just reviewed. Following Boolos (1975, 1984) and others, I think quantification into predicate positions is unavoidable (if we want to account for relevant facts) and unobjectionable (if understood appropriately). But it is worth being clear about this, since second-order quantification is still suspect in some quarters.

There are certainly reasons for being suspicious of second-order quantification as Frege understood it, in terms of quantification over sets (or Concepts, or their extensions). Russell's Paradox looms, given predicates like 'is not an element of itself'. And intuitively, mundane sentences like 'Three bottles are red' do not *logically* imply that the bottles (or the red things) form a *set*. I return to these points, which do suggest that something is wrong with the following two-part hypothesis: sentences of natural language have second-order logical forms; *and* second-order quantification should be construed set-theoretically. But why did anyone ever conclude that the problem lies with the first part?

Historically, there was—and I suspect still is—a tendency to think that logical forms are first-order because: logical forms are *regimentations* of natural sentences; and for purposes of regimentation, first-order formal languages are preferable to alternatives. This can even seem plausible, given certain assumptions. Suppose one thought that (Frege-Russell-Wittgenstein showed that) there are mismatches between logical and gramamtical form. Absent an account of how speakers associate utterances of sentences with the corresponding propositions, one might consider the skeptical view that speakers

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typically *don't* make such associations, especially if: there was a behaviorist trend in psychology anyway; negative facts like those discussed in §2.3 were not yet recognized as explananda; and language acquisition was assumed to be a matter of learning from experience. Talk of propositions might then seem to be (at best) a proposed *revision* of natural language. From this perspective, mismatches are expected, and first-order "analyses" should not be combined with the hypothesis that ordinary speakers somehow associate ordinary sentences with divergent mental structures. (See Quine [1950, 1951, 1960].)

On this view, when an ordinary speaker reports on the passing show, there is no fact of the matter about (which proposition is expressed or) which logical form is associated with the speaker's utterance. There may be constraints, like coherence, on interpretation; see Davidson (1984). And a speaker's causal relations to the environment, which includes other speakers, may also render specific interpretations implausible (perhaps to the point of unacceptability). But questions about logical form were said to be, at least largely, calls for *decisions* about how best to regiment natural language; where such decisions, about which logical structures to *impose* on verbal behavior, are to be made in terms of which schemes allow for the best "overall fit" between experience and theory. This raised the question of whether some formal languages are better than others for purposes of regimentation. And when assessing *logical* systems, avoiding paradoxes counts for a *lot*. Moreover, since the first-order fragment of Frege's logic was provably sound and complete, one could characterize proof-theoretic notions that capture some intuitions about entailment without reliance on suspect semantic notions. So one might think that the "study" of natural language semantics is really a project of first-order regimentation. [11]

Like Chomsky (2000b), I think this web of theses is unwarranted, given what the study of natural language has revealed. But for present purposes, let me just stress that a bias in favor first-order regimentation can obscure the evidence that endlessly many natural sentences have meanings that cannot be captured in first-order terms. The evidence is abundant, even setting aside certain expressions that might be regarded as somehow special in this regard ('most', 'only', 'outnumber', 'equinumerous', etc.)

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As Boolos (1984) notes, '∀x(Fx --> ∃yGy)' does not imply that there are at least as many Fs as Gs. But the English sentence (38) is naturally heard as having this kind of pessimistic implication.

(38) For every pleasure there is a pain

We can grasp the intended thought, which would be false given many pleasures and just one pain, and which could be true however many pleasures there are. One might reply that the relevant logical form is more like '∀y:Pleasurey(y){∃x:Pain(x)[For(x, y)]', where ∀x∀y∀z[For(x, y) & For(z, y) --> x = z]. But then one has to say what 'For(x, y)' means, and not just what it (allegedly) implies, without appealing to a variant of the following second-order characterization: ∃F∃G(Gx & Gy & |F| ≥ |G|); where '|F| ≥ |G|' means that the number of the Fs is greater than the number of the Gs (or that there is a one-to-one mapping between the Fs and some of the Gs). Similarly, given a hundred submissions, 'For every paper we accepted, we rejected nine' is naturally heard as a claim incompatible with ninety-one acceptances.[12]

Perhaps 'for' is a word requiring special treatment. But if instances of 'For every F, there is a G' are not firstorderizable, then using such instances to gloss '∀x(Fx --> ∃yGy)' is misleading, and (39)

(39) The dogs, most of which are brown, are equinumerous with the cats

seems less idiosyncratic. Moreover, Boolos offers other arithmetically inspired examples that are (given a technique due to David Kaplan) *provably* nonfirstorderizable: there are some horses that are all faster than Zev and also faster than the sire of any horse that is slower than all of them; and there are some gunslingers each of whom has shot the right foot of at least one of the others. As these examples suggest, plural pronouns and relative clauses provide endlessly many examples of nonfirstorderizability, given some elementary relations. Consider the following truism, corresponding to Frege's definition of precedence, with subscripts ('X' capitalized, 'y' not) indicating referential dependence: there are [some people]$_X$ who$_X$ include my parents and every parent of [each person]$_y$ who$_y$ they$_X$ include.

Boolos also discusses examples of "essential plurality" like (40).

(40) The rocks rained down on the mountain huts

No thing can rain down, not even if it is a *collection* of rocks. But if a first-order predicate is satisfied, it

is satisfied by some thing; and if it is satisfied by some things, it is satisfied by each of them. Hence, no

first-order predicate captures the meaning of 'rained down', which can only be satisified (plurally) by

many things. Likewise, Schein (1993) offers (41), noting that no single thing can be clustered anywhere.

(41) The elms are clustered in the middle of the forest

Theorists can *invent* terms like 'collustered', satisfied by collections, stipulating that a thing *is*

collustered in the middle of the forest iff its elements *are* clustered in the middle of the forest. And one

might say that regimentations of (40-41) need not have essentially plural constituents corresponding to

'rained down' or 'clustered'—especially if one is already inclined to say (implausibly, in my view) that

sentence meanings are importantly prior to word meanings. But if we reject first-order regimentalism,

why think that whenever a natural language predicate is satisfied, it is satisfied by some one thing?

This "singularist" hypothesis seems false. It is not forced on us by the very idea of predication;

see Yi (1999). As discussed below, it is not forced on us by the compositional semantics of natural

language. And note that relative clauses can impose nonfirstorderizable restrictions: the rocks *that* rained

down on the huts, *which* were clustered near the lakes, were abandoned by the people *whose* ancestors

had built them. So if Φ is a noun and Ψ is a relative clause, it is not enough (for theorists) to say that

something satisfies the complex predicate Φ^Ψ iff *it* satisifes both Φ and Ψ. For it is also true that some

thing*s* satisfy Φ^Ψ iff *they* satisfy both Φ and Ψ.

3.2 Regimentation or Plural Quantification

Nonfirstorderizability runs deep in natural language. But given a set-theoretic construal, second-order

analyses also seem to be regimentations, as opposed to reflections of propositions expressed with

ordinary sentences. And one does not need examples of essential plurality to make this point.

Prima facie, 'Every bottle is red' does not imply that *there is a set* whose members are the red

things. But '{x: x is red} ⊇ {x: x is a bottle}' does imply this, in the way that 'Brutus stabbed Caesar'

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implies that there is an indvidual who is Brutus. And as Quine (1950) notes, this kind of set-theoretic

regimentation can be recoded in explicitly first-order terms, without any suggestion that predicates are

names for extensions: $\exists\alpha\exists\beta[\forall x(x \in \alpha <\to x \text{ is red}) \& \forall x(x \in \beta <\to x \text{ is a bottle}) \& \forall x(x \in \beta \to x \in \alpha)]$.

So one might think that second-order logical forms just are first-order regimentations involving

quantification over special entities.

Set-theoretic construals also lead to difficulties with examples like (43) and (44).

(43) Every set is a set   (44) Some sets are not selfelemental

If (43) implies that $\exists\alpha\forall x[x \in \alpha <\to x \text{ is a set}]$, we must reject ZF set theory—and the "iterative"

conception of set—according to which $\neg\exists\alpha\forall x[x \in \alpha <\to x \text{ is a set}]$; see Boolos (1975, 1998) for

discussion. Correlatively, if one maintains a set-theoretic construal of second-order quantification while

holding that (43) does *not* imply the existence of a universal set, one might conclude that sentences of the

form 'Every Φ...' have first-order analyses (without covert quantification over sets). And with regard to

(44), one might say that its logical form is '$\exists x\exists y(Sx \& Sy \& x \notin x \& y \notin y)$', in order to avoid the claim

that (44) is true only if $\exists\alpha\forall x[x \in \alpha <\to x \notin x]$. But on this view, endlessly many sentences (including

'Most bottles are red' and 'Most sets have members') have mismatching logical forms, perhaps involving

type-restrictions and quantification over abstracta. So if one wants to avoid mismatches and first-order

regimentalism, in order to avoid the problems *they* breed, one might look for an alternative construal of

second-order quantification: ideally, a construal according to which predicate-variables range over

whatever first-order variables range over (thereby avoiding any special and unwanted ontological

implications), and predicates are not treated (implausibly) as names of a special sort.

This is what Boolos (1984, 1998) offers. Consider (45), an instance of '$\exists X\forall x(Xx <\to \Phi x)$'.

(45) $\exists X\forall x(Xx <\to x \notin x)$

If we interpret '$\exists X$' set-theoretically, with 'Xx' meaning that $x \in X$, then (45) is provably false. For there

is no *set* X such that for each thing x, x is *an element of* X iff x is not an element of x. But alternatively,

we can interpret '∃X' as a distinctively plural quantification over the very same things that 'x' ranges

over, with 'Xx' meaning that x is one of the Xs (*and not* that x ∈ X). Then (45) means that there *are*

some things such that for each thing, it is *one of them* iff it is not an element of itself. Using subscripts:

some things$_X$ are such that each thing$_x$ is one of them$_x$ iff it$_x$ is not an element of itself$_x$. On this

interpretation, (45) is true. For there are some things—like you, me, and at least many sets—each of

which is nonselfelemental. (Indeed, one might think that every thing is nonselfelemental.) This shows

that the plural construal differs from the set-theoretic construal. Only the former makes (45) true.

On this view,

> [N]either the use of plurals nor the employment of second-order logic commits us to the existence of extra items beyond those to which we already committed...We need not construe second-order quantifiers as ranging over anything other than the objects over which our first-order quantifiers range...a second-order quantifier needn't be taken to be a kind of first-order quantifier in disguise, having items of a special kind, collections, in its range. It is not as though there were two sorts of things in the world, individuals and collections of them, which our first- and second-order variables, respectively, denote. There are, rather, two (at least) different ways of referring to the same things, among which there may well be many, many collections (Boolos 1998, p.72).

Let me emphasize, as Boolos does in the last line, that the issue here is not about ontological *parsimony*.

I grant that there are many sets, each related in some intimate way to its elements. But this does not settle

the question of how to construe second-order quantification for purposes of representing logical forms

associated with sentences of natural language. The proposal is that we should invoke distinctively plural

quantification over singular entities, not singular quantification over distinctively plural entities.[18]

A technical question: does (45) imply *two* nonselfelemental things on the plural construal? We

can define '∃X' either way. But let's take it to be compatible with exactly one, using 'Plural(X)' to

signify that there is more than one. Then '∃X:Plural(X)[∀x(Xx <–> Φx)]' implies '∃X[∀x(Xx <–>

Φx)]', which is true iff there are one or more (i.e., more than zero) Φs; and '∃X:¬Plural(X)[∀x(Xx <–>

Φx)]' is true iff there is exactly one Φ. Hence, ∃x[Φx & ∀y(Φy --> y = x)] iff ∃X:¬Plural(X)[∀x(Xx <–>

Φx)].

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While the plural construal is well-known in philosophical logic, it is less familiar (than the set-theoretic alternative) to many philosophers and linguists. So let me note some of its independently attractive features. As we just saw, '∃X∀x[Xx <–> Φx]' is true *whenever* 'Φ' is satisfied. And we can define 'ancestor' in terms of 'parent', without quantifying over sets: Axy iff ¬∃X[∀z(Pzy –> Xz) & ∀z∀w(Xz & Pwz –> Xw) & ¬Xx]. That is, x is an ancestor of y iff the following claim is false: there are some things such that every parent of y is one of them, and every one of them is such that each of its parents is one of them, and x *isn't* one of them. These points bear on the study of logic and its relation to arithmetic. For it turns out that Frege (1903) used his infamous Axiom-V, which implied that every predicate has an extension, only to derive a generalization known as "Hume's Principle": the number of the Φs is also the number of the Ψs iff the Φs correspond one-to-one with the Ψs; see Wright (1983), Heck (1993), Demopolous (1994), Boolos (1998), and for a very helpful introduction, Zalta (2003). Frege proved that the Dedekind-Peano axioms follow from this one biconditional, given his second-order logic *without* the paradox-inducing Axiom-V. This remarkable result suggests that a plural construal of '∃X' is better than Frege's own construal in terms of quantification over extensions (of Concepts).

We can also capture certain patterns across tautologies like those expressed with (46-48)

(46) Nothing is both a linguist and not a linguist

(47) Nothing is both a philosopher and not a philosopher

(48) Nothing is both an avocado and not an avocado

with second-order generalizations that seem to be genuine laws of *logic* (as opposed to set theory); see Putnam (1971). Given a plural interpretation, '∃X∃x[Xx & ¬Xx]' is a contradiction, since it has the following meaning: there are some (i.e., one or more) things, the Xs, such that something is both one of them and not one of them. So '¬∃X∃x[Xx & ¬Xx]', which is definitionally equivalent to '∀X¬∃x[Xx & ¬Xx]', is a tautology: whatever the Xs are, nothing is both one of them and not one of them. Intuitively, this is a fully *general* truth that subsumes endlessly many "content-specific" tautologies

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whose predicative constitutents are (like 'linguist' or 'avocado') not satisfied by everything.[19]

By contrast, given a set-theoretic interpretation, '∀X¬∃x[Xx & ¬Xx]' is shorthand for

'∀α¬∃x[(x ∈ α) & ¬(x ∈ α)]'. And while this is a tautology—for each set, nothing is both a member of it

and not—it is not the "Law of Noncontradiction;" see Quine (1950). On the contrary, one might have

thought that this generalization over sets follows from the relevant logical principle (and not because

every claim follows from itself). Intuitively, a generalization over sets with the relational predicate '∈' is

still too content-specific to be a basic principle of *logic*. Put another way, one expects the relevant logical

principle to explain (or at least subsume) tautologies like those expressed with (46-48); and intuitively, a

generalization over sets cannot play this role, especially once we recognize that the generalization itself

is a relevant tautology.

There also seems to be a pattern across (46-48) and (49), which can be rendered as (49a).

(49) Nothing is both a successor of something and not a successor of it

(49a) ¬∃x∃y[Sxy & ¬Sxy]

But (49a) is not an instance of '∀X¬∃x[Xx & ¬Xx]'. And the plural construal does not provide an

interpretation for formalism like '∃ℜ∃x∃y(ℜxy & ¬ℜxy)'. There is, however, an obvious strategy for

extending the scope of plural construal: treat relational predicates (as Frege did) as predicates satisfied by

ordered pairs. With regard to (49a), we can introduce a monadic predicate '_S_' such that for every ordered

pair o, o satisfies '_S_' iff ∃x∃y[o = <x, y> & Sxy]. Then (49a) is equivalent to '¬∃o[_S_o & ¬_S_o]', relative

to the assumption that <x, y> exists if x and y exist.[20] For convenience, let's say that x is the external

element of <x, y>. Trivially, no ordered pair is such that its external element both is and isn't a successor

of its other (internal) element. So at least for relations characterizable in terms of predicates of ordered

pairs, we can capture the relevant generalization as follows: ¬∃O∃o(Oo & ¬Oo); where 'O' is a device

for plural quantification over the same things that 'o' ranges over. That is, there are no ordered pairs such

that some ordered pair is both one of them and not one of them.

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More generally, we can speak of ordered n-tuples or (to foreshadow) events and a finite number of participation relations like *being an Agent of*. So verbs like 'kicked' and 'gave' do not themselves present serious difficulties for Boolos-style second-order analyses. But there is still a potentially deep problem in the neighborhood. And this brings us back to our main topic.

3.3 A Threat of Collapse

If determiners like 'every' are associated with second-order *relations* (on the model of transitive verbs), how they can be treated as monadic predicates, without treating them as predicates satisfied by ordered pairs of *extensions*? Suppose we say that 'every' is a monadic predicate $\underline{\Delta}$, and that there is some "thing" o such that $\underline{\Delta}$(o) iff every bottle is red. What could o be, if not something like an ordered pair of sets? And what could $\underline{\Delta}$ be, if not a predicate satisfied by ordered pairs <X, Y> such that Y *is a subset of* X? But the plural construal was supposed to provide a way of avoiding these set-theoretic glosses (of quantificational claims and the meanings of determiners); cf. Larson and Segal (1995, ch. 8).

One might worry that all the apparent virtues of the plural construal are now about to evaporate, leaving us with a mess. For quantification and plurality are intimately linked, as indicated by (50-51).

(50) Every bottle is red if and only if every *one of the* bottle*s* is red

(51) If they$_X$ are the bottles, each bottle is red if and only if each *of them*$_X$ is red.

One expects a *unified* account of quantification and plurality. So if determiners turn out to be predicates satisfied by ordered pairs of sets, even on a plural construal of second-order quantification, one might think that 'the bottles' and 'them' really are devices for referring to (or quantifying over) collections. In which case, interpreting 'Xx' as 'one of them' does not avoid appeal to collections after all.

Unsurprisingly, the Frege-Montague conception of propositional architecture pushes in the same direction. Suppose we say that 'kicked' is satisfied by ordered pairs of individuals. (This is a notational variant of associating the transitive verb with a function of type <x, <x, **t**>>.) Then relative to a context in which 'He' and 'it' are associated with x and y respectively, the sentence 'He kicked it'—or using

indices, 'He$_x$ kicked it$_y$' —is true iff <x, y> satisfies 'kicked'. But this reinvites the idea that collective readings of sentences like 'They kicked them' arise from associating plural pronouns with collections as follows: 'They$_X$ kicked them$_Y$' is true, relative to a context that associates 'They$_X$' with (the collection) X and them$_Y$ with (the collection) Y, iff <X, Y>, satisfies 'kicked'. On this view, 'it is one of them' means something like 'it$_x$ is an element of that$_X$ (collection)', and 'Three linguists kicked five bottles' is true on its collective reading iff: there is a three-membered collection X of linguists, and a five-membered collection Y of bottles, such <X, Y> satisfies 'kicked'.[21]

Of course, all the difficulties attending such analyses remain. And as Schein (1993) argues, the triviality of examples like (50-51) tells *against* set-theoretic analyses: speakers regard it as obvious that something$_x$ is a Φ iff it$_x$ is one of the Φs; so if (speakers understand natural language so that) something$_x$ is *one of* the Φs iff it$_x$ is *an element of the set that is* the Φs, then (speakers understand natural language so that) something$_x$ is a Φ only if there is a set α such that $\forall x(x \in \alpha$ iff x is a $\Phi)$. But this condition on being a Φ seems to be false, and there is no independent reason for thinking that speakers tacitly believe it. For speakers can come to believe that while there are many nonselfelemental sets, there is no *set* of all such sets, just as no barber shaves all and only the barbers that do not shave themselves. But one can agree that analyses based on a set-theoretic construal of second-order quantification are unsatisfactory, while thinking that the plural construal is no better. Thus, one might conclude that mismatches between grammatical and logical form are unavoidable, even if this leads us to deny that ordinary speakers (as opposed to theorists) associate natural sentences with logical forms.

In my view, this would be a big mistake. The plural construal has too many virtues; and mismatch/regimentation conceptions of logical form have too many vices. But to really reconcile Fregean logic and natural language grammar, advocates of the plural construal must show how to accommodate sentences with determiners and plural noun-phrases, given a plausible conception of propositional structure and compositional semantics.[22]

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My preferred strategy is to treat determiners as Boolos-style second-order monadic predicates, satisfied by value-entity pairs of the form <**t**, x> or <**f**, x>. The idea, elaborated in section four, will be that 'Every bottle is red' is true iff there are some value-entity pairs that satisfy three conditions: every one of them associates its entity with **t**; they associate (all and only) the bottles with values; and each of them associates its entity with **t** iff its entity is red. But without independent motivation, this might seem *ad hoc*. And by itself, it says nothing about how to embed second-order quantification over value-entity pairs in an otherwise plausible theory that accommodates plural constructions. But luckily, the needed work has already been done (by various theorists) in the context of accounting for some facts that present difficulties for the Frege-Montague conception of propositional structure and compositional semantics.

3.4 Predicate-Conjunction and Events

Davidson (1967) famously argued that the logical form of an action report like (52) involves covert quantification over events, as in (52a), where 'Stabbed(x, y, e)' means that e was a stabbing of y by x.

(52) Brutus stabbed Caesar   (52a) $\exists e[\text{Stabbed(Brutus, Caesar, e)}]$

This allows for an attractive account for certain implications. If (53-55) have the indicated logical forms,

(53) Brutus stabbed Caesar in Rome

(54) Brutus stabbed Caesar on the Ides of March.

(55) Brutus stabbed Caesar in Rome on the Ides of March.

(53a) $\exists e[\text{Stabbed(Brutus, Caesar, e) \& In(e, Rome)}]$

(54a) $\exists e[\text{Stabbed(Brutus, Caesar, e) \& On(e, the Ides of March)}]$

(55a) $\exists e[\text{Stabbed(Brutus, Caesar, e) \& In(e, Rome) \& On(e, the Ides of March)}]$

then the corresponding valid inferences involving (52-55) are instances of conjunction-reduction.[23] Compare: Fido is a happy brown dog; so Fido is a brown dog; so Fido is a dog.

Following Castañeda (1967) and others, it has become standard to go further and say that logical forms also involve appeal to participation relations as in (52b).

(52b) ∃e[Agent(e, Brutus) & Past-Stabbing(e) & Theme(e, Caesar)]

For present purposes, the exact character (and number) of such relations does not matter; we could

replace 'Agent' and 'Theme' with 'Θ_{ext}' and 'Θ_{int}', corresponding to external and internal arguments of

action verbs. But there are constraints. For example, there is no verb in natural language such that

'Brutus VERB Caesar' means that there was a stabbing/kicking/pushing by Caesar of Brutus. Yet there is

nothing incoherent about a verb 'quabbed' satisfied by ordered triples <x, y, e> such that e was a

stabbing by y of x. Natural language evidently precludes certain ways of associating grammatical

arguments with participation relations. It is hard to see how one can describe (much less explain) these

negative facts without appealing to the relevant relations in a theory of how grammatical structure is

related to logical form.[24] But one can view (52b) in different ways: as a reflection of how the *verb*,

satisfied by ordered triples, can be analyzed; or as a reflection of how *grammatical relations* contribute

to meaning, along with a semantically monadic verb, satisifed by events.

One can encode the first view by saying that 'stabbed' is satisfied by <e, x, y> iff Agent(e, x) &

Past-Stabbing(e) & Theme(e, y); where the internal and external arguments of the verb are associated,

respectively, with the Theme and Agent variables. This preserves the Fregean idea that verbs are

semantically "saturated" by their arguments.[25] On the second view, 'stabbed' is satisfied by e iff e was an

event of stabbing; combining the verb with 'Caesar' creates a phrase satisfied by e iff e satisfies

'stabbed' *and* Caesar was the Theme of e; adding 'Brutus' creates a phrase satisfied by e iff Brutus was

the Agent of e *and* e satisfies 'stabbed Caesar'. Either way, the *sentence* (52) involves existential closure

of a monadic predicate as shown in (52b). But if action verbs are predicates of events, then combining

such verbs with grammatical subjects and objects corresponds to predicate-conjunction, much like

combining verb-phrases with prepositional-phrases as in (53-55). Indeed, one can encode the second

view by saying that grammatical relations like *being the external argument of* have the same kind of

semantic significance as prepositions. Correlatively, one can think of (52b) as a conclusion derived from

(52c) and two premises.

(52c) ∃e[External(e, Brutus) & Past-Stabbing(e) & Internal(e, Caesar)]

(56) ∀e:Past-Stabbing(e)[Θe]

(57) ∀e:Θe{∀x[External(e, x) <--> Agent(e, x)] & ∀x[Internal(e, x) <--> Theme(e, x)]}

The idea is that (52c) reflects the purely formal significance of certain grammatical relations, abstracting from the specific participation relations in question; while (56-57) reflect the mapping between grammatical relations and the participation relations that things can bear to events of the relevant kind.[26]

These are different hypotheses about natural language; see Parsons (1990), Schein (1993, 2001 2002, forthcoming), Herburger (2001), Pietroski (2002, forthcoming) for arguments in favor of the latter. One easy way to see the difference is by noting that if 'attacked the duke' is a predicate of events (and adverbial modification corresponds to predicate-conjunction), it follows immediately that 'attacked the duke from Essex' is a predicate of events (satisfied by e iff was an attack *and* the duke was Theme of e *and* e was from Essex). By contrast, if 'attacked the duke' is satisfied by ordered pairs <e, x>, one needs a further assumption to explain why 'from Essex' cannot be linked to the variable 'x'. And even if the further assumption is plausible in this case, the point (for now) is simply that one can adopt the predicate-conjunction perspective, according to which transitive action verbs are semantically monadic.[27]

Correlatively, one can agree that determiners combine with two grammatical arguments (of the right type) to form a sentence, while maintaining that determiners are semantically monadic. For one can think about the logical form of 'Every Φ is Ψ' on the model of (52c): ∃O[Δ(O) & Internal(O, Φ) & External(O, Ψ)]; see §4.1 below. On this view, determiners are indeed like transitive verbs, though not quite in the way that Frege suggested.[28]

We must also consider how to extend event analyses to plural constructions like (3) and (58).

(3) The guests brought four bottles   (58) They brought them

Suppose we say that the logical form of (58), on its collective reading, is as shown in (58a) or (58b)

(58a) $\exists e[\text{Brought(They, them, e)}]$

(58b) $\exists e[\text{Agent(e, They)} \ \& \ \text{Past-Bringing(e)} \ \& \ \text{Theme(e, them)}]$

This suggests that if (58) is true, it is because there was *an* event of bringing in which some individuals brought some things. Likewise, if 'They moved them' is used to correctly describe a situation in which many workers moved many pianos (each being moved by several workers), there must be *an* event of mass-piano-moving. But once we abandon first-order regimentalism, there is no reason to insist that covert quantification over events be first-order (especially not if overt quantification is second-order).

Following Schein (1993, 2002, forthcoming), we can replace (58b) with (58c),

(58c) $\exists E[\text{Agents(E, They}_X) \ \& \ \text{Past-Bringings(E)} \ \& \ \text{Themes(E, them}_Y)]$

intepreting the second-order variables plurally *a la Boolos*: for some events, the Es, they$_X$ were the Agents of the Es, and the Es were events of bringing, and the Themes of the Es were them$_Y$. Here, 'Agents(E, They$_X$)' is satisfied by the Es, relative to a context that associates some individual*s* with the indexed occurrence of 'they', iff the Agents of the Es are *those individuals*; and likewise for 'Themes(E, them$_Y$)'. Correspondingly, we can say that (3) is true on its collective reading iff: some events of bringing were such that their Agents were the guests, and their Themes were four bottles.

For simplicity, let 'Agents(E, X)' mean that the Xs are the Agents of the Es: x is one of the Xs iff x is the (or least an) Agent of one of the Es; $\forall x\{Xx \longleftrightarrow \exists e{:}Ee[\text{Agent(e, x)}]\}$. Let 'Themes(E, X)' mean that $\forall x\{Xx \longleftrightarrow \exists e{:}Ee[\text{Theme(e, x)}]\}$. The important point is that 'Agents(E, X)' and 'Themes(E, X)' can be cashed out in monadic terms. We can now say that (3) is true, on the relevant reading, iff $\exists E\{\text{The(X):Guests(X)[Agents(E, X)]} \ \& \ \text{Past-Bringings(E)} \ \& \ \text{Four(X):Bottles(X)[Themes(E, X)]}\}$. And 'Four(X):Bottles(X)' can be spelled out as '$\exists X{:}\text{Four(X)} \ \& \ \forall x(Xx \longrightarrow x \text{ is a bottle})$': the Xs are four, in the plural predicational sense that the apostles were twelve, and each of the Xs is a bottle. Similarly, 'The(X):Guests(X)' can be spelled out, ignoring (for simplicity) the context-sensitivity of 'the', as '$\exists X{:}\forall x(Xx \longleftrightarrow x \text{ is a guest})$'. As Schein argues, this provides a simple account of examples like (59),

(59) Five composers wrote ten musicals

which can be true if each musical was co-written but no composer collaborated with each of the others.[29] The idea is that (59) has the following reading: for some events, the Es, five composers were the Agents of the Es, and the Es were events of writing, and the Themes of the Es were ten musicals.

Schein (1993) also argues that this kind of analysis is required, to capture the reading of (60)

(60) Three instructors taught five students four theories

according to which: three instructors (together) did some teaching in which each of five students was taught four (perhaps different) theories; where this is compatible with each student encountering at most two instructors. We can represent this reading as follows:

$\exists$E{Three(X):Instructors(X)[Agents(E, X)] & Past-Teachings(E) &

Five(X):Students(X){$\forall$x:Xx

[$\exists$D: $\forall$e(De –> Ee){Recipient(D, x) & Four(Y):Theories(Y)[Themes(D, Y)]}]}};

there were some events whose Agents were three instructors, and which were events of teaching, and for each of five students, some of those events had that student as a recipient and also had four theories as their Themes. Appeal to collections will not capture this essentially plural meaning, according to which there are four Themes per Recipient, implying twenty events (of a student "receiving" a theory) from one or more of the instructors. Similar examples include (61-63).

(61) Three quarterbacks threw five receivers four passes

(62) Three parents read five children four stories

(63) Three doctors wrote five patients four prescriptions (each)

Schein (2002, forthcoming) provides further evidence for such logical forms; see also Pietroski (2002, forthcoming). But this kind of appeal to second-order quantification is at least a viable option when thinking about the logical forms of natural language sentences. Indeed, it may be the best option.

4. Putting the Pieces Together

We can now return to the proposal about determiners floated at the end of §3.3 above.

4.1 Value-Entity Pairs, not Pairs of Extensions

Frege understood second-order quantification in terms of the idea that every predicate has an extension. Boolos showed that this (bad) idea can be detached from appeals to second-order quantification. Given plural quantification, one can also recode the "relational" conception of determiners without appeal to extensions or relations, at least in so far as one is offering a theory about *natural* language.

Recall that for Frege, an expression like 'bottle' ('is red', 'prime number', 'is nonselfelemental', etc.) is not directly associated with a set or any other *entity*. An expression of this sort was said to express an "unsaturated" propositional constituent—a Concept that associates each entity x in the domain with one of two sentential values, $\mathbf{t}$ or $\mathbf{f}$; where each Concept is associated with a "course of values" and by extension, a higher-order entity X such that X includes x iff the Concept associates x with $\mathbf{t}$. It is often convenient to encode this view in set-theoretic terms, especially in light of Frege's claim that we cannot refer to (or quantify over) Concepts, at least not by using expressions of his *Begriffsschrift*. But appeal to special entities is unnecessary, given plural quantification over the value-entity pairs.

Once we say that a predicate like 'bottle' can be satisfied by some things, and that an open sentence like 't_x is red' associates each such thing with $\mathbf{t}$ or $\mathbf{f}$, we have all the *ontology* we need for a semantics of natural language quantificational constructions. We can think about the ordered pair of expressions <'t_x is red', 'bottle'>, with 'bottle' as the internal element of this pair, as a device for associating each bottle with <$\mathbf{t}$, x> or <$\mathbf{f}$, x> but not both; and plural quantification over such pairs is enough, without being too much. For the moment, let's not worry about the distinction between 'is red' and 't_x is red'. The idea is that 'Every bottle is red' is true iff some value-entity pairs satisfy three conditions: *every* one of them associates its entity with $\mathbf{t}$; their entities are (all and only) the *bottle*s; and each of them associates its entity with $\mathbf{t}$ iff its entity *is red*. For convenience, let 'Entity(o, x)' mean that x

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is the entity of o, and let 'Value(o, **t**)' mean that **t** is the value of o. If o is a value-entity pair, then by definition, Value(o, **f**) iff ¬Value(o, **t**). Then 'Every bottle is red' is true iff

$$\exists O\{EVERY(O) \ \& \ \forall x\{\exists o:Oo[Entity(o, x)] <\text{-->} Bottle(x)\} \ \&$$

$$\forall o:Oo\{Value(o, \textbf{t}) <\text{-->} \exists x:Entity(o, x)[Red(x)]\}\};$$

where EVERY(O) iff the Os are value-entity pairs every one of which has **t** as its value.[30]

The capitalization is a reminder that in the metalanguage, 'EVERY' is a second-order predicate that takes one argument. But just as event-theorists say (not that the Es stabbed, but rather) that the Es were past events of stabbing, so one can say (not that the Os are every, but rather) that the Os are value-entity pairs with the value **t**. And one can still specify the *lexical* meaning of 'every' in first-order terms, with a restricted quantifier: the Os satisfy 'every' iff EVERY(O); and EVERY(O) iff $\forall o:Oo[Value(o, \textbf{t})]$.

Predicates like 'set' or 'nonselfelemental' pose no special problem if we say that 'Every Φ is Ψ' is true iff $\exists O\{EVERY(O) \ \& \ \forall x\{\exists o:Oo[Entity(o, x)] <\text{-->} x$ satisfies 'Φ'$\} \ \& \ \forall o:Oo\{Value(o, \textbf{t}) <\text{-->} \exists x:Entity(o, x)[x$ satisfies 'Ψ'$]\}\}$. Trivially, every value-entity pair that has **t** as its value and a set as its entity also has **t** as its value iff its entity is a set. Similarly, every set is nonselfelemental iff: every value-entity pair that has **t** as its value and a set as its entity also has **t** as its value iff its entity is nonselfelemental. Given any set x, along with **t** and **f**, there are the value-entity pairs <**t**, x> and <**f**, x>. But this does *not* imply that given the sets, there is a set that has each of them as an element.

We can handle 'some', which can combine with plural or singular nouns, in various ways. But one option is the following: $SOME_{pl/\neg pl}(O)$ iff $\exists N:Plural/\neg Plural(N)\{\forall o[No <\text{–>} Oo \ \& \ Value(o, \textbf{t})]\}$; though one can also say that $SOME_{\neg pl}(O)$ iff $\exists o:Oo[Value(o, \textbf{t})]$. The determiners 'No' and 'The' can also be accommodated in various ways, in terms of negation and existential/universal quantification. But I take no stand here on just *how* the lexical meanings of quantificational words should be specified, so long as they are specified as monadic predicates satisfied by value-entity pairs.[31]

Most bottles are red is true iff there are some value-entity pairs, the Os, such that: most of them

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associate their entities with **t**; the bottles are the entities of the Os; and each of the Os associates its entity with **t** iff its entity is red. For finite domains, we can say that MOST(O) iff there is a one-to-one correspondence between *some of* the Os that have **t** as their value and the Os that have **f** as their value: ∃T∃F∃U∀o{[To <–> Oo & Value(o, **t**)] & [Fo <–> Oo & Value(o, **f**)] & [Uo –> To] & ¬[Uo <–> To] & one-to-one(U, F)}. For infinite domains, we must appeal to something like cardinalities: MOST(O) iff the Os such that Value(o, **t**) *outnumber* the Os such that Value(o, **f**). But even if the *lexical* meaning of 'most' must be specified relationally, or in terms of cardinalties, this is compatible with the proposal that 'most' is satisfied (plurally) by value-entity pairs.

Put another way, the hypothesis is that the *compositional* semantics for natural language treats 'most' as a monadic predicate imposing a condition on value-entity pairs. This leaves room for what seems to be the case: 'most' imposes a condition that differs in kind from the condition imposed by 'every' (and other determiners with first-orderizable lexical meanings). But we can and should resist the temptation to infer that the compositional semantics, which applies to both 'Every bottle is red' and 'Most bottles are red', involves covert quantification over extensions. We can rewrite Frege-Montague theories, in terms of plural quantification over value-entity pairs, without saying that each predicate is associated with some *thing* that somehow includes each satisfier of the predicate.[32]

4.2 A Potential Explanation for Conservativity

It would be nice, though, to have a reason independent of worries about extensions for treating determiners as semantically monadic plural predicates. One reason is that this is what we should expect, given quantifier-raising and the (Fregean) assumption that sentences are of type <**t**>. In the structure [[every bottle]$_x$ [t$_x$ [is red]]], it sure looks like 'every' combines with a predicate ('bottle') whose values are things that 'x' ranges over and an open sentence ('t$_x$ is red') whose value is **t** or **f** relative to an assignment of an individual to the variable/trace. And this suggests a kind of explanation for the much discussed negative fact that many second-order relations *cannot* be expressed by using determiners.

Nonbottles (even the red ones) are irrelevant to the truth or falsity of 'Every bottle is red'. Correlatively, biconditionals like (64-66) are trivial.

(64) Every bottle is red iff every bottle *is a bottle that* is red

(65) No bottle is red iff no bottle *is a bottle that* is red

(66) Most bottles are red iff most bottles *are bottles that* are red

The general pattern, ignoring tense and agreement, is: [DETERMINER NOUN] be PREDICATE iff [DETERMINER NOUN] be [NOUN that be PREDICATE]. The observation is ancient. But as Barwise and Cooper (1981) note, if determiners "live on" their internal arguments in this sense, there is evidently a *constraint* of some kind on determiner meanings. In particular, if determiners express relations between (extensions of) predicates, they express relations that satisfy the following "conservativity" condition: $\Re(X, Y)$ iff $\Re(Y \cap X, Y)$; where Y corresponds to the internal argument, '$\Re(X, Y)$' means that X bears $\Re$ to Y, and '$\cap$' signifies the (intuitively intersective/conjunctive) meaning of restricting a noun with a relative clause. Trivially, inclusion and intersection are conservative: $X \supseteq Y$ iff $(Y \cap X) \supseteq Y$; $X \cap Y$ iff $(Y \cap X) \cap Y$. And likewise for other second-order relations associated with determiners on Fregean view; see Higginbotham and May (1981), Keenan and Stavi (1986), Westerståhl (1984), Keenan (1996).

One might think that 'only' is an exception, since (67) is false.

(67) Only bottles are red iff only bottles *are bottles that* are red

For it isn't true that only bottles are red; but trivially, only bottles are bottles that are red. Correlatively, nonbottles are relevant to the truth of 'Only bottles are red'. But this is not a counterexample to the generalization, since 'only' is not a determiner. As the medievals recognized, it is a kind of sentential operator; see Herburger (2001). Unlike a real determiner, 'only' can appear before any word in (68),

(68) The student said that his teacher likes her job

with roughly the meaning that (68) is true *and* that some contrasting claim is false—that someone else said the same thing, that there are other students, etc. Still, 'only' is important.

For it reminds us that the converse of inclusion, being among, is not a conservative relation: trivially, $(Y \cap X) \subseteq Y$; but it doesn't follow that $X \subseteq Y$. Likewise, equinumerosity is nonconservative: it is false that the bottles correspond one-to-one with the red things iff the bottles correspond one-to-one with the red bottles. So if there *were* a determiner 'equi' that expressed the relation of equinumerosity, or a determiner 'ryev' that was the semantic converse of 'every', that *would* falsify the generalization about natural language determiners. That is, (69) and (70) would be false.

(69) Equi bottles are red iff equi bottles *are bottles that* are red

(70) Ryev bottles are red iff ryev bottles *are bottles that* are red

And the red nonbottles would be relevant. So it is a substantive (and interesting) fact that there are no such determiners.[33]

In one sense, this just highlights the point that natural language quantification is *restricted* quantification: things that don't satisfy a determiner's internal argument are irrelevant. But in another sense, this reveals a tension between saying that determiners express *relations* between (extensions of) predicates, and saying that natural language quantification is restricted. For if determiners express relations, why can they only express conservative relations? Why should natural language allow for a determiner ('every') that signifies inclusion, but not one ('ryev') that signifies the converse relation? The answer is surely not that the second-order relation *being among* is somehow more complex that *inclusion*, in a way that makes the former inexpressible with a single word. For we have the word 'only'. Indeed, one can put the question this way: why *don't* kids grow up to be adults who speak a language with a *determiner* that means what 'only' seems to mean?

Likewise, we can easily grasp the thought that the women outnumber the men. So why are we unable express to this thought by introducing a determiner 'evenmoor', corresponding to 'outnumber', and saying 'Evenmore women are men'? Of course, 'Evenmoor Φs are Ψs' would typically not have the same truth value as 'Evenmoor Φs are Φs that are Ψs'. It isn't generally true that the Φs outnumber the

Ψs iff the Φs outnumber the Φs that are Ψs. But this doesn't stop us from saying that there are the men, and there are even more women—or that most people are women.

One can speculate that natural language determiners are defined, in a way that preserves conservativity, from "basic" determiners that express conservative relations; see Keenan and Stavi (1986). But even if this is correct, one wants to know why the mechanism for defining determiners has the character it does, and why the linguistically basic determiners do not include 'ryev' or 'equi'. Perhaps it is in somehow arbitrary that we have 'every' but not 'ryev' (though not arbitrary that we don't have both). But one-to-one correspondence is arguably as simple as any second-order *relation*: as noted in section two, it seems to be foundational for arithmetic reasoning; and it is intuitively more basic the relation associated with 'most'. One can say that 'equi' fails to be a basic determiner because it would express a genuine relation between two predicates in the following sense: all the satisfiers of both predicates would be potentially relevant to the truth or falsity of 'Equi Φs are Ψs'. But this is, I think, to admit that (from a semantic perspective) determiners are fundamentally monadic. In which case, it is wrong to characterize the relevant negative facts in terms of a constraint on *which* second-order relations determiners can express. Moreover, a simpler explanation of these facts is available.

If determiners are monadic predicates satisfied by value-entity pairs, then natural language quantification *must* be restricted quantification, with the restrictor being whichever argument of the determiner specifies the entities. In natural language, this is the internal argument; and this is no surprise: Given quantifier raising, the external arguments of determiners are open *sentences*; and sentences are associated with sentential values, **t** or **f**, perhaps relative to assignments of entities to any variables. Internal arguments of determiners (nouns or noun-phrases) are expressions satisfied by individuals, not expressions that are themselves associated with **t** or **f**. In this sense, external arguments of determiners are especially apt for specifying values, while internal arguments are especially apt for specifying entities that can be values of a variable.

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We can, if we like, think of subsentential monadic predicates as expressions of *type* <x, **t**>; and we can, via sequence-relativization, treat an open sentence with one variable as a predicate satisfied by individuals (and perhaps associated with an extension). But we should not let these useful Frege-Tarski techniques, which let us treat determiners as devices for expressing second-order relations, blind us to the grammatical distinction between a word and a sentence from which an expression has been displaced. If the verb 'kicked' is satisfied by events, it differs significantly from the open sentence 't_x kicked t_y'; for the latter gets the value **t**, relative to an assignment of individuals to variables, iff there was an event of kicking whose Agent was x and whose Theme was y. Similarly, we should be alive to the distinction between the noun 'bottle' and 't_x is a bottle' or 't_x is red'. For if determiners are monadic predicates satisfied by value-entity pairs, one expects their external arguments to provide the values, while their internal arguments to proivide the entities. In which case, external arguments do not have the job of introducing their satisfiers for (second-order) comparison with the things that satisfy the internal argument. Rather, the semantic job of the external argument is to associate each relevant entity (determined by the internal argument) with **t** or **f**.

Consider now the proposed logical forms for 'Every Φ is Ψ' and 'Every Φ is a Φ that is Ψ'.

(71) ∃O{EVERY(O) & ∀x{∃o:Oo[Entity(o, x)] <--> Φx} &

∀o:Oo{Value(o, **t**) <--> ∃x:Entity(o, x)[Ψx]}}

(72) ∃O{EVERY(O) & ∀x{∃o:Oo[Entity(o, x)] <--> Φx} &

∀o:Oo{Value(o, **t**) <--> ∃x:Entity(o, x)[Φx & Ψx]}}

The only difference between (71) and (72) is that according to (71), each one of the Os has the value **t** iff its entity is a Ψ; while according to (72), each one of the Os has the value **t** iff its entity is a Φ that is Ψ. Both say that x is an entity of one of the Os iff x is a Φ; and given this restriction to the Φs, there is no difference (relevant to truth) between being a Ψ and being a Φ that is a Ψ. So without *any* assumptions about 'EVERY', we know that an instance of (71) is true iff the corresponding instance of (72) is true.

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Conservativity can thus be viewed as a consequence of semantic monadicity, given an asymmetry (induced via quantifier raising) between internal and external arguments of determiners; see Hornstein and Uriagerecka (1999) for a proposal about the grammaticalsource of this asymmetry. Absent independent support for the idea that determiners are satisfied by value-entity pairs, this might seem to be (at best) a redescription of the conservativity constraint. But as we have seen, there are independent motivations for taking determiners to be second-order/plural predicates satisfied by value-entity pairs. So it is of interest that this hypothesis also lets us account for a striking negative generalization about determiners, without positing any special constraint. Even if this is not yet a real *explanation* of the negative generalization, I think it is better than just saying that determiners cannot be used to express certain second-order relations. This provides a reason, independent of worries about extensions, for treating determiners (like transitive action verbs) as semantically monadic plural predicates.

4.3 Concluding Polemic

This is a good time for philosophers to be actively engaged in the study of natural language. Frege initiated about a hundred years of thinking that logical form diverges from grammatical form in theoretically important ways, even in the simple cases discussed here. The worst reactions to the alleged divergence are subsiding. And given the success of Chomsky's program, we can return to the ancient view that logic and grammar are intimately connected, now informed by significant advances in the study of both; see Ludlow (2002) for related discussion. There was always something odd about the analytic project of trying to study thought by studying language, given the assumption that grammar *masks* the structure of thought. But we need not renounce the "linguistic turn." Instead, we can view 1879 as the start of an unusual period in which the study of logic outpaced the study of grammar, due to Frege's genius and those he inspired. Chomsky and those he inspired closed the gap, by rejecting the idea that grammatical structure must be simple and more or less observable (since even children can recognize it), in favor of the idea that human grammars are transformational and subtly constrained (since children

have a certain kind of innate endowment).

Once we grant that grammatical structure can be as complicated as logical structure, and just as distant from audible features of word strings, we can approach the study of human cognition by combining the insights of modern logic (and not just its first-order fragment) and linguistics. Those deciding where to invest might want to compare this project, in terms of the results it has delivered and its potential for delivering more in the forseeable future, with alternative projects that philosophers of language and mind have been pursuing. My bias in this regard will be evident. Though a more dispassionate assessment might lead to much the same conclusion: for now, our best hope of learning something important about the structure of thought—and giving substance to the ancient idea of language as a mirror of the mind—lies with figuring out how Frege and Chomsky (Montague, Davidson, and many others) could each be importantly right, and no doubt wrong, about the *same* thing: namely, the *shared* syntactic/semantic structure of our sentences/thoughts. My suggestion has been that this structure is more conjunctive, monadic, and second-order than one might think.[34]

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<u>Notes</u>

1. Three caveats. First, I speak of propositions as (abstract) structured entities. But one can speak instead in terns of potential episodes of using sentences to perform speech acts, including (acts of assertion and) acts of advancing potential premises/conclusions for consideration; where some such acts are instances of presenting a valid argument. Second, since attending to use/mention ambiguities will not be especially important here, I often rely on the context to disambiguate, and use single (instead of corner) quotes throughout. Third, it will be harmless (here) to assume that sentence S has value **t** relative to context σ iff S is true relative to σ. But in my view, this is a *big* idealization; see Pietroski (2003, forthcoming-b).

2. See Ludlow (2002) for interesting discussion, drawing on Sánchez (1991), of *dictum de omni* and *dictum de nullo*; see also Kneale and Kneale (1966).

3. For example, letting 'α' and 'β' range over propositional constituents of the sort indicated with names: if α R β and β was one who was a Φ, then α R one who was a Φ.

4. Consider 'No man who *ate after* he found every (brown) dog saw a cat', which is ambiguous. Adding the italicized words to (17) reverses the direction of entailment again; though replacing 'after' with 'before' changes things yet again; etc. See Ladusaw (1979, 1980) and Ludlow (1995, 2002) for discussion.

5. Frege (1892, 1893, 1903) spoke in terms of Concepts and "courses of values;" see section four. But the basic idea has been developed in explicitly set-theoretic terms, with interesting consequences and applications; see Barwise and Cooper (1981), Higginbotham and May (1981), Keenan and Stavi (1986), Keenan (1996) and further reference there. Larson and Segal (1995) provide a helpful introduction.

6. And likewise for (9). This raises an important issue to which I return. One might stipulate that claims about logical form are claims about inferential relations *and nothing else*, in a way that makes it senseless to ask which of two logically equivalent formal sentences better reflects the logical form of (the proposition associated with) some natural sentence? But if we trying to get at whatever *explains* inferential relations—and whatever other explananda are (in fact) related to inferential relations—we must allow for the possibility of distinct but logically equivalent logical forms?

7. This correctly predicts that 'Was the child who in the garden ate' is nonsense, as opposed to the following yes/no question: did the child who was in the garden eat?

8. These points are also relevant to the evaluation of "deflationary" accounts of semantic composition, like the one urged by Horwich (1998); see Pietroski (2000). Perhaps there is a sense in which one can explain why 'attacked the duke from Gloucester' means what does simply by saying that (i) the constituent expressions mean what they do, and (ii) the relevant syntax makes the semantic contribution it makes. But this tells us nothing about why the phrase fails to be a predicate satisifed by individuals from Gloucester who attacked the duke.

9. Chomsky (1995, 2000a) urged a further theoretical reduction, in the context of his "minimalist" program: given constraints on LF, it may be possible to eliminate appeal to specific constraints on DS and SS, in favor of general constraints on how to "construct" expressions (that must be pronounced at some point and eventually interpreted). On this view, DS and SS are not independent "levels" of grammatical structure. Linguistic expressions are simply PF-LF pairs formed in accordance with

constraints that do *not* invoke stages of expression-construction not "visible" to the cognitive systems responsible for pronunication and interpretation. If this reduction is sustainable, it is a powerful argument for appeal to LF.

10. For relevant discussion, see Partee and Rooth (1983), Higginbotham (1985), Hornstein and Uriagereka (1999); for different textbook treatments, see Larson and Segal (1995), Heim and Kratzer (1998). I return to the idea that 'is red' is ambiguous as between a subsentential phrase and a (homophonous) open sentence. Of course, *other* examples—like 'The current administration intends to do nothing for the sake of the average American, who seems not to care'—*may* show that LFs diverge importantly from logical forms. But one needs to provide plausible (mismatching) hypotheses about grammatical and propositional structures, without relying on question-begging assumptions about how truth it is related to propositional structure; see Pietroski (2003b, forthcoming-b).

11. Although there is no reason for thinking that our notion of valid inference is essentially first-order. Boolos (1998, pp. 376-81) discusses a class of short, obviously valid inferences with first-order premises and conclusions such that proving their validity would require astronomically many steps—suggesting that we can reason about the relevant propositions in nonfirstorder terms. And as Boolos notes, while completeness is an interesting property for a formal system to have, it is just one such property among many. Why not prefer regimentation into a language that only allows for *decidable* inferences?

12. See also Higginbotham (1998). One can insist, implausibly, that these are pragmatic effects. But then the following argument is not valid: for every paper accepted, nine were rejected; finitely many papers were accepted; hence, most papers were rejected. And why insist that meanings are first-order if pragmatic implications are not? Perhaps (38) is true iff $\exists n\exists m[n \leq m$ & Numbers(n, the pleasures) & Numbers(m, the pains)]. But why think that (38) implies *as a matter of logic* that given some pleasures, *there is* a number that Numbers them? All (38) seems to imply is that the pleasures are such that: each of them can be associated with a pain not associated with any other one of them.

18. Of course, one must also deny that whenever a variable ranges over some things, some set is the domain over which the variable ranges. But like Boolos, I think we can and should deny this. One must also deny that plural reference and second-order quantification can be *reduced* to singular reference and first-order quantification. But such reduction is not the only theoretical virtue, especially if we are talking about theories of *meaning* for natural language. It is a hypothesis that speakers understand plural constructions in terms of singular reference and first-order quantification over plural entities. Examples like (22), and (14-17) above, tell against the hypothesis. But let me also grant that a plural interpretation of '$\exists X...X...$' may not be best for all purposes. (Logicians, who need not restrict attention to the logical forms of natural language, may prefer the set-theoretic construal for some of their projects.)

19. Perhaps '$\forall X\neg\exists x[Xx$ & $\neg Xx]$' also follows from something more general. But given what predication is, perhaps being one of the Xs just *is* being one of Xs *and not* one of the other things (if there are any). Note that '$\forall x\forall y[x = y <-> \forall X(Xx <-> Xy)]$' is also a truism, whether or not this formulation of Leibniz' Law (which makes no reference to properties) is a basic principle of logic. And we can also define 'ancestor' as follows: $\forall X[\forall z(Pzy -> Xz)$ & $\forall z\forall w(Xz$ & $Pwz -> Xw) -> Xx]$; whatever the Xs are, *if* every parent of y is one of them, and every parent of one them is one of them, *then* x is one of them.

20. Even if this is not guaranteed by logic *alone*, it is far more modest than the assumption that each predicate has an extension; see the appendix to Lewis (1991), cowritten with John Burgess and A.P. Hazen. And even if it does not provide the full resources of quantification into relational predicate

positions, the question here is whether we need more expressive power for natural language semantics. Note also that given sentences of natural language, there are at least ordered pairs of expressions.

21. See, for example, Link (1983, 1991, 1998), Landman (1989, 1996), Schwartzschild (1996). And if it is wrong to say that one collection kicked another, perhaps we should say that <X, Y> satisfies 'kicked' iff X collkicked Y (i.e., the elements of X together kicked the elements of Y).

22. One might hope to avoid the problem by refusing to treat determiners as expressions satisfied by anything, in favor of syncategorematic treatments according to which one says (simply) that each instance of 'Every Φ is Ψ' is true iff every individual that satisfies the instance of 'Φ' satisfies the instance of 'Ψ'. But I think the proposed alternative explains facts that syncategorematic treatements fail to explain; see §4.2 below. I also suspect that such treatments hide difficulties that should be squarely addressed. For example, if a theorist says that 'Most bottles are red' is true iff most individual*s* that satisfy 'bottle' satisfy 'red', one wants to know whether or not the theorist (covertly) quantified over plural entities by *using* the phrase 'most individuals'. Theorists are free to employ a metalanguage that allows for sentences like the following: 'Most bottles are red' is true iff for most x such that x is a bottle, x is red. But then one wants to know what 'most x such that x is a bottle'; means. While 'every' lulls us into not fussing about the distinction between 'every x (such that x is a bottle)' and 'every X (such that an individual is an X—i.e., one of the Xs—iff that individual is a bottle)', perhaps we should fuss, at least with regard to the distinction between 'most x' and 'most X' or 'most Xs'.

23. For reviews of other arguments for event analyses, see Taylor (1985), Parsons (1990), Higginbotham, Pianesi, and Varzi (200x), Pietroski (2003). An example due to Gareth Evans, discussed by Taylor (1985), shows how event analyses can also account for *non*implications. Suppose that Shem hit Shaun sharply with a red stick, and that Shem hit Shaun softly with a blue stick: ∃e[Hit(Shem, Shaun, e) & Sharp(e) & With(e, a red stick)]; ∃e[Hit(Shem, Shaun, e) & Soft(e) & With(e, a blue stick)]. Then as predicted, it *doesn't* follow that Shem hit Shaun sharply with a blue stick (or softly with a red stick). But all the predicted instances of conjunction-reduction are still intuitively valid. (Shem hit Shaun sharply, softly, with a red stick, and with a blue stick.)

24. See Baker (1988, 1997) and Pesetsky (1995) for discussion; see also Jackendoff (1972, 1987), Carlson (1984), Higginbotham (1985), Grimshaw (1990), Parsons (1990), Schein (1993, 2002), Tenny (1994), Levin and Rappaport (1995), Williams (1995) Higginbotham, Pianesi, and Varzi (1999), Hornstein (2002).

25. Higginbotham (1985) speakers in terms of theta-*binding*, contrasting this with theta-*linking*; where the latter corresponds to cases of adjunction, as in (53-55), in which an expression of a certain (thematic) type is extended with a modifier to form an expression of the same type.

26. While negative facts reveal constraints on this mapping—e.g., that internal arguments cannot represent Agents—this leaves room for the possibility that some verbs are satisfied by events with different kinds of participants (say Experiencers); see Pesetsky (1995), cf. Baker (1997). Other verbs may be satisfied by states (see Parsons [1990], Schein [2002]) or even ordered pairs in special cases like 'precedes' (in its mathematical sense); see Pietroski (forthcoming) for discussion.

27. One can say that transitive verbs result from conjoining a predicate satisfied by ordered pairs <e, x> such that x is the Theme of e with a predicate satisfied by ordered pairs <e, x> such that x is the Agent of e; cp. Kratzer (1996). But this is very close to the view urged in the text.

28. As we saw in §2.1, many examples suggest that modifying a noun with an adjective (or prepositional phrase like 'in the yard') typically corresponds to predicate-conjunction. So if combining verbs with their arguments also corresponds to predicate-conjunction, one cannot insist that combining determiners with their arguments corresponds to predicate-saturation. On the contrary, one must consider the possibility that concatenation signifies conjunction more generally; see Pietroski (forthcoming-a) for discussion. One can hypothesize instead that some words are semantically flexible; see Montague (1974), Partee and Rooth (1983). Perhaps 'brown' can indicate function B of type $<x, t>$ such that $B(x) = t$ iff x is brown, or a function B* from functions of type $<x, t>$ to functions of the same type; where $B*(F) = t$ iff $F(x) = t$ & $B(x) = t$. But with regard to examples like (31-33), the explanandum is that the inference goes from 'brown dog' to 'dog', and not *vice versa*. So it is no explanation to say that when 'brown' does not indicate a function of type $<x, t>$, it indicates a function such that inference goes from 'brown dog' to 'dog'. As suggested by the appearance of '&' in the specification of function B*, any explanation for the entailment lies with the (obvious) idea that 'brown dog' is a *conjunctive* predicate.

29. Imagine A working once with B, who worked once with B and C, who worked once with A and D, *etc*. Gillon (1987) tries to accommodate such examples by appealing to (ways of partitioning the class of) plural entities.

30. Note that '$\forall x\{\exists o:Oo[Entity(o, x)] <--> Bottle(x)\}$' is true iff: something$_x$ is the entity of one of the Os iff it$_x$ is a bottle; $\forall x:Bottle(x)\{\exists o:Oo[Entity(o, x)]\}$ & $\forall x:\{\exists o:Oo[Entity(o, x)]\}Bottle(x)\}$. And '$\forall o:Oo\{Value(o, t) <--> \exists x:Entity(o, x)[Red(x)]\}$' is true iff: each$_x$ O has the value t iff the entity of that$_o$ O is red.

31. There are many options for numerals. But following Frege, we can say that some things—apostles, eggs, or pairs of the form $<t, x>$—are *twelve* iff subtracting one of them leaves some things that are *eleven*, and so on; where some (i.e, one or more) things are *one* iff each of them is identical with the same thing. So we say that TWELVE(O) iff there are twelve Os with the value t: TWELVE(O) iff $\exists N:Twelve(N)\{\forall o:[No <-> Oo$ & $Entity(o, t)]\}$. Similarly, one can accommodate 'at least thirteen', 'fewer than eighty', 'between thirteen and eighty', etc.

32. See Pietroski (forthcoming-a) for details about how to deal with complex predicates. But in 'Every cook washed his pots quickly', 'washed his pots quickly' corresponds (given quantifier raising) to an open sentence that gets the value t relative to assignment of an individual x to the variable/trace iff x was the Agent of some quick events of washing whose Themes were the pots belonging to x. So the whole sentence is true iff the value-entity pairs that associate cooks with values all have the value t, and each has the value t iff its entity x was the Agent of some quick events of washing whose Themes were the pots belonging to x. Sentences with more than one variable can be dealt with by relativizing values to sequences in the usual way. For example, [[Every logician]$_x$ [[some bottle]$_y$ [t$_x$ kicked t$_y$]]] is true relative to a sequence σ iff $\exists O\{EVERY(O)$ & $\forall x\{\exists o:Oo[Entity(o, x)] <--> Logician(x)\}$ & $\forall o:Oo\{Value(o, t) <--> \exists x:Entity(o, x)[x$ satisfies '[some bottle]$_y$ [t$_x$ kicked t$_y$]' relative to a sequence σ* that differs from σ at most with regard to whatever σ associates with 't$_x$'; where x satisfies '[some bottle]$_y$ [t$_x$ kicked t$_y$]' relative to σ* iff $\exists O\{SOME_{pl}(O)$ & $\forall y\{\exists o:Oo[Entity(o, y)] <--> Bottle(y)\}$ & $\forall o:Oo\{Value(o, t) <-->$ $\exists y:Entity(o, y)[y$ satisfies '[t$_x$ kicked t$_y$]' relative to a sequence σ** that differs from σ* at most with regard to whatever σ* associates with 't$_y$']\}\}$; where y satisfies '[t$_x$ kicked t$_y$]' relative to σ** iff σ** associates y with 't$_y$', and whatever σ** associates with 't$_x$' (and hence, whatever σ* associates with 't$_x$') kicked y. At this point, one can say that [[Every Φ]$_x$ [...t$_x$...]] is true relative to σ iff $\exists O[EVERY(O)$ & Entities(O, Φ, σ) & Values(O, [...t$_x$...], σ)], defining 'Entities' and 'Values' accordingly.

33. Indeed, it is reminiscent of the fact that there is no verb 'quabbed' such that 'Brutus quabbed Caesar' means that there was a stabbing by Caesar of Brutus. It is also worth considering the relation allegedly associated with 'the': $\Re(X, Y)$ iff $(X \cap Y)$ & $|Y| = 1$. While this relation is conservative, the formally similar relation that imposes the cardinality restriction on the external argument—$\Re(X, Y)$ iff $(X \cap Y)$ & $|Y| = 1$—is not. Given a determiner 'gre' associated with this relation: 'gre bottle is red' would be true iff some bottle is red, and exactly one thing is red; yet 'gre bottle is a bottle that is red' would be true iff some bottle is a red bottle, and exactly one thing is a red bottle.