ABSTRACT

Title of dissertation: ON BEING THE RIGHT SIZE:
A FRAMEWORK FOR THE
ANALYTICAL STUDY OF
SCALE, ECONOMY, AND ECOSYSTEM

Deepak Malghan, Doctor of Philosophy, 2006

Dissertation directed by: Professor Herman Daly
School of Public Policy

If the economy is conceived as an open subsystem of the larger ecosystem, the physical size of the economy relative to the ecosystem that contains and sustains it becomes a salient feature of economic analysis. This key question of scale is therefore one of the central organizing principles of ecological economics. However, scale has mostly been used as a pedagogical device or a heuristic rather than as an empirical tool for environmental policy. The primary bottleneck has been the lack of well-defined theoretical frameworks to empirically measure scale, and to interpret measured values of scale.

Our overarching research question is: how can scale be measured at different levels of economic-geographic aggregations? The seemingly simple question of 'how large is the economy relative to the ecosystem' is fraught with several theoretical difficulties. We develop a novel theoretical framework for empirical measurement of scale based on a simple analytical representation of the economy-ecosystem interaction in terms of stock, flows, funds, and fluxes.

We also develop theoretical frameworks to determine "benchmark scale measures" that address the questions: how large *can* the economy be relative to the ecosystem, and how large *should* the economy be relative to the ecosystem? For scale measures to be useful as tools for environmental policy, a critical requirement, besides being able to empirically measure scale, is a consistent and objective ordinal ranking of two or more measured values of scale. Given two empirical measurements we need to be able to consistently rank the states of the world represented by the scale metric. We develop an axiomatic framework for consistent ordinal raking of scale measures.

The framework developed here helps identify theoretical problems with extant empirical assessments of the biophysical size of economic activity. The biophysical assessments that we review in detail include the Material Flow Analysis methodology, Human Appropriation of the Products of Photosynthesis, and the Ecological Footprint.

Keywords: *Ecological Economics and Environmental Economics*; *Sustainability*; *Empirical Measurement and Ordinal Ranking of Scale*; *Optimal Scale and Maximum Sustainable Scale*; *Axiomatic Properties of a Consistent Scale Metric*; *Material Flow Analysis and Industrial Ecology.*

ON BEING THE RIGHT SIZE: A FRAMEWORK FOR THE ANALYTICAL STUDY OF SCALE, ECONOMY, AND ECOSYSTEM

by

Deepak Malghan

Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2006

Advisory Committee:

Professor Herman Daly, Chair and Principal Advisor

Professor Joshua Farley

Professor Matthias Ruth

Professor Robert Ulanowicz

Professor Jeffrey Herrmann, Dean's Representative

Dedicated to my parents, who have sustained and enriched me
with their love, support, and sacrifice

and

to Nicholas Georgescu-Roegen, my academic grandfather.

ACKNOWLEDGEMENTS

I would like to thank Herman Daly for his unflinching support and guidance during these past five years. It has been a great privilege to have worked on this dissertation under his guidance. Even as Herman provided the most conducive environment for independently pursuing my research interests, his occasional red flags were crucial to the timely completion of this project. Beyond the obvious intellectual debts, Herman has been the single biggest influence on my evolution as a researcher. I have learnt from Herman, above everything else how good ideas outlive the vagaries of ivory tower fashion cycles.

Matthias Ruth has been part of this project from the very beginning. His efforts in the department to create a “Natural Economics Lab” provided me with opportunities to test several ideas presented in this dissertation during informal lab-seminars. Beyond the informal group meetings, I have benefited greatly from his own considered comments and criticisms. Appendix - A originated as an independent study project that Matthias supervised. Matthias has also been a great mentor outside of this dissertation and it was a pleasure being his teaching assistant for two semesters.

Joshua Farley provided important inputs. In particular, chapter - 5 draws upon several discussions that I have had with him. I would also like to thank Josh for hosting me on a visit to his lab. Josh was also generous with his time at three different conferences conferences, and those off-line discussions stimulated a lot of new ideas. I am also grateful to Josh for carefully proofreading my entire final draft.

Robert Ulanowicz provided many useful suggestions. Specifically, I would like to acknowledge his contributions to the discussion on aggregation in chapter - 3.

I would like to thank Jefferey Herrmann, the deans's representative on the committee for facilitating the smooth conduct of the oral defense.

Jack Santa-Barbara has been a constant source of support and encouragement. Some of the central pieces in this dissertation were worked out during a visit to his idyllic country home. I cannot thank Jack enough for those two weeks of enlightened leisure that resulted in the basic framework for empirical measurement of scale. I have also benefited greatly from Jack's insightful comments on several early chapter drafts.

I would like to acknowledge Robert Sprinkle for his critical reading of the dissertation prospectus. Mac Destler helped iron out several administrative bottlenecks as the director of the Ph.D. program. Mac also helped develop a non-technical abstract for the dissertation. Mark Lopez provided useful feedback on the econometric model used to illustrate disaggregated scale metrics.

Matthis Wackernagel helped clarify how the ecological footprint accounting framework measures “overshoot.” Matthis was also generous with his time at a conference where I had a very fruitful discussions with him on the theoretical aspects of ecological footprint.

I literally lived on two different continents during the last five years and the department’s administrative staff bore the brunt of my vagabond life. It is my pleasure to acknowledge Janet Anderson, Makeba Clay, Elisabeth El’Khodary, Zina Evans, Karen Logan, William Powers, Anthony Savia, and Sharon Tazelaar for their patience and support. I would like to make a special mention of Samuel Corvah, the department system administrator who helped with numerous, and often somewhat unreasonable computing demands.

The primary financial support for this research came form a graduate fellowship made possible by a grant from the the Charles Stuart Mott Foundation to my advisor. Additional support included fellowships from the University of Maryland Graduate School, and the School of Public Policy. Maryland Leadership Institute provided summer support.

Fellow graduate students in the department provided a collegial atmosphere. I would like to single out Matthias Deutsch, Manu Matthai, and Puja Jawahar for their special friendship. Haibing Ma and Michelle Parez were wonderful people to share the office with. Tim Gulden provided sagely advice on several occasions. Egor Kraev greatly enriched my graduate school experience by simply being the maverick genius that he is. I would like to thank Sean McCrossin for our qualifying exam study sessions at the Bagel Place. It was an honor to have intersected with Gary Flomonhoft at graduate school.

Aravind Sundaresan provided invaluable advice on all things computers. In particular, I would like to thank Aravind for installing, and maintaining the Gentoo operating system on my personal computer.

It would not be an exaggeration to say that Deepak "Deepu" Iyengar was my lifeline outside the school. Thanks, Deepu for just being there.

I am indebted to R. "Rams" Ramasubramanian for his patience and generosity. Rams endured great odds to grow our small rural electrification business in India that I helped cofound. I left the project at a very early stage to pursue my interests in theoretical ecological economics.

While Rams was ably supported by Ashok Rao and Punit Singh, my debts to Ashok and Punit go beyond their participation in what was supposed to have been a joint project. Ashok and Punit, in very different ways, have been my moral compass for several years. Their unwavering commitment to a just and egalitarian

India has helped reaffirm my hopes for the innate goodness of the human nature to wither the vicissitudes of history.

Venu Govindu has been my principal collaborator in the last five years. Venu's two visits to Maryland, and the time that I spent working with him in India have been some of my most enjoyable moments in the last five years. Our collaboration on a scholarly biography, has been an important source of nourishment for the completion of this research even if it meant stretching deadlines on my dissertation.

The support provided by my extended family has been an important source of sustenence. I would like to thank *Dada*, *Ajji*, *Mama*, *Mami*, *Kaka*, and *Kaku* for indulging me.

My debts to my sister, Varsha are too numerous to recount here. Varsha has been a source of unstinting support, and just knowing that she was only a days drive away helped me tide over difficult periods. I would also like to acknowledge Hemanth for his warmth and friendship. Little Shishir never failed to remind me about the most important things in life.

I thank Hema for her love, patience, and understanding. She, more than anybody else, has borne the brunt of my preoccupation with completing this project.

Last, but certainly not the least, I thank my parents for their love and affection.

My academic progress has been a direct product of their innumerable sacrifices. My biggest regret as I write this is that *Appa* did not live to see me complete my dissertation. In the three years that *Appa* has not been around, *Amma* has given so much, and yet, asked very little in return. I dedicate this dissertation to my parents.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter 1

Introduction

1.1 Preamble

From the time human societies were hunter-gatherers, maintaining life in its entire vicissitude has involved a continuous extraction of a diverse array of resources from the ecosystem. History bears witness to a long and chequered tale of how our species has interacted with the environment. This interaction has not been limited to providing for basic food, water, clothing, shelter, and energy but has included other cultural and material artifacts. One of the dominant themes in this interaction has been the struggle of civilizations to balance the resource needs of cultures on one hand, and the consequent pressures exerted by this resource demand on the natural resilience of ecosystems on the other[McNeill 2001, Ponting 1993]. The imperative to balance our resource needs and pressures on ecosystems is perhaps more salient in our contemporary world than at any other time in history. In the past, particular cultures (or sometimes even civilizations) have come under threat, or faced extinction because of the inability of those cultures and civilizations to strike an appropriate balance between resource needs and the pressures on the ecosystem. However in the present times, our social, cultural, and economic pursuits exert such unprecedented pressure on the biophysical system nurturing us that we have managed to jeopardize not just local and regional ecosystems but even planetary-scale processes like the global climate.

With the ability of the ecosystems to continue supporting different human pursuits under threat, myriad sections of the society ranging from the grassroots, the academia, and the policy community have tried to take stock of the problem and respond in creative ways. These responses are usually studied under the rubric of sustainability or sustainable development. The central question in any sustainability or conservation debate is the size of the human footprint on the biophysical environment. However, key debates that have animated the sustainability discourse have stopped short of acknowledging the centrality of this question. If the human economy is conceived as an open subsystem of a larger supporting ecosystem, the proportional relationship between the physical size of the economy and the supporting ecosystem, or *scale*, is perhaps the most important question to be asked about the relationship between the economy and ecosystem. The primary objective of this dissertation is to develop a theoretical framework for analytical and empirical study of this proportional relationship between the economy and the ecosystem.

The Oxford English Dictionary lists ten different ways in which the word ‘scale’ has been used in the English language. We are interested here in one of the most common usages whose connotation is that of ‘some desirable proportion.’ When we say “something is of the right scale” we are implicitly comparing magnitudes of two different things. By “right” scale we suggest that the two things that we are comparing are in some desirable proportion. Consider some familiar examples from everyday life: “that building is of just the right scale”; “I like the scale of this wedding because it is not very ostentatious”; “That business failed because its scale was too large to be run as a family venture." In each of these three examples we were comparing two related entities and drawing conclusions about how their relative magnitudes. In all three cases we were also asking if the relative sizes of building, wedding, or the business were right in relation to some optimal (or harmonious) proportion. Our focus here is of course a somewhat more involved (but conceptually homologous) problem than a well-designed building, a wedding, or a private business. We are interested in the physical size of the economy in relation to the biophysical system that contains and sustains it. Important as it may be, the study of the physical scale of the economy has received scant scholarly attention. Starting with the ecological economics' basic assumptions about the relationship between the economy and the ecosystem we develop a formal framework to empirically measure, and interpret scale.

More specifically, we develop a consistent framework to answer questions about scale at different levels of economic and geographic aggregations. Human activities impact the biophysical environment at multiple levels of geographic aggregation – starting from the local, to regional, national, and finally global. While the specific nature of how the human economy interacts with the ecosystem varies with the level of geographic aggregation, the central theme is nevertheless the question of the physical size of the economy relative to the ecosystem (appropriately aggregated). Is there a consistent way to measure scale across different levels of geographic aggregation? At each level of aggregation, how does one determine the maximum physical size of the economy that the ecosystem can sustain without losing its resilience?

What is the optimal scale of the economy at different levels of aggregation? This dissertation develops a framework to help answer such questions.

1.1.1 Economics in the Biophysical Dimension

Scale as the proportional relationship between the economy and the ecosystem is intimately tied to a particular conception of how the economy and the ecosystem interact. Empirical assessments of this proportional relationship are relevant for practical policy only if the economy is conceived as a subsystem of the larger ecosystem. This relationship between scale and the conception of the economy is central to the theoretical framework developed by this dissertation. Scale has little conceptual meaning if the ecosystem is conceived as one of the sectors of the larger economy. Following the ecological economics literature, we will consider the conception of the economy as a subsystem of the larger ecosystem to be "preanalytic." Schumpeter [1954] introduced the term "preanalytic vision" to refer to the "distinct set of coherent phenomena as a worthwhile object of our analytic effort." The ecological economics literature has used the term to represent a set of fundamental axioms and the interrelationships between those axioms. In Kuhnian terms the preanalytic vision represents the operating paradigm for a discipline.[1] Perhaps the single biggest achievement of ecological economics has been to provide reasonable evidence that the preanalytic vision of standard economics is incapable of account-

---

[1] See for example Daly [1991b] and Costanza et al. [1997b]. Also see the appendix for a more detailed discussion.

ing for certain well-established biophysical[2] facts that have a direct bearing on the human economic predicament. The ecological economics' preanalytic vision is also shared by sister disciplines like industrial ecology and material flow analysis whose methods we will use in this dissertation.

This dissertation is based on the understanding that some of the most important aspects of human economic predicament have a bearing on the biophysical environment that contains and supports the economy. In very broad terms, economics studies the relationship between physical commodities and human wants. The production and consumption of these commodities have a direct and discernable impact on the ecosystem. The biophysical system is the ultimate source of raw materials for the economy as well as the sink for all the waste products of production and consumption. Traditionally, economics has abstracted these source and sink functions of the ecosystem as being two sub-sectors of the economy, to be treated as any other parts of the economic organization. Economic analysis then involves determining the 'correct price' for the services provided by the ecosystem based on the market exchange value of such services. When the services rendered by the ecosystem are not accounted for in any market exchange, we see the familiar "externality" or the missing markets problem. Variations on the theme of determining socially optimal ways of correcting for these externalites has been the mainstay

---

[2] Ecological economics literature uses the term "biophysical system" to refer to the biological and physical components of the ecosystem that contains and sustains the human economy. As is the case with ecological economics literature, this dissertation will use the terms "biophysical system" and "ecosystem" interchangeably unless otherwise stated explicitly. This treatment is consistent with the understanding that ecology is the study of the relationships that different organisms share with the biotic *and* abiotic environment that they inhabit.

of economic analysis used for environmental policy-making.

In this dissertation we study the economy-ecosystem linkages in biophysical, rather than in monetary terms that is the staple of traditional economics. While the central subject of this dissertation, scale, can be studied only in the biophysical domain, we also propose ways to study the more traditional questions of allocation and distribution in the biophysical dimension. Scale is the physical size of the economy relative to the ecosystem that contains and supports it. Distribution of income is homologous to distribution of the physical throughput between different members of the society. Distribution of wealth can be studied by looking at distribution of ownership of natural stocks. Allocation is the study of how the physical throughput is divided between competing economic ends. We study distribution and allocation in the biophysical dimension because we are primarily interested in knowing how the structure of the economy as represented by access to natural resources (distribution) and use of the physical throughput by the different sectors of the economy (allocation) affect the physical scale of the economy.

Even while we try to build a framework for studying the economy in physical terms, we recognize that the conventional study of economics in monetary terms offers valuable insights. Indeed throughout this dissertation we suggest ways in which the insights from conventional economic analysis in the monetary dimension can be integrated with the analysis presented here. The object of carrying out economic analysis in the physical dimension is certainly not to reinvent the sophisticated models of the economy developed by modern neoclassical economics. Unlike the preanalytic vision of neoclassical economics, the preanalytic vision of ecological economics permits analysis in both monetary and biophysical dimensions. In neoclassical economics, the ecosystem is conceived as one of the sub-sectors of the economy. At a sufficiently large level of economic-geographic aggregation, one can (logically speaking) argue that it is indeed possible to study the economy in its physical dimension, including the scale of the economy starting with the neoclassical vision of how the economy and the ecosystem interact. Scale after all is the proportional relationship between two physical quantities, and in the case of neoclassical economics we could be studying the scale of the ecosystem relative to the economy. Not only is this an 'epicycles approach' to reconcile the empirical problems with neoclassical economics' preanalytic vision inelegant, we will show in this dissertation that it is not always possible (even logically) to study scale with neoclassical assumptions about how the economy and the ecosystem are related. In particular, it is not possible to abstract *all* the functions of the ecosystem that are relevant to the economy when it is conceived as being one of the sub-sectors of the economy. As an illustration, consider a simple stylized example of a forest that is the source of timber for two different industries – a paper mill and a construction company. Studying allocation in the biophysical dimension involves studying how the timber harvested from the forest is distributed between the two industries. Distribution is somewhat more involved. First, we need to account for the property rights – or who owns the timber from the forest? Second, timber is contained in paper products made by the paper mill and the houses built by the construction company, and studying distribution would also entail studying the how the timber embodied in the form of paper or a home is distributed between different members of the society.[3] The primary object of this exercise is to help characterize scale – or the size of the economy (the combined physical size of the paper-mill and the construction industry) relative to the ability of the ecosystem (forest) to support the economy. This treatment of the economy in no ways precludes the analysis of the monetary elements of the economy or traditional microeconomic studies starting with assumptions about consumers' preferences and technologies available for production. Indeed, the understanding of scale is incomplete without incorporating insights from traditional analysis.

In addition to directly studying the economy in the biophysical dimension, we will also employ several metaphors from the life sciences, and particularly ecology to shed light on the question of scale. For example, in our treatment of scale at various levels of economic-geographic aggregation we will draw parallels from the taxonomy-structure used in biology. When studying the scale of a single disaggregated economic unit we will compare and contrast the economic entities' interaction with the biophysical system with how an individual living organism exchanges matter and energy with the environment. The use of biophysical concepts to study the economy and economics concepts to study ecology has a long history.[4] Here, we only want to emphasize that while this dissertation makes contributions to applying ideas from physical and life sciences to economics, ideas from from economics have

---

[3] See Appendix - A for formal models of allocation and distribution in the biophysical domain.

[4] For example, see Martinez-Alier and Schlupmann [1987] and Cleveland [1987].

long been used in ecological studies. Etymology of the term "ecology" has its roots in economics — the Oxford English Dictionary defines ecology as "[t]he science of the economy of animals and plants." Thus this dissertation will at several places, also freely borrow analogies from traditional economics to illustrate ecological concepts that are relevant to the study of biophysical scale of the economy.

1.2 A Program for the Analytical Study of Scale

While several empirical contributions to ecological economics allude to the concept of scale, the lack of a well-developed framework to empirically measure scale has resulted in scale being used as a heuristic at best. If the empirical work has used scale mostly as a heuristic device, the ecological economics research that has tried to clarify fundamental conceptual issues has used scale as a metaphor, and a proxy for describing the biophysical limits on the physical size of the economy. As we will see in the next chapter, there is no one accepted analytical definition of scale. Thus while the concept of scale has served as one of the central and foundational concepts in recent ecological economics literature, its dominant use as a metaphorical and heuristic concept has prevented policy relevant empirical debates being cast in terms of the scale concept. It is somewhat paradoxical that relative to its importance as the central organizing theme in ecological economics, very few works *directly* explore the concept of scale.[5] The focus of this dissertation is to de-

---

[5] For example in a definitive anthology of the intellectual history of ecological economics the editors were able to include only three articles that explicitly explored the nuances of scale [Costanza et al. 1997b]. Even as there are very successful examples of scale-like measures(chapter - 4) in

velop a theoretical framework for the analytical study of scale. Further, even when used as a metaphor, scale has been studied at the global aggregate level – the physical size of the global economy in relation to planet's carrying capacity. We start with the understanding that at every level of economic-geographic aggregation, scale represents the proportional relationship between the physical size of the economy and the ecosystem. Our primary research question is: *how can scale be empirically determined at different levels of economic-geographic aggregation*?

In figure - 1.1 we have illustrated the program for analytical study of scale. The ultimate goal for the program is to be able to use the scale methodology for addressing practical environmental policy questions. There are four parts to the program. Any practical policy is contingent on empirical measurement. Thus the first and the most technically involved part of the scale program is a framework for empirical measurement of scale. After we have selected an appropriate scale measure and determined the empirical value of scale, policy interpretation of the measured value requires some benchmarks. Consider the example of a measured value of scale for the interaction between a logging industry (economy) and forest that supports the industry (ecosystem). Policy relevant questions in this specific example would include: is the logging industry physically sustainable? Beyond physical sustainability is the logging industry operating at a socially optimal level? Benchmark scale measures help answer these questions. From a scale perspective, there are two

---

related disciplines, there has been no research that tries to clarify the fundamental theoretical properties of these empirical measures of the biophysical size of the economy.

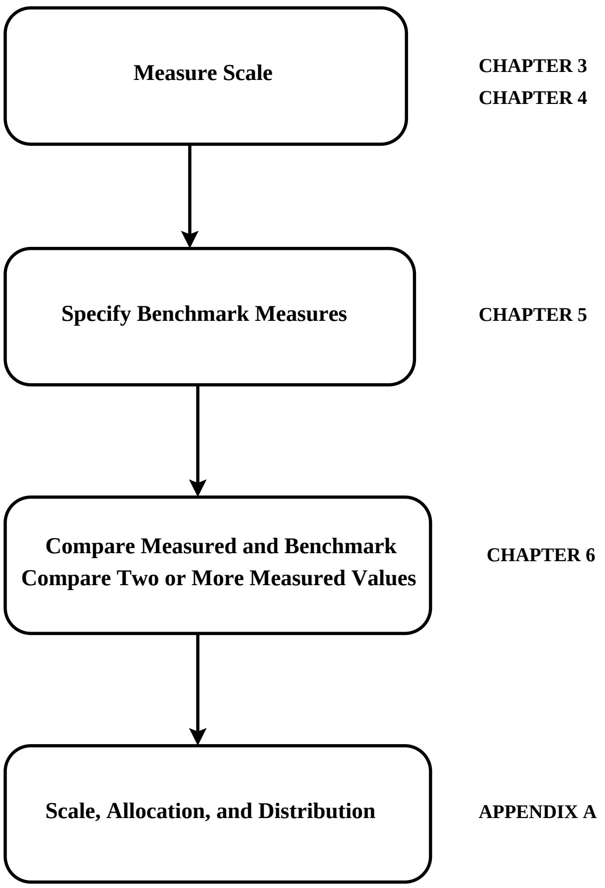

Figure 1.1: The Scale Program. The figure describes the various steps involved in the analytical study of scale along with the chapters in this dissertation that address a given topic.

different sets of benchmarks – one that concerns itself with physical sustainability and the other that is related to social choices. If empirical measurement of scale addressed itself to answering the question "how large is the economy relative to the ecosystem that contains and sustains it," benchmark measures of scale address two overarching policy questions:

1. How large *can* the economy be relative to the ecosystem that contains and sustains it?
2. How large *should* the economy be relative to the ecosystem?

Maximum scale and maximum sustainable scale are the two benchmarks that address the first question, and optimal scale defines the normative benchmark. The relationship between the positive *can* question and the normative *should* question is of central importance if biophysical sustainability is one of the goals of environmental policy.

After we have measured scale and specified benchmarks the next logical step would be to compare the measured value of scale with the benchmarks. For the scale methodology to be relevant for practical policy, we not only need to be able to compare empirically measured values with benchmarks but also be able to rank two or more measured values of scale. Continuing with our logging industry example, how should practical policy-making interpret changes in measured values of scale over time? In order to answer such questions, we need to to be able to rank two or more measured values of scale. Thus the next logical step in the scale program is a consistent framework to rank states of the world represented by two or more empirically measured values of a scale metric. Scale, as we discussed earlier is only one aspect of how the economy and ecosystem interact. A complete economic analysis would try and relate scale to more traditional concerns of allocation, and distribution. More specifically, we are interested in the allocation and distribution processes that result in a particular proportional relationship between the economy and the ecosystem.

1.3 Chapter Organization

Following this introductory chapter, in chapter - 2 we survey the extant literature on scale. The primary objective of the survey is to help contextualize the contributions of this dissertation. Chapter - 2 will also serve as a point of departure for a systematic review of three major biophysical assessments of the economy that we carry out in chapter - 4. Scale as a proportional relationship between the economy and the ecosystem can benefit from other disciplines that study proportional relationship between two entities. We briefly review some contributions from fields as diverse as classical music, architecture, and ethics. Indeed the title of this dissertation is plagiarized from a seminal essay by biologist J.B.S. Haldane that used metaphors from biology to mediate on social organization [Haldane 1995].

Chapter - 3 is the first of the five chapters where we sequentially develop the "scale program" discussed in figure - 1.1. There, we systematically develop a framework to answer the central research question for this dissertation: how can scale be measured at different levels of economic-geographic aggregation? Starting with the preanalytic vision of ecological economics we develop a coherent analytical representation of how the economy and the ecosystem interact.

Following the development of the basic framework for empirical measurement of scale, we validate the framework by applying it to study four well-known biophysical assessments of the economy. Chapter - 4, the longest chapter of this dissertation makes use of the framework developed in chapter - 3 to interpret biophysical assessments that use material flow analysis and accounting frameworks based on photosynthesis flows to measure the physical size of human activity. Besides helping refine the theoretical framework this exercise also serves to identify significant theoretical problems with some of the biophysical accounts that we review.

Chapter - 5 develops an analytical framework for benchmark measures. We treat physical sustainability as a question of maximum, and maximum sustainable scale. We also present a framework for optimal scale and show how optimal scale is related to maximum scale and maximum sustainable scale.

Chapter - 6 develops an axiomatic framework for ordinal ranking of scale measures. We develop a set of axioms to define a “consistent scale measure” and show that the consistency criteria that we develop are sufficient for meaningful policy relevant interpretation of scale measures. Following this framework for comparing two or more measured values scale-measure, we take up the question of how scale is related to allocation and distribution. Traditional environmental policy has primarily focussed on efficiency of allocation with distributional consequences snapped on mainly in a retrofit fashion. Ecological economics has emphasized the importance of the physical size of the economy. Our discussion in appendix - A will present a new framework to understand how scale is related to allocation and distribution. We conclude this dissertation with a summary of key results, significant lacunae, and a future program for research in chapter - 7.

Chapter 2

State of the Art: Literature Review

2.1 Introduction

Scale as a Proportional Relationship

The characteristic feature of any scale metric is that it is a measure of the *proportional* relationship between the economy and the ecosystem. If scale is conceived first and foremost as a proportional relationship, several other disciplines ranging from classical music to civil engineering have lessons to offer. While the focus of this dissertation is on scale that is related to the physical sizes of the economy and the ecosystem, we have borrowed some of the metaphors from diverse fields in the development of a framework to study the interaction between economy and the ecosystem. In chapter - 5 where we develop a framework for maximum scale and maximum sustainable scale, we borrow concepts from civil and structural engineering's study of proportional relationship between stress and strain. The plagiarized title of this dissertation comes from a seminal essay by one of the founders of population genetics, J.B.S. Haldane [Haldane 1995]. The concept of optimal scale or the most desirable scale is studied by various disciplines including art, architecture, aesthetics, and ethics. A good example is LeCorbusier's architecture classic, *The Modulator* [Jeanneret-Gris 1954]. A particularly relevant study from architecture that we use in the development of optimal scale is Peter Smith's exposition of the concept of "harmony" within architecture [Smith 1987]. On optimal scale, we also borrow from D'arcy Thmoson's magnum opus [Thompson 1992].

While it is tempting to review a broad swathe of literature relevant to the conception of scale as a proportional relationship, we focus our attention in this chapter on the relevant ecological economics literature relating to the biophysical scale of the economy. Contextualizing the ecological economics concept of scale that this dissertation develops within the larger literature about proportional measures is an important goal for the scale program that we hope to address elsewhere. Even as our primary focus is on a survey of how scale has been used within the ecological economics literature, we will take detours into allied disciplines like industrial ecology that are also centered around studying the economy as a metabolic process. Even within ecological economics, the goal of this chapter is a narrow survey rather than an annotated bibliographic essay. Our focus here is on the literature relevant to theoretical aspects of scale. Chapter - 4 builds on this brief chapter in the form of a systematic review of some specific scale-like metrics in the literature.

2.2 Scale in Ecological Economics

As we discussed in the introductory chapter, the ecological economics literature has limited itself to scale as a dialectical concept. The key reason for ecological economics not using scale for analytical and empirical research has been a lack of consensus definition for scale. The literature that we survey in this section helps uncover alternative interpretations of scale besides the notion of scale as a proportional relationship between the economy and the ecosystem.

The fact that there is no precise understanding of scale as an analytical concept in the extant literature is best illustrated by looking at how scale is treated in a recent introductory textbook coauthored by one of the principal progenitors of the scale concept in ecological economics. Daly and Farley, in their recent textbook define scale as:

> [t]he physical size of the economic subsystem relative to the ecosystem that contains and sustains it. It [scale] could be measured in its stock dimension of population and inventory of artifacts, or in its flow dimension of throughput required to maintain the stocks"[1]

The above 'textbook definition' of scale contains two different interpretations of what constitutes scale. If scale is the size of the economic subsystem *relative* to the ecosystem that contains and sustains it, scale cannot be measured in the stock dimension as "population and inventory of artifacts." Population or inventory of artifacts could be absolute but not relative measures of the physical size of the economy. This inconsistency is explained by the relative maturity of the literature that studies scale at the conceptual level on one hand, and the lack of well-defined frameworks for empirical studies of scale on the other. Scale as a proportional relationship between the economy and the ecosystem is clearly understood at the basic conceptual level – indeed one of the key sources of motivation for this dissertation comes from the seminal paper by Daly that for the first time clearly proposed scale

---

[1] Daly and Farley [2004, p.439]

as the physical size of the economy relative to the ecosystem, and showed why conceptually the question of scale is distinct from the more traditional questions of allocation and distribution [Daly 1992]. However, there have been no systematic efforts to analytically characterize scale as practical empirical tool to understand the relationship between the economy and the ecosystem.

In addition to scale being used to denote absolute, instead of relative physical size, it has also been used to refer to limits and thresholds. A phrase like "economy has exceeded scale" is encountered in many places in the literature.[2] Here, the terms "scale" and "optimal scale" have been used interchangeably. This rather lose use of the term "scale" amounts to a confusion between what *is* and what *should* (or *can*) be. As we illustrated in the introduction to this dissertation, scale is a positive metric that describes a particular aspect about the biophysical dimension of the economy while maximum sustainable scale and optimal scale are respectively physical and normative benchmarks.

Admittedly all three interpretations of scale are directly derived from the preanalytic vision of ecological economics – that of an economy as an open subsystem embedded within the larger biophysical system. The absolute physical size of the economy is of little importance to the human economic predicament if the economy were not an open subsystem of a larger system that is non-growing in material terms; and limits and thresholds make sense only when there are biophysical constraints

---

[2] See table - 2.1 below for specific instances.

on the physical size of the economy. While the ‘absolute-size’ and ‘limits-threshold’ interpretation of scale implicitly require the economy to be an open subsystem of the ecosystem, they do not explicitly recognize the existence (or lack thereof) of any kind of *proportional relationship* between the economy and the ecosystem.

The distinction between scale as a dialectical versus scale as an empirical and analytical concept only partly explains the three different interpretations of scale discussed above. Scale as the absolute physical size of the economy or as thresholds and limits is conceptually meaningful only at the global level because the global biophysical system is finite. At lower levels of economic-geographic aggregation, there is at least a theoretical case to be made for the possibility of expansion of the containing ecosystem. However, we show in this dissertation that even when the basic preanalytic vision of ecological economics allows for expansion of the ecosystem at lower levels of aggregation, with a stock-fund representation of the ecosystem this is seldom possible in practice — stocks and flows can cross economic geographic aggregations but funds and fluxes cannot. The primary motivation for studying the complex interactions between the economy and the ecosystem comes from the fact that there is broad recognition of a sense of ‘loss of proportion’ that manifests itself in myriad forms like pollution, congestion, sprawl at local and regional levels, to climate change and ozone depletion at the global level. To recollect, one of the central features of this dissertation is the study of scale at *multiple* levels of economic-geographic aggregation.

Multiple levels of economic geographic aggregation directly leads us to the fourth and the most common usage of the term “scale” in the literature. In addition to scale referring to the physical size of the economy (absolute size, relative size, or in the sense of limits and thresholds), it has also been used in a more common-sense definition of the word to refer to ecosystem trophic hierarchy in ecology, and to economic-geographic aggregation in the economic parts of the ecological economics literature.

2.2.1 Literature Review

In Table - 2.1, we summarize the four principal interpretations of scale in ecological economics. We have limited our summary here to research that explicitly uses at least one of the four interpretations of scale. This is not meant to be an exhaustive survey but is to be read as a review of how some influential contributions to the ecological economics literature have interpreted scale.

Table 2.1: ***Interpretation of Scale in Ecological Economics.*** *This table looks at recent ecological economics literature through the 'scale' lens. The table entries, 'YES/NO' represent research that is open to multiple interpretations (See main text for details).*

Cited Work	*Highlights*	*Interpretation of Scale*			
		Absolute Size	*Limits-Threshold*	*Proportional Relationship*	*Hierarchy or Level of Aggregation*
Vitousek et al. [1986]	Widely cited paper estimates the portion of terrestrial net primary productivity that is appropriated by the humans.	No	YES	YES	NO

continued on the next page

Table 2.1: ...*continued*

Daly [1991b]	One of the first papers to use the term "optimal scale." Makes a case for optimal scale as a goal for macroeconomic policy.	YES	YES	NO	NO
Daly [1992]	The seminal paper that introduced the concept of scale into ecological economics literature. Presents scale as being an economics question that is different from allocation and distribution.	YES	IMPLICIT	NO	NO
Schrder [1995]	Formal treatment of optimal scale as defined by Daly [1992] in terms of the elasticity of macroeconomic output *w.r.t.*energy and natural resources.	No	YES	YES	NO

continued on the next page

Table 2.1: *…continued*

Lawn and Sanders [1999]	Tries to determine optimal scale for Australia through marginal cost-benefit analysis of GDP growth.	YES	YES/NO	YES/NO	NO
Lawn [2001]	Tries to clarify the distinction between ‘scale’ and ‘allocation’ issues in ecological economics. Develops a heuristic model to measure “ecological economic efficiency”. Argues that the problem of achieving optimal scale can be reduced to constrained maximization problem with maximum sustainable scale providing the constraint.	YES	YES	YES/NO	NO

continued on the next page

Table 2.1: *…continued*

Gibson et al. [2000]	Looks at different spatial and temporal scales at which social and natural phenomenon intersect.	NO	NO	NO	YES
Jordan and Fortin [2002]	The paper argues that from a sustainability perspective, it is important to explicitly recognize the spatial and temporal dimensions of scale. In particular, the author argues that treatment of scale in ecological economics following Daly [1992] has not considered spatial topology in studying scale.	YES	YES	YES	YES

continued on the next page

Table 2.1: *...continued*

Daly and Farley [2004]	The first major 'principles' text for ecological economics.	YES	IMPLICIT	IMPLICIT	NO

continued on the next page

Table 2.1: *…continued*

Daly [1998]	This paper tries to defend the often-cited 'valuing nature' article (Costanza et al. [1997a])by suggesting that valuing the 'exchange' value of natural services is actually a proxy for the aggregate use value of nature that has been lost. Daly uses the logic developed by the Earl of Lauderdale who first suggested that there is a tension between creation of marginal value (private value) and loss of aggregate use value (public value).	IMPLICIT	NO	IMPLICIT	NO

continued on the next page

Table 2.1: *…continued*

Illich [1994]	This is part of a Feschrift Symposium for Leopold Kohr. Summarizes how Kohr looked at proportional relationships. Kohr's work that clearly lies outside the rubric of ecological economics is nevertheless directly related to our research here.	NO	NO	YES	NO

Chapter 3

Empirical Measurement of Scale: An Analytical Framework

3.1 The Ecological Economics Vision of the Economy

To motivate the development of a framework to empirically measure scale, we start by looking at the ecological economics' vision of the relationship between the economy and the ecosystem. Figure - 3.1 below is a simplified representation of the relationship between the economy and the ecosystem. The figure shows the economy embedded within the ecosystem. Also shown is the flow of matter and energy from the ecosystem into the economy and back to the ecosystem. A key point to note from the perspective of this dissertation is that figure - 3.1 represents the relationship between the economy and the ecosystem at every economic-geographic resolution.[1] The picture represents the global economy with a global ecosystem as much as it represents the relationship between a paper mill and the forest that supports the paper mill. At the global level the ecosystem is materially closed for all practical purposes with the only input into the system being incident solar radiation. The picture is somewhat more complicated at lower aggregations with diverse material and energy flows into the system. At higher resolutions, this basic picture in figure - 3.1 repeats itself — the larger global ecosystem contains smaller aggregations. While this conception of the economy is seemingly non-controversial from a biophysical perspective, mainstream environmental economics' ontological picture of the economy-ecosystem relationship is inverted with the ecosystem being

---

[1] We use the terms aggregation and resolution interchangeably to refer to the level of economic-geographic aggregation. A high resolution corresponds to low aggregation.

conceived as one of sub-sectors of the economy. Scale, the focus of this dissertation is a useful concept only within a particular ontological framework represented by figure - 3.1.[2]

The arrow representing the flow of degraded matter and energy from the economy to the ecosystem is 'broken' to illustrate the entropic nature of the physical throughput through the economy. The low-entropy matter and energy from the ecosystem is transformed into high-entropy waste.[3] In figure - 3.1 below, what we see most clearly is a snapshot in the material dimension. The size of the ellipse is indicative of the size of the ecosystem and the size of the rectangle represents the portion of ecosystem that is currently appropriated by the human economy.[4] Not immediately seen in the simplified picture is the fact that the size of stock can be changing in time. For example consider a paper mill (economy) drawing timber from a forest (ecosystem). The forest is continually regenerating timber while the paper mill extracts timber out of the forest through logging. Depending on the relative rates of regeneration and logging, the stock of wood contained in the forest is either growing, shrinking, or is in a steady-state.

---

[2] Also refer to the appendix and our previous discussion in chapter - **??**.

[3] We assume here that the system boundary for the analysis is drawn in such a way that an economic process involves entropic degradation. It is of course possible that processes within a smaller economic-aggregation actually results in lower entropy. For example, if we focussed on just the metal in a metal extracting industry, the high-entropy metal-ore is converted to low-entropy metal by the metal extraction industry. The more general point here is that at all levels of analysis, there is an entropic transformation of the natural throughput. Unless otherwise stated we implicitly assume an entropic degradation when we refer to any entropy change – an assumption that is consistent with our primary focus on scale.

[4] In section - 3.2.1, we will see how figure - 3.1 simultaneously represents the physical material stocks as well as "fund" of ecosystem services.

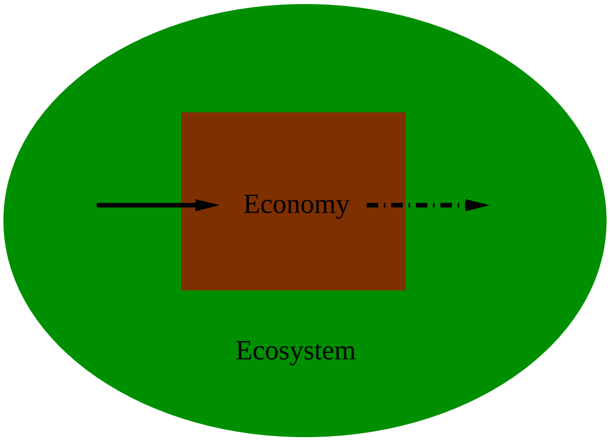

Figure 3.1: A Simplified Representation of Ecological Economics' Ontological Vision

Scale is a measure of the proportional relationship between the economy and the ecosystem that supports the economy. Our preliminary discussion immediately suggests that a complete understanding of this proportional relationship requires measuring scale on four different dimensions and interpreting their interrelationships. First, the scale of the economy may be measured either on the source-side or on the sink-side. Source-side of the economy represents the part of throughput from ecosystem to the economy and the sink-side of the economy represents the part of throughput from the economy back to the ecosystem. The entropy difference between the source-side and sink-side requires that we treat them independently. The ecosystem's ability to support a given throughput is different on source and sink sides of the economy. For example, consider the process of extracting and burning coal. On the source-side, scale is a measure of how the extractive industry is related to the ability of the ecosystem to support a given throughput. On the sink-side, scale is related to the ability of the ecosystems to absorb products of combustion including carbon-di-oxide and ash. Carbon in the form of coal is very different from carbon in the form of carbon-di-oxide.

Second, scale can be measured either in the stock dimension or the flow dimension. In the stock dimension, we get a snapshot of the proportional relationship between the economy and the ecosystem. Scale measured on the flow dimension describes how the physical size of the economy is evolving with respect to ecosystem's ability to support the physical throughput driving the change. Consider the paper mill example again where a paper mill (economy) uses timber from the forest (ecosystem). In the flow dimension we have the amount of timber harvested from the forest every year on the source-side; and the factory effluents, and discarded paper products (that embodies timber used by this mill) on the sink-side. Stock of standing timber in the forest as well as the stock of timber embodied in the products of the economy change over time. Scale in the flow dimension compares the natural regrowth of timber to the rate at which timber is harvested. On the sink-side, scale in the flow dimension will compare the rate at which timber from the forest is 'consumed' (either in making paper or from paper products that are discarded) and the ability of the ecosystem to sustain this waste throughput. Scale in the stock dimension will involve studying the *current* state of the forest as a source of timber and ecosystem as a sink for waste products of timber use.

The schematic in figure - 3.2 presents a rough taxonomy of scale measures that we develop in this dissertation. In addition to summarizing the introductory discussion, the schematic taxonomy also introduces one of the central themes for this dissertation – developing a framework to empirically measure scale that is consistent across different levels of economic-geographic aggregation. The scant literature on scale metrics treats the concept of scale at the global level. Several of the most important global ecological problems have ramifications at lower levels of economic-geographic aggregation. For instance, our coal burning example not only precipitates the well-known green house gas (GHG) induced global warming, but has impacts at the local level (mining and local air quality) as well as regional levels (regional air quality). Ecological integrity at the local level may be compromised before effects are felt at more larger geographic aggregations. Starting with this chapter and continuing in the following two chapters our primary goal is the systematic exposition of the taxonomy presented here. Central to understanding the proportional relationship between the economy and ecosystem are three questions: how large *is* the physical size of the economy relative to the ecosystem that sustains and supports it? How large *could* the economy be relative to the ecosystem? And finally, how large *should* the economy be relative to the ecosystem? The recurring theme in this dissertation is the theoretical ramifications of juxtaposing these three questions and the rough taxonomy for scale metrics presented here.

The rest of the chapter is organized as follows. In section - 3.2 we review some key concepts from ecological economics as an exercise in setting up the toolbox needed to study scale. In our toolbox-section, we expand on the ontological position of ecological economics described above from the perspective of measuring the physical size of the economy relative to the containing ecosystem. Of the three

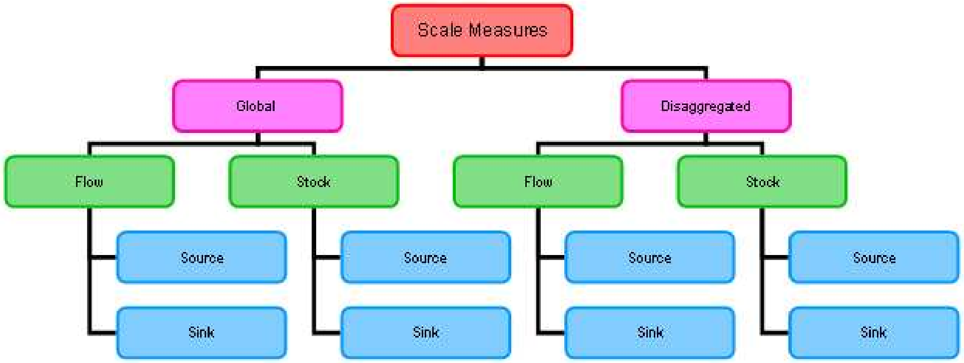

Figure 3.2: Scale-Measures: A Rough Taxonomy

central questions that punctuate the analytical study of scale, we will begin by developing a framework to answer the first question of how large *is* the economy relative to the ecosystem in the flow dimension. In section - 3.3, we develop a framework to empirically measure scale in the flow-dimension. In section - 3.4 we introduce a formalism for dimensionless metrics in the fund-flux space. Section - 3.5 delves into theoretical problems with construction of scale metrics in the stock dimension. Finally in section - 3.6 we look at role of temporal dimension empirical measurement of scale.

3.2 Setting up the Toolbox: Basic Concepts and Definitions

In this section we review some of the foundational concepts of ecological economics and refine them for our current purpose – empirical measurement of scale. We begin by clarifying how the ecosystem is simultaneously a *stock* that gives rise to the material flow and also a *fund* of ecosystem services. After we have clarified the essential differences between the stock and fund functions of the ecosystem, we look at how the concepts of natural capital and natural income are related to stock and fund functions of the ecosystem. We conclude this section by clarifying how natural income is treated on the source-side and the sink-side.

3.2.1 Stock-Flow and Fund-Flux

The economy as an open subsystem is connected to the larger ecosystem through two different ‘flows’ that are fundamentally different from each other. The first flow is the familiar material throughput. The ecosystem is the ultimate physical resource base for the economy and is also the ultimate sink for waste products that are an inevitable part of any economic process. We will use the term *resource flow* to denote this physical throughput.[5] *Resource flow* will be used to denote the material throughput on both the source-side as well as the sink-side. Thus we will use the term ‘resource flow’ to refer to both the amount of timber harvested by the paper mill in a given year and a particular effluent released by the factory in the same in the same time period. The relationship between stock of resources and the corresponding resource flow is readily understood on the source-side but needs careful and somewhat more nuanced analysis on the sink-side – the subject matter of the next section. Here it is sufficient to note that on both the source-side and the sink-side, material flows can accumulate into stocks and are depleted by outflows from the stock. On both the source-side and the sink-side, the stock at any given

---

[5] We argue elsewhere in this dissertation that the framework developed here is applicable to energy. Our primary focus however is on material flows.

time is given by:

$$x(\tilde{t}) = x(0) + \int_0^{\tilde{t}} (f_{in}(t) - f_{out}(t))\,\mathrm{d}t \tag{3.1}$$

where $f_{in}(t)$ is the flow into the stock at any time t and $f_{out}(t)$ is the outflow from the stock. $\tilde{t}$ is the current time period and $x(0)$ is the reference stock at time $t = 0$. Equation - 3.1, of course is the solution of the simple differential equation that is an accounting identity that holds good on both the source-side as well as the sink-side:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f_{in}(t) - f_{out}(t) \tag{3.2}$$

We make extensive use of this elementary relationship between material flows and stocks to develop a framework to measure scale.

The forest is not just a stock of timber but also provides valuable services like micro-climate stabilization. Unlike material flows, there is no way to write out an accounting identity like equation - 3.2 where a ‘flow’ of micro-climate stabilization service accumulates into any stock. Analytically intractable as they may be, these services provided by the ecosystem are vastly more crucial that the material flows derived from natural stocks. While several material flows can be replaced by flows derived from human-made stocks, substitution of critical services provided by the ecosystem is, in general, not feasible. Thus while timber from the forest can be replaced by a wood substitute, the micro-climate stabilization service provided by the forest is indispensable. There is of course a definite connection between the magnitude of material flows and the more abstract service like micro-climate stabilization. The stock of timber that is the source of material flows is after all one of the constituents of the forest. One of the key concerns of this dissertation is to develop tools that will help us discern the nature of the relationship between material flows from the ecosystem and valuable services like micro-climate stabilization. A fund is a special configuration of a given stock of material(s). For example a *special* configuration of the given stock of steel, aluminum, plastic, etc. constitutes the automobile which is a fund of transportation services. The operative words here are *special configuration* — the same stock of steel and aluminum in any other configuration does not constitute a fund of transportation services though they continue to contain the exact same amount of physical material. The most obvious example will be an automobile which has met with an accident and is completely mangled and on its way to the junk yard.

Thus while simple conservation laws (like the mass balance accounting identity in equation - 3.2) are sufficient to fully characterize the relationship between stocks and flows, the relationship between a fund and the service derived from it are more complex. The fund-service relationship is more appropriately characterized by laws that follow the spirit of the entropy law in thermodynamics.[6] The ability of the fund to provide a service is contingent on a *particular* configuration of the stocks that

---

[6] Georgescu-Roegen [1971] in addition to providing the original exposition of the concept of fund also speculated on a entropy law modeled on the Second Law of thermodynamics for matter. This so-called 'fourth law' has been hotly contested. For our purposes here, it is sufficient to note that conservations laws (like the First Law of thermodynamics) alone cannot completely describe a fund and we need to invoke some mechanism like the Second Law that allows for qualitative degradation of energy *and* matter.

constitute a fund. Like an automobile, the forest in its role as a fund of valuable services is contingent on a particular configuration of the stocks that make up the forest. Moreover the natural regeneration of the any constituent stock is dependent on the structure of the underlying configuration. Thus a captive plantation with the same standing stock of timber as an old growth forest will have different regeneration rates for the timber stock and the micro-climate stabilization service provided.

The service derived from a fund is not a physical flow like the material flow from the stock but is nevertheless to be treated as flow in the sense that it has a 'per unit time period' kind of dimension to it. The difference between flow and service on the time-dimension is crucial to understanding the difference between ecosystems as stocks and ecosystems as funds. Flow as represented by either f_{in} or f_{out} in equation - 3.2 represent rates of flow. For example with the timber example, f_{out} is the rate at which timber is harvested and could have the dimensions of $tons/year$. On the other hand it makes little sense in the case services derived from funds to talk about (*micro climate stabilization service/year*). In fact as Georgescu-Roegen [1971, p. 227] shows, the time dimension for a service is part of the amount of service and not the service rate. Automobile as a fund of transportation service has a life time of say ten years. In this time, the rate at which the service is delivered is fixed – some finite person-miles in any given time period. This is in stark contrast to automobile as a stock of steel. One could potentially 'use up' all the steel in the car in one instant or 'mine' the car for steel at say 10 grams a minute. The key fact is that for stocks and material flows, the time dimension is part of the flow rate and the amount of material is independent of time. For a fund and service, the amount of service has an irreducible time dimension to it while the service rate is independent of time. One of the corollaries of this difference in how the time-dimension is related to stocks and funds is that the human economy has little control over the rate at which a service can be extracted from the ecosystem in its role as a fund but ecosystem as a stock can (at least theoretically) be liquidated in an instant. However, the same physical system is simultaneously a stock as well as the fund. In the same way an automobile would no longer be a fund of transportation services if we took away all the steel embodied in the automobile, the forest will cease to be a fund of micro-climate stabilization if all the timber in the forest is harvested.

Services derived from the ecosystem in its role as a fund usually have very small 'rates of flow.' We will use the term *service flux* to distinguish service (derived from the fund function of the ecosystem) from the material flow (derived from the stock function). A *flux* unlike a *flow* is invisible but is nonetheless impressionable. A flux is not amenable to simple additive arithmetic of flows. The different service fluxes derived from various ecosystems are critical not just for the survival of the human economy but all biological life. In the ecological economics' ontological conception, the human economy can indeed be conceived as one of the groups of stocks that makes up a fund of ecosystem service fluxes.[7] The integrity of the fund is related to an harmonious balance between the different stocks that make up the fund. We will

---

[7] In an analytical study of the economy in the biophysical dimension the system boundary between economy and ecosystem can be a contentious issue. See discussion on material flow analysis in chapter - 4.

see how a fund needs to be constantly nourished by material flows to help maintain the essential "configuration" of the fund. Analytical study of scale in the main involves developing a coherent accounting framework to reconcile how the human economy generated material and energy flows affect the ecosystem in its role as a fund. The policy relevant questions of maximum sustainable scale and optimal scale, we will see, are analytically a problem of discerning the 'mapping relationship' between the stock-flow and fund-flux spaces.

3.2.2 Natural Capital and Natural Income

One of the highlights of development of ecological economics in the last two decades has been the ascendency of the the concept of "natural capital." Like the concept of "human capital," the concept of natural capital is rather amorphously defined. In the same way as human capital yields an income, natural capital yields a flow that is the "natural income." The use of the term 'capital' is based on the understanding that broadly defined, capital is anything that yields income and the traditional use of the term in economics to refer to manufactured equipment is only a specific example.[8] The concept of natural capital is amorphous because ecosystem is simultaneously a stock of material flows and a fund of service fluxes. Both the material flows as well as the service fluxes are constituent elements of natural income. Natural capital is an aggregate term used to simultaneously denote both

---

[8] This functional definition of capital as anything that yields a flow or income is the reason why capital is used in the context of human capital or natural capital. For example one of the early papers in ecological economics to discuss the concept of natural capital, Daly and Costanza [1992, p. 38] uses this functional definition like most of the new literature on economic growth that makes use of human capital.

the fund and stock functions of the ecosystem. The concept of natural capital was developed mainly in the context of the sustainability discourse. The goal of 'strong sustainability' is to maintain non-diminishing natural capital while that of 'weak sustainability' is to maintain a non-diminishing sum of natural and manufactured capital.

3.2.3 Source-Side and Sink-Side: Key Differences

We begin by looking at a simplified version of the basic conception of the economy presented in figure - 3.1. In figure - 3.3, we show a snapshot of the economy supported by a throughput which is $\dot{x}_i(t)$ on the source-side and $\dot{x}_o(t)$ on the sink-side with the subscripts i and o standing for input and output respectively. In general the throughput cannot be treated as a single flow because $\dot{x}_i(t) \lesseqgtr \dot{x}_o(t)$, with all three cases possible. Only in the steady-state case of $\dot{x}_i(t) = \dot{x}_o(t)$ can the throughput be represented by a single flow. When $\dot{x}_i(t) \lessgtr \dot{x}_o(t)$ we have the economy shrinking or growing with the change in the stock of of the economy as measured in the dimension of x is given by the the accounting identity, equation - 3.2. The concept of scale is of interest primarily when the economy is not in a steady state.[9]

Figure 3.3: Source and Sink Aspects of the Throughput

---

[9] We will see in chapter - 5 that concepts of optimal scale and maximum sustainable scale are indeed defined in steady-state terms. The point is that while the steady-state serves as a necessary

It is for this reason that the arrows representing $\dot{x}_i(t)$ and $\dot{x}_o(t)$ in figure - 3.3 are drawn of different thickness. Figure - 3.3 does not dilute the fact that the ecosystem besides being the ultimate source[10] is also the ultimate sink for the waste products of the economy. Everything that goes 'into' the economy does indeed 'come out.' However, figure - 3.3 represents a snapshot in time and at any given time, t, there is no reason for the two sides of the throughput to be necessarily of equal magnitude. For example all the wood contained in the paper made from timber in the forest will eventually be returned to the forest but in any given year the amount of timber harvested by the paper mill need not be equal to the paper discarded by the economy. If it were, the economy's stock of paper would remain in steady state. This simple idea of the possibility of the two sides of the throughput being quantitatively different will be central to the framework for measuring the physical scale of the economy.

The two sides of the throughput are not only quantitatively different but are qualitatively different. In figure - 3.3, the arrows representing inflow into the economy, $\dot{x}_i(t)$ and outflow from the economy, $\dot{x}_o(t)$ are not only of different thickness but also shaped differently. As we alluded to earlier, the throughput on the source side and sink side are qualitatively different because of the entropic nature of the economic process. Even when the two sides of the throughput are quantitatively

---

benchmark for defining what is a socially optimal scale and what is a physically sustainable scale, the scale-methodology itself is most useful to discern the relative magnitudes of the economy and the ecosystem for systems that are not in steady state – perhaps with the implicit normative policy goal of guiding the economy towards an optimal steady-state.

[10] Implicit here is the fact that solar energy is the 'ultimate' source in the strictest sense. However, for our purposes here, it is indeed proper to treat the terrestrial ecosystem as the ultimate source – especially of material flows.

similar, there is a necessary qualitative difference. Consider for instance the coal burning example. If the arrows in figure - 3.3 tracked the flow of carbon through the economy, it is a matter of simple mass conservation that a kilogram of carbon in coal will exactly be a kilogram of carbon in the combustion products of coal. However, the carbon in the form of coal is very different from carbon in the form of carbon-di-oxide. Indeed the biophysical dimension of the human economic predicament is directly linked to the fact that there is an entropy change associated with the every economic process. At an appropriately drawn system boundary, the sink-side of the throughput is more degraded than the source-side.

3.2.4 Modeling Regeneration

An important corollary of the qualitative and quantitative difference between the throughput on the source and sink sides is the readily observed fact that the stock that generates the throughput is physically distinct from the stock that makes up the sink. However the two stocks that make up the source and sink can indeed be part of a common fund. For example while the stock of wood in the forest and the stock of discarded paper products from the economy are clearly distinct stocks, the forest and the river can, and often are part of a common larger fund that is the source of service fluxes across the watershed. Figure - 3.4 below summarizes our discussion here. In the figure, the human economy derives benefit from all three stocks – the source, the sink and the economy. The benefit derived is shown as a utility flux. This flux is not physically quantifiable and is made up of three components –

the two ecosystem service fluxes from the source and sink, and the traditional utility flux from artifacts in the human economy.[11] The utility flux is of course related to the fund aspect of the three stocks. Utility is derived from an automobile as a fund of transportation service and not a stock of steel. Similarly, the contribution of the ecosystem to the utility flux is from the fund-role of the ecosystem. In figure - 3.4 utility, U consists of three parts – $\breve{Y}_{so}$, from the ecosystem fund in its role as a *source* of resource flow ($\dot{x}_i$); $\breve{Y}_{si}$, from the fund function of the ecosystem as the *sink* for waste flow ($\dot{x}_o$); and finally $\breve{Y}_{so}$ from the fund function of the material stock in the economy. The representation of the throughput is straightforward. The parameters t_i and t_o, a function of economic decision making influence the source-side and sink-side of the throughput, $\dot{x}_i$ and $\dot{x}_o$ respectively.

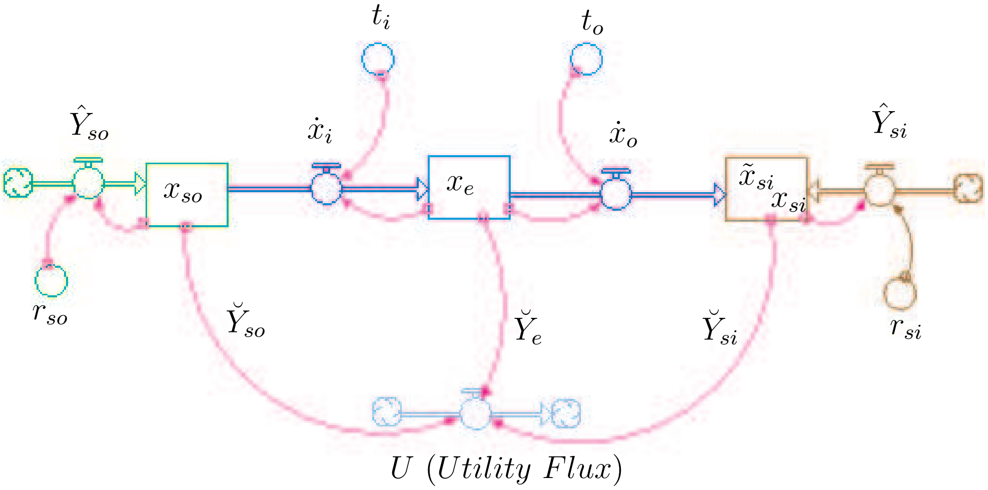

Figure 3.4: Source, Sink, Natural Capital, and Natural Income

---

[11] We recollect that this dissertation even while recognizing the intrinsic worth of ecosystem and the resource flows and service fluxes derived from it abstracts from all non-anthropogenic concerns as our primary purpose here is the measurement of the scale as the physical size of the *human economy* relative to the ecosystem. Also see section- -5.4, where we suggest ways in which non-anthropogenic concerns could be integrated into a normative framework to determine optimal scale.

We now turn to the regeneration portion of fig - 3.4. On the source-side, $\hat{Y}_{so}$ is the inflow associated with the stock x_{so} in every time period. This regeneration flow is a function of the standing stock (x_{so}) and the regeneration parameter, (r_{so}).[12] We will treat $\hat{Y}_{so}$ as one component of the natural income derived from the ecosystem on the source-side with the other component being the service flux $\breve{Y}_{so}$. Unlike the service flux, $\breve{Y}_{so}$ that is not readily quantified, the natural income on the resource flow dimension, $\hat{Y}_{so}$ has the same physical dimensions as $\dot{x}_i$. For instance, consider the 'timber from a forest' example – the annual income represented by $\hat{Y}_{so}$ is simply the amount of new wood that the forest adds each year and has the same dimensions as $\dot{x}_i$, the throughput of timber into the economy. For non-renewable resources like coal or oil, there is no income, $\hat{Y}_{so}$ as new coal or oil is not produced in time scales that are relevant to the human economic predicament. Thus on the source-side, we have two different kinds of income – one from the ecosystem in its role as a stock of raw materials ($\hat{Y}_{so}$) and the other from the ecosystem in its role as a fund of service flux ($\breve{Y}_{so}$). What are the corresponding flow and flux incomes on the sink-side? We turn next to modeling the aspects of natural income on the sink-side where things are somewhat less analytically tractable.

Our primary interest here is in the physical and tangible waste that is absorbed by the ecosystem in its sink function and not the various indirect service benefits like climate regulation that are indeed derived from the ability of ecosystems to absorb

---

[12] Figure - 3.4 show the regeneration rate represented by the parameter r_{so} to be independent of x_{so}. While we have not shown how r_{so} is related to the health of the ecosystem fund (x_{so}) the discussion here and elsewhere in the dissertation (especially section - 5.4 treats regeneration rate as being endogenously determined by the underlying fund structure of x_{so}.

wastes generated by economic processes. The service benefits are represented by $\breve{Y}_{si}$ in fig - 3.4 and is similar to $\breve{Y}_{so}$ on the source-side. Here we model the sink-analogue of the regenerative resource flow, $\hat{Y}_{so}$ on the stock dimension. In fig - 3.4, the waste stream from the economy, $\dot{x}_o$ flows into the sink. Waste absorption as physical process typically involves change in the chemical and/or physical structure of the waste. When acid rain falls on a lake, the lake water has the ability to neutralize some of the acid. River water similarly has ability to get rid of certain amount of sewage waste every year. The amount of waste that is physically or chemically absorbed and is no longer distinguishable as waste is the natural income from the ecosystem in its role as a sink. If new wood is what is regenerated as natural income on the source side in the timber example, the natural income on the sink side of our river example corresponds to regeneration of waste absorption capacity. Thus as an analogue of resource regeneration $\hat{Y}_{so}$ on the source-side, what is being regenerated on the sink-side is absorption capacity for the waste stream.

In fig - 3.4 this regeneration is represented by $\hat{Y}_{si}$. Unlike the regeneration $\hat{Y}_{so}$ on the source-side, $\hat{Y}_{si}$ is an abstraction from the actual physical process. The actual physical process would be represented by the waste stream, $\dot{x}_o$ flowing into the sink and a flow out of the sink. However the stock that represents the sink in fig - 3.4 has both the flows flowing *into* the box. The waste stream from the economy, $\dot{x}_o$, has the same physical dimensions as the source-side of the throughput, $\dot{x}_i$. However, the regenerative flow on the sink-side, $\hat{Y}_{si}$ represents a 'flow of new regenerative capacity.' To use an analogy from electrical engineering and physics, resource regeneration on the source-side and sink capacity regeneration are like electrons and holes. Holes are abstract theoretical constructs while electrons are a physical reality. Nevertheless it is useful to study regeneration of ecosystem's ability to absorb waste in the same way as it is useful to study positive charge in semiconductor physics as 'holes'. In fig - 3.4 we have the following simple accounting relating waste stock, x_{so} and holes, $\tilde{x}_{so}$.

$$\frac{\mathrm{d}x_{si}}{\mathrm{d}t} = -\left(\frac{\mathrm{d}\tilde{x}_{si}}{\mathrm{d}t}\right) = \dot{x_o} - \hat{Y}_{si} \tag{3.3}$$

As a consequence of this simplified representation on the flow-dimension, the stock representing the ecosystem in its sink function is more complex that the ecosystem stock on the source-side. On the sink side the ecosystem stock is represented by a combination of waste stock (x_{si}) and holes ($\tilde{x}_{si}$). The sink is in constant flux with new holes being created by the regeneration process, and holes 'consumed' by the waste-stream. If the waste flow ($\dot{x}_o$) exceeds the rate at which holes are created ($\hat{Y}_{si}$), there will be an accumulation of waste in the ecosystem and if the regeneration exceeds waste flow into the sink, there will be excess holes in the sink.[13]

3.2.5 Four Components of Natural Income

From the perspective of studying the scale of the economy, this representation of the sink-side is particularly useful because we can directly compare the throughput ($\dot{x}_o$) with regeneration ($\hat{Y}_{si}$). In the same way as we treat $\hat{Y}_{so}$ as the natural

---

[13] Equation - 3.3, is not a good description of a pristine sink ($\dot{x} = 0$). If we assume $\hat{Y}_{si} > 0$ (absorption capacity being continually regenerated), then equation - 3.3 implies that the 'stock' of holes or $\tilde{x}_{si}$ is monotonically increasing. This apparent anomaly is explained by the fact that our representation of the sink in fig - 3.4 does not account for 'natural death' of holes.

income on the source-side, $\hat{Y}_{si}$ represents a natural income on the sink-side. This natural income (measured as holes per unit time) is as physical and tangible as the regeneration of stock on the source-side – the regeneration of absorption capacity is as tangible as regeneration of timber.[14] Like on the source-side the magnitude of this income is determined by the size of the stock from which the income is derived. The size of the stock is a function of the economic throughput on both the source-side and the sink-side. When the timber withdrawal exceeds the forest's ability to regenerate new wood, the natural capital (in its role as a fund) is affected and there is subsequent reduction in new timber that is generated. Similarly, when the amount of the sewage flowing into a river exceeds the capacity of the river to absorb wastes, the natural capital (in its sink function) is affected and there is a reduction in the amount of waste that can be absorbed.

The four different natural incomes that we have introduced here ($\hat{Y}_{so}$; $\breve{Y}_{so}$; $\hat{Y}_{si}$; $\breve{Y}_{si}$) will form a key part of the framework to empirically measure scale. This disaggregation of natural income into four components is necessary because the economic throughput affects these four components differently (but not necessarily indepen-

---

[14] On the sink side there are certain important caveats that we will not discuss here. The present discussion is not an accurate description of the oceanic sinks for carbon-di-oxide for example. A more complete exposition will include physical features of the ecosystem that makes this logical abstraction of 'sinks as holes' possible. In particular, we will need to include the mechanisms through which the sink is able to generate new holes. Our characterization of natural income here suggests that sinks can be treated as biophysical systems—living and teeming with life. Regeneration is the distinguishing characteristic of life be it regenerating wood in the forest, or the capacity to absorb wastes in the form of "holes." However, the technical and philosophical issues surrounding treatment of source and sinks as living entities is beyond the scope of this dissertation. We present this model of regeneration as a convenient framework to facilitate an analytical study of scale and will be careful to point to the limitations of the model when we discuss specific empirical examples.

dently). We use the term “component” rather imprecisely. The four “components” do not add up to give the sum-total of natural income that is derived from the ecosystem but rather represent four facets of the natural income that are indeed separable. Two of these incomes are derived from the ecosystem in its role as a stock of resources (or holes in the case of sink) and two from the ecosystem in its role as a fund. We recollect that on both the source-side and the sink-side the stock and fund are the same physical entity and the two stocks (source and sink) can even be part of a larger common fund. Table - 3.1 below summarizes this discussion. Table - 3.1 and fig - 3.4 contain all the elements necessary to build a framework to empirically measure scale. We start this exercise in the next section by looking at a flow-measures of scale.

Natural Income, $\mathbf{Y_n}$			
Source, $\hat{\mathbf{Y}}_{\mathbf{so}}$		Sink, $\hat{\mathbf{Y}}_{\mathbf{si}}$	
Resource flow $\hat{Y}_{so}$	Service flux $\breve{Y}_{so}$	Flow of Holes $\hat{Y}_{si}$	Service flux $\breve{Y}_{si}$

Table 3.1: Four Components of Natural Income

A Note on Notation

We have carefully chosen the notation to represent various elements of natural capital, natural income, throughput. The notation used here is consistent with standard study of mass-transfer as well as standard economics literature. Thus a $\dot{x}$ represents flow out of stock x and Y represents some kind of income. We will reserve the bold-faced $\mathbf{Y}$ (with appropriate subscripts) to represent compound natural income that is made up of more than one of the four component incomes. Y with a regular typeface will be used to represent individual components. We summarize the notation for natural income in equation - 3.4 below.

$$\begin{aligned}
\mathbf{Y_{so}} &= \{\hat{Y}_{so}; \breve{Y}_{so}\} && \textit{Source Side} \\
\mathbf{Y_{si}} &= \{\hat{Y}_{si}; \breve{Y}_{si}\} && \textit{Sink Side} \\
\hat{\mathbf{Y}} &= \{\hat{Y}_{so}; \hat{Y}_{si}\} && \textit{Stock Income} \qquad (3.4) \\
\breve{\mathbf{Y}} &= \{\breve{Y}_{so}; \breve{Y}_{si}\} && \textit{Fund Income} \\
\mathbf{Y_n} &= \{\hat{Y}_{so}; \breve{Y}_{so}; \hat{Y}_{si}; \breve{Y}_{si}\} && \textit{Aggregate Natural Income}
\end{aligned}$$

We will read $\hat{Y}$ as "Y-stock" and $\breve{Y}$ as "Y-Fund." Following equation - 3.4, we can represent the economy generated throughput and ecosystem stocks as compound entities:

$$\mathbf{x} = \{x_{so}; x_{si}, \tilde{x}_{si}\} \quad \textit{Ecosystem Stock} \tag{3.5}$$

$$\dot{\mathbf{x}} = \{\dot{x}_i; \dot{x}_o\} \quad \textit{Throughput} \tag{3.6}$$

The notation chosen for the two components of throughput needs some explanation. We have used subscripts i and o instead of so and si because $\dot{x}_{so}$ or $\dot{x}_{si}$ mean 'rates of change' of the respective stocks in the standard notation. Rather, the rate of change of stock is the difference between the throughput and the corresponding natural income.

The 'natural income' used here closely mirrors Hicksian income that by definition is sustainable. While we will take up the conceptual difference between natural income defined here and the notion of income used in traditional neo-classical resource economics elsewhere, some preliminary comments are appropriate here. In terms of the notation used here, the traditional dynamic optimization problem where a discounted utility function is maximized over state variable represented by the resource-stock and the control variable, throughput can be written as:

$$\max \int\limits_{t=0}^{t=T^*} \left[e^{-rt} U\left(x_{so}(t), \dot{x}_{so}(t)\right)\right] \mathrm{d}t \tag{3.7}$$

In the traditional notation, the utility, U would have been a function of throughput that is denoted by Y or 'income.' Our income, $\hat{\mathbf{Y}}$ is different from the Y used in traditional resource economics to refer to withdrawal or throughput.

We close this brief section by emphasizing again that it makes no physical sense to write $\mathbf{Y_n} = \hat{Y}_{so} + \breve{Y}_{so} + \hat{Y}_{si} + \breve{Y}_{si}$ and certainly not $\dot{\mathbf{x}} = \dot{x}_i + \dot{x}_o$ or $\mathbf{x} = x_{so} + x_{si} + \tilde{x}_{si}$ The compound incomes, stocks and the throughput in the equations above are *not* a sum of constituent components. In the context of developing metrics for scale, we sometimes use the term "vector" a convenient conceptual placeholder to denote a compound entity. Not all of the compound quantities in the equations above are mathematically 'vectors.'

3.3 Flow Measure of Scale

We begin the development of the formal framework to measure scale by studying the relationship between the economy and the ecosystem in the flow dimension. In terms of the taxonomy presented in figure - 3.2, we will start by looking at conceptual elements that are invariant across levels of aggregation. We study specific aggregation issues surrounding scale metrics at varying resolutions in chapter - 4. Scale as a proportional relationship between the economy and the ecosystem is *measured* in the stock-flow space and not the fund-flux space. In fig - 3.4 the economy, represented by stock x_e is 'connected' to the economy through the throughput represented by $\dot{x}_i$ and $\dot{x}_o$. The quantum of throughput does affect the ecosystem as a fund, and consequently the service fluxes that are derived from the fund but this effect is a secondary consequence of the effect of the throughput on the stock. Most decision-making by the economic actors is informed by the relationship between the throughput and the level of ecosystem stock rather than the quality of ecosystem fund. The so-called 'externalities' result from economic agents not able to factor the effects of throughput on the ecosystem as a fund.

The fund-flux space, besides not being directly connected to the economy is also beyond empirical measurement. To recollect, scale is a measure of the proportional relationship between the economy and the ecosystem and answers the question: how large is the economy relative to the ecosystem? Empirical measurement of scale presupposes that the metric can be characterized by cardinal numbers.

While scale itself is measured in the stock-flow space, policy relevant questions of maximum sustainable scale and optimal scale indeed take into account the effect of the economy on the fund functions of the ecosystem. The empirically measured values of scale are compared against benchmarks of what is sustainable and what is socially optimal. This comparison requires only an ordinal ranking of different empirically measured values and in chapter - 6, we show how it is indeed possible to characterize a fund by ordinal indices rather than cardinal numbers, and also present a framework for ordinal ranking of scale. To recollect, the use of scale as a tool for environmental policy is thus a two-stage process. First, we empirically measure scale and them compare the measured value of the scale with benchmark values of scale.

3.3.1 Scale as a Ratio of Throughput and Natural Income

On the flow dimension, scale measures the magnitude of the economy generated throughput relative to the ability of the ecosystem to support that throughput. We have seen in the previous sections of this chapter why this measurement is to be carried out independently on the source-side and the sink-side. The most straightforward way to measure the proportional relationship between the economy and the ecosystem is to look at the ratio of throughput and the corresponding natural income. Thus the flow-measure of scale is defined as:

$$\mathbf{S} = \frac{\dot{\mathbf{x}}}{\hat{\mathbf{Y}}} \tag{3.8}$$

The key point to note from the definition of the flow-measure of scale, $\mathbf{S}$ is independent of any service flux as we consider only the natural income derived from the ecosystem in its role as a stock. This follows our argument that scale is to be measured in the stock-flow space and not the fund-flux space. Equation - 3.8 is to be read as a symbolic pseudo-equation, as the division of two vectors, $\dot{\mathbf{x}}$ and $\hat{\mathbf{Y}}$ has no mathematical meaning. The equation only suggests that scale in the flow dimension is a ratio of throughput ($\dot{\mathbf{x}}$) and natural income ($\hat{\mathbf{Y}}$). The vector scale measure $\mathbf{S}$ is actually evaluated as two different scalar measures for the source-side and the sink-side respectively.

$$\mathbf{S} = \begin{pmatrix} S_{so} \\ S_{si} \end{pmatrix} \tag{3.9}$$

The scalar measures of scale on the source-side (S_{so}) and the sink-side (S_{si}) are defined following equation - 3.8:

$$S_{so} = \frac{\dot{x}_i}{\hat{Y}_{so}} \tag{3.10}$$

$$S_{si} = \frac{\dot{x}_o}{\hat{Y}_{si}} \tag{3.11}$$

Equations 3.10 and 3.11 simply compare the ‘withdrawal’ by the economy relative to what the ecosystem is able to regenerate (income). The definition of scale on the flow-dimension is a simple accounting tool that compares two related flows in much the same way as one would compare withdrawals from a financial stock to the income or the yield from that stock. Instead of comparing two flows monetary flows measured in dollars, equations 3.10 and 3.11 compare the physical throughput (withdrawal) with the biophysical yield (income). The scale ratios on both the source-side and the sink-side are dimensionless numbers. On the source-side it is easy to see that the throughput, $\dot{x}_i$ and the natural income, $\hat{Y}_{so}$ have the same physical dimension and thus making S_{so} dimensionless. For example, if the scale ratio measured the scale of the timber industry relative to the forest that supplies the timber, $\dot{x}_i$ is the amount of timber harvested every year and $\hat{Y}_{so}$ is the amount of new timber that the forest regenerates every year. Thus both timber withdrawal and regeneration have the same physical dimension of say tons of wood per year. On the sink-side, we have seen that the natural income, $\hat{Y}_{si}$ represents the regeneration of 'holes' or the capacity to absorb waste. This regeneration can be measured in the same physical units as the throughput on the sink-side, $\dot{x}_o$. For example if we are measuring the capacity of a lake to neutralize acid rain (throughput of sulphur on the sink-side, $\hat{Y}_{si}$) and $\dot{x}_o$, the new capacity to absorb this waste can both be measured as tons of sulphur per year, once again making the scale ratio itself a dimensionless number.[15]

The scale measured on the source-side and the sink-side can be, and indeed are in most instances different. A complete answer to the central question of how large is the economy relative to the ecosystem takes into account the scale measured on both the source-side as well as the sink-side. Even while we study the source-side and the sink-side independently, it is important not to lose track of the fact that $\dot{x}_i$

---

[15] Also see discussion in section - 3.4.

and $\dot{x}_o$ are only two sides of a common underlying throughput. In particular, $\dot{x}_o$, the throughput on the sink-side is, among other things, a function of the throughput on the source-side, $\dot{x}_i$. Thus if it is determined that the scale on the sink-side exceeds what is sustainable or what is socially desirable, it is often possible to limit the throughput on the sink-side by limiting the throughput on the source-side. In terms of the representation of the economy in figure - 3.4 policy has greater control over t_i than t_o for the obvious reason that the throughput on the sink-side is more degraded than the throughput on the source-side – it is a lot easier to control how much coal is extracted from the mines rather than controlling the the combustion products of burning coal at every smoke stack. A lot of contemporary global scale ecological problems have to do with the scale of waste flows rather than scale of resource flows on the stock-side. The 'ozone hole' or the massive reduction of stratospheric ozone and climate change driven by rapid accumulation of green house gases in the atmosphere are good examples. The stratospheric ozone problem for example was addressed by limiting, and even eliminating the relevant the throughput on source-side.

While the simple scale-ratio, **S** developed here describes the relationship between the economy and the ecosystem in the flow-dimension at any level of economic-geographic aggregation, the representation of the sink-side in figure - 3.4 is a vast over simplification at all but the highest resolutions. Take the case of coal burning for example. While the source-side throughput can indeed be represented by a single x_i, measured in say tons of carbon per year, the sink-side throughput is more complicated because there are multiple sinks for combustion products of coal burning, that have different regeneration rates of absorption capacity. In figure - 3.5 we focus on the sink-side of figure - 3.4. Now, instead of a single stock that serves as a sink we now have several sinks for the waste flow, $\dot{x}_o$. In figure - 3.5 the waste flow, $\dot{x}_o$ has been split into several components that each flow into a different physical sink. Each of the flows have the same physical dimension as the throughput on the source-side and it is a matter of accounting that $\sum\limits_{k=1}^{k=n} \dot{x}_o^k = \dot{x}_o$ or that the components add up to the whole. In the coal burning example, the stocks representing the different sinks could be oceans, trees, soil, and the atmosphere. While formal modeling of disaggregated throughput is the subject matter of chapter - 4 the discussion here suggests that even at a given level of economic-geographic aggregation, we may need to disaggregate the throughput if multiple stocks are involved on the sink-side (or for that matter on the source-side). The techniques needed to formally tackle this problem are identical to ones that we will develop to study scale at multiple levels of economic-geographic aggregation. In particular, we need a consistent framework

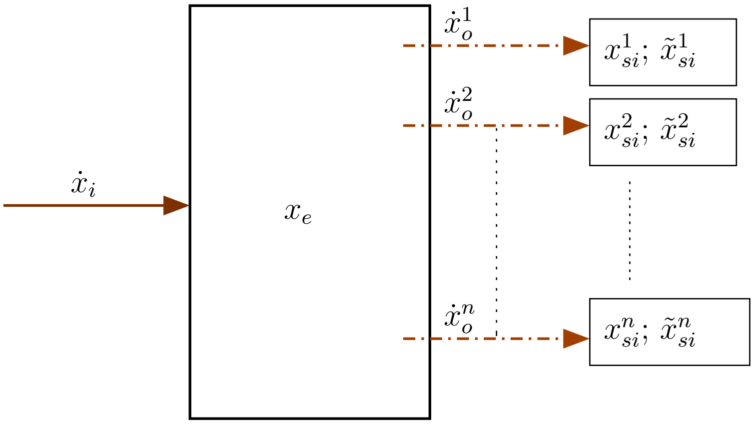

Figure 3.5: Multiple Components of Sink-Side Throughput

to reconcile the fact that in a representation like figure - 3.5 multiple values of scale are possible for a given throughput. Accounting for multiple stocks and interactions across different resolutions is key to any empirical work with scale.

3.3.2 Indeterminate Scale Ratio with $\hat{\mathbf{Y}} = 0$

In equations 3.10 and 3.11 the scale ratio is mathematically indeterminate when the natural income $\hat{Y}_{so}$ or $\hat{Y}_{si}$ is zero. On the source-side this corresponds to any throughput of a non-renewable resource. On the sink-side, zero-natural income corresponds to waste stream that cannot be processed by the ecosystem. Heavy metals, dioxins, or even simple non-biodegradable everyday plastic are all examples of wastes that cannot be absorbed the ecosystem. By "absorption" we refer to the ability of the ecosystem to regenerate relevant "holes." The stratospheric ozone depletion is a problem because there are no sinks for chlorine atoms in the upper atmosphere. Strictly speaking, $\hat{\mathbf{Y}} > 0$ for almost any throughput. For example, though the chlorine atoms from the CFC's are long-lived in the upper atmosphere they are indeed eventually destroyed. Fossil fuels are perhaps 'renewable' in geological time scales. By zero natural income or $\hat{\mathbf{Y}} = 0$ we refer to time scales that are relevant to: (a) the human economic predicament, and (b) to time scales where a positive throughput can be maintained without fundamentally altering the fund structure of the ecosystem and irrevocably alter the service flux, $\breve{\mathbf{Y}}$. In this sense we use natural income as an analogue of Hicksian monetary income and a Hicksian income is sustainable by definition as long as there is no change to the underlying stock. Even when the scale is indeterminate because natural income from the ecosystem in its stock function is zero, the ecosystem continues to be a source of service fluxes. For a throughput of non-renewable resource, there is no regeneration of the resource ($\hat{Y}_{so}$) but the mineral deposit is part of the fund that continues to yield the service flux, $\breve{Y}_{so}$. Policy implications of the scale-ratio being indeterminate are indeed complex. We devote substantial portion of chapter - 7 to discuss various options available for policy to address the important question of non-renewable sources and sinks.

3.3.3 Comparing Throughput Scales Measures

Using the scale methodology for environmental policy is a two-stage process. Besides being able to empirically measure scale, policy application of the scale methodology requires that we are able to compare two or more measured values of a scale-measure. This is the subject matter of chapter - 6 but this is a good place to make preliminary observations in the context of flow measure of scale. The first question to ask is: are the scale ratios determined independently on the source-side and the sink-side comparable? The answer, in general, is negative. S_{so} and S_{si} compare the economic throughput with two different, and independent regeneration rates. The throughput itself is quantitatively and qualitatively different on the source-side and the sink-side. For example, there is no physical meaning in comparing scale measured in extraction of coal (where we have seen the scale is indeterminate) to scale measured with respect to ability of oceans to absorb carbon from burning coal. As a corollary of the fact that the source-side and the sink-side are not comparable, two vector scales, say $\mathbf{S_1}$ and $\mathbf{S_2}$ are not comparable as the vector $\mathbf{S}$ includes both the source-side and the sink-side.

Is the flow measure of scale comparable across spatial and temporal dimensions, separately for the source-side and the sink-side? The answer is once again negative for spatial coordinates and we will see why the temporal question is not reducible to a dichotomous yes-no answer. Consider the now canonical forestry example on the source-side. Let us say that we have two different measured values of scale for the forestry industry: one in North America (say $S_{1_{so}}$) and other for Central America ($S_{2_{so}}$). Further assume the measured values for scale for the two places (measured during the same time-period) are given by $S_{1_{so}} = 1.05$ and $S_{2_{so}} = 1.2$. It would be erroneous to conclude that the economy of Central America is of a larger scale compared to that of North America because $S_{2_{so}} > S_{1_{so}}$. This comparison is erroneous because the two ecosystems that we are comparing are fundamentally different – tropical forest system versus a temperate forest. We recollect that the ecosystem is simultaneously a stock as well as a fund and even if the exact same kind of timber was being harvested from the two forests (unlikely in the present example), the fund that this stock of timber is a part of is very different in two places. Comparison of scale measures is physically meaningful only across spatial and temporal coordinates with comparable ecosystem funds.[16]

---

[16] Later in this dissertation we speculate on how trade between economic-geographic aggregations can be modeled using tools we develop for measuring scales at varying levels of resolution. In this chapter we have implicitly assumed complete autarky which in the present example means that the economies of North America and Central America do not trade with each other.

It is for this ‘constancy of fund’ reason, that an ordinal ranking of scale measures of the same physical system from two or more temporal coordinates is not always guaranteed. Now consider only the North American forest but with two measured values of scale from two different time periods – say $S_{t1_{so}}$ and $S_{t2_{so}}$ measured at times $t1$ and $t2$ respectively. Using the same set of hypothetical measured values as before, $S_{t1_{so}} = 1.2$ and $S_{t2_{so}} = 1.05$. Is an ordinal ranking of $S_{t1_{so}}$ and $S_{t2_{so}}$ possible? **S** is a flow measure of scale but the nature of the fund is primarily related to the stock. To be able to discern the structure of the fund at times $t1$ and $t2$ we need information about the state of stocks at those two times — $x_{t1_{so}}$ and $x_{t2_{so}}$. We know that at time $t1$ the measured value of scale was greater than one and thus in time period-1 there was a negative change in stock. Assuming that time-period $t2$ follows $t1$ and that the two time periods are contiguous, the measured value of scale at $t2$, is, everything else being equal, at a different fund level and at the new fund level (which typically would be less than at $t1$ with a reduction in stock), a lower measured value of scale ($S_{t2_{so}} = 1.05$ as opposed to $S_{t1_{so}} = 1.2$) can be better or worse than the scale measured at $t1$. Only information about stock and fund can resolve the ordinal ranking problem for a flow measure of scale.

Two measured values of scale are comparable across time only for systems that are close to steady-state. For systems at steady-state this comparison is trivial as the scale measures have to be equal for a system to be in steady state. For systems that are far from steady state and thus experiencing appreciable change in the fund-structure, comparison is somewhat complex and we present a strategy in chapter - 6. The framework includes formal specifications for optimal scale, maximum sustainable scale, and discerning the relationship between stock and flow measures of scale.

3.4 "Dimensionless" Measures

In figure - 3.4 x_{so}, x_e, and x_{si} all have the same physical dimension. For example, if $\dot{x}_i$ represents the flow of timber, then x_{so} is the stock of standing wood in the forest, x_e is the stock of artifacts containing timber in the economy, and finally x_{si} represents discarded timber products. While the three stocks in figure - 3.6 have the same physical dimension, they are qualitatively different. For example, standing wood in the forest (x_{so}), artifacts in the economy that use wood (x_e), and discarded wood products (x_{si}) are all qualitatively different. We have already seen how the two components of the throughput in figure - 3.4 are qualitatively different even when they have the same physical dimensions. Our purpose here is to clarify the meaning of "dimensionless" in the context of the stock-fund framework represented by figure - 3.4. The formalism developed here will form the basis for much of the discussion chapters 4 and 6 where we develop methodologies to study cross-section and time-series of different scale measures. Clarifying the nature of dimensional quantities under the stock-fund formulation will also inform the framework that we develop in section - 3.5 to measure scale in the stock dimension.

Dimensional quantities in the cardinal stock-flow space are well understood. A more complicated fund-flux space is needed to understand the qualitative transformation of the throughput as it flows from the source to the economy and back to the sink. The stock-flow space is governed by simple conservation of mass and energy principles. For example, the simple accounting identity in equation - 3.1 is sufficient to track the flow of carbon and hydrogen atoms that make up timber when figure - 3.6 is used to track the flow of wood from the forest to the economy and the flow of discarded artifacts from the economy containing wood. However each of the three stocks that hold wood in figure - 3.6 are dramatically different funds. A forest cannot be more different from a stock of paper products in the economy or certainly discarded artifacts in a landfill. A metric that is 'dimensionless' in the stock-flow space is not necessarily dimensionless in the fund-flux space. Consider for example a measure that is a ratio of two components of the throughput (Say $M = \frac{\dot{x}_i}{\dot{x}_o}$). The metric M is dimensionless in the stock-flow space – both $\dot{x}_i$ and $\dot{x}_o$ are measured in the same flow units, $tons/year$ of timber for example. However, in the fund-flux dimension, $tons(wood)/year$ and $tons(discarded\text{-}paper)/year$ have a different qualitative dimension. Thus what is dimensionless is determined by the scope of our study. M is dimensionless if we are interested in tracking the throughput of coal in terms of carbon atoms rather than as coal and carbon-di-oxide. However if we are interested in the qualitative transformation of the throughput, M is no longer a dimensionless scalar – indeed M now becomes a metric that characterizes the transformation efficiency of the economic process. We formalize this discussion of dimensioned quantities in the fund-flux space by defining three different categories of dimensionless measures.

Definition 3.4.1 ***Dimensionless Quantity***: *(Physical Dimension)*

A quantity is "dimensionless" if that quantity can be expressed as a scalar with no physical dimensions.

The metric M in the discussion above for example is dimensionless. Other examples of metrics derived from figure - 3.4 that are dimensionless include $\frac{x_e}{x_{si}}$ or even $\frac{x_e}{\tilde{x}_{si}}$.

Definition 3.4.2 ***Qualitatively Dimensionless Quantity***: *(Qualitative Dimension)*

In addition to being "dimensionless," "qualitatively dimensionless" quantities can be expressed as a scalar with no physical dimensions even after after accounting for the qualitative differences between the constituent elements of the metric.

As discussed above, a "qualitatively dimensionless" quantity is dimensionless in the fund-flux space, in addition to being dimensionless in the stock-flow space.

Definition 3.4.3 ***Strictly Dimensionless Quantity***: *(Scale Dimension, Departure from Benchmark Scale)*

Strictly dimensionless quantities derived from two or more qualitatively dimensionless quantities can be expressed as a scalar with no physical dimensions even after accounting for qualitative differences.

To motivate the need for strictly dimensionless quantities consider the following metric: $C = \frac{S_{so}(t_2) - S_{so}(t_1)}{S_{so}(t_1)}$ where t_1 and t_2 are two different time periods at which the flow measure of scale, S_{so} was measured for some throughput, $\dot{x}_i$. The metric C simply computes the percentage change in scale S_{so} between time t_1 to time t_2. To see why C is not qualitatively dimensionless, we only need to note that between time t_1 and t_2, the fund underlying the stock from which the throughput $\dot{x}_i$ is derived could have changed. As suggested earlier in this chapter, what is comparable across time periods (and across different regions) is departure from optimal scale (or in some cases other benchmark measures). Consider the metric $C_\delta = \frac{\delta(t_2) - \delta(t_1)}{\delta(t_1)}$ where $\delta(t)$ is the departure from optimal scale at time — $\delta(t) = S(t) - S^*(t)$ where $S^*(t)$ is the optimal scale at time t. The metric C_δ is strictly dimensionless because departures from optimal scale are comparable even in the qualitative dimension.

3.5 Stock Measure of Scale

In figure - 3.6, we reproduce stock and flow elements from the analytical representation of the economy-ecosystem interactions presented in figure - 3.4. Any scale-measure that measures scale directly in terms of empirically discernable stocks

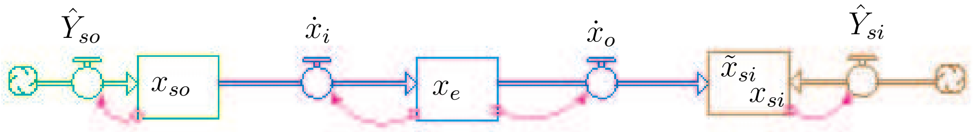

Figure 3.6: A Simplified Analytical Representation of the Economy-Ecosystem Interaction

and flows will be a function of two or more of the seven basic elements in figure - 3.6 (three stock elements and four flow elements). Each of the seven elements represent the physical size of the economy or the ecosystem. The flow measure of scale uses the flow elements, $\hat{Y}_{so}$, $\dot{x}_i$, $\dot{x}_o$, $\hat{Y}_{si}$. Any stock measure of scale will be a function of the three principal stocks in 3.6, x_{so}, x_e, and x_{si} (or $\tilde{x}_{si}$). In simple scale measures like S_{so} or S_{si} we compare one economy element with another element representing the ecosystem. This simple construction presents little difficulty when measuring scale in the flow dimension. The source-side and the sink-side have one flow element each corresponding to the economy and the ecosystem thus enabling independent measurement of scale on the source-side and sink-side. However there are only three stock elements in figure - 3.6 and it seems like it is impossible to measure scale on the stock dimension independently for the source-side and the sink-side – two stock elements are needed on source-side and sink-side to define a stock measure of scale in terms of the elements in figure - 3.6.

Figure - 3.6 represents a snapshot at any given time, t. We will show here that a snapshot representation of the economy is inadequate to defining scale in the stock dimension. The most meaningful definition of scale in the stock dimension is given by:

$$\sigma = \begin{pmatrix} \sigma_{so} \\ \sigma_{si} \end{pmatrix} \tag{3.12}$$

$$\sigma_{so} = \frac{x_{so}(t)}{x_{so}(t^*)} \tag{3.13}$$

$$\sigma_{si} = \frac{\tilde{x}_{si}(t)}{\tilde{x}_{si}(t^*)} \tag{3.14}$$

where t is the current time period and t^* represents some reference time period ($t \geq t^*$). σ is the vector scale in the stock dimension and σ_{so} and σ_{si} are scale measures on the source-side and sink-side respectively. On the sink-side we have defined scale in terms of holes ($\tilde{x}_{si}$), but can also be defined in terms of x_{si}. However, we argue below that a scale measure defined in terms of holes is analytically more desirable.

3.5.1 Candidate Measures

We will briefly review different possible candidates for scale measures in the stock dimension and discuss why scale defined in equations 3.13 and 3.14 represent the most desirable measures of scale in the stock dimension. The primary requirement of any scale-measure is that it convey quantitative information about the *proportional* relationship between the economy and the ecosystem. For the purposes of analytical tractability, it is also desirable that a scale-measure be dimensionless. A scale-measure that is not a dimensionless ratio is especially problematic when scale is used across multiple levels of economic-geographic aggregations. In chapter - 6 we discuss properties of scale-measures that enable a consistent ordinal ranking of different measured values of a given scale-measure. Here, we restrict ourselves to more basic considerations of analytical tractability.

The most commonly accepted notion of scale in the stock dimension involves measuring the inventory of economy generated artifacts. For example in their recently published undergraduate ecological economics text Daly and Farley define scale as:

> [t]he physical size of the economic subsystem relative to the ecosystem that contains and sustains it. It [scale] could be measured in its stock dimension of population and inventory of artifacts, or in its flow dimension of throughput required to maintain the stocks.[17]

"[I]nventory of artifacts" is indeed a measure of the physical size of the economy but does not measure the *proportional* relationship between the economy and the ecosystem. This is not to suggest that Daly and Farley [2004] present scale as an absolute measure of the physical size of the economy rather than as the size of the economy relative to the ecosystem. Their substantive treatment of scale, especially in the flow dimension (the above reference to absolute throughput notwithstanding) is clear on scale as a proportional relationship between the economy and the ecosystem. Rather, the confusion with the definition of scale reproduced above is related to the conceptual difficulties with representing the proportional relationship between the economy and the ecosystem in the stock dimension. A scale-measure helps answer the question: how large is the economy *relative* to the ecosystem that

---

[17] Daly and Farley [2004, p.439]

contains and sustains it? A potential scale-measure that uses inventory of artifacts (represented by x_e in figure - 3.6) would also use one of the other two stocks from figure - 3.6. For example a metric constructed as a ratio of x_e (stock of x in the economy) and x_{so} (the stock on source-side) is a potential scale-measure – at least to the extent that such a metric represents a proportional relationship between the economy and the ecosystem. Similarly a metric that is the ratio of x_e and x_{si} (the stock on the sink-side) is a potential scale-measure. We have already seen why both $\frac{x_e}{x_{so}}$ and $\frac{x_e}{x_{si}}$ cannot simultaneously be scale measures because both these metrics use x_e and thus do not represent independent measures of scale on the source-side and sink-side.

The measure σ defined in equation - 3.12 achieves independent measurement on the source-side and sink-side by using the two ecosystem stocks and only implicitly imputing the stock of the economy. The implicit assumption is that the difference between the current level of the stock and the reference stock is accounted for by human activity. Thus the definition of σ requires that most if not all of the difference between the current stock $x(t)$ and the reference stock $x(t^*)$ is attributable to human activity. However in cases where this assumption is not tenable, the basic representation of equation - 3.12 can still be retained by suitably modifying the choice of reference stock.[18] While σ overcomes the problem of source-side and sink-side independence on the stock dimension it does so only at the cost of not directly

---

[18] See also chapter - 5 for a discussion on the differences in interpretation in the fund-flux space in the presence of significant non-anthropogenic perturbations. In section - 4.2.4 we discuss a particular accounting framework that helps discern the anthropogenic contribution to total perturbation of the ecosystem stock.

taking into account the inventory of artifacts in the economy. There are important areas of practical environmental policy where a scale measure that directly measures the inventory in the economy (x_e) is useful even at the expense of strict distinction between the source-side and sink-side. In particular a scale measure like $\frac{x_e}{x_so}$ or $\frac{x_e}{x_si}$ is useful in characterizing society's use of non-renewable resources. On the source-side, consider the example of metals that are integral to the modern technological society. Knowing the current stock of the metal in the economy (x_e) relative to the current stock remaining untapped (x_{so}) is important because the metric $\frac{x_e}{x_{so}}$ is crucial to determining if the economy is on a physically sustainable course.[19] Similarly, on the sink side the ratio $\frac{x_e}{\tilde{x}_{si}}$ is important in case of stocks that have a non-regenerating ($\hat{Y}_{si} = 0$) sink capacity. Thus even when direct comparison of the economy-stock and the ecosystem-stocks are not attractive in the fund-flux dimension they are indispensable in the stock-flow dimension. For this reason, we will formally write down scale-measures that use the economy-stock, x_e:

$$\sigma^{\mathbf{e}} = \begin{pmatrix} \sigma^e_{so} \\ \\ \sigma^e_{si} \end{pmatrix} \tag{3.15}$$

$$\sigma^e_{so} = \frac{x_e(t)}{x_{so}(t)} \tag{3.16}$$

---

[19] See discussion on maximum scale in the stock-flow dimension in chapter - 5. A recent paper by researchers at Yale [Gordon et al. 2006] estimate the total stock of copper in North America and conclude that there is not enough extractable copper left in the ground if the rest of the world used copper at the same level of intensity (in per capita terms) as North Americans (see also section - 4.2.4). Here, we use 'physical sustainability' in a very restrictive stock-flow space.

$$\sigma_{si}^{e} = \frac{x_e(t)}{\tilde{x}_{si}(t)} \tag{3.17}$$

3.5.2 Relationship between σ, $\sigma^{\mathbf{e}}$, and $\mathbf{S}$

Having defined two different stock measures of scale, the next logical question to ask would be about the relationship between these new measures and the throughput scale measure, **S**. The stock measure of scale represents a snapshot – the relationship between the economy and the ecosystem at a particular point in time and the flow measure of scale characterizes the evolutionary relationship between the economy and the ecosystem. Looking at the relationship between snapshot and evolutionary measures of scale will also set the stage for a fuller understanding of the study of the temporal dimension in construction of scale measures. We consider three different cases based on the relative magnitudes of the throughput ($\dot{\mathbf{x}}$) and the regeneration flow ($\hat{\mathbf{Y}}$):

1. $\boxed{\dot{x} > \hat{Y}}$ When the economy generated throughput exceeds the natural rate of regeneration, it is fairly straightforward to see how the throughput scale, **S** is greater than unity. The behavior of σ is somewhat less obvious. σ gives a snapshot view of the scale of the economy and the snapshot in the current time-period is a product of history. The actual value that σ takes is a related to how long the economy generated throughput has exceeded the natural regeneration. When throughput exceeds regeneration, the natural-stock (x_{so} or $\tilde{x}_{si}$) is decreasing. In the limiting case of the natural stock being

reduced to zero the value of scale, σ will be zero irrespective of the reference level of stock. For this limiting case, the throughput scale, $\mathbf{S}$ is indeterminate as the natural income or regeneration, $\hat{\mathbf{Y}}$ is zero – there is no income if there is no capital. Further, on the source-side, there can be no throughput either if the stock, x_{so} is reduced to zero. The important point to note is the difference in values taken by σ and $\mathbf{S}$. The throughput scale, $\mathbf{S}$ is greater than unity while the stock measure of scale σ is likely going to be less than unity. The actual value of σ is of course determined by the reference level of stock, $\mathbf{x}^*$.

2. $\boxed{\dot{x} < \hat{Y}}$ When the economy generated throughput is less than the natural regeneration rate, the natural-stock (x_{so} or $\tilde{x}_{si}$) is increasing. The throughput scale measure, $\mathbf{S}$ is less than one. The exact value taken by the scale as measured in the stock dimension is a function of the reference stock. In the case where the level of stock vastly exceeds the reference stock (on the source-side, $x_{so} \gg x^*_{so}$) the value of scale, σ will be a large number. Again, the point to note is the difference in the values taken by flow and stock measures of scale. While $\mathbf{S}$ will take on a value less than one, the stock measure of scale, σ will likely be a large number. The actual value of σ will depend on the level of reference stock, and the relationship between the time elapsed from the reference time period $(t - t^*)$ and the duration of time for which natural regeneration has exceeded throughput. Neither $\mathbf{S}$ nor σ can fully characterize the relationship between the economy and the ecosystem. Consider the example of a previously destroyed forest that is now protected (if x represents timber, then currently

x_{so} is a small number but $\dot{x}_i < \hat{Y}_{so}$ or in case of an outright prohibition of logging, $\dot{x}_i = 0$). Looking only at the flow measure (S_{so}, equation - 3.10) would lead us to conclude that forest is on a possibly sustainable evolutionary path ($S_{so} < 1$).[20] However with an appropriate choice of the reference stock, x^*_{so}, the stock measure of scale σ_{so} will be a small number because the current level of the stock ($x_{so}(t)$) is much smaller than the historical reference stock, x^*_{so}. This simple example illustrates that our general conclusion between of σ being large when regeneration exceeds throughput is to be treated, at best, as a rule of thumb.

3. $\boxed{\dot{x} = \hat{Y}}$ This is the steady state case and arguably the easiest to analyze. When rate of withdrawal (throughput, T) equals the regeneration rate, the natural stock (x_{so} or $\tilde{x}_{si}$) is neither decreasing not increasing. The throughput scale, **S** which is simply the ratio of throughput and regeneration will be unity. On the other hand the value that the stock measure of scale, σ takes is a function of the reference level of stock. All we can say about σ in the steady state is that it will be a constant over a period of time.

3.5.3 Quasi-Stock Measure of Scale

The stock measure of scale defined in equation - 3.12 measures scale independently on the source-side and the sink-side by using the temporal variable t. An alternative to σ can be derived starting with the simple observation that stocks

---

[20] Also see previous discussion in section - 3.3 and chapter - 5 for a discussion on the relationship between stock and flow measures in the fund-flux space.

can be conceived as accumulated flows. In particular, the two components of the throughput ($\dot{x}_i$ and $\dot{x}_o$) determine the level of the economy-stock, x_e. The most significant drawback of the measure σ defined in equation - 3.12 is that it does not explicitly account for the economy-stock. This is a direct consequence of the fact that we wanted independent source-side and sink-side measures while being constrained by a single stock variable for the economy – x_e. Here we define a quasi-stock measure of scale that makes use of the independent source-side and sink-side representation of the economy in the flow dimension.

$$\sigma^f_{so} = \frac{\int\limits_0^T \dot{x}_i \, \mathrm{d}t}{x_{so}(0)} \tag{3.18}$$

$$\sigma^f_{si} = \frac{\int\limits_0^T \dot{x}_o \, \mathrm{d}t}{\tilde{x}_{si}(0)} \tag{3.19}$$

The measure σ^f defined on the source-side and sink-side in equations 3.18 and 3.19 respectively achieve an independent stock representation of the economy by representing the economy stock as accumulated flows. Interpretation of σ^f on the source-side is straightforward – σ^f_{so} simply represents the portion of the reference stock ($x_{so}(0)$) that is withdrawn in time period T. Interpretation is inextricably tied to the length of the time-period, T in relation to the magnitude of the regeneration flow, $\hat{Y}_{so}$. Consider for example the previously discussed example of fossil fuel extraction. Regeneration flow is meaningful only under geological timescales. If the stock $x_{so}(0)$ represents the stock of say oil at time $t = 0$ when human societies first started extracting oil, and $\int_0^T \dot{x}_i \, \mathrm{d}t$ represents the total withdrawal of oil in one year ($T = 1\,\text{year}$), what information does σ^f_{so} convey about the proportional relationship between the economy and the ecosystem? In this particular example it is evident that comparing the annual withdrawal of oil to an historical stock does not say much about the relationship between current withdrawal and the remaining stock – the most important concern in the stock-flow space. However, σ^f does tell us how much of the initial stock has been used up. There are at least two other ways in which equation 3.18 can be interpreted. First, if we let $t = 0$ denote the beginning of current time period instead some historical reference, then σ^f is a good metric that provides information about current consumption relative to current levels of stock. Second, we could use as our reference $x_{so}(T)$ instead of $x_{so}(0)$. This is the subject matter of the next section where we will also discuss the somewhat more involved sink-side interpretation of σ^f.

3.6 The Temporal Dimension

3.6.1 An Alternate Flow Measure of Scale

In the discussion of scale on the flow dimension we have implicitly assumed that all the relevant quantities are being measured at the same temporal coordinate. This follows from figure - 3.4 where all the stocks, flows, as well as the other parameters are measured at a single time instant. However, our discussion on the relationship between stock and flow measures of scale has shown that the temporal dimension is central to interpreting flow measures of scale. The primary motivation behind measuring **S**, the scale in the flow-dimension is to compare the relative magnitudes of throughput and ecosystem regeneration. The question that we ask here is: is throughput measured relative to instantaneous regeneration always the best possible measure of the proportional relationship between the economy and the ecosystem in the flow dimension?[21] To motivate the need for flow measures of scale that use regeneration rate from a time period that is different from that of throughput, consider two familiar examples - one on the source-side and the other on the sink-side. On the source-side, consider the timber industry that has been harvesting more than the annual biological yield of the forest for several years but due to some temporary exogenous shock in a downstream market, there was a collapse of the logging industry in the current time period. On the sink-side, consider a polluting industry that has been dumping toxic waste into a water system for several years, and always in excess of the regeneration rate ($\dot{x}_o > \hat{Y}_{si}$). What information does the throughput measure of scale, **S** convey in the case of these two stylized examples? For the source-side problem, the scale-measure ($S_{so} = \frac{\dot{x}_i}{\hat{Y}_{so}}$) will be less than unity assuming the current depressed logging rate (small $\dot{x}_i$) is less than the current regeneration rate ($\hat{Y}_{so}$). Without the exogenous shock that lead to the collapse of this logging industry, the throughput in the current time period would have exceeded regeneration. Further, following several years of excessive logging (in excess of regeneration), the current regeneration is much lower than the regenera-

---

[21] "Instantaneous regeneration" refers to the fact that throughput and regeneration are measured at the same instant.

tion when the logging began ($\hat{Y}_{so}(t) < \hat{Y}_{so}(t^*)$; where t is the current time period and t^* is when intensive logging began). An alternative to the measure S_{so} would be one that compared current regeneration rate with some historical reference (in the present example, this reference could be regeneration before intensive logging started). Similarly for our sink-side example, the current regeneration rate will be lower than historical regeneration rate before the polluting industry started dumping ($\hat{Y}_{si}(t) < \hat{Y}_{si}(t^*)$) its effluent. Here again comparing the two regeneration rates provides an alternative to the throughput measure, S_{si}. We will formally define this scale-measure here:

$$S^*_{so} = \frac{\hat{Y}_{so}(t)}{\hat{Y}_{so}(t^*)} \tag{3.20}$$

$$S^*_{si} = \frac{\hat{Y}_{si}(t)}{\hat{Y}_{si}(t^*)} \tag{3.21}$$

We will show in chapter - 5 that the flow measures defined in equations 3.20 and 3.21 are more useful than the simple throughput measure, **S** when a scale measure is primarily used to characterize the relationship between the economy and the supporting ecosystem in the fund-flux dimension. Our primary purpose here is to motivate the discussion in the following chapters by highlighting the centrality of the time dimension to construction of scale measures.

3.6.2 Choosing Reference Time Period, t^*

Our discussion of the stock measure of scale, σ, and the flow measure $\mathbf{S}^*$ have underscored the centrality of the temporal dimension. From a practical environmental policy perspective, the key question is one of selecting the appropriate reference time period, t^*. While the question of picking the reference time period can be posed in formal terms, we will show here that even in its simplest form the question has an irreducible normative component. Consider the stylized version of the problem where for all time prior to $t = 0$, the ecosystem stock x_{so} is in steady state with no economy generated throughput ($\dot{x}_i = 0$).[22] We will denote the current time-period as $t = \bar{t}$. Our problem now is to chose a reference time-period t^* from among the $\bar{t}+1$ possible values if we constrain our choices to exclude the future – after all the future value of the different stocks and flows in figure - 3.6 are unknown in the present time-period. Out of the $\bar{t} + 1$ candidates, we will signal out two special ones. First, there is a case to be made that the reference time-period be set at $t = 0$. Second, we can chose a reference time-period such that the scale measure under study is at its optimal value during the reference time-period.[23] Both of these choices are fraught with a variety of problems. In the case of setting the reference time-period to be $t^* = 0$, identifying the 'zero-point' is fraught with technical problems besides philosophical questions about what constitutes human appropriation of nature. Some of the technical issues have been addressed by various attempts to measure the size of

---

[22] For the purposes of exposition we will use the source-side example with the understanding that the discussion here is equally applicable to the sink-side of the economy.

[23] We implicitly assume here that the relationship between the economy and the ecosystem was optimal at some point in time before the current time-period. Also see chapters - 5 and 6 for further discussion on existence and determination of optimal scale.

economic activity in biophysical terms rather than monetary terms. For example attempts to measure human appropriation of photosynthesis (a methodology we review in chapter - 4) have estimated net loss in primary productivity due to human activity including agriculture. Setting t^* to be the time-period corresponding to optimal scale is fraught with its own theoretical difficulties arising from the fact that optimal scale can vary over time. In each of the next three chapters we treat some aspects of varying optimal scale.

Chapter 4

Biophysical Assessments as Scale Measures

4.1 Introduction

In this chapter we conduct a detailed review of popular biophysical assessments of the relationship between human activity and the supporting ecosystem. Our focus will be on assessing the strengths and weaknesses of some of the most important contributions in their role as scale measures. While chapter - 2 provided a general review of literature on biophysical measures of the physical size of the economy in relation to the sustaining capacity of the ecosystem, the specific focus of this chapter will be the application of the basic scale methodology developed in chapter - 3 to study three broad categories of biophysical assessments of human activity.

The various efforts to understand the biophysical basis of human activity can be classified into two broad categories. First, physical scientists and engineers have developed methodologies to track material flows between the ecosystem and the economy that have more recently recently evolved into interdisciplinary endeavors under the rubric of *industrial ecology* or *material flow analysis.* Second, ecologists have lead the efforts to understand systemic impacts on the ecosystem arising out of increasing pressures exerted by the economy on biophysical environment. These efforts have spawned entire new fields like *conservation biology.* In terms of our discussion in chapter - 3, these two seemingly disparate attempts at measuring the size of human activity in biophysical terms by physical scientists and ecologists corresponds respectively to analysis in the stock-flow space and the fund-flux space. Our review in this chapter suggests that the two seemingly divergent efforts are mostly complementary and taken together serve as a good foundation to build empirically measurable scale measures.

We will start by studying two attempts at measuring the size and composition of the material throughput supporting human activity. We will use Klee and Graedel [2004], Adriaanse et al. [1997a], and Matthews et al. [2000b] as specific examples representing the broader endeavor that has been organized under the rubric of material flow analysis (MFA). We have chosen these examples for the influence that they have had on the public discourse [Adriaanse et al. 1997a, Matthews et al. 2000b] and an attempt at completeness in Klee and Graedel [2004]. The authors of these studies have been at the forefront of the material flow analysis methodology, and in addition to important empirical contributions these studies have also made contributions in the form of novel theoretical frameworks. Following our review of these material flow methodologies, we will turn our attention to the hugely influential enterprise pioneered by Vitousek et al. [1986] that involves tracking the human appropriation of the net products of global photosynthesis(HANPP) as a measure of influence of human activity on natural systems. In the twenty years since the original invention of this methodology, several theoretical and empirical advancements have been made towards refining the use of the HANPP methodology and we will focus our attention on some key contributions from a ‘suitability as a scale-measure’ perspective. Finally, we analyze the currently popular ecological footprint methodology in detail. The analysis here will use the basic analytical representation of the economy in figure - 3.4. We will use the analytical representation of the economy that we developed through figure - 3.4 to both critique, and learn from the biophysical measures considered here. The chapter will conclude with object lessons from these varied efforts over the last two decades and how they can contribute to more refined framework to empirically measure scale.

4.2 Material Flow Analysis

As we discussed in chapter - 2, the distinguishing feature of modern economics that sets it apart from other social sciences is its focus on empirical measurement. No variable has received more attention than the flow of money between different sectors of the economy. However from the standpoint of figure - 3.4 – the analytical representation of the ecological economics vision of an economy embedded within the larger ecosystem, an exclusive focus on monetary flows ignores two other stocks that are relevant to the human economic predicament. Figure - 3.4 shows four flows – the two components of the throughput ($\dot{x}_i$ and $\dot{x}_o$) and the regenerative flows on the source-side and the sink-side ($\hat{Y}so$ and $\hat{Y}si$). The focus of this section will be on the efforts by engineers, chemists, and other physical scientists to characterize the throughput by empirically measuring the flows associated with the two components of the throughput.

To understand the relationship between study of physical material flows and monetary flows, we reproduce three figures from a report by the National Research Council (2004) reproduced here as figures 4.1, 4.2, and 4.3. In figure - 4.1 the thick rectangle represents the system boundary of the economic-geographic aggregation under study and the triangle is the economy contained with that boundary. For example the boundary could be a particular national economy being studied. On the left-hand side of the boundary are material flows coming into the region under study. These include flows from the rest of the world (imports) and from ecosystem source contained within the system boundary under study. On the right-hand side of figure - 4.1, are the outflows from the economy and include exports and waste flows back to the ecosystem. In figure - 4.2 we now have the physical input-output table superimposed on figure - 4.1. The physical input-output table like the more familiar economic input-output table records all the transactions in the economy in physical rather than monetary terms. For example as the authors of the original

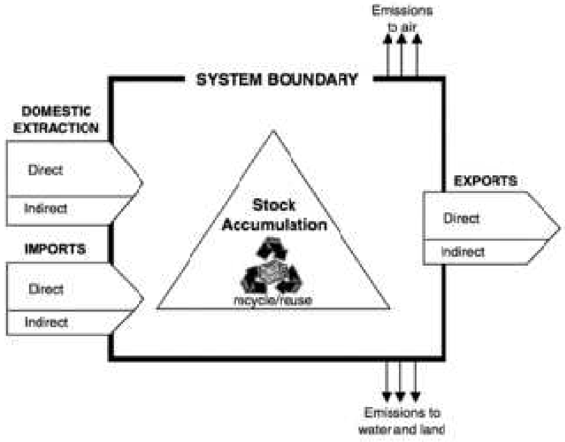

Figure 4.1: Basic elements of Material Flow Analysis used in Adriaanse et al. [1997a] and Matthews et al. [2000b]. Chart reproduced from National-Research-Council [2004, p.26]

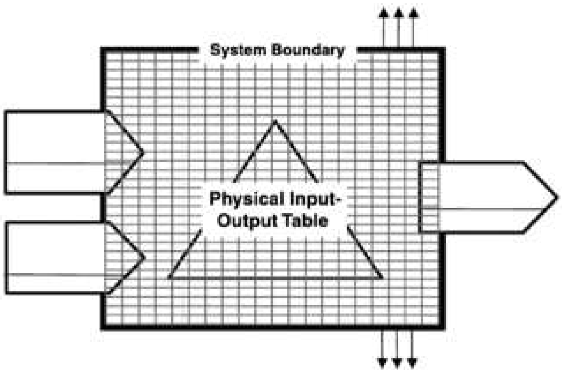

Figure 4.2: The grid lines in this figure, an extension of figure - 4.1, represent the physical input-output tables for the economy contained within the system boundary shown. Chart reproduced from National-Research-Council [2004, p.27]

report suggest, the physical input output table can for example show the flow of copper through the economy. Before the copper that is extracted and imported ends up as exports or in the ecosystem, it is processed within the economy and the physical input-output table helps identify the flow of copper within the economy. Our focus here will be on flows that cross the economy stock rather than the flows within the economy. Finally in figure - 4.3 the traditional economic input-output matrix

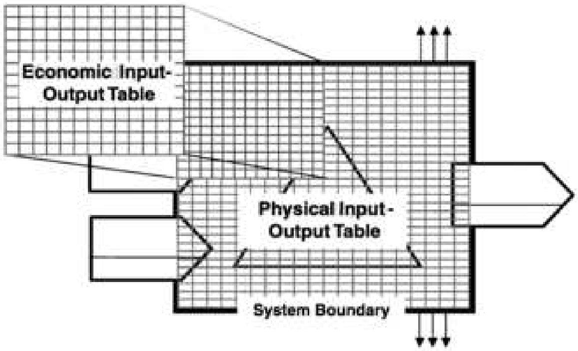

Figure 4.3: The monetary economy is now shown by superimposing the conventional economic input-output tables on figure- 4.2.Chart reproduced from National-Research-Council [2004, p.28]

is superimposed on the physical input-output table. The transactions within the economy are now represented using the exchange value of the materials involved in the transaction rather any physical material. Figure - 4.3 depicts how the material flow analysis is related to the more familiar measures of monetary flows within the economy.

With this conceptual framework for material flow analysis in place, we begin this section by reviewing two influential studies from the World Resources Institute and its partners in Germany and Japan [Adriaanse et al. 1997a, Matthews et al. 2000b]. The first part of the study published in 1997 (Adriaanse et al.) looked at the source-side of the throughput needed to sustain the human economy and the second study published in 2000 extended the methodology to the sink-side of the problem (Matthews et al.). As discussed in chapter - 2, these studies primarily looked at three advanced industrialized nations. However our interest here is in the methodology developed rather than the specific empirical findings of the studies. Following a review of these studies, we will study Klee and Graedel [2004] as a further example of material flow analysis applied to elemental cycles. We conclude this section by accessing the potential of these material flow analysis studies to be used as flow measures of scale.

4.2.1 The Material Basis of Industrial Economies: The Source-Side

Central to the material flow analysis of Adriaanse et al. is the concept of Total Material Requirement or TMR. Any extractive activity even when it involves harvesting a potentially renewable flow like sustainable production of timber entails flow of materials not directly related to the primary material that is being extracted. Mining metals is a good example where a large quantity of mineral and soil is extracted along with the primary ore of interest. Further, large portions of the concentrated ore are discarded during the process of purification of the ore. Ancillary Material Flow is "the material that must be removed from the natural environment, along with the desired material, to obtain the desired material." Distributed Material Flow is "material moved or distributed to obtain a natural resource, to create and maintain infrastructure"[Adriaanse et al. 1997a, p.8]. Together, the ancillary flows and distributed flows are termed Hidden Flows – hidden because these flows do not enter the monetary economy but are nevertheless needed to sustain the economy. Total Material Requirement (TMR) of an economy is the sum total of the material input to the economy (the Direct Material Input or the source-side of the throughput, $\dot{x}_i$, Ancillary Flow, and the Distributed Flow). Figure - 4.4 below illustrates the principal theoretical contribution of Adriaanse et al. [1997a] to the material flow accounting methodology. Figure - 4.4 reproduces the source-side of figure - 3.4 with two additional flows besides the familiar source-side of the throughput, $\dot{x}_i$. The two components of the hidden flow are depicted such that they flow directly from the

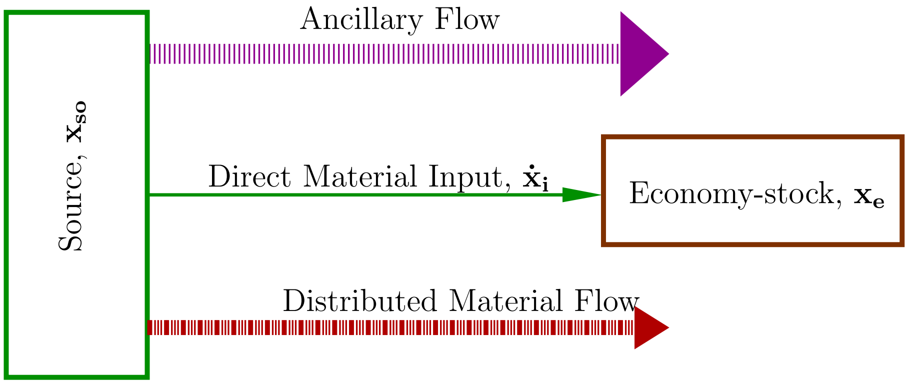

Figure 4.4: Different Components of the Total Material Requirement of an Economy

source to the sink, bypassing the economy. In terms of figure - 4.2, these hidden flows do not figure in the input-output matrix of the economy. Nevertheless, as authors of the Adriaanse et al. report show, hidden flows constitute a large portion of the total flows needed to sustain modern industrial economies. As seen in figure - 4.5, between 55 and 75 percent of the TMR generated by industrial economies are attributable to hidden flows that do not enter the monetary economy. While we will explore hidden flows and their implications for how we model the interface between the economy and the ecosystem in considerable detail, the pervasiveness of these flows provides a *prima facie* evidence that the so-called ‘externalities’ are the rule rather than the exception. The immediate lesson from the relative size of hidden flows within TMR is that an exclusive focus on monetary flows will exclude most of the physical flows that support the economy.

Before we turn to looking at TMR from a scale perspective, and investigate the information that TMR provides about the relationship between the economy and the ecosystem, we review the relationship between TMR and the most well-known time series in economics, the Gross Domestic Product (GDP). Figure - 4.6 depicts a time series for “overall material intensity” as measured by the ratio of TMR and GDP for four different industrial economies over a period of two decades. The figure suggests that industrial economies have, in general, grown more efficient in terms of their material use – it takes a smaller material base (as measured by TMR) to produce a dollar of GDP today than three decades ago.[1] While figure - 4.6 provides us with

---

[1] Though figure - 4.6 stops at 1993, this ‘dematerialization’ trend has largely continued through

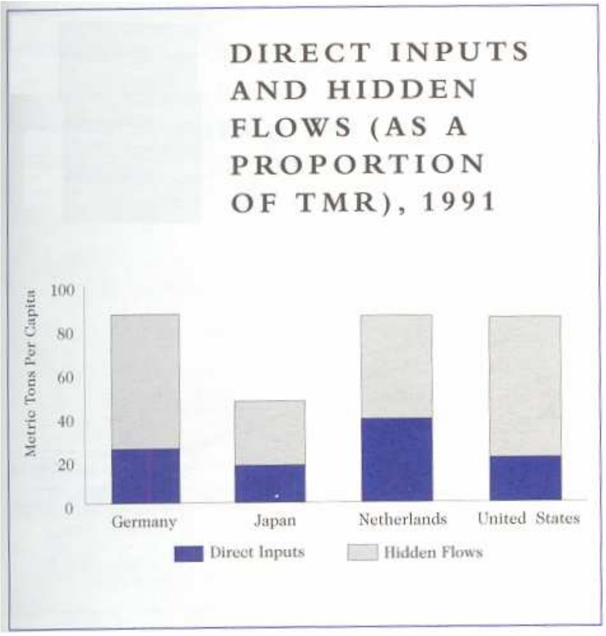

Figure 4.5: Two components of the total throughput. Chart reproduced from Adriaanse et al. [1997a, p.13]

important information about how efficiently industrial economies have been using the material throughput, it does not say anything about the proportional relationship between the economy and the ecosystem. The decades of improving efficiency coincided with a rapid increase in the aggregate economic output and as a result the aggregate TMR increased rather than decreased. From a scale perspective, it is the aggregate TMR that is important rather than how efficiently the society makes use of its aggregate material withdrawal. Indeed as we argue in appendix - A, improving material use efficiency (or in general thermodynamic efficiency) does not automatically guarantee either of the twin societal goals of interest here – physical sustainability or just distribution nor does improving efficiency result in an automatic improvement of aggregate well-being of the society.

---

the nineties.

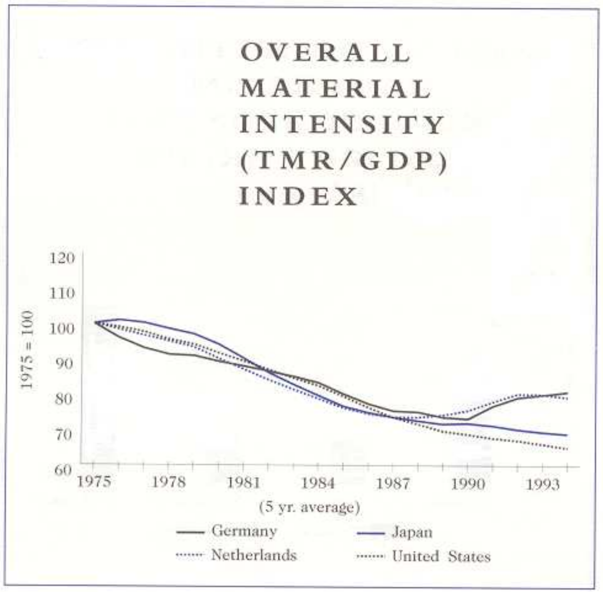

Figure 4.6: TMR/GDP continues to decline in industrialized countries even as total throughput has been increasing, suggesting that efficiency improvements have been outstripped by an increase in economic activity. Chart reproduced from Adriaanse et al. [1997a, p.13]

TMR and the Fund-Flux Space

The ecosystem stock, x_{so} is not only a stock of some material x that is of interest to the human economy but is simultaneously part of a larger fund that can consist of other stocks. A forest is not just a stock of timber but a fund where timber is only of the several stocks that make up the fund. The regenerative capacity of the ecosystem, broadly conceived, is a function of the overall state of the fund rather than any particular constituent stock. Focusing on Total Material Requirement rather than just the extractive flow of interest to the economy helps focus the attention on the entire fund rather than any particular stock that is of interest to the economy. Thus in modeling the relationship between a logging industry and the supporting forest, material flow analysis using TMR will include 'hidden flows' such as loss of soil due to increased erosion as a result of logging and non-timber products that have to be extracted along with the timber that is of primary economic interest.

A Flow Measure of Scale

TMR provides an assessment about the rate at which the entire fund is changing and not just the stock that is of direct interest to the human economy. However, as we have seen in section - 3.2.1, any empirical assessment of the scale of the economy relative to the ecosystem has to be carried out in terms of stocks and flows rather than funds and fluxes. Cardinal measurements are not possible on the fund-flux space. Here we investigate how a flow measure of scale can be constructed using empirical information about the three categories of flows that constitute the Total Material Requirement methodology – the Direct Material Input, the Ancillary Flow, and the Distributed Flow.

Figure - 4.7 below, shows how the flow measure of scale, S_{so} (equation - 3.10, page - 53) can be evaluated using the information contained in the three different constituents of TMR. To evaluate a flow measure of scale defined by equation - 3.10 we need empirical measurements for regeneration corresponding to the three flows that are part of TMR – the simple flow measure of scale, **S** simply compares the rate of withdrawal from the source to the rate to the regeneration rate. The basic analytical representation of the economy in figure - 3.4 looks at stocks and flows corresponding to a single material, x. Figure - 4.7 simply uses the representation in figure - 3.4 as a template and applies it to the three different flows that constitute the TMR methodology. Figure - 4.7 represents x_{so} as a fund rather than a stock to indicate that in addition to x that is of direct interest to the economy the fund contains stocks associated with the two hidden flows. For example x could be some metal that is part of a fund that includes soil and minerals. The two new flows into the fund, $\hat{Y}_{so_{anc}}$ and $\hat{Y}_{so_{dis}}$ represent the regeneration flows corresponding to respective hidden flows, $\dot{x}_{i_{anc}}$ and $\dot{x}_{i_{dis}}$. Following our metal extraction example, $\dot{x}_{i_{anc}}$ could represent the regeneration of minerals that are extracted along with the primary material of interest and $\dot{x}_{i_{dis}}$ the natural regeneration of soil.

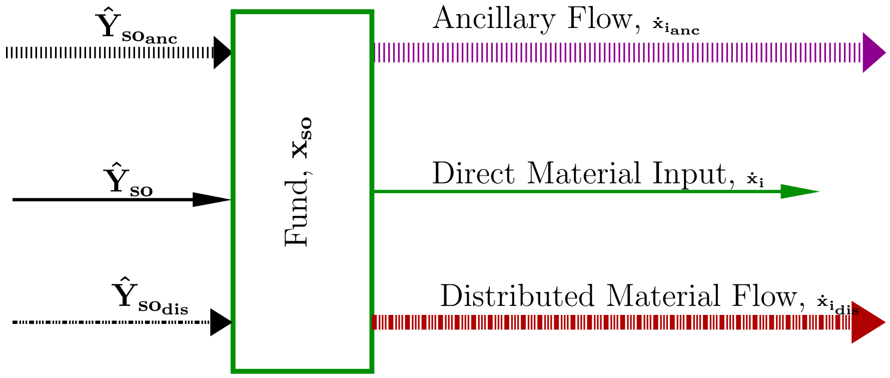

Figure 4.7: TMR as a Flow Measure of Scale. This figure is obtained by combining figure - 4.4 and figure - 3.4. Superimposed on the basic definition for TMR are the two additional regenerative or maintenance flows corresponding to ancillary flow and the distributed flow.

A flow measure of scale that uses the flows represented in figure - 4.7 will simply be a vector form of equation - 3.10:

$$\mathbf{S_{soTMR}} = \begin{pmatrix} S_{so} \\ \\ \\ S_{so_{anc}} \\ \\ \\ S_{so_{dis}} \end{pmatrix} = \begin{pmatrix} \frac{\dot{x}_i}{\hat{Y}_{so}} \\ \\ \\ \frac{\dot{x}_{i_{anc}}}{\hat{Y}_{so_{anc}}} \\ \\ \\ \frac{\dot{x}_{i_{dis}}}{\hat{Y}_{so_{dis}}} \end{pmatrix} \tag{4.1}$$

The vector representation of a flow measure using the TMR methodology, as specified by equation - 4.1 represents a significant improvement over the simple scalar measure that we introduced in equation - 3.10. Besides being a more complete empirical characterization of the flows in the source-side of the economy, combined with the complementary methodology for the sink-side (section - 4.2.2) offers a clear methodology for measuring scale in the flow-dimension using the stock-fund framework introduced in chapter - 3. First, the vector measure in equation - 4.1 provides an empirically measurable characterization of how the fund (of which the stock of material x is only a part) evolves over time. Ecosystem service fluxes that contribute to human welfare are related to the overall structure of the fund rather than the size of the individual stocks that make up the fund. However, any empirical characterization of a fund is fraught with difficulties because no direct cardinal measurement is possible in the fund-flux space. Equation - 4.1 provides a framework that can be used to construct ordinal measure of the state of the fund based on cardinal measurements of changes in the stocks that constitute the fund.[2]

---

[2] See chapter - 6 for a framework for ordinal ranking.

Secondly, besides helping with a fuller characterization of how the fund changes with time, the vector measure of scale in equation - 4.1 helps define a scale measure when a single scalar measure is not defined. Given our focus here on the source-side, the most familiar example would be extraction of non-renewable materials. As discussed in section - 3.3.2 the flow measure of scale is mathematically indeterminate when the regeneration is zero. However the vector measure in equation - 4.1 is defined even if only one of the three regenerations is non-zero.

Interpreting $\mathbf{S_{so_{TMR}}}$

The flow measure of scale introduced in equation - 4.1 encapsulates three different scalar measures corresponding to the three different flows that are part of TMR. Before we consider $S_{so_{TMR}}$ as a vector measure, the interrelationship between the three constituent measures, S_{so}, $S_{so_{anc}}$, and $S_{so_{dis}}$ needs to be investigated. The six flows that are used to obtain the three scale measures all related through the common fund, x_{so}, and all three scale measures contain partial information about how the fund is changing with time. What additional information do the scale measures corresponding to the hidden flows add to the simple scalar scale flow-measure that we developed in equation - 3.10? The two new scalar measures, $S_{so_{anc}}$, and $S_{so_{dis}}$ convey information about the proportional relationship between the economy and the ecosystem that supports the economy because even as the two hidden flows do not enter the monetary economy, they are attributable to economic activity. The two hidden flows, $\dot{x}_{i_{anc}}$ and $\dot{x}_{i_{dis}}$ would not exist absent economic activity. The relative importance of the three scale measures or the question of the additional information provided by the hidden flows is contingent on how each of the three scalars are related to the state of the fund, x_{so}. The state of the fund affects a fund's ability to provide services. Here we are interested in the service flux from the ecosystem in its role as the source of materials for the economy ($\breve{Y}_{so}$). The three different constituents of the vector measure in equation - 4.1 indirectly measure the tradeoff involved in increasing the service flux that can be derived from the economy-stock ($\breve{Y}_e$) and the possible decrease in the service flux from the ecosystem ($\breve{Y}_{so}$). The hidden flows do not contribute to the economy-stock and have no bearing on the service derived from the economy-stock. Thus a correct metric to estimate the relative importance of the three scalar measures would be to look at how the service flux associated with the ecosystem changes as the three different regeneration flows change. Regeneration is a function of the state of the overall fund besides the size of individual stocks. To determine which of the three scalar measures is most relevant we look at the following vector:

$$\delta_{\mathbf{TMR}} = \begin{pmatrix} \frac{\partial \breve{Y}_{so}}{\partial \hat{Y}_{so}} \\ \\ \frac{\partial \breve{Y}_{so}}{\partial \hat{Y}_{so_{anc}}} \\ \\ \frac{\partial \breve{Y}_{so}}{\partial \hat{Y}_{so_{dis}}} \end{pmatrix} \tag{4.2}$$

Each element of the vector $\delta_{\mathbf{TMR}}$ describes how the service flux derived from the ecosystem-stock responds to change in regeneration rates of the three stocks that constitute the fund under the TMR methodology. Of the three scale-measures that constitute $\mathbf{S_{so_{TMR}}}$ in equation - 4.1 the empirically relevant ones will be those where the service flux $\breve{Y}_{so}$ are significantly responsive to changes in corresponding regenerative flows. Thus to interpret the flow measure of scale using the TMR methodology, we will have to use both equation - 4.1, and equation - 4.2. While equation - 4.1 is an actual measure of the scale in the flow dimension, equation - 4.2 tells us how the three-pronged measurement of scale using the TMR methodology is an improvement over a simple scalar measure of scale.

While $\delta_{\mathbf{TMR}}$ in equation - 4.2 encapsulates the process of interpreting a flow measure of scale based on the TMR methodology, there are significant measurement problems – both methodological and empirical that need to be addressed. First, the flux $\breve{Y}_{so}$ is not a physical flow, and as discussed in chapter - 3, represents an ordinal index rather than a cardinal measure. Secondly, evaluating the three partial derivatives that make up $\delta_{\mathbf{TMR}}$ assumes that it is possible in practice to invoke the *ceteris paribus* conditions required to evaluate the partial derivatives. However given that $\hat{Y}_{so}$, $\hat{Y}_{so_{anc}}$, and $\hat{Y}_{so_{dis}}$ are all related to the health of a common fund. By definition the hidden flow, and the ancillary flow in particular is part and parcel of direct material flow and it is impossible even in principle to separate out the effects of Direct Material Input ($\dot{x}_i$) and the ancillary flow ($\dot{x}_{i_{anc}}$). Thus $\delta_{\mathbf{TMR}}$ is a convenient heuristic rather than a metric that can be empirically evaluated. However even as a rough heuristic, $\delta_{\mathbf{TMR}}$ can help in the interpretation of the scale measure $\mathbf{S}_{\mathbf{so}_{\mathbf{TMR}}}$ defined in equation - 4.1 because even if precise evaluation of $\delta_{\mathbf{TMR}}$ is not possible, it may be possible to obtain an estimate of $\delta_{\mathbf{TMR}}$ for the purposes of obtaining a simple rank-ordering of the three elements of $\delta_{\mathbf{TMR}}$.

4.2.2 The Weight of Nations: The Sink-Side

We now take up the sink-side accounting of material flows that builds on the concept of hidden flows introduced in Adriaanse et al. [1997a]. Three years after the source-side accounting was published, Matthews et al. [2000b] extended the analysis to the sink-side. While Matthews et al. [2000b] retain the theoretical framework introduced by Adriaanse et al. [1997a], it makes important additions that reflect the inherent differences between the source-side and sink-side of the economy. Corresponding to the concept of TMR on the source-side is the concept of Total Domestic Output (TDO). The Total Domestic Output is sum of Domestic Processed Output (DPO) and the hidden flows that we encountered in section - 4.2.1. The Domestic Processed Output, the sink-side counterpart of Direct Material Input is the "the total weight of materials, extracted from the domestic environment and imported from other countries, which have been *used in the domestic economy*, then flow to the domestic environment" [Matthews et al. 2000b, p.7, emphasis in original]. Figure - 4.8 below illustrates the basic components of the material flow analysis on the sink-side using the TDO methodology. Figure - 4.8 is homologous to the source-side representation of the economy in figure - 4.4. However the three flows that constitute the sink-side of the throughput (TDO) have somewhat different meanings than the corresponding flows that we encountered in section - 4.2.1, especially in terms of how flows from outside the economic-geographic aggregation under study are treated. In figure - 4.8, the sink is contained within the boundaries of the economic-geographic aggregation under study unlike the source in figure - 4.4 that included resource input to the economy under study from all over the world. This modification helps simplify the accounting procedure on the sink-side. The difference between the source-side and sink-side accounting is analogous to the difference between the definition of Gross National Product (GNP) and the Gross Domestic Product (GDP). The source-side accounting corresponds to the GNP where all the inputs that feed into the economy under study, including foreign inputs are counted in TMR while the TDO like the GDP counts waste that is generated within the boundaries of the economy under study. Thus in figure - 4.8 Domestic Processed Output (DPO, $\dot{x}_o$) includes all the waste flowing out the economy under study, including waste resulting from consumption of foreign goods. Processed goods that are exported get counted in the economy where they eventually find their way to biophysical sinks as waste flows. In addition to DPO, figure - 4.8 shows the two components of hidden

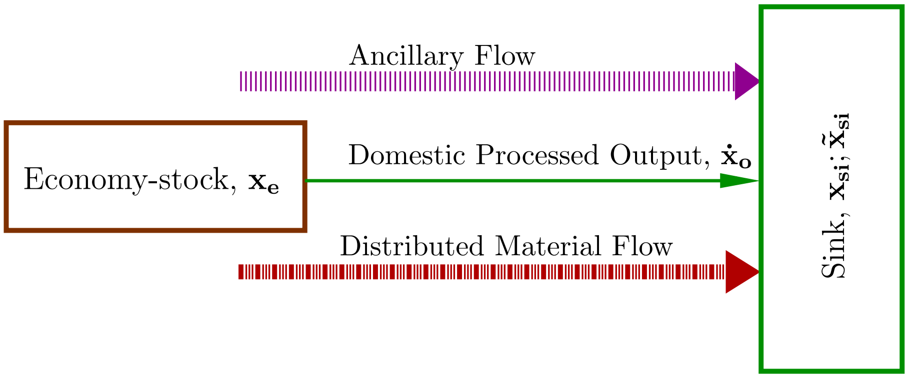

Figure 4.8: Different Components of the Total Domestic Output of an Economy

flows. These hidden flows are identical to ones depicted in figure - 4.4 except that foreign hidden flows have been subtracted out. Thus the portion of hidden flows in TMR that were generated within the borders of the economic-geographic aggregation under study are included in the TDO as well. Hidden flows account for a significant portion of the TDO as seen in figure - 4.9.

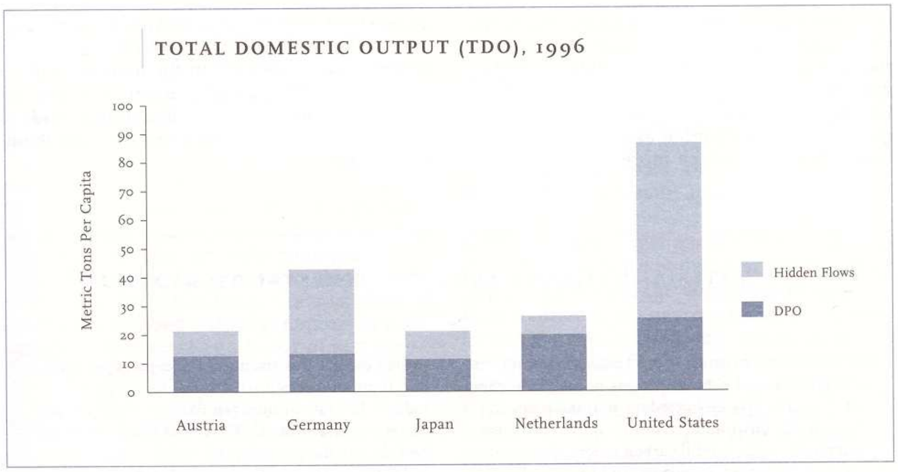

Figure 4.9: Two components of the total sink-side throughput. Chart reproduced from Matthews et al. [2000b, p.16]

Scale on the Sink-Side

A simple flow measure of scale on the sink-side can be constructed on the same lines as our treatment on the source side (equation - 4.1). We begin by recollecting the basic analytical representation of the sink-side of the economy. The ecosystem-sink is represented as a stock of 'holes' that can potentially regenerate. The sink also consists of a stock of waste this is in excess of what the holes can 'absorb.' Thus the sink in figure - 4.10 is shown to contain both holes ($\tilde{x}_{si}$) and waste (x_{si}). Also shown in figure - 4.10 are the three regenerative flows. These flows represent regeneration of holes corresponding to the three flows that make up the throughput on the sink-side. $\hat{Y}_{si}$ is the rate of regeneration of holes that absorb the waste flow represented by DPO or the Domestic Processed Output. $\hat{Y}_{si_{anc}}$ and $\hat{Y}_{si_{dis}}$ are regeneration flows corresponding to the respective 'hidden' components of the throughput. As with the source-side of the economy, any of these regenerative flows may not exist – zero regeneration simply means that the sink does not have any capacity to absorb a particular waste. A flow measure of scale that compares throughput and the corresponding regeneration is specified in equation - 4.3 below.

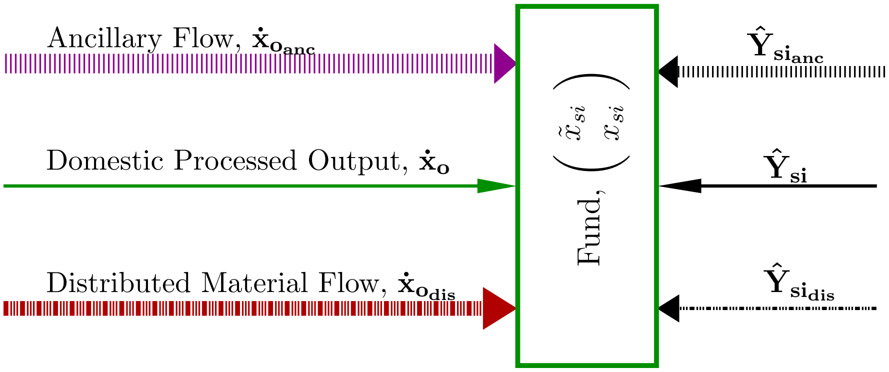

Figure 4.10: TDO as a Flow Measure of Scale. This figure is obtained by combining figure - 4.8 and figure - 3.4. Superimposed on the basic definition for TDO are the two additional regeneration or maintenance flows corresponding to ancillary flow and the distributed flow. This representation is the sink counterpart of the source-side using the TMR methodology in figure - 4.7

$$\mathbf{S_{si_{TDO}}} = \begin{pmatrix} S_{si} \\ \\ S_{si_{anc}} \\ \\ S_{si_{dis}} \end{pmatrix} = \begin{pmatrix} \frac{\dot{x}_{o}}{\hat{Y}_{si}} \\ \\ \frac{\dot{x}_{o_{anc}}}{\hat{Y}_{si_{anc}}} \\ \\ \frac{\dot{x}_{o_{dis}}}{\hat{Y}_{si_{dis}}} \end{pmatrix} \tag{4.3}$$

Equation - 4.3 together with $\mathbf{S_{so_{TMR}}}$ constitutes a complete specification of scale in the flow dimension. Following equation - 3.8 we can combine $\mathbf{S_{so_{TMR}}}$ and $\mathbf{S_{si_{TDO}}}$ as:

$$\mathbf{S_{TMR-TDO}} = \begin{pmatrix} S_{so} & S_{si} \\ \\ S_{so_{anc}} & S_{si_{anc}} \\ \\ S_{so_{dis}} & S_{si_{dis}} \end{pmatrix} \tag{4.4}$$

Equation - 4.4 once again focuses attention on the fact that the TMR and TDO methodologies, theoretical difficulties notwithstanding, represents a scale measure that uses simple accounting of flows to characterize the underlying fund. In equation - 4.4, this is most directly seen in the source-side and sink-side differences in the four scalar measures derived from hidden flows. These differences are not immediately apparent if we looked only at the magnitude of hidden flows without reference to corresponding regenerative flows. The hidden flows or more accurately the domestic component of hidden flows appear on both the source-side and the sink-side. However, the two scale measures on the source-side, $S_{so_{anc}}$ and $S_{so_{dis}}$ can be very different from the corresponding sink-side measures of $S_{si_{anc}}$ and $S_{si_{dis}}$.

Interpreting $\mathbf{S_{si_{TDO}}}$

Equation - 4.5 is homologous to the source-side definition of $\delta_{\mathbf{TMR}}$ in equation - 4.2. The relative importance of the hidden flows on the sink-side is determined by the impact that the regenerative flows associated with the hidden flows ($\hat{Y}_{si_{anc}}$ and $\hat{Y}_{si_{dis}}$) have on the service-flux that is derived from the sink ($\breve{Y}_{si}$).

$$\delta_{\mathbf{TDO}} = \begin{pmatrix} \frac{\partial \breve{Y}_{si}}{\partial \hat{Y}_{si}} \\ \\ \frac{\partial \breve{Y}_{si}}{\partial \hat{Y}_{si_{anc}}} \\ \\ \frac{\partial \breve{Y}_{si}}{\partial \hat{Y}_{si_{dis}}} \end{pmatrix} \tag{4.5}$$

As with the source-side, interpreting the scale measure based on the TDO methodology requires the use of both equation - 4.3 that defines the scale-measure and equation - 4.5 that assesses the relative importance of the three scalar measures that constitute $\mathbf{S_{si_{TDO}}}$. We emphasize again that $\delta_{\mathbf{TDO}}$ is a convenient heuristic rather than a metric that can be empirically evaluated — at least not as an ordinal metric.

Hidden Flows: Relative Impacts on Source and Sink

Extending the logic of equations 4.2 and 4.5, one is tempted to construct a similar heuristic to help compare the relative impact of hidden flows on the source-side and sink-side. Given that the domestic component of the hidden flow is counted on both the source-side and the sink-side, the question of differential impact of these hidden flows on the source-side and the sink-side is an important question. Of the four different scale measures derived from the hidden flows ($S_{so_{anc}}$, $S_{si_{anc}}$, $S_{so_{dis}}$, $S_{si_{dis}}$), which one is the most pertinent in a given economy-ecosystem interaction?

The two vectors, one each for the two components of the hidden flow in equations 4.6 and 4.7 that are similar to the definitions for $\delta_{\mathbf{TMR}}$ and $\delta_{\mathbf{TDO}}$ are not useful to understanding the source-side and sink-side impacts of hidden flows.

$$\delta_{\mathbf{ANC}} = \begin{pmatrix} \frac{\partial \breve{Y}_{so}}{\partial \hat{Y}_{so_{anc}}} \\ \\ \\ \frac{\partial \breve{Y}_{si}}{\partial \hat{Y}_{si_{anc}}} \end{pmatrix} \tag{4.6}$$

$$\delta_{\mathbf{DIS}} = \begin{pmatrix} \frac{\partial \breve{Y}_{so}}{\partial \hat{Y}_{so_{dis}}} \\ \\ \\ \frac{\partial \breve{Y}_{si}}{\partial \hat{Y}_{si_{dis}}} \end{pmatrix} \tag{4.7}$$

The vectors δ_{ANC} and δ_{DIS} are defined such that a comparison of the elements of either δ_{ANC} or δ_{DIS} makes little physical sense. For example in comparing $\partial \breve{Y}_{so}/\partial \hat{Y}_{so_{dis}}$ and $\partial \breve{Y}_{si}/\partial \hat{Y}_{si_{dis}}$, we are comparing two funds that are not necessarily part of the same physical system. The relative importance of the hidden flows on the source-side and sink-side is best captured by examining the relative contributions of the service fluxes derived from the source-side and the sink-side. δ_U in equation - 4.8 contains this information.

$$\delta_{\mathbf{U}} = \begin{pmatrix} \frac{\partial U}{\partial \breve{Y}_{so}} \\ \\ \\ \frac{\partial U}{\partial \breve{Y}_{si}} \end{pmatrix} \tag{4.8}$$

We can further combine equation - 4.8 with equations 4.6 and 4.7 to see how the two components of the hidden flow contribute to the final flux, U. It is useful to reiterate again that U, or $\breve{\mathbf{Y}}$ are fluxes and there are no cardinal measures of these fluxes.

$$\delta_{\mathbf{U_{ANC}}} = \begin{pmatrix} \frac{\partial U}{\partial \breve{Y}_{so}} \frac{\partial \breve{Y}_{so}}{\partial \hat{Y}_{so_{anc}}} \\ \\ \\ \frac{\partial U}{\partial \breve{Y}_{si}} \frac{\partial \breve{Y}_{si}}{\partial \hat{Y}_{si_{anc}}} \end{pmatrix} \tag{4.9}$$

$$\delta_{\mathbf{U_{DIS}}} = \begin{pmatrix} \frac{\partial U}{\partial \breve{Y}_{so}} \frac{\partial \breve{Y}_{so}}{\partial \hat{Y}_{so_{dis}}} \\ \\ \\ \frac{\partial U}{\partial \breve{Y}_{si}} \frac{\partial \breve{Y}_{si}}{\partial \hat{Y}_{si_{dis}}} \end{pmatrix} \tag{4.10}$$

Equations 4.9 and 4.10 that define $\delta_{\mathbf{U_{ANC}}}$ and $\delta_{\mathbf{U_{DIS}}}$ in terms of respective regeneration rates, following the logic of $\delta_{\mathbf{TMR}}$ or $\delta_{\mathbf{TDO}}$ are indirect measures of the differential impact of the various components of the throughput on ecosystem conceived as a fund. We use regeneration as a proxy for the health of the ecosystem.

Alternatives to $\mathbf{S_{so_{TMR}}}$ and $\mathbf{S_{si_{TDO}}}$

We will briefly consider alternative flow measures of scale that can be defined using the TMR or the TDO methodologies. These alternatives to $\mathbf{S_{so_{TMR}}}$ and $\mathbf{S_{si_{TDO}}}$ capture the proportional relationship between the economy and the supporting ecosystem without the need to account for regeneration flows. Consider:

$$\mathbf{S_{MFA1}} = \begin{pmatrix} S_{MFA1_{so}} \\ \\ S_{MFA1_{si}} \end{pmatrix} = \begin{pmatrix} \frac{\dot{x}_i}{\dot{x}_{i_{anc}} + \dot{x}_{i_{dis}}} \\ \\ \frac{\dot{x}_o}{\dot{x}_{o_{anc}} + \dot{x}_{o_{dis}}} \end{pmatrix} \tag{4.11}$$

$\mathbf{S_{MFA1}}$ defined in equation - 4.11 defines a flow measure of scale as the ratio of direct material input and hidden flows on the source-side and the ratio of domestic processed output and domestic hidden flows on the sink-side. As depicted in figure - 4.5 hidden flows constitute the bulk of the economy generated throughput, and a related scale measure can be constructed directly from figure - 4.5:

$$\mathbf{S_{MFA2}} = \begin{pmatrix} S_{MFA2_{so}} \\ \\ S_{MFA2_{si}} \end{pmatrix} = \begin{pmatrix} \frac{\dot{x}_i}{\dot{x}_i + \dot{x}_{i_{anc}} + \dot{x}_{i_{dis}}} \\ \\ \frac{\dot{x}_o}{\dot{x}_o + \dot{x}_{o_{anc}} + \dot{x}_{o_{dis}}} \end{pmatrix} \tag{4.12}$$

There are two distinct advantages in using a measure like $\mathbf{S_{MFA1}}$ instead of a measure constructed as a ratio of throughput to regeneration. First, in the context of the material flow analysis accounting with the TMR and TDO methodologies, scale measured as the ratio of throughput and regeneration is empirically difficult to determine in the case of hidden flows. Second, even with direct material input (or domestic processed output on the sink-side), as seen in section - 3.3.2, scale measured as the ratio of throughput and regeneration is indeterminate when a regeneration flow does not exist.

4.2.3 Aggregation Problems in TMR and TDO Representations: A Stock-Fund Perspective

In this section we review some of the most important theoretical problems associated with the TMR and TDO methodologies. The critical shortcoming of the TMR and TDO methodologies is related to the dimensional consistency of the twelve flows represented in figures 4.7 and 4.10. All the scale measures that we have constructed are different functions of two or more of the twelve flows that constitute the TMR and TDO methodologies.[3] The dimensional integrity of the scale measures are contingent on the dimensional consistency of the underlying flows. A desirable feature of any scale measure is that it be "dimensionless." Dimensionless scale measures facilitate cross-sectional and time-series comparison of a scale measure. In section - 3.4 we showed how a scale measure that is dimensionless in some stock-flow space may not be dimensionless in the fund-flux space. The twelve different flows that define the TMR and TDO methodologies are all aggregates. The stocks and funds represented in figures 4.7 and 4.10 are abstract stocks corresponding to the aggregate flows. On the source-side, the Direct Material Input ($\dot{x}_i$) in figure - 4.4 is a sum of all the inputs used by an economy. Symbolically, for n different direct inputs that an economy uses,

$$\dot{\mathbf{x}}_{\mathbf{i}} = \sum_{j=1}^{n} \dot{x}_i^j \tag{4.13}$$

---

[3] The original analysis presented in Adriaanse et al. [1997a], Matthews et al. [2000b] do not contain the six regeneration flows that we have introduced to construct flow measures of scale.

The sum in equation - 4.13 that is central to the TMR methodology is defined only in the abstract stock-flow space that the methodology uses. In the abstract stock-flow space stocks are measured in kilograms and flows in kilogram/year. In this abstract space, kilograms of copper can be added to kilograms of aluminum. Indeed, as suggested by equation - 4.13, $\dot{\mathbf{x}}_{\mathbf{i}}$ under the TMR methodology sums up not just weight of two metals used by the economy but n different (and diverse) material inputs used by the economy under study. The stock or fund, x_{so} represented in figure - 4.4 is an abstract aggregate that is the source of n different constituents of the direct material input as well as the associated hidden flows. As such x_{so} as represented in figure - 4.4 does not exist in nature – it is an abstraction that aids the construction of scale measures using the TMR and TDO accounts. The focus of this section is to examine the theoretical difficulties associated with this abstraction.

The purpose of any empirical exercise using scale-measures is to characterize the proportional relationship between the economy and the ecosystem. The scale methodology is a tool to understand how a given quantum of economic activity, measured in biophysical terms, impinges on the supporting ecosystem. The most important requirement of any good scale measure is to be able to link a given economic activity to the impact that economic activity has on the ecosystem. While the quantum of economic activity is necessarily measured in the stock-flow space, the health of the ecosystem is a function of the structure of the fund. Thus two questions follow: how does a scale measure account for economic activity in the stock-flow space, and what information does the scale measure provide about impact of economic activity on the ecosystem fund? We take up each of these questions in the context of material flow analysis based on the TMR-TDO methodology.

Measuring aggregate economic activity in biophysical terms presents considerable theoretical difficulty that escapes monetary aggregation. An aggregate measure of economic activity like the Gross Domestic Product (GDP) aggregates exchange value of all economic activity. All forms of value addition are reduced to a common monetary denominator – the dollar for example. Monetary aggregation has a straightforward interpretation – it represents the aggregate exchange value of economic activity. Biophysical measures of economic activity try to aggregate diverse economic activity in terms of physical stocks and flows. Aggregate biophysical measures of economic activity are difficult to interpret because reducing the multitude of physical flows that feed the economy to a common denominator is a wholly different exercise from that of aggregating the exchange value of all economic activity in an economy. The TMR-TDO framework for material flow analysis provides an especially fertile ground to investigate some of the difficulties involved. We begin by focusing again on equation - 4.13. Expressing direct material input as a sum of n different throughputs supporting the economy is based on two important assumptions. First is the assumption that it is possible to express the summation in equation - 4.13 in terms of a certain common physical attribute. Secondly an assumption is made that the chosen physical aggregation has a consistent meaning relevant to the analysis of how the economy and the ecosystem interact. We show here that the TMR-TDO framework at best satisfies these assumptions in a very restricted sense.

In the context of the stock-flow space, the question of whether it is possible to aggregate economic activity in terms of some common attribute is rather moot. All matter has mass and is embedded within space-time.[4] Thus it is essentially possible to aggregate economic activity in material terms using any combination of length, mass, and time. Thus in principle, any arbitrary combination of meter, kilogram, and seconds will represent some aggregation. The metric used by the TMR-TDO framework reduces all economic activity to the single dimension of mass (kilograms). The pertinent question then is that of representation – what does an aggregate like TMR or TDO represent in terms of economy-ecosystem interactions. Before we take up the economic question of primary interest here – that of scale, it is instructive to briefly discuss how an aggregate measure like the TMR can be used to discuss the more traditional economic concerns of allocation and distribution.[5] Looking at the representation question in the context of allocation and distribution will help motivate our discussion on scale measures derived from aggregate biophysical measures in general and aggregate material flow measures like the TMR or TDO in particular. This brief detour away from our central concern of scale will aid in bringing to fore the importance of the representation question – what kind of economic analysis is possible with aggregate biophysical measures? This question directly impinges

---

[4] As discussed in chapter - 1, while bulk of the discussion in this dissertation is applicable to matter and energy, the primary focus of this dissertation is on stock and flows of materials rather than energy flows that support the economy.

[5] We will present a more detailed discussion on aggregate biophysical measures and allocation-distribution questions in section - 4.4 that investigates the ecological footprint. Also see appendix - A.

on scale, as scale metrics are a basic tool for economic analysis if the economy is conceived according to the ecological economics' ontological vision represented in figure - 3.1.

We begin by looking at the meaning of "distribution" in biophysical terms. Conceptually, distribution of income refers to how the throughput, $\dot{\mathbf{x}}_\mathbf{i}$ in figure - 3.4 is divided up between different people or groups in a society. Distribution of wealth refers to control of the productive stocks x_{so} and x_e. A monetary metric represents this distribution in terms of the exchange value of the throughput (income) or the stocks (wealth). As a positive representation of how the throughput is distributed between different members of the society, we can directly use a physical attribute of the throughput rather than the exchange value of the throughput.[6] In case of the TMR methodology under study, the physical metric would be the weight of the throughput. For the sake of simplicity, we will disregard the hidden flows and focus our attention on the direct material inputs. Direct Material Input is simply the total weight of the throughput used by the economy – it is a simple sum of all the material flows represented by equation - 4.13. Distribution measured in terms of aggregate material input will give a distribution of the total weight of the throughput appropriated by each individual in a society. The first observation that we make is that distributions as measured by a monetary metric and the physical metric will, in general, not coincide – for example if we superimposed a Lorenz Curve based

---

[6] In the discussion here, we will focus on income distribution rather than wealth distribution. Our motivation here is not theoretical completeness rather it is to try and use distribution as an expository device to motivate similar questions surrounding scale.

on income measured in dollars with a another curve based on income measured in kilograms, the curves will not coincide.

If the two distribution curves do not coincide, the question of which representation is more useful for economic analysis becomes salient. This question is somewhat straightforward to resolve in the context of studying income distribution but a similar dilemma that we will soon encounter in studying TMR as a scale measure is far from being trivial. Here, we only need to ask what can an individual *do* with income that is dollar denominated; or with income denominated in kilograms of aggregate throughput. The aggregate throughput represents a sum of all material flows into the economy that is agnostic about the qualitative differences between the various constituents of the throughput. This agnostic valuation of the different constituents of the throughput leaves an aggregate measure like the TMR lacking in any economic meaning as a measure of distribution. Of course the TMR was not developed to be a measure of distribution – a kilogram of nickel cannot be worth the same as a kilogram of gold. The point is not about how TMR could never be a measure of distribution because of its agnostic aggregation of all throughput supporting the economy but the fact that the usefulness of an aggregate measure is contingent on how the measure abstracts from fundamental features of an economy.[7] If a proper valuation (marginal or otherwise) of the throughput is important to distribution, an aggregate measure of scale needs to abstract the economy in

---

[7] In appendix - A we argue that a framework that allows for simultaneous interpretation of scale, allocation, and distribution in biophysical terms can make a significant advance over independent frameworks to understand scale, allocation, or distribution.

a way that enables a consistent interpretation of the linkages between the physical size of economic activity and the health of the ecosystem that contains the economy.

We started this section by showing how the analysis of an aggregate scale measure consists of two parts. First, we look at at how the scale measure aggregates economic activity and second the impact of that economic activity on the supporting ecosystem. We start by extending the logic of our discussion of TMR as a distribution measure to TMR as an aggregate measure of the physical size of economic activity. In particular we are interested in the economic meaning contained in TMR as an aggregate physical measure of economic activity. A good place to start is the physical input-output representation of the economy that we used to introduce the concept of material flow analysis (figure - 4.2). The physical input-output matrix shows the flow of different components of the throughput that make up Direct Material Input. The input-output matrix defines the physical structure of the economy under study – for any given good in the economy one can discern all the physical inputs that go into making that good. Alternatively, if one is interested in how a given material flow is used, say for example, copper, the row corresponding to copper shows how much copper is used by the different sectors of the economy. The aggregate copper used by the economy is simply a row sum. Our focus here is on the question of how (if) these different row-sums can be aggregated. The thick arrows in figures 4.1 through 4.2 that cross the system boundaries are not a single aggregate flow like the TMR but represent all the materials that make up the input-output matrix. Ignoring hidden flows, the thick arrows in figure - 4.1 can symbolically be represented as:

$$\dot{\mathbf{x}}_{\mathbf{i}} = \begin{pmatrix} \dot{x}_i^1 \\ \vdots \\ \dot{x}_i^j \\ \vdots \\ \dot{x}_i^n \end{pmatrix} \tag{4.14}$$

Equation - 4.13 reduces the vector $\dot{\mathbf{x}}_{\mathbf{i}}$ above to a simple scalar aggregate by simply summing up the different elements of $\dot{\mathbf{x}}_{\mathbf{i}}$. We can now pose the representation question that we are interested in as: what information is lost when we use $\dot{x}_i$ from equation - 4.13 instead of the vector $\dot{\mathbf{x}}_{\mathbf{i}}$ in equation - 4.14? The answer to this question is contingent on how the individual $\dot{x}_i^j$ are measured. If the n elements of $\dot{\mathbf{x}}_{\mathbf{i}}$ are represented in dollars, the sum in equation - 4.13 is dimensionally consistent and the sum merely represents the sum total of economic activity of a given economy, measured in dollar terms. However, if the n elements of $\dot{\mathbf{x}}_{\mathbf{i}}$ are represented in physical terms, the sum in equation - 4.13 is dimensionally consistent in a very limited sense. While one can add n different weights (corresponding to the weights of each element of $\dot{\mathbf{x}}_{\mathbf{i}}$), the resulting aggregate sum has at best very limited economic meaning, as a measure of the physical basis for economic activity. Even disregarding the biophysical funds from which the various flows are derived (to which we will turn to shortly), the aggregation of all the flows supporting the economy is a representation that contributes little to understanding the physical basis of the economy. The input-output matrix helps clarify why aggregating economic activity based on a single physical attribute like the weight of the throughput that is used by the TMR-TDO framework offers very little insights into the physical structure of the economy. The economy produces various goods and services using real material flows of oil, copper, timber, etc., and not using kilograms of some some abstract aggregate flow.

One could of course argue that the aggregate throughput measured by the TMR-TDO framework is only meant to study throughput crossing the economy either as raw material inputs or as waste from economic activity. This argument cannot be sustained for two reasons. First, we will show shortly that even if we are interested in the aggregate abstract flow represented by either the TMR or the TDO as a proxy for impact on the ecosystem from all economic activity, an aggregation based on a single physical characteristic does not capture important aspects of how the economy and the ecosystem interact. Secondly, there is an irreducible relationship between flows that cross the system boundary and flows inside the boundary, and a consistent representation of these two sets of flows is crucial to understanding the relationship between economic activity and the supporting ecosystem. When aggregate flows are measured in monetary terms, the monetary input output matrix describes how the aggregate economic product is allocated between various sectors of the economy. Individual sector flows are measured in dollars and the total product of the economy is measured in dollars as well.[8] A consistent metric that describes flows entering (or exiting) an economic-geographic aggregation as well as

---

[8] A monetary aggregate typically looks only at the source-side of the physical economy. The sink-side of the economy, by definition consists of ‘waste flows’ that are have no economic value. Monetary aggregates, again by definition, do not consider hidden flows.

the flows within a given economic-geographic aggregation opens up possibility for ecological macroeconomics. If aggregate flows are measured with a metric that has no economic meaning, it becomes impossible to empirically validate any theoretical linkages between economic activity and the material basis for that economic activity. Practical policy making is contingent on being able to establish causal linkages between the physical structure of the economy and the aggregate material flows that support the economy. The best that can be achieved using an aggregate measure like the TMR is a study of correlations in terms of simple efficiency measures like the relationship between TMR and GDP that is presented in figure - 4.6. Finally, as we show in appendix - A it is desirable to measure economic activity using a metric that can be used to describe all three different aspects of the economic process that is of interest to ecological economics – scale, allocation, and distribution.

Having discussed the principal problems with the TMR-TDO methodology as a measure of aggregate economic activity, we turn to a discussion of flow measures of scale derived from the TMR-TDO accounts. A flow measure of scale such as the one represented in equation - 4.4 is simply the ratio of throughput to regeneration. The throughput is a physical measure of the size of economic activity and regeneration, as we have seen is at least in part related to the overall health of the supporting ecosystem. The flow measure of scale then is one of the ways to characterize the proportional relationship between the economy and the ecosystem. Scale is measured in the stock-flow dimension but the ultimate goal of any empirical work with scale is to discern the relationship between the economy and the ecosystem in the fund-flux space. While in chapter - 5 we will lay out a detailed framework on how empirical measurements in the stock-flow space map onto to the fund-flux space, the goal here is to study some very basic features of the TMR-TDO framework. The primary interest here is understanding the TMR-TDO accounting framework from a fund-flux perspective.

$\mathbf{S}^{\mathsf{T}}_{\mathbf{so_{TMR}}}$ and $\mathbf{S}^{\mathsf{T}}_{\mathbf{si_{TDO}}}$ in the Fund-Flux Space

The fundamental problem with the scale measures defined using the TMR-TDO framework is that they are not "qualitatively dimensionless" (section - 3.4). The sum defined in equation - 4.13 is not defined in the fund-flux space. Even when the sum is mathematically defined in the abstract stock-flow space of the TMR-TDO framework it is a simple non-weighted sum and does not convey the true state of the ecosystem even in the stock-flow space. Consider the chart in figure - 4.11 reproduced from Matthews et al. [2000b]. Figure - 4.11 shows the contribution of CO_2 emissions from fossil fuels to aggregate DPO. It shows that between 70 and 90% of the DPO is accounted for by CO_2 from fossil fuel burning. On the source-side, water (as part of the hidden flows) trumps everything else in the aggregate TMR.

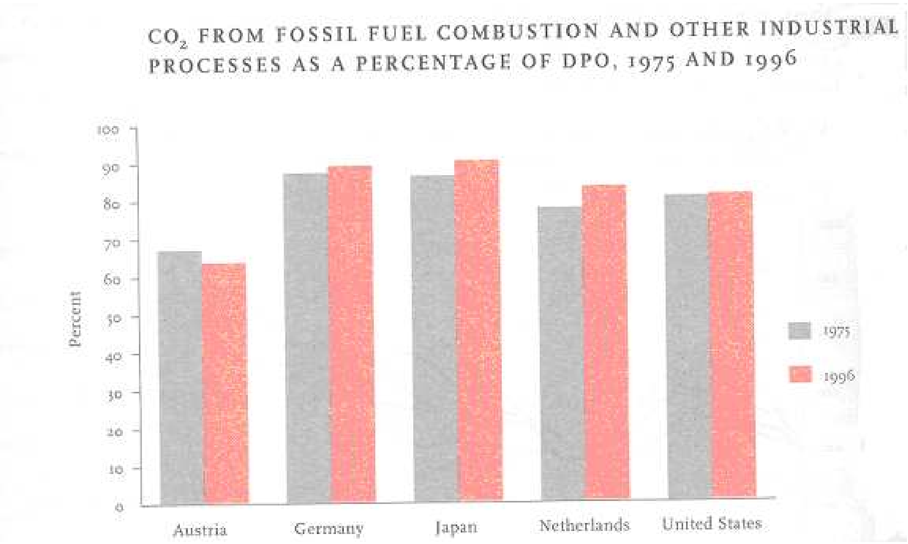

Figure 4.11: Carbon-di-oxide Component of DPO. The data does not include emissions from biomass combustion, that would add approximately 15% to the share of CO_2 in the total DPO. Chart reproduced from Matthews et al. [2000b, p.22]

Modifying TMR-TDO Scale Measures

We modify the scale measure on the source-side from equation - 4.1 to account for the n different constituents of the direct material input, $\dot{x}_i$:

$$\mathbf{S_{so}} = \begin{pmatrix} \frac{\dot{x}_i^1}{\hat{Y}_{so}^1} \\ \vdots \\ \frac{\dot{x}_i^j}{\hat{Y}_{so}^j} \\ \vdots \\ \frac{\dot{x}_i^n}{\hat{Y}_{so}^n} \end{pmatrix} \tag{4.15}$$

Equation - 4.15 is simply the disaggregated representation of S_{so}. We can replicate equation - 4.15 on the sink side:

$$\mathbf{S_{si}} = \begin{pmatrix} \frac{\dot{x}_o^1}{\hat{Y}_{si}^1} \\ \vdots \\ \frac{\dot{x}_o^j}{\hat{Y}_{si}^j} \\ \vdots \\ \frac{\dot{x}_o^n}{\hat{Y}_{si}^n} \end{pmatrix} \tag{4.16}$$

Equations 4.15 and 4.16 account for only the direct flows. The simplest way to incorporate hidden flows would be to simply add more elements corresponding to ancillary and distributed flows to the vectors in equations 4.15 and 4.16. If we assume that there are m different kinds of ancillary flows and k different kinds of distributed flows, $\mathbf{S}_{\mathbf{so}_{\mathbf{TMR}}}$ and $\mathbf{S}_{\mathbf{si}_{\mathbf{TDO}}}$ can be rewritten as:

$$\mathbf{S}_{\mathbf{so}_{\mathbf{TMR}}} = \begin{pmatrix} \frac{\dot{x}_i^1}{\hat{Y}_{so}^1} \\ \vdots \\ \frac{\dot{x}_i^n}{\hat{Y}_{so}^n} \\ \\ \frac{\dot{x}_{i_{anc}}^1}{\hat{Y}_{so_{anc}}^1} \\ \vdots \\ \frac{\dot{x}_{i_{anc}}^m}{\hat{Y}_{so_{anc}}^m} \\ \\ \frac{\dot{x}_{i_{dis}}^1}{\hat{Y}_{so_{dis}}^1} \\ \vdots \\ \frac{\dot{x}_{i_{dis}}^k}{\hat{Y}_{so_{dis}}^k} \end{pmatrix} \qquad \mathbf{S}_{\mathbf{si}_{\mathbf{TDO}}} = \begin{pmatrix} \frac{\dot{x}_o^1}{\hat{Y}_{si}^1} \\ \vdots \\ \frac{\dot{x}_o^n}{\hat{Y}_{si}^n} \\ \\ \frac{\dot{x}_{o_{anc}}^1}{\hat{Y}_{si_{anc}}^1} \\ \vdots \\ \frac{\dot{x}_{o_{anc}}^m}{\hat{Y}_{si_{anc}}^m} \\ \\ \frac{\dot{x}_{o_{dis}}^1}{\hat{Y}_{si_{dis}}^1} \\ \vdots \\ \frac{\dot{x}_{o_{dis}}^k}{\hat{Y}_{si_{dis}}^k} \end{pmatrix} \tag{4.17}$$

However, the representation in equation - 4.17 is not fully faithful to how the two components of hidden flows are defined. The hidden flows are not independent of the direct material input or domestic processed output that are actually used by the economy. A more useful representation of the hidden flows would be to partition the $m + k$ hidden flows among the n direct flows. For example if soil is a hidden flow associated with extraction of both oil and copper, the hidden flow of soil will be apportioned between copper and oil in direct proportion to contribution to the hidden flow of soil from each of the two extractive processes.

$$\mathbf{S}^{\mathsf{T}}_{\mathbf{so_{TMR}}} = \begin{pmatrix} S_{so} & S_{so_{anc}} & S_{so_{dis}} \end{pmatrix} = \begin{pmatrix} \frac{\dot{x}^1_i}{\hat{Y}^1_{so}} & \frac{\dot{x}^1_{i_{anc}}}{\hat{Y}^1_{so_{anc}}} & \frac{\dot{x}^1_{i_{dis}}}{\hat{Y}^1_{so_{dis}}} \\ \vdots & \vdots & \vdots \\ \frac{\dot{x}^j_i}{\hat{Y}^j_{so}} & \frac{\dot{x}^j_{i_{anc}}}{\hat{Y}^j_{so_{anc}}} & \frac{\dot{x}^j_{i_{dis}}}{\hat{Y}^j_{so_{dis}}} \\ \vdots & \vdots & \vdots \\ \frac{\dot{x}^n_i}{\hat{Y}^n_{so}} & \frac{\dot{x}^n_{i_{anc}}}{\hat{Y}^n_{so_{anc}}} & \frac{\dot{x}^n_{i_{dis}}}{\hat{Y}^n_{so_{dis}}} \end{pmatrix} \tag{4.18}$$

We have used $\mathbf{S}^{\mathsf{T}}_{\mathbf{so_{TMR}}}$ instead of $\mathbf{S}_{\mathbf{so_{TMR}}}$ for the purposes of simpler exposition. The transpose representation also results in computational efficiency when used for empirical analysis. We can similarly represent the scale measure $\mathbf{S}_{\mathbf{si_{TDO}}}$ on the sink-side as:

$$\mathbf{S}^{\mathsf{T}}_{\mathbf{si_{TDO}}} = \begin{pmatrix} S_{si} & S_{si_{anc}} & S_{si_{dis}} \end{pmatrix} = \begin{pmatrix} \frac{\dot{x}^1_o}{\hat{Y}^1_{si}} & \frac{\dot{x}^1_{o_{anc}}}{\hat{Y}^1_{si_{anc}}} & \frac{\dot{x}^1_{o_{dis}}}{\hat{Y}^1_{si_{dis}}} \\ \vdots & \vdots & \vdots \\ \frac{\dot{x}^j_o}{\hat{Y}^j_{si}} & \frac{\dot{x}^j_{o_{anc}}}{\hat{Y}^j_{si_{anc}}} & \frac{\dot{x}^j_{o_{dis}}}{\hat{Y}^j_{si_{dis}}} \\ \vdots & \vdots & \vdots \\ \frac{\dot{x}^n_o}{\hat{Y}^n_{si}} & \frac{\dot{x}^n_{o_{anc}}}{\hat{Y}^n_{si_{anc}}} & \frac{\dot{x}^n_{o_{dis}}}{\hat{Y}^n_{si_{dis}}} \end{pmatrix} \tag{4.19}$$

We reiterate that hidden flows and the regeneration corresponding to each component of the hidden flows have different meanings in equations 4.18 and 4.19 above than in equation - 4.17. Any particular component of the hidden flow, $\dot{x}^j_{i_{anc}}$, for example in equation - 4.18 represents the ancillary flow associated with the j^{th} component of direct material input. However in equation - 4.17, $\dot{x}^j_{i_{anc}}$ would simply represent the j^{th} kind of ancillary flow on the source-side. In general $\dot{x}^j_{i_{anc}}$ in equation - 4.17 will be greater than the $\dot{x}^j_{i_{anc}}$ in equation - 4.18 because the former represents an aggregate of a given hidden flow that could be associated with more than one direct input. Given the importance of this reformulation to any empirical work that uses the TMR-TDO framework to derive flow measures of scale, we will formalize this modification in how the hidden flows are to be accounted. We begin by recollecting that there are m different kinds of ancillary flows and k different kinds of distributed flows associated with n direct flows. For reasons discussed earlier, we include only the domestic flows in the formalism here as our primary motivation is a framework to use the empirical data from the TMR-TDO methodology to construct flow measures of scale rather than study varying resource use between different economies. Thus $m + k + n$ direct and hidden flows in equations do not include any flows that are not generated from within the economic-geographic aggregation under study.[9] Setting aside imported flows also makes it easier to combine the source-side and sink-side treatment under the TMR-TDO methodology into a single framework.[10] For ancillary flows on the source-side, the m different kinds of

---

[9] Refer to earlier discussion in this section on the difficulties involved in delineating system boundaries in the context of material flows that transcend simple aggregations like a national economy.

[10] It is useful to reiterate that for reasons stated in chapter - 3, scale is measured independently on the source-side and sink-side. For our purposes here m, k, and n take on different values on the source-side and sink-side. Equations 4.18 and 4.19 use the same set of symbols only for the purposes of maintaining expository consistency between the source-side and sink-side. We have avoided cluttering the notation with more subscripts – k, m, and n are to be read as short-cut representations for k_{so}, m_{so}, n_{so} on the source-side and k_{si}, m_{si}, n_{si} on the sink-side.

flows can be expressed in terms of n direct flows as:

$$\dot{x}^{\theta}_{i_{anc}} = \sum_{j=1}^{n} \mathcal{X}^{\theta^j}_{i_{anc}} \tag{4.20}$$

In equation - 4.20 $\mathcal{X}^{\theta}_{i_{anc}}$ represents one of the m different kinds of ancillary flows on the source-side. Also note that $\mathcal{X}^{\theta}_{i_{anc}}$ could be zero for several of the n direct flows. This is to say that a given ancillary flow, $\mathcal{X}^{\theta}_{i_{anc}}$ is associated with only some and not all of the direct flows. The extraction of oil from the ground will involve ancillary flows that are not present during the harvest of timber or even extraction of natural gas. Writing down a relationship similar to equation - 4.20 for each of the m different ancillary flows we get:

$$\dot{\mathbf{x}}_{\mathbf{i_{anc}}} = \begin{pmatrix} \dot{x}^{1}_{i_{anc}} \\ \vdots \\ \dot{x}^{\theta}_{i_{anc}} \\ \vdots \\ \dot{x}^{m}_{i_{anc}} \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{n} \mathcal{X}^{1^j}_{i_{anc}} \\ \vdots \\ \sum_{j=1}^{n} \mathcal{X}^{\theta^j}_{i_{anc}} \\ \vdots \\ \sum_{j=1}^{n} \mathcal{X}^{m^j}_{i_{anc}} \end{pmatrix} \tag{4.21}$$

Following the structure of equation - 4.21 for k different distributed flows, we can express the distributed flow vector as:

$$\dot{\mathbf{x}}_{\mathbf{i_{dis}}} = \begin{pmatrix} \dot{x}^{1}_{i_{dis}} \\ \vdots \\ \dot{x}^{\theta}_{i_{dis}} \\ \vdots \\ \dot{x}^{k}_{i_{dis}} \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{n} \mathcal{X}^{1^j}_{i_{dis}} \\ \vdots \\ \sum_{j=1}^{n} \mathcal{X}^{\theta^j}_{i_{dis}} \\ \vdots \\ \sum_{j=1}^{n} \mathcal{X}^{k^j}_{i_{dis}} \end{pmatrix} \tag{4.22}$$

In both equations 4.21 and 4.22 there is no fixed relationship between the magnitudes of m, k, and n:

$$m \lesseqqgtr k \lesseqqgtr n \tag{4.23}$$

However empirically speaking, the number of hidden flows $(m + k)$ far exceed the material flows that actually get counted (n). Also, as noted earlier, m, k, and n take on different values on the source-side and the sink-side. Equations 4.21 and 4.22 can be reproduced on the sink-side. Risking repetition for the sake of completeness, we write the sink-side counterparts of equations 4.21 and 4.22:

$$\dot{\mathbf{x}}_{\mathbf{o_{anc}}} = \begin{pmatrix} \dot{x}^{1}_{o_{anc}} \\ \vdots \\ \dot{x}^{\theta}_{o_{anc}} \\ \vdots \\ \dot{x}^{m}_{o_{anc}} \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{n} \mathcal{X}^{1^j}_{o_{anc}} \\ \vdots \\ \sum_{j=1}^{n} \mathcal{X}^{\theta^j}_{o_{anc}} \\ \vdots \\ \sum_{j=1}^{n} \mathcal{X}^{m^j}_{o_{anc}} \end{pmatrix} \tag{4.24}$$

Similarly, for the distributed flow, we have:

$$\mathbf{\dot{x}_{o_{dis}}} = \begin{pmatrix} \dot{x}^1_{o_{dis}} \\ \vdots \\ \dot{x}^\theta_{o_{dis}} \\ \vdots \\ \dot{x}^k_{o_{dis}} \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^n \mathcal{X}^{1^j}_{o_{dis}} \\ \vdots \\ \sum_{j=1}^n \mathcal{X}^{\theta^j}_{o_{dis}} \\ \vdots \\ \sum_{j=1}^n \mathcal{X}^{k^j}_{o_{dis}} \end{pmatrix} \tag{4.25}$$

The four vectors in equations 4.21, 4.22, 4.24, and 4.25 cannot be combined into a single matrix for hidden flows because the source-side and sink-side vectors, as discussed earlier, are in general not of the same dimensions. Equations 4.21, 4.22, 4.24, and 4.25 show how the different components of the throughput are accounted for in our revision of the TMR-TDO accounting framework. The revised scale measures in equations 4.18 and 4.19 are ratios of throughput and the corresponding regeneration flows. While much of the accounting structure for throughput can be directly translated to regeneration flows as well, the treatment of regeneration, and especially regeneration corresponding to hidden flows ($\hat{Y}^j_{so_{anc}}$,$\hat{Y}^j_{so_{dis}}$ on the source-side and $\hat{Y}^j_{si_{anc}}$,$\hat{Y}^j_{si_{dis}}$ on the sink-side) deserve further explanation. There is also a significant source-side and sink-side difference in how regeneration is interpreted on the source-side and sink-side. In the stock-flow space, we can write down an equation similar to equation - 4.20. Once again using the ancillary flow as the example:

$$\mathbf{\hat{Y}^\theta_{so_{anc}}} = \sum_{j=1}^{n} \hat{\mathcal{Y}}^{\theta^j}_{so_{anc}} \tag{4.26}$$

Suppose $\hat{\mathbf{Y}}^{\theta}_{\mathbf{so_{anc}}}$ in equation - 4.26 represents the regeneration of soil, then the sum simply represents the sum of regeneration of soil associated with different extractive flows in the economy. While equation - 4.26 is dimensionally consistent in the stock-flow space, it represents several problems in the fund-flux space that we will consider in the next subsection – the most obvious problem being that from the perspective ecosystem health, adding regeneration flows from different sources obliterates (for example soil at vastly different sites) the qualitative differences between the various components of the regenerative flow. However, our goal here is to simply provide a complete empirically computable framework for every element in equations 4.18 and 4.19 that define the source-side and sink-side scale measures respectively. The scale measures introduced here, $\mathbf{S}^{\mathsf{T}}_{\mathbf{so_{TMR}}}$ and $\mathbf{S}^{\mathsf{T}}_{\mathbf{si_{TDO}}}$ in the context of the fund-flux space will apply to special cases where the economic-geographic aggregation coincides with the geographic extent of the ecosystem fund of interest. The scalar sum in equation - 4.26 is repeated m and k times respectively for following the logic of equations 4.21, 4.22, 4.24, and 4.25.

$$\hat{\mathbf{Y}}_{\mathbf{so_{anc}}} = \begin{pmatrix} \hat{Y}^{1}_{so_{anc}} \\ \vdots \\ \hat{Y}^{\theta}_{so_{anc}} \\ \vdots \\ \hat{Y}^{m}_{so_{anc}} \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{n} \hat{\mathcal{Y}}^{1^j}_{so_{anc}} \\ \vdots \\ \sum_{j=1}^{n} \hat{\mathcal{Y}}^{\theta^j}_{so_{anc}} \\ \vdots \\ \sum_{j=1}^{n} \hat{\mathcal{Y}}^{m^j}_{so_{anc}} \end{pmatrix} \tag{4.27}$$

In case of regeneration associated with the distributed flows we have:

$$\hat{\mathbf{Y}}_{\mathbf{so_{dis}}} = \begin{pmatrix} \hat{Y}^{1}_{so_{dis}} \\ \vdots \\ \hat{Y}^{\theta}_{so_{dis}} \\ \vdots \\ \hat{Y}^{m}_{so_{dis}} \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{n} \hat{\mathcal{Y}}^{1^j}_{so_{dis}} \\ \vdots \\ \sum_{j=1}^{n} \hat{\mathcal{Y}}^{\theta^j}_{so_{dis}} \\ \vdots \\ \sum_{j=1}^{n} \hat{\mathcal{Y}}^{m^j}_{so_{dis}} \end{pmatrix} \tag{4.28}$$

Again, for the sake of completeness we reproduce the above equations for regeneration on the sink-side as well. On the sink-side, regeneration refers to the regeneration of “holes” that we introduced in chapter - 3.

$$\hat{\mathbf{Y}}_{\mathbf{si_{anc}}} = \begin{pmatrix} \hat{Y}^{1}_{si_{anc}} \\ \vdots \\ \hat{Y}^{\theta}_{si_{anc}} \\ \vdots \\ \hat{Y}^{m}_{si_{anc}} \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{n} \hat{\mathcal{Y}}^{1^j}_{si_{anc}} \\ \vdots \\ \sum_{j=1}^{n} \hat{\mathcal{Y}}^{\theta^j}_{si_{anc}} \\ \vdots \\ \sum_{j=1}^{n} \hat{\mathcal{Y}}^{m^j}_{si_{anc}} \end{pmatrix} \tag{4.29}$$

Finally for the regeneration associated with distributed flows on the sink-side, we have:

$$\hat{\mathbf{Y}}_{\mathbf{si_{dis}}} = \begin{pmatrix} \hat{Y}^{1}_{si_{dis}} \\ \vdots \\ \hat{Y}^{\theta}_{si_{dis}} \\ \vdots \\ \hat{Y}^{m}_{si_{dis}} \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{n} \hat{\mathcal{Y}}^{1^j}_{si_{dis}} \\ \vdots \\ \sum_{j=1}^{n} \hat{\mathcal{Y}}^{\theta^j}_{si_{dis}} \\ \vdots \\ \sum_{j=1}^{n} \hat{\mathcal{Y}}^{m^j}_{si_{dis}} \end{pmatrix} \tag{4.30}$$

Scale measures $\mathbf{S}^{\mathsf{T}}_{\mathbf{so_{TMR}}}$ and $\mathbf{S}^{\mathsf{T}}_{\mathbf{si_{TDO}}}$ constructed using the eight aggregation equations above are dimensionally consistent in the stock-flow space. The essential modification that we have made from the original specification (for example **S** in equation - 4.4) is that instead of adding apples and oranges we aggregate all apples together and all oranges together. More usefully, our new dimensionally consistent aggregation aggregates all sources of copper into the economy but not copper and aluminum. However, this framework is still inadequate to aggregate copper that is mined from more than one source, especially when the two sources happen to be part of vastly different ecosystem funds, or the case where there is a qualitative difference in a given material flow. Questions about differences in ecosystem structure or qualitative difference in flows belong to the fund-flux space. In chapter - 5 we present a framework for mapping between the stock-flow space and the fund-flux space. Here, we end this section by rewriting equations 4.11 and 4.12 using the stock-flow consistent disaggregated flows introduced here:

$$\mathbf{S}^{\mathsf{T}}_{\mathbf{MFA1}} = \begin{pmatrix} S_{MFA1_{so}} & S_{MFA1_{si}} \end{pmatrix} = \begin{pmatrix} \frac{\dot{x}_i^1}{\dot{x}^1_{i_{anc}} + \dot{x}^1_{i_{dis}}} & \frac{\dot{x}_o^1}{\dot{x}^1_{o_{anc}} + \dot{x}^1_{o_{dis}}} \\ \vdots & \vdots \\ \frac{\dot{x}_i^j}{\dot{x}^j_{i_{anc}} + \dot{x}^j_{i_{dis}}} & \frac{\dot{x}_o^j}{\dot{x}^j_{o_{anc}} + \dot{x}^j_{o_{dis}}} \\ \vdots & \vdots \\ \frac{\dot{x}_i^n}{\dot{x}^n_{i_{anc}} + \dot{x}^n_{i_{dis}}} & \frac{\dot{x}_o^n}{\dot{x}^n_{o_{anc}} + \dot{x}^n_{o_{dis}}} \end{pmatrix} \tag{4.31}$$

Similarly, we can rewrite $\mathbf{S}^{\mathsf{T}}_{\mathbf{MFA2}}$ as:

$$\mathbf{S}^{\mathsf{T}}_{\mathbf{MFA2}} = \begin{pmatrix} S_{MFA1_{so}} & S_{MFA1_{si}} \end{pmatrix} = \begin{pmatrix} \frac{\dot{x}_i^1}{\dot{x}_i^1+\dot{x}_{i_{anc}}^1+\dot{x}_{i_{dis}}^1} & \frac{\dot{x}_o^1}{\dot{x}_o^1+\dot{x}_{o_{anc}}^1+\dot{x}_{o_{dis}}^1} \\ \vdots & \vdots \\ \frac{\dot{x}_i^j}{\dot{x}_i^j+\dot{x}_{i_{anc}}^j+\dot{x}_{i_{dis}}^j} & \frac{\dot{x}_o^j}{\dot{x}_o^j+\dot{x}_{o_{anc}}^j+\dot{x}_{o_{dis}}^j} \\ \vdots & \vdots \\ \frac{\dot{x}_i^n}{\dot{x}_i^n+\dot{x}_{i_{anc}}^n+\dot{x}_{i_{dis}}^n} & \frac{\dot{x}_o^n}{\dot{x}_o^n+\dot{x}_{o_{anc}}^n+\dot{x}_{o_{dis}}^n} \end{pmatrix} \tag{4.32}$$

Equations 4.31 and 4.32 will form the starting point for our analysis when we study anthropogenic contribution to elemental mobilization in section - 4.2.4. There we show how a scale measure constructed following the logic of equations 4.31 and 4.32, can, in certain cases, complement a flow measure of scale that compares throughput and regeneration.

4.2.4 Anthropogenic Contribution to Elemental Mobilization

In this section we review a recent contribution to the burgeoning field of industrial ecology that tries to quantify anthropogenic contribution to mobilization of various elements. Klee and Graedel [2004] compare natural and anthropogenic mobilization for seventy-seven elements of the periodic table. They conclude that "human activities likely dominate or strongly perturb the cycles of most of the elements other than alkalis, alkali earth, and halogens." The primary purpose of our review here is to look at how this effort to build a comprehensive accounting system based on elemental mobilization can contribute to construction of flow measures of scale. The basic components of the elemental cycle are represented in figures

4.12 and 4.13. Figure - 4.12 represents an idealized closed cycle absent any anthropogenic intervention. Klee and Graedel classify stocks and flows into two different kinds – mobilization and sequestration. Stocks that contain "majority of a given element" are termed sequestration reservoirs and the "reservoir or reservoirs into which this material is transferred, typically for much shorter periods of time, are termed mobilization reservoirs."[11] Mobilization flows flow from the sequestration reservoir to the mobilization reservoir and sequestration flows flow from mobilization reservoirs back to the sequestration reservoirs. In figure - 4.12, F_μ represents the mobilization flow from the sequestration reservoir R_σ to the mobilization reservoir, R_μ and F_σ represents the sequestration flow. The simple representation in figure - 4.12 is extended to account for anthropogenic contributions to mobilization of various elements. In figure - 4.13 the basic elements of the anthropogenic cycle are represented. The source-side throughput $\dot{x}_i$ is represented as two different flows – F_1

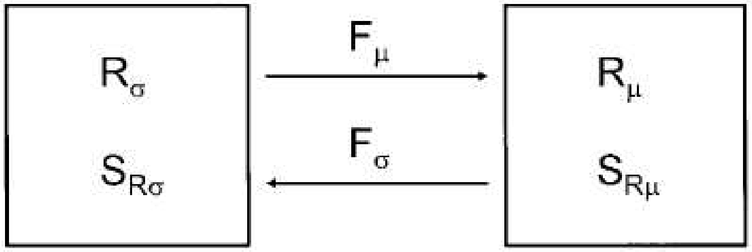

Figure 4.12: Basic elements of a Closed Material Flow Cycle. μ represents mobilization and σ, sequestration. $S_{R\sigma}$ is the stock contained in reservoir R_σ, and $S_{R\mu}$ is the stock contained in reservoir R_μ. Chart reproduced from Klee and Graedel [2004, p.72]

---

[11] Klee and Graedel [2004, p.71]. The authors use the term "stock" somewhat differently from how we have used the term. We have used the term stock to denote both the logical compartment in which a material can accumulate as well as the actual amount of material in a given compartment. Klee and Graedel use the term "reservoir" to represent the logical compartment enclosing a particular element, and the term "stock" is reserved for the actual amount of material in the reservoir.

and F_2. The sink-side throughput is represented as F_3. The figure also accounts for some amount of recycling in the form of F_4. Klee and Graedel use the term "landfill" as a convenient label for all kinds of sinks including actual physical landfills. In figure - 4.14 we have modified figure - 4.13 using familiar notations introduced used in the previous sections. The stocks R_2 and R_3 have been subsumed under the economy-stock, x_e. The two components of throughput, $\dot{x}_i$ and $\dot{x}_o$ represent the anthropogenic mobilization, $F_\mu(a)$. Also shown in figure - 4.14 is the natural mobilization, $F_\mu(n)$. Note that the flow representing $F_\mu(n)$ is shown to bypass the economy to indicate that there is no purposeful human agency that controls the magnitude of natural mobilization.[12]

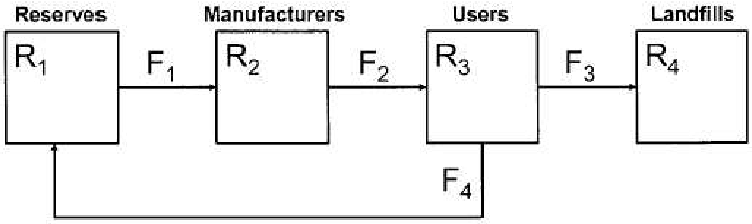

Figure 4.13: Basic elements of an Open Anthropogenic Material Flow Cycle. Chart reproduced from Klee and Graedel [2004, p.72]

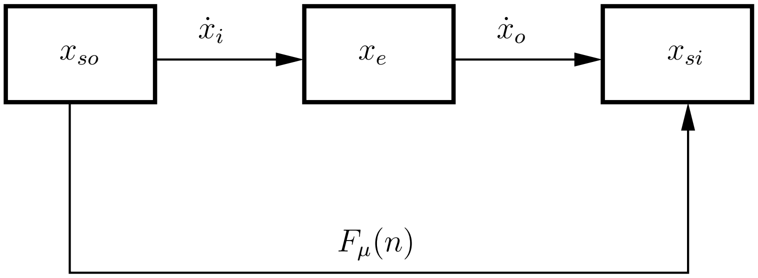

Figure 4.14: Anthropogenic Contribution to Elemental Mobilization. A simple flow measure of scale is the ratio of anthropogenic mobilization, $\dot{x}_i$ or $F_\mu(a)$ and natural mobilization, $F_\mu(n)$.

---

[12] This representation is admittedly a simplification because it ignores any possibility for coupling and feedback between natural and anthropogenic flows. However, as Klee and Graedel show the

A Flow Measure of Scale

A flow measure of scale derived from figure - 4.14 simply compares natural and anthropogenic contributions to elemental mobilization.

$$S_{MFA3} = \frac{F_\mu(a)}{F_\mu(n)} = \frac{\dot{x}_i + \dot{x}_o}{F_\mu(n)} \tag{4.33}$$

$$S_{MFA4} = \frac{F_\mu(a)}{F_\mu(a) + F_\mu(n)} = \frac{\dot{x}_i + \dot{x}_0}{\dot{x}_i + \dot{x}_o + F_\mu(n)} \tag{4.34}$$

Equation - 4.33 is a simple ratio of anthropogenic and natural mobilization of various elements and equation - 4.34 computes the contribution of anthropogenic mobilization to the total. The periodic table below, reproduced from the original study shows the range of anthropogenic domination as represented by S_{MFA4} in equation - 4.34. As seen from the periodic table, for a vast majority of elements, anthropogenic mobilization constitutes more than half of the total mobilization. The accounting methodology used to determine anthropogenic contribution to elemental mobilization deserves some explanation. Equations 4.33 and 4.34 add up both source-side and sink-side components of the throughput as contributing to anthropogenic mobilization. At first glance this appears to be double accounting – for example it makes little sense to count both the carbon in the coal when it is mined (source-side) and the carbon that is released when coal is burnt (sink-side). The short-hand represen-

---

representation in figures 4.13 or 4.14 is adequate for the purposes of meaningfully measuring anthropogenic contribution to elemental mobilization.

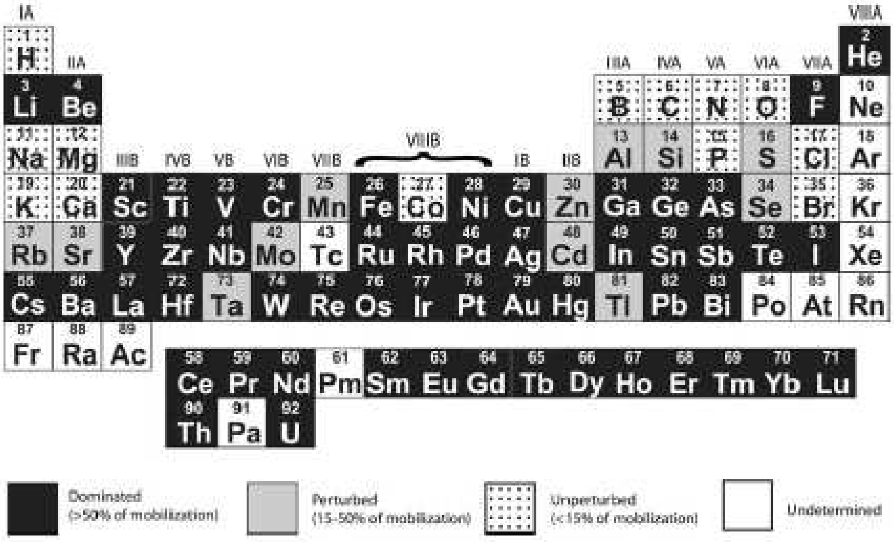

Figure 4.15: Human Domination of Elemental Mobilization. Reproduced from Klee and Graedel [2004, p.94]

tation used in equations 4.33 and 4.34 is to blame and the authors of the original study have carefully avoided any double accounting. Equations 4.33 and 4.34 represent a template for each of the seventy-two elements for which the equations are computed. Different sources contribute to mobilization of any one element and for each source only the source-side or the sink-side is included in the accounting. For example, sodium is mobilized by mining, and burning of fossil fuels. Total anthropogenic mobilization of sodium is obtained by adding sodium that is mined (only the source-side for this component) and sodium that is released when fossil fuels are burnt (only the sink-side for this component). We clarify this accounting procedure below by expanding the simple sum $F_\mu = \dot{x}_i + \dot{x}_o$ that is used in equations 4.33 and 4.34.

$$F_\mu^\theta(a) = \dot{x}_i + \dot{x}_o = \sum_{j=1}^{n} \dot{\mathcal{X}}_i^{\theta j} + \sum_{j=1}^{m} \dot{\mathcal{X}}_o^{\theta j} \tag{4.35}$$

$$\dot{\mathcal{X}}_o^{\theta j} = 0 \; \forall \; \dot{\mathcal{X}}_i^{\theta j} \neq 0 \tag{4.36}$$

In equation - 4.35 $F_{\mu}^{\theta}(a)$ represents the anthropogenic mobilization of element θ – one of the seventy-two elements of the periodic table considered by the original study. Each mobilization flow consists of n source-side components $(\dot{\mathcal{X}}_i^{\theta 1} \ldots \dot{\mathcal{X}}_i^{\theta n})$ and m sink-side components $(\dot{\mathcal{X}}_o^{\theta 1} \ldots \dot{\mathcal{X}}_o^{\theta m})$. Equation - 4.36 simply states that the sum in equation - 4.35 can include only the source-side or the sink-side for any one constituent (j) of $F_{\mu}^{\theta}(a)$. In general, the sum $\sum_{j=1}^{n} \dot{\mathcal{X}}_i^{\theta j}$ includes all primary extraction of the element and the sum $\sum_{j=1}^{m} \dot{\mathcal{X}}_o^{\theta j}$ includes all incidental mobilization of an element. For example, primary mercury extracted appears on the source-side of the throughput ($\dot{x}_i = \sum_{j=1}^{n} \dot{\mathcal{X}}_i^{\theta j}$ with n different sources from which mercury is mined) and the incidental mobilization from sources like fossil fuel burning appear on the sink-side ($\dot{x}_o = \sum_{j=1}^{m} \dot{\mathcal{X}}_o^{\theta j}$ with m different sources from which mercury is released as part of some other throughput stream).

The flow measures of scale defined in equations 4.33 and 4.34 are similar in structure to scale measures $\mathbf{S_{MFA1}}$ and $\mathbf{S_{MFA2}}$ that we derived in equations 4.11 and 4.12 using the TMR-TDO methodology. Like $\mathbf{S_{MFA1}}$ and $\mathbf{S_{MFA2}}$, scale measures S_{MFA3} and S_{MFA4} do not make use of any regeneration flows. If $\mathbf{S_{MFA1}}$ and $\mathbf{S_{MFA2}}$ compare direct flows to hidden flows, S_{MFA3} and S_{MFA4} compare anthropogenic and natural components of elemental mobilization. Like their TMR-TDO counterparts, the scale measures introduced here can complement a flow measure of scale that compares throughput and regeneration. A scale measure that uses regeneration is mathematically not defined in the absence of regeneration flows. While S_{MFA3} and S_{MFA4} that use elemental mobilization do not suffer from the dimensional consistency problems associated with the scale measures derived from the aggregate TMR-TDO methodology, a significant drawback of the accounting framework used to compute anthropogenic contribution to elemental mobilization is that it does not allow for independent evaluation of scale on the source-side and the sink-side. We have shown previously in chapter - 3 that being able to independently determine scale on the source-side and sink-side is central to using the scale methodology as a practical tool for environmental policy. Here we discuss how the accounting framework used for determining anthropogenic contribution to elemental mobilization can be modified to enable independent source-side and sink-side assessments.

We begin by commenting on two relevant aspects of the model in figure - 4.14 that we have used to define scale measures S_{MFA3} and S_{MFA4}. First note that independent evaluation of scale on the source-side and sink-side was possible using a scale measure that took into account regeneration flows because, there were two flows on both the source-side and the sink-side. To recollect, our basic analytical representation of the economy-ecosystem interaction depicted in figure - 3.4 contains four flows – $\dot{x}_i$, $\hat{Y}_{so}$ on the source-side; and $\dot{x}_o$, $\hat{Y}_{si}$ on the sink-side. Second, note that even as figure - 4.14 shows the two flows $\dot{x}_o$ (the sink-side throughput) and $F_\mu(n)$ terminating in a common stock (x_{si}), these two flows are qualitatively very different flows. In general the natural mobilization flow, $F_\mu(n)$ is also qualitatively different from the source-side of the throughput, $\dot{x}_i$. It is easy to see why $F_\mu(n)$ is qualitatively different from the sink component of the throughput in figure - 4.14. For example, if x_e is the stock of coal in the economy, $\dot{x}_o$ represents carbon in the combustion products of coal, $F_\mu(n)$ that originates in x_{so} cannot represent carbon in the same form as $\dot{x}_o$. The central problem with the representation in figure - 4.14 is that natural mobilization is represented by a single flow that spans both the source-side and sink-side of the economy. This representation problem is directly related to the fundamental differences between natural and anthropogenic material cycles. The natural cycle is a closed cycle (figure - 4.12 and the anthropogenic cycle is an open cycle (figure - 4.13). Figure - 4.14 is an approximation that reconciles the essential differences between the natural and anthropogenic material cycles. In terms of the terminology of Klee and Graedel [2004], all the stocks in the open anthropogenic cycle of figure - 4.13 are mobilization reservoirs. Our representation of the economic process that forms the basis of figure - 4.14 essentially consists of three "mobilization reservoirs." The absence of any "sequestration reservoir" in figure - 4.14 is primarily responsible for the inadequate representation of F_μ, the natural mobilization flow.

Figure - 4.14 is derived from the basic representation of the economic process in chapter - 3. There, in figure - 3.4 the sequestration reservoirs are implicitly represented in the form of sources for the two regeneration flows. The correct interpretation of the natural mobilization flow is to treat it as part of the process that contributes to the regeneration flows that we have thus far used to construct flow measures of scale. Indeed the three different categories of flows considered by

Klee and Graedel – seaspray, crustal weathering, and plant primary production all contribute to regeneration on both the source-side as well as the sink-side. Figure - 4.16 is drawn such that the natural mobilization flow now originates and terminates in the implicit regeneration stocks that have been outside our system boundary in chapter - 3. This representation underscores the fundamental difference between natural and anthropogenic mobilization – while the natural mobilization flow contributes to regeneration flows, anthropogenic flows are simply the throughput that sustains human activity. Natural mobilization flows go into maintaining the fund (represented in our model by x_{so} and x_{si}) and anthropogenic mobilization flow is used to maintain the economy-fund (represented by the economy stock, x_e). Thus symbolically, the two regeneration flows can be represented as some function of the natural mobilization flow:

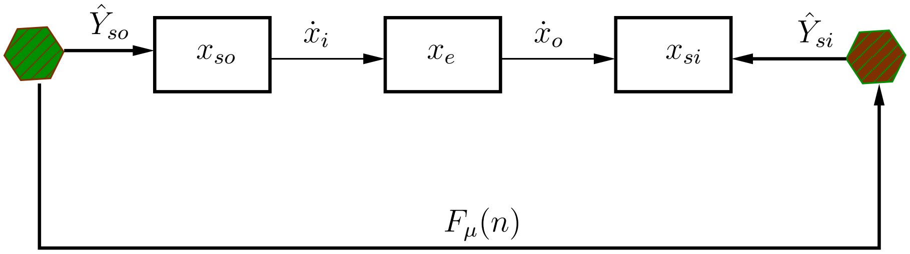

Figure 4.16: Natural Mobilization and Regeneration. Source-side and sink-side regeneration flows (first introduced in figure - 3.4) have been added to the model in figure - 4.14.

$$\hat{Y}_{so} = f_{so}\left(F_\mu(n), \hat{\mathcal{Y}}_{so}\right) \tag{4.37}$$

$$\hat{Y}_{si} = f_{si}\left(F_\mu(n), \hat{\mathcal{Y}}_{si}\right) \tag{4.38}$$

In equations 4.37 and 4.38 $\hat{\mathcal{Y}}$ represents all other factors besides the natural mobilization flow that affects regeneration. With this interpretation of the natural mobilization flow, the scale measures presented in this sections, S_{MFA3} and S_{MFA4} indirectly compare throughput and regeneration. Equation - 4.36 ensures that there is an implicit separation of the source-side and the sink-side if not an explicit one. In summary, accounts based on elemental mobilization besides being an important contribution to material flow accounting, can also be used to construct meaningful flow measures of scale. This accounting framework, as we discussed earlier, is especially useful when the simpler scale measures that use throughput and regeneration are not mathematically defined.

4.3 Human Appropriation of Products of Photosynthesis

The sun is the ultimate source of energy that powers all biogeochemical cycles that are the basis for all processes on earth. One of the primary objectives of the scale methodology is to develop accounting frameworks that quantify the impact of human activity on these. Perhaps the most significant of these processes is photosynthesis. All life depends of the ability of autotrophs to capture the flow of solar energy and synthesize organic food from inorganic inputs. All heterotrophs including including humans draw upon surplus products of photosynthesis produced by autotrophs. In this section as well as in the next we will review two influential sets of studies that can be used to construct scale measures. Here we review research, that following Vitousek et al. [1986], has tried to estimate Human Appropriation of Net Primary Production (HANPP); and in the next section we will review an ingenious measure of photosynthesis in terms of land area as used by the ecological footprint methodology. The scale measures that can be built using an accounting framework for tracking human appropriation of net primary production are rooted in a systems ecology framework and complement the material flow analysis based scale measures that were rooted in physical sciences and engineering. In particular, the HANPP framework that we discuss here provides an attractive avenue to incorporate energy analysis into our frame for empirical assessment of scale that has hitherto focussed almost exclusively on material flows (as depicted in the central model in figure - 3.4).

Conceptually, the HANPP framework is the simplest among the different methodologies reviewed in this chapter. In figure - 4.17 below, we have reproduced the source-side from figure - 3.4. Typically, the two flows as measured as grams of carbon per year. The regeneration-flow, $\hat{Y}_{so}$ is net of what is required for maintaining the given stock of autotroph population represented by x_{so}. However, figure - 4.17 is incomplete in that only anthropogenic withdrawal of NPP is shown. A flow measure of scale using NPP accounting is easily defined following equation - 3.10:

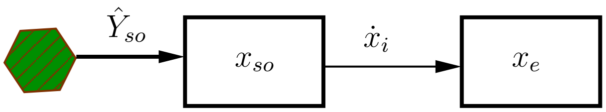

Figure 4.17: Human Appropriation of NPP

$$S_{HANPP} = \frac{\dot{x}_{HANPP}}{\hat{Y}_{NPP}} \tag{4.39}$$

While conceptually simple, the actual empirical estimation of equation - 4.39 is fraught with significant uncertainty. Vitousek et al. [1986] estimated that about a third of terrestrial NPP is appropriated by human activity – that number has been subject to scrutiny by several later studies including authors of the original 1986 study. Figure - 4.18 below, reproduced from Rojstaczer et al. [2001] is a Monte-Carlo simulation for the possible range of human appropriation of terrestrial net primary production (HTNPP). However, Rojstaczer et al. [2001] even while pointing to the significant uncertainties in estimates of human appropriation of NPP acknowledge that "it is clear that human impact on TNPP is significant." The lower bound of the 95% confidence interval from their monte carlo simulations for human appropriation of terrestrial NPP (figure - 4.18) is six Peta-grams of Carbon which suggests that "humans have had more impact on biological resources than any single species of the megafauna known over the history of Earth." Our focus here is not to review in detail the measurement problems associated with the HANPP accounting framework but to look at conceptual issues surrounding how this framework could be used to construct meaningful scale measures. We will only gloss over some of the key issues

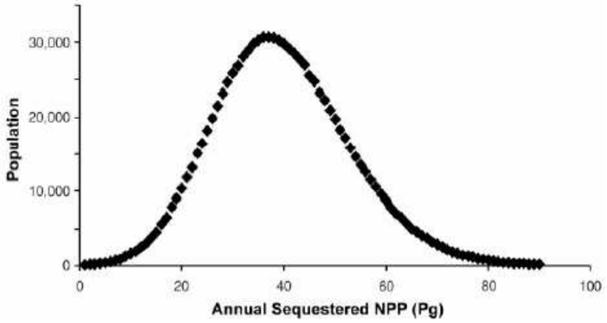

Figure 4.18: Uncertainty in Estimates of Human Appropriation of Terrestrial Net Primary Productivity. The vertical axis refers to the number of estimates in the monte-carlo simulation that allowed every parameter used to estimate equation - 4.39 for terrestrial stock to vary fully within published ranges for each parameter. Chart reproduced from Rojstaczer et al. [2001, p.2550].

that underlie an accurate empirical estimation of HANPP.[13] First, there has been some controversy about what constitutes 'human appropriation.' Thus starting with the original Vitousek et al. [1986] study, the different studies have each reported a range of estimates for HANPP – the different estimates corresponding to varying assumptions about what can counted as human appropriation. If only direct consumption (food, clothing, shelter) are included, the human appropriation of NPP is about 4%. At the other extreme if one estimates HANPP based on potential NPP in the complete absence of humans, human appropriation is at least 40% of terrestrial NPP.

Second and perhaps more importantly, human appropriation of products of photosynthesis is only one of the different flows that supports human activity. In figure - 4.17 the stocks x_{so} and x_e are also simultaneously funds. In particular the maintenance of the economy, x_e requires several other flows besides $\dot{x}_{HANPP}$. Thus if HANPP accounting is to serve as a reliable indicator of the proportional relationship between human activity and the supporting ecosystem, we have to account for how HANPP is influenced by other components of the aggregate throughput, and by how HANPP influences those other components. It would of course be impossible to account for all the myriad interactions in the fund-flux space but it is nevertheless necessary to account for some of the most significant interactions for the HANPP accounting to be useful as a scale measure. These effects become par-

---

[13] For a good review on this subject see Field [2001]. Roy et al. [2001] provides a detailed look at key issues involved in estimating terrestrial NPP.

ticularly important when we consider questions of maximum scale and optimal scale in chapter - 5. Field [2001] documents how land use change from human activity (mainly in the form of agriculture) has influenced photosynthesis. Figure - 4.19, reproduced from Field [2001] shows how land use change has affected net primary production around the world. There has been a 5% reduction in in global NPP as a direct result of human induced land used changes. While there has been a 5% reduction in the aggregate the figure shows that some of the land use modification has contributed to an increase in primary productivity. Figure - 4.19 treats NPP in the stock-flow space — in the main, it shows the effects of replacing natural growth with human-managed agricultural stock.

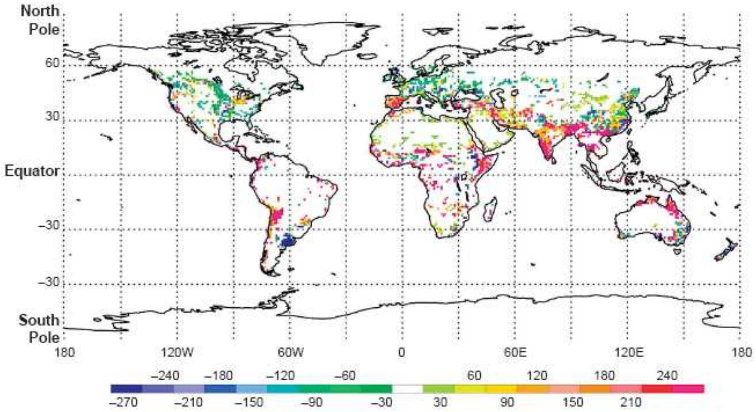

Figure 4.19: Change in NPP from land cover conversions. Positive changes indicate land use modification that has led to decreased NPP. NPP values are shown as grams of Carbon per square-meter per year. Chart reproduced from Field [2001, p.2491].

4.3.1 HANPP as Scale Measure

In this section we review some of the properties of the basic flow measure of scale based on the HANPP accounts (equation - 4.39) as well as some recent attempts to modify the accounts in ways that makes construction of stock measures of scale a possibility. While the basic scale measure, S_{HANPP} described in equation - 4.39 has mostly been used at global planetary scale, there are few theoretical barriers (unlike with scale measures that use the material flow accounts) preventing a more geographically disaggregated use of the HANPP accounting framework. In a recent paper, Imhoff et al. [2004] computed a disaggregated balance sheet of local NPP availability and demand. Figure - 4.20, reproduced from the Imhoff et al. study

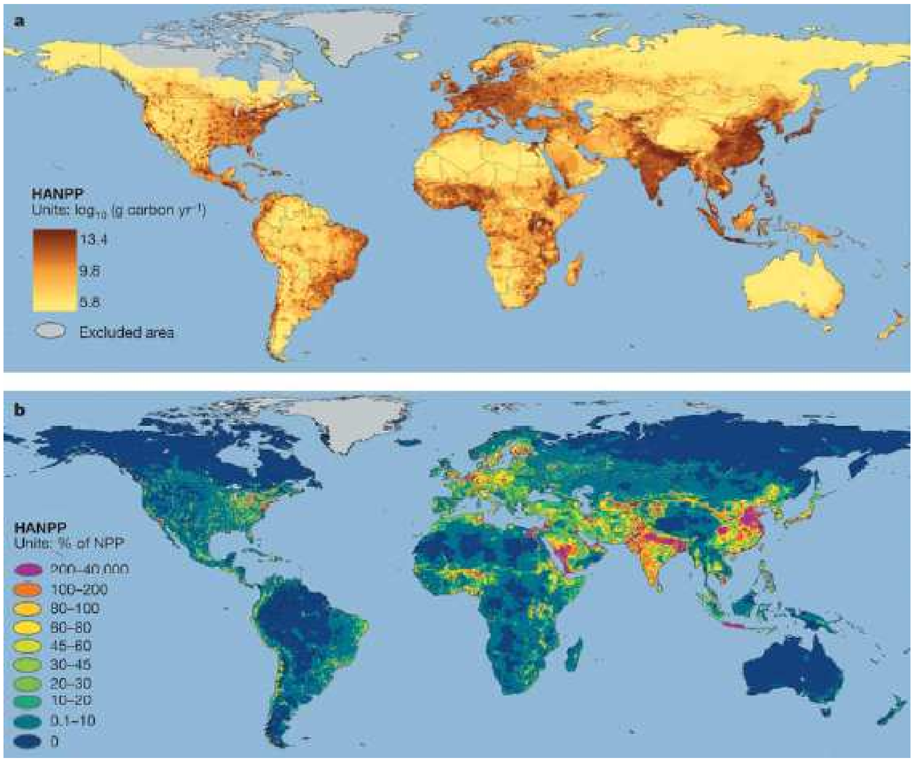

Figure 4.20: Spatial Distribution of Human Appropriation of NPP. Chart-**a** shows spatial distribution of HANPP and chart-**b** shows HANPP as a percentage of local NPP. Charts reproduced from Imhoff et al. [2004, p.871].

shows the variation in HANPP across the globe. The charts assume an uniform per captia consumption of HANPP within a given country. The data on NPP available is on a much finer scale – 0.25 degree spatial resolution. While the assumption of uniform per capita consumption of HANPP within a country is problematic in several instances, this first attempt at deriving disaggregated HANPP numbers across the globe is nevertheless a definite improvement over a single aggregate number for the entire world. The Imhoff et al. study is especially significant because the total global estimate for HANPP obtained by summing up disaggregated data closely matches earlier studies that used different methodologies to estimate HANPP.

While a flow measure of scale based on the HANPP accounting framework is easy to construct, a stock measure of scale is more involved. In figure - 4.17 x_e represents the stock of all artifacts in the economy that contain accumulated flows of NPP. However in practice, this stock is much harder to empirically estimate than the throughput, $\dot{x}_i$ used in a flow measure of scale. Beyond, difficulties with empirical estimation of the economy-stock, there are conceptual difficulties with using the economy-stock to construct a stock measure of scale as discussed previously in section - 3.5. We have shown in section - 3.5 that the most meaningful stock measure of scale is one that compares the ecosystem stock (x_{so} in this case) with a reference stock (equation - 3.13). While we will see how a stock measure of scale can be constructed using the HANPP accounting in the next section on ecological footprint, here we review a recent research that tries sheds new light on a particular stock of accumulated NPP that is central to modern industrial societies – fossil fuels. Dukes

[2003] calculate human consumption of "buried sunshine" by estimating embedded "paleo-NPP" in modern fossil fuels.

In addition to providing a quasi-stock-measure of scale using the HANPP framework, Dukes [2003] also provides a flow measure of scale on the source-side for non-renewable fossil fuels. The simple flow measure of scale that compares throughput and regeneration is not mathematically defined (on the source-side) for non-renewable resources. In section - 3.3.2 we suggested an alternative would be to work with sink-side measures – by looking at absorptive capacities for the various combustion products. The various material flow based accounting methodologies that compared throughput to natural mobilization, as we discussed in the previous section offered yet another alternative. Dukes [2003] provide a more direct source-side flow measure of scale. By estimating the amount of ancient plant matter contained in current fossil fuel use, Dukes concluded that the more than four-hundred times the current annual total net primary productivity was contained in fossil fuel that was consumed globally in 1997. Figure - 4.21 summarizes the main conclusions reached by Duke. While there are a range of estimates in the literature (indicated by the grey-band around the time series for annual consumption of "paleoproductivity"), the aggregate annual consumption of NPP embedded in fossil fuels even under the lowest estimates in the literature is more than the current annual aggregate net primary productivity. Figure- - 4.21 also shows a time series (starting at 1980) of the amount of biomass that would be needed to completely replace fossil fuel use. At current rates of energy consumption about 10% of the

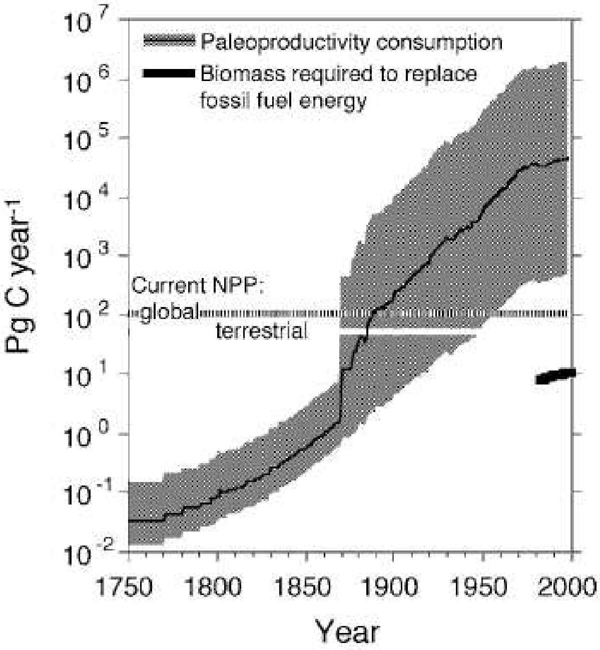

Figure 4.21: Human Consumption of Paleoproductivity. The grey-band contains the high and low estimates and the main schedule in the chart, within this band represents the best estimates. There is an abrupt jump in 1870 when the data from oil consumption was added. Data on replacement biomass starts at 1980. Chart reproduced from Dukes [2003, p.40].

current NPP will be needed to replenish the NPP contained in fossil fuels. Lest we understate the centrality of fossil fuels to modern industrial societies, we clarify the true import of this substitution possibility that at first glance suggests an easy transition from a fossil fuel based economy to a biomass based economy. First, if we considered only terrestrial NPP, over 20% of the terrestrial NPP would be needed to replace fossil fuel consumption with biomass based fuels. Given that humans currently appropriate around a third of terrestrial NPP, this would represents a significant additional appropriation. Increasing current withdrawal of terrestrial NPP by over 50% entails significant changes in land-use patterns around the world. Land-use pattern not only affects plant productivity (figure - 4.19) but also has impact on the larger ecosystem-fund. The impacts in the fund-flux space of a 50% increase in human appropriation of terrestrial NPP will remain unknown. Second, fossil fuels are highly concentrated relative to plant biomass. As discussed previously over four-hundred times the planet's current NPP is contained in the world's annual fossil fuel use. While 20% of current terrestrial NPP can replace the current consumption of fossil fuel, the energy-densities of fossil fuels that were formed from plant matter over geological time scales are vastly greater than biomass fuels.[14] In figure - 4.22, reproduced from Dukes [2003] illustrates the geological processes involved in transformation of low-density plant matter into fossil fuels. As illustrated in figure - 4.22, only 0.001% (or 100 parts per million) of the original feedstock from

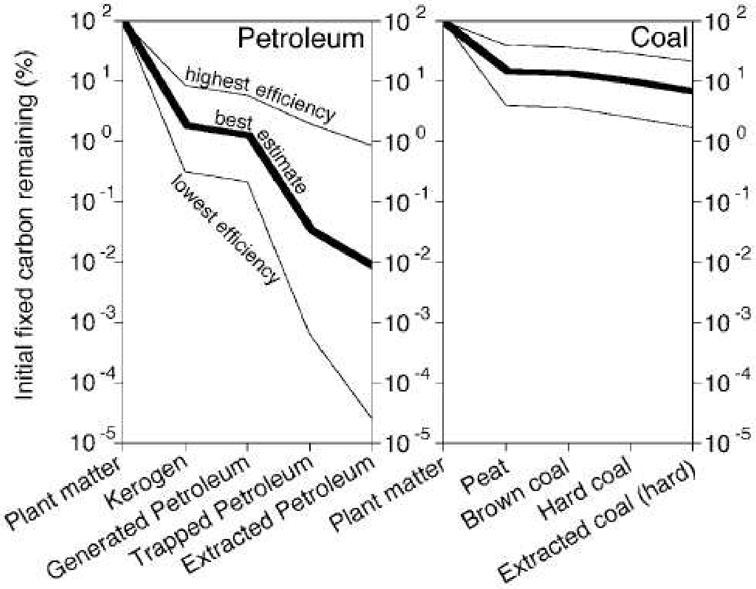

Figure 4.22: Geological-Timescale Conversion Efficiencies. The figures show conversion efficiencies over geological timescales in terms of percentage of initial paleo-carbon remaining at various stages of the during the conversion of fossilized plant matter into extractable fossil fuel. Each panel shows best estimates (think lines) as well as upper and lower bounds. Charts reproduced from Dukes [2003, p.35].

---

[14] Crude oil has an energy density of about 42 MJ/kg compared to energy densities for different kinds of biomass that range from 10 to 18 MJ/Kg.

paleo-photosynthesis is recovered as petroleum.

4.4 Ecological Footprint

The concept of ecological footprint, first developed by Rees [1992] and further refined by Wackernagel and Rees [1996], measures "how much biologically productive land and water area a given population occupies to produce all the resources it consumes and to absorb its waste, using prevailing technology." In this section we investigate basic properties of ecological footprint as a scale measure using the formalism that we developed in chapter - 3.

4.4.1 The Mechanics of Footprint

Much has been written about ecological footprint and we do not propose to present a review of footprint the literature but instead directly delve into the conceptual apparatus of ecological footprint that is most relevant for our purposes here.[15] In figure - 4.23, we present the central idea behind ecological footprint. The ecological footprint methodology is best understood as a transfer function. The basic step in calculating ecological footprint is to convert every throughput on both the source-side as well as sink-side into an area required to support that throughput. The footprint methodology indirectly measures the amount of photosynthesis products needed to support a given throughput. The area required to support a given

---

[15] For example see Ayres [2000], Deutsch et al. [2000], Herendeen [2000], and Rees [2000]. Wackernagel and Silverstein [2000] looks at how the footprint addresses the "scale imperative." The widely cited Wackernagel et al. [2002] used the footprint accounting framework to determine if the global economy is in overshoot.

throughput is a function of bioproductivity. The productivity is determined among other things by the extant technology and can change with technological progress. Productivity is ultimately a function of ecosystem health, and past damages to the ecosystem undermines productivity. The footprint methodology compares the land area available with the land area required to support a given quantum of throughput.

The area used in footprint accounting is measured in 'global hectares.' This abstract unit of area represents a weighted average of the different biologically productive areas on the surface of the earth. In figure -4.23, the ecological footprint methodology 'translates' any throughput, $\dot{\mathbf{x}}(t)$ into a corresponding area measured in abstract global hectares, $a(t)$ where t is a simple time subscript to keep track of the temporal dimension. An important point to note is that the transfer function heuristic that we have used here applies to both the source-side and the sink-side but with an important difference. "Sustainability" is built into how the transfer function of figure - 4.23 computes $a(t)$ on the sink-side. For example in computing the land area required for absorbing the combustion products of burning fossil

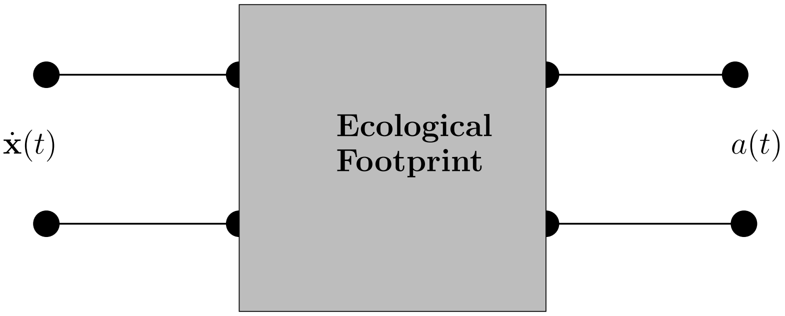

Figure 4.23: Ecological Footprint as a Transfer Function

fuels, it is assumed that all the emitted carbon is terrestrially absorbed.[16] On the source-side, no such assumption is possible. For example, continuing with the fossil fuel burning example, consider the case of oil. There is a finite stock of oil and increasing the land-area (global hectares) set aside to drill oil out of the ground is not going to alter the stock of oil that can be recovered. While the footprint accounting includes non-renewable sources, it excludes waste throughput for which no sink capacity exists. For example toxic waste stream for which there are no known biophysical sinks ($\hat{Y}_{si} = 0$) are excluded from the footprint accounting framework. The footprint calculations currently only include only those components of the aggregate throughput that is potentially sustainable at least on the sink-side. Fossil fuel burning that contributes to nearly half of the global footprint is assumed to be potentially sustainable — all the carbon from fossil fuel burning can *potentially* be absorbed by terrestrial sinks. These asymmetries between how the footprint, seen as a transfer function, treats the sources and sinks on one hand and non-renewable and potentially renewable throughput on the other will prove important when we attempt to analyze ecological footprint as a scale measure.

Looking Inside the Black-box Transfer Function

We begin by looking at how the output of the transfer function, global hectares or *gha* is defined. We follow the notation used by the latest footprint methodology presented in Wackernagel et al. [2004]. The surface of the earth is divided into k

---

[16] The notion of "sustainability" used here is different from our discussion on maximum scale or maximum sustainable scale. The "sustainable" sink-side assumption of ecological footprint is equivalent to saying $S_{si} = 1$ or that the throughput equals absorption capacity ($\dot{x}_o = \hat{Y}_{si}$).

(for 2004, $k = 9$) broad aggregate categories of land/water area (crop land, pasture, forest, fisheries, etc.) Let $P_i(t)$ be the total "bioproductive" land area available of type i in time period t. $P_i(t)$ is measured in standard hectares (not the abstract global hectare). "Bioproductive land" has sustainability as well as an anthropocentric bias built into the definition of P_i. Only the "usable portion of biomass [or equivalent] that can be renewably harvested *and* is valuable to people" is considered [Wackernagel et al. 2004, emphasis added]. $H(t)$ is the sum-total of bioproductive land available at a given time, t such that:

$$H(t) = \sum_{i=1}^{k} P_i(t) \tag{4.40}$$

$H(t)$ is measured in hectares but we need to convert *hectares* into *global-heactares*. This is achieved by defining a "equivalence factor," $E_i(t)$ corresponding to each of the k categories of productive areas.

$$E_i(t) = \frac{b_i(t)}{\bar{b}(t)} \tag{4.41}$$

where b_i is the productivity of the area of type i, currently measured by the "suitability index" published by FAO and $\bar{b}(t)$ is the average productivity for a given year[17]:

$$\bar{b}(t) = \frac{\sum_{i=1}^{k} b_i(t)P_i(t)}{\sum_{i=1}^{k} P_i(t)} \tag{4.42}$$

We are now ready to define the global hectare, $G_i(t)$ corresponding to the actual productive area, $P_i(t)$

$$G_i(t) = P_i(t)E_i(t) \tag{4.43}$$

It is important to note that among other things E_i and P_i are driven by the extant technology. A technological progress could make a new land or water area to be counted in P_i that influences yields, and thus E_i. For the earth as a whole, the global hectares are normalized such that the sum-total of of global hectares is the same as the total hectares of actual productive area. This scaled global hectare, A is defined such that

$$\sum_{i=1}^{k} A_i(t) = \sum_{i=1}^{k} P_i(t) = H(t) \tag{4.44}$$

---

[17] The suitability index is a discrete ranking of of different land types based on the physical characteristic of the land as well as the end use of the land. It takes into account physical characteristics of the land such as soil-type, slope of the land, climatic conditions, etc. For a detailed account of this standard methodology to determine land productivity, see FAO [1976, 1979, 1983] and FAO's 1998 report titled *World reference base for soil resources.*

and for each productive category, $A_i(t)$ is given by:

$$A_i(t) \quad = \quad G_i(t) \left(\frac{\sum_{i=1}^{k} P_i(t)}{\sum_{i=1}^{k} G_i(t)} \right) \tag{4.45}$$

An important thing to recollect is that in our discussion of the ecological footprint transfer function, each of the quantities measured has a time subscript, t to indicate that the total productive area and therefore the standardized global hectare changes every year but for any given year, t these two are the same for the earth as a whole and is illustrated in figure - 4.24, reproduced from Wackernagel et al. [2004].

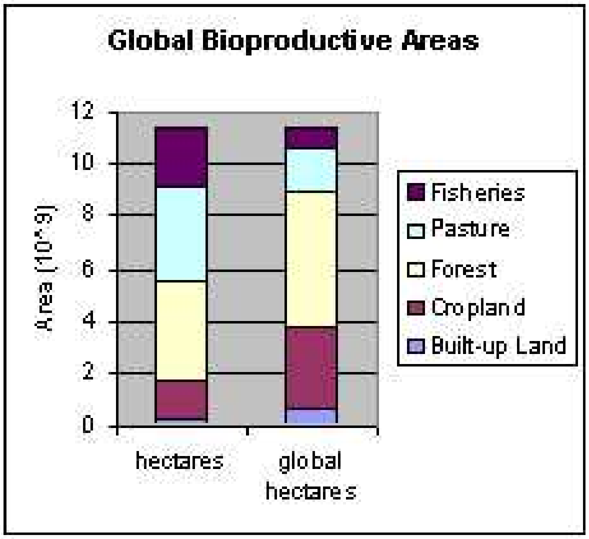

Figure 4.24: Quantity of global hectares (A_i) and actual hectares (P_i) for 2001. Area is measured in billions of hectares. Note that while $A_i \neq P_i$, $\sum A_i = \sum P_i = H$. Chart reproduced from Wackernagel et al. [2004, p.11]

The definition of $G_i(t)$ is dimensionally consistent in the stock-flow space but not in the fund-flux space. Equation - 4.43 can be written as:

$$G_i(t) = b_i(t)P_i(t)\left(\frac{\sum_{i=1}^{k} P_i(t)}{\sum_{i=1}^{k} b_i(t)P_i(t)}\right) \tag{4.46}$$

The sum $\sum_{i=1}^{k} P_i$ is defined in the stock-flow space but not in the fund-flux space. In the fund-flux space, each of the individual bioproductive areas, P_i have different dimensions, say $acre_i$. Thus $\sum_{i=1}^{k} P_i$ involves adding k items with different dimensions – $acre_1 + acre_2 + \ldots acre_k$. Each of the terms in this sum is a fund, and arithmetic summation of funds and has no real physical meaning, unlike an arithmetic addition of stocks. Thus the global hectare of type - i defined by equation - 4.43 is an abstract fund corresponding to an abstract stock – abstract because there is no physical $G_i(t)$. Defining the central fund in abstract terms helps comparison across regions with vastly varying real physical funds (P_i). While contributing to the power of the footprint methodology, this abstract aggregation is also, as we will see, the methodology's primary drawback as well.

4.4.2 Ecological Footprint as a Scale Measure

Stock dimension

At the global-level, the stock measure of scale using the ecological footprint methodology involves comparing the size of the bioproductive land area appropriated by the economy and the total bioproductive land area that is available across the globe in that same time-period. The 'stock' of land area used by the economy and the land area available are not measured in physical hectares but in abstract global hectares. At the global level, the total area available as measured in global hectares is the same as physical hectares available (we showed in the previous section that $\sum A(t) = \sum P(t) = H(t)$). At less than global levels of economic-geographic aggregations, the total productive area within that aggregation (for example, country) is computed by using "yield factors" for individual area-types (P_i) to take into account the fact that the productivity of each area-type contained within an aggregation like a nation is in general different from the global average. If $e_i(t)$ is the yield factor for area-type i, then equation - 4.43 can be rewritten for for lesser-than-global aggregations as:

$$G_i(t) \quad = \quad P_i(t)E_i(t)e_i(t) \qquad (4.47)$$

For the globe as a whole $e_i(t) = 1$. Thus the yield factor e_i is simply a correction factor to correct for equivalent factors, that are averaged over the globe. A scale-measure in the stock dimension compares an economy-stock with an ecosystem stock. The ecosystem stock in case of the footprint methodology is simply the total productive area available ($\sum A_i(t)$).

The economy-stock is the land-area required to support all human activity within the aggregation represented by $\sum A_i(t)$. The demand made by the economy is calculated in the same way as the total available productive areas of different kinds were aggregated into a common abstract metric of global hectares. All throughput is divided into k categories corresponding to the k different types of productive areas that the footprint methodology considers. For each of these categories, the equivalent land area is calculated as using the heuristic in figure - 4.23:

$$a_j(t) \quad = \quad \mathbf{f}(\dot{x}_j(t)) \tag{4.48}$$

The function **f** essentially consists of first evaluating the land area needed in raw hectares, and then transforming it into global hectares (using equation - 4.47) corresponding to throughput $\dot{x}_j(t)$ for land area of type j.

$$p_i(t) \quad = \quad \dot{x}_i(t)e_i(t) \tag{4.49}$$

$$g_i(t) \quad = \quad p_i(t)E_i(t) \tag{4.50}$$

where $p_i(t)$ represents the land area (measured in standard hectares) corresponding to the throughput $\dot{x}_i(t)$, for throughput and land area of type i. $E_i(t)$ and $e_i(t)$

are the familiar equivalence factor and yield factor respectively. The yield factor in equation - 4.49 is not dimensionless like the one used in equation - 4.47 because the yield factor here converts a flow into an equivalent area.

We are now fully equipped to define a scale measure in the stock dimension based on the footprint methodology. The scale metric is simply the ratio of land area required to sustain a given level of throughput and the land area that is available.

$$\sigma_{\mathbf{fp}} = \frac{\sum_{i=1}^{k} g_i(t)}{\sum_{i=1}^{k} G_i(t)} \tag{4.51}$$

Flow Dimension

The abstract acres or hectares used in calculating the ecological footprint can be represented using our simplified representation of the economy-ecosystem interaction presented in figure - 3.4. Both the source and sink in the figure are now measured in terms of abstract hectares. In the context of ecological footprint a modified version of figure - 3.4 is presented below as figure - 4.25. In this new figure, both the source-side and sink-side of of the ecosystem are represented by the abstract hectares of land used by the footprint measure. The two constituents of the throughput are now represented by a bidirectional flow in and out of the ecosystem that is represented by a stock measured in global-hectares of land. This land is a living system is regenerating at a rate determined by *regenrate*. The quantum of economic throughput, *throughput* is determined by rates that control the source and sink aspects of this throughput. There are obvious problems with the representation

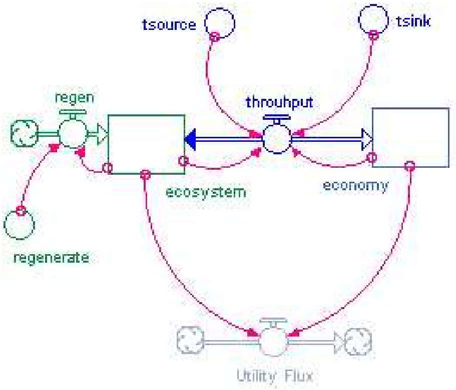

Figure 4.25: Stock-Flow Representation of Ecological Footprint

of the economy-ecosystem relationship in figure - 4.25. Given that the stocks are measured in terms of 'global hectares,' it makes little physical sense to be talking about a flow of global hectares from the economy back to the ecosystem, as figure - 4.25 suggests. Figure - 4.25 only represents that fact the footprint methodology uses a physically identical source and sink. The stocks and flows in figure - 4.25 are still measured in real material dimensions – for example tons of carbon and tons of carbon per year rather than in terms of abstract global hectares and global hectares per year. Figure - 4.25 simply illustrates the fact that the footprint framework does not provide independent accounts for the source-side and sink-side.

To use the footprint measure as a scale-measure we need to abstract from figure - 4.25 and represent the stocks and flows in terms of global hectares. Figure - 4.26 does just this. Here the ecosystem is represented by a stock of productive area measured in global hectares and denoted by $A_g(t)$ with the subscript t used to keep track of the time dimension. This stock has a natural regeneration rate, $\hat{\mathbf{Y}}(t)$, and $a(t)$ represents the economic throughput measured in terms of 'global hectares demanded per unit time.' We recollect that $a(t)$ is computed using the heuristic presented in figure - 4.23.

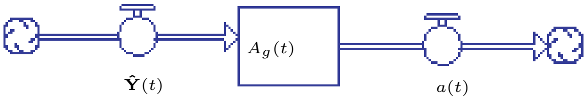

Figure 4.26: Abstract Flow Representation of Ecological Footprint

Figure - 4.26 is an abstract representation of the interaction between the economy and the ecosystem at all levels of economic-geographic aggregation. At different levels, the interpretation of $A_g(t)$, $\hat{\mathbf{Y}}(t)$ will be determined by the size of the geographic-economic entity being studied. However, the basic 'structure' of figure - 4.26 will remain invariant at every resolution.

Flow Measure of Scale

We begin our analysis here by looking at flow measure of scale at the global level. A flow measure of would compare the relative magnitudes of economic throughput and the rate at which the stock supporting this throughput is regenerating. Using the footprint framework in equation - 3.8 with $\dot{\mathbf{x}}$ set to $a(t)$:

$$\mathbf{S_{fp}} = \frac{a(t)}{\hat{\mathbf{Y}}(t)} \tag{4.52}$$

Since we are measuring this at the global-scale, $a(t)$ represents an aggregate of all throughput on the global-scale.

$$a(t) = \sum_i a_i(t) \quad \forall\, i \tag{4.53}$$

The regeneration flow, $\hat{\mathbf{Y}}$ is derived from the current stock of productive area, $A_g(t)$ as indicated in figure - 4.26.

$$\hat{\mathbf{Y}}(t) = r(t) * A_g(t) \tag{4.54}$$

where $r(t)$ is the rate at which the stock regenerates. The presence of the time-subscript indicates that the rate of regeneration is not constant.[18]

What is the relationship between the flow measure $\mathbf{S_{fp}}$ defined in equation - 4.52 and the stock measure of scale, $\sigma_{\mathbf{fp}}$ defined earlier in equation - 4.51? The most significant finding of the ecological footprint literature is that $\sigma_{\mathbf{fp}} \approx 1.2$ – the aggregate global demand for productive land exceeds supply by about 20%. Figure - 4.27 shows a 30-year time trend for $\sigma_{\mathbf{fp}}$. It is immediately obvious that figures 4.27 and 4.26 are (at least apparently) not consistent with each other. We developed 4.26 to represent the economy-ecosystem interaction in terms of global hectares used by footprint accounting. Figure - 4.27 shows that the economy has been in overshoot ($\sigma_{\mathbf{fp}} > 1$) for about twenty-five years.[19] If we have been in overshoot for twenty five

---

[18] Among other things the value of $r(t)$ is a function of the health of the fund underlying the abstract stock represented by $A_g(t)$.

[19] The overshoot trend has continued beyond 1999 [Loh 2002, Loh and Wackernagel 2004].

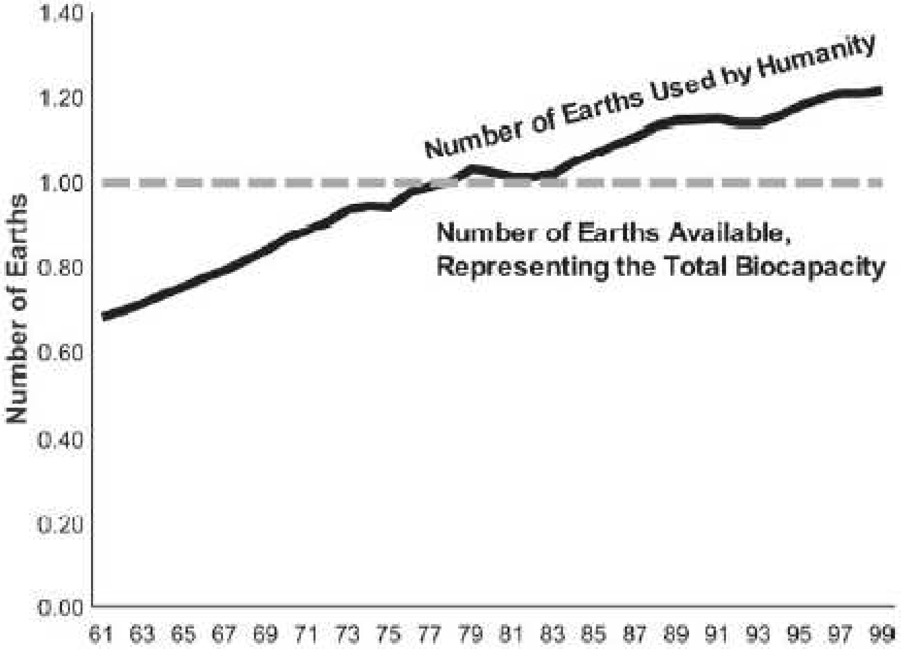

Figure 4.27: The vertical axis represents the regenerative capacity of the planet for a given year. At the present time, the footprint accounts estimate that the aggregate global demand exceeds regeneration by 20 %. Chart reproduced from Wackernagel et al. [2002, p.9269]

years, why has the stock A_g not been run down?

The answer to resolving the conundrum above lies in distinguishing between how the footprint framework defines "biocapacity" and its relationship to ecosystem's regenerative capacity. For the purposes of footprint accounting, biocapacity is defined as:

> [t]he amount of useful (for humans) biological material in a region or country that could be harvested using current schemes and extraction technologies. Material that the human economy used in a given year is defined as "useful." Hence what is considered "useful" can change from year to year. The biocapacity of an area [measured in global hectares]

> is calculated by multiplying the actual physical area by the yield factor and the appropriate equivalence factor.[20]

Regenerative capacity represents the ability of the ecosystem to renew itself. While biocapacity is a stock concept, regenerative capacity belongs to fund-flux space. In figure - 4.26, the magnitude of the regeneration flow, $\hat{\mathbf{Y}}$ is not only a function of the size of the stock A_g but also the health of the fund that the stock A_g represents. While biocapacity is a cardinal quantity (biocapacity is simply $\sum\limits_i G_i(t)$ as we saw in equation - 4.51), regenerative capacity is only an ordinal quantity. The stock measure of scale, $\sigma_{\mathbf{fp}}$ compares two quantities that requires $A_g(t)$ in figure - 4.26 to only be a stock. However, the flow measure of scale uses $\hat{\mathbf{Y}}$ which is a function of $A_g(t)$ as a fund. The dimensional inconsistencies that we identified with the footprint accounting framework in section - 4.4.1, account for the inconsistency between the stock and flow measures of scale derived from the footprint accounting framework.

The representation of the footprint methodology in figure - 4.26 is fraught with several problems related to how the ecological footprint accounting framework aggregates real physical throughput into the abstract global stock, $A_g(t)$. This global aggregation is an abstraction for which the regeneration flow $\hat{\mathbf{Y}}(t)$ has no real physical meaning. Formally, no real $\hat{\mathbf{Y}}(t)$ can be found such that $A_g(0) + \int_0^t \{\hat{\mathbf{Y}}(t) - a(t)\}\mathrm{d}t = A_g(t)$. The ecological footprint is a snapshot measure and no meaningful flow measure of scale can be constructed using the footprint methodology. Thus the question of consistency between stock and flow measures of

---

[20] Global Footprint Network (2006, p.28)

scale is moot. The representation of $a(t)$ as throughput in figure - 4.26 is not tenable.

This elaborate exposition of the impossibility of building a flow measure of scale that compares throughput and regeneration was meant to illustrate one of the most serious limitations of the footprint accounting framework. The footprint methodology was primarily developed to empirically determine if the global ecosystem (or the ecosystem contained in some smaller aggregation) is in overshoot, and is not particularly suited to quantitatively characterize the proportional relationship between the economy and the ecosystem.[21] In particular, the discussion here shows that the intuitive interpretation of overshoot implying a necessary drawing down of the stock is misplaced. Overshoot is a fund-flux phenomenon and the consequence of overshoot in the stock-flow space is the decrease, and the possible eventual collapse of regeneration flows. In the next question we will investigate overshoot in the fund-flux space and the footprint methodology's characterization of overshoot.

Overshoot in the Fund-Flux Space

As we discussed above, the primary effect of ecological overshoot that is of economic interest is the reduction in the regenerative capacity of natural capital, and even a possible collapse. The pertinent question to ask here is: what *specific* overshoot is captured by the footprint methodology? Policy response to any overshoot is contingent on being able to identify specific ecosystem funds that are in

---

[21] Wackernagel et al. [2002], Ayres [2000], Wackernagel et al. [2004], and personal communication from Matthis Wackernagel, July 2005.

overshoot. To motivate this exercise, consider the pictorial representation of how the footprint methodology calculates ecological demand on the global scale. Figure - 4.28 presents the time-series for the six principal components that make up the global ecological demand. The energy component is obtained by calculating

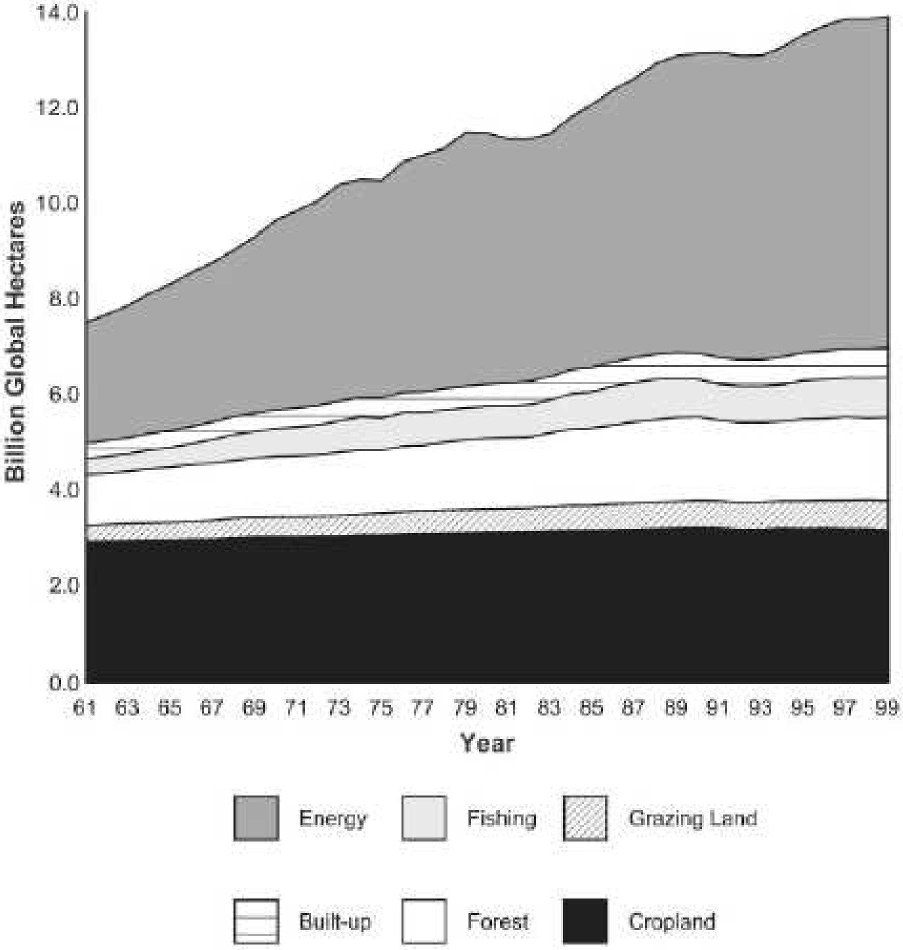

Figure 4.28: Six Components of Ecological Demand. On the vertical axis, global hectares use year-specific global average bioproductivity. Chart reproduced from Wackernagel et al. [2002, p.9270]

the land area that would be needed to terrestrially absorb all the carbon emitted from fossil fuel burning. The sum total of the productive area demanded by all six major categories of human activity currently exceeds the total available global productive area by about 20%. The footprint methodology therefore concludes that the global ecosystem is in overshoot. The aggregate global biophysical system is meaningful only in the stock-flow space. As we showed earlier, this aggregation is not dimensionally consistent in the fund-flux space, and yet overshoot is essentially a phenomenon of the fund-flux space. In terms of stock-flow space, overshoot signifies a stock that has grown beyond the carrying capacity.

To illustrate the problem with how the footprint framework aggregates different human activities, consider the largest contributor to the total ecological demand – fossil fuel burning. Fossil fuel burning results in increased concentration of atmospheric carbon that drives the greenhouse effect driven climate change. Estimates about what concentration should be considered an "overshoot" vary from 300 ppm to 450 ppm. Beyond this overshoot concentration, carbon in the atmosphere is over the 'carrying capacity' and the fund (which regulates global climate) suffers damage. While empirically the abstract overshoot measured in terms of global hectares in the footprint framework may coincide with an actual overshoot in physical terms, (measured in atmospheric concentration of carbon in ppm), there is no theoretical reason to support such a correlation. The footprint methodology asks the question: how much land area is required to terrestrially absorb all the carbon from fossil fuel burning? The answer to the preceding question is then compared with the bioproductive area actually available. Thus all that the footprint exercise can conclude is that a system is in overshoot in terms of bioproductive area availability that may empirically coincide with some real physical overshoot. Even with this theoretical problem, the footprint only answers a yes/no question: is the system in overshoot? The footprint does not say anything about the exact nature of the overshoot. Reducing carbon concentration from 550 ppm to 500 ppm will still leave the system in overshoot but the nature of the overshoot at those two concentrations is likely going to be different. The footprint methodology cannot discern between the two states of the world.

As a more dramatic illustration of the problem, consider a thought experiment where the five other components of footprint, are reduced to zero. The footprint methodology will conclude that there is enough biocapacity for terrestrial absorption of carbon from fossil fuel burning. The effect of this thought experiment on the actual physical ecosystem is contingent on the state of the world before all other components of footprint were reduced to zero. If the actual physical system was already in overshoot, we know that it will take over a century before the actual climate system driven by atmospheric carbon concentration is below overshoot. The snapshot view (and as we discussed earlier the footprint *only* gives a snapshot view) of the footprint methodology would have erroneously concluded that the climate system was below overshoot. This is only a thought experiment as it is clearly impossible in reality to burn any fossil fuel without some additional throughput. However this illustration nevertheless points to the theoretical problems with aggregation in the fund-flux space. Similar arguments could be made for other components of the footprint as well.

4.5 Object Lesson

The primary object lesson from our review of five different sets of biophysical assessment is that aggregation of throughput is fraught with important theoretical difficulties. From a scale perspective, the primary purpose of biophysical assessments is to empirically discern the relationship between the economy and the ecosystem in terms of physical stocks and flows. A complete answer to the question of "how large is the economy relative to the ecosystem that contains and sustains it" includes accounting for the fund-flux space that is not amenable to cardinal arithmetic. If physical sustainability is a policy goal at multiple levels of economic-geographic aggregation aggregation differences between the stock-flow space and the fund-flux space become salient. For example aggregating the throughput at the national level following the TMR-TDO methodology assumes that there is actually an ecosystem fund corresponding to the aggregation at the national level. Total weight of the throughput at the national level is physical metric, but not very useful if the objective of constructing such a metric is to aid decision making about the biophysical sustainability of the multiple stocks from which the aggregate throughput is derived. Similarly we showed that the ecological footprint accounting framework has important theoretical problems in the fund-flux space.

Despite important lacunae that we identified, each of the biophysical accounting frameworks reviewed in this chapter provide important points of departure for future efforts to construct scale metrics that even wile being stock-flow consistent allow for useful policy interpretation in the fund-flux space. In the next two chapters of this dissertation we present a framework for the analytical study of the fund-flux space. Chapter - 5 introduces benchmark scale measures that are ultimately related to the health of the ecosystem fund, and chapter - 6 develops a framework for consistent ordinal ranking in the fund-flux space.

Chapter 5

A Framework for Benchmark Measures

5.1 Introduction

Our focus thus far has been on constructing and validating a framework to empirically measure scale. For scale measures to be useful as tools for environmental policy, a critical requirement, besides being able to empirically measure scale, is a consistent and objective ranking of measured values of scale. Of particular importance to practical policy is comparing empirically measured values of scale with policy relevant benchmarks. In this chapter we focus our attention on deriving two different benchmarks. Maximum scale, and maximum sustainable scale are benchmarks that are related to the question: how large *can* the economy be relative to the ecosystem? Implicit in this "*can*" question is the constraint imposed by physical sustainability: how large can the economy be relative to the ecosystem without overwhelming the ecosystem?[1] Optimal scale is a normative benchmark that answers the question: how large *should* the economy be relative to the ecosystem.

The primary goal for this chapter is to establish a framework to interpret measured values of scale in the fund-flux dimension. All scale measures are defined in terms of seven stocks and flows in figure - 3.6 that are all cardinal variables. The scale benchmarks, on the other hand are most usefully defined in the fund-flux

---

[1] Note that this notion of what constitutes biophysical sustainability is different from the more common normative notions of sustainability including leaving enough for future generations. We will treat normative questions surrounding the physical size of the economy as questions related to *optimal scale*.

space. In case of maximum, or maximum sustainable scale, ecosystem resilience is essentially a function of the state of the ecosystem fund. The optimal scale problem involves normative questions about the tension between the service flux from the ecosystem ($\breve{\mathbf{Y}}$) and the service flux derived from the artifacts in the human economy ($\breve{Y}_e$). We develop a framework that helps 'map' measured values of stock and flow measures onto the fund-flux space. This mapping will serve as the basis for formal axiomatic framework for comparing two or more measured values of scale (chapter - 6). Consider an illustrative example: say we have a time series of flow measure of scale, S_{so} for a logging industry that has been harvesting timber from a forest. Physical sustainability in the stock-flow space is trivial – the forest system will eventually collapse if throughput exceeds regeneration year after year ($\dot{x}_i > \hat{Y}_{so}$). Thus in the stock-flow space maximum sustainable scale is given by $S_{so} = 1$. The fund-flux space is somewhat more complicated. Suppose the logging industry is actually harvesting timber at a sustainable rate in the stock-flow space ($S_{so} \leq 1$) from a forest that is recovering from some historical destruction, what would be the value of maximum sustainable scale, taking into account biophysical sustainability in the fund-flux dimension? The regeneration flow, $\hat{Y}_{so}$ is not only a function of the stock but also the health of the fund. In particular $\hat{\mathbf{Y}}$ could be decreasing even when $\mathbf{S} < 1$. If maximum sustainable scale is a benchmark that sheds light on resilience of the ecosystem in the fund flux space, optimal scale is a benchmark that addresses question of social choice. We will see how optimal scale like maximum sustainable scale is a benchmark that is best conceived in the fund-flux space. We will show in this chapter that optimal scale describes the societal tradeoffs between service flux from the economy stock ($\breve{Y}_e$) and the service flux derived from the ecosystem ($\breve{\mathbf{Y}}$).

5.2 Maximum Sustainable Scale

5.2.1 Relationship between Maximum Sustainable Scale and Maximum Scale

Before we present a framework for maximum sustainable scale, we will briefly clarify the relationship between maximum sustainable scale and maximum scale. While the two benchmarks have been used synonymously in the literature, the distinction between the two is crucial to understanding the scale methodology's relationship to the phenomenon of overshoot. To motivate the discussion, we invoke a rather crude analogy: consider an airplane that is designed to cruise at a certain maximum altitude – this is the altitude that can be maintained over a sustained period of time. However, there is some finite resilience built into the design of the aircraft that allows it exceed this design altitude over short periods of time. However, there is an upper bound on how high the aircraft can safely climb even for short durations of time. Maximum sustainable scale, like the design altitude of the aircraft, describes the maximum throughput that can be sustained over long periods of time and maximum scale describes the size of the economy that is tenable (even if not sustainable) over short periods of time. There is of course a difference of several order of magnitudes in terms of what constitutes 'short' and 'long' periods of time for an aircraft, and for complex ecosystems.

Both maximum scale, and maximum sustainable scale are fundamentally related to the fund function of the ecosystem. Given the complex nature of the fund and the impossibility of an ordinal characterization of the fund-flux space, maximum scale or maximum sustainable scale cannot be precisely identified. However, it is often possible to identify a 'region' for maximum sustainable scale and speculate on the range for maximum scale. Continuing with the mechanical analogy of an airplane, we will denote the region (in the fund-flux space) where the maximum sustainable scale falls as the "yield point region," and the region corresponding to maximum scale as the "buckle point region." The region between yield point region and the buckle point region will correspond to various forms of ecological overshoot [Catton 1982, Wackernagel et al. 2002]. We merely present a formal heuristic for maximum and maximum sustainable scale – a black box view of how the economy generated throughput impacts the ecosystem funds. It is impossible to generalize the complex dynamics underlying the fund, but even an attempt to develop a general framework to describe fund dynamics is outside the scope of this study.[2]

5.2.2 Mapping between Stock-Flow and Fund-Flux Spaces

Ecosystem is simultaneously a fund of service fluxes and a stock of resource flows.[3] The resilience of the ecosystem is primarily a function of the state of the

---

[2] Within the ecological economics literature, two different efforts are worth noting here. First, the development of the concept of "critical natural capital" that tries to identify critical and irreplaceable services derived from funds [Elkins et al. 2003a,b, Elkins 2003]; and second, the ambitious attempt pioneered by C.S. Holling to understand long-term dynamics of ecological and social systems under the rubric of "panarchy" [Holling et al. 2002].

[3] Here and elsewhere in this chapter, we use "resource flows" to refer to both the source-side and the sink-side of the eocnomy. On the source-side "resource flows" refers to the source-side of

fund rather than the mere size of the stock. A managed captive plantation with as much timber stock as an old growth forest is not a 'resilient ecosystem' like the natural forest. Maximum sustainable scale is that point where any further increase in the throughput compromises ecosystem's resilience. From an anthropogenic perspective, maximum sustainable scale of the economy, is the point where the ability of the ecosystem to generate critical service fluxes ($\check{\mathbf{Y}}$) is irrevocably destroyed. Further, regeneration flows ($\hat{Y}_{so}$ and $\hat{Y}_{si}$) are related to the resilience of the ecosystem.

We present the framework for maximum sustainable scale as a two-stage process. In the first stage, we determine a point on the service flux continuum that corresponds to the critical service flux – this is also the level at which the integrity of the underlying ecosystem fund is not irrevocably compromised. In figure - 5.1, this is shown as the maximum sustainable scale region — it is a region rather than a point because there is considerable uncertainty involved in determining this critical point. In the second state of the two-stage framework for maximum sustainable scale, we map this critical region to the resource flow continuum. This mapping is important from both theoretical and practical policy perspectives. Cardinal measurements are possible only in the stock-flow space and maximum sustainable scale which is fundamentally defined in the fund-flux space has to be mapped to a empirically measurable quantity. Further, a society's economic decision making happens in the stock-flow space (markets for funds are difficult to design because in most

---

the throughput, $\dot{x}_i$ and on the sink-side, to the waste flow, $\dot{x}_o$. Unless otherwise specifically noted, all our discussion here holds good for both the source-side and sink-side of the economy.

instances, funds are pure public goods). Thus from a policy perspective it is important to understand the linkages between how regulating throughput impacts the fund-flux space.

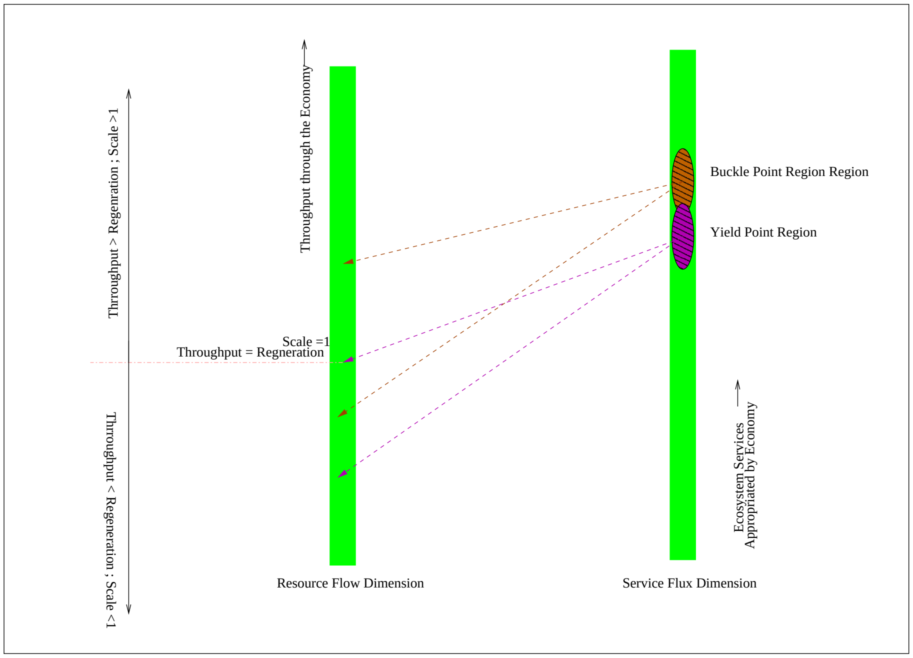

Figure 5.1: Maximum Sustainable Scale: Mapping Resource Flows to Service Fluxes

The mapping is shown in figure - 5.1. The service-flux continuum on the right-hand-side of the figure is mapped on to the resource-flow-continuum on the left-hand-side of the figure. Before we discuss the mapping process in detail, it is important to understand that figure - 5.1 is for single time period. The mapping is not independent of the level of stock (and hence the fund) at any given time. Figure - 5.1 describes the mapping in the flow-dimension, as maximum sustainable scale, as discussed above, is most usefully defined in terms of the throughput scale measure, **S**. The largest value maximum sustainable scale can take is of course a unity scale ratio. At $\mathbf{S} = 1$, the economy generated throughput is equal to natural income on the resource flow continuum and the system is in steady state.[4] For $\mathbf{S} > 1$, throughput exceeds the regeneration capacity of the ecosystem and a throughput at this level will eventually run down the stock. As we are interested in determining the level of throughput that can be sustained over a long time periods the scale cannot be any greater than the steady state scale of $\mathbf{S} = 1$. However, $\mathbf{S} \leq 1$ is only a *necessary* condition for throughput to be below maximum sustainable scale but is not a *sufficient* condition. This is reflected in the mapping on the figure below. The lower bound for maximum sustainable scale is less than one.

The last point needs some explanation. Under what circumstances can we have maximum sustainable scale be below stock-flow space steady state scale of $\mathbf{S} = 1$? Why would the scale be over maximum sustainable scale even as the stock is actually growing as it would be when $\mathbf{S} < 1$? At $\mathbf{S} < 1$ the resource *stock* is indeed 'sustainable' in some restricted sense but maximum sustainable scale is primarily related to the fund and not to the stock. In some metaphorical sense, maximum sustainable scale represents a 'steady state' in the fund-flux, rather than the stock-flow space. Consider the forestry example again where timber is being harvested at some rate below the rate of natural regeneration. Though timber harvest as a resource flow is apparently sustainable it is possible that the process of harvesting timber at this level has fundamentally altered the forest ecosystem in its fund role. Further, even at this apparently sustainable rate, the regeneration rate, $\hat{\mathbf{Y}}$ could be decreasing

---

[4] In the stock-flow space.

and thus leading to the eventual collapse if throughput remains unchanged. As a more obvious example consider timber that comes from a managed captive plantation that was set up by clearing a natural forest. The captive plantation, though a 'sustainable' source of timber, obviously does not provide all the services that the natural forest provides. The important point is that the fund is more than just the stock – it is a *particular* arrangement of multiple stocks. This 'arrangement' can disintegrate even when the stock itself is sustainable. From a policy perspective, it is prudent to define maximum sustainable scale with reference to the fund function of the ecosystem rather than the stock function of the ecosystem because the ecosystem in its fund role has no real substitutes in most instances, and should be the basis of the absolute limits on the physical size of the human economy. In the next section the discussion on maximum scale will further underscore this point.

5.3 Maximum Scale

As discussed in section - 5.2.1, if maximum sustainable scale of the economy determines the throughput that is physically sustainable in the long-run, maximum scale determines the physical size of the economy that cannot be exceeded even in the short run. Maximum scale is fundamentally related to the size and health of the fund. Maximum scale refers to that physical size of the economy which irrevocably destroys the special configuration of stocks that make up the fund. This destruction of the special configuration can happen in three different ways:

1. The stock runs down to zero
2. The stock is positive, but smaller than the minimum stock size required to maintain the fund configuration. For biological systems, this corresponds to the minimum stock size required for positive a regeneration rate.
3. Destruction of the fund configuration due to some 'surge throughput' (the resulting stock size here can be greater than in (2))

For the purposes of analysis, (1) can be treated as a special case of (2) where the critical stock size from (2) is zero. Thus we have two boundary values – one in the flow dimension (given by (3) above) and one in the stock dimension. Formally, we can write down the two boundary conditions for maximum scale as:

$$\mathbf{Scale_{max}} : \begin{cases} \mathbf{x(t)} & \leq & \mathbf{x_{critical}} \\ \mathbf{\dot{x}(t)} & \geq & \mathbf{\dot{x}_{critical}} \end{cases} \tag{5.1}$$

While the stock condition in equation - 5.1 is self evident, the flow condition needs some explanation. Before we take up the flow condition ($\mathbf{\dot{x}(t)} \geq \mathbf{\dot{x}_{critical}}$) it is proper to note that the stock condition is merely the cumulative effects of throughput over maximum sustainable scale over several time periods. The stock at any time t is given by:

$$\mathbf{x}(t) = \mathbf{x}(0) + \int_{0}^{\tilde{t}} \left(\dot{\mathbf{x}}(t) - \hat{\mathbf{Y}}(t)\right)\, \mathrm{d}t \tag{5.2}$$

where $\dot{\mathbf{x}}(t)$ is the throughput at time t and $\hat{\mathbf{Y}}(t)$ is the regeneration flow at time t. $\tilde{t}$ is the current time period and t_0 is some time period corresponding to the reference stock level such that $\mathbf{x}(0) > \mathbf{x_{critical}}$, and maximum scale is breached when: t is given by:

$$\mathbf{x}(0) + \int_{0}^{\tilde{t}} \left(\dot{\mathbf{x}}(t) - \hat{\mathbf{Y}}(t) \right) \mathrm{d}t \leq \mathbf{x_{critical}} \tag{5.3}$$

Throughput Condition

The throughput condition in equation - 5.1 is related to how the fund configuration is maintained. Recalling than a fund is a particular configuration of the stock that enables it to serve as a source of service fluxes. This special configuration is maintained by a complex 'maintenance flows.' Maintenance flows are outside the system boundary of our analytical picture of the economy-ecosystem interaction in figure - 3.4. For example consider the familiar example of timber in the forest. The analytical representation only shows regeneration of the particular material (timber here) under study. However the regeneration flow, $\hat{Y}_{so}$ is not only the function of the level of stock (x_{so}) but also the quality of fund that the stock is part of. A given stock of timber in a tropical rain forest regenerates at a different rate than the same stock of timber that is part of a temperate managed-plantation. Maintenance flows account for the difference – material and energy cycles in the two ecosystems are different. For the most part, the human economy has no control over this background rate of 'maintenance flows.' The throughput condition in equation - 5.1 states that beyond a certain critical throughput, the fund configuration is destroyed irrespective of the level of maintenance flows in subsequent time periods. As a rather stark example consider a badly mangled car that is beyond repair (car, we recall is a fund of transportation services).

We have so far not investigated the determinants of the critical throughput, $\dot{\mathbf{x}}_{\mathbf{critical}}$. An obvious lacuna in the representation of critical throughput in equation - 5.1 is that $\dot{\mathbf{x}}_{\mathbf{critical}}$ is shown to be independent of time. The critical throughput level is obviously a function of the level of stock at any given time. Thus except when the stock is in steady state, this implies that $\dot{\mathbf{x}}_{\mathbf{critical}}$ is a function of time. Thus a more accurate representation of the flow condition in equation - 5.1 would be:

$$\mathbf{Scale}_{\mathbf{max}} : \left\{ \dot{\mathbf{x}}(\mathbf{t}) \quad \geq \quad \dot{\mathbf{x}}_{\mathbf{critical}}(\mathbf{x}(\mathbf{t})) \right. \tag{5.4}$$

Further, even in the representation in equation - 5.4 the critical throughput is assumed to be exogenous to our analysis. First, with technological progress, it is possible that $\dot{\mathbf{x}}_{\mathbf{critical}}$ will change. However, for our purposes here, it is sufficient to note that with any kind of technical progress, $\dot{\mathbf{x}}_{\mathbf{critical}}$ is still a finite number. More importantly, a fund is composed of more than one stock. In the simplest case, suppose there are two resources x and y that map on to a common fund – the critical throughput $\dot{\mathbf{x}}_{\mathbf{critical}}$ is a function of the critical throughput associated with the other stock ($\dot{\mathbf{y}}_{\mathbf{critical}}$).

Representation in the Stock-Fund Space and the Flow-Flux Space

In figures 5.2 and 5.3 we have attempted to represent maximum, and maximum sustainable scale in the flow and stock dimensions. For the flow representation (figure - 5.2) we show how throughput is related to the service flux from the ecosystem, and for the stock representation, we depict the relationship between the ecosystem as a stock and ecosystem as a fund. In the flow representation, the horizontal axis represents the throughput and the vertical axis the amount of service flux that is lost. In the absence of any throughput, there is no service flux that is lost. The $\mathbf{S} = 1$ line represents the point where throughput equals the regeneration flow $(\hat{\mathbf{Y}})$. The schedule is not linear and is shown with increasing slope because, as throughput increases, the underlying fund is likely disturbed at an increasing rate − in general empirical evidence supports the assumption of an increasing marginal rate of service flux loss. The maximum sustainable scale, following the analogy we introduced earlier is shown as the yield point. The throughput corresponding to yield point is

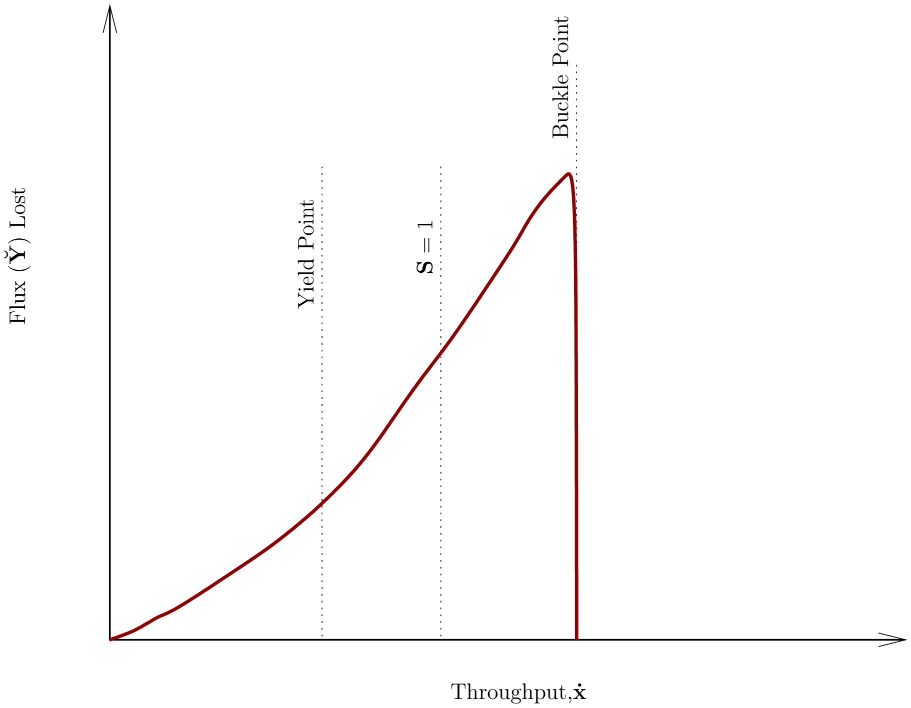

Figure 5.2: The Flow-Flux Space. The figure shows the relationship between throughput and lost service flux. The yield point is a function of the standing stock, an can occur even when the throughput it less than regeneration.

shown to be lower than the throughput where regeneration and the throughput are the same ($\mathbf{S} = 1$). This is consistent with our discussion about how maximum sustainable throughput can be lower than the throughput corresponding to $\mathbf{S} = 1$ but no greater than that throughput. Between yield point and buckle point (throughput corresponding to maximum scale) the underlying fund is not irrevocably destroyed if the overshoot is temporary. At buckle point, corresponding to the flow-condition for maximum scale (equation - 5.4), there is the collapse of the ecosystem fund and this is depicted by the fact that the service flux lost abruptly drops to zero – there is no more service flux to be lost as the fund has been destroyed irrevocably. It must be emphasized again that the flux-flow schedule shown in figure - 5.2 is for a *given* level of stock. For systems that are not in steady state, we will in theory need a new figure - 5.2 for every time-period – which is for most systems for very few systems are in steady state.

Figure - 5.3 represents maximum scale in the stock-fund space. Maximum sustainable scale is fundamentally a measure of the throughput and cannot be represented in the stock-fund space. The horizontal axis in figure - 5.3 shows standing stock (x_{so} on the source-side; and holes, $\tilde{x}_{si}$ on the sink-side). The vertical axis shows the amount of fund that is lost to the economy. The fund is measured in terms of an index variable. At high levels of standing stock, there is no loss in the fund. At the critical level of the stock, corresponding to the stock condition for maximum scale in equation - 5.4, the fund breaks down. The relationship between the flow picture in figure - 5.2 and the stock picture in figure - 5.3 is easy to interpret if we are only

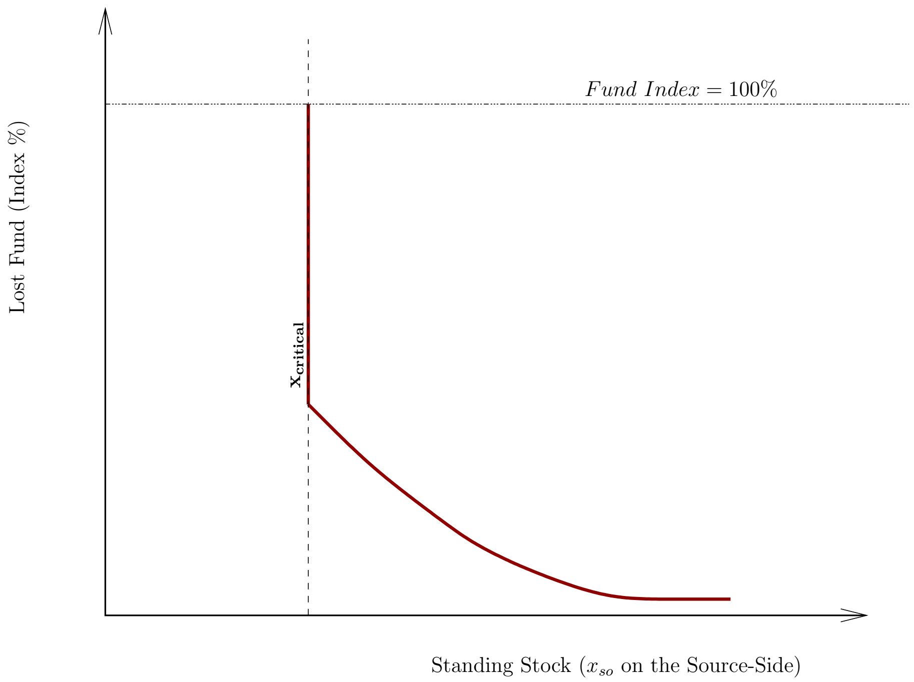

Figure 5.3: The Stock-Fund Space. The figure shows relationship between stock and fund. The vertical axis measures the integrity of the fund in terms of an ordinal index. The ordinal index is normalized to a 0-100% scale with a index value of 100% representing a fully functional fund. As the standing stock decreases, the ordinal index is shown to decrease, and at $\mathbf{x_{critical}}$, the fund is irrevocably destroyed.

concerned with service fluxes from the fund that benefit the human society. From this narrow anthropogenic perspective, the fund index can be interpreted as amount of service flux derived from the fund.

5.4 Optimal Scale

The systematic study of scale is motivated by questions about the actual proportional relationship between the economy and the ecosystem; the limits on the physical size of the economy relative to the ecosystem's ability to support the economy; and the normative notion of a 'harmonious' proportional relationship between the economy and the ecosystem. From a policy perspective, the normative question of how large *should* the economy be relative to the ecosystem is perhaps the most important question. The most desirable of the possible proportional relationships between the economy and the ecosystem will define the "optimal scale" of the economy. In this section, we will use the framework developed to study the positive questions about scale to shed light on the largely normative question of optimal scale. Our focus here will be to develop a broad set of criteria rather any specific methodology for determining optimal scale. This is consistent with the fact that optimal scale is ultimately a social, political, cultural choice. Indeed in appendix - A we show how questions about optimal scale are not unlike questions about what constitutes a just distribution of wealth or income.

5.4.1 Stock and Flow Dimensions

Optimal scale is the most *desirable* proportional relationship between the economy and the ecosystem. We have seen how the complete characterization of a proportional relationship between the economy and the ecosystem requires specification of scale in both the stock and flow dimensions. Here we investigate how optimal scale can be specified in the stock and flow dimensions with a particular emphasis on the key differences between stock and flow specifications of optimal scale. In the basic stock-flow representation of how the economy and ecosystem intersect (figure - 3.4), any normative notion of a 'desirable scale' is ultimately related to the relative magnitudes of the two components (economy and ecosystem) of the utility flux, $\breve{Y}_e$ and $\breve{\mathbf{Y}}$. However, the utility flux, U is a subjective experience and concrete environmental policy has to be ultimately cast in objectively measurable stocks and flows of figure - 3.4. Thus our goal here is to derive possible frameworks to 'translate' a normative conception of optimal scale in the fund-flux space to the objective and cardinal stock-flow space.[5] The question of how large *should* the economy be relative to the ecosystem is fundamental to the ecological economics vision of the economic predicament because of the finitude of the ecosystem capacity represented by stocks x_{so} and $\tilde{x}_{si}$. Solar energy is the ultimate source of energy driving the ecosystem and the fixed rate of solar insolation incident on the earth's surface imposes finite limits on regenerative rates, r_{so} and r_{si}. Implicit in the discussion of optimal scale is the assumption that societies strive for some kind of sustainability that at the very least preserves adequate economic opportunities for future generations. Indeed most questions surrounding scale will be moot if we did not care about sustainability. As we discuss below, specifics about how a society conceives sustainability has important implications for how optimal scale could be specified, but the key point to note is that some notion of sustainability is integral to any discussion about optimal scale. From a practical policy perspective, in most cases, the ecosystem stocks x_{so} and $\tilde{x}_{si}$, are large enough to last a single generation that a systematic study of the proportional relationship between the economy and the ecosystem becomes moot if some notion of sustainability were not important.

In utilitarian terms, the society's goal is to maximize the aggregate utility flux,

---

[5] As we have discussed in section - 3.2.1, cardinal measurements are possible only on the stock-flow space. The fund-flux space is limited to ordinal rankings.

U. However, this utility flux is a subjective experience and a framework for optimal scale that can be operationalized in practice has to be specified in physically measurable stocks and flows. Optimal scale consists of specifying the relative sizes of the three stocks and two flows in figure - 3.4. From a policy perspective cast in narrow anthropogenic terms, the society's objective becomes one of determining in every time-period the magnitude of the two components of throughput, $\dot{x}_i$ and $\dot{x}_o$ so that the overall 'wellbeing' of the society is maximized within the constraints imposed by concerns of physical sustainability. The problem of optimal scale is straightforward (at least in theory) – any increase in the utility flux derived from the economy is predicated on increasing the economy-stock, x_e and the corresponding throughput needed to maintain this stock. Increasing throughput ($\dot{x}_i$) and the economy-stock results in increased $\breve{Y}_e$. However increasing $\dot{x}_i$ or the withdrawal from the source is also fraught with two other consequences that result in the decrease of the fund-flux component of natural income, $\breve{\mathbf{Y}}$. First, increasing withdrawal from the source also means that the waste component of the throughput increases.[6] This increased waste flow results in in the decrease of available waste absorption capacity (the stock of 'holes,' $\tilde{x}_{si}$ goes down). The decreased waste absorption capacity results in a decrease of service flux that is derived from the ecosystem in the sink-side. Decrease in sink-side service flux, $\breve{Y}_{si}$ contributes to a reduction in the overall utility flux, U. Second, increasing withdrawal from the source can result in the decrease of service

---

[6] It is a matter of simple accounting identity that $\frac{dx_e}{dt} = \dot{x}_i(t) - \dot{x}_o(t)$. The withdrawals from the source cannot be perpetually stocked in the economy-stock, x_e and the inevitable entropic degradation of x_e means that the increased withdrawal from the source will eventually find its way into the waste stream, $\dot{x}_o(t)$. However, in a given time-period it is possible not to observe a positive correlation between the two components of the throughput because the economy-stock can serve as a buffer.

flux derived on the source-side. The ecosystem stock, x_{so} is not only a stock for the physical throughput, $\dot{x}_i$ but is also a fund from which the service flux $\breve{Y}_{so}$ is derived. Any withdrawal not only changes the size of the stock but also alters the nature of the fund.[7] For example when a logging company extracts timber from forest, it not only alters the stock of standing wood in the forest but also the forest's ability to provide services like erosion prevention. Thus increasing throughput has, in general, an indeterminate effect on the aggregate utility flux, U. While increasing the throughput ostensibly increases the amount of service flux that can be derived from the economy-stock it can just as well decrease the service flux derived from the ecosystem.

Having laid out the basic scheme for specifying optimal scale, we are now equipped to delve into the idiosyncrasies involved in the practical environmental policy relevant application of optimal scale. Of particular importance is the difference between how optimal scale can be specified on the stock, and flow dimensions. As seen from figure - 3.4 the three components that contribute to the utility flux, U are all derived from the stocks rather than flows. This feature of the analytical representation of the ecological economics vision, discussed in section - 3.2.1 deserves some repetition here. Though the ecosystem service flux ($\breve{\mathbf{Y}}$) as well as the that from the economy are shown to be derived from the stocks, the three stocks of figure - 3.4

---

[7] The size of the stock is always altered except in the 'stead state' case when the withdrawal, $\dot{x}_o$ is exactly matched by regeneration, $\hat{Y}_{so}$. Arguably, even under steady state withdrawal, the nature of the underlying fund is altered even as the stock remains unchanged. Indeed as we have argued elsewhere in this chapter, the nature of the fund is potentially altered even when the withdrawal is less than regeneration.

are simultaneously also 'funds.' The throughput, $\dot{x}_i$ does not directly contribute to human welfare but is only a means to building and replenishing the standing stock of artifacts in the society, represented in figure - 3.4 by the economy-stock, x_e. For example, we derive satisfaction from artifacts (x_e) in the society that make use of the timber harvested in a given year ($\dot{x}_i$) rather than timber itself. The service flux derived from the economy, $\breve{Y}_e$ is related to the stock (or more accurately fund) of timber products in the economy (furniture for example) rather than the raw timber that is harvested from the forest in a given year.[8] Figure - 3.4 represents a snapshot in time at *a particular* time resolution. At a different time-resolution the designation as 'stocks' and 'flows' will change – at a lower resolution, stocks from a higher resolution may have to be treated as flows. A family's kitchen pantry is a 'stock' at a time resolution of one-month but a flow at a lower resolution of say one-year. In similar vein, timber harvested from a forest is both a stock and a flow depending on the time resolution chosen. The choice of time resolution used to depict figure - 3.4 is particularly relevant in the discussion of optimal scale. Given the fact that some notion of sustainability is implicit to the discussion of optimal scale, an important normative question becomes that of what is to be sustained, and for how long?[9] Society's answer to these normative questions determine the time resolution that is relevant in interpreting figure - 3.4. Our purpose here is not to delve into questions of how societies can or should answer these normative questions but to lay out a

---

[8] The most commonly used monetary measure of economic activity, the Gross Domestic Product or the GDP simply measures the monetary value of the aggregate throughput.

[9] We argue elsewhere in this dissertation, and most notably in the discussion of maximum sustainable scale that questions of sustainability are ultimately normative questions rather than technical positive questions.

general framework to interpret the answers to these questions in terms of physical stocks and flows.

The first step in specifying optimal scale is to determine the relative sizes of the three stocks. Varying the sizes of the three stocks results in a change in the three service fluxes derived from the respective stocks. The problem of optimal scale is to rank different states of the world resulting from a change in one or more of the three service fluxes. In particular, the problem is one of specifying the most desirable state of the world amongst the infinitely many that are possible. In terms of the formalism of figure - 3.4, societies have to chose between states of the world $A = \left[\breve{Y}_{so}(x_{so}^{A}), \breve{Y}_{si}(\tilde{x}_{si}^{A}), \breve{Y}_{e}(x_{e}^{A})\right]; B = \left[\breve{Y}_{so}(x_{so}^{B}), \breve{Y}_{si}(\tilde{x}_{si}^{B}), \breve{Y}_{e}(x_{e}^{B})\right] \ldots$ In chapter - 6, we investigate how any two states of the world A and B can be ranked but for now it is sufficient to note that there is a state of world represented by $\left[\breve{Y}_{so}(x_{so}^{*}), \breve{Y}_{si}(\tilde{x}_{si}^{*}), \breve{Y}_{e}(x_{e}^{*})\right]$ that is preferred to any other state of the world. On the stock dimension, $\sigma(x_{so}^{*}, x_{e}^{*}, \tilde{x}_{si}^{*})$ represents the optimal scale. In our representation here, we have shown the service fluxes to be functions of the three stocks which are also simultaneously funds. Implicit in this representation is the assumption of *ceteris paribus.* The abstraction in figure - 3.4 that looks at only three stocks assumes that all other stocks and flows having a bearing on the funds represented by the three stocks under study are not changing.

For any given set of four flows in figure - 3.4 the scale-measure **S** is completely determined.[10] However, optimal scale in the flow dimension cannot be specified independent of the magnitude of stocks. Consider the familiar example of the logging industry. To determine the flow measure of scale on the source-side we only need information about the rate at which timber is harvested from the forest ($\dot{x}_i$) and the rate at which it is regenerated ($\hat{Y}_{so}$). Given these two flows, how does the flow measure of scale determined by equation - 3.10 compare with optimal scale, S^*_{so}? Consider two different scenarios with vastly different standing stock of trees in the forest but with the same regeneration and withdrawal rates. A withdrawal rate that is optimal in the context of a mature forest may not be 'optimal' in case of a newly regenerating forest even when the measured values of scale on the flow dimension are the same in both cases. In particular, even a measured of value of scale with $S_{so} < 1$ that is theoretically 'sustainable' with withdrawal less than regeneration can be larger than optimal scale if the size of the stock (x_{so}) is different from that which corresponds to optimal stock size, x^*_{so}. Optimal scale is properly defined only on the stock dimension because optimal scale is ultimately related to the relative contributions of the three service fluxes, and these fluxes are most directly related to the health of the respective funds.

---

[10] See equation - 3.8 on page - 52.

5.4.2 Operationalizing Optimal Scale

Optimal Scale and Marginal Valuation

Optimal scale is the benchmark that is most relevant from an environmental policy perspective. While optimal scale is ultimately a normative benchmark that depends on social, political, and even moral choices of a society, physical sustainability is ostensibly an important concern. The standard utilitarian heuristic of comparing marginal benefits and marginal costs from increasing the physical size of the economy has been widely used in ecological economics to illustrate how societies can determine the most desirable physical size of the macro-economy and thus optimal scale [Daly 2005]. In this section we review the key ideas behind a utilitarian conception of optimal scale. We find that while useful as an expository device, utilitarian definitions of optimal scale cannot be operationalized when sustainability is a central concern of environmental policy.

The central idea behind an utilitarian framework to determine optimal scale is depicted in fig - 5.4, reproduced from Daly [2005]. Economic growth involves increase in the physical size of the economy. A larger physical size implies a larger scale because the economy is growing relative to a fixed ecosystem. Increasing the physical size of the economy not only provides for increased consumption of goods and services that results in increasing utility but has a cost or ‘disutility’ associated with it. The marginal utility curve (drawn in solid blue) is shown downward sloping and is consistent with the assumption of decreasing marginal utility of consumption.

Increasing consumption also has costs associated with it, and the broken-red line in the figure is the marginal disutility or the marginal cost associated with increasing consumption. The net utility (or disutility) enjoyed by a society is the area between the utility and disutility curves. The optimal scale is the point where marginal utility equals marginal disutility or marginal cost, and at this point the net utility is maximized. Any expansion beyond this optimal point, as is familiar from basic marginal analysis of microeconomics, results in a net loss of utility and has therefore been termed "uneconomic." The distinguishing feature of the heuristic in fig - 5.4 is that the marginal cost-benefit analysis presented there is not for a single economic agent – a firm or an individual, or even for a single market but it is an attempt to apply marginal cost-benefit analysis to the macro-economy as a whole. Our purpose in here is to critically examine the implications of extending marginal analysis to the macro-economy especially in the context to using marginal analysis to determine the optimal physical scale of the economy.

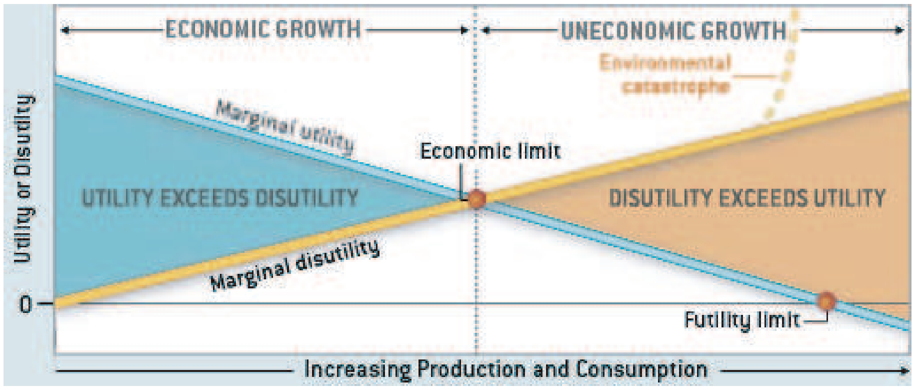

Figure 5.4: An Utilitarian Framework to Determine Optimal Scale. Reproduced from Daly [2005]

There are three related questions that directly follow from the use of a cost-benefit calculus to determine the optimal physical size of the economy. First, what does the utility (or disutilty) curve represent when used to aggregate more than one economic agent? Second, what is the most general specification of the utility function in fig - 5.4 that will ensure that the maximizing net utility is consistent with physical sustainability and distributive justice? Finally, how can (if) one operationalize the utilitarian framework for the purposes of determining optimal scale? The first question is the staple of social choice theory and has received adequate attention in the literature. The second question is a question that has received some attention in the literature too, if only in the form of individual preferences that lead to sustainability,[11] and our primary focus here will be to answer the third question.

Without going into theoretical difficulties involved in aggregating individual preferences into society-wide 'social welfare functions,' we assume that the marginal utility curve presented in fig - 5.4 is derived from an underlying social welfare function. The marginal disutility schedule simply represents the true social marginal cost rather than private marginal cost. While it may be impossible to exactly specify the social marginal cost curve, it nevertheless presents no fundamental theoretical or conceptual difficulty. Operationally however, it is impossible to fully specify a social marginal cost because it would entail 'internalizing' all of the 'externalities.' Thus the 'Economic limit' in fig - 5.4 represents a society-wide efficiency in allocation of *all* resources used in the economy. Even from an ecological economics perspective

---

[11] See for example Chichilnisky [1996]

of economy as an open subsystem of a finite ecosystem the point that represents the ‘economic limit’ can change over time because technological progress can move the marginal disutility curve in at least two ways. First, changes in technology can, at the margin, increase or decrease the direct tangible costs associated with further economic expansion. Second, technological progress can have an indirect effect through changes in mitigation and adaptation options that are available, to cope with ecological habitat destruction for example. Thus even if we assumed that the underlying social welfare function does not change with time, the optimal ‘economic limit’ shifts with time.

The fact that the optimal point where marginal cost equals marginal benefit merely points to an efficient allocation of resources rather than a biophysical basis for a sustainable economy begs the question of whether the optimal ‘economic limit’ can coincide with conditions for biophysical sustainability. A related question would be about the relationship between the ‘economic limit’ determined using an utilitarian framework and normative notions of just distribution. A society can be at the optimal point where the net utility is maximized but the distribution of the economic product among different economic agents is grossly unfair. The relationship between the nature of social welfare functions (from which the marginal utility curve is derived) and normative conceptions of fairness and justice has received considerable attention in the literature. Our focus here will be to discuss specifications of the social welfare function in the context of relationship between physical sustainability and the utilitarian framework for determining the optimal physical size of the economy.

For our purposes here the most important question is the relationship between 'economic limit' obtained through an utilitarian framework and optimal scale. To recollect, optimal scale is a normative benchmark that answers the question: how large *should* the economy be relative to the ecosystem? It is a social, political question rather than a technical question. It is possible in theory that a society could arrive at an 'optimal scale' that is not sustainable in biophysical terms. However, given that one of the primary objectives of systematically studying scale is to inform debates surrounding questions of biophysical sustainability, we will assume that a society's conception of optimal scale includes sustainability. Thus optimal scale can at most be as large as maximum sustainable scale (section - 5.2). We have already seen that the utilitarian framework does not shed any light on biophysical sustainability and that the 'economic limit' is sustainable only under certain specifications of the society-wide utility function. Even while not addressing the sustainability question, can the utilitarian calculus contribute to characterizing the proportional relationship between the economy and the ecosystem?

The straightforward interpretation of the marginal utility curve is that it represents marginal benefit derived from increasing the dollar value of the aggregate economic product. The aggregate exchange value of a society's economic product though related to the physical size of the economy does not directly say anything about the that size in proportion to the size of the ecosystem. This is easily seen from the fact that the utilitarian framework is not contingent on the basic preanalytic vision for ecological economics (figure - 3.1). Scale – the proportional relationship between the economy and the ecosystem is part of the economic predicament only when the economy is conceived as a subsystem of the larger ecosystem. The marginal disutility curve is related to the scarcity of 'means' relative to 'ends' but is not necessarily related to the scarcity of ecological space that directly follows from the ecological economics vision of how the economy and the ecosystem are related. For example, as alluded to by Daly [2005] the utility and disutility curves may be derived from a simple labor-leisure model of labor supply: finite time budgets mean that people have to balance the utility from additional income gained by working an extra hour with lost leisure.

A Stock-Fund Approach

Under the ecological economics conception of the economy, the utility flux consists of two different aspects as depicted in figure - 3.4. Both natural capital (x_{so}, x_{si}) and human-made capital (x_e) in their role as funds of service fluxes contribute to the utility flux. If we conceived of utility in terms of flows rather than stocks (as the utilitarian framework to determine optimal scale does in figure - 5.4), we will need to somewhat modify the representation in figure - 3.4. In biophysical terms, utility is simply a function of throughput, $\dot{x}_i$. Given the finitude of the ecosystem, there is an inherent tension between increasing throughput, $\dot{x}_i$ and maintaining the integrity of ecosystem funds. In terms of figure - 3.4, $\breve{Y}_e$ increases with increasing throughput

$(\partial\breve{Y}_e/\partial\dot{x}_i \geq 0)$ while the service flux derived from the ecosystem decreases (on the source-side, $\partial\breve{Y}_{so}/\partial\dot{x}_i \leq 0$). For example a paper mill in a small town whose primary economic activity is logging for paper pulp can increase its economic welfare (as measured by the proxy of increasing utility) by increasing the throughput (rate at which trees are felled for converting to pulp) but increasing throughput also contributes to some discomfort (or 'disutility') from loss of ecosystem services like micro-climate stabilization or even just reduced aesthetic value of the forest. One of the extensions of the basic complementary relationship between natural capital and human-made capital is the complementary contribution of monetary income and ecosystem services to an utility function.[12] Following the notations from equation - 3.4, a simple form of the aggregate utility function, W could then be specified as:

$$W = U(I, \breve{\mathbf{Y}}) \tag{5.5}$$

where I is the average per-capita income and $\breve{\mathbf{Y}}$ is the total service flux derived from the ecosystem. The exact functional forms of U as well the production function underlying the production of the aggregate economic product, I will determine the relationship between the 'economic limit' determined by the utilitarian framework, and optimal scale. Optimal scale is the most desirable (proportional) relationship between the physical size of the economy and the ecosystem. While chapter - 6 discusses valuation of scale measures with respect to optimal scale in full detail, our treatment here is limited to investigating how (if) the utilitarian framework can

---

[12] See Kraev [2002] for a formal treatment.

help locate optimal scale.

One of the basic tenets of ecological economics has been that there are limited possibilities for substitution between natural capital and human-made capital. The fundamental relationship between natural capital and human-made capital is one of complementarity rather than substitutability [Daly 2005, Daly and Farley 2004, Daly 1997, Kraev 2002]. Thus ecological economists have called for maintaining a certain level of natural capital as the goal for sustainability. This is in contrast to the more widely used notion of sustainability in economics that calls for maintaining the sum of natural and human-made capital intact assuming natural capital and human-made capital are substitutable. This difference between the 'strong sustainability' approach of ecological economics and the 'weak sustainability'of mainstream economics has a bearing on the question of the relationship between the optimal 'economic limit' determined using an utilitarian calculus and physical sustainability. The complementarity relationship between natural capital and human-made capital has its origins in the stock-fund duality. In many instances natural capital and human-made capital are substitutable in the stock-flow space but not in the fund-flux space. Our canonical forestry example is once again illustrative. While timber can be substituted, the services derived fromm the fund (of which timber is only one of the constituent stock) are not easily substituted. The fund-flux space has remained beyond the pale of what constitutes the core of modern economic analysis. Economics treats most fund related functions under the rubric of "public good" (hence no competitive market is possible by definition) or as one of the many instances of markets failing (the well-known externality problem).

The fundamental problem in operationalizing optimal scale is one of reconciling stock and fund aspects of ecosystem. Even within an utilitarian framework, the representation in equation - 5.5 suggests that maximizing W involves aggregating over the fund-flux space. Given that markets cannot be relied upon to aggregate preferences in the fund-flux space, operationalizing optimal scale is contingent on being able to device other ways of aggregating preferences. A possible solution in a democratic society is through the ballot process at local, state, or national levels depending on the aggregation involved. However, a democratic aggregation will not necessarily result in an optimal scale that is less than or equal to maximum sustainable scale. Any scale greater than maximum sustainable scale is not physically sustainable. It is beyond the scope of the present study to speculate on the possibility of a democratic processes resulting in an optimal scale that is greater than maximum sustainable scale.[13] Our purpose here was to merely scope out the irreducible preference aggregation problem in the fund-flux space that is at the heart of operationalizing optimal scale.

---

[13] Another increasingly important area where a balloting process will not work is where a fund transcends national borders. The global buildup of atmospheric carbon is a marquee illustration of the difficulties that trans-boundary problems present

Chapter 6

Axioms for Consistent Ordinal Ranking

6.1 Introduction

In this chapter, we develop an axiomatic framework to define “consistent” scale measures, and develop an axiomatic framework for ordinal ranking of woo or more measured values of scale. Interpretation of a given scale-measure involves ranking different empirical values of the scale-measure under study. These measured values are from different spatial and temporal coordinates that are physically comparable. For example, if we had a time series of a scale-measure that describes the relationship between a paper industry and the forest that feeds the industry, we ask the question: what are the necessary and sufficient conditions that will enable a consistent *objective* ranking of different points in the time series? The key operative word here is “objective” – we are interested in axiomatic properties scale measures that are independent of the person ranking. Scale could never be used as a tool for environmental policy if the ranking of scale measures was a function of the subjective individual preference of the person carrying out the ranking. While two people can disagree about the about how they feel about the paper mill using timber from the forest, they will need to be able to objectively rank the scale of the paper mill (economy) in relation to the forest’s ability to support the mill (ecosystem) – at any rate the exposition here assumes that it is indeed possible to come up with an objective ranking scheme.

We do not review familiar problems with aggregating individual preferences but assume that this preference aggregation has been achieved. We start with the assumption that there is some desirable proportional relationship between the physical size of the economy and the ecosystem supporting the economy. This harmonious proportion, that defines the *optimal scale* of the economy will form the basis for our analysis here. In particular, we are interested in a consistent ranking criteria when the empirically determined value of a scale-measure is different from the relevant optimal scale. We begin this chapter with the assumption that for a given scale-measure the relevant optimal scale is invariant. This is a rather restrictive assumption that will be relaxed later in the chapter. We build a framework that will enable consistent ordinal ranking of both flow and stock measures of scale.

Before we start building the formal framework it is useful to briefly recollect recollect how ordinal ranking of scale measures is a necessary part of being able to use scale measures for environmental policy. The entire scale methodology for environmental policy may broadly be conceived as a three-stage process. In the first stage, we empirically determine scale using an appropriate scale-measure (the subject matter of chapters 3 and 4). Empirical measurement of scale answers the question: how large is the economy relative to the ecosystem? Second, we determine the relevant benchmark measures (chapter - 5). Benchmark scales answer the questions: how large can the economy be relative to the ecosystem, and how large should the economy be relative to the ecosystem. For scale measures to be useful for practical policy, we need to able to meaningfully compare two or more measured values of scale and in particular compare the measured values of scale to benchmark measures. Formally, for scale measures to be useful for practical policy we need to develop a framework for objective ordinal ranking of measured values of scale.

The next section will develop axiomatic properties of consistent scale-measures. We are interested in the most general properties of a scale-measure that hold good irrespective of whether the measurement is being made on the flow or stock dimensions. The axiomatic properties developed here are also independent of the nature of the interaction between the economy and the ecosystem – the consistency criteria will hold for the ecosystem in its role as the source of raw materials for the economy as much as in its role as the sink for waste products of the economy. While we speculate on specific issues relating to ranking scale measures that differ across different levels of economic and geographic aggregations, our focus here is on developing a set of criteria that hold at all levels of aggregations.

6.2 Axiomatic Properties of a Consistent Scale Measure

Preliminary Definitions

Let Ω be any scale measure. Ω can either be a stock or flow measure of scale. Further let us assume we have n different data points on Ω given by $\Omega_1, \Omega_2, \ldots, \Omega_i, \Omega_j, \ldots, \Omega_n$ and that the n different points are physically comparable[1].

---

[1] For a detailed discussion on the physical possibility of comparing two different observations of a scale measure see section - 3.3.

Let ω_i be the *objective* valuation of Ω_i. Corresponding to each of the n points in the Ω set there is a ω.[2]

$$\omega_i = \omega(\Omega_i) \tag{6.1}$$

ω is a continuous function defined for all values that Ω can take. ω assigns an ordinal index to each of the measured values of the scale-measure, Ω.

Objective-Preference Relationship on Ω

The primary purpose of the present exercise is to rank the n different observations of given scale-measure, Ω. This ranking is developed symbolically in terms of the objective-preference relationships, $\succ$ and $\sim$. It is useful to reiterate again that $\succ$ and $\sim$ represent *objective* preferences about different values that the scale-measure can take. This preference relationship is independent of the person ranking the scale-measure. $\succ$ or $\sim$ represent a binary relation between pairs of scale-measures, Ω_i, $\Omega_j \in \Omega$. $\Omega_i \succ \Omega_j$ is read as "scale-measure Ω_i is preferred to scale measure Ω_j" and $\Omega_i \sim \Omega_j$ is interpreted as "indifference between scale measures Ω_i and Ω_j." The composite preference relationship, $\succsim$ is not defined on Ω. It makes little physical sense to say "scale-measure Ω_i is at least as good as the scale-measure Ω_j – implied by $\Omega_i \succsim \Omega_j$.

---

[2] In this chapter, we will use the term "valuation" and "ranking" interchangeably.

The objective of the present exercise is to clearly spell out the ranking procedure for the n elements of Ω. We are interested in the properties of the ranking function, ω. This will form the basis for developing the notion of a "consistent scale measure." Consistency is related to how Ω maps on to ω. However before we begin, the following three points made earlier deserve to be repeated:

1. While the scale measures themselves are cardinal quantities, their valuations are only ordinal. Thus the elements of the valuation set, ω are ordinal variables while the elements of, Ω are cardinal variables. It does not make physical sense to talk about $\omega_i - \omega_j$ while it is perfectly legitimate to refer to $\Omega_i - \Omega_j$. In the latter case the subtraction simply gives the difference between two empirical observations of the scale-measure, Ω. However it is valid to talk about $\omega_i \lesseqgtr \omega_j$ and indeed this will form the basis of much of the discussion here.

2. It is important to remember that ω represents an *objective* valuation of the corresponding Ω. Some of the axioms for ω that we develop here will seem familiar from the preference theory that is foundational to standard microeconomics. Our goal here is to derive general properties of consistent mapping from the cardinal Ω to ordinal ω. We are interested in consistency and not abstract notions of 'rational preference' as in microeconomics. We are looking at objectively discernable valuations as opposed to any particular individual's subjective valuation. The object of mapping from Ω to ω is to inform public policy through empirical measurement of scale. Policy relevant decisions cannot be linked to any subjective valuation of Ω.

3. Our analysis of consistent scale measures implicitly invokes the *ceteris paribus* assumption. Our analysis here is based on ranking states of the world on a single dimension – a particular scale-measure, Ω. We assume everything else is held constant when we rank our scale-measure.

Completeness Assumptions

Before we develop consistency requirements for the $\Omega \rightarrow \omega$ mapping, we need to formally characterize the mapping. In particular, we need to make familiar assumptions about completeness of ranking.

Completeness of the Mapping

We assume that for every distinct element in the set Ω there exists a unique element in set ω. In other words, equation - 6.1 above is defined for all values of i.

Completeness of Ranking

We assume that it is possible to completely rank all the points in set ω. This ranking, as was discussed earlier, is only ordinal. Formally, we can uniquely determine $\omega_i \lesseqgtr \omega_j$ for every pair of i and j in ω.

Objective-Preference Map is Completely Defined

The preference map is completely defined if for every pair of points in ω there is a corresponding objective preference ordering on Ω.

$$\Omega_i \succ \Omega_j \quad \forall \quad \omega_i > \omega_j \tag{6.2}$$

$$\Omega_i \sim \Omega_j \quad \forall \quad \omega_i = \omega_j \tag{6.3}$$

$\Omega_i \succ \Omega_j$ has the usual interpretation of Ω_i is preferred to Ω_j and $\Omega_i \sim \Omega_j$ denotes indifference between Ω_i and Ω_j.

Objective-Preference Ranking is Transitive

This is similar to the transitive preference condition from preference theory in microeconomics and it is a formal requirement for the validity of the consistency conditions that we develop here.

$$\Omega_p \succ \Omega_r \quad \forall \quad \omega_p > \omega_q \text{ and } \omega_q > \omega_r \tag{6.4}$$

Existence of Unique Optimal Scale

We assume that there is exists a point in Ω, Ω^* which is the optimal scale. Optimal scale is a normative concept (informed by physical realities) whose exact value is determined by social, political, ethical, and perhaps even moral considerations. We have devoted an entire section (5.4) to the study of optimal scale, but for now it is sufficient to note that ω^* the ordinal ranking corresponding to optimal scale has the highest valuation among all the elements in ω.

$$\left.\begin{array}{rcl}\Omega^{*} & \succ & \Omega_i \\ \omega^{*} & > & \omega_i\end{array}\right\} \forall\ \Omega_i \neq \Omega^{*} \tag{6.5}$$

For our discussion here, we make a restrictive assumption that optimal scale, Ω^* is invariant. In other words when we compare two empirically measured values of Ω, Ω_i and Ω_j, we assume that the value of optimal scale has not changed as we move from i to j. We will relax this assumption after we have presented a formal discussion of optimal scale in section - 5.4. In our rigorous discussion of optimal scale we saw how optimal scale could vary with time for realistic non-steady state systems. For our purposes here, it is sufficient to recollect that optimal scale answers the question of: how large *should* the economy be relative to the ecosystem that contains and sustains it? Thus optimal scales can change even in a steady state system with no apparent change in the physical system if the normative definition of what constitutes optimality changes. For example, non-anthropocentric concerns like ecological space for other species can make an economy that was optimal in terms of it physical size exceed the optimal scale even when the actual physical size of the economy has not changed. The difficulties in analysis arising from relaxing the invariant optimal scale assumption will use the material presented here as the point of departure and much of our general discussion will structurally follow the arguments presented here for the restricted case.

6.2.1 Consistency

The three completeness requirements, the transitive property, together with the existence of a unique optimal-scale constitute the formal requirements for any scale-measure and its corresponding valuation. These formal criteria represent the necessary but not sufficient conditions for a scale-measure being "consistent." We are now ready to look at what makes a scale measure consistent. All the formal requirements above codify the ordinal relationship on Ω. Elements of Ω are of course cardinal quantities and we need a set of consistency axioms to tie up our discussion about ordinal measures with cardinal measures of elements in Ω. We will develop our consistency conditions by taking two arbitrary elements in Ω, Ω_i and Ω_j. We will specify how the ordinal relationship between the two elements is related to the cardinal relationship.

Mapping from Ω to ω is Consistent

For any scale measure to be consistent, two similar data points have the same valuation:

$$\omega_i = \omega_j \quad \forall \quad \Omega_i = \Omega_j \tag{6.6}$$

Equation - 6.6 represents the consistency condition only when Ω^* is invariant. When the optimal scale Ω^* is different at i and j, Ω_i and Ω_j are not directly comparable.

Mapping of Departures from the Optimum is Consistent

In equation - 6.6 above, we were able to only specify the consistency condition for $\Omega_i = \Omega_j$, but not when $\Omega_i \gtrless \Omega_j$. The latter requires us to consider three different ways in which the two arbitrarily chosen points in Ω, Ω_i and Ω_j are different from the optimal scale, Ω^*. In each of the three cases, consistency requires that we rank a scale that is closer to optimal scale higher than the scale that is farther away from optimal scale. The three cases are presented below:

1. $\mathbf{\Omega_i < \Omega_j < \Omega^*}$

   In this case, both the measured values of scale are less than optimal scale. For a fixed ecosystem size it means that the economy at both points i and j is smaller than the optimal size. For a scale measure to be consistent, we want to rank the larger among i and j higher because that point will be closer to the optimum.

   $$\left.\begin{aligned} \Omega_j &\succ \Omega_i \\ \omega_j &> \omega_i \end{aligned}\right\} \forall\ \Omega_i < \Omega_j < \Omega^* \tag{6.7}$$

   For example, let's say the optimal scale has the value 0.8 or $\Omega^* = 0.8$ and let's say we have two measured values of scale of $\Omega_i = 0.6$ and $\Omega_j = 0.7$. Consistency requires that $\Omega_j \succ \Omega_i$ because $\Omega_j = 0.7$ is closer to optimal scale than $\Omega_i = 0.6$.

2. $\mathbf{\Omega_i > \Omega_j > \Omega^*}$

Here, at both points i and j, the measured scale is larger than the optimal scale of the economy. For consistency sake, we want to rank the smaller among i and j higher because that point will be closer to the optimum.

$$\left.\begin{array}{rcl}\Omega_j & \succ & \Omega_i \\ \omega_j & > & \omega_i\end{array}\right\}\forall\ \Omega_i > \Omega_j > \Omega^* \tag{6.8}$$

Consider the example from the previous case where the optimal scale is still $\Omega^* = 0.8$. Now the two measured values of scale are greater than 0.8. Let's say $\Omega_i = 0.9$ and $\Omega_j = 1.2$. Consistency requires that we rank 0.9 ahead of 1.2 or $\Omega_j \succ \Omega_i$.

3. $\mathbf{\Omega_i > \Omega^* > \Omega_j}$

   Here we have the points i and j on either side of the optimum. There is no straightforward way to rank the two values of the scale-measure. We make an assumption that exceeding optimal scale is more problematic than being below optimal scale. This is also consistent with scale-measures as a tool for environmental policy. Thus consistency requires that we 'penalize' exceeding optimal scale more than we do being below optimal scale. We define consistency conditions below for equal departures on either side of optimal scale.[3]

$$\left.\begin{array}{rcl}\Omega_j & \succ & \Omega_i \\ \omega_j & > & \omega_i\end{array}\right\}\forall\ \Omega_i > \Omega^* > \Omega_j,\ \Omega_i - \Omega^* = \Omega^* - \Omega_j \tag{6.9}$$

---

[3] For a more general discussion with unequal departures from optimal scale, see section on scale-efficiency (page - 249).

> Continuing with our example of optimal scale, $\Omega^* = 0.8$, We now have the two measured values of scale are on either side of optimal scale. Let's say $\Omega_i = 0.9$ and $\Omega_j = 0.7$. Consistency requires that we rank 0.7 ahead of 0.9 or $\Omega_j \succ \Omega_i$.

Figure - 6.1 summarizes our discussion on consistency here. In the figure, the vertical axis is the valuation, ω; and the horizontal axis is the measured value of the scale-measure, Ω. The measured value of scale, Ω cannot take on negative values and the horizontal axis on the left of the origin stops at 0. Negative values for a scale-measure have not physical meaning – scale-measure describes the proportional relationship between the physical size of the economy and the ecosystem. This proportion cannot take on a negative value. The figure is also drawn such that ω takes on only positive values, but negative values for ω are not ruled out by any of our criteria above. We can after all only comment on the ordinal relationship between the different valuations of the scale-measure. However, in this dissertation we will assume that ω takes on only positive values to help simplify our exposition without any loss of generality. In the figure the origin is at optimal-scale, ω. The Ω - ω space is split into two regions. To the right of the origin, we have measured values of scale that exceed optimal-scale and on the left the measured value of the scale is less than optimal-scale. The essential feature of of fig - 6.1 is that the comparison of two measured values of scale is always with respect to an origin defined by the optimal scale. It makes no physical sense to talk about $\Omega_i \gtrless \Omega_j$ without reference to the optimal scale, Ω^*. Given two measured values of a scale-measure, it is not possible assign a consistent ranking, ω in the most general case of $\Omega_i \neq \Omega_j$. On the right

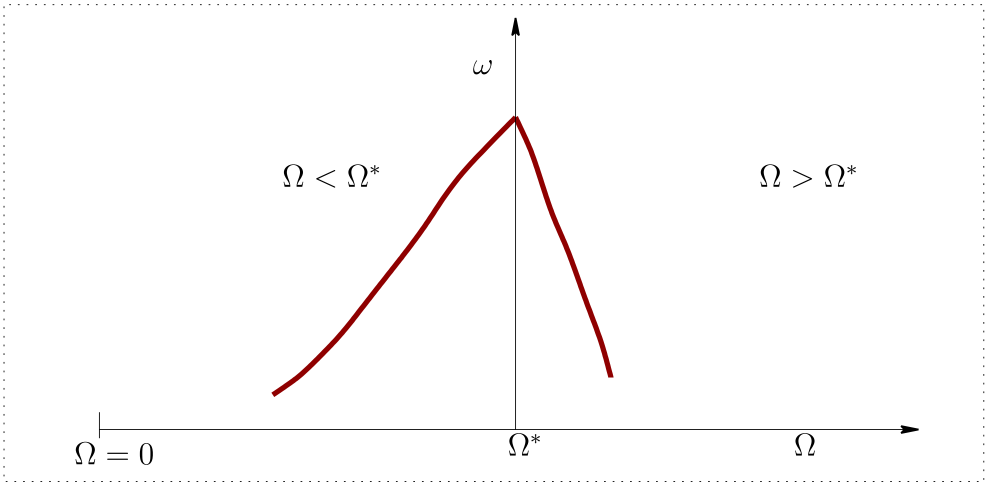

Figure 6.1: A Schematic of a Consistent Scale-Measure, Ω and its Valuation, ω

side of the origin, we see that ω decreases as Ω increases. However, in the left-hand quadrant where scale is less than optimal, we see that ω increases as Ω increases. ω, the valuation decreases more rapidly for departures from optimal scale that exceed optimal scale. This is seen from the fact that the schedule for ω has a steeper slope in the right had quadrant relative to the left hand quadrant. Any scale measure that has a $\Omega - \omega$ characteristic similar to the one in figure - 6.1 will be termed "consistent." For a given scale-measure, Ω if there exists no real valued function, ω that looks like the one in fig- -6.1 then we say that the scale-measure Ω is "not consistent."

6.3 Varying Optimal Scale

In our discussion of consistency between any two arbitrary points in Ω, we have assumed that the optimal scale is a constant — Ω^* did not have a subscript i or j associated with it. However in our discussion of optimal scale in section - 5.4 we have seen how optimal scale could change across time and certainly across different spatial coordinates. First, optimal scale for a flow measure of scale is a function of the ecosystem stock (and hence fund). For example what was optimal when a timber company first started logging a virgin forest is different from when that same forest has been degraded. Similarly the optimal scale (on both the stock and flow dimension) is different for a forest in the tropics and temperate latitudes. In summary, optimal scale in its physical dimension is related to the nature of the fund. Hence, if the quality of the fund is varying, our consistency criteria developed above will have to be suitably modified to to account for different optimal scales at Ω_i and Ω_j.

A change of optimal scale in moving from Ω_i to Ω_j does not amount to comparing apples and oranges but now the ranking exercise acknowledges that the two apples (or oranges), Ω_i and Ω_j are from different orchards and that fruits from different orchards have different innate qualities. In general when $\Omega_i^* \lesseqgtr \Omega_j^*$ (of which $\Omega_i^* = \Omega_j^* = \Omega^*$ is a special case) we define consistency by looking at departures of Ω_i and Ω_j from their respective optimal scales. Let δ be the departure from optimal scale for any scale measure Ω and corresponding optimal scale, Ω^*.

$$\delta_i = \Omega_i - \Omega_i^* \tag{6.10}$$

$$\delta_j = \Omega_j - \Omega_j^* \tag{6.11}$$

Now in terms of departure from optimal scale, the best possible departure is obviously no departure at all:

$$\delta^* = 0 \quad \forall \quad i, j \tag{6.12}$$

We are now ready to define consistency in terms of δ, the departure from optimal scale. Following the general logic of the special case above, we consider three different possibilities.

1. $\boldsymbol{\delta_i < \delta_j < 0}$

   Here both Ω_i and Ω_j are less than their respective optimal scales. Following the arguments of equation - 6.7, consistency requires that we rank the scale measure with the smallest departure from respective optimal scales higher than the one with the larger departure. Thus we have a condition that mirrors equation - 6.7.

$$\left.\begin{array}{ccc} \Omega_j & \succ & \Omega_i \\ \omega_j & > & \omega_i \end{array}\right\} \forall \; \delta_i < \delta_j < 0 \tag{6.13}$$

2. $\boldsymbol{\delta_i > \delta_j > 0}$

   Here both Ω_i and Ω_j are greater than their respective optimal scales. Following the arguments of equation - 6.8, we assign a higher to rank the smaller departure among i and j.

$$\left.\begin{array}{lcl}\Omega_j & \succ & \Omega_i \\ \omega_j & > & \omega_i\end{array}\right\} \forall\ \delta_i > \delta_j > 0 \tag{6.14}$$

3. $\boldsymbol{\delta_i > 0 > \delta_j}$

   Here we have Ω_i that is greater than optimal scale Ω_i^* and Ω_j that is less than its corresponding optimal scale, Ω_j^*. As we discussed in equation - 6.9 there is no straightforward way to rank the scale measures when their deviations from respective optimal scales are in opposite direction. Following our earlier arguments, we impose the condition that for a given magnitude of departure from optimal scale, a scale below the optimal is to be preferred to a scale that is above the optimal scale.

$$\left.\begin{array}{lcl}\Omega_j & \succ & \Omega_i \\ \omega_j & > & \omega_i\end{array}\right\} \forall\ \delta_i\delta_j < 0,\ \ |\delta_i| = |\delta_j| \tag{6.15}$$

Summary

In general, we will use δ instead of Ω to determine the ordinal rank, ω. We can translate the special case represented by figure - 6.1 by simply setting the origin as $\delta^* = 0$ instead of $\Omega = \Omega^*$. This is accomplished in figure - 6.2. While ω evaluated directly from the scale measure is a special case of the more general methodology that uses departure from optimal scale, δ, there is no one-to-one correspondence between the three consistency conditions derived in the general case and the conditions used when optimal scale is constant. For example in equations - 6.7 and 6.13,

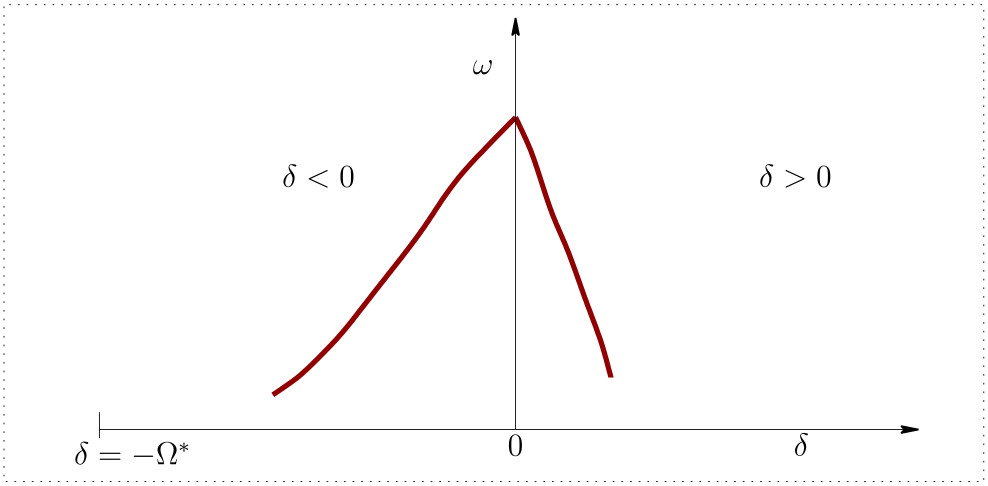

Figure 6.2: A Schematic of a Consistent Scale-Measure with Variable Optimal Scale

the condition of $\delta_i < \delta_j$ can be true even when $\Omega_i \geqslant \Omega_j$. In our discussion of consistency, we have thus far treated the scale-measure Ω in the abstract without reference to any real stocks and flows. Here we are interested in how the consistency axioms map to real stocks and flows. In terms of our analytical representation of the economy-ecology interaction in figure - 3.4, Ω can be written as a function of the three different stocks, the throughput, and the natural income from the stock-function of the ecosystem:

$$\Omega = \mathbf{\Omega}(\hat{Y}_{so}, \hat{Y}_{si}, x_{so}, x_{si}, \tilde{x}_{si}, \dot{x}_i, \dot{x}_o) \tag{6.16}$$

Equation - 6.16, encompasses stock and flow dimensions on both the source-side as well as the sink-side. In practice we will only deal with a subset of the arguments in equation - 6.16.[4] Of particular interest is the consistency properties of $\frac{1}{\Omega}$. Scale measures the proportional relationship between the economy and the ecosystem.

---

[4] In terms of the notation used in this section the bold-faced $\mathbf{\Omega}$ is the function and the Ω in regular typeface is the scale-measure.

Ω is often measured as a ratio of two physical quantities representing the physical sizes of the economy and the ecosystem. When Ω is measured as a ratio, $\frac{1}{\Omega}$ represents the proportional relationship between the economy and the ecosystem just as Ω does. For example the ratio of natural income to throughput, or $\frac{1}{\mathbf{S}}$ is a proportional relationship between the economy generated throughput and the relevant natural income. What can we say about the consistency properties of $\frac{1}{\Omega}$ given the consistency properties of Ω? We believe the framework presented here is general enough for future research to answer theoretical questions like this. Additionally, the framework presented here is also open to empirical work in the form of testing candidate scale measures for consistency.

6.4 Mapping between Stock-Flow and Fund-Flux Spaces

The cornerstone of the framework for measurement and interpretation of scale has been the relationship between the cardinal stock-flow space and the ordinal fund-flux space. In this section, we show how the axioms for consistent scale metrics developed here can be used to operationalize the mapping between the stock-flow space and the fund-flux space. The axioms for consistent scale metric formalize the relationship between the empirically measured value of scale, Ω_i and the corresponding ordinal rank, ω_i. We show here how that formalism can be adapted to achieve a mapping between the stock-flow and fund-flux spaces.

Before we turn to a formal discussion on the mapping between the stock-flow space and the fund-flux space, a word of caution is in order. The axiomatic framework presented in this chapter is to be treated as just that – a framework, and not a substitute for for the specific empirical details involved in mapping cardinal stock-flow measurements to the ordinal fund-flux space:

> It is one thing to have a census (or birth and death rates) of trees and birds. It is something else to describe how certain trees and their detritus provide habitat for certain birds, and how certain birds disseminate the seeds of trees which will not germinate unless passed through the digestive tract of the birds, etc. The information content of the fund -flux space is vastly greater than that in the stock-flow space. This does not make the mapping less necessary, but it inspires humility and a certain distrust of symbolic discussions at a high level of abstraction from the millions of specific brute facts and seemingly arbitrary interconnections that could be interrupted by importing and imposing a set of stock-flow relations into the fund-flux space.[5]

Given the practical impossibility of discerning the stock-flow mechanism behind every fund-flux process, we present a framework that relies on democratically aggregated social and political preferences to achieving an ordinal ranking on the fund-flux space. Given the uncertainties and complexities involved in a complete characterization of the fund-flux space, no technocratic central planner can possibly fix the optimal scale for a given economy. This however does not mean that we have eliminated the role for science in understanding how the complex ecosystem processes

---

[5] Herman Daly, personal communication, April 2006

work. Indeed as we discussed in chapter - 5, a democratic process for determining optimal scale can result in the optimal scale being set above maximum sustainable scale even when the society otherwise cares about biophysical sustainability. If biophysical sustainability is a goal for public policy, an alternative institutional mechanism would be for a central agency to determine maximum sustainable scale through science and let the society democratically determine optimal scale at some level below maximum sustainable scale. The various empirical attempts that we reviewed in chapter - 4 despite serious theoretical problems represent the kind of science that can support public policy that has sustainability as one of its goals.

In chapter - 5 we showed how the basic problem for environmental policy using the scale program is best understood in terms of the tradeoffs involved in determining optimal scale. Optimal scale as well as the two physical benchmarks of maximum and maximum sustainable scale are defined on the fund-flux space while scale itself is measured in the stock-flow space. Any scale metric is a function of two or more of the seven stock and flow elements in figure - 3.6.[6] Thus while scale is *measured* in the stock-flow space, policy relevant interpretation involves ranking states of the world in the fund-flux space. We will illustrate how the mapping between the stock-flow space and fund-flux space works in practice for each of the major scale metric that we have developed in this dissertation. For every metric we will show how the axioms for consistent scale metric enable the mapping between the stock-flow and

---

[6] In chapter - 4, our discussion of material flow analysis introduced three additional flows – the two hidden flows and the natural mobilization flow. However, figure - 3.6, as discussed previously can be easily modified to accommodate hidden flows and mobilization flows.

fund-flux spaces.

6.4.1 Flow Measures of Scale

We begin by looking at the flow measure of scale, $\mathbf{S}$ that compared throughput and regeneration (equation - 3.8). The optimal scale for a flow measure of scale is contingent on the nature of the underlying stock. Thus $\mathbf{S}^*(\mathbf{t})$ is a function of the quality of the source-side fund ($x_{so}(t)$) or the sink-side fund, $x_{si}(t)$. For the purposes of illustration we consider the source-side example of a logging industry that uses timber from the forest. Further let us assume that the optimal scale $\mathbf{S}^*$ does not change between two measured points, S_{so_i} and S_{so_j} (we will relax this assumption later). Given the current state of x_{so}, let us say the optimal scale is determined as $S^*_{so} = 0.7$. We are now interested in mapping the two measured values of scale, S_{so_i} and S_{so_j} onto the fund-flux space. Let us assume that $S_{so_i} = 0.75$ and $S_{so_j} = 0.8$. Mapping measured values of scale onto the fund-flux space is equivalent to an ordinal ranking of the states of the world represented by the scale metric. The consistency axioms require that the ranking of the states of the world represented by these scale measures be such that $S_{so_i} \succ S_{so_j}$. For a simple measure like $\mathbf{S}$ that is the ratio of regeneration and throughput, it is reasonable to expect that the ordinal ranking of the state of the world will monotonically decrease for departures above optimal scale.

Recall that among other things, optimal scale represents the tradeoff between the stock and fund aspect of ecosystem — $\dot{\mathbf{x}}$ (throughput), and $\check{\mathbf{Y}}$ (the fund income).

In the present example, $S^{*}_{so} = 0.7$ represents the physical size of the economy at which the society has chosen as the most desirable tradeoff between the throughput $\dot{x}_i$ and service flux, $\breve{Y}_{so}$. Given that we have assumed optimal scale to be constant between i and j, the regeneration rate, $\hat{Y}_{so}$ is also constant and the change in the value of scale is due to change in throughput — $\dot{x}_i(j) > \dot{x}_i(i)$. The regeneration is constant in our particular example because the ecosystem retains some of its resilience between maximum sustainable scale and maximum scale (when the system is in overshoot); the regeneration is not only a function of the size of the stock but also the overall health of the fund that contains the stock. If we assume that the society incorporated physical sustainability considerations into its choice of optimal scale, a move from i to j represents a loss of welfare even when the quantum of regeneration flow has not changed because a higher level of throughput will eventually result in reduced regeneration in a later time period. If welfare, U is a function of throughput, $\dot{x}_i$ and the service flux from the fund-function of the ecosystem-stock, $\breve{Y}_{so}$, in the present example we have, $U(j) < U(i)$ even when $\hat{Y}_{so}(j) = \hat{Y}_{so}(i)$ because $\breve{Y}_{so}(j) < \breve{Y}_{so}(i)$. Service flux, as we discussed in chapter - 5 can change even when the regeneration does not change.

Unlike **S**, not all the flow measures of scale that we developed satisfy the consistency axioms developed here. Scale metrics that do not satisfy the consistency axioms cannot be used as a device for making inferences about the fund-flux space because it is not possible to define a consistent mapping scheme between the stock flow space and the fund-flux space. Even when a consistent mapping is not possible, a scale metric could nevertheless be useful to characterize the proportional relationship between the economy and the ecosystem in the stock-flow space. As an illustration we consider the scale measures S_{MFA3} and S_{MFA4} that we developed using the accounting framework that Klee and Graedel [2004] use to measure anthropogenic contribution to elemental mobilization (section - 4.2.4).

Figure - 4.15 shows how natural mobilization of carbon trumps anthropogenic mobilization — $S_{MFA4} < 0.15$. However, even at this level of mobilization human perturbation of the carbon cycle has resulted in a discernable influence on the global climate. It is evident that the optimal scale for global carbon mobilization, as measured by metric S_{MFA4} is less than 9%, the current portion of the total carbon mobilization that can be attributed to anthropogenic sources.[7] The question of interest here is whether optimal scale can be defined for S_{MFA4}. A flow measure of scale like $\mathbf{S}$ has a clearly defined optimal scale for any given state of the ecosystem fund because the metric includes regeneration flows. Regeneration flow, $\hat{\mathbf{Y}}$ is not only a function of the level of the stock but is also determined by the health of the fund. Unlike $\mathbf{S}$, the metric S_{MFA4} does not have any variable that is a *direct* function of the health of the fund.[8] The apparent anomaly of a small value of S_{MFA4} and the obvious impact of this 'low' value of scale as measured by anthropogenic perturbation is simply a consequence of the fact that it is not possible to define an optimal scale for a flow metric like S_{MFA4} — the mapping between the stock-flow space

---

[7] Klee and Graedel [2004, p.81]

[8] Also see the previous discussion surrounding the model that we introduced in figure - 4.16 to study the relationship between natural mobilization flows and regeneration flows (page-134).

and the fund-flux space is contingent the existence of unique optimal scale for a given metric. The axiomatic framework that we have developed in this chapter uses optimal scale as the origin for ordinal comparison of two states of the world as represented by a scale metric. A metric like S_{MFA4} does not have a unique optimal scale because it does not contain enough 'information' about the state of the world for the society to be able to make the $\dot{\mathbf{x}} - \hat{\mathbf{Y}}$ tradeoffs involved in determining optimal scale.

Among the flow measures of scale that we reviewed in chapter - 4, S_{HANPP} based on the accounting framework that calculates the human appropriation of net primary productivity is the only one that that directly makes use of a regeneration flow. It is for this reason that we suggested that S_{HANPP} is most useful for characterizing the fund-flux space (page - 80). In figure - 4.19 we showed how land use change affects NPP, which in the current context is to be interpreted as the effect of the fund structure on the regeneration flow ($\hat{Y}_{NPP}$). Conceptually, S_{HANPP} is a particular example of a scale metric that compares throughput and regeneration, and thus a mapping between the stock-flow space and fund-flux space is possible using the axioms for consistent scale metrics developed in this chapter. For example, on a global level, the current estimates of S_{HANPP} range between 20% and 30%.[9] The mapping between the stock-flow and fund-flux spaces can be used to study the impact of increasing terrestrial NPP withdrawals by 50% if we replaced the current global consumption of fossil with energy derived from biomass sources.

---

[9] See section - 4.3 for a detailed discussion on the empirical assessments of S_{HANPP}.

6.4.2 Stock Measures of Scale

To the extent that optimal scale is more meaningful in the stock dimension than in the flow dimension,[10] the mapping between the stock-flow space and the fund-flux space is best studied in the stock dimension. In section - 5.4, we discussed theoretical problems with operationalizing optimal scale. We will focus our discussion here on the ordinal ranking of states of the world with optimal scale as the reference point. As discussed with the flow measure of scale, the ordinal rank, ω_i corresponding to the measured value of scale, Ω_i is the society's valuation of the of the state of the world at point i as represented by the scale metric, Ω.

An operational program for mapping a measured stock metric onto the fund-flux space can be conceived as a two-part process. First, we determine if it is possible to define an optimal scale based on the information contained in the scale metric. Second, if optimal scale exists, we determine if the metric satisfies the consistency axioms developed in this chapter. We illustrate this with two different stock measures of scale developed in section - 3.5.

We begin by looking at the metric $\sigma^{\mathbf{e}}$ that we developed in equation - 3.15 (page - 69). The metric $\sigma^{\mathbf{e}}$ is a simple ratio of the economy-stock and the ecosystem stock. Does $\sigma^{\mathbf{e}}$ contain sufficient information to enable determination of optimal scale? We noted in chapter - 3 that $\sigma^{\mathbf{e}}$ is a useful characterization of the proportional relationship between the economy and the ecosystem in the stock-flow space

---

[10] See section - 5.4

but not in the fund-flux space. We consider two specific examples here. First let us study σ_{so}^{e} for timber products in the economy. Here, $\sigma_{so}^{e} = \frac{x_e(t)}{x_{so}(t)}$ where $x_e(t)$ is the stock of timber in the various artifacts in the economy, and $x_{so}(t)$ is the standing stock of timber in the forest.[11] To see why an optimal scale is not defined for $\sigma^{\mathbf{e}}$, consider plausible numerical examples. Let us start by assuming that optimal scale for the timber problem under consideration can indeed be defined, and further say $\sigma_{so}^{e^*} = 0.6$. The problem with this assumption is that $\sigma_{so}^{e^*} = 0.6$ does not represent a tradeoff between $\breve{Y}_e$ and $\breve{Y}_{so}$, the economy and ecosystem components respectively of the total welfare flux. σ_{so}^{e} is a metric that is useful to characterize the proportional relationship between the economy and the ecosystem in the stock-flow space but not in the fund-flux space. The size of the economy-stock, in addition to historical levels of the source-side of the throughput (the rate at which timber was withdrawn from the forest, $\dot{x}_i$) is also a function of the waste flow, $\dot{x}_o$. While in general, the waste flow can indirectly affect the health of the ecosystem fund on the source-side, this effect is only indirect, as reflected in our basic analytical representation of the economy-ecosystem interaction in figure - 3.4.[12] Even when $\sigma^{\mathbf{e}}$ cannot be used to operationalize the mapping between the stock-flow space and the fund-flux space, it can nevertheless convey important information about how the economy and the ecosystem interact in the stock-flow space.[13]

---

[11] Specifically, $x_e(t)$ represents timber from *a* particular forest whose current standing stock is $x_{so}(t)$.

[12] The sink-side of the throughput can have a bearing on the fund on the stock-side because, often the source and the sink components of the ecosystem are subsystems of a larger fund. For example see chapter - 3 for a detailed discussion on how the source-side and sink-side can be interconnected in case of a paper mill withdrawing timber from the forest and dumping effluents in the river that flows through the forest ecosystem.

[13] See chapter - 3 for our discussion on copper extraction using $\sigma^{\mathbf{e}}$.

Now consider another stock metric that we developed in chapter - 3, σ. Unlike $\sigma^{\mathbf{e}}$, the metric σ measures scale independently on the source-side and the sink-side. Continuing with the 'timber from the forest' example, we can write $\sigma_{so} = \frac{x_{so}(t)}{x_{so}(t^*)}$ where $x_{so}(t)$ is the current standing stock of timber in the forest and $x_{so}(t^*)$ is the stock of timber at some historical reference time-period, t^*. The simplest way to achieve a mapping between stock-flow and fund-flux spaces using σ is to chose the reference time-period, t^* such that the stock in that time period represents optimal stock. Optimal stock is simply that stock that represents the optimal tradeoff between timber that can harvested and the service flux lost due to timber withdrawal (figures 5.2 and 5.3). Now $\sigma_{so} - 1 = \left(\frac{x_{so}(t) - x_{so}(t^*)}{x_{so}(t^*)}\right)$ represents the percentage deviation of the current stock from the optimal stock. Given that we can identify some unique optimal scale as measured by the metric σ, we only need to verify that σ satisfies the consistency axioms developed here to conclude that this metric can be used to map empirical measurements in the stock-flow space onto the fund-flux space. We illustrate this with a numerical example. For the forest under study, let us assume that the optimal scale is defined at $\sigma^*_{so} = 0.3$. Now consider two points i and j where the measured value of $\sigma_{so} \neq \sigma^*_{so}$. We will first consider the two cases where i and j are both simultaneously less than or greater than optimal scale. Consider $\sigma_{so}(i) = 0.5$ and $\sigma_{so}(j) = 0.4$. The optimal scale is 'exceeded' in both cases but the point j is closer to optimal scale than point i — $\sigma_{so}(i) > \sigma_{so}(j) > \sigma^*_{so}$. The society will indeed rank $\sigma_{so}(j) \succ \sigma_{so}(i)$, and $\omega_j > \omega_i$, as required by the consistency axioms. There is a welfare loss at both i and j as they represent a movement away from the stock-fund tradeoff that is implicit in the optimal scale calculus. It is straightforward to see why the metric σ passes the consistency test. Without loss of generality, let us normalize the standing stock in the forest so that the reference stock, $x_{so}(t^*)$ is set to unity — $x_{so}(t^*) = 1$. Now the numerical value of the scale metric simply represents the extent of the current stock. The society has determined that maintaining the stock at 30% of the reference stock level represents the optimal tradeoff between welfare from timber and welfare from other ecosystem services offered by standing timber as part of the fund. At both points i and j the standing stock is greater than 30%. Thus while in both cases, the society derives increased welfare from the greater service flux derived from a larger standing stock, this welfare gain does not offset the welfare loss associated with withdrawing lesser quantity of timber. The society will prefer to be at point j rather than point i because the net welfare loss is smaller at j. We can similarly show how σ satisfies the consistency axiom when $\sigma_{so}(i) < \sigma_{so}(j) < \sigma^*_{so}$. There, the society is not able to offset the welfare lost in terms of service flux by chopping down more timber than that is optimal.

We note here, an important difference between the flow measure of scale, **S** and the stock metric σ. In the case of the flow metric, a scale value greater than optimal scale represents a net welfare loss associated with increased throughput but reduced ecosystem services. The converse is true for the stock metric – when the measured value of scale is less than the optimal scale, there is a net welfare loss associated with increased throughput but reduced ecosystem services. This apparent anomaly follows from the fact that the flow measure of scale is built around throughput and the stock measure is constructed around standing stock — increasing throughput decreases standing stock. As suggested in the previous section, we could use $\frac{1}{\sigma}$ instead so that both stock and flow measures of scale 'move' in the same direction. Following the logic used for σ we can show that $\frac{1}{\sigma}$ is also a consistent scale metric that allows for a mapping between the stock-flow space and the fund-flux space.

Chapter 7

Conclusion

7.1 Introduction and Dissertation Summary

In this concluding chapter we summarize the key contributions of the research presented in this dissertation, and also lay out a road map for future research. As the plagiarized title of this dissertation suggests, the central theme of this research has been to establish an analytical framework for empirical measurement of scale. If the economy is conceived as open subsystem of the larger ecosystem, the problem of “being the right size” assumes theoretical and practical importance.

The primary contribution of this dissertation has been to clarify what *size* means in the context of economy-ecosystem interactions. The framework to empirically measure scale and to interpret scale measures for practical policy was developed in four stages. First, in chapter - 3 we developed a framework to empirically measure scale. We dealt with some of the theoretical difficulties in answering a seemingly straightforward positive question: how large is the economy relative to ecosystem? The central feature of the framework was a simple analytical representation of how the economy and ecosystem interact (figure - 3.4). we developed a clear analytical distinction between the stock-flow space the fund-flux space. Chapter - 3 also developed a taxonomy for scale measures (figure - 3.2) and showed how scale is to be measured in the flow and stock dimensions. Last but not the least, the framework that we developed in chapter - 3 demonstrated the need for independent measurement of scale on the source-side and the sink-side.

Chapter - 4 used the framework developed in chapter - 3 to critically examine four well-known biophysical assessments of human impact on the ecosystem. The primary purpose of the systematic review was to validate our framework for measuring scale at different levels of economic-geographic aggregation. Our review of the TMR-TDO methodology highlighted the theoretical pitfalls with aggregating throughput. The framework using mobilization and sequestration flows provides an important extension to measuring scale in the flow dimension in the absence of regeneration flows. In addition to engineering approaches in the from of material flow analysis, we also reviewed two contrasting and widely used methods to study human society as a significant heterotroph species. Estimates of Human Appropriation of Net Primary Productivity (HANPP) and the ecological footprint represent two different ways of measuring how societies access our ultimate energy source – solar energy. We once again identified aggregation problems with both these methodologies.

After having established a framework to empirically measure scale, chapter - 5 developed a framework for benchmark measures. For the scale methodology to be useful for policy, in addition to the *what is* question we need to answer the *what can be* and *what should be* questions. Maximum sustainable scale and maximum scale answer the “what can be” question, and optimal scale answers the normative question. We developed a clear distinction between maximum scale and maximum sustainable scale. The primary insight from the framework for benchmark measures is that any reasonable framework for benchmark scales is contingent on a consistent mapping between the fund-flux space and the stock-flow space. In our discussion on optimal scale we illustrated the possible incongruence between optimal scale determined through the market or the political process, and biophysical sustainability.

Chapter - 6 developed a framework for ordinal ranking of scale measures. In addition to being able to answer the *what is*, *what can be*, and the *what should be* questions,the use of scale metrics for practical policy requires a consistent ranking of the states of the world represented by two or more measured values of scale. In particular it is important to be able to compare the measured value of scale to optimal scale, or the normative benchmark. The problem is homologous to development of consistent inequality measures – in particular, being able to consistently interpret changes in income or wealth distribution by looking at two measured values of a given inequality index. We developed an axiomatic framework that described the properties of a consistent scale measure. These properties provide a blueprint for designing disaggregated scale measures that are policy relevant.

The disconnect between the ontological conception of ecological economics and the extant epistemic toolbox is perhaps nowhere more apparent than in lack of a well developed framework to fully characterize allocation and distribution in biophysical terms. Not only is a biophysical characterization of allocation and distribution central to understanding the physical scale of the economy, this approach also provides a framework to link the analyses in biophysical and monetary dimensions.

Scale is only one of three fundamental aspects of the economic process studied by ecological economics [Daly 1992]. In appendix - A we presented a simple dynamic model to illustrate the linkages between scale, allocation, and distribution. The model helps distinguish between positive questions surrounding scale, allocation, and distribution from normative questions of optimal scale, efficient allocation, and just distribution. While the model itself was intended to be a heuristic rather than a predictive model, it developed the necessary formalism for application to actual empirical problems.

This dissertation is intimately tied to the preanalytic vision of ecological economics of the economy being an open subsystem of the larger ecosystem. The preanalytic vision was itself not interrogated except for a brief section in the introductory chapter. Scale as a relevant tool for characterizing the relationship between the economy and the ecosystem is intimately tied to the preanalytic vision of ecological economics – proportional relationship between the economy and the ecosystem is relevant only if the economy is conceived as a subsystem of the larger ecosystem. In a brief appendix to the dissertation we describe how different preanalytic visions for economics can be understood in terms of more fundamental beliefs about ends and means.

The remainder of this chapter is organized as follows. In the next section we highlight some of the key theoretical contributions of this dissertation. Section - 7.3 points out practical environmental policy implications that directly follow from the theoretical framework developed here. In the concluding section, we discuss the most important lacunae and suggest a road map for future theoretical and policy research based on the findings of this dissertation.

7.2 Theoretical Contributions

This is a theoretical dissertation whose primary contribution is the development of a framework for analytical study of scale. While we have summarized the development of this framework in the summary above, some central features of the framework, and their place in theoretical ecological economics are highlighted here. We begin by contextualizing this dissertation's contribution within the extant ecological economics theory.

7.2.1 The 'Science' of Ecological Economics

Two dominant research strategies help map the broad contours of extant ecological economics literature. On one hand, ecological economics research in the last two decades has gathered myriad empirical facts about how the economy and the ecosystem interact. On the other hand, research has tried to clarify fundamental preanalytic visions surrounding the relationship between the economy and the ecosystem. With a few notable exceptions, there has been little effort to systematically connect these two streams of research.[1] There are several reasons that help

---

[1] Some of these exceptions were discussed in chapters 1 and 4.

explain why the two principal research strategies adopted by ecological economics have remained disparate. In very broad terms, the epistemological apparatus of ecological economics has not kept pace with the evolution in ecological economics' ontological understanding of how the economy and the ecosystem interact.[2] The theoretical research dealing with foundations and basic organizing principles of ecological economics has focussed on fleshing out elements of the preanalytic vision as an *a priori* conception of reality. There has been a relative paucity of research centered on ways of knowing and understanding that reality. Even when methodological innovation has been the focus of research, the impact of such research has largely been on the ontological dimension rather than on the epistemic dimension. In particular, the study of material and energy flows between the economy and the ecosystem, one of the highlights of theoretical research in ecological economics, has contributed significantly to the understanding of the basic preanalytic vision of ecological economics – the human economy conceived as an open subsystem of the larger ecosystem with continuous exchange of matter and energy between the economy and the ecosystem supporting it.[3] However, we find that much of the empirical work within ecological economics has not taken cognizance of this foundational insight.

Ecological economics' failure to develop epistemic tools consistent with its on-

---

[2] We use "ontology" in a very limited sense of economy-ecosystem interaction. This somewhat metaphorical usage of the term 'ontology' is not be confused with its more traditional understanding in philosophy.

[3] For a sampling of this theoretical research see the papers contained in Part -III of Costanza et al. [1997b].

tology, has been the single most important barrier to developing a consistent analytic framework to organize the surfeit of empirical data that describes how the human economy intersects with the biophysical environment. Unfortunately, even as the literature has done much to clarify the ontological position of ecological economics, its vision of how the economy and the ecosystem interact does not carry wide currency among academic and policy orthodoxies. While the normative and political implications of ecological economics' basic insight are the primary reasons, the lack of a consistent analytical framework to organize empirical data is a significant reason for ecological economics' ontological conception not being widely accepted. Thus we have a vicious spiral where the disconnect between the epistemic tools and the ontological conception leads to a blurring of our ontological understanding and contributes to furthering the original incongruency. A good example is the burgeoning of studies that purport to use the ontological vision of ecological economics but are limited by their use of tools from neoclassical environmental economics, including subsuming the scale question under the rubric of "externality." A key contribution of this dissertation is to demonstrate that scale is fundamentally a biophysical concept that is not amenable to the epistemic apparatus of standard environmental economics.

7.2.2 Key Results

Representation of Stock-Flow and Fund-Flux Spacess

The common analytical thread that runs throughout this dissertation is the simultaneous representation of the ecosystem as a stock and a fund. The concept of funds, first introduced by Georgescu-Roegen [1971] was adapted in this dissertation to develop a clear analytical representation of how the economy and the ecosystem interact. This analytical representation (figure - 3.4) represents the economy in both the stock-flow space as well as the fund-flux space.

Analytical Representation of Natural Capital and Natural Income

An important part of the framework for measuring scale is the representation of natural capital and natural income in both the stock-flow and fund-flux spaces. The framework that we developed clearly distinguishes between flow and flux components of natural income ($\hat{\mathbf{Y}}$ and $\check{\mathbf{Y}}$ respectively). The simplified analytical representation of natural income that we developed, helps clear the confusion in the literature between flow and flux components of natural income. The clear distinction between the source-side and sink-side will likely prove important in understanding the metabolic flow that sustains the economy. Even when the source-side of the throughput is physically sustainable, the sink-side may not be sustainable.

Dimensionless Metrics

Scale measures are proportional measures and we showed why dimensionless metrics are especially good candidates as consistent scale measures. In addition to dimensionless metrics in the stock-flow space, we developed and defined dimensionless measures in fund-flux space.

Mapping between Stock-Flow and Fund-Flux Spaces

The central feature of scale methodology developed here is a framework for consistent mapping between the stock-flow space and the fund-flux space. The mapping between the cardinal quantities in the stock-flow space (including scale measures) and ordinal variables in fund-flux space will likely find applications beyond development of benchmark scale measures.

Ordinal Ranking around Optimal Scale

The axiomatic properties of a consistent scale measure that we developed in chapter - 6 have applications beyond ordinal ranking for scale measures. Similar properties could conceivably be adapted to metrics measuring allocation and distribution in the biophysical domain (the distribution and allocation efficiency discussed in appendix - A for example). The simple framework allows for a consistent comparison of any metric that can take values on either side of a benchmark measure.

Material Flow Analysis

Among the various biophysical assessments that we reviewed in chapter - 4, the most significant was the examination of methodologies that used material flow analysis. Our analysis systematically uncovered the problems with aggregation in the fund-flux space. In particular, we established the dimensional inconsistencies in the aggregating throughput from different sectors of the economy.

A Syntax for Analytical Study of Scale

The central theoretical contribution of this dissertation has been to clearly spell out a framework for analytical study of scale. We believe that even where incomplete (ordinal ranking or benchmark scale measures for example), the framework presented in this dissertation contains the core 'syntax' for future biophysical studies of how the economy and the ecosystem interact.

7.2.3 Object Lessons

Having summarized the specific theoretical contributions made by this dissertation, we briefly highlight two most important object lessons for ecological economics theory – beyond a framework for analytical study of scale. These observations directly follow from the stock-fund representation of economy-ecosystem interaction.

Disaggregated Biophysical Assessments

The primary lesson from our review of biophysical assessments in chapter - 4 was that even when the aggregation is meaningful in the stock-flow space, it is fraught with difficulties in the fund-flux space. If the ultimate objective of biophysical assessment is to inform practical policy on the connections between economic drivers and *specific* ecosystem stocks, aggregate measures alone will not suffice. As we discussed in section - 4.2.2 biophysical aggregation is entirely different from monetary aggregation. While it is tempting to develop biophysical accounts that mirror monetary accounts at the levels of aggregation used in traditional system of national accounts, such aggregations are likely devoid of any biophysical meaning.

The Irreducible Normative Dimension

While the bulk of this dissertation has focussed on the positive *what is* question, one of the subtle but important lesson from our research is that the seemingly straightforward positive question of “how large is the economy relative to the ecosystem that contains it” has irreducible normative components to it. First, the choice of system boundary in figure - 3.4 is more than just a technical issue. It is motivated in part by the ultimate purpose of analysis. For example, if physical sustainability of the ecosystem is the question of interest, the system boundary is determined in part by normative considerations of what is to be sustained, and for how long. Another place where we saw normative considerations play a central role was in the treatment of the temporal dimension. The choice of reference time-period, t^* as we saw in section - 3.5 was at at least in part a normative choice.

7.3 Lessons for Policy

The overarching lesson for policy is that important aspects of economy-ecosystem interaction have an irreducible biophysical dimension. Scale is a biophysical concept directly related to the conception of how the economy and the ecosystem interact. By systematically studying various aspects of the biophysical scale of the economy we were able to show why the extant treatment of the ecosystem in economic sciences is inadequate. Scale, we showed is intimately tied to the ecological economics' conception of the economy. To the extent that the primary contribution of this dissertation is an analytical framework for study of scale, the policy prescriptions that follow have much in common with the policy consequences of ecological economics. We present here some very specific lessons for practical policy that follow directly from the framework that we developed rather than repeat general prescriptions that arise from biophysical constraints imposed on an economy that is contained within a larger ecosystem.

Regeneration Flow, $\hat{\mathbf{Y}} = 0$

In our discussion about flow measure of scale, we noted that the scale measure $\mathbf{S}$ is indeterminate in the absence of regeneration flows. $\mathbf{S}$ is indeterminate on the source-side for non-renewable resources; and on the sink-side, for waste flows with no sink capacity. We have discussed elsewhere in this dissertation the implications of our framework for economics and policy of non-renewable resource extraction. We will focus here on the sink-side. Some of the most pressing of contemporary ecological problems are related to sink-capacity rather than resource scarcity including the green house gas induced climate change at the global level and toxic waste disposal at more local levels. The modern industrial economy is predicated on the use of a variety of synthetic chemicals with no known natural sources or sinks. The stock-fund abstraction of the sink in terms of 'holes' that we presented in section - 3.2.4 suggests that waste flows with no sink capacity need particular policy attention. Absence of sink-capacity means that policy has to deal exclusively with the fund-flux space rather than a combination of stock-flow and fund-flux spaces. Stock-flow space is 'countable' and thus predictable whereas the fund-flux space is not. The determination of benchmark scale measures on the sink-side is especially difficult when there is no sink capacity as the framework suggested in figure - 5.1 is no longer operative.

There can be no better illustration of the need for, and the usefulness of this "precautionary principle" in the absence of any known sinks than how the scientific and policy communities dealt with stratospheric ozone depleting chemicals. Accepting the Nobel Prize in 1995 for his role in the discovery of ozone destruction, Sherwood Rowland recounted the original motivation for his research group's interest in chloroflurocarbons. It was motivated by the fact that at the time there were no known natural sinks for chloroflurocarbons:

> [t]he starting point for that work [CFC- stratospheric ozone problem] was the discovery by Jim Lovelock that the molecule CCl_3F, a substance for which no natural sources have been found, was present in the Earth's atmosphere in quantities roughly comparable to the total amount manufactured to that date. ... The appearance in the atmosphere of a new, man-made molecule provided a scientific challenge: Was enough known about the physico-chemical behavior under atmospheric conditions of molecules such as CCl_3F to allow prediction of its fate, once released into the environment?[4]

James Lovelock's study was published in 1973 and only a few months later, Molina and Rowland worked out the how chlorine from chloroflurocarbons can destroy stratospheric ozone [Lovelock et al. 1973, Molina and Rowland 1974]. The international scientific community decisively acted on the Molina-Rowland findings more than fifteen years later in the from of the Montreal Protocol. The "ozone hole" has been expanding is not slated to start recovering before the end of this decade. The policy response to stratospheric-ozone depleting substances is perhaps deservedly hailed as an example for international cooperation, we suggest a different lesson to draw from the history of science and policy surrounding the Montreal Protocol.[5] We suggest that the international response to the ozone depleting chemicals would have been swifter and decisive if policy makers were as alert to the problem of 'zero sink capacity' as the scientific community. The framework that we developed in chapters

---

[4] Rowland [1995, p.273-274]

[5] For a detailed and authoritative account of the history of Montreal Protocol, see Benedick [1998].

3 and 5 helps bridge the gap between science and policy.

Relationship between Optimal Scale and Maximum Sustainable Scale

The most important lesson from the framework we developed for benchmark scales is about the relationship between optimal scale and maximum sustainable scale. Specifically, if optimal scale is the scale that is determined to be the most socially desirable scale, how do we ensure that the optimal scale chosen is also physically sustainable. If the optimal scale is chosen in some democratic fashion, there is no theoretical guarantee that the optimal scale would be less than maximum sustainable scale. We also showed that optimal scale that is determined through a market allocation process is not necessarily less than maximum sustainable scale. From a policy perspective, this once again underscores the irreducible normative dimension that is integral to the scale methodology. If distribution is ultimately a normative question about justice within a given generation, scale and sustainability are normative questions about inter-generational justice. If physical sustainability is indeed a societal concern, the framework developed in this dissertation can help make economic policy choices that can help put society on a physically sustainable path. The framework for maximum sustainable scale that we developed in chapter - 5 recognizes that it is impossible to precisely determine maximum sustainable scale — the fund-flux space after all is not cardinal. We thus defined maximum sustainable scale in terms of a “yield point region” rather than as a single point. The uncertainty that is integral to the yield point region is nevertheless discernable through a judicious mix of empiricism and careful applications biophysical first principles.

Varying Levels of Economic Geographic Aggregation

One of the most important lessons from this dissertation is the demonstration that the scale methodology can be used to study the economy-ecosystem interactions at multiple levels of economic-geographic aggregation. The stock-fund framework developed in this dissertation is as applicable to studying *a particular* material flow, as it is to the study of aggregate throughput. In chapter - 4 we developed a consistent aggregation framework as part of our review of material flow methodologies.

A particular practical application would be to use departures from benchmarks to achieve "eco-labeling." Consider the example of timber again. Typically a particular variety of timber comes from a variety of different sources (forests in different ecological zones, managed plantation for example). Irrespective of where the timber comes from, departure from optimal scale ($\delta = \Omega - \Omega^*$) is comparable across regions, and over time. If physical sustainability is the objective of the labeling policy, a flow measure of scale like S_{so} is to be used rather than a stock measure as maximum sustainable scale is defined only in the flow dimension. While δ is a useful metric for the purposes of defining a generalized consistent scale measure, it is somewhat confusing for purposes of eco-labeling. The best possible score a timber product in our example here can achieve is $\delta = 0$ when $\Omega = \Omega^*$ – that is when the actual scale is the same as optimal scale. The labeling is best achieved on a easy to understand 0–100 scale with a score of 100 representing optimal scale. This can easily be achieved using some transformation function – for example using the framework used to define the scale efficiency function, ψ in appendix - A.

7.4 Directions for Future Research

This dissertation, perhaps like most research of this nature has raised more questions than it has been able to provide conclusive answers to. In this brief section we identify some of the obvious lacunae in the research presented in this dissertation and suggest a program for future research.

7.4.1 Theoretical Research

Scale as a Proportional Relationship

In the introduction to this dissertation we discussed how scale as the proportional relationship between the economy and the ecosystem has much to gain from other disciplines that make use of proportional quantities. In particular we believe that further refinement to the frameworks for maximum sustainable scale and optimal scale will benefit from a more substantive study of those diverse fields. If scale is a proportional relationship, optimal scale simply represents the most ‘harmonious’ relationship between the economy and the ecosystem.

Scale in a Non-Anthropogenic Framework

From a normative standpoint, perhaps the biggest lacuna in the framework developed by this dissertation has been the singular focus on anthropogenic concerns. Admittedly a framework for scale as the relationship between the *human* economy and the ecosystem is bound to be centered on anthropogenic concerns. However, the normative question of optimal scale is closely linked to society's normative beliefs about the relationship between humans and other species.

Incorporating Energy

The biggest drawback in the framework that we have developed to measure scale is that it does not explicitly suggest how energy could be incorporated in the framework. The basic premise of scale as a proportional relationship between the economy and the ecosystem is valid in the energy dimension as much as it is in the material dimension. The stock-fund description of the ecosystem needs several extensions before it can be used for empirical measurement of scale in energy terms. The most significant theoretical barrier is the characterization of the ecosystem in terms of energy – there are no immediate parallels to regeneration flows that describe the material dimension. Ecologists have long studied energy balance across trophic levels and the systems ecology literature has evolved useful accounting frameworks to track energy flows in the ecosystem. Future research that incorporates energy into our framework will also help shed light on our black-box model for ecosystem as a fund.

Ecological Economics of Resource Extraction

The most obvious extension of the stock-fund framework that we developed in this dissertation would be its application to resource economics – economics of renewable resources in particular. Under a stock-fund framework, the problem of optimal harvest can explicitly include ecosystem services lost to resource extraction. As we discussed in section - 5.4 $\partial\breve{\mathbf{Y}}/\partial\dot{\mathbf{x}} < 0$ — increasing throughput degrades the ecosystem fund (even when $\mathbf{S} < 1$). The ordinal ranking framework developed in chapter - 6 can be used to quantify the resource optimization problem that combines flows and fluxes. Beyond the economics of resource extraction, the stock-fund framework used in this dissertation also holds possibilities for a novel treatment of environmental public goods.

Open Economy

In much of the theoretical discussion we have implicitly assumed a closed economy. While the basic framework that we have developed here can accommodate throughput that cuts across a given economic-geographic aggregation, future research will have to explicitly specify these trans-boundary flows. The TMR-TDO methodology that we discussed in chapter - 4 incorporates a rudimentary model of the open economy that can serve as a point of departure despite the theoretical problems that we identified with that framework.

7.4.2 Empirical and Policy Research

By quantifying the proportional relationship between the economy and the ecosystem, scale measures help achieve a novel and policy relevant empirical characterization of how the economy and ecosystem interact. This dissertation was focussed on theoretical frameworks that enable a measurement, and interpretation of scale. We have not been able to use the framework developed in this dissertation to develop scale relationships for any *particular* empirical case. In chapter - 4 we identified important theoretical problems with well-known biophysical assessments of human activity. The logical next step in the research program that uses the scale methodology developed in this dissertation would be to use the framework developed here to refine the empirical understanding gleaned from the biophysical assessments that we have reviewed.

Appendix A

A Dynamic Model of Scale, Allocation, and Distribution

A.1 Introduction

While the core of this dissertation is focussed on developing an analytical super structure to formally study the concept of scale, the focus here is on understanding the relationship between the ecological economics concept of scale and the more traditional questions of 'allocation' and 'distribution.'

Much of the work in trying to understand the relationship between scale, allocation, and distribution has its origins in two essays by Daly[Daly 1991b, 1992]. In these essays Daly developed the now commonly accepted notion of the hierarchical relationship between scale, allocation, and distribution with scale at the top of the hierarchy and allocation at the bottom. The basic point that Daly was trying to make in his seminal 1992 article was to show how scale, allocation, and distribution are *independent* concepts. All critical responses to Daly [1992] tried to argue that scale, allocation, and distribution were not really independent but were simultaneously determined. By scale, allocation, and distribution being independent, Daly was suggesting that the three are independent variables in the mathematical sense. We need a system of three equations to determine them all and in a system of three simultaneous equations the three variables are obviously related to each other [1] [Daly 1992, Stewen 1998, Daly 1999].

---

[1] The original 1992 essay by Daly did not allude to the possibility that scale, allocation, and distribution could indeed be part of a system of simultaneous equations. Daly [1999] clarifies this position.

The debate that followed publication of Daly's scale-allocation-distribution article is only a symptom of the larger confusion within the ecological economics literature between "scale" and "optimal scale." We have shown in Chapter - 5 of this dissertation how "scale" is a positive concept while "optimal scale" is a normative concept. Scale is the answer to the question: what is the physical size of the economy relative to the ecosystem? Optimal scale answers the question: how large *should* the economy be (in physical terms) relative to the ecosystem? The former question is a positive question — at least in principle. The latter question is a social, political,ethical, and perhaps even a moral question. One of the primary reasons there has been very little work on formalizing the relationship between scale, allocation, and distribution is that the ecological economics literature has not been careful to distinguish between the positive concept of 'scale' and the normative concept of 'optimal scale'.

We will start by adding to our discussion on the problems in ecological economics literature from the failure to distinguish between optimal scale and "scale." After we have clearly delineated the positive and normative spaces for analysis of scale, we develop a framework to understand the relationship between scale, allocation, and distribution. We build our model in the "efficiency space" captures departures from optima for each scale, allocation, and distribution. The primary motivation for this chapter in the context of this dissertation is to demonstrate how scale can help cast the traditional questions of allocation and distribution in biophysical terms. Understanding allocation and distribution in biophysical terms

A.2 The Positive-Normative Distinction

The starting point in trying to develop a formal framework to study the relationship between scale, allocation, and distribution is to recognize that for each of the three aspects of economic analysis, the positive and normative concepts have to be treated separately. In case of scale we have already seen the distinction between "scale" and "optimal scale". In case of allocation, "optimal allocation" would coincide with the familiar "Pareto-efficient allocation." Once again, like with "optimal scale," "optimal allocation" is a normative concept—it answers the question:what *should* a good allocation be? "Optimal distribution" will be determined by normative considerations about what constitutes a just distribution.[2]

There are three broad ways in which we can study the relationship between scale, allocation, and distribution.[3] First, we need to study how normative concepts of "optimal scale", "optimal allocation", and "optimal distribution" are related to each other. Next we need to study how the positive concepts of "scale", "allocation", and "distribution" are related. Finally, it is possible to study scale, allocation, and distribution by combining the normative and positive aspects. In particular we will

---

[2] For our purposes here, we do not need to go into the definitional issues with distribution or theories of just distribution. A simple description of wealth and income distribution will prove sufficient.

[3] In this chapter, unless otherwise mentioned, we will use scale, allocation, or distribution without quotation marks to represent respective general concepts. Quotation marks will be used to denote respective positive quantities.

be interested in studying the departure of "scale", "allocation", and "distribution" from their respective optima. Thus the study of the interrelationship between scale, allocation, and distribution is three stage process. We will briefly discuss the first two stages here but the primary focus here will be on developing a computable dynamic model to study how the three departures from optima are related to each other.

The hierarchical relationship between scale, allocation, and distribution suggested by Daly (1991) describes the relationship between scale, allocation, and distribution in the normative space. It describes how the three optima are related to each other. It is fairly straightforward to see why "optimal scale" and "optimal distribution" have to be determined before one can specify "optimal allocation". "Optimal allocation" is a function of of a *given* "optimal scale" and "optimal distribution". Following Daly the ecological economics literature has stuck to the hierarchical relationship between "optimal scale" and "optimal distribution" as well. However unlike optimal allocation that presupposes a given optimal distribution, the definition of "optimal distribution" does not (logically) need an *a priori* specification of "optimal scale". Daly [1992] uses the example of tradable pollution permits to argue for the strictly hierarchical relationship between scale, allocation, and distribution. Setting aside the confusion between positive and normative spaces, the suggested hierarchical relationship between scale, allocation, and distribution can be thought of as a ranking exercise of the different states of the world on the normative space. In each of the three cases we are asking : what *ought* to be a desirable state of the world

— be it sustainable scale (optimal scale), just distribution (optimal distribution), or efficient allocation (optimal allocation). The question that we are interested in here is if it is (in theory) possible to talk about just distribution without reference to sustainable scale. We have already seen how it is not possible (even in theory) to talk about efficient allocation without reference to some given optima for scale and distribution. However the same is not true for distribution as suggested by Daly and others. One can speculate about desirable states of the world in the normative-distribution space without reference to any particular notion of sustainable scale[4]

A.2.1 Departure from Optima: The Efficiency Space

Of even greater interest is to study how each of scale, allocation, and distribution are different from their respective optima. We will study the coevolutinary dynamic relationship between scale, allocation, and distribution in terms of respective "efficiencies" that measure departure form the optimum.

A.2.2 Summary

Before we move on to a formal model in the efficiency space, it would be useful to summarize the three-tier approach to studying the relationship between scale, allocation, and distribution.

---

[4] Also see section - 5.4.

Analysis Space	**Objects of Analysis**
Positive Space	*Scale, Allocation, Distribution*
Normative Space	*Optimal Scale, Optimal Allocation, Optimal Distribution*
Efficiency Space	*Scale Efficiency, Allocation Efficiency, Distribution Efficiency*

Table A.1: Positive and Normative Aspects of Scale, Allocation, and Distribution

A.3 A Dynamic Model of Scale, Allocation and Distribution in the Efficiency Space

A.3.1 Basic Definitions

Scale Efficiency

The scale efficiency is defined following the basic ecological economics insight of economy being an open subsystem of a finite ecosystem. For the model that we develop here, we look at the economy and the ecosystem in the stock dimension. As discussed in chapter - 5, optimal scale is most meaningfully defined in the stock dimension. We being by looking at figure - 3.1, that represents the basic ontology of ecological economics. Let R represent the ellipse (ecosystem); and C, the rectangle (economy). Further, let R and C represent the areas of the ellipse and the rectangle respectively. When drawn to scale, R represents an index of ecosystem services; and C represents the portion of ecosystem services appropriated by the economy.[5] Now we define a parameter, α to the difference between R and C:

$$\alpha = R - C \tag{A.1}$$

The index α simply represents ecosystem service not appropriated by the economy. In terms of α, the existence of optimal scale implies that there is an α^* that is

---

[5] The definition of R and c mirror the logic of figure - 5.3.

preferred to any other α. We now define departure from the optimum:

$$\delta = \alpha^* - \alpha \tag{A.2}$$

We define scale-efficiency, η^S in terms of this deviation, δ. We use α instead of C to define scale efficiency despite the direct use of C being simpler because we want to underscore the fact that optimal scale is ultimately a normative benchmark.

$$\eta^s = \psi(\delta) \tag{A.3}$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}\delta} < 0 \ \forall \alpha \neq \alpha^* \tag{A.4}$$

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}\psi^2} < 0 \ \forall \alpha > \alpha * \tag{A.5}$$

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}\psi^2} > 0 \ \forall \alpha < \alpha * \tag{A.6}$$

The function ψ is defined such that any deviation from optimal scale is penalized, but as the economy grows bigger than the optimum scale, this penalty increases at an increasing rate as indicated by the second derivative in the case of $\alpha < \alpha*$. This specification of ψ follows the properties of a consistent scale metric that we developed in chapter - 6.

Allocation Efficiency

Allocation efficiency is simply the ratio of missed trade opportunities to the maximum social product that is possible when all available opportunities for trade are exhausted.

$$\eta^A = 1 - \frac{\theta}{\theta^*} \tag{A.7}$$

In the above equation, θ represents the social product corresponding to missed trade opportunities and θ^* is the social product that is achievable when all opportunities for trade have been exhausted. Measurement problems notwithstanding, θ^* in the above equation includes both inter and intra-generational trade opportunities.

Distribution Efficiency

Distributive efficiency is simply derived from the Gini coefficient. We will use a combined Gini that is a linear combination of wealth and income Gini's. Admittedly a simplification, this simple modeling of wealth and income distribution is sufficient for our primary purpose here – which is to illustrate how scale is related to allocation and distribution.

$$G = \gamma G_W + (1-\gamma) G_Y \;\; ;0 \le \gamma \le 1 \tag{A.8}$$

$$\eta^D = 1 - G \tag{A.9}$$

In the above equations, G_W is the wealth Gini, and G_Y is the income Gini. γ assigns the weight wealth and income Gini. The distribution efficiency, η^D simply converts the weighted Gini to a 0 –1 scale so that distribution efficiency is consistent with scale and allocation efficiency.

A.4 The Dynamic Feedback Structure

We are now ready to develop a model to study the relationship between scale, allocation, and distribution in the efficiency space. The three equations below represent the most general form of the feedback structure between the three efficiencies. In this model, the determinants of the three efficiencies are contingent on all three efficiencies.

$$\alpha = \Omega^S(\eta^A, \eta^D, \phi^S; t) \tag{A.10}$$

$$\theta = \Omega^A(\eta^D, \eta^S, \phi^A; t) \tag{A.11}$$

$$G = \Omega^D(\eta^S, \eta^A, \phi^D; t) \tag{A.12}$$

The functions $\Omega^i(i = S, A, D)$ are all continuous functions of the policy parameters, ϕ, the relevant efficiencies and time, t. The parameter ϕ captures relevant directly targeted policies. The equations of the basic model represent the time paths for the three efficiencies. We have specified ϕ such that it is endogenous — policy targets scale, allocation, and distribution in a way so that it tries to steer all three towards respective optimal values (to recall, optimal values are determined by normative, political considerations and are exogenous to the feedback structure).

A.4.1 The Dynamic Model

To obtain the dynamic coevolutionary path for the three efficiencies, we simply take the total time derivatives of the last three equations.

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \left(\frac{\partial\Omega^S}{\partial\eta^A}\right)\left(\frac{\mathrm{d}\eta^A}{\mathrm{d}t}\right) + \left(\frac{\partial\Omega^S}{\partial\eta^D}\right)\left(\frac{\mathrm{d}\eta^D}{\mathrm{d}t}\right) + \left(\frac{\partial\Omega^S}{\partial\phi^S}\right)\left(\frac{\mathrm{d}\phi^S}{\mathrm{d}t}\right) + \frac{\partial\Omega^S}{\partial t} \tag{A.13}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \left(\frac{\partial\Omega^A}{\partial\eta^D}\right)\left(\frac{\mathrm{d}\eta^D}{\mathrm{d}t}\right) + \left(\frac{\partial\Omega^A}{\partial\eta^S}\right)\left(\frac{\mathrm{d}\eta^S}{\mathrm{d}t}\right) + \left(\frac{\partial\Omega^A}{\partial\phi^A}\right)\left(\frac{\mathrm{d}\phi^A}{\mathrm{d}t}\right) + \frac{\partial\Omega^A}{\partial t} \tag{A.14}$$

$$\frac{\mathrm{d}G}{\mathrm{d}t} = \left(\frac{\partial\Omega^D}{\partial\eta^S}\right)\left(\frac{\mathrm{d}\eta^S}{\mathrm{d}t}\right) + \left(\frac{\partial\Omega^D}{\partial\eta^A}\right)\left(\frac{\mathrm{d}\eta^A}{\mathrm{d}t}\right) + \left(\frac{\partial\Omega^D}{\partial\phi^D}\right)\left(\frac{\mathrm{d}\phi^D}{\mathrm{d}t}\right) + \frac{\partial\Omega^D}{\partial t} \tag{A.15}$$

Time Paths for Software Simulation

For the purposes of software simulation of the dynamic model above, it is convenient to use total differentials instead of total time derivatives:

$$\mathrm{d}\alpha = L_{SA}(\mathrm{d}\eta^{A}) + L_{SD}(\mathrm{d}\eta^{D}) + K_{SS}(\mathrm{d}\phi^{S}) \tag{A.16}$$

$$\mathrm{d}\theta = L_{AD}(\mathrm{d}\eta^{D}) + L_{AS}(\mathrm{d}\eta^{S}) + K_{AA}(\mathrm{d}\phi^{A}) \tag{A.17}$$

$$\mathrm{d}G = L_{DS}(\mathrm{d}\eta^{A}) + L_{DA}(\mathrm{d}\eta^{A}) + K_{DD}(\mathrm{d}\phi^{D}) \tag{A.18}$$

In the equations above the coefficients follow the following notation:

$$\begin{cases} L_{ij} = \frac{\partial\Omega^{i}}{\partial\eta^{j}} \quad \forall i \neq j \\ \\ K_{ii} = \frac{\partial\Omega^{i}}{\partial\phi^{i}} \end{cases}$$

The above equation can be written in a matrix form as:

$$\begin{pmatrix} \mathrm{d}\alpha \\ \mathrm{d}\theta \\ \mathrm{d}G \end{pmatrix} = \begin{pmatrix} 0 & L_{SA} & L_{SD} & K_{SS} & 0 & 0 \\ L_{AS} & 0 & L_{AD} & 0 & K_{AA} & 0 \\ L_{DS} & L_{DA} & 0 & 0 & 0 & K_{DD} \end{pmatrix} \begin{pmatrix} \mathrm{d}\eta^{S} \\ \mathrm{d}\eta^{A} \\ \mathrm{d}\eta^{D} \\ \mathrm{d}\phi^{S} \\ \mathrm{d}\phi^{A} \\ \mathrm{d}\phi^{D} \end{pmatrix} \tag{A.19}$$

We will define the following matrices that will simplify the notation.

$$\mathbf{\Delta} = \begin{pmatrix} \mathrm{d}\alpha \\ \mathrm{d}\theta \\ \mathrm{d}G \end{pmatrix} \tag{A.20}$$

$$\tilde{\eta} = \begin{pmatrix} \mathrm{d}\eta^S \\ \mathrm{d}\eta^A \\ \mathrm{d}\eta^D \end{pmatrix} \tag{A.21}$$

$$\mathbf{\Phi} = \begin{pmatrix} \mathrm{d}\phi^S \\ \mathrm{d}\phi^A \\ \mathrm{d}\phi^D \end{pmatrix} \tag{A.22}$$

$$\mathbf{L} = \begin{pmatrix} 0 & L_{SA} & L_{SD} \\ L_{AS} & 0 & L_{AD} \\ L_{DS} & L_{DA} & 0 \end{pmatrix} \tag{A.23}$$

$$\mathbf{K} = \begin{pmatrix} K_{SS} & 0 & 0 \\ 0 & K_{AA} & 0 \\ 0 & 0 & K_{DD} \end{pmatrix} \tag{A.24}$$

In the notation that we have just defined, we can write out the matrix equation as:

$$\mathbf{\Delta} = \mathbf{L}\tilde{\eta} + \mathbf{K}\mathbf{\Phi} \tag{A.25}$$

The primary objective of software simulation is to completely characterize the two structural matrices above, **L** and **K**. We will use several calibration strategies to obtain the coefficients of **L** and **K** under a variety of conditions. These matrices will likely be different for different levels of economic aggregation as well as for different places and times.

A.5 Simulation Results

The software simulation was carried out using the mutually coupled feedback model in equation - A.19 as well as by turning off the feedback (which was achieved by setting $L_{ij} = 0 \, \forall i \neq j$). Given that only published data available are for distribution efficiency (in the form of various Gini Coefficients), the results of the simulation could well be artifacts of the particular inspired guesses for η^S and η^A. However our primary purpose is illustration of the method rather than empirical measurement. Figures A.1, A.2, and A.3 present the dynamic time paths for scale, allocation and distribution efficiencies respectively. We have superimposed two sets of time paths for each of the efficiencies. The first graph in each figure is without feedback from the other two efficiencies. The first thing that is apparent from the results of simulation is that there is not much difference between the time paths of the three efficiencies with and without feedback. This seems to be counterintuitive especially in light of the original motivation for developing the model presented here. The results here perhaps reflect the uncertainties in the calibration of the model as well larger uncertainties in specification of the sector specific driver functions, ϕ^i ($i = S, A, D$). There are also uncertainties associated with the initial values of allocation improving trade opportunities, θ and size of the economy, C. The model behavior was observed to be particularly sensitive to initial condition of C. The qualitative relationship between feedback and non-feedback cases was also sensitive to specification of the optimal size of economy, α^*, and the functional specification of scale efficiency, ψ. In

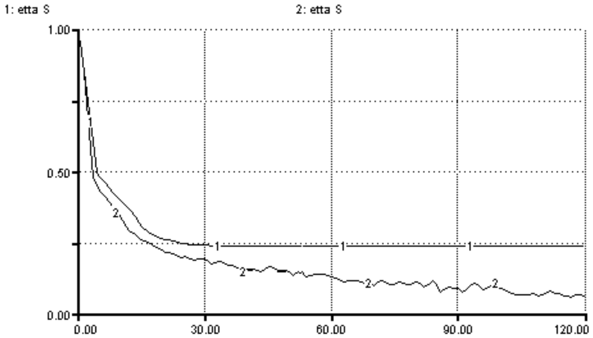

Figure A.1: Scale Efficiency, η^S. Graph-1 is without feedback and graph-2 is the mutually coupled model. η^S is normalized to a 0–1 scale. The horizontal axis is time, t in steps of $\mathrm{d}t = 1\,\text{year}$.

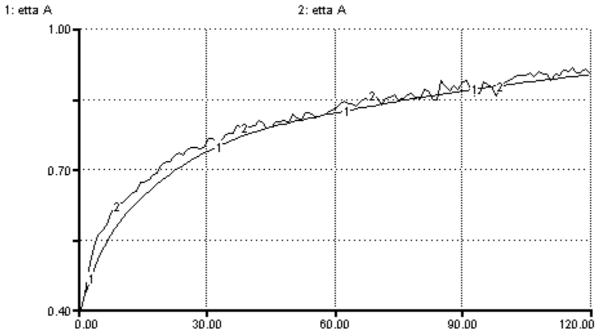

Figure A.2: Allocation Efficiency, η^a. Graph-1 is without feedback and graph-2 is the mutually coupled model. η^S is normalized to a 0–1 scale. The horizontal axis is time, t in steps of $\mathrm{d}t = 1\,\text{year}$.

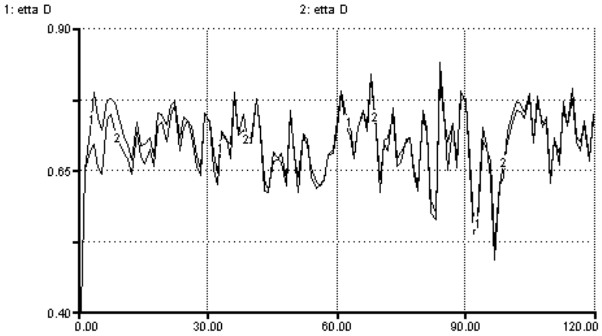

Figure A.3: Distribution Efficiency, η^D. Graph-1 is without feedback and graph-2 is the mutually coupled model. η^D is normalized to a 0–1 scale. The horizontal axis is time, t in steps of $\mathrm{d}t = 1\,\text{year}$.

particular it is worth recollecting that α^* is at least partly determined by normative concerns. Simulation results were also sensitive to functional specifications of L_{ij}. The results presented here use a simple linear step relationships.

Figures A.4 and A.5 present the relationship between the three efficiencies. The relationship between distributive and scale efficiency deserves some explanation. At very low levels of distributive efficiency (high levels of inequality), there is a significant increase in allocation efficiency. This improvement in allocation efficiency leads to an increase in the size of the economy and a decrease in scale

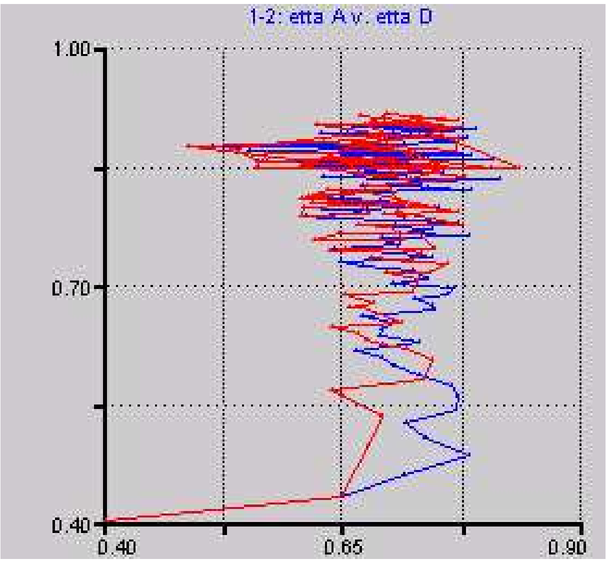

Figure A.4: Allocation Efficiency v. Distribution Efficiency ($\eta^A v.\eta^D$). The blue graph is without any feedback, and the red graph is the mutually coupled model. η^A is on the vertical axis and η^D is on the horizontal axis. Both η^A and η^D are normalized to a 0–1 scale. The simulation is run for $t = 120$ *years* in steps of $\mathrm{d}t = 1\,\mathrm{year}$.

efficiency. At higher levels of distributive efficiency, the relationship between scale and distributive efficiencies is not very clear, especially in the feedback model. This difference between scale-distribution relationship as a function of distribution efficiency is less pronounced as would be expected.

Figure - A.6 shows the sensitivity of the scale-distribution relationship to initial size of the economy, C. As was discussed earlier, the calibration of model did not explicitly take into account scale data and this sensitivity analysis indicates that a more careful calibration than has been done here is required to fully understand the linkages between scale and distribution.

The model used in the simulation here was directly derived from the analytical model that we developed. Adding more 'flesh and blood' to the model did not

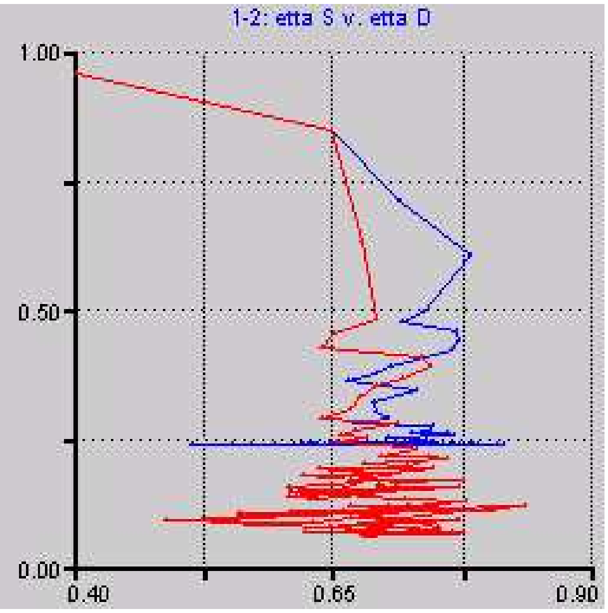

Figure A.5: Scale Efficiency v. Distribution Efficiency ($\eta^S v.\eta^D$). The blue graph is without any feedback, and the red graph is the mutually coupled model. η^S is on the vertical axis and η^D is on the horizontal axis. Both η^S and η^D are normalized to a 0–1 scale. The simulation is run for $t = 120$ *years* in steps of $\mathrm{d}t = 1\,\text{year}$.

change the basic behavior observed. The first modification that we made was to let ϕ^j ($j = S$, A, D) be functions of η^j instead of the constant values they take in the model above. The model behavior was most sensitive to changes in the policy target for inequality level. Instead of the Gini coefficient of 0.3 that was assumed as the policy target I ran the model for three values of target Gini coefficient and the result for allocation efficiency is presented in figure - A.7.This once again illustrates the need to fully understand the complex interactions between scale, allocation and distribution. The way the model is set up with the feedback structure, the question of ranking 'states of the world' takes new salience. Suppose the triplet (η^S, η^A, η^D) represents, at any given time the values of three efficiencies. It is fairly trivial to rank $A = (0.4, 0.3, 0.5)$ and $B = (0.4, 0.3, 0.8)$. The state of the world B is unambiguously preferred to A as moving from A to B at least one of the efficiencies has

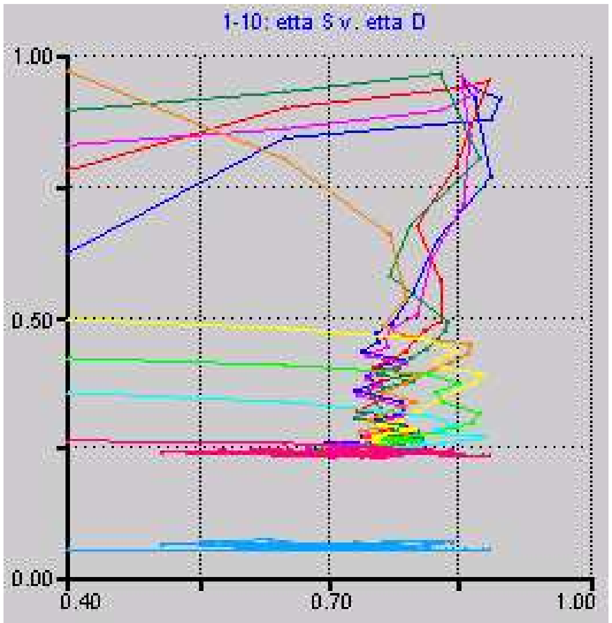

Figure A.6: Scale Efficiency v. Distribution Efficiency ($\eta^S v. \eta^D$). Sensitivity to initial size of the economy. All runs mutually coupled model. η^S is on the vertical axis and η^D is on the horizontal axis. Both η^S and η^D are normalized to a 0–1 scale. The simulation is run for $t = 120$ *years* in steps of $\mathrm{d}t = 1\,\text{year}$.

improved without changing the other two. We can write $A \succ B$. Now consider $C = (0.3.0.5, 0.4)$ and $D = (0.2, 0.8, 0.3)$ How do we rank C and D ? A general ranking mechanism (be it through the much abused social welfare function or otherwise) is important because the way the model is set up, the directly targeted policy $(\phi^i; i = S, A, D)$ is a function of departure of the actual efficiencies from policy targets.

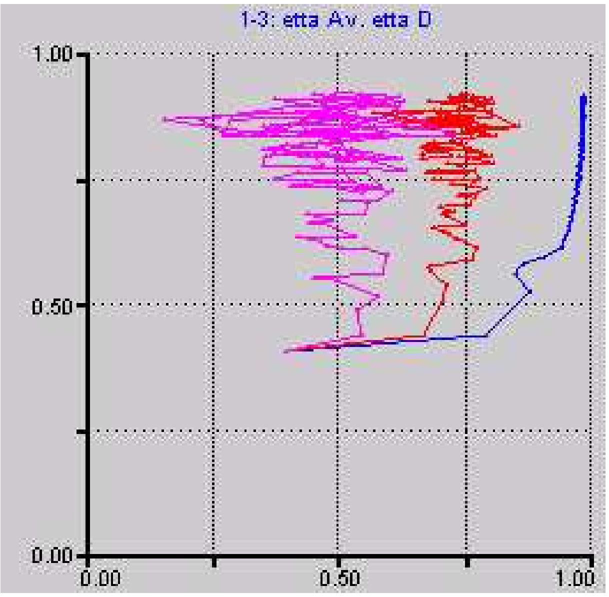

Figure A.7: Scale Efficiency v. Distribution Efficiency ($\eta^S v.\eta^D$). Sensitivity to distribution policy. Target Gini coefficient varied from blue ($G = 0$), red ($G = 0.25$), and purple ($G = 0.5$). All runs mutually coupled model. η^A is on the vertical axis and η^D is on the horizontal axis. Both η^A and η^D are normalized to a 0–1 scale. The simulation is run for $t = 120$ *years* in steps of $\mathrm{d}t = 1\,\mathrm{year}$.

Appendix B

Preanalytic Visions in Economics

B.1 Introduction

Perhaps the biggest achievement of ecological economics has been to provide reasonable evidence that the ‘preanalytic vision’ of standard economics is incapable of explaining certain well-established biophysical facts that have a bearing on the human economic predicament. Analytical study of scale, the focus of this dissertation is closely tied to the ecological economics’ vision of how the economy and the ecosystem interact. Indeed we started building our framework for empirical measurement of scale by describing the preanalytic vision of ecological economics (figure - 3.1). We attempt to understand the origins of that vision in this brief appendix. Given that the ecological economics literature emphasizes the need to debate on the preanalytic visions for the economy, we ask the following question:*what are the determinants of preanalytic visions in economics*? In answering this question, we find that what are considered ‘preanalytic’ visions of economics (the standard “neoclassical” as well as the ecological variants) can in fact be *analytically* deduced from our beliefs about “ultimate means” and “ultimate ends.” The purpose of this essay is to review the specific meanings for “ultimate means” and “ultimate ends” as used in the ecological economics literature and show how preanalytic visions for the economy are related to fundamental beliefs about ultimate means and ends.

B.2 Ends and Means

The most important of the predicaments studied by traditional economics is that of scarcity. Scarcity arises because human wants are not satiable but the means to fulfill these infinite wants are finite – consumers and producers alike have limited means. The normative goal of economics is to maximize the aggregate welfare (as measured by fulfilled wants) in a society, and to this extent economics becomes the study of allocation of scarce means among competing ends. Thus as Daly [1993] suggests, an understanding of the nature of these 'competing ends' and 'scarce means' is fundamental to understanding the human economic predicament. We have reproduced below, a scheme for studying ends and means from Daly [1993, p.20]. The ends and means are represented on an ends-means spectrum in figure - B.1.

At one end of the spectrum, we have the *Ultimate End*. For our purposes here, we do not need to define Ultimate End in teleological terms. *Telos*, or otherwise, the need for an Ultimate End in an ends-means spectrum is directly related to the human economic predicament of 'infinite competing ends.' Given that we cannot fulfill all our competing ends (even in principle), the Ultimate End provides a mechanism for ranking our competing ends. Given the scarcity of means relative to the ends, every society has to chose between competing ends. Ultimate End is not contingent on any *particular* moral principle, but at the same time it is not devoid of *any* moral content. Ultimate End helps make choices between competing ends, and choosing between *ends* is inherently a moral exercise, unlike merely satisfying preferences –

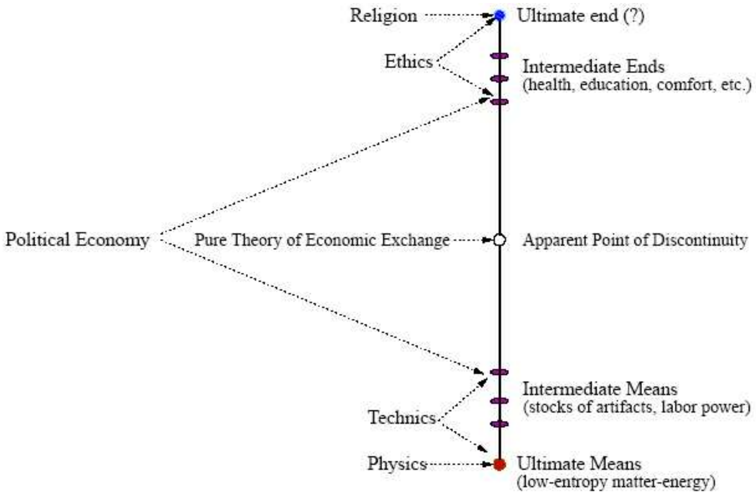

Figure B.1: The ends-means spectrum. Reproduced with minor changes from Daly [1993, p.20]

.

choosing between national defense and public education has an irreducible moral element unlike picking between two different kinds of toothpaste. The ends-means spectrum in figure - B.1 represents the spectrum for the society as a whole but can be adapted to an individual economic agent. Continuing down the spectrum from Ultimate End, we have Intermediate Ends.

At the opposite end of the spectrum from the Ultimate End is Ultimate Means. Ultimate Means is the terrestrial stock of low-entropy matter-energy and the incident solar radiation. The human agency (even in theory) cannot alter the nature of Ultimate Means at the global level. As we move up the spectrum from Ultimate Means towards Ultimate End, every point is a means towards achieving ends represented by points above a chosen point, and is also the end achieved by using the means at points below it. Thus Intermediate Means are ‘ends’ achieved using Ultimate Means and Intermediate Ends are a means to achieving the Ultimate End.

Thus the ends-means spectrum is a continuum of points that are simultaneously a means for some higher order end and the end achieved by a lower order means. The movement along the continuum involves purposeful action of the human agency.

As indicated in figure - B.1 the ends-means continuum is finite, terminating at Ultimate Means and Ultimate End. Ultimate Means cannot be a product of purposeful human agency and is therefore not an end. Ultimate End by definition cannot be the means for any higher-order end. In terms of the ends-means continuum in figure - B.1 the ultimate goal for economics is to use Ultimate Means in the satisfaction of the Ultimate End. The various disciplines that study different parts of the ends-means spectrum are shown on the left of the spectrum. Given that the human economic predicament defined in terms of ends and means intersects a variety of disciplines across the ends-means spectrum, the traditional focus of political economy on the middle portion of the spectrum warrants a serious enquiry. As Daly [1993] points out, one of the most important consequences of the exclusive focus of political economy on the intermediate portion of the ends-means spectrum is that it does not recognize any absolute limits imposed by either Ultimate Means or the Ultimate End. Daly originally developed this spectrum to explain the reasons for, and the consequences of modern economics' obsession with economic growth by laying out a framework to study the desirability and possibility of continued economic growth.

Unlike traditional political economy, modern economics that focuses on pure theory of exchange does not consider any kind of ends. In what is a significant departure from even its antecedents in utilitarian ethics, contemporary microeconomic theory is based on subjective individual *preferences.* The utility maximization exercise, that is the bedrock of consumer theory, is reduced to an ordinal preference ranking exercise rather than fulfillment of a set of wants. Preferences and ends belong to distinct axiological categories. In particular, preferences lie outside the realm of a moral discourse while moral disputes are integral to ends. Thus while the neoclassical price theory is able to resolve the problems associated with classical value theory, it ironically achieves this by eliminating any moral possibilities for the participating economics agents. Thus in figure - B.1, there is an apparent point of discontinuity in the ends-means continuum. This is depicted in the figure as a point on the ends-means spectrum. We have three distinct points on the spectrum – Ultimate Means (shown in red); Ultimate End (shown in blue); and the point of discontinuity in the middle. Every other point is part of the continuum (and hence shown as magenta, combining red and blue). The point in the middle is studied by pure theory of economic exchange. We have labeled this point as *apparent* point of discontinuity because it exists only in the idealized theory of pure exchange. One only needs to look at how macroeconomics is organized – modern macroeconomic theory was indeed developed as a means to achieving a very tangible end of pulling the world out of the Great Depression. In the next section we briefly elaborate on the main features of an "embedded" economy before illustrating how the embedded economy is directly related to the ends-means spectrum discussed here.

B.3 An Embedded Economy

The primary normative goal of economic organization from the time of industrial revolution has been the establishment of a fully autonomous and self regulating market system. This goal of economic organization also forms the basis of neoclassical economics, perhaps best captured by the familiar picture of the circular flow of goods and money that is the staple of any introductory macroeconomics textbook. In this circular-flow ontology, the economy is unaffected by social, political processes as well as the physical environment – the economy is simultaneously dis-embedded from both the social-political fabric and the larger physical environment. In his seminal work, Polanyi marshals a large swathe of historical evidence to suggest that a completely dis-embedded economy is an utopian ideal whose pursuit results in tragic consequences [Polanyi 2001]. The pioneering critique by Polanyi has evolved in two distinct discourses. Historians, sociologists, political scientists, and anthropologists have fleshed out the consequences of the *great transformation*, and studied continuing efforts to create large-scale self regulating markets. If these various social sciences have studied the disconnect between the economy and the larger social-political fabric, ecologists and other physical scientists have focussed on the material foundations of the economy. Prompted initially by the energy and resource crises of the seventies and later by the growing problems with finding a place to safely dispose the various waste products of our prodigal resource use, this discourse has tried to anchor the economy to its material foundations. Ecological economics is a direct offshoot of this discourse whose principal precept is that the economy is to be studied as an open subsystem of the larger biophysical system that contains and sustains it.

If an economy that is dis-embedded form either the physical environment or the larger social fabric is an empirical impossibility, what explains the enduring appeal of an autonomous, self regulating market as the dominant goal for economic organization, and neoclassical economics as the theory that studies such an economic organization? Political, sociological, historical, and even cultural explanations are indeed central to a nuanced understanding of our complex economic organization. Our goal here is much more modest – at the expense of a refined understanding that is possible with political or historical studies, we seek a more direct explanation in terms of the fundamental constituents of the human economic predicament. The human economic predicament has been traditionally defined in terms of ends, and the means to satisfy those ends – economics is a study of means to satisfy competing ends. We suggest here that varying assumptions about how the ends and means are related to each other directly lead to the different "preanalytic visions" of the economy. We find that the preanalytic vision of ecological economics as well as that of standard neoclassical economics can both be derived from assumptions about means and ends. Thus what are "preanalytic visions" are actually working visions derived from a preanalytic conception of the human economic predicament cast in terms of ends and means.

While ecological economics alludes to how the economy is not only embedded within the larger biophysical system but also within the surrounding social, political, and cultural fabric, there is no well established framework within ecological economics that can address questions at the intersection of the economy, polity, and society. The ends-means framework presented here suggests that the relative neglect of ends in the formal models of ecological economics results in a significant lacuna in the ecological economics' understanding of an embedded economy. Studying the human economic predicament in terms of ends and means helps uncover how the economy is embedded in the larger physical environment as well as in the social-political fabric, but does not offer direct clues on how such an embedded economy could be empirically studied. We suggest here that the concept of scale or the proportional relationship between two entities holds the key to understanding an embedded economy. In biophysical terms, scale is a measure of the proportional relationship between the physical size of the economy and the ecosystem that contains and sustains it. Scale as we have seen, answers the question: how large is the economy relative to the ecosystem that contains and sustains it?

B.4 Constraints on an Embedded Economy

The most important feature of any embedded economy is that it is *constrained* in one or more dimensions. There is a constraint on the physical size of the economy when the economy is embedded within the larger ecosystem – the economy cannot be larger than the ecosystem. Social, political constraints limit both the physical size of the economy as well as the qualitative structure of the economy. Our goal here is to understand the nature of these constraints in terms of the basic constituents of the human economic predicament – competing ends and scarce means. The basic model is presented in figure - B.2. **UM** represents Ultimate Means, **UE** , Ultimate End;

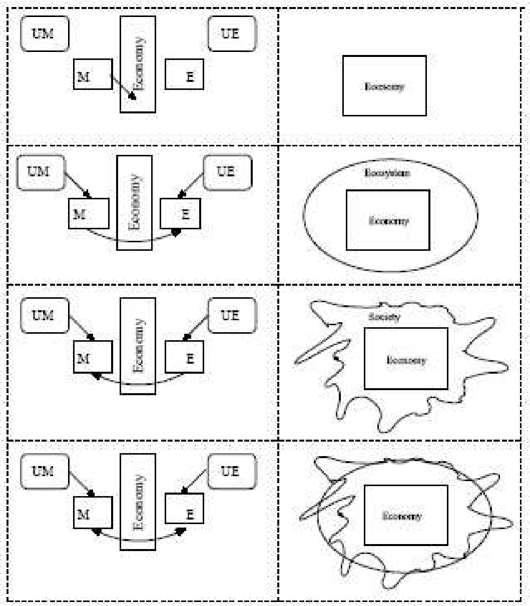

Figure B.2: Ends, Means, and Constraints. The left column shows the four possible ways in which Ultimate End and Ultimate Means interact. The corresponding columns on the right show the 'working vision' for the respective economy that is derived from the assumptions about the interactions between Ultimate End and Ultimate Means. Arrows on the left hand side column represent flow of information – a cause effect relationship in the directions indicated.

and **M**, **E** represent intermediate means and ends respectively. **UM**, **UE**, **M**, and **E** are treated as possibility sets. Thus **UM** represents all states of the world that are permitted by known biophysical laws. From an economics perspective, **UM** consists of terrestrial sources of low-entropy matter and energy as well as the incident solar radiation. **M** is always some subset of **UM**. As discussed in section - B.2 unlike **UM**, it is not possible to precisely define **UE**. We will use two different broad definitions of **UE** to illustrate the connection between an embedded economy and the ends-means dialectic. First we will consider a minimalist definition of **UE** where the Ultimate End only helps with the ordinal ordering of lower-order ends. At the other end of the spectrum, We will study **UE** with a teleological definition. The arrows on the left-hand side panel of figure - B.2 represent flow of information between the various possibility sets with the direction of the arrows indicating the direction of the flow of information. When multiple arrows are incident on any set, that set is the intersection of the two sets from which the arrows originate.[1] For example, in the second row of figure - B.2, **E** is the intersection of **UE** and **M**. When a possibility set has only one arrow incident on it, it is a subset of the set from which the arrow originates.

Figure - B.2 shows four different ways in which the possibility sets **M** and **E** are related to Ultimate Means and Ultimate End on one hand, and the economy on the other. The different interaction between ends and means is shown on the left-hand side and the resulting constraints (or lack thereof) on the economy are depicted on the right-hand side column. In the top row, we have the picture of the economy as a stand-alone entity composed of several self regulating markets. The economy is unaffected by any ends. Preferences are different from ends and are unaffected by any kind of Ultimate End. Ultimate Means has no bearing on the more intermediate means used to satisfy preferences. In terms of the notation introduced here, both **M** and **E** are not subsets but derived independently without

---

[1] We use "intersection" in the usual set theoretic sense. Thus if $C = A \cap B$ then set C contains elements that are common to both sets A and B.

reference to **UM** and **UE** respectively. **E** is a null set because there are no ends in this treatment of the economy, and **M** is an infinite set.

In the second row, we have the ecological economics vision of the economy where the economy is embedded within the larger ecosystem. Here both **M** and **E** are subsets of **UM** and **UE** respectively. More importantly, there is an interaction between the means and the ends. While the means are derived independent of any ends, the ends are contingent on means. **E** is contingent on both **UE** and **M**. Accounting for the fact that **M** is itself derived from **UM**, ends are now directly or indirectly contingent on both Ultimate End as well as Ultimate Means. For example one can reasonably argue that under a certain specification of **UE** continuous economic growth may not be excluded from **E** but continuous economic growth is not an end in ecological economics because it is excluded from Ultimate Means; and Ultimate Means contributes to how ends for an economy are determined within ecological economics. Thus the ultimate constraint on the economy in ecological economics comes from Ultimate Means.

The third row of figure - B.2 shows an economy embedded within the larger social, political fabric rather than the biophysical environment. This conception of the economy, now famous as Polanyi's original "embedded economy" can again be explained in terms of the underlying relationship between means and ends. Unlike the ecological economics vision of the economy, **E** is determined only by the Ultimate End. However, **M** is now jointly determined by both Ultimate Means as well as **E**.

Here, the ultimate constraint on the economy is not the Ultimate Means but the Ultimate End. A particularly stark example to illustrate how means are constrained is that of slave labor. Retooling the society in the interest of thermodynamic thrift (which is a major policy prescription that emerges from the ecological economics vision above) conflicts with the moral, ethical choices of the society. Slave labor is absent in **M** not because it is not part of Ultimate Means but because it is not part of any end that the society considers legitimate.

The nature of the constraint on an economy embedded in the larger social, political fabric is much more complex than the biophysical constraints in the embedded economy of ecological economics. While the nature of Ultimate Means is clearly discernable (at least in theory), it may well be impossible to completely describe the nature of Ultimate End. The social, political constraint derived from the Ultimate End is therefore depicted in figure - B.2 as an amoeba rather than as a regular ellipse that was used for the biophysical constraint. A complete discussion of how the nature of the Ultimate End determines the social, political constraint is beyond the scope of this brief essay.

BIBLIOGRAPHY

Albert Adriaanse, Stefan Bringezu, Allen Hammond, Yuichi Moriguchi, Eric Rodenburg, Donald Rogich, and Helmut Schtz. *Resource Flows: The Material Basis of Industrial Economies*. World Resources Institute, 1997a.

Albert Adriaanse, Stefan Bringezu, Allen Hammond, Yuichi Moriguchi, Donald Rogich, and Helmut Schltz. Resource flows: the material basis of industrial economies. Technical report, World Resources Institute, Wuppertal Institute, Netherlands Ministry of Housing, and National Institute for Environmental Studies, Japan, 1997b.

Robert U. Ayres. Commentary on the utility of the ecological footprint concept. *Ecological Economics*, 32(3):347–49, 2000.

Richard Elliot Benedick. *Ozone Diplomacy : New Directions in Safeguarding the Planet, Enlarged Edition*. Harvard University Press, 1998.

Kenneth.E. Boulding. The economics of the coming spaceship earth. In Henry Jarrett, editor, Environmental Quality in a Growing Economy, pages 3–14. Resources for the Future / Johns Hopkins University Press, 1966.

William R. Jr. Catton. *Overshoot: The Ecological Basis of Revolutionary Change*. University of Illinois Press, 1982.

Graciela Chichilnisky. An axiomatic approach to sustainable development. *Social Choice and Welfare*, 13:231–257, 1996.

Paul. P Chirstensen. Historical roots for ecological economics. *Ecological Economics*, 1:17–36, 1989.

Cutler Cleveland. Biophysical economics: Historical perspective and current research trends. *Ecological Modeling*, 38:47–73, 1987.

Cutler J. Cleveland and Matthias Ruth. Indicators of dematerialization and the materials intensity of use. *Journal of Industrial Ecology*, 2(3):15–50, 1999.

Robert Costanza. What is ecological economics. *Ecological Economics*, 1(1):1–7, 1989.

Robert Costanza. The dynamics of the ecological footprint concept. *Ecological Economics*, 32(3):341–45, 2000.

Robert Costanza, Ralph d'Agre, Rudolf de Groot, Stephen Farber, Monica Grasso, Bruce Hannon, Karin Limburg, Shahid Naeem, Robert.V. O'Neill, Jose Paruelo, Robert.G. Raskin, Paul Sutton, and Marjan van den Belt. The value of the world's ecosystem services and natural capital. *Nature*, 387:253–260, 1997a.

Robert Costanza and Herman Daly. Towards an ecological economics. *Ecological Modelling*, 38, 1987.

Robert Costanza, Charles Perings, and Cutler Cleveland. Historical roots and motivations: Introduction. In Robert Costanza, Charles Perings, and Cutler Cleveland, editors, The Development of Ecological Economics, chapter i, pages xiii–xxix. Edward Elgar Publishing, 1997b.

Robert Costanza, Charles Perings, and Cutler Cleveland. Historical roots and motivations: Introduction. In Robert Costanza, Charles Perings, and Cutler Cleveland, editors, The Development of Ecological Economics, pages xiii–xxix. Edward Elgar Publishing, 1997c.

H.E. Daly. Toward a macroeconomics of environment. In *ISEE Conference on Down to Earth: Practical Applications of Ecological Economics*, San Jose, Costa Rica, 1994.

Herman Daly. On economics as a life science. *Journal of Political Economy*, 76: 392–406, 1968.

Herman Daly. Elements of environmental macroeconomics. In R. Costanza, editor, Ecological Economics: The Science and Management of Sustainability, pages 32–46. Columbia, 1991a.

Herman Daly. Towards an environmental macroeconomics. *Land Economics*, 67(2): 255–59, May 1991b.

Herman Daly. Allocation, distribution, and scale: towards an economics that is efficient, just, and sustainable. *Ecological Economics*, 6:185–193, 1992.

Herman Daly. Introduction to essays toward a steady-state economy. In Herman Daly, editor, Valuing the Earth. MIT Press, 1993.

Herman Daly. Georgescu-roegen vs. solow/stiglitz. *Ecological Economics*, 22, 1997.

Herman Daly. The return of lauderdale's paradox. *Ecological Economics*, 25:21–23, 1998.

Herman Daly. Letters to the editor: Reply to marcus stewen. *Ecological Economics*, 30:1–2, 1999.

Herman Daly. Economics in a full world. *Scientific American*, September 2005, 2005.

Herman Daly and Robert Costanza. Natural captial and sustainable development. *Conservation Biology*, 6:37–46, 1992.

Herman E. Daly and Joshua Farley. *Ecological Economics. Principles and Applications*. Island Press, 1st edition, 2004.

Lisa Deutsch, sa Jansson, Max Troell, Rnnbck, Carl Folke, and Nils Kautsky. The ecological footprint: communicating human dependence on nature s work. *Ecological Economics*, 32(3):351–55, 2000.

Faye Duchin and Edgar Hertwich. Industrial ecology. Technical report, ISEE Online encylopaedia of ecological economics, 2004.

Jeffrey S. Dukes. Burning burried sunshine: Human consumption of ancient solar energy. *CLimate Change*, 61:31–44, 2003.

Gareth Edwards-Jones, Ben Davies, and Salman Hussain. *Ecological Economics: An Introduction*. Blackwell Science, 1st edition, 2000.

Paul Elkins. Identifying critical natural capital: Conclusions about critical natural capital. *Ecological Economics*, 44:277–292, 2003.

Paul Elkins, Carl Folke, and Rudolf DeGroot. A framework for the practical application of the concepts of critical natural capital and strong sustinability. *Ecological Economics*, 44:165–185, 2003a.

Paul Elkins, Sandrine Simon, Lisa Deutsch, Carl Folke, and Rudolf DeGroot. A framework for the practical application of the concepts of critical natural capital and strong sustinability. *Ecological Economics*, 44:165–185, 2003b.

M Faber, H Nimes, and G Stephan. *Entropy, Environement and Resources : An Essay in Physico-Economics*. Springer-Verlag, 2nd edition, 1995.

FAO. A framework for land evaluation. *Soils Bulletin*, 32, 1976.

FAO. Soil survey investigation for irrigation. *Soils Bulletin*, 42, 1979.

FAO. Guidelines: land evaluation for rainfed agriculture. *Soils Bulletin*, 52, 1983.

Christopher Field. Sharing the garden. *Science*, 294:2490–2491, 2001.

Nicholas Georgescu-Roegen. *The Entropy Law and the Economic Process*. Harvard University Press, 2nd reprint, 1999 edition, 1971.

Mario Giampietro. *Multi-Scale Integrated Analysis of Agroecosystems*. CRC Press, 2004.

Mario Giampietro and Kozo Mayumi. Multiple-scale integrated assessment of societal metabolism:introducing the approach. *Population and Environment*, 22:109, 2000.

Clark Gibson, Elinor Ostrom, and T.K. Ahn. The concept of scale and the human dimensions of global change: a survey. *Ecological Economics*, 32:217–239, 2000.

Eli Ginzberg and George Vojta. *Beyond Human Scale : The Large Corporation at Risk*. Basic Books, 1985.

H.E. Goeller and Alvin M. Weinberg. The age of substitutability: What do we do when the mercury runs out? *Science*, 191(4228):683–689, February 1976.

R.B. Gordon, M Bertram, and T.E. Graedel. Metal stocks and sustainability. *Proceedings of the National Academy of Sciences*, 103:1209–1214, 2006.

J.B.S. Haldane. On being the right size. In John Mynard Smith, editor, On Being the Right Size and Other Essays. Oxford University Press, 1995.

F.A. Hayek. Maintaining capital intact: A reply. *Economica*, 8:276–280, 1941a.

F.A. Hayek. *The Pure Theory of Capital*. Midway Reprints in Economics. Univeristy of Chicago Press, 1975 reprint edition, 1941b.

Robert A. Herendeen. Ecological footprint is a vivid indicator of indirect effects. *Ecological Economics*, 32(3):357–58, 2000.

J.R. Hicks. Maintaining capital intact: A further suggestion. *Economica*, 9:174–179, 1942.

C.S Holling, Lance H. Gunderson, and Donald Ludwig. In quest of a theory of adaptive change. In Lance Gunderson and C.S. Holling, editors, Panarchy: Understanding Transformations in Human and Natural Systems, chapter 1. Island Press, 2002.

H. Hotelling. The economics of exhaustable resources. *Journal of Political Economy*, 39:137–175, 1979.

William DeWitt Hyde. *Self Measurement: A scale of human values with directions for personal application.* B.W.Huebsch, 6th printing, 1916 edition, 1908.

Ivan Illich. The wisdom of leopold kohr. *Annual Schumacher Lecuture*, 14, 1994.

Marc Imhoff, Lahouari Bounoua, Taylor Ricketts, Colby Loucks, Robert Harriss, and William Lawrence. Global patterns in human consumption of net primary production. *Nature*, 429:870–873, 2004.

Charles Edouard Jeanneret-Gris. *The Modulor: A harmonious measure to the human scale universally applicable to architecture and mechanics.* Harvard University Press, english translation by peter de francia and anna bostock edition, 1954.

W.Stanley Jevons. *The Coal Question.* Macmillan, 1865.

Geraldine.J Jordan and Marie-Jose Fortin. Scale and topology in the ecological economics sustainability paradigm. *Ecological Economics*, 41:361–366, 2002.

R.J Klee and T.E. Graedel. Elemental cycles: A status report on human or natural dominance. *Annu. Rev. Environ. Resour.*, 29:69–107, 2004.

Allen Kneese, Robert U. Ayres, and Ralph C. d'Arge. *Economics and the Environment: A Materials Balance Approach*. Resources for the Future, 1971.

T.C. Koopmans. Economics among the sciences. *American Economic Review*, 69: 1–13, 1979.

Egor Kraev. Stocks, flows and complemantarity: formalizing a basic insight of ecological economics. *Ecological Economics*, 43:277–286, 2002.

Thomas Kuhn. *The Structure of Scientific Revolutions*. Chicago University Press, 1962.

Thomas Kuhn. *The Copernican Revolution: Planetary Astronomy in the Development of Western Thought*. Harvard University Press, 1992.

Philip.A Lawn. Scale,prices and biophysical assessments. *Ecological Economics*, 38: 369–382, 2001.

Philip.A Lawn and Richard.D Sanders. Has australia surpassed its optimal macroeconomic scale? finding out with the aid of 'benefit' and 'cost' accounts and a sustainable net benefit index. *Ecological Economics*, 28:213–229, 1999.

Jonathan Loh. Living planet report 2004. Technical report, WWF- World Wild Fund for Nature, 2002.

Jonathan Loh and Matthis Wackernagel. Living planet report 2004. Technical report, WWF- World Wild Fund for Nature, 2004.

J.E. Lovelock, M.J. Maggs, and R.J Wade. Halogenated hydrocarbons in and over the atlantic. *Nature*, 241:194–196, 1973.

Juan Martinez-Alier and Klaus Schlupmann. *Ecological Economics*. Basil Blackwell, 1987.

Emily Matthews, Christoff Amamm, Stefan Bringezu, Walter Httler, R Kleijn, Yuichi Moriguchi, Christian Ottke, Eric Rodenburg, Don Rogich, Heinz Schandl, Helmut Schltz, and Helga Van der Voet, Ester ad Weisz. The weight of nations: material outflows from industrial economies. Technical report, World Resources Institute, 2000a.

Emily Matthews, Christof Amann, Stefan Bringezu, Marina Fischer-Kowalski, Walter Httler, Rene Kleijn, Yuichi Moriguchi, Christian Ottke, Eric Rodenburg, Don Rogich, Heinz Schandl, Helmut Schtz, Ester Van Der Voet, and Helga Weisz. *The Weight of Nations: Material Outflows from Industrial Economies*. World Resources Institute, 2000b.

John Robert McNeill. *Something New Under the Sun: An Environmental History of the Twentieth-Century World*. W.W. Norton, 2001.

Ian Moffatt. Ecological footprints and sustainable development. *Ecological Economics*, 32(3):359–62, 2000.

M.J. Molina and F.S. Rowland. Stratospheric sink for chlorofluromethanes: Chlorine atom-catalysed destruction of ozone. *Nature*, 249:810–812, 1974.

Robin Naidoo. Economic growth and liquidation of natural capital: The case of forest clearence. *Ambio*, 80:194–208, 2004.

National-Research-Council. *Materials Count: The Case for Material Flow Analysis.* The National Academies Press, 2004.

Global Footprint Network. Ecological footprint standards 1.0: Comment draft 1. Technical report, Global Footprint Network, 2006.

Richard Norgaard. The case for methodological pluralism. *Ecological Economics*, 1: 35–37, 1989.

A.C. Pigou. Maintaining capital intact. *Economica*, 8:271–275, 1941.

Karl Polanyi. *The Great Transformation: The political and economic origins of our time.* Beacon Press, Boston, 2nd paperback edition, 2001.

Clive Ponting. *A Green History of the World: The Environment and the Collapse of Great Civilizations.* Penguin USA, reprint paper edition edition, April 1993.

D.J. Rapport. Ecological footprints and ecosystem health: complementary approaches to a sustainable future. *Ecological Economics*, 32(3):359–62, 2000.

William Rees. Eco-footprint analysis: merits and brickbats. *Ecological Economics*, 32(3):371–74, 2000.

William E. Rees. Ecological footprints and appropriated carrying capacity: what urban economics leaves out. *Environment and Urbanization*, 4(2):121–30, 1992.

Stuart Rojstaczer, Shannon Sterling, and Nathan Moore. Human appropriation of photosynthesis products. *Science*, 294:2549–2552, 2001.

Sherwood F. Rowland. Nobel lecture in chemsitry. Technical report, Nobel Prize in Chemistry, 1995.

J Roy, B Saugier, and H.A. Mooney. *Terrestrial Global Productivity.* Academic Press, 2001.

Thmoas Schrder. Daly's optimal scale of economic activity. *Ecological Economics*, 14:163–164, 1995.

Joseph Schumpeter. *History of Economic Analysis.* Oxford University Press, 1954.

Maurice Scott. Maintaining capital intact. *Oxford Economic Papers*, 36:59–73, 1984.

Peter.F. Smith. *Architecture and the principle of harmony.* RIBA Publications, 1987.

Marcus Stewen. The interdependence of allocation, distribution, scale and stability – a comment on herman e. daly's vision of an economics that is efficient, jsut and sustainable. *Ecological Economics*, 27:119–130, 1998.

Marcus Stewen. Letters to the editor: Response to herman daly. *Ecological Economics*, 30:2–3, 1999.

D'Arcy Wentworth Thompson. *On Growth and Form: The Complete Revised Edition.* Dover Paperback, 1992.

G. Cornelis van Kooten and Erwin H. Bulte. Externalities, subsidies, and the ecological footprint: an empirical analysis. *Ecological Economics*, 32(3):381–83, 2000.

Peter M. Vitousek, Paul R. Ehrilch, Anne H. Ehrlich, and Pamela A. Matson. Human appropriation of the products of photosyntheis. *Bioscience*, 36:368–373, 1986.

M Wackernagel and W Rees. *Our ecological footprint: reducing human impact on the earth*. New society, 1996.

Mathis Wackernagel. Why sustainability analyses must include biophysical assessments. *Ecological Economics*, 29(1):13–14, 1999.

Mathis Wackernagel, Chad Monfreda, Dan Moran, Steve Goldfinger, Diana Deumling, and Michael Murray. National footprint and biocapacity accounts 2004: the underlying calculation method. Technical report, Global Footprint Network, 2004.

Mathis Wackernagel, Niels Schulz, Diana Deumling, Alejandro Callejas Linares, Martin Jenkins, Valerie Kapos, Chad Monfreda, Jonathan Loh, Norman Myers, Richard Norgaard, and Jorgen Randers. Tracking the ecological overshoot of the human economy. *Proceedings of the National Academy of Science, USA*, 99(14): 9266–9271, 2002.

Mathis Wackernagel and Judith Silverstein. Big things first: focusing on the scale imperative with the ecological footprint. *Ecological Economics*, 32(3):391–94, 2000.

I.K. Wernick and J.H. Ausubel. National material metrics for industrial ecology. *Resources Policy*, 21:189–98, 1995.

George M. Woodwell. On purpose in science, conservation and government. *Ambio*, 31:432–36, 2002.