ABSTRACT

Title of dissertation: Minimal SUSY SO(10) Model and Neutrino Oscillations

Hock Seng Goh, Doctor of Philosophy, 2004

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Many neutrino experiments in the last few years have reported their large statistic data which all converge to the conclusion that the three known neutrinos have masses and mix among themselves. The mixing angles in the quark sector are known to be very small, whereas that for neutrinos are large. Understanding this difference between quarks and leptons is a major challenge of theoretical particle physics. This is especially acute in the framework of Grand Unified Theories (GUT) which unifies quarks and leptons. In this thesis, we show that a very simple supersymmetric SO(10) model predicts a large atmospheric mixing angle (θ_{23}), as well as a large solar angle (θ_{12}) as required to fit observations and a small but non-vanishing $\sin\theta_{13} \equiv U_{e3}$ without any extra assumption. The small neutrino masses are provided by the seesaw mechanism which is also one of the key ingredients of the model. This is the first extensive analysis that shows this model can have the correct predictions for the two mixing angles as well as the mass differences $\triangle m^2_{\odot}$ and $\triangle m^2_A$ required to explain the oscillation data. The prediction of the third angle "θ_{13}" can be tested in ongoing and planned experiments.

This model has a number of other predictions; in particular, we have deduced the predictions of the model for proton decay. We find the upper bounds on the partial lifetime for the modes $\tau(n \to \pi^0\bar{\nu}) = 2\tau(p \to \pi^+\bar{\nu}) \leq (5.7 - 13) \times 10^{32}$ yrs and $\tau(n \to K^0\bar{\nu}) \leq$

2.97×10^{33} yrs. These results can also be used to test the model.

The specific form of the seesaw mechanism that we need to make our prediction imply constraints on the physics at the GUT scale. We find that (i) SO(10) must break to SU(5) before breaking to the standard model; (ii) $B-L$ symmetry must break at the time of SO(10) breaking and (iii) constraints of unification seem to require that the minimal model must have a **54** dimensional Higgs field together with the minimal set of $\{\mathbf{210}, \mathbf{10}, \mathbf{126}, \overline{\mathbf{126}}\}$.

Minimal SUSY SO(10) Model and Neutrino Oscillations

by

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Dedication

To my lovely wife and to the memory of my parents

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Finally, I owe my greatest appreciation to my wife sylvia for her constant support and sacrificing so much keeping me accompany.

I keep in my heart the love from my parents which I can never repay.

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Chapter 1: Introduction

1.1 Neutrino Oscillations

Neutrino was first introduced in 1930 by Wolfgang Pauli to ensure energy-momentum conservation in beta decay. For a long time it was assumed that neutrinos are massless since there was no evidence to the contrary. However, neutrinos are in many ways similar to quarks and charged leptons. It is natural to assume that the neutrinos have mass like all known matter fields. Therefore, experiments have been conducted over the years searching for neutrino masses. An important feature of neutrino mass is oscillations between different types of neutrinos. In 1958, the possibility of neutrino-antineutrino oscillations was first suggested by Pontecorvo following the analogy to kaon oscillations and later in 1967, the concept was expanded to flavor oscillations[1] after ν_μ was discovered in 1962 in Brookhaven. The observation of such neutrino oscillations was suggested to be an effective method to search for neutrino masses compared to the usual method of nuclear β decay. Two kinds of neutrino oscillation searches have been carried out since the 1960's. One uses the neutrinos from the atmosphere and the other uses neutrinos from the sun. These experiments have culminated in an abundance of experimental evidence for neutrino masses in the last five years.

Atmospheric neutrinos are produced by high energy cosmic particles stopped by the atmosphere through the following processes

$$p(N..) \rightarrow \pi^{\pm}(K..) \rightarrow \mu^{\pm} + \nu_\mu(\bar{\nu}_\mu) \tag{1.1}$$

$$\mu^{\pm} \rightarrow e^{\pm} + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu)$$

The above process predicts that the number ratio of the muon-neutrinos (ν_μ) and the electron-neutrinos (ν_e) is close to 2 at sub-GeV energy. For higher energy events the ratio will be different because high energy muons $\mu^{\pm}$ can reach the earth before they decay. The discrepancy between the observed and the predicted atmospheric neutrino fluxes is called the atmospheric neutrino anomaly. Atmospheric neutrinos were first detected in 1965 by F. Reines et al. and H. Achar et al. [2]. Since 1998 when Super-Kamiokande presented their conclusive evidence of atmospheric neutrino oscillations by measuring the double ratio $\frac{(\nu_\mu/\nu_e)_{data}}{(\nu_\mu/\nu_e)_{predict}}$ which was found to be around 0.6, more data has been accumulated with very high statistic. Now it is a widely accepted fact that neutrino is massive.

Solar neutrinos are purely ν_e's produced by nuclear fusion in the core of the sun. The flux of ν_e's can be predicted by the standard solar model (SSM)[3]. Fig. (1.1) shows the ν_e fluxes and energy of solar neutrinos from different nuclear fusions. In 1970, Davis and his group in Homestake reported a deficit in the detection rate of ν_e's from the sun compared to the expected flux. This was confirmed by Super-Kamiokande in 1998[5], SAGE[6], GALLEX/GNO[7, 8] and SNO in 2002[9]. This are evidences for oscillations involving solar neutrino. The oscillations of $\overline{\nu}_e$'s from reactors was observed in the KamLAND experiment recently (2003)[10]. Before 2001, the parameter region which is compatible with experimental data within 3σ confidence level (C.L.) can be divided into four regions. They are denoted as small mixing angle (SMA), large mixing angle (LMA I and II), low mass (LOW) and vacuum oscillations (VO). Recent improvements of the solar neutrino data have reduced the allowed region of the oscillation parameter space . They are summarized below

- (2001) SNO(CC)+SK(ES) excludes (VO) and (SMA) solutions[11].
- (2002) SNO(NC) phase I (D_2O) refines the Boron flux and disfavor the LOW solution[9].

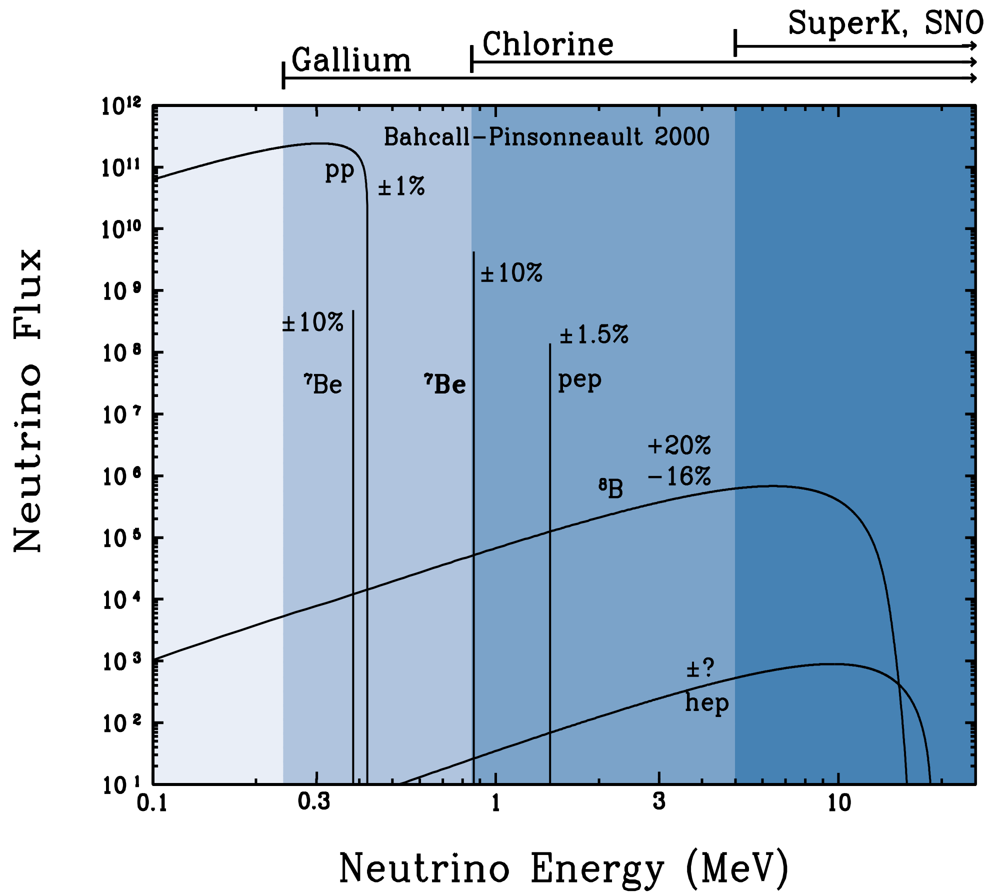

Figure 1.1: Solar neutrino spectrum and the theoretical estimated flux and error. The thresholds of different detector are indicated on the top of the diagram.[4].

- (2003) KamLAND rule out LOW at 5 σ, leave only LMA I (including the best fit point) and LMA II (at 99% CL)[10].
- (2003) after SNO phase II (salt phase), LMA II is allowed only at 3σ, maximum angle is allowed only at 5σ[12, 13].
- (2004) LMA II ruled out at 3σ.(766 Ty KamLAND Spectrum)[14]

These results strongly reinforce the neutrino oscillation interpretations of observa-

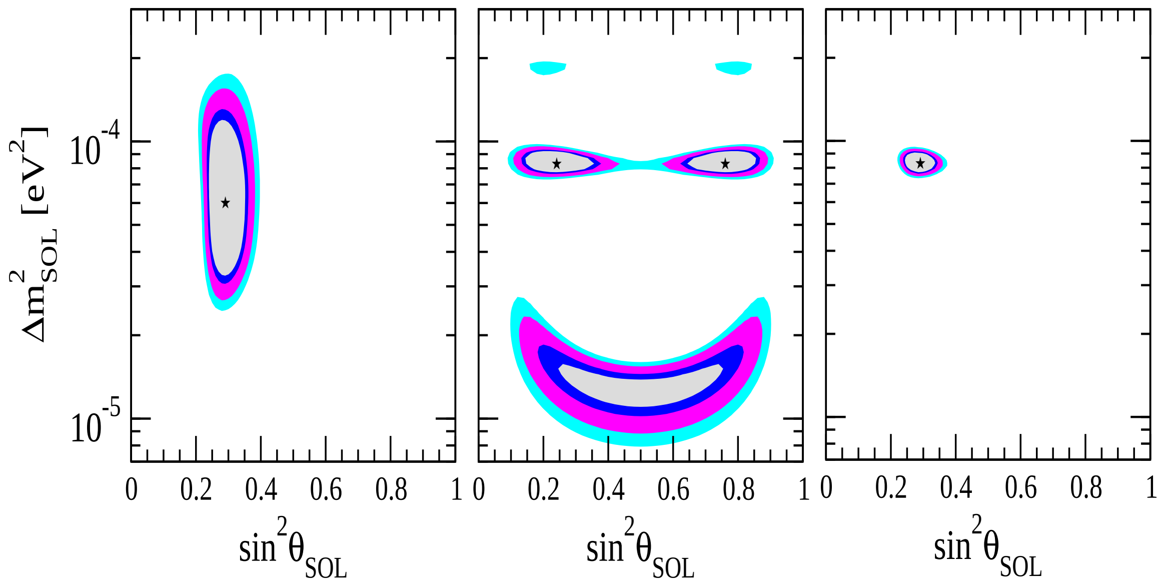

Figure 1.2: 90%, 95%, 99% and 99.73% C.L. allowed regions of the neutrino oscillation parameters from the analysis of the latest solar data (left panel), the 766.3 ton-yr KamLAND data (middle panel) and from the combined analysis (right panel).[15]

tions and lead to the conclusion that at least two of the three known neutrinos have to be massive. Recent analysis of the solar and the atmospheric neutrino oscillations is shown in fig.(1.2) and fig.(1.3).

All of the data from these experiments can be understood in the framework of three neutrino oscillations, which in turn can be parameterized by three mixing angles θ_{12}, θ_{23}, θ_{13} and two mass-squared differences : $\triangle^2_{32} = m^2_3 - m^2_2$ and $\triangle^2_{21} = m^2_2 - m^2_1$. $m_{1,2,3}$ are the masses of the three neutrinos $\nu_{1,2,3}$. In general, $\nu_{1,2,3}$ (mass eigenstates) do not coincide with the three neutrinos $\nu_{e,\mu.\tau}$ (weak eigenstates) which pair up with e, μ, τ in the isospin doublets. The mixing angles are defined as the transformation matrix between mass and weak eigenstates similar to the CKM matrix in quark sector and can be written in the

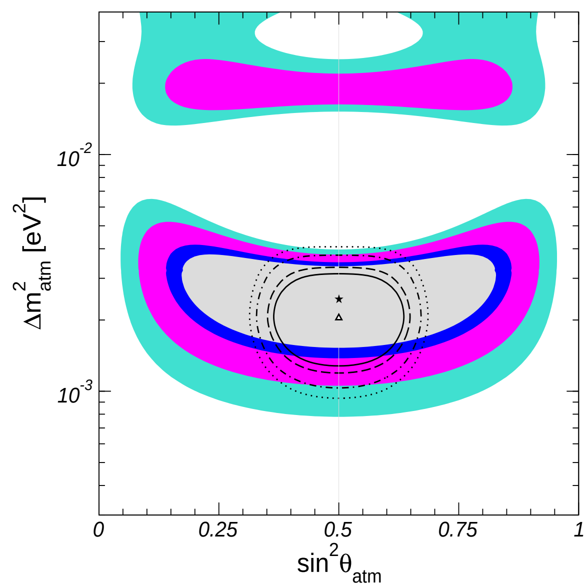

Figure 1.3: This figure shows the allowed K2K regions in the ($\sin^2\theta_{atm}$, $\triangle m^2_{atm}$) plane at 90%, 95%, 99%, and 3σ C.L. The hollow lines delimit the region determined from the atmospheric data only. The star (triangle) corresponds to the K2K (atmospheric) best fit point.[16]

conventional form

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\varphi} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\varphi} & 0 & c13 \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\rho_1} & 0 \\ 0 & 0 & e^{i\rho_2} \end{pmatrix} \tag{1.2}$$

with the relation $\nu_\alpha = U_{\alpha i}\nu_i$. α(i) is the weak(mass) eigenvector index and c_{ij}, s_{ij} are the short form of $\cos\theta_{ij}$ and $\sin\theta_{ij}$ respectively. There are three different complex phases in the matrix. $\rho_{1,2}$ are called Majorana phases which appear only when the neutrinos are Majorana particles and hence correspond to the violation of total lepton number $L = L_e + L_\mu + L_\tau$. The φ is called the Dirac phase analogous to the phase in the CKM matrix.

Neutrinos can only be produced and detected through the weak interaction, however, the energy of neutrinos has to be defined in the mass eigenstates. This results in the phenomenon of oscillations when the neutrinos travel through space. Under the approximation of $p \gg m$ [20], the transition probabilities $P_{\beta\alpha}$ of finding ν_β in the detector located at a distance L from the source of ν_α can be written as

$$\begin{aligned} P_{\beta\alpha} &= <\nu_\beta|\nu_\alpha> = \sum_{i,j} U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}exp(-i\frac{\triangle m^2_{ij}L}{2E_\nu}) \\ P_{\alpha\alpha} &= <\nu_\alpha|\nu_\alpha> = 1 - 2\sum_{i<j}|U_{\alpha i}|^2|U_{\alpha j}|^2\sin^2(\frac{\triangle m^2_{ij}L}{4E_\nu}) \end{aligned} \tag{1.3}$$

By fitting the experimental data, three mixing angles in $U_{\alpha i}$ and two $\triangle m^2$ can be determined. It is very easy to see that the two Majorana phases ρ_1 and ρ_2 in $U_{\alpha i}$ have no effect on the transition probabilities $P_{\beta\alpha}$. One nice way to understand this is that oscillations between ν_α and ν_β violate individual lepton number but conserve the total lepton number. The Majorana phases can not be probed by the L conserving process. We can ignore these phases safely in our discussion on neutrino oscillations. There is no experimental data about φ due to the fact that θ_{13} is small. In the case of two neutrino oscillations, the survival probability formula is simplified to the form

$$P_{\alpha\alpha} \;=\; 1 - \frac{1}{2}\sin^2 2\theta \sin^2(\frac{\triangle m^2 L}{4E_\nu}) \tag{1.4}$$

We should also note that when the neutrinos travel through matter instead of vacuum, the interaction with matter will significantly change the transition probabilities. Within matter, due to the weak interaction, the Hamiltonian is given by

$$H = \frac{MM^T}{2E} + \frac{1}{2E}\begin{pmatrix} A + A' & 0 & 0 \\ 0 & A' & 0 \\ 0 & 0 & A' \end{pmatrix} \tag{1.5}$$

where M is the non-diagonal mass matrix of the neutrinos. The quantities A and A' are contributions from charged-current and neutral-current scattering with electrons in matter. Taking into account the matter effect, one has to replace the mixing matrix and the m^2 in the transition probabilities formula by a new mixing matrix that diagonalizes H and the eigenvalues of H respectively. A' does not change the mixing angles because this part itself is just a term proportional to unit matrix. It does shift the m^2 by A'. However as can be seen from the transition probabilities formula, only the difference in m^2 makes a difference. In conclusion, the matter effect on the neutrino oscillations comes solely from A which is given by

$$A = \pm\frac{2\sqrt{2}G_F Y \rho E}{m_n} \tag{1.6}$$

Other observations involving neutrinos come from cosmology. We now summarize the results (or constraints) on the oscillation parameters from these experiments at the 3 σ C.L.

- $\sin^2 2\theta_{23} > 0.87$
- $0.7 < \sin^2 2\theta_{12} < 0.95$
- $1.2 \times 10^{-3} eV^2 < \triangle m^2_{32} < 3.4 \times 10^{-3} eV^2$ (sign unknown)
- $5.4 \times 10^{-5} eV^2 < \triangle m^2_{21} < 9.5 \times 10^{-5} eV^2$
- $\sin \theta_{13} < 0.23$
- $m_{ee} < 0.3$ eV ($\beta\beta_{0\nu}$)
- $\sqrt{\sum_i |U_{ei}|^2 m_i^2} < 2.2$ eV (beta decay)
- $\sum m_i < 0.7 - 2$ eV (WMAP)
- $\sum m_i < 0.42$ eV (SDSS at 90 % CL)
- $|m_\nu| \sim 1 - 2$ eV (if neutrino is the most significant dark matter)
- leptonic CP phases - unknown

Evidence of $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations was claimed by the LSND collaboration in 1994[17]. Their experimental fit of data requires the mass-squared difference to be $\sim 1\ eV^2$ as compared to $10^{-5}\ eV^2$ for solar type experiments and $10^{-3}\ eV^2$ for atmospheric type experiments. Their result, when combined with the result of atmospheric and solar neutrino, can not be simultaneously explained with only three neutrinos. If LSND is confirmed by the future experiment MiniBooNE, other extensions of three neutrino oscillations will be necessary. At this point, we will not include the LSND result into our analysis and work within the framework of the simple three neutrino oscillations.

1.2 Theory of Neutrino Masses and mixings-Grand Unified Theory

The discovery of neutrino oscillations provides the first evidence of “new physics” beyond the standard model which predicts the existent of three families of neutrinos and they are all massless. In order to find a way to give neutrino a mass, we have to understand why it was predicted to be massless in the first place. In the Standard Model (SM), fermion mass terms are prevented by the chiral SU(2)×U(1) (electroweak) symmetry. The left-handed particles are SU(2) doublets and the right-handed particles are singlets. Lorentz invariance requires that the mass terms of the fermions can only be of the form $\bar{L}R$, $\bar{R}L$, L^TCL or R^TCR where C is the charge conjugation operator and L, R represent left-handed and right-handed particle. The first two terms are complex conjugate to each other and they are called the Dirac mass terms and the last two are called Majorana mass terms. The fact that L and R transform differently under the SU(2) and the SU(2) singlet R in the SM has non-zero U(1) charge prevents any gauge and Lorentz invariant mass term. Masses can aries only after the spontaneous breaking of the electroweak (EW)symmetry. The gauge symmetry after EW symmetry breaking is SU(3)×$U(1)_{EM}$. The Dirac mass terms can be written down for L and R because they have the same charge under the $SU(3) \times U(1_{EM})$. The Majorana mass terms still require the particle to be singlet. Because the right-handed neutrinos are absent in the SM, the only possible neutrino mass term consistent with the symmetry is of Majorana type. However, in the SM where only renormalizable terms are included, B-L is an anomaly free accidental symmetry. This symmetry prevents the Majorana masses of neutrinos to aries because the Majorana mass term breaks B-L by 2 units. As a result, the neutrino masses are zero in the SM.

Here we see that the reasons of vanishing neutrino masses in the SM are the conservation of B-L and the absence of the right handed neutrino. If the standard model is treated as an effective theory, the left-handed Majorana neutrino masses can arise in nonrenormalizable lepton number violating terms coming from integrating out the hypothetical heavy particle at UV scale. These terms are suppressed by the heavy mass scale where new physics become important. If the right-handed neutrino is also added into the theory, one can write down the general neutrino masses in terms of a 4-component Dirac spinor ν

$$L = m_D \overline{\nu}\nu + \frac{1}{2} m_M \nu^T C \nu + h.c. \tag{1.7}$$

where $C = i\gamma^0\gamma^2$ is charge conjugation operator. In terms of the two-component spinor, $\nu = \begin{pmatrix} \chi_\alpha \\ i\bar{\phi}^{\dot{\alpha}} \end{pmatrix}$ where χ is the left-handed neutrino and $\bar{\phi}$ is the right-handed neutrino (and so ϕ is the right-handed antineutrino), the mass terms can be rewritten as

$$L = m_D(\chi\phi + \bar{\chi}\bar{\phi}) + \frac{1}{2} m_L(\chi\chi + \bar{\chi}\bar{\chi}) + \frac{1}{2} m_R(\phi\phi + \bar{\phi}\bar{\phi}) \tag{1.8}$$

In general, $m_R \neq m_L$ in a theory with parity violation. If the neutrinos are a purely Majorana particle, we have $\chi = \phi$. In matrix form, it looks like

$$\frac{1}{2} \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \tag{1.9}$$

If $m_R >> m_D$ and m_L, the light Majorana masses are given by

$$m_\nu = m_L - \frac{m_D^2}{m_R} \tag{1.10}$$

In the special case where $m_L = 0$, this is the well known seesaw mechanism used to explain the smallness of neutrino mass [18] and is now called type-I seesaw in the literature. The most general form with $m_L \neq 0$ is called the type-II seesaw[19]. This formula is a simplified version of the seesaw formula of three neutrinos where m_L, m_R and m_D are 3×3 matrices. In the full matrix form the equation is

$$m_\nu = m_L - m_D^T m_R^{-1} m_D \tag{1.11}$$

If the seesaw mechanism is the reason why neutrinos are light, we need m_R or the mass of the heavy particle that generate m_L to be of order 10^{15} GeV in order to have the heaviest neutrino mass be larger than 0.05 eV which is required to explain the observed $\triangle m_A^2 \sim 10^{-3}\ eV^2$. This implies clearly the existence of a new scale beyond the electroweak scale ($\sim$ 100 GeV). On the other hand, in the Minimal Supersymmetric Standard Model(MSSM), the three coupling constants, when extrapolated up in energy scale, intersect at 2×10^{16} GeV which is close to 10^{15} GeV. This strongly suggests that the UV theory which gives rise to the seesaw mechanism will probably be some kind of Grand Unified Theories.

The first task in the Grand Unified Theory is embedding the standard model into a simple group. This includes finding a simple group that contains SU(3)×SU(2)×U(1) as a subgroup and representations that contain fields in the SM as submultiplets. Among all classical simple groups, SU(5) is the smallest group that contains the SM group as a subgroup. The SM can then be embedded in SU(5) or any larger group that contains SU(5); for example, SO(10) or E6. Some of the reasons that make SO(10) models so attractive as grand unification theories of nature are the following: (i) all fermions in one family can be part of a single spinor representation; (ii) it contains the left-right symmetric unification group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$[21] which provides a more satisfactory way to understand the origin of parity violation in Nature; (iii) the single spinor representation discussed above also contains the right handed neutrino needed in implementing the seesaw mechanism and (iv) they are automatically anomaly free.

There is one more compelling reason for the SO(10) model: The right-handed(RH) neutrino has mass $M_R \leq 10^{15}$ GeV which is considerably smaller than the Planck mass and therefore one is faced with a new hierarchy problem similar to the corresponding problem of the standard model. However, it was pointed out long ago[22] that the Majorana mass of the RH neutrino owes its origin to the breaking of local B-L symmetry which implies that $M_R \simeq M_{B-L}$. Local B-L symmetry therefore provides a natural way to understand the smallness of the RH neutrino mass as compared to $M_{P\ell}$. What is very interesting is that SO(10) group also contains the local B-L as a subgroup.

1.3 The Minimal SO(10) Model

The results from the neutrino oscillation experiments not only show that the neutrinos have small masses, they also show the differences between Quarks and leptons: The two mixing angles in the lepton sector are large while all of the mixing angles in the quark sector are small. The SO(10) model, despite its attractiveness for understanding the overall scale of neutrino masses with the seesaw mechanism, runs into a potential trouble in providing an understanding of the observed mixings. The problem arises from the fact that SO(10) unifies the quarks and leptons into one single spinor representation as mentioned in the previous section. In the simplest approximation this type of model leads to equal quark and lepton mixing angles. Resolving this difference within the framework of GUT is a big challenge in model building.

One approach is to make further assumptions to get a handle on the mixings[27]. An obvious conceptual problem is that if one of these models is ruled out by data, one would not be able to tell whether it is the SO(10) unification which is "at fault" or it is one of the assumptions used to derive neutrino mixings. A different approach, which we will be following, to this issue was taken in ref.[28]. The idea is to avoid the use of any symmetries beyond the gauge symmetry, in this case SO(10), and use the minimal set of Higgs fields that can break the group down to the standard model and give mass to the fermions. The minimal set of Higgs field should contain the following : (i) For SO(10) multiplets not bigger than 126 dimensions, in order to break the group down to standard model, we need at least one multiplet from each of the following two sets of Higgs fields : $\{\mathbf{16},\mathbf{126}\}$[23] and $\{\mathbf{45},\mathbf{54},\mathbf{210}\}$. Note that the numbers used to name the different Higgs multiplets are the number of components in the multiplets. The reason for this requirement is that, in the language of SU(5)×U(1) (one of the two maximal subgroups of SO(10)), the vacuum expectation value (vev) of each element in the first set breaks the U(1) (and so it also breaks B-L) but conserves the full SU(5) and the vev of Higgs in the second set conserves U(1) but has a component which breaks SU(5) to the standard model (for the branching rules of SO(10) see [76]). The intersection of the resulting subgroups SU(5) and SM$\bigotimes$U(1) is the standard model while each one of the Higgs vev's preserve a bigger gauge group. And (ii), at the renormalizable level, at lease two of the $\{\mathbf{10},\ \mathbf{126},\mathbf{120}\}$ are needed in order to get sensible mass relations among quarks and charged leptons. The matter field in one family of the SM can be packed into one spinor representation $\mathbf{16}_f$ of SO(10). The masses of fermions are generated from the Yukawa terms $\mathbf{16}_f\mathbf{16}_f H$ where H can only be either $\mathbf{10}$, $\overline{\mathbf{126}}$ or $\mathbf{210}$. With only one of these Higgs fields couple to matter, all of the mass matrices are proportional to the same Yukawa matrix and this results in the same mass hierarchy and same mixings, which are actually vanished, in the quark and lepton sector. This is of course contradicting the observations.

There are reasons why we have picked a certain set of Higgs fields for our minimal model. It was observed in ref.[28] that in the model with $\mathbf{10}$ and $\overline{\mathbf{126}}$, the neutrino masses and mixings are completely predicted up to an overall scale, when one uses the seesaw mechanism which is part of the SO(10) model[1]. An appealing feature of breaking B-L

[1]This is to be contrasted with the SU(5) case where the minimal Higgs set needed to break the gauge

symmetry of SUSY SO(10) by a **126**, as opposed to by **16** Higgs, is that it automatically leaves R-parity as an exact symmetry and thereby explains why the neutralino is stable and can be a candidate for dark matter[24, 25]. This is because the submultiplet of **126** that breaks B-L carries B-L =2. The R-parity (defined by $Rp = (-1)^{3(B-L)+2S}$) quantum number of this field is even and therefore, its vacuum expectation value(vev) leaves R-parity unbroken. In contrast,in the models where B-L is broken by a **16**-plet of Higgs, the B-L symmetry is broken by one unit and without any additional symmetries (e.g. matter parity), the neutralino is unstable and cannot therefore serve as a dark matter[26]. Of course, if a fundamental theory e.g. a superstring theory that led to an SO(10) model with appropriate additional symmetries that guarantee the stability of the neutralino was known, then the above objection to an **16** Higgs would not apply. For the above reasons, we decide to choose $\{\mathbf{10},\overline{\mathbf{126}},\mathbf{126}\}$ as the minimal set of Higgs to break B-L and give masses to fermions. Finally, we have to pick one multiplet from the set $\{\mathbf{45},\mathbf{54},\mathbf{210}\}$ to complete the symmetry breaking. It turns out that at the renormalizable level, if we pick a **45**, the supersymmetric vev of **45** vanished and the resulting group becomes SU(5); If we pick a **54**, the supersymmetric vev of **126** vanished and the resulting group is SM×U(1). Although the combination of **45** and **54** will work, we choose a **210** instead to follow the criteria of a minimal number of multiplets. Our final choice for the minimal set of Higgs multiplets is therefore $\{\mathbf{10},\ \mathbf{126},\ \overline{\mathbf{126}},\mathbf{210}\}$.

The question now is whether this minimal model can have large mixing angles in the neutrino sector and correct fermion masses ? This is the question we want to answer in the following chapters.

symmetry i.e. **5**+**24** Higgses lead to the mass relation $m_e/m_\mu = m_d/m_s$ that is in contradiction with observations.

1.4 Outline of Thesis

In Chap. 2, we present the effective theory of the minimal SUSY SO(10) Model below the GUT scale. This effective theory is useful in the analysis of fermion masses and mixing without going into the details of symmetry breaking. We show how the mass formulae of all the quarks and leptons can be derived from the group structure and how the masses of leptons can be written purely in terms of the known quark masses and the CKM matrix plus two effective parameters. These relations show that the model is totally predictive in the neutrino sector. We then analyze the mass formulae by using both perturbative methods and numerical scanning to find the predictions of neutrino masses and mixings.

In Chap. 3, we write down the general Higgsino mediated dimension 5 operator which leads to proton decay. We use the result from the chapter 2 to make the prediction for proton decay through different modes.

In Chap. 4, we found a specific "minimal" model that satisfies the constraints imposed on the effective parameters which were used in chapter 2. In this chapter, we explicitly calculate the vacuum expectation values and the masses of all heavy Higgs particles. We argue how this model works consistently with the requirement of coupling unification even though there are some extra light Higgs multiplets other than the two doublets in MSSM.

Chapter 5 provides the conclusion to the thesis.

In the appendix, we include some tools for calculating the SUSY SO(10) superpotential in term of SU(5) irreducible representations. Explicit tensor representations are also given.

Chapter 2: Predictive SO(10) Model and Understanding Neutrino Masses and Mixings

The SUSY SO(10) model that we will work with has the following features: It contains three spinor superfields **16** each contain one whole family of the matter fields; two sets of Higgs fields, one contains $\overline{\mathbf{126}}$ ($\overline{\Sigma}$) and **10** (H) that couple directly to the matter fields. Another set contains **126** (Σ),**210** (Φ) and **54**(S) that only couple to the matter fields through the nonrenormalizable terms. We constraint ourself to include only renormalizable terms in the general superpotential which is consistent with the symmetry group SO(10). On the stage of symmetry breaking, both **126**,$\overline{\mathbf{126}}$ and **210** play the role of breaking the symmetry down to SU(5); **210** can further break the symmetry down to Standard Model. Because **10** and $\overline{\mathbf{126}}$ couple to the matter fields, after symmetry breaking they give rise to all the fermion masses including neutrinos. **54** is not necessary as far as symmetry breaking in concerned[35]. The inclusion of **54** is however important for the reason we will discuss later in chapter 4.

2.1 The Mass Sumrules For Minimal SO(10)

It is the $\mathbf{10}\oplus\overline{\mathbf{126}}$ Higgs which is crucial to our discussion of fermion masses. The first stage of the symmetry breaking that break SO(10) down to SM at the GUT scale could have been accomplished by **210**,**126**, $\overline{\mathbf{126}}$ and any other Higgs without effecting our results. We will get back to the detail of the symmetry breaking in chapter 4. For now, we assume that the symmetry breaking is accomplished and the theory right below the GUT scale contain only two Higgs doublets as in MSSM. As has been noted earlier[28, 26], the set $\mathbf{10}+\overline{\mathbf{126}}$ which couple to matter contains two pairs of MSSM Higgs doublets belonging to

(2,2,1) and (2,2,15) submultiplets (under $SU(2)_L \times SU(2)_R \times SU(4)_c$ subgroup of SO(10)). The **210** also have a pair of doublets which has the same quantum number as the MSSM Higgs pair (h_u, h_d). In the language of SU(5), **5** of H, Φ, Σ and **45** of $\overline{\Sigma}$ have the same quantum number as h_u; $\overline{\mathbf{5}}$ of H, Φ, $\overline{\Sigma}$ and $\overline{\mathbf{45}}$ of Σ has the same quantum number as h_d. At the GUT scale, by some doublet-triplet splitting mechanism the MSSM Higgs pair can be expressed in terms of linear combination of these fields as follows:

$$\begin{aligned} h_u &= \alpha_u^1 H_u + \alpha_u^2 \bar{\Sigma}_u + \alpha_u^3 \Phi_u + \alpha_u^4 \Sigma_u^{45} \\ h_d &= \alpha_d^1 H_d + \alpha_d^2 \bar{\Sigma}_d^{45} + \alpha_d^3 \Phi_d + \alpha_d^4 \Sigma_d \end{aligned} \quad (2.1)$$

In order to discuss fermion masses in this model, we start with the SO(10) invariant superpotential giving the Yukawa couplings of the **16** dimensional matter spinor ψ_i (where i, j denote generations) with the Higgs fields H and $\overline{\Sigma}$.

$$W_Y = h_{ij}\psi_i\psi_j H + f_{ij}\psi_i\psi_j\bar{\Sigma} \quad (2.2)$$

SO(10) invariance implies that h and f are symmetric matrices. We ignore the small effects coming from the higher dimensional operators. Below the GUT scale, we can write the superpotential terms for the charged fermion Yukawa couplings as:

$$W_0 = Y_u Q h_u u^c + Y_d Q h_d d^c + Y_e L h_d e^c + Y_\nu L h_d \nu^c + v_{B-L}\nu^c\nu^c + \frac{\kappa}{M_T} f(Lh_u)(Lh_u) \quad (2.3)$$

As in the case of MSSM, we will assume that the Higgs doublets $h_{u,d}$ have the vevs $< h_u^0 >= v\sin\beta$ and $< h_d^0 >= v\cos\beta$, which then leads us to the mass formulae for quarks and leptons at the GUT scale as:

$$\begin{aligned} M_u &= \bar{h} + \bar{f} \\ M_d &= \bar{h} r_1 + \bar{f} r_2 \\ M_e &= \bar{h} r_1 - 3 r_2 \bar{f} \end{aligned} \quad (2.4)$$

$$
\begin{aligned}
M_{\nu^D} &= \bar{h} - 3\bar{f} \\
M_{N^R} &= v_{B-L} f \\
M_{\nu} &= v_L f = \frac{\kappa v^2 \sin^2\beta}{M_T} f
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{h} &= 2hv\overline{\alpha}_u^1 \sin\beta \\
\bar{f} &= \frac{1}{\sqrt{6}} f v \overline{\alpha}_u^2 \sin\beta \\
r_1 &= \frac{\overline{\alpha}_d^1}{\overline{\alpha}_u^1} \cot\beta \\
r_2 &= -\frac{2\overline{\alpha}_d^2}{\sqrt{3}\overline{\alpha}_u^2} \cot\beta
\end{aligned}
\tag{2.5}
$$

In general $r_1 \neq r_2$ and this difference is responsible for nonzero CKM mixing angles. We assume here everything are real and the CP violations come from SUSY breaking sector. To count the number of parameters describing the fermion sector, we choose a basis where $\bar{h}$ is diagonal. Since $\bar{f}$ is symmetric, we have a total of nine parameters from the couplings and including $r_{1,2}$ and β gives us a total of twelve parameters. All these parameters can be determined by fitting the the six quark masses, three lepton masses and three CKM angles. This enables a complete determination of the neutrino masses up to an overall scale related to the B-L symmetry breaking and the three mixing angles. The model is therefore completely predictive in the neutrino sector.

In order to determine the neutrino masses and mixings, one uses the seesaw mechanism as noted in the introduction. In this model, we have the left handed Majorana term induced from $SU(2)_L$ triplet vev v_T or from higher dimensional terms involving left doublets which can implement the general seesaw mechanism (called type II here and in the literature)[19] :

$$
\mathcal{M}_\nu = f v_L - M_{\nu^D} M_{N_R}^{-1} M_{\nu^D}^T \tag{2.6}
$$

where $M_{N_R} = fv_{B-L}$. If the first term is small compare to the second, the analysis is similar to that of type I which have been studied by many people[28, 29, 30, 31, 32, 33]. The conclusion now appears to be that one needs CP violating phases to achieve this goal, as noted in [33]. Even though, the ratio $\frac{\triangle m_{\odot}^2}{\triangle m_A^2}$ is still big compared with experiment. A way out of this problem is to use the type II seesaw mechanism, as was initially done in [31]. A very interesting point about this approach has been noted in a recent paper[34], where it has been shown that if we restrict ourselves to the 2-3 sector of the model and use the type II seesaw mechanism with only the first term, then the $b-\tau$ unification of supersymmetric grand unified theories leads to a neutrino Majorana mass matrix which explains the large $\nu_\mu - \nu_\tau$ mixing angle needed to understand atmospheric neutrino data. The important point is that no symmetries are needed to get this result. To understand this, first note that if the first term dominate, as was shown in [31] and can be easily derived from eq. 2.5, one gets a sumrule

$$\mathcal{M}_\nu \; = \; a(\mathcal{M}_\ell - \mathcal{M}_d) \tag{2.7}$$

since this relation is valid at the GUT scale, one must use the extrapolated quark and lepton masses in the formula. The fact that at or near the GUT scale $m_b/m_\tau \; \simeq 1 - 1.2$ depending on the value of $\tan\beta$, implies that the 3-3 element of the $\mathcal{M}_\nu$ which is proportional to $m_b - m_\tau$ has value of order o.2. If the off diagonal elements of the $\mathcal{M}_{nu}$ in the 2-3 subsector has also the same order , this resulting matrix leads to the largeness of the atmospheric mixing angle without any further assumptions.

It is however essential to do a complete three generation analysis of this model if this important observation is to lead to a realistic SO(10) model for understanding all neutrino mixings. In fact, since the model has no free parameters, it is a priori not obvious that within this framework one would simultaneously get a large solar mixing angle and a small

$\sin\theta_{13} \equiv U_{e3}$ as well as the correct value for the ratio $\Delta m^2_\odot/\Delta m^2_A$. It is the goal of this chapter to analyze this question. We will leave the detail in comparing the first and the second term in the seesaw formula until chapter 4. and simply assume now for certain range of parameters, the induced triplet vev term can dominate the neutrino mass matrix.

2.2 Details of Calculation

In this section, we outline our method for determining the neutrino mixing parameters. For this purpose, we first note that the matrices $\bar{h}$ and $\bar{f}$ in Eq. (2.5) can be eliminated in terms the mass matrices $M_{u,d}$ so that we have a sumrule involving the three mass matrices $M_{u,d,\ell}$. Before giving the sum rule, we note that we will work in a basis where M_d is diagonal and M_u is given by $M_u = V^T \cdot M_u^D \cdot V$ (where M_u^D is the diagonal mass matrix of up type quark and V is the CKM mixing matrix). This can be done without any loss of generality. We also introduce a new set of matrices $\tilde{M}_{l,u,d}$ where $\tilde{M} \equiv \frac{M}{m_3}$, m_3 being the third family mass for the corresponding flavor. The sumrule for charged lepton matrices is given by:

$$k\tilde{M}_l = r\tilde{M}_d + \tilde{M}_u \tag{2.8}$$

where k and r are functions of $r_{1,2}$ and fermion masses as follows:

$$k = \frac{r_2 - r_1}{4r_1 r_2}\frac{m_\tau}{m_t} \tag{2.9}$$

$$r = -\frac{r_2 + 3r_1}{4r_1 r_2}\frac{m_b}{m_t} \tag{2.10}$$

and left handed Majorana mass matrix is

$$\mathcal{M}_\nu \ \propto \ (\frac{m_b}{m_\tau}\tilde{M}_d - \tilde{M}_l) \tag{2.11}$$

The proportional constant is not important at this step while we will only be fitting $\frac{\triangle m^2_\odot}{\triangle m^2_A}$. This constant will be needed finally to get the correct $\triangle m^2$ and will be discussed in chapter

4. Note that these relations are valid only at the GUT scale.

The advantage of working with $\tilde{M}$ rather than M is that the 33 elements of all $\tilde{M}_{l,u,d}$ matrices are either one or of order one; so we expect solutions for k and r also of order one. Furthermore since the formula for $\mathcal{M}_\nu$ involves only M_ℓ and M_d, $b-\tau$ unification helps to see the cancellation in the 33 element of $\mathcal{M}_\nu$ somewhat more easily. At the same time the 23 element of $\mathcal{M}_\nu$ receives only one contribution from M_ℓ since in our basis M_d is diagonal. These two results lead to atmospheric mixing angle being large[34].

To carry out the calculations, we have to solve for the two unknowns k and r using the low energy inputs from the quark and charged lepton sectors. To obtain a perturbative estimation of these parameters, we decompose $r\tilde{M}_d + \tilde{M}_u$ as:

$$\begin{pmatrix} x & 0 & 0 \\ 0 & y & \epsilon_2 \\ 0 & \epsilon_2 & z \end{pmatrix} + \begin{pmatrix} 0 & \epsilon_1 & a \\ \epsilon_1 & 0 & 0 \\ a & 0 & 0 \end{pmatrix} \equiv r \begin{pmatrix} d & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} u & \epsilon_1 & a \\ \epsilon_1 & c & \epsilon_2 \\ a & \epsilon_2 & 1 \end{pmatrix} \tag{2.12}$$

where $\epsilon_i, a \ll 1$ as are x and y. In this analytical approach, our procedure will be to find the eigenvalues of (2.12) by perturbation method and match them to the known leptonic masses at the GUT scale. The advantage of this decomposition is that it allows a nice perturbative determination of the eigenvalues analytically without having to resort to immediate numerical analysis. We will compare our results with the numerical evaluation using Mathematica.

The i^{th} eigenvalue $\lambda_i = \lambda_i^{(0)} + \lambda_i^{(2)}$ is found to be

$$\begin{aligned} \lambda_1^{(0)} &= x \\ \lambda_2^{(0)} &= \frac{y+z-\sqrt{(z-y)^2+4\epsilon_2^2}}{2} \\ \lambda_3^{(0)} &= \frac{y+z+\sqrt{(z-y)^2+4\epsilon_2^2}}{2} \sim z + \frac{\epsilon_2^2}{z} + zO(10^{-2}) \end{aligned} \tag{2.13}$$

$$\lambda_2^{(2)} \simeq \frac{(z\epsilon_1 - a\epsilon_2)^2}{\lambda_2^{(0)} z^2} \simeq O(10^{-2})\lambda_2^{(0)}$$

$$\lambda_3^{(2)} \simeq \frac{a^2}{\lambda_3^{(0)}} \simeq O(10^{-2})\lambda_3^{(0)}$$

$$\lambda_1^{(2)} = -(\lambda_1^{(2)} + \lambda_3^{(2)})$$

We consider only cases where $y \simeq 10^{-2}$ and $z > 0.1$. Within this regime, the unperturbed 2nd and 3rd lepton masses are accurate up to a few %. However, the higher order electron mass correction is big and so the perturbation formula breaks down for this case. We therefore use the perturbation technique for the second and third generation masses but use the determinant to find that for the first generation. As mentioned, we will check the validity of perturbation result using numerical methods.

Taking determinant of the above equation 2.12, we find that the three charged lepton masses are related as follows:

$$k^3 \tilde{m}_e \tilde{m}_\mu = xyz - x\epsilon_2^2 - ya^2 - z\epsilon_1^2 + 2a\epsilon_1\epsilon_2 \tag{2.14}$$

$$k\tilde{m}_\mu = \lambda_2 \simeq \lambda_2^{(0)} \tag{2.15}$$

$$k = \lambda_3 \simeq \lambda_3^{(0)} \simeq z + \frac{\epsilon_2^2}{z} \tag{2.16}$$

We now solve the above equation by substituting x,y,z,a,ϵ_1,ϵ_2 with the corresponding elements in the matrix $rM_d + M_u$. From eq.(2.14), we find

$$k(1 + \tilde{m}_\mu) = y + z \tag{2.17}$$

$$k = z + \frac{\epsilon_2^2}{z} \tag{2.18}$$

Since Eq.2.12 tells us that $z = 1 + r$ and $y = rs + c$, we can use the above two equations to determine the parameters k and r, which we can then use to find neutrino masses and mixings. We find k and r to be

$$r = \frac{(s + c - 2\tilde{m}_\mu) \pm \sqrt{(s-c)^2 - 4(\tilde{m}_\mu - s)(1 + \tilde{m}_\mu)\epsilon_2^2}}{2(\tilde{m}_\mu - s)} \tag{2.19}$$

$$k = \frac{(1+s)r + 1 + c}{1 + \tilde{m}_\mu}$$

and a consistency relation for the d-quark mass

$$d = \frac{k^3 \tilde{m}_e \tilde{m}_\mu + z\epsilon_1^2 + ya^2 - 2a\epsilon_1\epsilon_2 - u(yz - \epsilon_2^2)}{r(yz - \epsilon_2^2)} \qquad (2.20)$$

In order to get a rough feeling for the way the maximal neutrino mixings arise, let us diagonalize the charged lepton mass matrix given in Eq. 2.8 and write the neutrino mass matrix in this basis:

$$\mathcal{M}_\nu = \; a(\frac{m_b}{m_\tau} U_l^\dagger \tilde{M}_d U_\ell - \tilde{M}_l^D) \qquad (2.21)$$

Where $\tilde{M}_l^D$ is the diagonal charged lepton mass matrix with τ mass is 1. U_l is the rotation matrix diagonalize charged lepton mass. U_l can be written approximately as

$$U_l \simeq \begin{pmatrix} 1 & \delta_1 & \delta_2 \\ \Delta_1 & \cos\phi & \sin\phi \\ \Delta_2 & -\sin\phi & \cos\phi \end{pmatrix}, \qquad (2.22)$$

where

$$\tan\phi = \frac{\epsilon_2}{z - \lambda_2^{(0)}}. \qquad (2.23)$$

The parameters δ_i and Δ_i are given to lowest order in perturbation theory by

$$\delta_1 = \frac{\epsilon_1 \cos\phi - a\sin\phi}{k\tilde{m}_\mu - x} \qquad (2.24)$$

$$\delta_2 = \frac{\epsilon_1 \sin\phi + a\cos\phi}{k - x}$$

$$\Delta_1 = -\delta_1 \cos\phi - \delta_2 \sin\phi$$

$$\Delta_2 = \delta_1 \sin\phi - \delta_2 \cos\phi$$

Using these parameters and neglecting small terms due to δ_1 and δ_2 multiplying light quark masses, we find that

$$M_\nu \simeq \begin{pmatrix} m_d - m_e + m_s\Delta_1^2 + m_b\Delta_2^2 & m_s\Delta_1\cos\phi - m_b\Delta_2\sin\phi & m_s\Delta_1\sin\phi + m_b\Delta_2\cos\phi \\ m_s\Delta_1\cos\phi - m_b\Delta_2\sin\phi & m_s - m_\mu + m_b\sin^2\phi & -m_b\sin\phi \\ m_s\Delta_1\sin\phi + m_b\Delta_2\cos\phi & -m_b\sin\phi & -m_b\sin^2\phi + m_b - m_\tau \end{pmatrix} \tag{2.25}$$

We now find the following analytic expression for the atmospheric mixing angle from Eq. 2.25 to leading order ignoring small terms to be:

$$\tan\theta_A \simeq \frac{2}{q + \sqrt{q^2+4}} \tag{2.26}$$

$$q = \frac{2m_b\sin^2\phi + (m_\tau - m_b) + (m_s - m_\mu)}{m_b\sin\phi}$$

For $|q| \leq 1$, we get $\sin^2 2\theta_A \geq 0.8$. We see that $b - \tau$ unification i.e. $m_b \simeq m_\tau$ and $m_b\sin\phi \simeq (m_b - m_\tau)$ are important to get a large θ_A. Also we need to have $m_s < 0$ and $m_\mu > 0$.

2.3 Predictions For Neutrino Masses And Mixings

In order to obtain the predictions for neutrino masses and mixings in our model, we will need the values of quark masses and mixings at the GUT scale. Experiments determine these input parameters near the GeV scale and they need to be extrapolated to the GUT scale which is 2×10^{16} GeV where our equations (2.5) are valid. Taking the values for the quark masses and mixings at the GUT scale we can determine k and r approximately. We will use this determination of k and r to solve for neutrino masses and mixings using the relation in Eq.2.11. We will also compare our results with a direct numerical scan of the Eq. 2.8 i.e. not using perturbation method to obtain k and r. Results obtained by both methods are in agreement.

In our model, the theory below the GUT breaking scale is the MSSM whose effect on fermion mass extrapolation is a well studied problem[48]. We will use the two loop analysis in the paper by Das and Parida[48] and take the values of the quark masses at the scale 2×10^{16} GeV in our analysis. In Table I, we give the input values of masses and mixings for values of the MSSM parameter $\tan\beta = 10$ and 55.

input observable	$\tan\beta = 10$	$\tan\beta = 55$
m_u (MeV)	$0.72^{+0.13}_{-0.14}$	$0.72^{+0.12}_{-0.14}$
m_c(MeV)	$210.32^{+19.00}_{-21.22}$	$210.50^{+15.10}_{-21.15}$
m_t(GeV)	$82.43^{+30.26}_{-14.76}$	$95.14^{+69.28}_{-20.65}$
m_d (MeV)	$1.50^{+0.42}_{-0.23}$	$1.49^{+0.41}_{-0.22}$
m_s (MeV)	$29.94^{+4.30}_{-4.54}$	$29.81^{+4.17}_{-4.49}$
m_b (GeV)	$1.06^{+0.14}_{-0.08}$	$1.41^{+0.48}_{-0.19}$
m_e (MeV)	0.3585	0.3565
m_μ(MeV)	$75.6715^{+0.0578}_{-0.0501}$	$75.2938^{+1912}_{-0.0515}$
m_τ (GeV)	$1.2922^{+0.0013}_{-0.0012}$	$1.6292^{+0.0443}_{-0.0294}$

Table I: The extrapolated values of quark and lepton masses at the GUT scale from the last reference in [48]. We have kept the errors to only two significant figures in the quark masses.

The quark and lepton masses and the uncertainties at the the scale $\mu = M_Z$ used in [48] are

$$m_u = 2.33^{+0.42}_{-0.45} MeV \tag{2.27}$$

$$m_c = 6.77^{+56}_{-61} MeV$$

$$m_b = 181 \pm 13 GeV$$

$$m_d = 4.69^{+0.6}_{-0.66} MeV$$

$$m_s = 93.4^{+11.8}_{-13.0} MeV$$

$$m_b = 3.00 \pm 0.11 GeV$$

$$m_e = 0.48684727 \pm 0.00000014 MeV$$

$$m_\mu = 102.75138 \pm 0.00033 MeV$$

$$m_\tau = 1.74669^{+0.00030}_{-0.00027} GeV$$

and the SUSY breaking scale M_s is assumed to be 1 TeV. For the mixing angles at GUT scale, we take:

$$V_{CKM} = \begin{pmatrix} 0.974836 & 0.222899 & -0.00319129 \\ -0.222638 & 0.974217 & 0.0365224 \\ 0.0112498 & -0.0348928 & 0.999328 \end{pmatrix} \quad (2.28)$$

In the first perturbation method, we use the above input values to obtain k and r using Eq. 2.19 and search for values around them that give a good fit to charged lepton masses and then use them in Eq.2.11 to derive the neutrino masses and the three mixing angles: $\sin^2 2\theta_\odot$, $\sin^2 2\theta_A$ and U_{e3}. The best fit range for k, r are $-.78 \leq r \leq -.74$ and $0.23 \leq k \leq .26$. We also do a direct numerical solution. Both the results are in agreement. (We ignore CP violation in this work.)

Note that the sign of a fermion is not physical, which leads to several choices for the sign of fermion masses that we have put into our search for solutions. The only choice we found our solutions correspond to $m_{e,\mu,\tau,b,t} > 0$ and $m_{c,d,s} < 0$ up to an overall sign.

Our results are displayed in Fig. 1-3 for the case of the supersymmetry parameter $\tan\beta = 10$. In these figures, we have restricted ourselves to the range of quark masses for which the atmospheric mixing angle $\sin^2 2\theta_A \geq 0.8$. (For presently preferred range of values of $\sin^2 2\theta_A$ from experiments, see [49]). We then present the predictions for $\sin^2 2\theta_\odot$, $\Delta m^2_\odot$ and U_{e3} for the allowed range $\sin^2 2\theta_A$ in Fig.1, 2 and 3 respectively. The spread in the predictions come from uncertainties in the s, c and the b-quark masses. Note two important predictions: (i) $\sin^2 2\theta_\odot \geq 0.91$ and $U_{e3} \sim \pm 0.16$. The present allowed range for the solar mixing angle is $0.7 \leq \sin^2 2\theta_\odot \leq 0.95$ at 3σ level[49, 50]. The solutions for the neutrino mixing angles are sensitive to the b quark mass.

It is important to note that this model predicts the U_{e3} value very close to the present experimentally allowed upper limit and can therefore be tested in the planned long base line experiments which are expected to probe U_{e3} down to the level of ~ 0.05[51, 52]. Our model would also prefer a value of the $\sin^2 2\theta_A$ below 0.9, which can also be used to test the model. For instance, the JHF-Kamioka neutrino experiment[52] is projecting a possible accuracy in the measurement of $\sin^2 2\theta_A$ down to the level of 0.01 and can provide a test of this model.

We have also checked that as $\tan\beta$ increases, the allowed values for the neutrino mixings and masses fall into an even narrower range. The result is disfavored by the experimental data. Note that, since renormalization played an important role in obtaining this result, one must ask what happens to the neutrino mixings once they are extrapolated to the weak scale[36]. It is well known[36] that for the case of normal hierarchy for neutrino masses as is the case here, the MSSM RGE's do not change the mixing angles very much and the GUT scale result persists at the weak scale with only minor changes.

From the result of above, we extract all the Yukawa parameters $\bar{h}_{ij}$ and $\bar{f}_{ij}$ corresponding to viable neutrino oscillation prediction. h and f on the other hand can be obtained if α_u^2 is known. A typical set of values for h's and f's in this range if the mixing

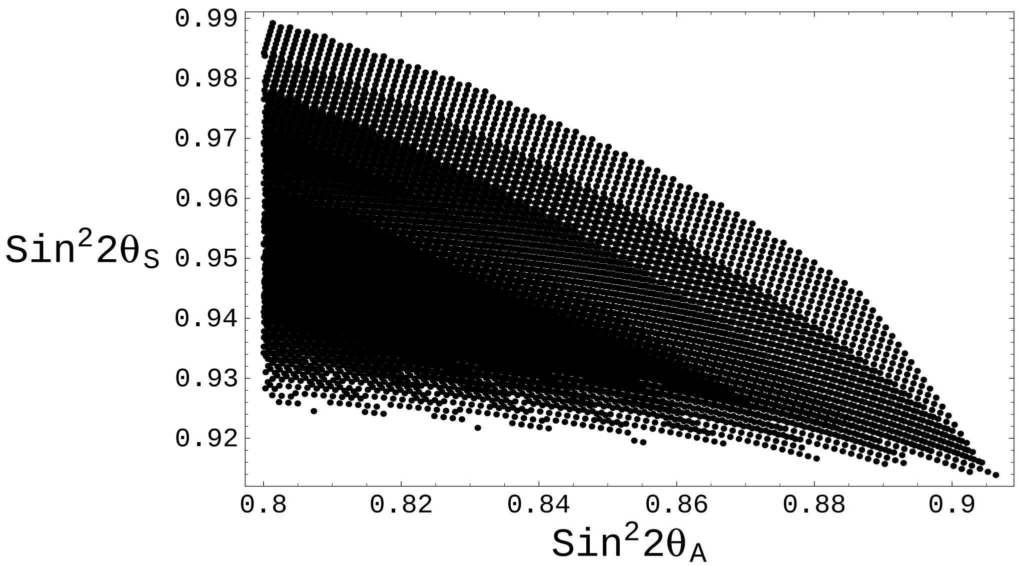

Figure 2.1: The figure shows the predictions for $\sin^2 2\theta_\odot$ and $\sin^2 2\theta_A$ for the range of quark masses in table I. Note that $\sin^2 2\theta_\odot \geq 0.9$ and $\sin^2 2\theta_A \leq 0.9$

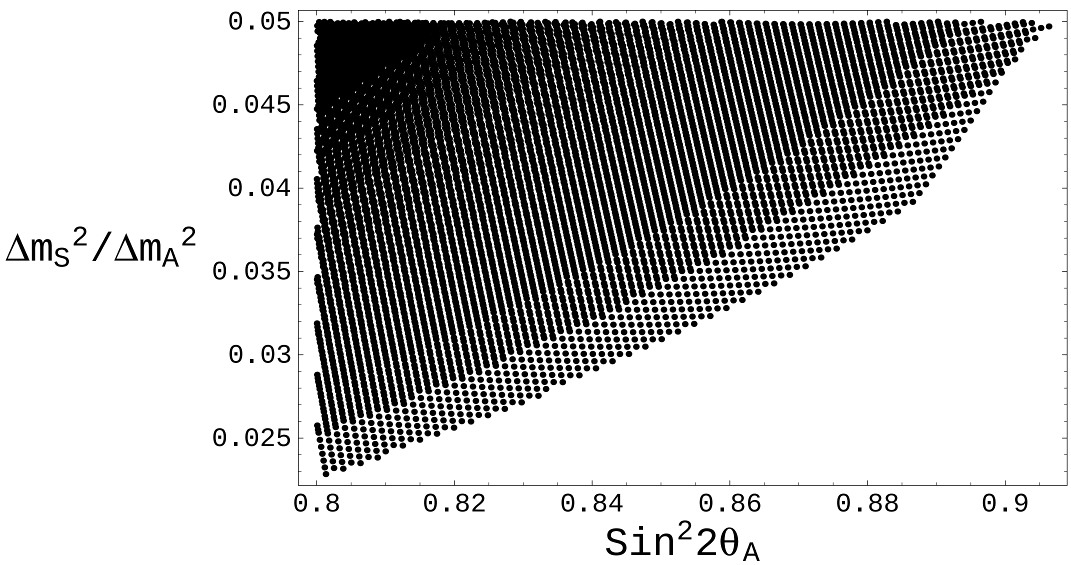

Figure 2.2: The figure shows the predictions for $\sin^2 2\theta_A$ and $\Delta m^2_\odot/\Delta m^2_A$ for the range of quark masses and mixings that fit charged lepton masses.

angle α_u^2 is chosen to be 0.7 is:

$$h = \begin{pmatrix} 3.26\times10^{-6} & 1.50\times10^{-4} & 5.51\times10^{-3} \\ 1.50\times10^{-4} & -2.40\times10^{-4} & -0.0178 \\ 5.51\times10^{-3} & -0.0178 & 0.473 \end{pmatrix} \tag{2.29}$$

and

$$f = \begin{pmatrix} -7.04\times10^{-5} & -2.05\times10^{-5} & -7.53\times10^{-4} \\ -2.05\times10^{-5} & -1.85\times10^{-3} & 2.43\times10^{-3} \\ -7.53\times10^{-4} & 2.43\times10^{-3} & -1.64\times10^{-3} \end{pmatrix}. \tag{2.30}$$

The typical value r_1 and r_2 are found to be $r_1 \sim 0.014$ and $r_2 \sim 0.15$.

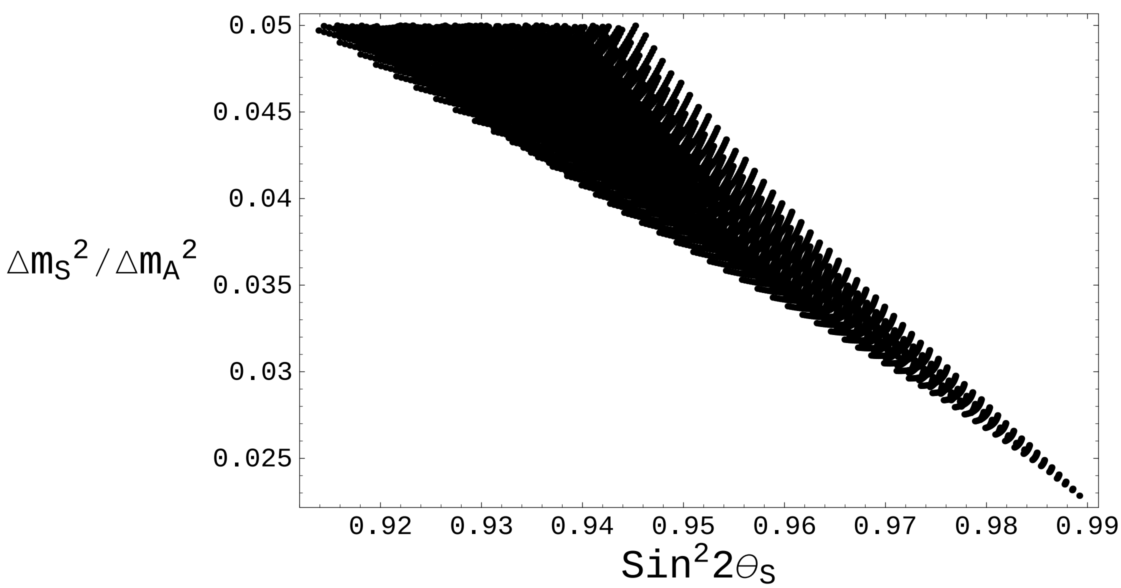

Figure 2.3: The figure shows the predictions for $\sin^2 2\theta_\odot$ and $\Delta m^2_\odot/\Delta m^2_A$ for the range of quark masses and mixings that fit charged lepton masses.

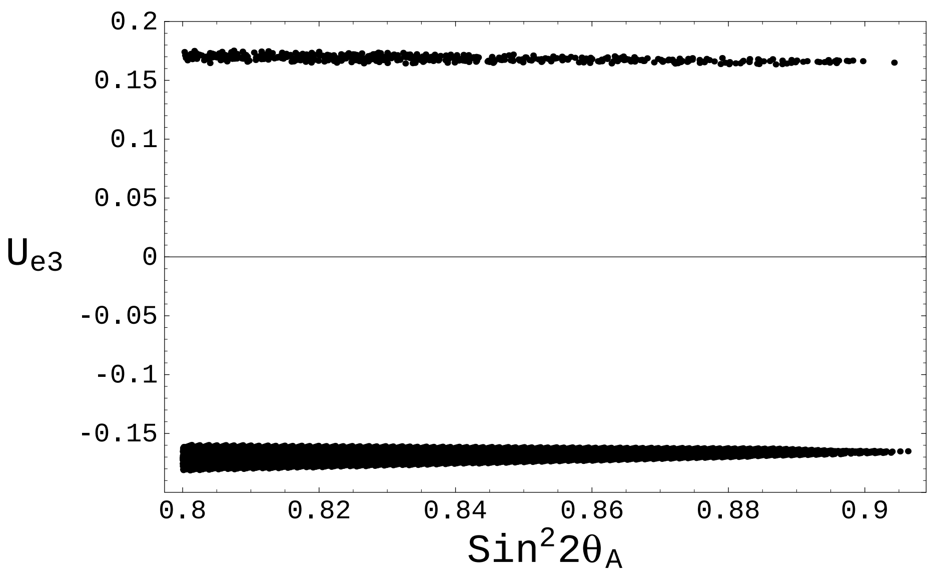

Figure 2.4: The figure shows the predictions of the model for $\sin^2 2\theta_A$ and U_{e3} for the allowed range of parameters in the model. Note that U_{e3} is very close to the upper limit allowed by the existing reactor experiments.

Chapter 3: Prediction for Proton Decay of The Model

3.1 Effective Operators for Proton Decay

In SUSY SU(5) model, the dominant decay of proton occurs via dimension five operators involving color triplet Higgsino exchange leading to the dominant decay mode [37] $p \to K^+\bar{\nu}$. The predictions of the minimal renormalizable SU(5) model for this mode has been discussed in many papers[38, 39][1]. The present experimental lower limit on this mode[40] is 1.9×10^{33} yrs, which is an order of magnitude larger than prediction of the minimal renormalizable SUSY SU(5) model. Therefore this model is ruled out. It has been shown[39] that if one includes nonrenormalizable terms in the superpotential[41], one can get somewhat higher lifetimes for this decay mode and the SU(5) model can still be consistent with experiments[2].

In our model, there are four supersymmetric graphs that contribute to $\Delta B = 1$ operator. They are given in Fig. 1 and involve the exchange of **10**, $\overline{\mathbf{126}}$[43] Higgs multiplets and two mixed $10 - 126$ diagrams. They will lead to both LLLL as well as RRRR type contributions given by the following effective superpotential:

$$\mathcal{W}_{\Delta B=1} \;=\; M_T^{-1}[C_{ijkl}\epsilon_{\alpha\beta\gamma}Q_i^\alpha Q_j^\beta Q_k^\gamma L_l \;+\; D_{ijkl}\epsilon_{\alpha\beta\gamma}u_i^{c,\alpha} d_j^{c,\beta} u_k^{c,\gamma} e_l^c] \tag{3.1}$$

where M_T is the effective mass of color triplet field.

Note that one could in principle, diagonalize the mass matrix involving the color triplet superfields and write the Feynman diagrams in that basis. It is not hard to convince one self that the final result in this case will also have four parameters- an effective mass

[1]By renormalizable, we mean a theory where only renormalizable terms are included in the superpotential.

[2]For proton decay in string theories with SU(5) GUT, see [42].

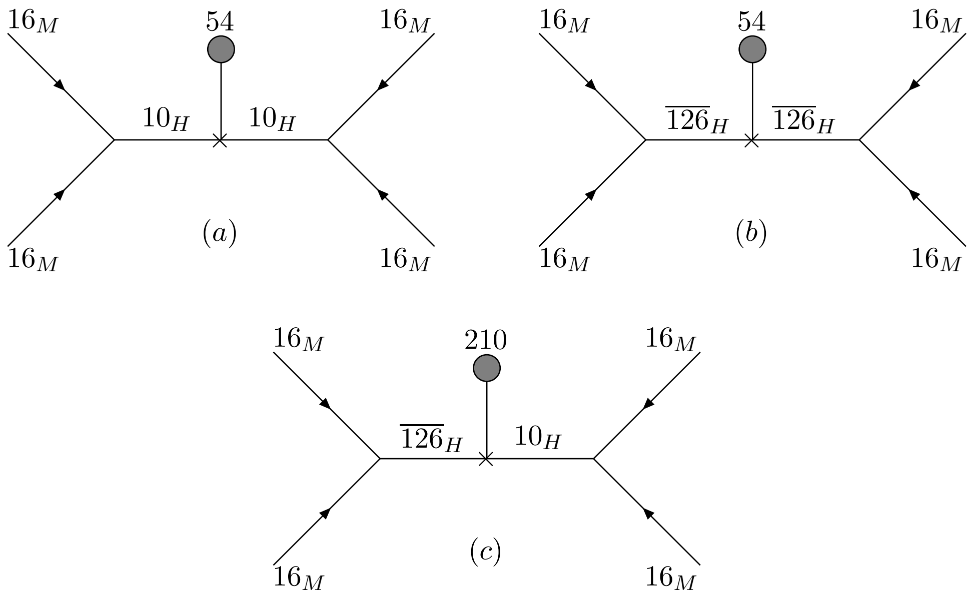

Figure 3.1: Superfield Feynman graphs that give rise to $d = 5$ effective proton decay operators.

and three products of mixing angles. So by considering the above parametrization, we have not lost any information. This supersymmetric operator leads to effective dimension five operators that involve two quark (or quark-lepton) fields and two superpartner fields. In order for these operators to lead to a Four Fermi operator for proton decay, they must be "dressed" via the exchange of gluinos, winos, binos etc. Before we discuss this, let us first note that these operators must be antisymmetrized in flavor indices and then we get for the LLLL term

$$\mathcal{W}_{\Delta B=1} = \epsilon_{\alpha\beta\gamma} M_T^{-1} [(C_{ijkl} - C_{kjil}) u_i^\alpha d_j^\beta u_k^\gamma e_l - (C_{ijkl} - C_{ikjl}) u_i^\alpha d_j^\beta d_k^\gamma \nu_l \quad (3.2)$$

There is a similar operator for the RRRR terms. As has been argued by various authors[44, 45], for small to moderate $\tan\beta$ region of the supersymmetry parameter space, these contributions are smaller than the LLLL contributions. We also find this to be the case in our model. We will show this later; for the time being therefore, we will focus on the

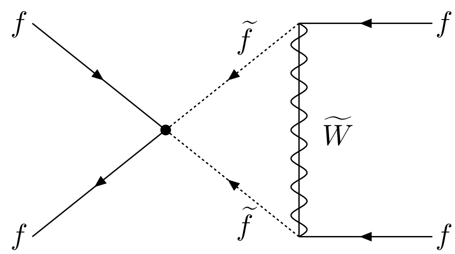

Figure 3.2: Generic Feynman graph for dressing of $d = 5$ effective proton decay operators via gluino, Wino, Bino and Higgsinos.

LLLL operator.

The effective four fermion operator responsible for proton decay can arise the gluino, bino and wino dressing of the above operators. The coefficient C_{ijkl} associated with the LLLL terms is expressible in terms of the products of the Yukawa couplings h and f which have already been determined by the neutrino and other fermion masses:

$$C_{ijkl} = h_{ij}h_{kl} + xf_{ij}f_{kl} + yh_{ij}f_{kl} + zf_{ij}h_{kl} \tag{3.3}$$

where x, y, z are the ratios of the color triplet masses and mixings. As already noted, we do not need to know the detailed form for these parameters (x, y, z) in terms of these masses and mixings. In the end we will vary these parameters to get the maximum value for the partial lifetimes for the various decay modes.

We now discuss the dressing of the various terms. The typical diagrams are shown in Fig.2.

3.1.1 Gluino dressing

It has been pointed out in several papers[46] in the limit of all squark masses being same as in mSUGRA type models, these contributions to the effective four-Fermi operator for proton decay vanishes. It results from the use of Fierz identity for two component spinors which states

$$(\phi_1\phi_2)(\phi_3\phi_4) + (\phi_1\phi_3)(\phi_2\phi_4) + (\phi_1\phi_4)(\phi_2\phi_3) = 0 \tag{3.4}$$

ϕ_i are the chiral two component spinors representing quarks and leptons and $(AB) = A^\alpha B_\alpha \equiv \epsilon^{\alpha\beta} A_\alpha B_\beta$ where α and β are the spinor indices $(\alpha, \beta = 1, 2)$. Since satisfying the flavor changing neutral current (FCNC) constraints allow only very small deviations from universality of squark masses, the gluino diagrams should be small (proportional to $\delta_{LL,ij}$ in standard notation[47]) in realistic models. We will therefore ignore these contributions.

The same results hold also for the RRRR operators.

3.1.2 Neutral Wino and Bino Contribution

To analyze the contribution from $\widetilde{W}^o$ and $\widetilde{B}$, we choose the the operator $\Omega_e = U_i^\alpha D_j^\beta U_k^\gamma E_l$ as an example. Note that we can use $\widetilde{W}^o$ and $\widetilde{B}$ in the loop instead of the the superpartners of Z boson and photon is because they are both mass eigenstates due the the assumption of the universal mass.

$\widetilde{B}$ dressing

There are 6 different dressings of the operator Ω_e through $\widetilde{B}$. We can split them into two groups. One group involves the lepton and the other one does not. Within each group, the product the hypercharges from the two vertices are same. Each of these groups then gives zero due to the Fierz identity as in the case of gluino dressing. This show that the $\widetilde{B}$ dressing is zero by the same Fierzing argument as the gluino case in the limit of universal squark masses.

$\widetilde{W}^o$ Dressing

For the $\widetilde{W}^o$ case, the vertex involving the lepton is same as that of quarks but different by a negative sign between up type and down type particles. The dressing of uu and dd are then different from that of ud by a negative sign. Because the $\widetilde{W}^o$ are lepton/quark blind and the dressing does not change anything except from boson to fermion, the two groups we used in the $\widetilde{B}$ analysis are the same. So after dressing, we have

$$\Omega_e \rightarrow 2(-(u_i^\alpha d_j^\beta)(u_k^\gamma e_l) - (u_i^\alpha e_l)(u_k^\gamma d_j^\beta) + (u_i^\alpha u_k^\gamma)(d_j^\beta e_l)) \tag{3.5}$$

By the Fierz identity, the sum of the first two terms is equal to the third and so we have

$$\Omega_e \rightarrow 4(u_i^\alpha u_k^\gamma)(d_j^\beta e_l) \tag{3.6}$$

Due to the antisymmetry of this expression in the color indices, it is antisymmetric in the interchange of i and j. This implies that i must be different from k and so the two up quarks belong to different family. This antisymmetry remains true even after we pass to the mass eigenstates basis, as is easily checked. The result is simply due to $(u_i^\alpha u_k^\gamma) = -(u_k^\alpha u_i^\gamma)$. The conclusion is that there is no $K^0 + e_l^+$ or $\pi^o + e_l^+$ decay mode from the $\widetilde{W}^o$ dressing. For the same analysis, the operator $U_i^\alpha D_j^\beta D_k^\gamma \nu_l$ gives $4(d_j^\beta d_k^\gamma)(u_i^\alpha \nu_l)$ and so it only contributes to $K^+ + \bar{\nu}_l$ decay mode.

3.1.3 Wino Contribution

In view of the discussion just given the dominant contribution to proton decay arises from charged wino exchange converting the two sfermions to fermions. These diagrams have been evaluated in earlier works[38, 39]; we will assume that all scalar superpartners have the same mass. This leads to the following effective Hamiltonian:

$$\mathcal{L}_{\Delta B=1} \ = \ 2I\epsilon_{\alpha\beta\gamma}(C_{kjil} - C_{ijkl})[u_k^{\alpha,T} C d_j^\beta d_i^{\gamma,T} C \nu_l + u_j^{\beta,T} C d_k^\gamma u_i^{\alpha,T} C e_l] \tag{3.7}$$

where I is given by $I = \frac{\alpha_2}{4\pi}\frac{m_{\widetilde{W}}}{M^2_{\widetilde{f}}}$ Using this expression and adding a similar contribution from $\tilde{W}^0$ exchange, we can now write down the C coefficients for the different proton decay operators. Table I lists the total contributions to the different operators in the leading order:

Table I

Operator	C-coefficient
$\mathbf{udd}\nu_\ell$	$2I\sin\theta_c(C_{211l} - C_{112l})$
$\mathbf{usd}\nu_\ell$	$2I(C_{112l} - C_{121l})$
$\mathbf{uds}\nu_\ell$	$2I\sin\theta_c(C_{221l} - C_{212l})$
$\mathbf{udue}_\ell$	$2I\sin\theta_c(C_{211l} - C_{112l})$
$\mathbf{usde}_\ell$	$2I(C_{112l} - C_{121l})$

Table Caption: The coefficients for various $\Delta B = 1$ operators from the GUT theory. The C's are products of the Yukawa couplings in the superpotential as in Eq. (12).

3.1.4 Estimates of The RRRR Operators

In this subsection, we give an estimate of the RRRR operators and confirm that they are indeed negligible compared to to the LLLL operator contributions for moderate $\tan\beta$ region that we are interested in. First we note that the gluino dressing graphs are zero in the limit of all squark and slepton masses being equal, by the same argument as for the LLLL operators. Secondly, since all superfields in this operator are $SU(2)_L$ singlets, there are no wino contribution to leading order. The only contributions are therefore from the bino exchange and the Higgsino exchange.

Bino exchange generates a four Fermi operator of the form $\epsilon_{\alpha\beta\gamma}u^{c\beta,T}_{j}Cd^{c\gamma}_{k}u^{c\alpha,T}_{i}Ce_{l}$. (where c in the superscript stands for charge conjugate). This operator in the flavor basis must be antisymmetric in the exchange of the two flavor indices i and j. Once they are antisymmetric in the flavor basis, they have to involve charm quark in the mass basis since uu terms will then be zero. Thus the to leading order the bino contribution also vanishes. The Higgsino exchange leads to an effective operator of the form:

$$I\epsilon_{\alpha\beta\gamma}(D_{kji'l'})X_{i'i,l'l}[u^{c\alpha,T}_{k}Cd^{c\beta}_{j}(d^{\gamma,*}_{i}C\nu^{*}_{l})+u^{\beta,T}_{j}Cd^{\gamma}_{k}u^{\alpha,T}_{i}Ce_{l}] \tag{3.8}$$

where $X_{i'i,l'l}\simeq\frac{1}{16\pi^2 v\sin\beta\cos\beta}M_{u,i'i}M_{\ell,l'l}$. Since $1/\sin\beta\cos\beta\sim\tan\beta$ for large values of $\tan\beta$, this contribution grows with $\tan\beta$. It is clear from inspection that the largest value for this amplitude comes from $\widetilde{t}$ intermediate states and we estimate the largest contribution to be of order $C_{1323}\frac{m_t V_{ub} m_\tau}{v^2_{wk}16\pi^2}\simeq 10^{-10}$ as compared to the LLLL contribution which are of order $C_{1123}\frac{\alpha_2}{4\pi}\sim 10^{-9}$. Therefore, we can ignore the RRRR contribution in our discussion.

3.2 Predictions for Proton Decay

Let us first note that the operators with s quark lead to p-decay final states with K meson whereas the ones without s lead to π final states. Also generally speaking the amplitude for nonstrange final states are down by a factor of Cabibbo angle (~ 0.22) compared to the strange final states as in the case of SU(5) model. However, as we will see, we need to do a fine tuning among the parameters x, y, z to make the $p\to K^{+}+\bar{\nu}$ compatible with experiments. The same fine tuning however does not simultaneously lower the amplitudes with nonstrange final states. As a result for some domain of the allowed parameter space, one can have the $p\to\pi^{+}+\bar{\nu}$ mode as the dominant mode. This is very different from the minimal SU(5) case.

In order to proceed to the calculation of proton lifetime, we must extrapolate the above operators defined at the GUT scale first to the M_S and then to the one GeV scale. These extrapolation factor have been calculated in the literature for MSSM and we take these values. The required factors are: $A_L A_S$[44] and are given numerically to be $A_L = 0.4$ (SUSY to one GeV scale) and $A_S = 0.9 - 1.0$ (GUT to M_S scale).

The next step in the calculation is to go from three quarks to proton. The parameter is denoted in the literature by β and has units of $(\mathrm{GeV})^3$. This has been calculated using lattice as well as other methods and the number appears to be: $\beta \sim 0.007 - 0.028$[53]. We find that for our choice of the average superpartner masses, for $\beta \geq 0.01$, there is no range for the parameters (x, y, z) where all decay modes have lifetimes above the present lower limits. Of course as the superpartner masses increase, larger β values become acceptable. For instance, we note that a change $\delta m^2_{\tilde{q}}/m^2_{\tilde{q}}$ by 10% allows a 20% higher value in β. We confine ourselves to the domain $0.007 \leq (\beta/GeV^3) \leq 0.01$ and find that for all choices of the free parameters allowed by the present lower limits, lifetimes for the decay modes $p \rightarrow \pi^+ \bar{\nu}$ and $n \rightarrow \pi^0 \bar{\nu}$ have upper limits, which can therefore be used to test the model (see below).

Finally, in a detailed evaluation of proton decay rate to different final states, we take into account the chiral symmetry breaking effects following a chiral Lagrangian model (the first two papers of Ref.[53]), where the chiral symmetry breaking effects are parameterized by two parameters D and F. These are usually chosen to be the same as the analogous parameters in weak semileptonic decays[54].

For this case, we find the rate for proton decay to a particular decay mode $P\ell$ (P

is the meson and ℓ denotes the lepton) to have the form:

$$\Gamma_p(P\ell) \simeq \frac{m_p}{32\pi f_\pi^2 M_T^2}|\beta|^2 \left(\frac{M_{\widetilde{W}}}{M_{\widetilde{f}}^2}\right)^2 \left(\frac{\alpha_2}{4\pi}\right)^2 |A_L A_S|^2 4|C|^2|f(F,D)|^2 \tag{3.9}$$

$$\simeq 2.7\times 10^{-50}|C|^2\left(\frac{2\times 10^{16}GeV}{M_T}\right)^2\left(\frac{M_{\widetilde{W}}}{200\ GeV}\right)^2\left(\frac{TeV}{M_{\widetilde{f}}}\right)^4|f(F,D)|^2\ GeV$$

where $f(F,D)$ is a factor that depends on the hadronic parameters F and D and we have used $\beta = 0.01$ GeV^3 in the last expression. We now discuss the evaluation of the parameter $|C|^2$ which determines the partial proton decay lifetimes for various modes. The relevant modes are $p \to K^+\bar{\nu},\ K^0\mu^+,\ K^0e^+,\ \pi e^+,\ \pi\mu^+$. The present lower limits (including $n \to \pi\nu, K\nu$ modes) on these modes are:

Table II

mode	lifetimes ($\times 10^{32}$ yrs)
$p \to K^+\bar{\nu}$	19
$p \to K^0e^+$	5.4
$p \to K^0\mu^+$	10
$p \to \pi^+\bar{\nu}$	0.2[55], 0.16[56]
$p \to \pi^0e^+$	50
$p \to \pi^0\mu^+$	37
$n \to \pi^0\bar{\nu}$	4.4
$n \to K^0\bar{\nu}$	1.8

Table caption: Present experimental lower limits on the relevant proton decay modes from Super-Kamiokande and Kamiokande experiments.

To proceed with this discussion, first note that C's are products of the known Yukawa coupling parameters h and f and the four GUT scale parameters as already discussed in Eq.(12). The GUT scale values of h and f are those obtained from neutrino fits in the last section.

As far as the GUT scale parameters go, we will keep the overall mass parameter to be the GUT scale i.e. 2×10^{16} GeV. We have diagonalized the mass matrix of the color triplet GUT scale Higgs fields in **10**, **126** etc and we find that they also lead to the same parametrization as we have given here. The meaning of the overall mass scale is then that it represents a product of one of the mass eigenstates with the determinant. We have checked that for the allowed range of parameters, the value of the determinant, given by $|x - yz|$ is around 0.25 or so, so that none of the mass eigenstates is too much higher than the GUT scale. As a result the threshold effects on the gauge coupling unification is minimal.

We then adopt the strategy that we vary the parameters x, y, z in such a way that the nucleon decay rate to the $p \to K^+\bar{\nu}$ mode (summed over all the final neutrino final states) is consistent with the present experimental lower limit. Since there are three final states which add incoherently, this narrows the space of the x, y, z to a small domain. In this domain we pick a point (call it (x_0, y_0, z_0)), where all other modes also satisfy their present experimental constraints as in Table II. We then vary the (x, y, z) parameters around (x_0, y_0, z_0) until the lifetime for a mode goes below its present experimental lower limit. We find that dependence on the parameter z is much stronger than the others. In Fig. 3 and 4, we give the allowed domain of the parameters (x, y) consistent with the various experimental lower limits on the partial lifetimes for an optimum value of z. The boundary of the domain is determined by the lower limit on the the $p \to K^+\bar{\nu}$. Inside this domain the $\tau(p \to K^+\bar{\nu})$ is higher than its present lower limit. The maximum value of the $p \to \pi^+\bar{\nu}$ and $n \to \pi^0\bar{\nu}$ occurs at the boundary. We find that $\tau(n \to \pi^0\bar{\nu}) = 2\tau(p \to \pi^+\bar{\nu})$ has an upper bound of $(5.7 - 13) \times 10^{32}$ yrs depending on whether $\beta = 0.01 - 0.007$ GeV^3.

At a different point in the parameter space, $\tau(n \to K\bar{\nu})$ acquires its maximum value of 2.9×10^{33} years. The predictions for the partial lifetimes of other modes are given in Table III for both these cases. These values are accessible to the next round of proton decay searches.

Table III

mode	$\tau/10^{32}$ yrs $\beta = 0.01$	$\tau/10^{32}$ yrs: $\beta = 0.007$	$\tau/10^{32}$ yrs: $\beta = 0.007$
	$\tau(n \to \pi\bar{\nu})$ maximized	$\tau(n \to \pi\bar{\nu})$ maximized	$\tau(n \to K\bar{\nu})$ maximized
$p \to K^+\bar{\nu}$	19	19	19
$p \to K^0 e^+$	1793	2848	188
$p \to K^0 \mu^+$	184	303	28
$p \to \pi^+\bar{\nu}$	2.87	6.5	2.59
$n \to \pi^0\bar{\nu}$	5.7	13	5.18
$p \to \pi^0 e^+$	2452	3857	243
$p \to \pi^0 \mu^+$	263	430	37
$n \to K^0\bar{\nu}$	1.9	3.1	29.7

Table caption: Predictions for various nucleon decay modes for the case when the lifetime for the mode $n \to \pi^0 + \bar{\nu}$ attains its maximum value. The units for β parameter (i.e. GeV^3) has been omitted in the table. In column 4, we give the lifetimes for the case when $\tau(n \to K\bar{\nu})$ is maximized.

We check the above results adopting an alternative strategy where we express the three parameters (x, y, z) in terms of three partial life times and plot the other lifetimes as a function of these partial life times. It turns out that if we pick a certain value for the partial life time of the $p \to K\mu$ mode and use it as an input, the other two input values get very restricted. This allows us to use only the $p \to K^0\mu^+$ mode as a variable and give the others as a prediction. In Fig. 5 and 6 we present the allowed values for various partial lifetimes as a function of the partial lifetime for the mode $p \to K^0 \mu^+$. There is a slight spread around the various lines. We first find that the lifetime for the mode $K^+ \bar{\nu}$ can be arbitrarily large as can be seen from Fig. 5. Also, from Fig. 5, we see that modes $n \to \pi^0 \bar{\nu}$ and $n \to K^0 \bar{\nu}$ have upper bounds which are same as the ones derived previously.

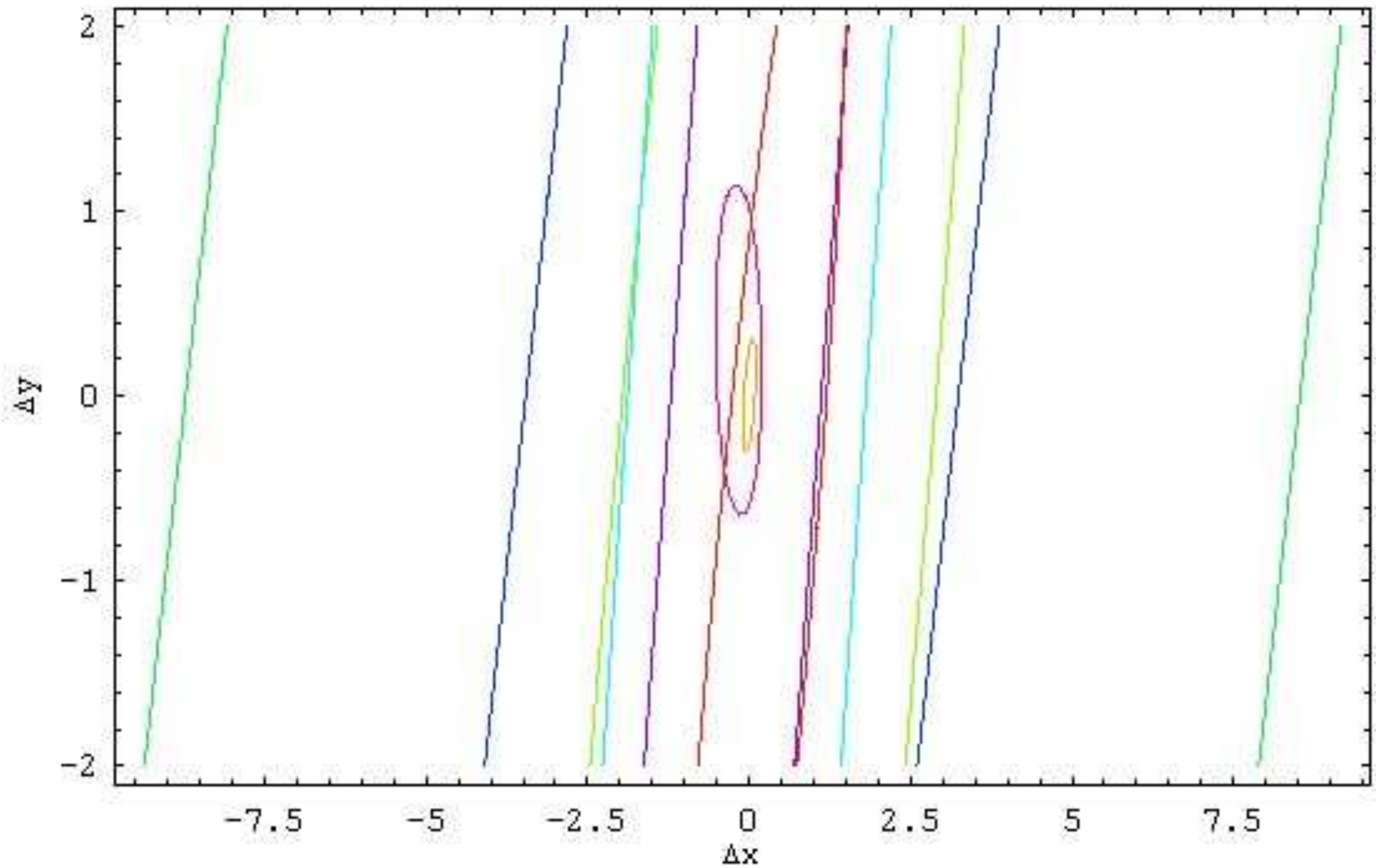

Figure 3.3: Allowed Region for (x, y) coming from experimental lower limits on lifetimes for different decay modes for $z = 0.329$. The point $(\Delta x, \Delta y) = (0, 0)$ corresponds to $(x, y) = (-0.036, 0.387)$. Note that the region is most constrained by $p \to K + \bar{\nu}$ mode.

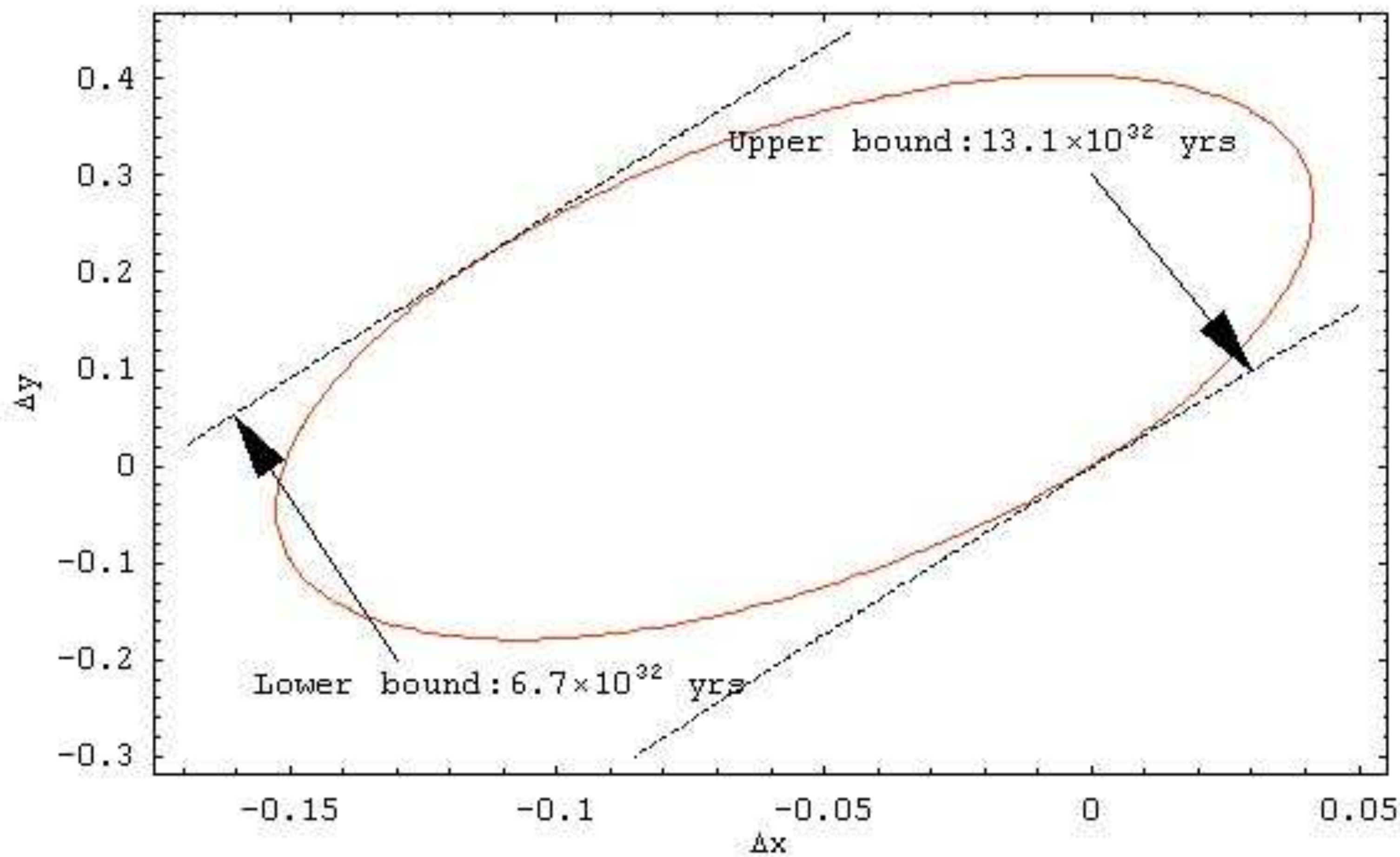

Figure 3.4: Upper limit on the $n \rightarrow \pi + \bar{\nu}$ partial lifetime while satisfying bounds on the lifetimes of all other modes. The point $(\Delta x, \Delta y) = (0, 0)$ corresponds to $(x, y, z) = (-0.132, 0.347, 0.306)$.

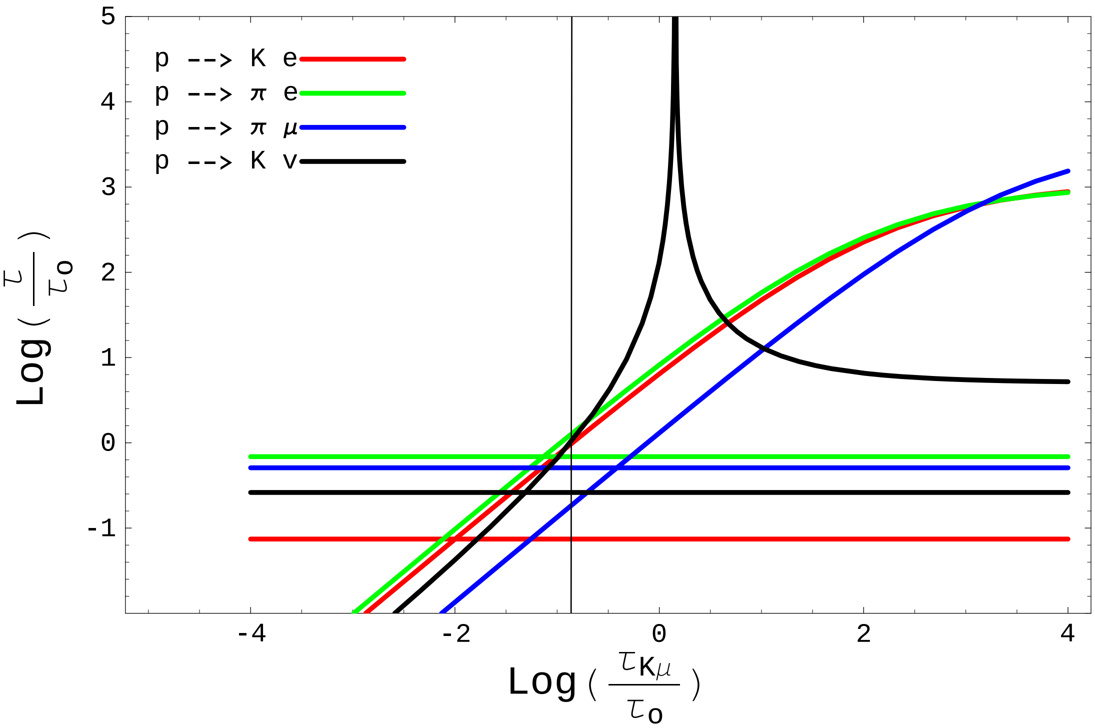

Figure 3.5: This figure gives the values of the lifetimes for different proton decay modes as a function of the lifetime of the $p \to K^0\mu^+$ mode (represented here by $log_{10}\frac{\tau_{K\mu}}{\tau_0}$ where $\tau_0 = 14.6 \times 10^{33}$ years) when $\tau(p \to K^+\bar{\nu})$ mode is at its maximum value. The vertical and horizontal lines indicate the experimental bound of the various decay modes. This figure displays the values for one range of (x, y, z) that correspond to positive amplitude of $p \to K^0\mu^+$ mode.

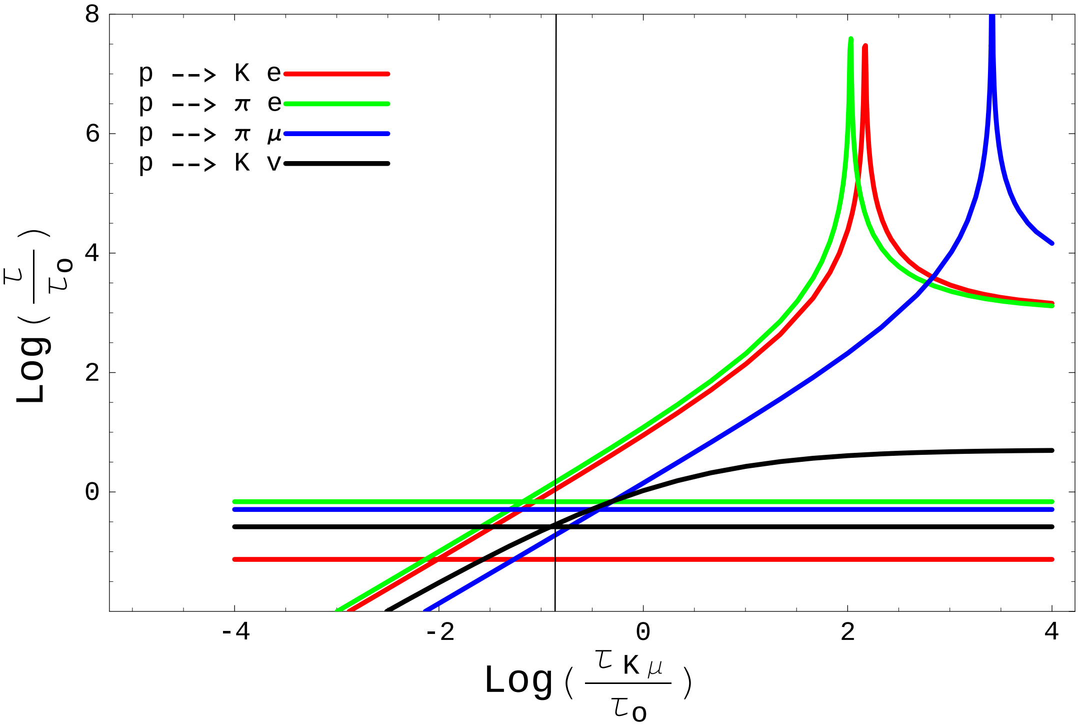

Figure 3.6: The same display as Fig. 3.5 but for a complementary range for the parameters (x, y, z) which correspond to the negative amplitude.

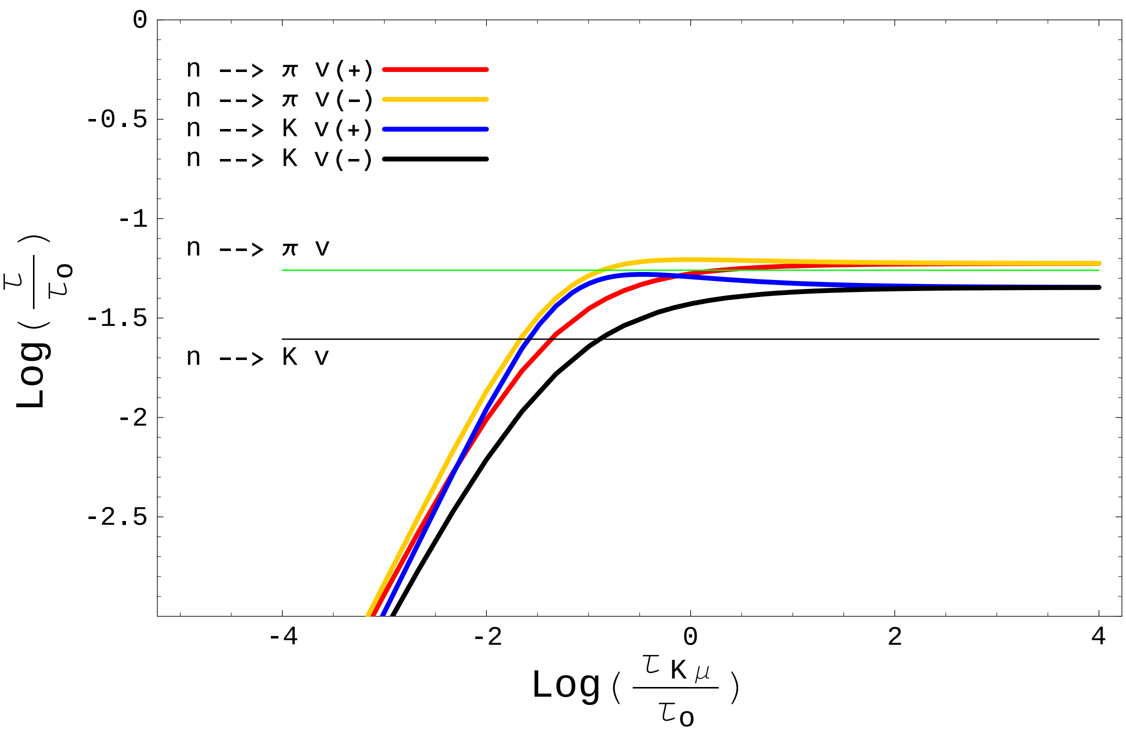

Figure 3.7: This figure show the upper limit of the two nucleon decay modes. Again, the horizontal line indicate the experimental bound and the "+" and "-" are the signs of the amplitude of $p \to K^0\mu^+$ decay mode

Chapter 4: Validity of The Type-II Seesaw Assumption in The Minimal SO(10) Model

In chapter 2, we have seen that if we assume the type II seesaw formula[19] for neutrino masses and that the triplet term dominates, then the solar mass squared difference $\triangle m^2_\odot$ and the two large mixing angles θ_{12}, θ_{23} are predicted to be in the right range and θ_{13} is predicted to be 0.18 which is slightly below the present CHOOZ-Palo-Verde upper limit. Crucial to the success of the model is the assumption that the triplet term in the seesaw formula dominates.

In this chapter we discuss the conditions under which the triplet term in the type II seesaw formula dominates the neutrino mass formula. We find that it imposes constraints on the way SO(10) symmetry breaks down to the standard model. In particular, we find that the minimal SO(10) model with the Higgs structure **10**, **126** pair and a **210**[35] needs to be extended by the addition of a **54** multiplet.

4.1 The Origin of The Triplet Term and The Conditions of Dominating

The triplet term arises from the higher dimensional operator which are obtained by integrating out the massive $SU_L(2)$ triplet in **15** of SU(5). In the minimal model, the only **15** comes from $\overline{\mathbf{126}}$ of SO(10). The leading diagram is given in fig.(4.1) and the effective operator has the form

$$(\overline{\mathbf{5}}_f \mathbf{5}_\Phi)(\overline{\mathbf{5}}_f \mathbf{5}_{H,\overline{\Sigma}}) \to \frac{\kappa}{M_T} f(Lh_u)(Lh_u) \tag{4.1}$$

whereκ is combination of some of the parameters that appear in the superpotential, M_T is the mass of the triplet Higgs field and f is the Yukawa coupling constants defined

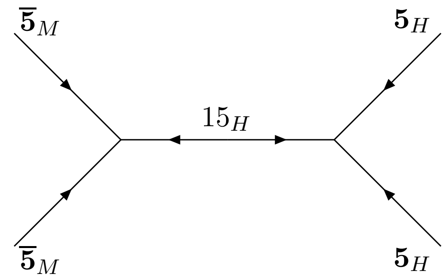

Figure 4.1: Superfield Feynman graph that gives rise to the triplet term of type-II seesaw.

in chapter 2. When the physical Higgs get a vev, the left handed neutrinos gain their Majorana masses which is given by $m_L \sim \kappa f \frac{10^4 GeV^2}{M_T}$. The numerical analysis in chapter 2 indicates that $f_{33} \sim 10^{-3}$ if the mixing angle α_u^2 is order of 1. For the first term to give the whole $\sqrt{\Delta m_A^2} \sim 0.05$, the 3-3 element of m_L has to be of order 0.05 eV. If κ is of order 1, the mass of the color singlet, $SU_L(2)$ triplet Higgs field M_T should be of the order 10^{13} GeV. On the other hand, the second term (the canonical seesaw term)is controlled by different parameter, namely the $B-L$ breaking scale v_{B-L}. From this we estimate that the biggest contribution to neutrino mass from the second term to be about $\sim \frac{10^6}{v_{B-L}}$ GeV, which for $v_{B-L} \simeq M_U \simeq 2 \times 10^{16}$ gives $\sqrt{\Delta m_A^2} \simeq 0.1$ eV which is slightly bigger than the experimental value. If the first term is to dominate, this must be smaller than (say) 0.02 eV i.e. it requires a value $v_{B-L} \geq 10^{17}$ GeV. This estimation tell us that if the mixing angle α_u^2 is of order 0.1, we need the triplet to be much lighter than the GUT scale and the B-L scale to be at least one order of magnitude bigger than the GUT scale. This implies that we need a two-step symmetry breaking.

Clearly the light triplet with mass of 10^{13} GeV is going to affect unification of couplings. To reduce the influence on the coupling unification, there are two things we can do:

(i) Increase the values of the matrix elements f so that the triplet Higgs boson mass could be larger, thus making unification possible. This would happen if the mixing angle α_u^2 are of order 10^{-4} or less. The reason for this is that the effective parameters we defined in the mass formula, which we call $\overline{f}$ in chapter 2, are the combination of f and the mixing angle α_u^2. Reducing the mixing angle and increasing f will not affect the result of the fermion masses and mixings. If the mixing angle is reduced by factor of 10^{-4}, f_{33} will increase by the same factor and the triplet mass can be brought up to 10^{16} GeV.

(ii) There is a whole multiplet of **15** at the 10^{12} GeV scale so the coupling unification is not disturbed. The only thing that is affected is the unified coupling constant at the same unification scale.

The difficulties is that making the triplet of **15** light by finetuning always leave other components off tuning and heavy. There is no natural reason why the whole SU(5) multiplet has to have the same mass when SU(5) is broken. On the other hand, due to the fit that requires $r_1 \sim 0.014$, $r_2 \sim 0.15$ and $\tan\beta = 10$, we need $\alpha_d^2 \sim \alpha_u^2 \sim 0.0001$ and $\alpha_d^1 \sim 0.1$ in order to get the required triplet mass up to GUT scale. This requires multiple fine tunings within one matrix. As can be seen in detail later, these requirements can be satisfied by introducing the new **54** Higgs and requiring that SO(10) first break at the scale of 10^{18} GeV down to SU(5) and then, to the standard model at the GUT scale, $(2 \times 10^{16}\text{GeV})$.

4.2 Breaking SO(10) to Standard Model via SU(5)

In order to demonstrate the need for **54**, let us start with the minimal Higgs fields $H(\mathbf{10})$, $\Phi(\mathbf{210})$, $\Sigma(\mathbf{126})$ and $\overline{\Sigma}(\overline{\mathbf{126}})$ and write down the most general renormalizable superpotential:

$$W = \frac{m_\Phi}{2\times 4!}\Phi^2 + \frac{m_\Sigma}{5!}\Sigma\overline{\Sigma} + \frac{m_H}{2}H^2 + \frac{\lambda}{4!}\Phi^3 + \frac{\eta}{4!}\Phi\Sigma\overline{\Sigma} + \frac{1}{4!}\Phi H(\alpha\Sigma + \overline{\alpha}\overline{\Sigma}) \tag{4.2}$$

We then extract the various SU(5) submultiplets from each of the SO(10) Higgs multiplets and rewrite the superpotential in terms of these fields. Extensive discussion of the decomposition of SO(10) multiplets in terms of SU(2)×SU(2)×SU(4) and SU(2)×U(1)×SU(3) exists in the literature[57, 58, 60, 59]. We have derived the SU(5) decomposition of various SO(10) invariant couplings and use them in this chapter. For this purpose, note that

$$\mathbf{210} = \mathbf{1}_0 \oplus \mathbf{5}_{-8} \oplus \overline{\mathbf{5}}_{+8} \oplus \mathbf{10}_4 \oplus \overline{\mathbf{10}}_{-4} \oplus \mathbf{24}_0 \oplus \mathbf{75}_0 \oplus \mathbf{40}_{-4} \oplus \overline{\mathbf{40}}_{+4} \tag{4.3}$$

$$\mathbf{126} = \mathbf{1}_{-10} \oplus \overline{\mathbf{5}}_{-2} \oplus \mathbf{10}_{-6} \oplus \overline{\mathbf{15}}_{+6} \oplus \mathbf{45}_2 \oplus \overline{\mathbf{50}}_{-2} \tag{4.4}$$

In terms of the properly normalized SU(5) submultiplets we first rewrite the bilinear terms and then the trilinear terms in the superpotential (See appendix for the details).

$$\begin{aligned} L_B &= m_H H_a H^a + m_\Phi\{(\phi^5)_a(\phi^5)^a + \frac{1}{3!}(\Phi^{40})_{abc}{}^d(\Phi^{40})^{abc}{}_d \\ &+ \frac{1}{2}(\phi^{10})_{ab}(\phi^{10})^{ab} + \frac{1}{8}(\phi^{75})_{ab}{}^{cd}(\phi^{75})_{cd}{}^{ab} + \frac{1}{2}(\phi^{24})_a{}^b(\phi^{24})_b{}^a + \frac{1}{2}\phi_o^2\} \\ &+ m_\sigma\{\sigma_o\overline{\sigma}_o + \frac{1}{4!}(\Sigma^{15})_a{}^{bcde}(\overline{\Sigma}^{15})^a{}_{bcde} + \frac{1}{3!}(\Sigma^{10})_{abc}(\overline{\Sigma}^{10})^{abc} \\ &+ \frac{1}{12}(\Sigma^{50})_{abc}{}^{de}(\overline{\Sigma}^{50})^{abc}{}_{de} + \frac{1}{2}(\Sigma^{45})_a{}^{bc}(\overline{\Sigma}^{45})^a{}_{bc} + (\Sigma^5)_a(\overline{\Sigma}^5)^a\} \end{aligned} \tag{4.5}$$

and the trilinear terms become

$$L_T = \frac{\lambda}{\sqrt{10}}\phi_o\{12(\phi^5)^a(\phi^5)_a + 3(\phi^{10})_{ab}(\phi^{10})^{ab} + (\phi^{24})_a{}^b(\phi^{24})_b{}^a - \frac{1}{2}(\phi^{75})_{ab}{}^{cd}(\phi^{75})_{cd}{}^{ab}\} \tag{4.6}$$

$$
\begin{aligned}
&+ \quad \alpha\sigma_o H^a(\phi^5)_a + \alpha\sqrt{\frac{3}{5}}\phi_o H^a(\Sigma^5)_a + \overline{\alpha}\overline{\sigma}_o H_a(\phi^5)^a + \overline{\alpha}\sqrt{\frac{3}{5}}\phi_o H_a(\overline{\Sigma}^5)^a \\
&+ \quad \sqrt{6}\eta\sigma_o\{(\phi^5)_a(\overline{\Sigma}^5)^a + \frac{1}{12}\epsilon_{abcde}(\phi^{10})^{ab}(\overline{\Sigma}^{10})^{cde}\} + c.c. \\
&+ \quad \frac{\eta}{\sqrt{10}}\phi_o\{\frac{2}{3}(\Sigma^{10})_{abc}(\overline{\Sigma}^{10})^{abc} + \frac{1}{12}(\Sigma^{15})_a{}^{bcde}(\overline{\Sigma}^{15})^a{}_{bcde} - \frac{1}{3!}(\Sigma^{50})_{abc}{}^{de}(\overline{\Sigma}^{50})^{abc}{}_{de} + 4(\Sigma^5)_a(\overline{\Sigma}^5)^a\} \\
&+ \quad \frac{2\lambda}{\sqrt{10}}\phi_o^3 + \eta\sqrt{10}\phi_o\sigma_o\overline{\sigma}_o
\end{aligned}
$$

4.2.1 Supersymmetric Vacuum without **54**

We can now discuss SO(10) breaking to SU(5). There are two SU(5) singlets: one in each of the **126** pair and one in **210**. The SU(5) singlets in the **126** pair have nonzero B-L and therefore B-L breaking scale is same as the SO(10) scale. Since supersymmetry must remain unbroken all the way down to the weak scale, we set the F-terms to zero. These F-term conditions give the following constraints on the parameters of the superpotential and the vacuum expectation values:

$$
F_{\phi_o} = m_\Phi\widetilde{\phi}_o + 6\lambda\widetilde{\phi}_o^2 + \eta\sigma_o\overline{\sigma}_o = 0 \tag{4.7}
$$

$$
F_{\overline{\sigma}_o} = \sigma_o(m_\sigma + 10\eta\widetilde{\phi}_o) = 0
$$

where $\widetilde{\phi}_o = \frac{\phi_o}{\sqrt{10}}$. The solution is

$$
\widetilde{\phi}_o = -\frac{m_\sigma}{10\eta} \tag{4.8}
$$

$$
\sigma_o\overline{\sigma}_o = \frac{m_\sigma}{10\eta^2}(m_\Phi - \frac{3\lambda m_\sigma}{5\eta})
$$

4.2.2 Masses of SU(5) Sub-multiplets

From the Lagrangian found above, we can easily write down the masses of the various SU(5) submultiplets and we list them in Table I.

40	m_ϕ
75	$m_\phi - 4\lambda\widetilde{\phi}_o$
24	$m_\phi + 2\lambda\widetilde{\phi}_o$
15	$\frac{4}{5}m_\sigma$
50	$\frac{6}{5}m_\sigma$
45	m_σ

Table caption This table gives the masses of the various SU(5) multiplets in the SO(10) multiplets of the minimal model.

The mass matrix for the SU(5) singlets in the basis $(\phi_0, \sigma_0, \overline{\sigma}_0)$ is found to be:

$$\begin{pmatrix} m_\phi + 12\lambda\widetilde{\phi}_o & \sqrt{10}\eta\overline{\sigma}_o & \sqrt{10}\eta\sigma_o \\ \sqrt{10}\eta\overline{\sigma}_o & 0 & 0 \\ \sqrt{10}\eta\sigma_o & 0 & 0 \end{pmatrix} \tag{4.9}$$

One of the combination of the singlets has zero mass and is the Goldstone boson corresponding to the breaking of B-L. As can be seen from the above matrix, the corresponding field is a linear combination of the fields $\overline{\sigma_0}$ and σ_0. Neglecting the Goldstone bason, we find the remaining 2×2 mass matrix to be:

$$\begin{pmatrix} m_\phi + 12\lambda\widetilde{\phi}_o & \sqrt{10}\eta\sqrt{\overline{\sigma}_o^2 + \sigma_o^2} \\ \sqrt{10}\eta\sqrt{\overline{\sigma}_o^2 + \sigma_o^2} & 0 \end{pmatrix} \tag{4.10}$$

The mass eigenvalues are given by:

$$m_{singlet} = \frac{m_\phi + 12\lambda\widetilde{\phi}_o \pm \sqrt{(m_\phi + 12\lambda\widetilde{\phi}_o)^2 + 40\eta^2(\overline{\sigma}_o^2 + \sigma_o^2)}}{2} \tag{4.11}$$

The mass matrix for the **10** is

$$\begin{pmatrix} m_\phi + 6\lambda\widetilde{\phi}_o & \sqrt{6}\eta\overline{\sigma}_o \\ \sqrt{6}\eta\sigma_o & -6\eta\widetilde{\phi}_o \end{pmatrix} \tag{4.12}$$

This mass matrix has a zero eigenvalue and the associated eigenstate field is the Goldstone boson corresponding to the breaking of SO(10) down to SU(5). The massive combination has a mass of

$$m_{10} \;=\; -\frac{\eta}{\widetilde{\phi}_o}(|\overline{\sigma}_o\sigma_o| + 6\widetilde{\phi}_o^2) \tag{4.13}$$

The mass matrix for 5-plet Higgs is

$$\begin{pmatrix} m_H & \overline{\alpha}\overline{\sigma}_o & \sqrt{6}\overline{\alpha}\widetilde{\phi}_o \\ \alpha\sigma_o & m_\phi + 12\lambda\widetilde{\phi}_o & \sqrt{6}\eta\sigma_o \\ \sqrt{6}\alpha\widetilde{\phi}_o & \sqrt{6}\eta\overline{\sigma}_o & -6\eta\widetilde{\phi}_o \end{pmatrix} \tag{4.14}$$

The first point we note from the Table I is that the three submultiplets **15**, **50** and **45** are all proportional to the same parameter m_σ in the superpotential. As a result, when one of them is at a lower scale, all others are. From the table, it is also clear that by adjusting m_ϕ and $\lambda\tilde{\phi}^0$, we can keep the **40** and the **75** Higgs field at the SO(10) scale while the **24** is light.

4.3 Necessity of **54** Higgs Field

It is now clear that in the minimal model with **10**, **126** pair and a **210**, the masses of the SU(5) submultiplets **15**, **50** and **45** are all of the same scale. Therefore, if we want to enforce the type II seesaw formula with the triplet vev dominating, we would have to have **15** and also the **45** pair and the **50** pair at the SU(5) scale (M_5). When SU(5) is broken by some vevs of order M_5, the triplet in the **15** can be tuned to below M_5. As have been discussed earlier, we want the whole **15** to be light in order to maintain unification. In this model with a single **15**, the sub-multiplets of **15** are always split by the vev $\sim M_5$. This means we can not have all of them getting masses of order 10^{12} GeV by tuning the parameters. On the other hand, if two-step symmetry breaking through SU(5) is assumed, the triplet itself can have mass of 10^{12} GeV only if some of the parameters in the superpotential are complex (see the mass formulae given in [58]). Even in this complex case, it is very likely to have the same difficulty making the whole **15** light. We will not consider this possibility. Furthermore, there is another reason why we have to make modifications to this model. We have many high dimensional multiplets like **75**, **50** and **45** in the model. The problem of having these high dimensional multiplets is that it causes the coupling constant to run too fast toward the strong regime (i.e. $\alpha_U \geq 1$). In our case, if all of our multiplets have masses of 10^{16} GeV (GUT scale), the strong coupling will be reached at the energy scale around 7×10^{16} GeV which is below the expected string scale and the Plank scale. Although this is not a compelling reason, we would like to keep the picture perturbative all the way to 10^{18} GeV. The only way to do this is, of course, by making most of the multiplets heavy. However, the light triplet require that the masses of **50** pair and **45** pair will be less than or equal to 10^{16} GeV. Of course, we also need **24** to break SU(5) and $\mathbf{5}\ \overline{\mathbf{5}}$ pair to give Higgs doublets to MSSM. This means the coupling constant will reach the strong regime at about 10^{17} GeV, which is still below the string scale. We therefore need a way to split only the **15** dimensional field from the others and stabilize the **15** at the scale below M_5. These are the reasons why we need to add to the model an additional **54** dimensional Higgs field which contain **15** and **24** as submultiplets.

In the presence of the **54** Higgs field (denoted by S), the superpotential of Eq. (4.2) has the following additional terms:

$$\begin{aligned} W_{54} &= \frac{m_{15}}{4} S_{ab} S_{ab} + \frac{\lambda_1}{3!} S_{ab} S_{bc} S_{ca} + \frac{\lambda_2}{2} S_{ab} H_a H_b \\ &+ \frac{\chi}{2 \times 4!} S_{ab} \Sigma_{acdef} \Sigma_{bcdef} + \frac{\overline{\chi}}{2 \times 4!} S_{ab} \overline{\Sigma}_{acdef} \overline{\Sigma}_{bcdef} + \frac{\rho}{12} S_{ab} \Phi_{acde} \Phi_{bcde} \end{aligned} \tag{4.15}$$

Note that $\mathbf{54} = \mathbf{15}_4 + \overline{\mathbf{15}}_{-4} + \mathbf{24}_0$ under SU(5). Therefore when $\sigma_0 = \bar{\sigma}_0 = v_{B-L}$, the **15** multiplets have a 2×2 mass matrix of the form:

$$M = \frac{m_{15}}{2} S_{ab} S^{ab} + \frac{m_{15}}{2} S_a{}^b S^b{}_a + (\sqrt{\frac{6}{5}} \rho \phi_o) S_a{}^b (\phi^{24})_b{}^a + [(\sqrt{2}\chi\sigma_o) \frac{1}{2} S^{ab} (\Sigma^{15})_{ab} + h.c.] \quad (4.16)$$

So the mass matrix for the **15** Higgs fields is

$$\begin{pmatrix} \frac{4}{5} m_\sigma & \sqrt{2}\chi\sigma_o \\ \sqrt{2}\overline{\chi\sigma}_o & m_{15} \end{pmatrix} \quad (4.17)$$

Similarly the **24**'s in 210 and in **54** mix and we have the following mass matrix for the **24** Higgses:

$$\begin{pmatrix} m_\phi + 2\lambda\widetilde{\phi}_o & \sqrt{\frac{6}{5}}\rho\phi_o \\ \sqrt{\frac{6}{5}}\rho\phi_o & m_{15} \end{pmatrix} \quad (4.18)$$

There is no effect on the **45** and **50** Higgs masses. We can now finetune the **15** mass matrix to get one **15**+$\overline{\mathbf{15}}$ lower mass (at 10^{13} GeV), while keeping the other pair at the SO(10) scale. We could not have done this without the **54** field. Furthermore, we finetune the parameters in **24** mass matrix to keep only one **24** at the SU(5) scale. Since the parameters in the **24** and **15** mass matrices are different, the two fine tunings can be done independently.

We thus conclude that in the minimal SO(10) model for the triplet term to dominate type II seesaw formula, the minimal Higgs set required are: **10**, **126**-pair, **210** and **54** dimensional. We believe this result is interesting with important implications for SO(10) model building. We also set the two scales $M_5 = 2 \times 10^{16}$ Gev and $M_{10} = 10^{18}$ GeV. In the following section we study the model in more detail when the SU(5) is broken down to standard model. We will show how the model stabilizes the whole **15** at some lower scale and how the mixing angles can be reduced.

4.4 Model with **54** Higgs

4.4.1 Supersymmetric Vacuum with **54**

When SU(5) is broken, the F-term equations with broken B-L are

$$m_\phi S_- + 6\lambda S_o^2 + \eta\sigma_o\overline{\sigma}_o - 2\rho S S_- = 0 \tag{4.19}$$
$$m_\phi S_+ + 2\lambda(S_+^2 + 2S_o^2) + \eta\sigma_o\overline{\sigma}_o - \frac{4}{3}\rho S S_+ = 0$$
$$m_\phi S_o + 2\lambda(S_- + 2S_+)S_o + \eta\sigma_o\overline{\sigma}_o + \frac{1}{3}\rho S S_o = 0$$
$$m_\Sigma + \eta(S_- + 3S_+ + 6S_o) = 0$$
$$\frac{5}{6}m_{15}S + \frac{5}{36}\lambda_1 S^2 + 2\rho(S_o^2 + S_-^2 - 2S_+^2) = 0$$

where $S_{(\pm,o)}$ are the three liner combinations of the three SM singlets in **1**, **24** and **75** of SU(5), and S is the singlet in the new added **54** Higgs. In order for the two-step symmetry breaking to happen, we need to have $S_i = \widetilde{\phi}_o + m_\phi s_i$ and $S = \frac{m_\phi}{\rho x}s$ with all s_i and s much less than 1 (they are order of 10^{-2} in our scheme). x is defined by $x = \frac{\widetilde{\phi}_o}{m_\phi}$. Up to linear order in s_i and s these equations become

$$x + 6\lambda x^2 + \eta\frac{\sigma_o\overline{\sigma}_o}{m_\phi^2} + s_- + 12\lambda s_o x + 2s = 0 \tag{4.20}$$
$$s_+ - s_- + 4\lambda x(s_+ - s_o) - \frac{10}{3}s = 0$$
$$s_o - s_- + 2\lambda x(s_- + 2s_+ - 3s_o) - \frac{5}{3}s = 0$$
$$s_- + 3s_+ + 6s_o = 0$$
$$\frac{5}{6}ms + 2\rho(S_o + S_- - 2S_+) = 0$$

Where $m = \frac{m_{15}}{m_\phi}\frac{1}{(\rho x)^2}$.

The first equation in 4.20 can be satisfied by solving $\sigma_o\overline{\sigma}_o$. In order to have non-vanishing solution of s's, we require the condition

$$Det \begin{pmatrix} 1+4\lambda x & -4\lambda x & -1 & -4 \\ 4\lambda x & 1-6\lambda x & -1+2\lambda x & -2 \\ 3 & 6 & 1 & 0 \\ -4 & 2 & 2 & m \end{pmatrix} = 0 \tag{4.21}$$

and this gives $x\lambda = \frac{1}{4}$ or $m = \frac{12}{1+2\lambda x}$. In the first case, the **75** Higgs will be light and has a huge contribution to the RG running above the SU(5) scale. This will bring the coupling constant to the strong regime below 10^{18} GeV. We will exclude this and set $x\lambda \neq (-\frac{1}{2}$ or $\frac{1}{4})$ from now on. The solution of s_i is given by

$$\begin{pmatrix} s_+ \\ s_o \\ s_- \end{pmatrix} = \frac{s}{3(1+2\lambda x)} \begin{pmatrix} 4 \\ -1 \\ -6 \end{pmatrix} \tag{4.22}$$

4.4.2 Masses of **15**

In the minimal model, there is no mixing between components of **10** and **15** but the mixing terms appear when we include **54** in the model. As we have already seen, one linear combination of the **10**'s is the Goldstone boson. With the appearance of the mixing term, the Goldstone boson is now the mixture of **10** and **15** with the mixing angle going like $\frac{M_5}{M_{10}} = 10^{-2}$ which we will neglect in the following analysis. With only M_{10}, the masses of **10** is given in eq. 4.12 and masses of **15** can be written in two parts: ${M_{15}}^{(10)} + {M_{15}}^{(5)}$. ${M_{15}}^{(10)}$ is induced by vevs at M_{10} and the correction ${M_{15}}^{(5)}$ is due to the breaking of SU(5). We found

$${M_{15}}^{(10)} = \begin{pmatrix} \frac{4}{5}m_\Sigma & \sqrt{2}\chi\sigma_o \\ \sqrt{2}\overline{\chi\sigma}_o & m_{15} \end{pmatrix} \tag{4.23}$$

In order to have one light **15**, we require $\frac{4}{5}m_\Sigma m_{15} = 2\chi\overline{\chi}\sigma_o\overline{\sigma}_o$. When **24** get a vev $< 24 >$, the mass matrix of $(\Sigma_{15}, S_{15}, \phi_{10}, \overline{\Sigma}_{10})$ has the form

$$
{M_{15}}^{(5)} = \begin{pmatrix} -3\eta\phi^{24} & 0 & \\ & & m_{12} \\ 0 & \lambda_1 S^{24} & \\ & & \\ & m_{21} & m_{22} \end{pmatrix} \tag{4.24}
$$

one of the **10**'s is Goldstone boson and the other has the mass of scale M_{10}. We can integrate out the heavy **10** and get the effective ${M_{15}}^{(5)}$. We go to the light **15** basis and find

$$
{M_{15}}^{(5)} \simeq \begin{pmatrix} -3\eta\phi^{24} + \frac{2\chi\overline{\chi}\lambda_1\sigma_o\overline{\sigma}_o}{m_{15}^2}S^{24} + \epsilon_{11} & \sqrt{2}\chi(\lambda_1\sigma_o S^{24} + 3\eta\overline{\sigma}_o\phi^{24}) + \epsilon_{12} \\ \sqrt{2}\overline{\chi}(\lambda_1\overline{\sigma}_o S^{24} + 3\eta\sigma_o\phi^{24}) + \epsilon_{21} & \lambda_1 S^{24} - \frac{6\chi\overline{\chi}\eta\sigma_o\overline{\sigma}_o}{m_{15}^2}\phi^{24} + \epsilon_{22} \end{pmatrix} \tag{4.25}
$$

where $\epsilon_{ij} \sim 10^{-2} < 24 >$. If we can fine tune the parameters to make the 1-1 element of the matrix vanish, the determinant of the **15** mass matrix $\sim < 24 >^2$ and therefore all of the light Higgs components will have masses of order $\frac{<24>^2}{m_\phi} \sim 10^{14}$ GeV. This will approximately satisfy the requirement that the whole **15** is light. This requires the following condition

$$
\frac{< \phi^{24} >}{< S_{24} >} = \frac{2\chi\overline{\chi}\lambda_1\sigma_o\overline{\sigma}_o}{3\eta m_{15}^2} \tag{4.26}
$$

Getting **15** mass to $\sim 10^{14}$ GeV is only half the story. To complete the full requirements, we still have to have the two mixing angles $\alpha_{u,d}^2 \sim 0.01$ and $\alpha_d^1 \sim 0.1$. To see how this can happen we have to analyze the Higgs doublets which come form **5**, $\overline{\mathbf{5}}$ and **45**, $\overline{\mathbf{45}}$.

4.4.3 Masses of **5**

In our scenario, **45** is heavy and so its contribution to the fermion masses comes through higher dimensional operators. The mixing angle α_d^2 which characterizes this higher dimensional contribution can be estimated to be of order $\frac{M_5}{M_{10}} \sim 10^{-2}$. One of the three conditions is then satisfied automatically. We now analyze the physical Higgs doublet from **5**'s under the approximation of SU(5) symmetry. The mass matrix is given in eq. 4.14. Again, the determinant of the matrix has to be zero in order to have light doublets. This requirement gives the the following equation

$$\frac{\lambda}{\eta} = \frac{2\alpha\overline{\alpha}\sigma_o\overline{\sigma}_o}{\eta m_H\widetilde{\phi}_o + \alpha\overline{\alpha}\widetilde{\phi}_o^2} \tag{4.27}$$

The small mixing of Σ_5 ($\alpha_u^2 \sim 10^{-2}$) requires

$$\frac{2\sqrt{6}\alpha\eta\widetilde{\phi}}{\alpha\overline{\alpha}\widetilde{\phi}_o - \eta m_H} = 100 \tag{4.28}$$

and $\alpha_d^1 \sim 0.1$ requires

$$\frac{\alpha}{\overline{\alpha}} = 10^3 \tag{4.29}$$

or

$$\frac{\alpha\widetilde{\phi}_o}{\eta\overline{\sigma}_o} = 10 \tag{4.30}$$

To summarize, we collect all conditions from the arguments above.

$$\sigma_o\overline{\sigma}_o = -(x + 6\lambda x^2 + s_- + 12\lambda s_o x + 2s)\frac{m_\phi^2}{\eta} \tag{4.31}$$

$$m = \frac{12}{1 + 2\lambda x} \tag{4.32}$$

$$x\lambda \neq -\frac{1}{2}, \frac{1}{4} \tag{4.33}$$

$$\begin{pmatrix} s_+ \\ s_o \\ s_- \end{pmatrix} = \frac{s}{3(1+2\lambda x)} \begin{pmatrix} 4 \\ -1 \\ -6 \end{pmatrix} \tag{4.34}$$

$$\frac{4}{5} m_\Sigma m_{15} = 2\chi\overline{\chi}\sigma_o\overline{\sigma}_o \tag{4.35}$$

$$\frac{< \phi^{24} >}{< S_{24} >} = \frac{4\lambda_1 m_\Sigma}{15\eta m_{15}} \tag{4.36}$$

$$\frac{\lambda}{\eta} = \frac{2\alpha\overline{\alpha}\sigma_o\overline{\sigma}_o}{\eta m_H \widetilde{\phi}_o + \alpha\overline{\alpha}\widetilde{\phi}_o^2} \tag{4.37}$$

$$\frac{2\sqrt{6}\alpha\eta\widetilde{\phi}}{\alpha\overline{\alpha}\widetilde{\phi}_o - \eta m_H} = 100 \tag{4.38}$$

$$\frac{\alpha\widetilde{\phi}_o}{\eta\overline{\sigma}_o} = 10 \tag{4.39}$$

When these conditions are all satisfied, we have the required type-II seesaw and the triplet term dominates. Because m, λ_1 and χ are free parameters, equation (4.32), (4.35) and (4.36) can be satisfied by assigning the correct value to these three parameters. Equation (4.34) is just the solution of $s_{\pm,o}$. Equation (4.38) can be satisfied by tuning the denominator. We simply set the denominator of equation (4.38) to zero and find that

$$\begin{aligned} \frac{\alpha\widetilde{\phi}_o}{\eta\overline{\sigma}_o} &= 10 \\ \alpha\overline{\alpha}\widetilde{\phi}_o &= \eta m_H \\ \frac{\lambda}{\eta} &= \frac{\sigma_o\overline{\sigma}_o}{\widetilde{\phi}_o^2} \\ \sigma_o\overline{\sigma}_o &= -x(1+6\lambda x)\frac{m_\phi^2}{\eta} \end{aligned} \tag{4.40}$$

where we have simplified equation (4.31) by including only the leading order terms. Note that $x = \frac{\widetilde{\phi}_o}{m_\phi}$. We found from the equations above that $\lambda x = -\frac{1}{7}$. If $m_H \sim m_\phi$, we have also found the relations $\frac{\alpha}{\overline{\alpha}} = \frac{100}{7}$ and $\frac{\alpha}{\sqrt{|\lambda\eta|}} = 10$. There are enough free parameters in the model to allow the above equations to be satisfied simultaneously. In this model we have $f_{33} \sim 0.1$, $v_{B-L} = 10^{18}$ GeV, the triplet mass $M_T \sim 10^{14}$ GeV and the GUT scale remains at 2×10^{16} GeV.

4.5 Gauge Unification and SO(10) Scale

Given the above multiplet structure, we can now check the strength of coupling at the SO(10) unification scale. For this purpose, let us assume that the theory below the SU(5) scale is MSSM with an extra **15** pair at 10^{14} GeV. To see the impact of this on the unification scale, first note that the a full multiplet does not change the unification scale. Therefore we still have $M_5 \simeq 2 \times 10^{16}$ GeV. However, the coupling constant changes. To see this we first write the new unification formula in the presence of light **15**'s:

$$\alpha_5^{-1} \;=\; \alpha_{MSSM}^{-1} - \frac{7}{2\pi} ln\left(\frac{M_5}{M_T}\right) \tag{4.41}$$

$\alpha_{MSSM}^{-1} \approx 23.7$ is the unified coupling constant predicted by MSSM. The new unified coupling constant at M_5 is found to be $\alpha_5^{-1} \simeq 17$. The RGE of the unified coupling constant above M_5 and below M_{10} is given by

$$\alpha_{10}^{-1} \;=\; \alpha_5^{-1} - \frac{\beta}{2\pi} ln\left(\frac{M_{10}}{M_5}\right) \tag{4.42}$$

At this energy scale , we have one **15** pair, one or two **24** and up to three **5** pairs. If only one **24** and one pair of **5** are below the SO(10) scale M_{10}, we found $\beta = 19$ and $M_{10} \simeq 5.5 \times 10^{18}$ GeV for $\alpha_{10} \simeq 1$. If we assume maximal light Higgs : two **24** and three pairs of **5** below M_{10}, β will be 26 and $M_{10} \simeq 1.2 \times 10^{18}$. Either case would be sufficient to make our model work without getting to the strong regime.

4.6 Summary

In this chapter we found a SO(10) model which break into MSSM with an extra pair of **15** Higgs. The GUT scale remain to be 2×10^{16} GeV. The mass of the heaviest right-handed neutrino in the model is 10^{17} GeV which is big enough such that its contribution to the neutrino masses is negligible. The triplet in the **15** with a mass of order $\sim 10^{14}$ GeV can then dominate the type-II seesaw as needed to explain the neutrino masses and mixings. The explicit RGE calculations show that the model is perturbative all the way to the SO(10) scale which is 10^{18} GeV and so our two-step symmetry breaking analysis is consistent.

Chapter 5: Conclusion

The main new results of this thesis are that a minimal SO(10) model with single **10** and $\overline{\mathbf{126}}$ Higgs coupling to matter is completely predictive for neutrino masses and mixings and can provide an excellent description of the presently favored data without any additional assumption except that the $SU(2)_L$ triplet vev dominates the neutrino masses. The model predicts a hierarchical mass pattern for neutrinos and a value of $U_{e3} \simeq 0.16$, both of which can be tested in the upcoming long baseline neutrino experiments. The atmospheric mixing angle is found to be around 0.9 which is also a testable prediction of the model. In our model, the Yukawa matrices have a hierarchical pattern. A rough understanding of which could come from introducing a local horizontal $U(2)_H$ symmetry under which the first two families transform as a doublet. This result has stimulated many others to look into different aspects of the model such as the inclusion of a CP phase[68], leptogenesis [69], the extension of the model with heavy **120** Higgs.[70] and including both terms in the type-II seesaw [71].

We have also discussed the predictions for nucleon decay in the model. For the range of the parameters that are allowed by the neutrino data, we vary the GUT scale parameters (unrelated to the neutrino sector) so as to satisfy the stringent experimental bounds for the decay mode $p \to K^+ + \bar{\nu}$. We then predict an upper limit for the lifetimes for the modes $\tau(n \to \pi^0 \bar{\nu}) = 2\tau(p \to \pi^+ \bar{\nu}) \leq 5.7 - 13 \times 10^{32}$ years and $\tau(n \to K\bar{\nu}) \leq 2.9 \times 10^{33}$ yrs for the wino masses of 200 GeV and squark and slepton masses of 1 TeV. This should provide motivation for a new search for proton decay, more specifically, for these decay modes in question.

Finally, we show explicitly that the triplet dominated type-II seesaw, which is a crucial ingredient in our work, can actually be realized in a specific SUSY GUT model. Some finetuning have to be enforced as expected in a minimal model. We find that, SO(10) must be broken down to SU(5) at the energy scale of 10^{18} GeV and to the standard model at the normal SUSY GUT scale which is 2×10^{16} GeV. The whole **15** multiplet of SU(5) has a mass of order 10^{14} GeV and $f_{33} \sim 0.1$ due to small mixing angles among the doublets. Although it is effectively SU(5) at the GUT scale with **5**, $\bar{\mathbf{5}}$ and **45** coupling to matter and a new pair of **15**, the requirement of SO(10) invariance at the more fundamental level is still essential. Without SO(10), the mass formulae given in chapter 2. will be totally different and the triplet dominated type II seesaw term will be a free parameter and unrelated to any of the Dirac mass matrices. The great predictive power we have shown due to the SO(10) symmetry cannot be provided by SU(5) alone without introducing extra symmetry.

The predictions on the neutrino oscillation parameters and the upper limit of the two neutron partial lifetimes can be used to test the model. Our analysis do not take into account the radiative corrections, which are expected to be small, to both the oscillation parameters and dimension five operators. This theoretical uncertainty will become important at the point when the experiments are probing the limit. Besides the radiative effect, the uncertainties in the quark masses are also important. When more reliable quark masses are obtained, this analysis have to be revised in order to provide a more precise test to this minimal SO(10) model. This thesis is based on a series of papers by the author and his collaborators[72].

Appendix A: Group and Representation

A.1 SO(10)and Its Subgroup

SO(10) is rank 5 compact classical group. It belong to the infinite series D_n of the classification theory with n=5. The Lie Algebra of SO(10) is denoted by the dynkin diagram as shown below

$$\begin{array}{ccccccc} & & & & & & 2 \\ 5 & - & 4 & - & 3 & < & \\ & & & & & & 1 \end{array} \tag{A.1}$$

Here we use **H** to be the Cartan sub-Algebra and α_i (i=1,2,3,4,5) to be the positive simple roots in the Cartan basis. The dual vectors w_i are defined by

$$2\frac{(w_i, \alpha_j)}{(\alpha_j, \alpha_j)} = \delta_{ij} \tag{A.2}$$

From the knowledge of the inner product between α_i we found that w_i can be written as

$$\begin{aligned} w_1 &= \frac{1}{4}(5\alpha_1 + 3\alpha_2 + 6\alpha_3 + 4\alpha_4 + 2\alpha_5) \\ w_2 &= \frac{1}{4}(3\alpha_1 + 5\alpha_2 + 6\alpha_3 + 4\alpha_4 + 2\alpha_5) \\ w_3 &= \frac{1}{2}(3\alpha_1 + 3\alpha_2 + 6\alpha_3 + 4\alpha_4 + 2\alpha_5) \\ w_4 &= (\alpha_1 + \alpha_2 + 2\alpha_3 + 2\alpha_4 + \alpha_5) \\ w_5 &= \frac{1}{2}(\alpha_1 + \alpha_2 + 2\alpha_3 + 2\alpha_4 + 2\alpha_5) \end{aligned} \tag{A.3}$$

Taken away α_1 will break the group down to SU(5)×U(1) and the U(1) charge is given by $\frac{\overrightarrow{w}_1.\overrightarrow{H}}{|\overrightarrow{w}_1|} = \sqrt{\frac{8}{5}}\overrightarrow{w}_1.\overrightarrow{H}$. For Example, $16 = 1(-5\sqrt{\frac{1}{40}}) + 10(\sqrt{\frac{1}{40}}) + 5(-3\sqrt{\frac{1}{40}})$.

The extended dynkin diagram is given by

$$\begin{array}{ccccccc} 5 & & & & & & 2 \\ & > & 4 & - & 3 & < & \\ x & & & & & & 1 \end{array} \tag{A.4}$$

where x has root vector equal to $-\frac{\overrightarrow{w}_4}{|\overrightarrow{w}_4|} = -\overrightarrow{w}_4$. Taken away α_4 will break the group down to $SU_L(2) \times SU_R(2) \times SU_c(4)$ with positive roots given by $(\alpha_5), (\alpha_x), (\alpha_{1,2,3})$ respectively. Further taken away α_1 break the $SU_c(4)$ down to $SU_c(3) \times U_{B-L}(1)$ with the U(1) charge given by $\sqrt{\frac{2}{3}}(2w_1 - w_4)$.

A.1.1 SU(4)

Although taking the roots $\alpha_{1,2,3}$ from the diagram of SO(10) form the diagram of SU(4), the dual vectors are different because those dual vector of the full SO(10) root space take into account α_4 which has non-trivial overlap with α_3. Using the same index, dual vectors are given by

$$\begin{aligned} w_1 &= \frac{1}{4}(3\alpha_1 + \alpha_2 + 2\alpha_3) \\ w_2 &= \frac{1}{4}(\alpha_1 + 3\alpha_2 + 2\alpha_3) \\ w_3 &= \frac{1}{2}(\alpha_1 + \alpha_2 + 2\alpha_3) \end{aligned} \tag{A.5}$$

The subgroup of SU(4) with color is SU(3)$\times U_{B-L}(1)$. In the language of weight and root, $U_{B-L}(1) = \widehat{w}_1.\overrightarrow{H}$. For example, the irrep **6** which has the highest weight w_3 can be decomposed into $3(\sqrt{\frac{1}{6}})$ and $\overline{3}(-\sqrt{\frac{1}{6}})$.

A.1.2 SU(5)

Using the index of SO(10) diagram, dual vectors of SU(5) are given by

$$
\begin{aligned}
w_2 &= \frac{1}{5}(4\alpha_2 + 3\alpha_3 + 2\alpha_4 + \alpha_5) \\
w_3 &= \frac{1}{5}(3\alpha_2 + 6\alpha_3 + 4\alpha_4 + 2\alpha_5) \\
w_4 &= \frac{1}{5}(2\alpha_2 + 4\alpha_3 + 6\alpha_4 + 3\alpha_5) \\
w_5 &= \frac{1}{5}(\alpha_2 + 2\alpha_3 + 3\alpha_4 + 4\alpha_5)
\end{aligned} \tag{A.6}
$$

A.2 Decomposition

The fundamental 10 can be expressed by ϕ_a or ψ_m with $a \in \{1, 2, .., 10\}$ is index of **10** of SO(10) and $m \in \{1, .., 5\} \bigcup \{1^*, .., 5^*\}$ is index of **5** and $\bar{5}$ of SU(5). Their can be transform from one to another by

$$
\phi_a = S_a^m \psi_m \tag{A.7}
$$

explicitly, the relation can be written as

$$
\begin{aligned}
\phi_{1,2} &= \frac{\sigma_{21}^{1,2}}{\sqrt{2}}(\psi_1 \pm \psi_{1^*}) \\
\phi_{3,4} &= \frac{\sigma_{21}^{1,2}}{\sqrt{2}}(\psi_2 \pm \psi_{2^*}) \\
\phi_{5,6} &= \frac{\sigma_{21}^{1,2}}{\sqrt{2}}(\psi_3 \pm \psi_{3^*}) \\
\phi_{7,8} &= \frac{\sigma_{21}^{1,2}}{\sqrt{2}}(\psi_4 \pm \psi_{4^*}) \\
\phi_{9,0} &= \frac{\sigma_{21}^{1,2}}{\sqrt{2}}(\psi_5 \pm \psi_{5^*})
\end{aligned} \tag{A.8}
$$

We can use the transformation matrix S_i^m given above to change the index of tensor representation of SO(10) into index of SU(5). Note that we have $\sum_i S_i^m S_i^n = \delta_{n,m^*}$. We use the lower index to be 1,2,..,5 and the upper index to be 1^*,..,5^*. The rule of the transformation is replacing the contracted pair of SO(10) index by SU(5) index with one of them up and the other down, and sum over all the possible configuration. In the language of SU(5), the Higgs multiples $\Phi(\mathbf{210})$ and Σ ($\mathbf{126}$) can be decomposed in the following irreducible representation

$$\begin{aligned} (\Phi^{40})_{abc}{}^{d} &= \Phi_{abc}{}^{d} - \frac{1}{6}((\Phi^{10})_{ab}\delta_c{}^d + (abc)) \\ (\Phi^{75})_{ab}{}^{cd} &= \Phi_{ab}{}^{cd} - \frac{1}{3}((\Phi^{24})_a{}^c\delta_b{}^d + (ab)(cd)) + (\Phi_o\delta_a{}^c\delta_b{}^d + (ab)) \\ (\Phi^{24})_a{}^b &= \Phi_{af}{}^{bf} - 4(\Phi_o\delta_a{}^b) \end{aligned} \tag{A.9}$$

and

$$\begin{aligned} 0 &= \overline{\Sigma}_{abcd}{}^{e} - \frac{1}{12}((\overline{\Sigma}^{10})_{abc}\delta_d{}^e + (abcd)) \\ (\overline{\Sigma}^{50})_{abc}{}^{de} &= \overline{\Sigma}_{abc}{}^{de} - \frac{1}{12}((\overline{\Sigma}^{5})_a\delta_b{}^d\delta_c{}^e + (abc)) \\ 0 &= \overline{\Sigma}_{ab}{}^{cde} - \frac{1}{4}((\overline{\Sigma}^{45})_a{}^{cd}\delta_b{}^e + (ab)(cde)) \\ (\overline{\Sigma}^{15})_a{}^{bcde} &= \overline{\Sigma}_a{}^{bcde} \\ 0 &= \overline{\Sigma}_{abf}{}^{df} - \frac{1}{4}((\overline{\Sigma}^{5})_a\delta_b{}^d + (ab)) \\ (\overline{\Sigma}^{45})_a{}^{cd} &= \overline{\Sigma}_{af}{}^{cdf} \end{aligned} \tag{A.10}$$

Where we have use the notation

$$\begin{aligned} \Phi_{abcd} &= \epsilon_{abcde}(\Phi^5)^e \\ \Phi_{abf}{}^{f} &= (\Phi^{10})_{ab} \end{aligned} \tag{A.11}$$

$$
\begin{aligned}
\Phi_{fg}{}^{fg} &= 20\Phi_o \\
\overline{\Sigma}^{abcde} &= 0 \\
\overline{\Sigma}_{abcde} &= \epsilon_{abcde}\sigma_o \\
\overline{\Sigma}_{abcf}{}^{f} &= (\overline{\Sigma}^{10})_{abc} \\
\overline{\Sigma}_{afg}{}^{fg} &= (\overline{\Sigma}^{5})_{a}
\end{aligned}
$$

It can be shown explicitly that all of the irreducible fields above (with the number of dimension denoted by the number in the parenthesis) are traceless as it should be, i.e. vanished when contract any one upper index with any lower index. the notation (abc) is used to include all the permutation of a,b, and c to make the field total anti symmetric. The decomposition of $\overline{\Sigma}$ can be obtained by taking the complex conjugate of Σ given above. So these are our irreducible representation in SU(5).

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