Capacity and Variability Analysis of the IEEE 802.11 MAC Protocol

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***Abstract*—*Packet error* in the IEEE 802.11 network is one source of performance degradation and its variability. Most of the previous works study how *collision avoidance* and *hidden terminals* affect 802.11 performance metrics, such as probability of a collision and saturation throughput. In this paper we focus on the effect of packet errors on capacity and variability of the 802.11 MAC protocol. We develope a new analytical model, called p_e-Model, by extending the existing model (Tay and Chua's model) to incorporate *packet error probability* p_e. With p_e-Model, we successfully analyze capacity and variability of the 802.11 MAC protocol. The variability analysis shows that increasing packet error probability by Δp_e has more effect on saturation throughput, than adding $0.5W\Delta p_e$ stations, where W is the minimum contention window size, We also show the numerical validation of p_e-Model with 802.11 MAC-level simulator.**

I. INTRODUCTION

With the popularity of the IEEE 802.11 [1] based wireless network, it has become increasingly important to analyze and predict the performance of the IEEE 802.11 protocol and its variability accurately. Carrier Sensing Multiple Access/Collision Avoidance (CSMA/CA) and *hidden terminals* [3], [4] have been considered to be important in performance analysis of the IEEE 802.11 PHY/MAC protocol.

CSMA/CA exploits *binary exponential backoff* mechanism to avoid collisions among the wireless stations contending for the medium as follows. Before sending a packet, a wireless station first senses the medium for the duration T_{DIFS} (DIFS is *Distributed Interframe Space*). If the medium is free for the duration, the wireless station starts sending the packet immediately. Otherwise, if the wireless station detects the medium was busy for the duration, the wireless station backs off for a multiple of time slots (T_{slot}). The multiple is randomly chosen between $[0, 2^i W]$ ($i = 0, 1, 2, \cdots, m$). W is called *minimum contention window (CW) size*, which is set to the same value for all the wireless stations. If the wireless station transmitted a packet and received ACK frame correctly, then i is set to 0. If the wireless station failed to receive ACK frame, i is incremented by 1. i can be incremented up to m, therefore the maximum CW size is $2^m W$.

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A pair of wireless stations in the same wireless LAN is referred to as being *hidden* from each other if the transmissions from one station cannot be heard by the other. Presence of such hidden terminals is another important consideration, which has been studied in many literatures [3], [4], [5].

In this paper we focus on the effect of *packet errors* on performance of the 802.11 MAC protocol. By packet errors in this paper, we mean the *packet transmission failures* between a pair of wireless stations, which are due to *other than* collisions. Packe errors can cause *reception errors* at receiver station.

A. Causes and Effects of Packet Errors

Packet errors usually occur due to *non-ideal channel condition* [7]. Partition loss in the building and multipath fading, combined with ambient noise, decrease SNR (Signal-to-Noise Ratio), therefore cause packet errors. Co-channel and adjacent channel interferences also cause packet errors.

Wireless device variability is another source of packet errors. Different devices have different output power, receive sensitivity and firmware, which may incur packet errors. Such errors would not occur if different pair of cards were used. In the experimental study on wireless monitoring [9], they observe that some firmwares send data at multirates (1, 2, 5.5 and 11 Mbps), but other firmwares receive them correctly only at 11 Mbps, generate packet errors at other rates. This is an example of the packet errors due to wireless device variability.

The impact of packet errors on performance metrics of the IEEE 802.11 protocol is different from that of hidden terminals. Hidden terminal problem focuses on unreliable carrier sensing ability, while packet error analysis concerns unreliable packet transmission ability. Another difference is that packet errors cause packet retransmissions at sender station and reception errors at receiver station, which is not the effect of hidden terminals. Packet retransmissions incur delay and packet loss at sender station. Reception errors incur additional delay at *receiver* station. When a reception error occurs, the receiver station waits for T_{EIFS} (*Extended Interframe Space*, usually more than $7 \times T_{DIFS}$) instead of T_{DIFS} so that the sender can have enough time to know that a reception error may have occurred.

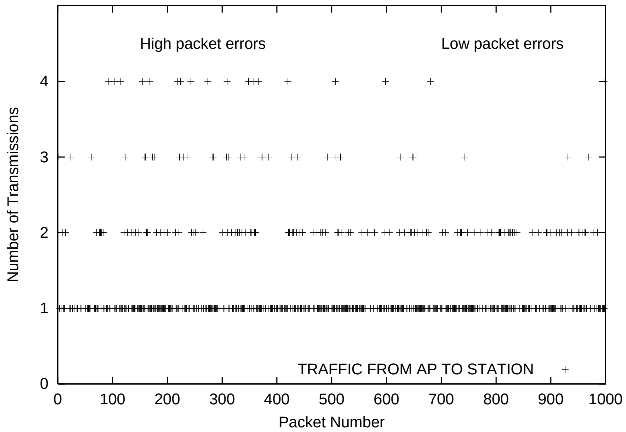

Fig. 1. Number of transmissions of distinct packet varies over time. Distinct packets are identified with Packet Number. Packets are sent at 200 packets/second. Number of retransmissions is limited to 3.

B. Variability of Packet Errors

If the channel condition varies over time, packet errors should have the corresponding variability. In this section we show the presence of such variability with an experimental evidence.

To observe such variability in real wireless network, we conduct *active measurements* on wireless traffic. We make only one wireless station, say station A, associated with an Access Point (AP), which has a distinct *essid* (wireless network identifier) from other APs. In this experiment a *wired* station sends UDP packets to A through the AP at 200 packets/second. At the same time we place a wireless *sniffer* between A and the AP to capture the traffic *from the AP to* A. By analyzing the sniffer traces, we observe many retransmissions due to packet errors.

In Fig.1, the number of transmissions of each distinct packet (identified with "Packet Number") varies over time. Because there are no other stations in the network, no collisions occur during the experiments. Therefore the variability of the transmission numbers reflect the variability of packet errors. As shown in Fig.1, packet errors increase until around 200th packet, then keep decreasing until around 800th packet. This observation provides an experimental evidence for presence of packet errors and their variability. Such packet error variability causes the variability of the IEEE 802.11 protocol performance, which we will go over in Section V.

In this work we have extensively studied the effect of packet errors on capacity and variability of the 802.11 MAC protocol. To quantify packet errors, we define a *packet error probability* p_e to be the probability with which each packet transmission experiences a packet error. We extend the existing analytical model (Tay and Chua's model [8]) to incorporate packet error probability p_e. The new model is called p_e*-Model* in this paper.

In Section II we overview the existing models and approaches, including Tay and Chua's model [8]. p_e-Model is described in detail in Section III. Capacity and variability analysis based on p_e-Model are followed in Section IV and V respectively. With p_e-Model, we successfully analyze capacity and variability of the 802.11 MAC protocol. The variability analysis shows that increasing packet error probability by Δp_e has more effect on saturation throughput, than adding $0.5W\Delta p_e$ stations to the current network. In Section VI we show the numerical results on validation and analysis of p_e-Model with 802.11 MAC-level simulator.

II. Performance Models of the IEEE 802.11 Protocol

In this section we overview the existing models and anaysis techniques. For our model is based on Tay and Chua's mathematical model [8] , We explain it in detail followingly.

A. Overview of the Performance Models

One of the issues in the analysis of the IEEE 802.11 protocol has been to devise an analytical model which can predict the collision probability and its effect on the performance metrics in consideration of CSMA/CD and hidden terminals. Bianchi [2] conducts Marcov chain analysis for calculating collision probability and saturation throughput. Similarly Ho and Chen [4] derive the throughput and average delay by use of 2-dimensional Markovian analys. Hidden terminals have been considered in the literatures [3], [4], [5].

While some of these studies use the stochastic analysis [2], [4], Tay and Chua [8] use the mathematical approximations by means of average values. This model provides closed-form expressions for the probability of a collision and the saturation throughput. Tay and Chua's model is based on our p_e-Model, will be described in more detail in the following section.

The models mentioned so far assume ideal channel conditions, where packet error does not occur. Qiao and Choi [6] assume additive white Gaussian noise channel (AWGN) and calculate packet error probability, then derive the goodput performance of PHY/MAC protocol analytically. But their MAC model is oversimplfied. They assume that there are only two stations (one sender and one receiver) therefore no collisions occur. In our model we consider both packet errors and the collisions among n stations.

B. Capacity Metrics and Parameters

Before describing Tay and Chua's model, we define important metrics and parameters used in capacity analysis of the IEEE 802.11 MAC protocol. Table I lists the all variables used in Tay and Chua's analysis and our analysis, described in the following sections.

The capacity metrics include *transmission failure probability* p_f, *collision probaility* p_c, *total channel utilization* u_{total} and *saturation throughput* S. Each transmision has the probability of transmission failure p_f and the probability of collision p_c. Total channel utilization u_{total} is the fraction of non-idle period of the channel. Saturation throughput S is the fraction of channel bandwidth that is used to successfully transmit payload bits if every station's buffer is always occupied, i.e. if the network is under *saturation* condition.

The parameters used in the capacity analysis of the IEEE 802.11 protocol include minimum window size W, maximum

n	the number of stations in a wireless cell
W	minimum window size
m	maximum size is $2^m W$
$W_{backoff}$	(average) contention window (CW) size
T_{slot}	slot time
T_{SIFS}	time duration of short interframe space
T_{DIFS}	time duration of distributed interframe space
T_{EIFS}	time duration of extended interframe space
$T_{payload}$	time to transmit payload bits
$T_{physical}$	time to transmit packet (including headers)
T_{ACK}	transmission time for an acknowledgement
T_{cycle}	time between the start of two payload transmissions
$r_{success}$	rate of successful transmissions
$r_{collision}$	rate of collisions
r_{pkterr}	rate of packet errors
r_{xmit}	rate of transmissions
p_f	probability of a transmission failure
p_c	probability of a collision
p_e	probability of a packet error
S	channel utilization by successful transmission of payload bits
u_{total}	total channel utilization (including collisions)

TABLE I

GLOSSARY (REVISED FROM TAY AND CHUA MODEL)

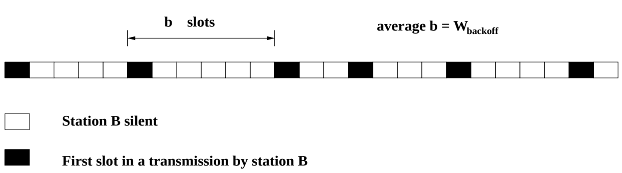

Fig. 2. Station B's activity as seen on station A's time-line (excerpted from [8])

window size $2^m W$, T_{slot}, T_{DIFS}, T_{EIFS}, which were defined in Section I. T_{SIFS} is the duration for which a receiver station waits before sending ACK frame to the sender station. Other parameters are described in Table I.

C. Tay and Chua's Model

Tay and Chua's model [8] assumes that packet tranmission fails only when collision occurs. Therefore each transmission fails with the probability of $p_c (= p_f)$. Average CW size $W_{backoff}$ is calculated as the sum of possible average CW size $2^i W/2 (i = 0, 1, \cdots, m)$ times the corresponding probability.

$$\begin{aligned} W_{backoff} &= \sum_{i=0}^{m} ((2^i W/2) p_c^i (1 - p_c)) + p_c^{m+1} (2^m W)/2 \\ &= \frac{1 - p_c - p_c (2p_c)^m}{1 - 2p_c} \frac{W}{2}. \end{aligned} \quad (1)$$

Suppose station A sees the packet transmissions by station B under *saturation* condition. Every time A detects any B's transmission, A's backoff timer is suspended untill B's transmission completes [1], and resumes after the completion, as illustrated in Fig 2. Therefore collision occurs only if A starts transmission at *black* slots, in Fig. 2. Because average interval between black slots equals to average CW size, $W_{backoff}$, A's probability of colliding with B is $1/W_{backoff}$. Therefore p_c, the probability of colliding with $n - 1$ stations is calculated as

$$\begin{aligned} p_c &= 1 - (1 - \frac{1}{W_{backoff}})^{n-1} \\ &= 1 - (1 - \frac{2(1 - 2p_c)}{1 - p_c - p_c (2p_c)^m} \frac{1}{W})^{n-1}. \end{aligned} \quad (2)$$

Let $T_{TX} = T_{physical} + T_{SIFS} + T_{ACK} + T_{DIFS}$, then saturation throughput S is derived based on p_c as

$$S = \frac{2(1 - p_c)}{2 - p_c} \times \frac{T_{payload}}{T_{TX} + WTslot/(n + 1)}.$$

The followings are the capacity analysis results in [8].

- Collision independence: the probability of a collision does not depend on packet length, the latency in crossing the MAC/PHY layers, the acknowledgement timeout, the interframe spaces and the slot size; it only depends on W, m, and n.
- Inverse gap dependence: When n is large but smaller than $2^{m-1} W$, the protocol's performance depends on W and n only through a new parameter $q = \frac{n-1}{W}$, called *an inverse gap*.
- Maximum window size effect: suppose $p_c < 0.5$. The choice of maximum window size has minimal effect (namely $O(p_c (2p_c)^m)$) on the collision rate and saturation throughput.
- Approximation of p_c and S: for large n the collision probability p_c and the saturation throughput S can be approximated by

$$\begin{aligned} p_c &= \frac{1}{2}(1 + 4q - \sqrt{1 + 4q^2}), \\ S &= \frac{2(1 - p_c)}{2 - p_c} \times \frac{T_{payload}}{T_{TX} + T_{slot}/q}. \end{aligned}$$

- Maximum throughput: suppose $T_{TX} \gg 4T_{slot}$. The saturation throughput is maximum when $W = \sqrt{\frac{T_{TX}}{T_{slot}}}(n - 1)$.

III. p_e-MODEL: A PACKET ERROR EXTENSION OF TAY AND CHUA MODEL

A. Assumptions

p_e-Model is an extension of Tay and Chua model by incorporating probability of packet error p_e. The model assumes that transmission fails when either collisions or packet errors occurs. The two events are assumed to be independent. Therefore transmission failure probability p_f is given by

$$p_f = p_c + p_e - p_c p_e \approx p_c + p_e. \quad (3)$$

For simplicity we ignore the product of p_c and p_e. We will show in Section VI the model does not lose much in the prediction accuracy due to this approximation. We also assume that packet errors *always* incur reception errors at the receiver station. Every time packet error occurs, therefore the sender experiences retransmission or loss of the packet, and the receiver suffers additional delay of $(T_{EIFS} - T_{DIFS})$. Another assumption is that n wireless statoins all have the same packet error probability, i.e. p_e is assumed to be a global parameter for all the stations.

B. p_e-Model

We can calculate the collision probability p_c in similar way as in (2), except that in p_e-Model p_f is different from p_c, therefore p_f is used to calculate $W_{backoff}$ in (1).

$$p_c = 1-(1-\frac{2(1-2p_f)}{1-p_f-p_f(2p_f)^m}\frac{1}{W})^{n-1} \quad (4)$$

By (3), transmission failure probability p_f is given by

$$\begin{aligned} p_f &= p_c+p_e \\ &= 1+p_e-(1-\frac{2(1-2p_f)}{1-p_f-p_f(2p_f)^m}\frac{1}{W})^{n-1} \quad (5) \end{aligned}$$

r_{xmit} and $r_{success}$ denote *the rate of transmissions* (including failures) and *the rate of successful transmissions* respectively. $(1-p_f)$ is the probability of successful transmission, therefore we have

$$\frac{r_{success}}{r_{xmit}} = 1-p_f. \quad (6)$$

Let $r_{collision}$ and r_{pkterr} to be *the rate of collisions* and *the rate of packet errors* respectively. We count multiple transmissions that collide as one collision. Approximately each collision is between just two transmissions, therefore $2r_{collision}$ contributes to the rate of transmission failures. Transmission failures are also due to packet errors, therefore we have

$$r_{xmit}-r_{success} = 2r_{collision}+r_{pkterr} \quad (7)$$

$$\frac{2r_{collsion}}{r_{pkterr}} = \frac{p_c}{p_e}. \quad (8)$$

We define T_{cycle} to be the average time between the starts of two payload transmissions under saturation condition. Collided transmissions occur at the same time, therefore $r_{collision}$ (*not* $2r_{collision}$) contributes to $1/T_{cycle}$ as follows.

$$\frac{1}{T_{cycle}} = r_{success}+r_{collision}+r_{pkterr}. \quad (9)$$

Soving the above equations (6), (7), (8) and (9), we can express r_{xmit}, $r_{success}$, r_{pkterr} and $r_{collision}$ in terms of p_f and p_e as follows.

$$r_{collision} = \frac{p_f-p_e}{2-p_f+p_e}\frac{1}{T_{cycle}}. \quad (10)$$

$$r_{success} = \frac{2(1-p_f)}{2-p_f+p_e}\frac{1}{T_{cycle}}. \quad (11)$$

$$r_{pkterr} = \frac{2p_e}{2-p_f+p_e}\frac{1}{T_{cycle}}. \quad (12)$$

$$r_{xmit} = \frac{2}{2-p_f+p_e}\frac{1}{T_{cycle}}. \quad (13)$$

Total utilization u_{total}, the fraction of non-idle period of the channel, is given by

$$\begin{aligned} u_{total} &= r_{success}(T_{physical}+T_{ACK}) \\ &+ (r_{collision}+r_{pkterr})T_{physical}. \quad (14) \end{aligned}$$

T_{cycle} consists of packet transmission time, T_{SIFS}, ACK transmission time, carrier sensing time(T_{DIFS} or T_{EIFS}) and contention period before any station obtains the medium. If all n stations experiences reception errors (due to packet errors), then T_{EIFS} is used for carrier sensing time. Otherwise there exists at least one station that waits for T_{DIFS} and the station obtains the medium. Therefore the average carrier sensing time is $(1-p_e^n)T_{DIFS}+p_e{}^nT_{EIFS}$. There exist $(1-p_f)n$ stations whose CW is W. When those stations uniformly choose a time in W, then the earliest slot will be $W/((1-p_f)n+1)$ slots, which is the average contention period. For simplicity, we approximate the average contention period to be $W/(n+1)$ slots. We discuss the effect of this approximation in SectionVI. T_{cycle} therefore is given by

$$\begin{aligned} T_{cycle} &= T_{physical}+T_{SIFS}+T_{ACK}+(1-p_e^n)T_{DIFS} \\ &+ p_e^nT_{EIFS}+\frac{W}{n+1}T_{slot}. \quad (15) \end{aligned}$$

Saturation throughput S is given by

$$S = r_{success}\times T_{payload} = \frac{2(1-p_f)}{2-p_f+p_e}\times\frac{T_{payload}}{T_{cycle}}. \quad (16)$$

IV. Capacity Analysis

Claim 1: When n is large (say, $n \geq 5$) but smaller than $2^{m-2}W$ and $p_e < 0.5$, the protocol's performance metrics(p_f, $r_{success}, r_{xmit}, r_{collision}, r_{pkterr}, u_{total}$, S) depends on W and n only through the inverse gap $q = \frac{n-1}{W}$.

Proof: From equation (5), taking first order approximation, we get

$$p_f = \frac{2(1-2p_f)}{1-p_f-p_f(2p_f)^m}\frac{n-1}{W}+p_e. \quad (17)$$

Now let

$$\begin{aligned} f(x) &= (x-p_e)\frac{1-x-x(2x)^m}{2(1-2x)}-q \quad (18) \\ &= \frac{x-p_e}{2}(x\sum_{k=0}^{m-1}(2x)^k+1)-q. \quad (19) \end{aligned}$$

where

$$0 \leq p_e \leq x \leq 1, p_e < \frac{1}{2}$$

$$0 \leq q = \frac{n-1}{W} < 2^{m-2}$$

Then, if $p_f \neq \frac{1}{2}$, then $f(x)$ in (19) is increasing and continuous function. Also by substituting 0 and 1 for x in (18) we get

$$\begin{aligned} f(0) &= -\frac{1}{2}p_e-q < 0 \\ f(1) &= (1-p_e)2^{m-1}-q \geq 2^{m-2}-\frac{n-1}{W} > 0. \end{aligned}$$

Thus, $f(x)=0$ has exactly one root in (0,1), which is a valid and unique value for p_f. p_f depends on n and W only through q. Furthermore for large n, we can approximate (15) by

$$\begin{aligned} T_{cycle} &= T_{physical}+T_{SIFS}+T_{ACK}+(1-p_e^n)T_{DIFS} \\ &+ p_e^n T_{EIFS}+\frac{W}{n-1}T_{slot}. \\ &= T_{physical}+T_{SIFS}+T_{ACK}+T_{DIFS}+\frac{T_{slot}}{q}. \end{aligned}$$

Therefore T_{cycle} depends on q. According to equations (10), (11), (12), (13), (14) and (16), $r_{success}$, r_{xmit}, $r_{collision}$, r_{pkterr}, u_{total} and S depend on q also. ■

Claim 2: Suppose $p_f < 0.5$. The choice of maximum window size has minimal effect (namely $O(p_f(2p_f)^m)$) on the transmission failure probability p_f and saturation throughput S.

Proof: Suppose

$$(p_m-p_e)\frac{1-p_m-p_m(2p_m)^m}{1-2p_m}=2q=(p_\infty-p_e)\frac{1-p_\infty}{1-2p_\infty}$$

p_m is the root of (17) for maximum window size 2^mW and p_∞ is the root for unbounded window size (using $2p_f<1$, so $\lim_{m\to\infty}(2p_f)^m=0$). Let $\Delta_{p_f}=\frac{(p_\infty-p_m)}{p_m}$. Ignoring the term $\Delta_{p_f}^2$, this gives

$$\Delta_{p_f}=\frac{(2p_m)^m(p_m-p_e)(1-2p_m)}{(p_m-p_e)(2p_m)^{m+1}+(1-2p_e)p_m-(1-2p_e)}$$

Now let the denominator as $g(x)$,

$$g(x)=(x-p_e)(2x)^{m+1}+(1-2p_e)x-(1-2p_e).$$
$$\text{where } 0\le p_e\le x<\frac{1}{2}.$$

$$g'(x)=(2x)^{m+1}+2(x-p_e)(m+1)(2x)^m>0.$$
$$g(p_e)=(1-2p_e)(p_e-1)\le g(x)<(2p_e-1)=g(\frac{1}{2}).$$
$$(1-2p_e)<|g(x)|\le(1-2p_e)(p_e-1).$$

$$|\Delta_{p_f}|<\frac{(2p_m)^m(p_m-p_e)(1-2p_m)}{1-2p_e}<(p_m-p_e)(2p_m)^m$$

Therefore the effect of m is bounded by $O(p_f(2p_f)^m)$.

Similarly, if S_m and S_∞ are the corresponding saturation throughputs and $\Delta_S=(S_\infty-S_m)/S_m$, then we get from (16)

$$\begin{aligned} \Delta_S &= -\frac{2(1-p_m(1+\Delta_{p_f}))}{2+p_e-p_m(1+\Delta_{p_f})}\frac{(2+p_e-p_m)}{2(1-p_m)}-1 \\ &= \frac{(1+p_e)p_m\Delta_{p_f}}{(1-p_m)(2+p_e-p_m(1+\Delta_{p_f}))} \end{aligned}$$

Since $\Delta_{p_f}<0$ and $0\le p_e\le p_f<0.5$, we have

$$\begin{aligned} |\Delta_S| &< \frac{(1+p_e)p_m|\Delta_{p_f}|}{(1-p_m)(2+p_e-p_m)} \\ &< \frac{(1+p_e)|\Delta_{p_f}|}{1.5+p_e}\le\frac{2}{3}|\Delta_{p_f}|=O(p_f(2p_f)^m). \end{aligned}$$

■

Claim 3: For large n the saturation throughput can be approximated by

$$S=\frac{2(1-p_f)}{2-p_f+p_e}\times \frac{T_{payload}}{T_{physical}+T_{SIFS}+T_{ACK}+T_{DIFS}+\frac{T_{slot}}{q}}, \quad (20)$$

where

$$\begin{aligned} p_f &= \frac{1}{2}(1+p_e+4q-\sqrt{1+(p_e+4q)^2-2p_e}), \\ q &= \frac{n-1}{W}. \end{aligned}$$

Proof: Since the choice of m has minimal impact on p_f, we can approximate (17) by

$$\frac{(p_f-p_e)(1-p_f)}{1-2p_f}=2q. \quad (21)$$

This has solution

$$p_f=\frac{1}{2}\left(1+p_e+4q-\sqrt{(p_e+4q)^2+1-2p_e}\right). \quad (22)$$

(The positive square root gives $p_f>1$, which is impossible.) The claim follows from (16) and (20). ■

Claim 4: Suppose $T_{physical}+T_{SIFS}+T_{ACK}+T_{DIFS}\gg 4T_{slot}$. The saturation throughput is maximum when

$$q = \frac{n-1}{W}=\frac{(1-p_e)^2}{(1-2p_e)\sqrt{(1+p_e)c}-4(1-p_e)},$$

where

$$c = (T_{physical}+T_{SIFS}+T_{ACK}+T_{DIFS})/T_{slot}$$

and

$$p_e+\frac{2(1-p_e)}{\sqrt{(1+p_e)c}}<0.5.$$

Proof: From (20),

$$\frac{dS}{dq}=\frac{-2T_{payload}}{T_{slot}(2-p_f+p_e)^2} \left(q(1+p_e)\frac{dp_f}{dq}-\frac{(1-p_f)(2-p_f+p_e)}{cq+1}\right).$$

so the maximum occurs when

$$\frac{dp_f}{dq}=\frac{(1-p_f)(2-p_f+p_e)}{(1+p_e)q(cq+1)}.$$

By (21), we have

$$\frac{dp_f}{dq}=\frac{2(1-2p_f)^2}{2p_f^2-2p_f+1-p_e}.$$

These two equations give

$$c = \frac{4(1-(3+2p_e+p_e^2)p_f+(4+2p_e)p_f^2-p_f^3)}{(1+p_e)(p_f-p_e)^2(1-p_f)}$$

$$
\begin{aligned}
&\approx \frac{1}{1+p_e}(\frac{2(1-p_f)}{p_f-p_e})^2, \text{ i.e.} \\
p_f &\approx \frac{pf\sqrt{(1+p_e)c}+2}{\sqrt{(1+p_e)c}+2}, \text{ since } \sqrt{c} \gg 2, \\
&\approx p_e + \frac{2(1-p_e)}{\sqrt{(1+p_e)c}} < 0.5.
\end{aligned}
$$

By this equation and (21) we get

$$
\begin{aligned}
2q &= 2\frac{n-1}{W} \\
&= \frac{\frac{2(1-p_e)}{\sqrt{(1+p_e)c}}(1-p_e-\frac{2(1-p_e)}{\sqrt{(1+p_e)c}})}{1-2(p_e+\frac{2(1-p_e)}{\sqrt{(1+p_e)c}})}, \text{ since } \sqrt{c} \gg 2 \\
&\approx \frac{2(1-p_e)^2}{(1-2p_e)(\sqrt{(1+p_e)c}-4(1-p_e)}.
\end{aligned}
$$

■

V. Variability Analysis

A. Variability of p_f

By (22),

$$
\frac{dp_f}{dq} = 2\left(1-\frac{p_e+4q}{\sqrt{(p_e+4q)^2+1-2p_e}}\right), \tag{23}
$$

$$
\frac{dp_f}{dp_e} = \frac{1}{2}\left(1-\frac{p_e+4q-1}{\sqrt{(p_e+4q)^2+1-2p_e}}\right) \tag{24}
$$

From (23) and (24), and let $D = \sqrt{(p_e+4q)^2+1-2p_e} > 0$,

$$
\frac{d}{dq}\left(\frac{dp_f}{dq}\right) = \frac{2}{D}\left(\frac{D^2-(1-2p_e)}{D^2}-1\right) < 0.
$$

dp_f/dq is decreasing in terms of q, and $0 \le p_e < 0.5$, we have

$$
\begin{aligned}
\left[\frac{dp_f}{dq}\right]_{q=1} &= 2(1-\frac{p_e+4}{\sqrt{p_e^2+6p_e+17}}) > 0, \\
\left[\frac{dp_f}{dq}\right]_{q=0} &= 2(1-\frac{p_e}{1-p_e}) < 2
\end{aligned}
$$

$$
\text{Therefore,} \quad 0 < \frac{dp_f}{dq} < 2. \tag{25}
$$

$$
\frac{d}{dp_e}\left(\frac{dp_f}{dp_e}\right) = \frac{1}{2D}\left(\frac{D^2-8q}{D^2}-1\right) < 0.
$$

dp_f/dp_e is decreasing in terms of p_e, and $0 < q \le 1$, we have

$$
\begin{aligned}
\left[\frac{dp_f}{dp_e}\right]_{p_e=0.5} &= \frac{1}{2}(1-\frac{8q-1}{8q+1}) > \frac{1}{9}, \\
\left[\frac{dp_f}{dp_e}\right]_{p_e=0} &= \frac{1}{2}(1-\frac{4q-1}{1+16q^2}) < 1
\end{aligned}
$$

$$
\text{Therefore,} \quad \frac{1}{9} < \frac{dp_f}{dp_e} < 1. \tag{26}
$$

We compare the effects of q's variability and p_e's variability on trasmission failure error p_f.

Let $D = \sqrt{(p_e+4q)^2+1-2p_e} > 0$,

$$
\begin{aligned}
\left[\frac{dp_f/dq}{dp_f/dp_e}\right] &= F(p_e,q) = 4\left(1-\frac{1}{D-(p_e+4q-1)}\right) \\
\frac{dF}{dq} &= 4\frac{dD/dq-4}{(D-(pf+4q-1))^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4}{(D-(p_e+4q-1))^2}\left(\frac{4(p_e+4q)}{D}-4\right) \\
&= \frac{16}{(D-(p_e+4q-1))^2}\left(\frac{p_e+4q}{\sqrt{(p_e+4q)^2+1-2p_e}}-1\right) \\
&< 0
\end{aligned}
$$

Therefore dF/dq is decreasing in terms of q. $0 < q \le 1$ and $0 \le p_e < 0.5$, so

$$
\begin{aligned}
[F(p_e,q)]_{q=1} &= 4(1-\frac{\sqrt{p_e^2+6p_e+17}+p_e+3}{8}) > 0 \\
[F(p_e,q)]_{q=0} &= 4(1-\frac{1}{2(1-p_e)}) \le 2
\end{aligned}
$$

$$
\text{Therefore,} \quad 0 < \left[\frac{dp_f/dq}{dp_f/dp_e}\right] \le 2. \tag{27}
$$

Claim 5: With W fixed, increasing packet error probability by Δp_e causes at least the same effect as adding $0.5W\Delta p_e$ stations, on transmission failure probability p_f.

Proof: From (27), for the same change of p_f (Δp_f), Δp_e and Δq have the following inequality.

$$
\begin{aligned}
\frac{\Delta p_f}{\Delta q} &\le 2\frac{\Delta p_f}{\Delta p_e}, \\
\Delta q &= \frac{\Delta(n-1)}{W} \ge 0.5\Delta p_e, \text{ therefore,} \\
\Delta n &\ge 0.5W\Delta p_e
\end{aligned}
$$

■

B. Variability of S

Restating (16) for reading convenience,

$$
S = \frac{2(1-p_f)}{2-p_f+p_e} \times \frac{qT_{payload}}{q(b+p_e^n(T_{EIFS}-T_{DIFS}))+T_{slot}}.
$$

where

$$
q = \frac{n-1}{W}, b = T_{physical}+T_{SIFS}+T_{ACK}+T_{DIFS}.
$$

S can be restated as $S = A \times B$ where

$$
\begin{aligned}
A &= \frac{2(1-p_f)}{2-p_f+p_e}, B = \frac{qT_{payload}}{C} \text{ and,} \\
C &= q(b+p_e(T_{EIFS}-T_{DIFS}))+T_{slot}.
\end{aligned}
$$

We now compare the effects of q's variability and p_e's variability on throughput S.

$$\left[\frac{dS/dq}{dS/dp_e}\right] = \frac{\frac{dA}{dp_f}\frac{dp_f}{dq}B + A\frac{dB}{dq}}{\frac{dA}{dp_f}\frac{dp_f}{dp_e}B + A\frac{dB}{dp_e}}$$

Let $\frac{dA}{dp_f} = A'$,

$$= \frac{qC\frac{dp_f}{dq} + \frac{A}{A'}T_{slot}}{qC\frac{dp_f}{dp_e} - \frac{A}{A'}q^2(T_{EIFS} - T_{DIFS})np_e^{n-1}}$$

To get the bounds of $A/A' = -G(p_e, q)$,

$$\begin{aligned} \frac{A}{A'} &= -G(p_e, q) = -\frac{(1-p_f)(2-p_f+p_e)}{1+p_e}, \\ \frac{dG}{dq} &= \frac{1}{1+p_e}\frac{dp_f}{dq}(2(p-1) - (p_e+1)) < 0. \end{aligned}$$

$G(p_e, q)$ is decreasing. With (28) and (22), we have

$$\begin{aligned} [G(p_e,q)]_{q=1} &= \frac{p_e + 15 - 3\sqrt{p_e^2 + 6p_e + 17}}{2(1+p_e)} \\ &> \frac{(p_e+1)+14}{2(1+p_e)} - 3\frac{\sqrt{p_e^2+6p_e+17}}{2(1+p_e)} \\ &= \frac{1}{2} + \frac{14}{3} - \frac{9}{2} > \frac{2}{3} \text{ (at } p_e = \tfrac{1}{2}\text{)}, \\ [G(p_e,q)]_{q=0} &= \frac{2(1-p_e)}{1+p_e} < 2, \end{aligned}$$

$$\text{Therefore,} \quad \frac{2}{3} < G\left(= -\frac{A}{A'}\right) < 2.$$

For simplicity, we use T_p for $T_{payload}$, T_s for T_s, T_E for T_E and T_D for T_D.

$$\begin{aligned} \left[\frac{dS/dq}{dS/dp_e}\right] &= \frac{qC\frac{dp_f}{dq} - GT_s}{qC\frac{dp_f}{dp_e} + Gq^2(T_E - T_D)np_e^{n-1}} \\ &< \frac{qC\frac{dp_f}{dq} - \frac{2}{3}T_s}{qC\frac{dp_f}{dp_e} + \frac{2}{3}q^2(T_E - T_D)np_e^{n-1}} \\ &< \frac{qC\frac{dp_f}{dq} - \frac{2}{3}qT_s}{qC\frac{dp_f}{dp_e} + \frac{2}{3}q^2(T_E - T_D)np_e^{n-1}} \\ &= \frac{C\frac{dp_f}{dq} - \frac{2}{3}T_s}{C\frac{dp_f}{dp_e} + \frac{2}{3}q(T_E - T_D)np_e^{n-1}} \\ &< \frac{C_{max}\frac{dp_f}{dq} - \frac{2}{3}T_s}{C_{max}\frac{dp_f}{dp_e} + \frac{2}{3}q(T_E - T_D)np_e^{n-1}} \end{aligned}$$

where

$$C_{max} = b + 0.5(T_E - T_D) + T_s$$

Let $R(p_e, q)$ to be the last expression and $D = \sqrt{(p_e + 4q)^2 + 1 - 2p_e}$, then from (23) and (24) we have,

$$\begin{aligned} R(p_e, q) &= \frac{C_{max}\frac{dp_f}{dq} - \frac{2}{3}T_s}{C_{max}\frac{dp_f}{dp_e} + \frac{2}{3}q(T_E - T_D)np_e^{n-1}} \\ &= \frac{C_{max}2(1 - \frac{p_e+4q}{D}) - \frac{2}{3}T_s}{C_{max}0.5(1 - \frac{p_e+4q-1}{D}) + \frac{2}{3}q(T_E - T_D)np_e^{n-1}} \end{aligned}$$

$$\begin{aligned} \frac{dR}{dq} = \frac{1}{(C_{max}\frac{dp_f}{dp_e} + \frac{2}{3}q(T_E - T_D)np_e^{n-1})^2} \times (\\ 4C_{max}^2\frac{p_e + 4q - D}{D^3} \\ -\frac{16}{3}q(T_E - T_D)np_e^{n-1}(\frac{1}{D} - \frac{(p_e+4q)^2}{D^3}) \\ +\frac{4}{3}(T_E - T_D)np_e^{n-1}(T_s - C_{max}(1 - \frac{p_e+4q}{D})) \\ -\frac{4}{3}T_sC_{max}(\frac{1-2p_e}{D^3} + \frac{p_e+4q}{D^3})) \end{aligned}$$

All the four terms in the numerator are negative, so $(dR/dq) < 0$. $R(p_e, q)$ is decreasing in terms of q, therefore $R(p_e, q)$ has the maximum when $q = 0$, hence, $D = 1 - p_e$.

$$\begin{aligned} \left[\frac{dS/dq}{dS/dp_e}\right] &< R(p_e, q=0) \\ &= \frac{C_{max}2\frac{1-2p_e}{1-p_e} - \frac{2}{3}T_s}{C_{max}}, \\ &< \frac{C_{max}2 - \frac{2}{3}T_s}{C_{max}}, \\ &< 2. \end{aligned} \tag{28}$$

Claim 6: With W fixed, increasing packet error probability by Δp_e causes at least the same effect as adding $0.5W\Delta p_e$ stations, on throughput S.

Proof: From (28), for the same change of S (ΔS), Δp_e and Δq have the following inequality.

$$\begin{aligned} \frac{\Delta S}{\Delta q} &< 2\frac{\Delta S}{\Delta p_e}, \\ \Delta q &= \frac{\Delta(n-1)}{W} > 0.5\Delta p_e, \text{ therefore,} \\ \Delta n &> 0.5W\Delta p_e \end{aligned}$$

■

VI. Numerical Results

We use DCF simulator [2] by Bianchi *et al.* for numerical validation and analysis. We modify the simulator to add the behaviors of packet errors and the delay due to the packet errors (T_{EIFS}). We obtain the results by varying the simulation factors, which are m, W, n and p_e, Other simulation parameters are summarized in Table II.

A. Numerical Validation of p_e-Model

Comparing our approximation of p_f in (22) (lines in Fig. 3) with the simulation results (points in Fig. 3), we observe that p_e-Model makes substantially accurate predictions of p_f.

packet payload	8184 bits
MAC header	272 bits
PHY header	128 bits
ACK length	112 bits
Channel Bit Rate	1 Mbps/sec
Propagation Delay	1 μsec
RxTx_Trunaround_Time Delay	20 μsec
Busy_Detect_Time	29 μsec
SIFS	8 μsec
DIFS	110 μsec
EIFS	1142 μsec
ACK_Timeout	280 μsec
Slot Time	51 μsec
Maximum Cycle(DATA-ACK) Duration	9570 μsec
Maximum Packet Rate for Single Station	10.449 pack/sec
Packet Rate for Single Station	12.219 pack/sec

TABLE II

PACKET FORMAT AND TIMING PARAMETERS USED IN THE SIMULATION

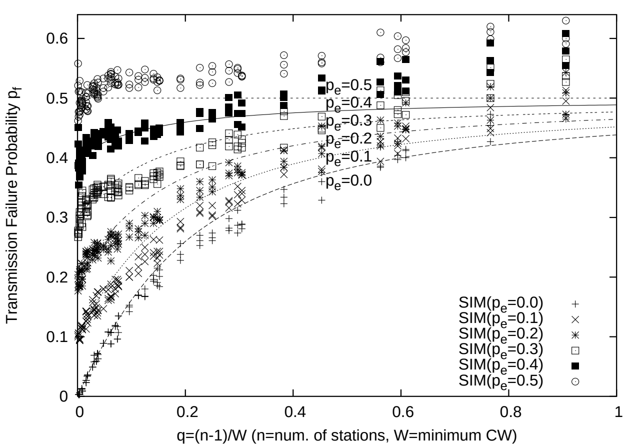

Fig. 3. Validation results for p_f: the simulation results obtained with th e combinations of the variable factors: $m = 3, 4, 5$, $W = 16, 32, 64, \cdots, 1024$, $n = 2, 5, 10, 20, 30, 40, 50$, $p_e = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5$

In p_e-Model we have used many approximations, which might affect the prediction accuracy of p_f. We made an approximation by ignoring $p_c \times p_e$ in (3), which we call *P-APPROX*. P-APPROX can make p_f in the model greater than the simulation results, i.e. incur *positive errors*. P-APPROX's positive errors become higher as p_e and n increase. First-order approximation used in (17), called *F-APPROX*, can introduce positive errors. The F-APPROX's positive errors increase as n increases. Ignoring $(2p_f)^m$ term in (21) (called *M-APPROX*) can cause *negative errors* on p_f. As p_e increases and m decreases, M-APPROX causes more negative errors.

In Fig. 4 M-APPROX's negative errors are higher for $p_e = 0.4$ than for $p_e = 0.1$. As m changes from 2 to 10, M-APPROX's negative errors are reduced significantly. For high p_e (e.g. $= 0.4$) negative errors are dominant due to M-APPROX,

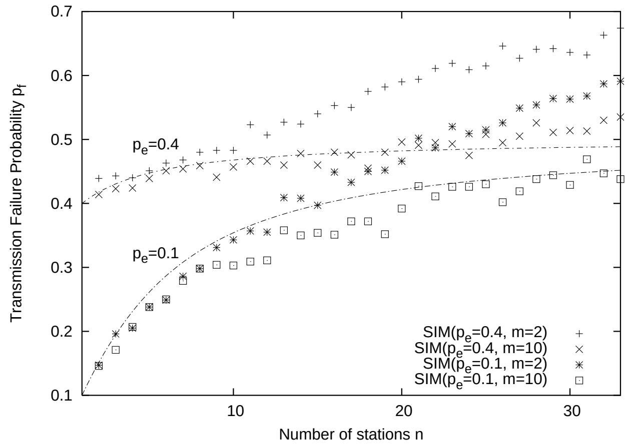

Fig. 4. The effect of m,n and p_e on p_f: p_e-Model makes more accurate prediction of p_f with greater m and smaller n,p_e.

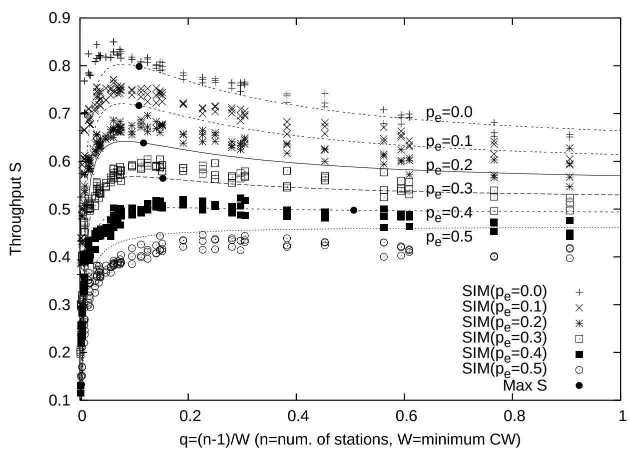

Fig. 5. Validation results for S: the simulation results obtained with the combinations of the variable factors: $m = 3, 4, 5$, $W = 16, 32, 64, \cdots, 1024$, $n = 2, 5, 10, 20, 30, 40, 50$, $p_e = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5$

as shown In Fig. 3. p_f is not much affected by P-APPROX and F-APPROX, even for high p_e and n.

For low p_e (e.g. $= 0.1$) errors change from positive to negative as n increases. For small n F-APPROX's positive errors are dominant. As p_f increases with n, M-APPROX's negative errors become dominant.

Fig. 5 shows that p_e-Model accurately predicts S also, comparing S in (20) (lines in Fig. 5) with the simulation results (points in Fig. 5). In (20) errors on p_f introduce the errors on S in the *opposite* sign, i.e. positive p_f errors incur negative S errors. Fig. 3 and Fig. 5 show that the p_f errors are *negatively* reflected in the S errors.

B. Numerical Results for Variability Analysis

To validate Claim 6 we run the simulator with $W = 32$ and $m = 5$, which are the typical setup specified in the standard [1]. As shown in Fig. 7, change of p_e from 0.1 to 0.3 and that from 0.3 to 0.5 have the same effect on S as adding 38 and 66 stations

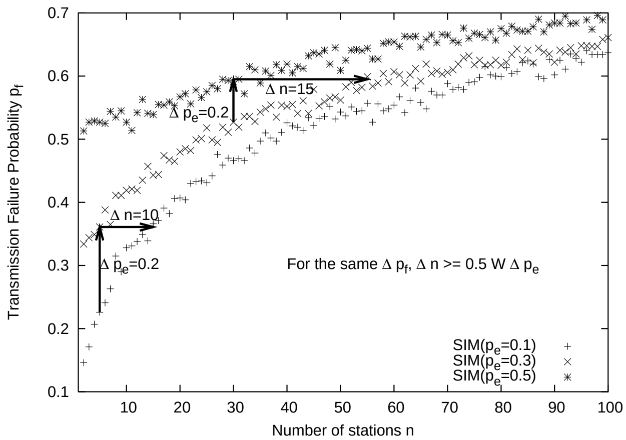

Fig. 6. Variability results for transmission failure probability: increasing p_e by Δp_e has more effect on p_f, than adding $0.5W\Delta p_e$ stations.

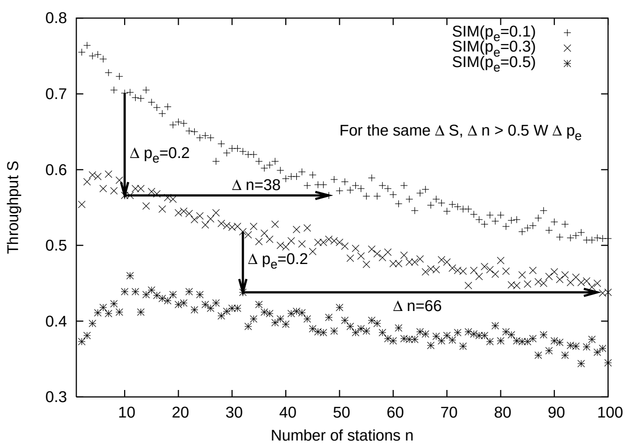

Fig. 7. Variability results for saturation throughput: increasing p_e by Δp_e has more effect on S, than adding $0.5W\Delta p_e$ stations.

respectively. 38 and 66 both are greater than 4 ($\approx 0.5W\Delta p_e = 0.5 \times 32 \times 0.2$), thus Claim 6 is validated.

VII. Conclusion

In this work we have extensively studied the effect of packet errors on capacity and variability of the 802.11 MAC protocol. We develop p_e-Model and successfully model transmission failure probability p_f and saturation throughput S in terms of packet error probability p_e and $q = (n-1)/W$.

Furthermore, introduction of p_e in the model enables us to make variability analysis on the effect of p_e on the performance metrics, such as p_f and S.

Numerical results show that our model can accurately predict capacity and variability of the real-world wireless LAN, where packet errors are common due to non-ideal channel condition and device variability.

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