Indian Journal of Animal Sciences **93** (3): 314–317, March 2023/Article
https://doi.org/10.56093/ijans.v93i3.122949

t-designs involving treatment combinations for animal experimentation

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Received: 1 April 2022 ; Accepted: 13 January 2023

ABSTRACT

Small and marginal farmers in India prefer an Integrated Farming System (IFS) approach i.e. a mixture of crop and livestock that enables them to maintain a stable income throughout the year. Identifying the best location-specific t-component crop-livestock combination to generate maximum profit is the major concern. t-designs can play a vital role in bringing out those best combinations. However, in many situations, combinations derived from specific components may not be practically feasible in a certain locality. Partially balanced t-designs with few selected combinations may prove to be useful in such situations. t-designs are special type of partially balanced incomplete block (PBIB) designs with some additional properties. Here, two series of partially balanced t-designs with some interesting characterization properties have been obtained by using the triangular association scheme. These designs find high application potential in crop and animal experimentation, especially in IFS research involving both crop and livestock components.

Keywords: 3-designs, Compound cattle feed, Incomplete block designs, Integrated farming system, Partially balanced, Triangular association scheme

India is mainly dominated by small and marginal farmers. Growing crops alone may not be a safe livelihood option for small farmers as in case of crop failures, due to weather vagaries and other natural calamities, there would be no alternative source of income. Integrated Farming System (IFS), incorporating appropriate livestock components along with crops can be a more stable choice for ensuring periodic income to the farmers throughout the year. Singh *et al*. (2009) suggested that the farming system approach is one of the solutions for increasing employment and income of rural population engaged in farming activities. Farmers can be encouraged for commercializing of dairy activity along with goatery, piggery, bee-keeping, etc. Importance of IFS with ducks as compulsory component has been explained by Naik *et al.* (2022).

It is pivotal to identify and adopt appropriate statistical techniques for choosing the best t-component crop-livestock combination, based on availability of resources, in order to generate optimum income for the farmers. Ravisankar *et al.* (2022) developed a sustainable livelihood security index for selecting the best improved IFS compared to benchmark farming for semiarid regions. However, t-designs, introduced by Calvin (1954), can save the resources and reduce the computational difficulties in identifying the best resources. These designs can also be advantageously used in animal feed production. Compound cattle feed consists of multiple components (multiple grain, brans, protein meals/cakes, locally available pulses, vitamins, etc.) out of which, one may have to choose the best combination of components that gives the maximum nutrition. In such situations, t-designs prove to be very useful.

But as the number of components increase, the number of possible t-component combinations will also increase. Most often, combinations derived from specific components are only practically feasible, for example, in a specific locality, farmers may not be interested in keeping a pig or goat and hence combinations involving these may not be of any use in that locality. In such situations, partially balanced t-designs, introduced by Karmakar *et al.* (2021), may be useful. These designs belong to a general class of partially balanced incomplete block designs with some additional properties and constraints.

Hedayat and Kageyama (1980) defined several types of t-designs that include Steiner triple system, Steiner quadruple system, Hadamard 3-design, Symmetric 2-design and Quasi-symmetric t-design. Kageyama and Hedayat (1983) provided tables of assorted t-designs. Chee *et al*. (1990) presented a set of tables of t-designs of small order having no repeated blocks for treatment number less than 30. David (2017) constructed some resolvable t-designs by identifying BIB designs which are also t-designs. Trung (2017, 2018) gave recursive methods of construction for simple t-designs satisfying a certain set of inequalities of the ingredient designs. Further, Trung (2019) gave a

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method of construction for s-resolvable t-designs for 1 < s < t.

Triangular association scheme is very useful for obtaining designs for animal experimentation. In this study, two series of partially balanced t-designs have been constructed using this association scheme.

MATERIALS AND METHODS

Karmakar *et al.* (2021) defined balanced t-designs and partially balanced t-designs as follows:

Balanced t-design: An IBD for v treatments is said to be a balanced t-design, if the experimental material can be divided into b blocks of size k ($1 < t < k < v$) such that each of the treatments occur in r blocks; any pair of treatments occur together in exactly λ blocks and any t-subset of distinct treatments appear together in exactly δ blocks.

The numbers v, b, r, k, λ and δ are called the parameters of the design.

If any t-subset of distinct treatments are appearing together in δ blocks, it implies that any lower order subsets [2 to (t-1)] of distinct treatments are appearing together in a constant number (may not be equal to δ) of blocks, indicating that a balanced t-design is also a balanced (t-1) design (for any $t > 2$).

Partially balanced t-design: In similar lines to partially balanced incomplete block designs by Bose and Nair (1939), partially balanced t-designs can be defined by incorporating more restrictions. Besides being incomplete, equi-replicate and proper, it has the following additional conditions to be satisfied:

(i) There exists an abstract relation between treatments satisfying:
- two treatments are either 1st, 2nd, …, or mth associates, the relation of association being symmetrical i.e., if treatment α is the ith associate of β, then β is also the ith associate of α (i = 1, 2, …, m),
- each treatment has exactly n_i, ith associates, and
- given any two treatments that are mutually ith associates, the number of treatments common to jth associates of the first and kth associates of the second is (i, j, k = 1, 2, …, m).

(ii) Two treatments that are mutually ith associates occur together in exactly λ_i blocks.

(iii) Any t subset of distinct treatments appear together in δ_p ($p > 0$, not a constant value) blocks.

In this study we consider two types of δ_p only, taking values 0 and a constant value.

Model and experimental set-up: The usual linear additive fixed effects model for block designs can be used as the experimental model for partially balanced t-designs for v treatments:

$$y_{ijk}=\mu+\tau_i+\beta_j+\varepsilon_{ijk}; \qquad i = 1\ (1)\ v,\ j = 1\ (1)\ b$$

where y_{ijk} is the observation corresponding to the jth block receiving the ith treatment, μ is the general mean effect, τ_i is the ith treatment effect, β_j is the jth block effect and ε_{ijk} is the random error component which follows iid $N(0, \sigma^2)$.

Partially balanced t-designs have been constructed based on two-associate class triangular association scheme (Shrikhande 1952, Clathworthy 1973), which is defined below.

Triangular association scheme: A triangular association scheme with $\binom{n}{2}$ treatments, where n is a positive integer (≥ 5) is the arrangement of v treatments in an array such that the principal diagonal is left blank. The $\binom{n}{2}$ positions above the principal diagonal are filled by $\binom{n}{2}$ treatments in a natural order. The positions below the principal diagonal are filled in such a way that the overall arrangement is symmetric about the diagonal. Every treatment appears twice in the association arrangement. Now, two treatments are said to be first associates if they belong to the same row or same column of the array, else they are said to be second associates.

Example 1: Consider the arrangement of $v=\binom{n}{2}=\binom{6}{2}=15$ (where n = 6) treatments in a 6×6 triangular association scheme is:

	i	ii	iii	iv	v	vi
i	*	1	2	3	4	5
ii	1	*	6	7	8	9
iii	2	6	*	10	11	12
iv	3	7	10	*	13	14
v	4	8	11	13	*	15
vi	5	9	12	14	15	*

Now, various associates of Treatment 1 are:

1st associates	**2nd associates**
2, 3, 4, 5, 6, 7, 8, 9	10, 11, 12, 13, 14, 15

RESULTS AND DISCUSSION

Now, steps involved in the construction of two series of partially balanced t-designs using the triangular association scheme have been described.

Series I

Step 1: Consider the v treatments in the upper triangular positions of the triangular association scheme. Obtain all possible distinct pairs (i, j) of treatments that can be formed from each row.

Step 2: Obtain $\binom{n}{3}$ blocks of size 3 each by augmenting a third element to each pair. This element is the intersecting element of the other row in which treatment i occurs and column in which treatment j occurs. The resultant design will be a partially balanced 3-designs with parameters $v=\binom{n}{2}$, $b=\frac{n(n-1)(n-2)}{6}$, $r = (n-2)$, $k = 3$, $\lambda_1 = 1$, $\lambda_2 = 0$, $\delta_1 = 1$, $\delta_2 = 0$, $n_1 = 2(n-2)$, $n_2 = \frac{(n-2)(n-3)}{2}$, $\eta_1 = b$ and $\eta_2 = \binom{v}{3} - b$.

Example 2: Consider the treatments in the upper triangular positions of the triangular association scheme given in Example 1. Obtain all possible distinct pairs of treatments that can be formed from each row. For the first row the distinct pairs are (1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5) and (4,5). For the pair (1,2), the intersecting element of the 1st row and 2nd column is 6. Therefore, the first block is (1,2,6). Proceeding in a similar manner for rest of the 5 rows in the array, 20 blocks, each of size 3, can be obtained as shown:

Blocks	Treatments		
B1	1	2	6
B2	1	3	7
B3	1	4	8
B4	1	5	9
B5	2	3	10
B6	2	4	11
B7	2	5	12
B8	3	4	13
B9	3	5	14
B10	4	5	15

Blocks	Treatments		
B11	6	7	10
B12	6	8	11
B13	6	9	12
B14	7	8	13
B15	7	9	14
B16	8	9	15
B17	10	11	13
B18	10	12	14
B19	11	12	15
B20	13	14	15

The resultant design thus obtained is a partially balanced 3-design having parameters v = 15, b = 20, r = 4, k = 3, λ_1=1, λ_2= 0, δ_1=1, δ_2= 0, n_1= 8, n_2= 6, η_1 = 20 and η_2 = 435.

The information matrix (**C**-matrix) pertaining to treatment effects for this class of designs is:

$$C = c_0 I_v - c_1 \mathbf{A}_v - c_2 \mathbf{B}_v$$

Here, $\mathbf{I}_v$ is an identity matrix of order v, $\mathbf{A}_v$ and $\mathbf{B}_v$ are called association matrices with $\mathbf{A}_v = \{a_{jl}\}$, a symmetric matrix of order with elements 0's and 1's where a_{jl} = if the j^{th} and l^{th} combinations are first associates and a_{jl} otherwise, and $\mathbf{B}_v = \{a_{jl}\}$, a symmetric matrix of order v with elements 0's and 1's where b_{jl} = 1 if the j^{th} and l^{th} combinations are second associates and b_{jl} = 0 otherwise, $c_0\left(r - \frac{r}{k}\right)$, $c_1 = \frac{\lambda_1}{k}$ and $c_2 = \frac{\lambda_2}{k}$. For the design given in Example 2.1, c_0 = 2.67, c_1=0.33 and c_2= 0.

Series II

A series of partially balanced 3-designs can be obtained using the triangular association scheme by developing a balanced incomplete block (BIB) design (choose one with smaller block size) using each row contents, of the above array and juxtaposing the blocks thus obtained, one below the other. As an unreduced BIB design exists for all values of n-1, a t-design is always possible. The parameters of the resultant series of design are $v = \binom{n}{2}$, $b = n.\binom{n}{k}$, $r = 2\binom{n-2}{k-1}$ k, $\lambda_1 = \binom{n-3}{k-2}$, λ_2=0 , $\delta_1 = \binom{n-2}{k-3}$, δ_2=0, n_1=2(n–2), $n_2 = \frac{(n-2)(n-3)}{2}$ $\eta_1 = \frac{b}{\delta_1} \cdot \binom{k}{3}$, and $\eta_2 = \binom{v}{3} - \eta_1$ Here is the number of triplets appearing times and is the number of triplets that are not appearing.

Example 3: Let n = 6. Then v = 15. Consider the (6 × 6) array for 15 treatments in Example 1. Obtain $\binom{5}{4}$= 5 blocks, each of size 4 from each of the rows of the (6 × 6) array. The 30 blocks thus obtained are:

Blocks	Treatments			
B1	1	2	3	4
B2	1	2	3	5
B3	1	2	4	5
B4	1	3	4	5
B5	2	3	4	5
B6	1	6	7	8
B7	1	6	7	9
B8	1	6	8	9
B9	1	7	8	9
B10	6	7	8	9
B11	2	6	10	11
B12	2	6	10	12
B13	2	6	11	12
B14	2	10	11	12
B15	6	10	11	12

Blocks	Treatments			
B16	3	7	10	13
B17	3	7	10	14
B18	3	7	13	14
B19	3	10	13	14
B20	7	10	13	14
B21	4	8	11	13
B22	4	8	11	15
B23	4	8	13	15
B24	4	11	13	15
B25	8	11	13	15
B26	5	9	12	14
B27	5	9	12	15
B28	5	9	14	15
B29	5	12	14	15
B30	9	12	14	15

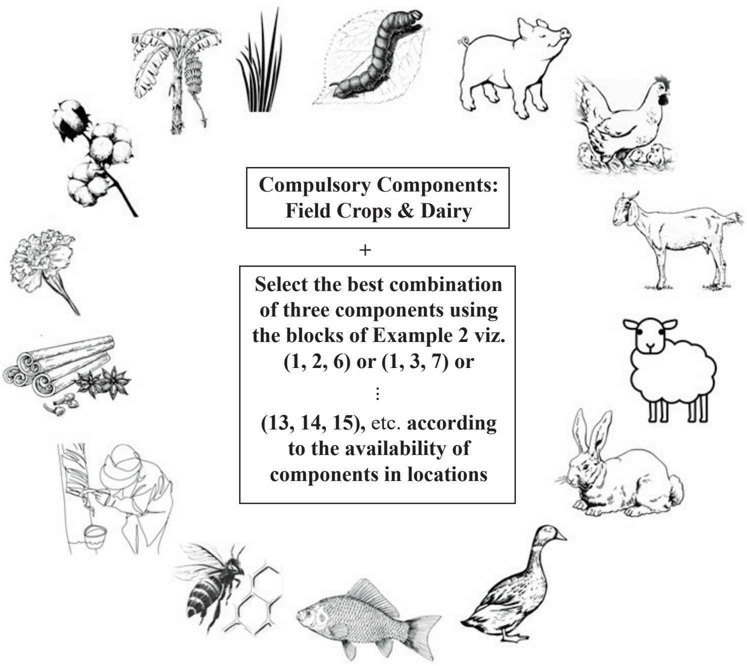

Fig. 1. Identification of best possible combination of IFS components using a 3-design.

The resultant design thus obtained is a partially balanced 3-design with having parameters v=15, b= 30, r = 8, k = 4, $\lambda_1 = 3$, $\lambda_2 = 0$, $\delta_1 = 2$, $\delta_2 = 0$, $n_1 = 8$, $n_2 = 6$, $\eta_1 = 60$, and $\eta_2 = 395$. The information matrix (**C**-matrix) pertaining to treatment effects for this for this particular design is-

$$\mathbf{C} = c_0\mathbf{I}_v - c_1\mathbf{A}_v - c_2\mathbf{B}_v = \left(r - \frac{r}{k}\right)\mathbf{I}_v - \frac{\lambda_1}{k}\mathbf{A}_v - \frac{\lambda_2}{k}\mathbf{B}_v$$

where $c_0 = 6$, $c_1 = 0.75$ and $c_2=0$.

Illustration

In Example 2.1, if we consider the 15 treatments as components of IFS, say, Sericulture, Piggery, Poultry, Goatery, Sheep rearing, Cuniculture, Duckery, Fishery, Apiculture, Rubber plantation, Spices, Floriculture, Cotton cultivation, Horticulture, Fodder plantation. Figure 1 explains the steps for selecting the best combination of components to ensure sustainable livelihood security of small and marginal farmers.

In this study, two series of partially balanced 3-designs are obtained using triangular association scheme. These designs are easy to construct and exist for number of treatments is of the type v = ${}^{n}C_2$. As these designs are constructed based on triangular association scheme, they involve a small number of combinations arising from few selected components. Thus, they have high application potential in animal experimentation, where a best 3-component combination from v components has to be chosen for further recommendation. These designs help to reduce the resource requirements to a great extent in comparison to a full factorial or even a full set of 3-component combinations. Availability of wide range of parametric combinations of these designs will encourage researchers for adoption of these designs, thus enhancing their utility.

ACKNOWLEDGEMENTS

The first author is grateful for the necessary facilities provided by Post Graduate School, ICAR-IARI, New Delhi and the financial assistance received from UGC, India.

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