1989

Little people learn numbers

Nancy A. Murphy
University of Northern Iowa

Little people learn numbers

Abstract

The importance of early childhood education to the balance of a child's academic career has been the subject of myriad articles and studies in recent decades. Numerous school districts are operating all-day kindergarten programs, pre-kindergarten programs, and preschool programs to ensure a proper beginning to a child's challenging journey through the public school system. All subject areas have been scrutinized, but the area of number learning in young children has been particularly popular.

LITTLE PEOPLE LEARN NUMBERS

A Research Paper

Submitted to

The Department of Curriculum and Instruction

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts in Education

by

Nancy A. Murphy

May, 1989

This Research Paper by: Nancy A. Murphy

Entitled: Little People Learn Numbers

has been approved as meeting the research paper requirement

for the Degree of Master of Arts in Education.

June 27, 1989	Charles R. May
Date Approved	Director of Research Paper
June 27, 1989	Charles R. May
Date Approved	Graduate Faculty Adviser
June 27, 1989	Mary Nan Aldridge
Date Approved	Graduate Faculty Reader
June 27, 1989	Roger A. Kueter
Date Approved	Head, Department of Curriculum and Instruction

ACKNOWLEDGMENTS

I would like to thank my graduate faculty adviser, Dr. Charles May, for his guidance to me during the process of writing this paper, as well as for the early childhood expertise that he has shared with me throughout a variety of courses. Thank you to Dr. Thomas Reifschneider for his assistance with the research paper style of writing and study design.

I am indebted to my husband, Robert, for the "crash course" in word-processing, and for his love, support, and patience throughout my graduate years at the University of Northern Iowa, 1986-1989.

Table of Contents

CHAPTER I

INTRODUCTION

The importance of early childhood education to the balance of a child's academic career has been the subject of myriad articles and studies in recent decades. Numerous school districts are operating all-day kindergarten programs, pre-kindergarten programs, and preschool programs to ensure a proper beginning to a child's challenging journey through the public school system. All subject areas have been scrutinized, but the area of number learning in young children has been particularly popular.

Theories regarding the development of number knowledge in young children have come and gone. The work of Swiss genetic epistemologist, Jean Piaget, however, has reigned over all other learning theorists for the past fifty years. As Bauch and Huei-hsin (1988) stated, "We now have fifty years of experience with Piaget's theory and a rapidly accumulating body of specific research-based knowledge about how children actually learn mathematics" (p. 9).

A study done by Rawl and O'Tuel (1983) compared three approaches to the teaching of mathematics. These three methods exemplified teaching practices that have commonly been utilized in early childhood classrooms for years. The methods included: (a) a behavioristic method utilizing drill and worksheets, (b) a cognitive developmental method stressing the use of manipulatives and games, and (c) an eclectic approach that involves a combination of worksheets, drill, games, and manipulatives. Rawl and O'Tuel found that children who learned through games and manipulatives scored much higher on mathematics achievement tests than the other two groups. This study is just one example of many studies completed over the last five decades that support the Piagetian theory of learning in young children.

The National Association for the Education of Young Children (NAEYC), an organization of educators and administrators involved in early childhood education, has issued a position statement of appropriate practices in programs for four- and five-year-olds (NAEYC, 1986). The practices outlined in this document are also consistent with Piagetian theory.

It was not the intent of this study to compare and contrast the various learning theories which describe the development of number concepts in young children. The reader should be aware that there are theories that contradict the teachings of Piaget. This paper accepts the well-researched claim that Piagetian theory has become a strong influence on early childhood mathematics programs. One cannot deny its influence on curriculum development, materials, and the role of teachers. Kamii (1985) discussed the significance of Piaget's work:

> Piaget's theory is more useful for education than any other theory of child development for two reasons. First, he is the only person who studied children's development of knowledge in great detail, including mathematics . . . Second, his theory called constructivism is solidly scientific, and has been verified by cross-cultural research all over the world. (p. 6)

<u>Statement of the Purpose</u>

Rather than making comparisons among the various learning theories, this study focused upon the widely-accepted Piagetian theory and its implications for early childhood number programs. The purpose of the study was to review and analyze the literature in the areas of early childhood mathematics programs and number learning in young children. The following questions were addressed:

1. What are the major components of Piagetian theory that are relevant to mathematics learning in young children?

2. What are the implications of Piagetian theory on early childhood mathematics programs? What are some concrete applications of Piagetian theory for classroom use?

Significance of the Study

The preponderance of literature in the field of early childhood mathematics boasts of the legitimacy of Piaget's claims from decades past. The fact remains, however, that many young children do not enjoy the benefits of Piaget's teachings in their classrooms. As Kamii (1985) stated, "Piaget [Piaget and Szeminska, 1941] showed more than 40 years ago that number concepts are constructed by each child, but math education is still going on as if Piaget had never published The Child's Conception of Number and many other volumes" (p. 8). Kamii's striking statement caused me to wonder why this contradiction between theory and practice exists.

One reason for this contradiction may be that volumes have been written about Piagetian theory itself, but there is less literature regarding concrete classroom implications of the theory. It is possible that teachers buy it, but are uncomfortable with the implications. The publication of the National Association for the Education of Young Children (NAEYC, 1986) position statements on appropriate teaching practices of four- and five-year-olds, as well as the recent surge of publicly-funded early childhood programs demonstrate that more early childhood authors are stressing practices rather than theory. The focus of this paper is to review the basic components of

Piaget's learning theory. This effort will emphasize concrete suggestions of recent authors on how to implement the theory.

Limitations

This study was not intended to be a comparison of theories in the field of early childhood number knowledge. It instead, will describe the major components of Piagetian theory and analyze its practical implications for classroom use. This study does not underestimate the teachings of other renowned theorists in the field.

The major portion of literature analysis is based upon what has been written within the last five years in order to describe current ideas regarding this area of early childhood education.

Definition of Terms

The terms that are used in this paper will be defined in the following way:

Concept. "A concept is all the elements in a class that can be represented by one label" (Schminke, Maertens, & Arnold, 1978, p. 14).

Constructivism. "Constructivism is the theory according to which each child builds his own knowledge from the inside, through his own mental activity, in action with the environment" (Kamii, 1985, p. 6).

Development. "Development means a long-term process of unfolding or maturation from inside the child, like a flower that develops out of a bud" (Kamii, 1985, p. 7).

Discovery learning. "The ability to 'discover' ideas themselves as a result of their interaction with the environment" (Maffei & Buckley, 1980, p. 42).

Drill. "Drill is the teaching strategy that uses repetition to develop precision in learning and to fix facts for efficient recall" (Driscoll, 1983, p. 72).

Logico-mathematical knowledge. This concept "consists of relationships constructed by each individual" (Kamii, 1985, p. 7).

Manipulatives. "Manipulatives are objects which appeal to several senses and which a student is able to touch, handle, and move" (Driscoll, 1983, p. 73).

Physical knowledge. "Physical knowledge is knowledge of objects in external reality" (Kamii, 1985, p. 7).

Preoperational stage of development. The preoperational stage follows the sensorimotor period in Piaget's theory of intellectual development. According to Schminke et al. (1985), the thinking of preoperational children is characterized by their need to manipulate concrete objects. It is difficult for this two- to seven-year-old child to mentally reverse actions and to focus upon more than one aspect of a situation at a time.

Relationships. "Relationships are rules or agreements by which we associate one object or abstraction with another" (Cruikshank, Fitzgerald, & Jensen, 1980, p. 30).

Social knowledge. "The ultimate source of social knowledge is conventions made by people" (Kamii, 1985, p. 7).

Structure. "Structure is a system or pattern of relationships" (Cruikshank et al., 1980, p. 30).

CHAPTER II

RELATED LITERATURE

Components of Piaget's Theory

Stages of intellectual development. Many contemporary writers in the field of early childhood mathematics begin their work with a discussion of the foundation of Jean Piaget's theory, his stages of intellectual development. Brainerd (1982) wrote that these stages attempt to describe "the cornerstone of Piagetian theory, which is that the main task of the intelligent mind is to construct logical means for structuring and, thereby, understanding reality" (p. 94). According to several descriptions (Barron, 1979; Brainerd, 1982; Copeland, 1979; Maffei & Buckley, 1980; Schminke et al., 1985), there are four major levels of development: (a) Sensorimotor Period (zero to two years), (b) Preoperational Period (two to seven years), (c) Concrete Operations Period (seven to eleven years), and (d) Formal Operations Period (eleven years and above). All children pass through these levels of thinking at approximately the same ages. The stages represent levels of conceptualizing about objects and experiences in a child's world. All learning is affected by the level of intellectual development that has been achieved by the child.

Feeney, Christensen, and Moravcik (1983) described each of Piaget's stages of cognitive development. Their descriptions are paraphrased as follows:

<u>Sensorimotor Period, Birth to Two Years</u> During this stage the child changes from a reflex organism to one capable of thought and language. Behavior is primarily motor and the child is dependent on physical manipulation to gain information about the world.

<u>Preoperational Period, Two to Seven Years</u> This developmental stage is characterized by language acquisition and rapid conceptual development. Children learn labels for experience, develop the ability to substitute a symbol for an object or an event that is not present, and make judgments according to how an object looks to them. It is during this stage that children gradually develop the concept of conservation. Conservation is the child's realization that the amount or quantity of a substance stays the same even when its shape or location changes.

<u>Concrete Operations Period, Seven to Eleven Years</u> During this period. children develop the ability

to apply logical thought to concrete problems. Formal thought processes become more stable and reasonable even though children still have to think things out in advance and try them out through direct manipulation.

Formal Operations Period, Eleven to Fifteen Years
During this stage, children's cognitive structures reach their highest level of development, and they become able to apply logic to all classes of problems. Children can weigh a situation mentally to deduce the relationships without having to try it out.

The characteristics of these stages of development influence all areas of concept-learning, including mathematical concepts.

Preoperational Stage of Development

The subjects of this study, children aged two through seven, are functioning at the preoperational stage of intellectual development. To an adult who is able to think abstractly in the formal operations stage, the preoperational child perceives objects and relationships in a skewed manner. Driscoll (1983) wrote: "In essence, the preoperative child finds it difficult to monitor her own thoughts and distinguish between what's 'out there' [her perceptions] and the order and organization which her mind gives to those perceptions [her abstractions]" (p. 12).

Children functioning at the preoperational level have difficulty with both reversing actions mentally and focusing attention on more than one aspect of an object or situation at one time.

> Piaget's classic conservation of number task can be used as an example. In the task, preoperational children believe that the number of counters in a row changes if they are rearranged. Why does a child think this way? The most common explanation of theorists and researchers is that the reasoning is bound to perceptual input, and is based upon how long or how short each row of counters appears to be. However, the child does not consider both the length and density of each row. (Ashlock, Johnson, Wilson, & Jones, 1983, p. 43)

The conservation task appears to be so blatantly obvious to adults. Preoperative children, however, are not able to look back at a particular thought process and discover the contradictions in logic that they have made.

Factors that Influence the Stages of Intellectual Development

Piaget's teachings described four factors that have a direct bearing upon how quickly a child is able to progress from one level of logical thinking to the next. These elements are: (a) maturation, (b) experience, (c) social interaction, and (d) equilibration (Maffei & Buckley, 1980). The first three elements are self-explanatory. The fourth element, equilibration, involves a child considering new experiences (assimilation) and rearranging their current mental structure (accomodation) to make sense of these new experiences. The two processes of assimilation and accomodation work together to create a balance in the child's mental functioning called equilibrium. With each new experience, the processes of assimilation and accomodation work together to process the new knowledge and create mental balance.

Categories of Knowledge

According to several authors (Copeland, 1979; Kamii, 1985; Kamii & DeClark, 1985), Piaget distinguished between three categories of knowledge: (a) physical knowledge, (b) social knowledge, and (c) logico-mathematical knowledge. Physical knowledge is the knowledge of objects in external reality. Such things as the color and weight of a block are examples of physical knowledge.

Social knowledge involves conventions made by people. The number names, for example, are examples of social knowledge. Social knowledge is quite arbitrary in nature.

Logico-mathematical knowledge consists of relationships that are constructed in the minds of each individual. A child who observes the difference between a green block and a yellow block is utilizing logio-mathematical knowledge. The child had to formulate a relationship about the two blocks in his/her mind in order to compare the two.

Piaget placed early number concept development in the logico-mathematical category of knowledge. Kamii and DeClark (1985) concluded that "number is a mental structure that each child constructs out of a natural ability to think rather than learn from environment" (p. 3). Kamii and DeClark further-categorized Piaget's types of knowledge as having sources that are either eternal or internal. Both physical and social knowledge are external sources of knowledge. "The source of logico-mathematical knowledge, by contrast, is internal" (p. 8). There is nothing arbitrary in the domain of logico-mathematical knowledge, the child, himself, is the source of knowledge.

The three sources of knowledge develop simultaneously in the maturing child. "Logicomathematical knowledge and physical knowledge depend on each other and develop together. As children's logicomathematical frameworks become better structured, they develop more precise and better organized physical knowledge, and vice versa" (Williams & Kamii, 1986, p. 24). It would be impossible for a child to recognize a blue ball as being blue (physical knowledge) without comparing the blue ball with balls of other colors in a relationship situation (logicomathematical knowledge).

Constructivism

Piaget explained the acquisition of number knowledge in the preoperative child with his theory of constructivism. The young child "constructs" knowledge by putting objects into relationships with each other. As Kamii (1985) explained, "Piaget clearly differentiated maturation, which is a biological process like the baby's becoming able to walk, from the construction of knowledge through children's own mental activity. While people are passive in biological maturation, they are mentally very active when they construct knowledge" (p. 7). Active mental activity, therefore, is essential to knowledge-acquisition in constructivism.

Another salient feature of constructivism is manipulation. According to Williams and Kamii (1986), Piaget taught that the acquisition of knowledge is linked to the senses. If sensory information is essential to the process of learning, it follows "that children can obtain sensory information only when they act on an object physically and mentally" (p. 25). Knowledge is acquired when children "handle objects and observe how they react" (p. 24).

A third major component of constructivism involves growing through recognizing errors. Children develop knowledge by going through levels of being logically wrong. As they develop early number concepts, children are actively involved in problem-solving.

Piaget (1963) taught that children construct number concepts in their own minds. He further observed that social interaction is indispensable to the child as he/she develops logic. Social interaction, then, is yet another essential characteristic of constructivism. Kamii and DeClark (1985) further described the necessity of interaction. "Isolating children in order to pour knowledge systematically and efficiently into their heads is undesirable. In the logico-mathematical realm, the confrontation of points of view serves to enhance children's ability to reason at increasingly higher levels" (p. 36).

Piaget stressed number concept acquisition prior to the introduction of number operations. Classification, seriation, and one-to-one correspondence are concepts that children must acquire before dealing with number skills. Clements and Callahan (1983) explained that Piaget considered number skill instruction before concept development to be meaningless, and possibly harmful to the child. Thus, Piaget stressed that number skills must be built upon a strong concept foundation.

Supporting learning theories. Barron (1979) and Schminke et al. (1978) included a discussion of learning theories that support Piaget's levels of intellectual development. These supporting theories include the works of Robert Gagne and Jerome Bruner.

Robert Gagne identified four levels at which children acquire mathematical knowledge. These include the following: (a) associative learning, (b) concept learning, (c) principle learning, and (d) problem solving. According Schminke et al. (1978), associative learning occurs when "children focus on memorization and mastery" (p. 13). This is considered to be the lowest level of learning in Gagne's hierarchy.

Children learn concepts when they categorize, classify, order, and label. "Concept learning occurs when children attempt to identify characteristics that determine inclusion in or exclusion from a set or class" (Schminke, et al., 1978, p. 14).

When children relate objects, generalize, analyze, and synthesize, they are involved in principle learning. "Principle learning results when children attempt to relate ideas" (Schminke, et al., 1978, p. 14).

Gagne proposed problem solving as the highest level of learning. "In its broadest sense, problem solving occurs when children employ principles to achieve a goal" (Schminke, et al., 1978, p. 14).

The theory of concept and principle learning by Jerome Bruner also supports Piaget's approach to learning. According to Barron (1979) and Schminke et al. (1978), Bruner proposed that individuals learn concepts in one of three modes: (a) enactive (concrete), (b) ikonic (illustrative), and (c) symbolic (abstract). In the enactive stage, children learn through the manipulation of objects. As children enter the ikonic stage, they begin to learn through illustrations and pictures. The symbolic stage involves concept development through symbols.

The levels of development in the theories of Gagne and Bruner are parallel to Piaget's stages of intellectual development. The three theorists stressed the importance of children acting upon objects as they formulate early concepts. Children are more able to deal with abstract information as they enter later stages of development.

CHAPTER III

REVIEW OF PRACTICAL IMPLICATIONS OF PIAGETIAN THEORY

Practical Implications of Piagetian Theory on Early Childhood Mathematics Programs

Manipulative use. According to Piaget's preoperative, Gagne's concept-learning, and Bruner's enactive stages of intellectual development, young children learn to place objects in relationships and formulate concepts about the world around them by manipulating concrete objects. Mathematical concept development in young children also necessitates this active- manipulation process. As Barnett and Young (1982) explained

> Children should learn mathematics through hands-on experiences that are problem-solving oriented. They need to experience mathematics in an environment in which they are encouraged to manipulate physical objects. Such an environment fosters the growth of mathematical concepts and the understanding of mathematical relationships and processes that are later represented symbolically. (p. 6)

It is not until later in their intellectual development that children will be able to learn mathematics on a more abstract, symbolic level. "The mathematical world of an adult often deals with abstractions, whereas the mathematical world of the young child involves physical realities from which abstractions will ultimately be formulated" (Mueller, 1985, p. 8).

Driscoll (1983) stressed the daily active use of manipulatives as a major implication of Piaget's theory on early childhood mathematics programs. Manipulatives are objects that appeal to the senses and can be touched and moved. Cruikshank et al. (1980) suggested "stacking blocks, cardboard boxes, pattern blocks, construction bricks, logical blocks, tinker toys, string and beads, puzzles, Cuisenaire rods, sand, clay, water, and various containers" (p. 1), as examples of manipulatives that should be found in every early childhood classroom. Mueller (1985) suggested other items that are not necessarily costly or commercial. These alternatives include: sand, rice, soybeans, cornmeal, buttons, bottle caps, pebbles, and popsicle sticks.

According to Elkind (1986), a mathematics program that encourages the use of manipulatives is a program that encourages the unique mode in which children learn. Manipulatives appeal to the natural curiosity in children and to their sense of touch. Miller and Harsh (1984) listed several ways that manipulative-use can benefit the early childhood mathematics programs. Manipulatives are: (a) motivating to children, (b) create positive attitudes, (c) give teachers insights about a child's strengths and weaknesses, and (d) lend themselves to a variety of teaching methods.

Maffei and Buckley (1980) discouraged the use of workbooks and worksheets by young children as these materials represent "the least effective learning modes" (p. 39). There have been numerous studies done in recent decades that support the use of manipulatives rather than pencil-and-paper tasks in mathematics programs for young children. Suydam and Higgins (1977) surveyed research on the effects of various teaching materials on number concept learning. Their findings supported the following claims:

1. Lessons that use manipulatives have a higher probability of producing greater mathematical achievement than do lessons that use workbooks and worksheets.
2. The use of manipulatives appears to be effective with children at all socioeconomic levels, intellectual levels, and levels of achievement.

According to Driscoll (1983), "In the last three decades many studies have probed the whys, whens, hows, and whats of manipulative use. The overriding consensus is that manipulatives can help children to understand and use mathematical concepts" (p. 27).

Utilizing abstractly-oriented workbook pages and worksheets is not only less effective than using manipulatives with young children, it is viewed as harmful by some researchers (Schultz, 1986, p. 53). "Children are at a disadvantage when they are expected to perform symbolically before they are ready" (p. 53).

Since the conclusions of numerous studies in early childhood mathematics dissuade the use of pencil-and-paper tasks, it is difficult to comprehend why workbooks and worksheets continue to be so popular. Kamii (1985) and Stone (1987) suggested several beliefs as to why worksheets are still widely-used in early number programs, despite their poor reputation. These beliefs include:

1. Worksheets are less expensive and more convenient than manipulatives.
2. Worksheets are more conducive to a quiet learning environment than manipulatives.
3. Worksheets are more accountable to parents as to what their child is learning than manipulatives.

The implications of Piagetian theory on early number learning also discourage the use of instructional television programs such as "Sesame Street" because children who watch television are not actively involved with concrete objects. According to Maffei and Buckley (1980), "Any curriculum that teaches children by means of pictures [whether they be from a workbook or on TV] only, and allows no room for active manipulation of everyday objects, is not meeting the developmental needs of the . . . child, according to Piaget" (p. 40).

Meaningful Activities

Another major implication of Piagetian theory on early number programs is the necessity of meaningful instructional activities. Driscoll (1983) described meaningful activities as tasks that are socially significant to the lives of children. Fifty years ago, according to Driscoll, there was considerable support for teaching by drill. Recently, researchers have found that children need to see sense and meaning in what is learned before drill can be effective. "One message that shines forth clearly from the research literature of the last few decades is the superiority of meaningful instruction over rote instruction for long-term retention of mathematical learning" (p. 69).

Meaningful activities, according to Tischler (1988), are those that "come out of children's real interests" (p. 42). Ashlock et al. (1983) linked a positive attitude about mathematics with the use of these interesting, meaningful activities.

> Positive feelings towards school, other children, and mathematics can be developed if we plan the

right kinds of experiences for each individual. A child who enjoys success with mathematics activities she understands and finds interesting will develop a positive attitude. She will come to value mathematics if it can be used to solve relevant, everyday problems. (p. 44)

Both Harsh (1987) and Tischler (1988) suggested utilizing children's literature as a means of bringing meaning into mathematics for young children. Routledge (1985) encouraged cooking with children to facilitate the meaningful learning of measurement, computation, and problem-solving. According to McClintic (1988), games and rhymes facilitate the learning of mathematical concepts.

Driscoll (1983) noted that there are several ways in which a teacher can create a meaningful environment that is conducive to learning mathematical concepts. These methods are paraphrased as follows:

1. making children recognize patterns when they arise
2. making mathematical generalizations with the children whenever it is both possible and appropriate
3. pointing to rules and principles of mathematics whenever it is both possible and appropriate
4. regularly asking mathematically-oriented

questions

5. alerting the children to the appearance of mathematics in their everyday world

Curriculum

Piagetian theory affects curriculum-planning for early childhood mathematics programs. Ashlock et al. (1983) suggested keeping the following two ideas in mind when planning number curriculum for young children:

> 1. Research has shown that children before the age of seven have somewhat fuzzy ideas about number and number relations, and about the invariant properties of number. Usually they cannot conserve these relationships.
>
> 2. Number ideas develop slowly. They grow from an experience base; that is, through physical and social experiences each child moves from an intuitive to a more formal meaning of number. (p. 49)

A child enters school with an early sense of numbers and numerical concepts that he/she was able to glean from meaningful experiences. Piaget taught of the importance of beginning formal instruction at the level in which children enter school. As Driscoll (1983) described,

> It is imperative-if we are not to shortchange or

> confuse young children-that we look carefully at what they bring by way of mathematical intuition and experience when they begin school, and then look at what we can offer them in formal training and curriculum. Will it allow them to continue to develop mathematically as they have already begun to develop on their own? (p. 7)

A primary curriculum implication of Piaget's work is that children learn mathematical concepts prior to learning arithmetic operations. Driscoll (1983) suggested that children should experience practice with classifying, comparing and ordering without numbers before children can be expected to relate these skills to numbers. Baruch and Huei-hsin (1988) cited seriation, one-to-one-correspondence, rational counting, and the recognition and comprehension of cardinal numerals as additional numerical concepts to be taught prior to arithmetic instruction. Miller and Harsh (1984) identified and described 11 pre-number skills that should be incorporated in an early childhood mathematics program (see Appendix for their complete description).

Barnett and Young (1982) suggested that early mathematics programs should focus on language acquisition in addition to instruction on concept-formation. "Oral language goes hand-in-hand with all mathematical experiences, and children must acquire a working knowledge of terms like more, shorter, between, the same, and smaller before these terms are used in the context of numbers" (p. 3).

The inclusion of problem-solving activities in mathematics programs for young children is another curriculum-related implication of Piaget's theory. Solving problems, according to Barnett and Young (1982) "allows children to see the relevance of mathematics in everyday life. . . .Numerical concepts and computational processes are introduced and reinforced through the solving of problems so that children learn to apply mathematics" (p. vii). Solving such problems as how to divide the snacks or how many scissors are necessary per table are examples of meaningful problem-solving tasks.

Grouping

Several authors (Barnett & Young, 1982; Copeland, 1979; Kamii & DeClark, 1985; Wadsworth, 1971) discussed the implications of Piagetian theory on grouping children for mathematics instruction. The consensus was that it is beneficial for children to interact with others as they develop mathematics concepts. "There is a necessity of action with people as well as action upon objects in the educational process. In lesson planning, provision should be made for group activity, which encourages questions and the interchange of ideas" (Copeland, 1979, p. 17). According to Kamii and DeClark (1985), "the confrontation of points of view serves to enhance children's ability to reason at increasingly higher levels" (p. 36).

Instruction in a group situation allows a child to compare his/her personal perceptions with those of the group. Wadsworth (1971) discussed the danger of limiting mathematics instruction to an individual process when he stated that "without interchange of thought and cooperation with others the individual would never come to group his operations into a coherent whole" (p. 24).

Barnett and Young (1982) encouraged a balance among individual, small group, and large group activities in mathematics instruction. They listed several advantages of teaching hands-on mathematics in smaller-sized groups:

*Fewer materials are needed per classroom.

*More student-teacher and student-student interaction is possible.

*Assessment of student understanding is easier.

*Learning is more personalized and thus more enjoyable for students. (p. x)

Teacher's Role

The implications of Piagetian theory advocate a change in roles for the classroom teacher. The role of the early childhood teacher differs greatly from that played by the upper-elementary teacher. An upper-elementary teacher gives direct instructional lessons to children. Maffei and Buckley (1980) described the early childhood teacher as one who organizes situations that will support the learning of concepts by children. Halperin (1985) stated that "teachers of young children should help focus, not dictate, the learner's attempt to give structure to what is observed" (p. 20).

Williams and Kamii (1986) cautioned that the role a teacher chooses to play in instruction has a direct effect on the success of a mathematics program. "When we tell or present knowledge to young children, we stifle their initiative and diminish their confidence" (p. 26). Children do not learn mathematics by being told about it, they learn by doing.

Baratta-Lorton (1976) described two approaches toward mathematics instruction. In one approach, mathematics is oriented toward skill development to mastery and the teacher acts as diagnostician. In the second approach, mathematics is oriented toward concept development. This approach finds no pressure for skill mastery and the teacher acts as an observer and a guide to learning. Early childhood mathematics, according to the teachings of Piaget, can be best-described by the latter approach.

Barnett and Young (1982) suggested several ways that the teacher can enhance the learning of children. These include: helping children become problem-solvers and "encouraging their physical, intellectual, and verbal involvement in mathematics". They encouraged teachers to question children to help clarify their thinking and assist in discovering reasoning flaws.

The characteristics of preoperative children should dictate the role played by their teacher. Young children, according to Miller and Harsh (1984), are active, so the teacher should emphasize the use of manipulatives. These children are curious about their environment, so the teacher should motivate children by making the environment attractive and stimulating. Young children are egocentric, so the teacher must provide experiences which encompass the world of children. Lastly, preoperative children are very verbal, so oral communication between teacher and child should be an important instructional tool.

The National Association for the Education of Young Children (NAEYC, 1986) issued a position statement on developmentally appropriate teaching practices in programs for four- and- five-year-olds. The statement outlined several practices that directly relate to the role that is played by the teacher:

*Teachers prepare the environment for children to learn through active exploration and interaction with adults, other children, and materials.

*Children work individually or in small, informal groups most of the time.

*Children are provided concrete learning activities with materials and people relevant to their own life experiences.

*Teachers accept that there is often more than one right answer. Teachers recognize that children learn from self-directed problem solving and experimentation.

*Different levels of ability, development, and learning styles are expected, accepted, and used to design appropriate activities.

*Interactions and activities are designed to develop children's self-esteem and positive feelings toward learning. (pp. 6-10)

Miller and Harsh (1984) described some of the major characteristics shared by what they considered to be effective mathematics teachers. Their idea of a good teacher is one who:

1. has knowledge of mathematics and effective teaching methods;

2. fosters a positive atmosphere;

3. is sensitive to the needs of individual students;

4. matches learning activities to the needs of the group and the individuals within that group;

5. is accepting of errors;

6. teaches through the use of questioning.

CHAPTER IV

SUMMARY AND CONCLUSIONS

Based upon this review of related literature, it is concluded that the work of Jean Piaget in the area of mathematics concept development in young children has been supported in research. Young children develop number concepts quite differently than do older children. They are functioning at Piaget's preoperational stage of development. They have difficulty reversing actions mentally and focusing attention on more than one aspect at a time.

Young children construct knowledge individually as they manipulate concrete objects and put them into relationships with each other. Piaget found that children must develop concepts such as classification and seriation prior to number operations instruction. Effective early childhood mathematics programs do not focus upon number skill acquisition prior to mathematical concept development.

Preoperational children must have daily active practice with manipulatives. It is not until later in a child's development that he/she will be able to learn mathematics on a more symbolic level as with worksheets, workbooks, and instructional television programs.

Meaningful instructional activities that focus upon problem-solving practice are an essential implication of Piagetian theory. Practice that results in the interests of children relate mathematics to concrete experiences. Children then develop an awareness of the importance of mathematics to their present and future lives.

Another practical implication of Piagetian theory is that children need to relate to other people as much as they need to manipulate objects in effective early childhood mathematics programs. When children interact with their peers, they verbalize their thought processes and strengthen their emerging number concepts. Limiting mathematics instruction to an individual process will limit development. Quiet early childhood classrooms, therefore, are not conducive to learning.

An early childhood mathematics teacher who accepts the principles of Piaget acts as a facilitator and guides children in active, hands-on learning activities. He/She does not lecture, but provides materials and stands back as children discover. The effective mathematics teacher will ask probing questions that invite further exploration by the child.

With the recent emphasis on appropriate early childhood teaching practices, greater numbers of teachers are implementing a Piagetian-based mathematics program in their own classrooms. They are no longer cranking-out worksheets to preserve tradition.

> Constructivist ideas clash squarely with pre-Piagetian notions, and individuals who operate within the boundaries of tradition and the established reward system are protected by them. That an increasing number of people are willing to stand up against the established views is a hopeful sign that attests to human beings' urge to go beyond the past. (Kamii & DeClark, 1985, p. 250)

REFERENCES

Ashlock, R. B., Johnson, M. L., Wilson, J. W., & Jones, W. L. (1983). Guiding each child's learning of mathematics: A diagnostic approach to instruction. Columbus: Charles E. Merrill.

Baratta-Lorton, M. (1976). Mathematics their way. Menlo Park, California: Addison-Wesley.

Barnett, C. S., & Young, S. (1982). Teaching kids math: Problem-solving activities to help young children learn and enjoy mathematics. Englewood Cliffs, New Jersey: Prentice-Hall.

Barron, L. (1979). Mathematics experiences for the early childhood years. Columbus: Charles E. Merrill.

Bauch, J. P., & Huei-hsin, J. H. (1988). Montessori: Right or wrong about number concepts? Arithmetic Teacher, 35 (6), 8-11.

Brainerd, C. J. (Ed.) (1982). Children's logical and mathematical cognition: Progress in cognitive development research. New York: Springer-Verlag.

Clements, D. H., & Callahanan, L. G. (1983). Number or prenumber foundational experiences for young children: Must we choose? Arithmetic Teacher, 31 (3), 34-37.

Copeland, R. W. (1979). How children learn mathematics: Teaching implications of Piaget's research (3rd ed.). New York: Macmillan.

Cruikshank, D. E., Fitzgerald, D. L., & Jensen, L. R. (1980). Young children learning mathematics. Boston: Allyn and Bacon.

Driscoll, M. J. (1983). Research within reach: Elementary school mathematics. Reston, Virginia: National Council of Teachers of Mathematics.

Elkind, D. (1986). Formal education and early childhood education: An essential difference. Phi Delta Kappan, 67, 631-636.

Feeney, S., Christensen, D., Moravcik, E. (1983). Who am I in the lives of children? Columbus: Charles E. Merrill.

Halperin, D. (1985). A staff development program to increase teacher competency in mathematics education for the early years. Florida: Nova University. (ERIC Document Reproduction Service No. ED 262 917)

Kamii, C. (1985, February). Can there be excellence in education without knowledge of child development? (Report No. PS 014 954). Paper presented at the annual conference of the Chicago Association for the Education of Young Children, Chicago, IL. (ERIC Document Reproduction Service No. ED 254 333)

Kamii, C. (1985). Leading primary education toward excellence: Beyond worksheets and drill. Young Children, 40 (6), 3-9.

Kamii, C. K., & DeClark, G. (1985). Young children reinvent arithmetic: Implications of Piaget's theory. New York:

Teachers College Press.

Maffei, A. C., & Buckley, P. (1980). Teaching preschool math: Foundations and activities. New York: Human Sciences Press.

McClintic, S. V. (1988). Conservation--a meaningful gauge for assessment. Arithmetic Teacher, 35 (6), 13.

Miller, B. B., & Harsh, A. (1984, March). M & M's--Mathematics and manipulatives, m-m-m-good! (Report No. PS 014 380). Paper presented at the annual conference of the Southern Association of Children Under Six, Lexington, KY. (ERIC Document Reproduction Service No. ED 245 813)

Mueller, D. W. (1985). Building a scope and sequence for early childhood mathematics. Arithmetic Teacher, 33 (2), 8-11.

National Association for the Education of Young Children (NAEYC). (1986). Position statement on developmentally appropriate practice in programs for 4- and 5-year olds. Young Children, 4, 20-29.

Piaget, J. (1963). The psychology of intelligence. Patterson, NJ: Littlefield, Adams, & Co.

Piaget, J., & Szeminska, A. (1965). The child's conception of number. New York: Norton (first published in 1941).

Rawl, R. K., & O'Tuel, F. S. (1983, March). Mathematics achievement in young children is increased with a cognitively oriented curriculum. Paper presented at the annual meeting of the Southern Association on Children Under Six, Charleston, SC. (ERIC Document Reproduction Service No. ED 227 933)

Routledge, J. (1985). What's cooking? Arithmetic Teacher, 32 (2), 14-15.

Schminke, C. W., Maertens, N., & Arnold, W. (1978). Teaching the child mathematics (2nd ed.). New York: Holt, Rinehart, and Winston.

Schultz, K. A. (1986). Representational models from the learners' perspective. Arithmetic Teacher, 33 (2), 52-55.

Stone, J. I. (1987). Early childhood math: Make it manipulative! Young children, 42 (6), 16-23.

Suydam, M. N., & Higgins, J. L. (1977). Activity-based learning in elementary school mathematics: Recommendations from research. Columbus, Ohio: Ohio State University. (ERIC Document Reproduction Service No. ED 144 840)

Tischler, R. W. (1988). Mathematics from children's literature. Arithmetic Teacher, 35 (6), 8-11.

Williams, C. K., & Kamii, C. (1986). How do children learn by handling objects? Young Children, 42 (1), 23-26.

Appendix

1. Classifying is the process of sorting objects according to an attribute or characteristic such as size, shape or color. Classification skill helps the child think analytically and express thoughts clearly, both of which are important to good mathematical reasoning. Classification skill is necessary when the child attempts to add apples and oranges and realizes he must reclassify them as fruit to complete the operation.

2. Comparing is the process by which the child establishes a relationship between two objects or sets on the basis of some specific characteristic or attribute like height, weight or length. The ability to compare contributes to mathematical understanding which prepares the child for the comparison of numbers using such terms as "more than", "less than", and "equal to".

3. Ordering builds on comparing. It involves comparing more than two things or two sets and placing them in a sequence according to some rule such as arranging them shortest to tallest or smallest to largest. The ability to arrange things in a particular way is a prerequisite for counting.

4. Patterning is a process of recognizing a pattern visually, aurally, or physically; analyzing that pattern; and then duplicating or continuing it. This skill has been called the underlying theme of mathematics. Patterning skill aids in development of the meaning of numerical symbols. As children learn to recognize dot patterns for numerals, they can look at the patterns and know without counting what each represents.

5. One-to-one correspondence is the process of matching one object to another such as one cookie to one child or one sock to one shoe. It has been called the most basic component of number. One-to-one correspondence is essential to rational counting by which the child matches objects in a set with the set of counting numbers.

6. Conservation of number is the process of holding in memory a set of objects that do not change in value but do change in position. It is believed that conservation of number is a necessary prerequisite skill to success with mathematical operations such as addition and subtraction (Charlesworth & Radeloff, 1978).

7. Rational counting (meaningful counting) involves attaching a numeral name in order to a series of objects in a group. It is simply a higher level of matching or one-to-one correspondence. This skill is basic to calculating cardinality and ordinality of sets.

8. Recognizing cardinal numbers involves rational counting to determine how many objects are in a set. This recognition is important in performing addition and subtraction operations on the concrete and semi-concrete level.

9. Recognizing ordinal numbers is an extension of rational counting. It refers to the order of an object in a set using terms such as "first", "second", "third." It answers the question "which one?" Ordinal number is a useful skill in problem solving.

10. Number recognition is the process of recognizing the symbol as well as understanding the meaning (value) of it. This skill is a prerequisite to all mathematical operations. A child needs to recognize numbers in a meaningful way so that mathematics will not be a rote, inefficient process.

11. Writing numerals is the process of recognizing numerical symbols and then being able to trace, copy, and finally independently produce them from memory. Writing numerals is a necessary skill for mathematical paper and pencil activities.