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A GENERAL MODEL OF THE RESISTIVE WALL INSTABILITY IN LINEAR ACCELERATORS *

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Abstract

A general model for wakefield-generated instabilities in linear accelerators, originally developed for cumulative beam breakup [1], is applied to the resistive wall instability. The general solution for various bunch charge distributions and application to various accelerator configurations are presented.

INTRODUCTION

The beam breakup instability caused by the resistive wall impedance has been studied for the cases of a uniform single bunch and of a point-like bunch train [2] in the asymptotic limit of strong coupling. However, in the final focus of a linear collider or in light sources, the beam current profile is typically non uniform and the coupling, while not negligible, is relatively modest. The formalism developed previously for cumulative beam breakup [1] is applied here to investigate the resistive wall instability for arbitrary beam current profile and arbitrary strength of the wake field

FORMULATION AND SOLUTION

In a continuum approximation, the transverse motion of a relativistic beam under the influence of focusing and transverse wakefield can be modeled by [1]

$$\left[\frac{1}{\gamma}\frac{\partial}{\partial s}\left(\gamma\frac{\partial}{\partial s}\right)+\kappa^2\right] x(s,\zeta) = \varepsilon\int_0^{\zeta} d\zeta_1\, w(\zeta-\zeta_1)F(\zeta_1)x(s,\zeta_1) \tag{1}$$

where γ is the usual energy parameter; s is the distance from the front of the accelerator; κ is the focusing wave number; $\zeta = t - s/c$, is the time measured after the arrival of the head of the beam at location s; $F(\zeta) = I(\zeta)/\overline{I}$, the current form factor, is the instantaneous current divided by the average current; $w(\zeta)$ is the wake function; ε is the coupling strength between the beam and the deflecting field, and includes properties of the beam and the transport channel. If the wakefield source is the resistive-wall of a cylindrical pipe, then the long-range wake field is [3]

$$W(\tau) = \frac{\varepsilon}{\sqrt{\tau}}, \tag{2}$$

where

$$\varepsilon = (4c^2\overline{I})/(c\gamma b^3 I_A)\sqrt{\varepsilon_0/\pi\sigma_c}\,. \tag{3}$$

In the above expression, c is the speed of light, γ is the Lorentz factor, b is the radius of the pipe, $I_A = 17{,}045$ Amp is the Alfven current, ε_0 is the vacuum permittivity and σ_c is the pipe conductivity. With this definition we have

$$w(\zeta) = \zeta^{-1/2} \quad \text{for} \quad \zeta > 0\,. \tag{4}$$

While Eq. (1) assumes a perfectly aligned accelerator, misalignment of the cavities and focusing elements can also be included in the following analysis in a straightforward fashion [1].

Without loss of generality, we will assume a coasting beam. As shown in [1], the analytical results can be extended to an accelerated beam by suitable coordinate and variable transformations. We will also assume that the beam is injected parallel to the axis with a time-independent offset x_0. Under these assumptions, the equation of motion becomes [1]

$$\frac{\partial^2}{\partial s^2}x(s,\zeta)+\kappa^2 x(s,\zeta)= \varepsilon\int_0^{\zeta} d\zeta_1\, w(\zeta-\zeta_1)\, F(\zeta_1)x(s,\zeta_1). \tag{5}$$

and the solution is

$$x(s,\zeta) = x_0\sum_{n=0}^{\infty}\varepsilon^n\, h_n(\zeta)\, j_n(\kappa,s)\,, \tag{6}$$

with

$$h_0(\zeta)=1; \qquad h_{n+1}(\zeta)=\int_0^{\zeta} d\zeta_1\, w(\zeta-\zeta_1)\, F(\zeta_1)$$
$$j_n(\kappa,s)=\frac{1}{n!}\left(\frac{s}{2\kappa}\right)^n\sqrt{\frac{\pi\kappa s}{2}}J_{n-1/2}(\kappa s) \tag{7}$$

Once the profile of the charge distribution in the bunch is determined, the functions $h_n(\zeta)$ can be calculated and then the shape and position of the bunch at any location s determined form Eqs. (6) and (7).

UNIFORM CHARGE DISTRIBUTION

In the case of a uniform charge distribution, the functions $h_n(\zeta)$ can be calculated in closed form and are

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$$h_n(\zeta) = \frac{[\pi\zeta]^{n/2}}{\Gamma(\frac{n}{2}+1)}. \tag{8}$$

If the coupling constant ε is small only a few terms in the series are sufficient and the bunch shape is given by

$$\frac{x(s,\zeta)}{x_0} = 1 + \varepsilon s^2 \zeta^{1/2} + \frac{\pi}{24}\left(\varepsilon s^2 \zeta^{1/2}\right)^2 + \frac{\pi}{540}\left(\varepsilon s^2 \zeta^{1/2}\right)^3 + \dots \tag{9}$$

in the absence of focusing, and by

$$\begin{aligned}\frac{x(s,\zeta)}{x_0} = \cos\kappa s\left[1 - \frac{\pi}{8}\left(\frac{\varepsilon s\zeta^{1/2}}{\kappa}\right)^2 + \frac{\pi^2}{768}\left(\frac{\varepsilon s\zeta^{1/2}}{\kappa}\right)^4 + \dots\right] \\ + \sin\kappa s\left[\frac{\varepsilon s\zeta^{1/2}}{\kappa} - \frac{\pi}{36}\left(\frac{\varepsilon s\zeta^{1/2}}{\kappa}\right)^3 + \dots\right]\end{aligned} \tag{10}$$

in the presence of focusing.

If the coupling is strong, then the displacement can be evaluated by the method of steepest descent and is given by

$$\begin{aligned}\frac{x(s,\zeta)}{x_0} &\sim \frac{1}{\sqrt{2\pi}} X^{-1/10}\exp\left(X^{1/5}\right) \\ X &= \left(\frac{5}{2}\right)^5 \frac{\pi\varepsilon^2 s^4 \zeta}{8}\end{aligned} \tag{11}$$

in the absence of focusing and by

$$\begin{aligned}\frac{x(s,\zeta)}{x_0} &\sim \frac{X^{-1/6}}{\sqrt{2\pi}}\exp\left(X^{1/3}\right)\cos\left(\kappa s + \frac{\pi}{6} - \sqrt{3}X^{1/3}\right) \\ X &= \left(\frac{3}{4}\right)^3 \frac{\pi\varepsilon^2 s^2 \zeta}{2\kappa^2}\end{aligned} \tag{12}$$

in the presence of focusing.

NONUNIFORM CHARGE DISTRIBUTION

Parabolic charge distribution

In the case of a single bunch of length ζ_b with a parabolic current distribution, the current distribution is of the form

$$F(\zeta) = \alpha\left(\frac{\zeta}{\zeta_b} - \frac{1}{2}\right)^2 + 1 - \frac{\alpha}{2}, \tag{13}$$

where α is a free parameter that is used to model the shape of the current distribution. In this case the functions $h_n(\zeta)$ cannot be obtained in closed form but can be calculated to arbitrary order through the recursion relations (7). The usual parabolic shape is obtained with $\alpha = -6$ which, together with the first functions $h_n(\zeta)$ is shown in Fig. 1. In certain circumstances, such as light sources, the charge is concentrated at the head and tail of the bunch, which can be modeled by $\alpha > 0$ as shown in Fig. 2.

Gaussian charge distribution

The form factor of a gaussian bunch truncated at $\pm 2\sigma$ can accurately be modeled by

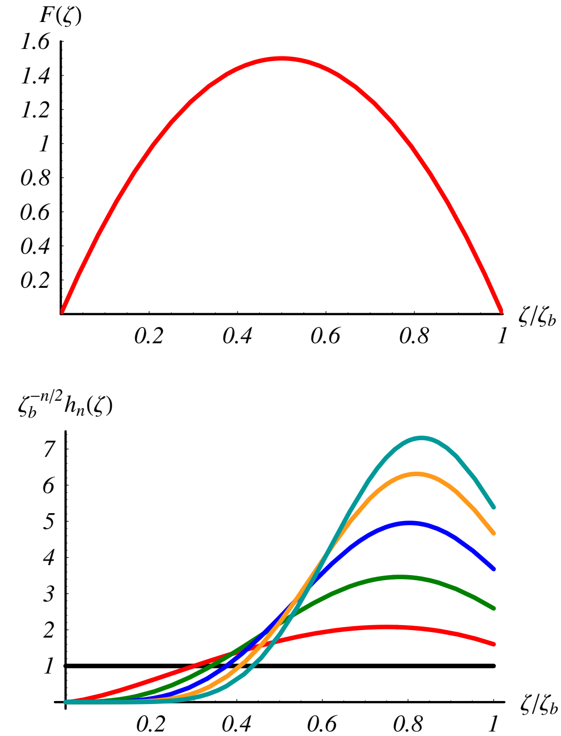

Fig. 1: Parabolic charge distribution from Eq. (13) with $\alpha = -6$ and functions $h_0(\zeta)$ to $h_6(\zeta)$.

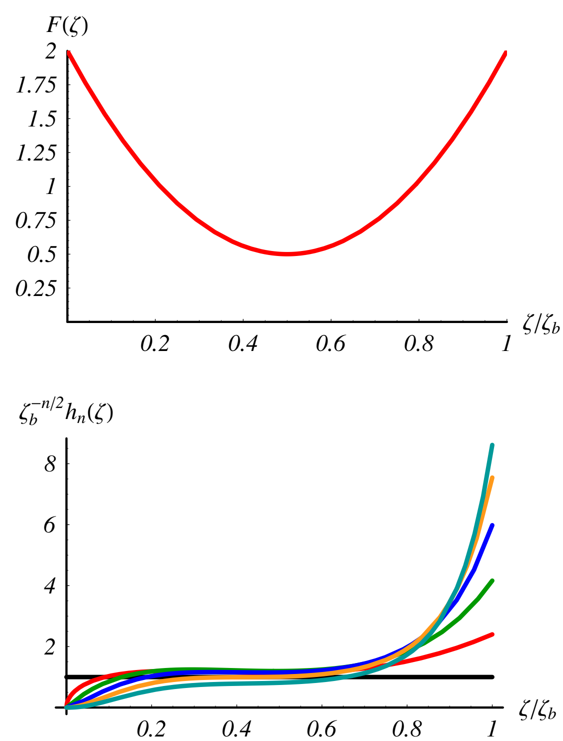

Fig. 2: Parabolic charge distribution from Eq. (13) with $\alpha = 6$ and functions $h_0(\zeta)$ to $h_6(\zeta)$.

$$F(\zeta)=\frac{5}{3}\left[1-\frac{168}{25}\left(\frac{\zeta}{\zeta_b}-\frac{1}{2}\right)^2+\frac{64}{5}\left(\frac{\zeta}{\zeta_b}-\frac{1}{2}\right)^4\right] . \quad (14)$$

Such a charge distribution and the associated functions $h_n(\zeta)$ are shown in Fig. 3.

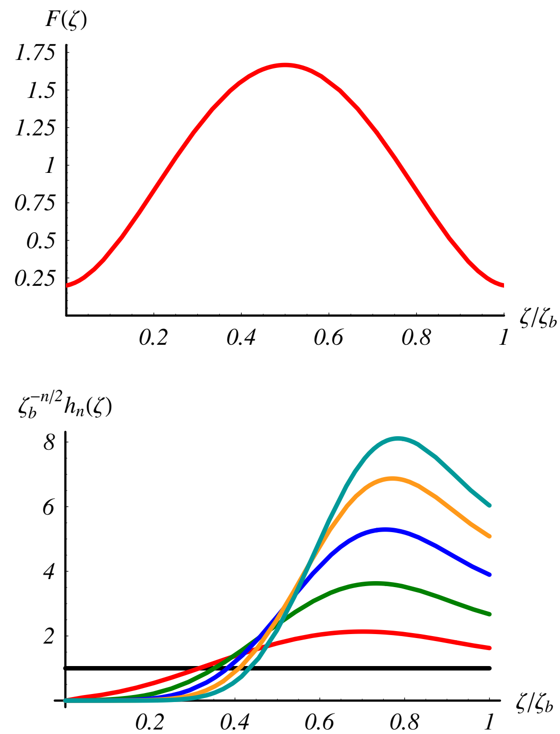

Fig. 3: Gaussian charge distribution from Eq. (14) and functions $h_0(\zeta)$ to $h_6(\zeta)$.

APPLICATIONS

The displacement and distortion of the bunch as it travels along the beam line under the influence of the resistive wall wakefield given by Eq. (6) is of the form

$$x(s,\zeta)=x_0\mathbf{A}(s,\zeta;\varepsilon) . \quad (15)$$

Table 1: Parameters representative of LCLS and ILC.

	LCLS	ILC
Bunch duration (ps)	.077	1
Pipe radius (cm)	0.3	2
Pipe length (m)	150	300
Conductivity ($10^7\Omega^{-1}m^{-1}$)	6	3.47
κ (m^{-1})	1/18	1/12500
Bunch charge (pC)	1	3.2
ε	104	$6.5\ 10^{-3}$

If focusing is present, then it can be separated in two parts:

$$\mathbf{A}(s,\zeta;\varepsilon)=\mathbf{A}_c(s,\zeta;\varepsilon)\cos\kappa s+\mathbf{A}_s(s,\zeta;\varepsilon)\sin\kappa s . \quad (16)$$

As examples of the application of these results to various accelerator configurations, Fig. 3 shows the functions $\mathbf{A}_c(s,\zeta;\varepsilon)$ and $\mathbf{A}_s(s,\zeta;\varepsilon)$ for LCLS with charge distribution of Fig. 2 and $\mathbf{A}(s,\zeta;\varepsilon)$ for ILC with charge distribution of Fig. 1. Typical parameters for these two accelerators are shown in Table 1.

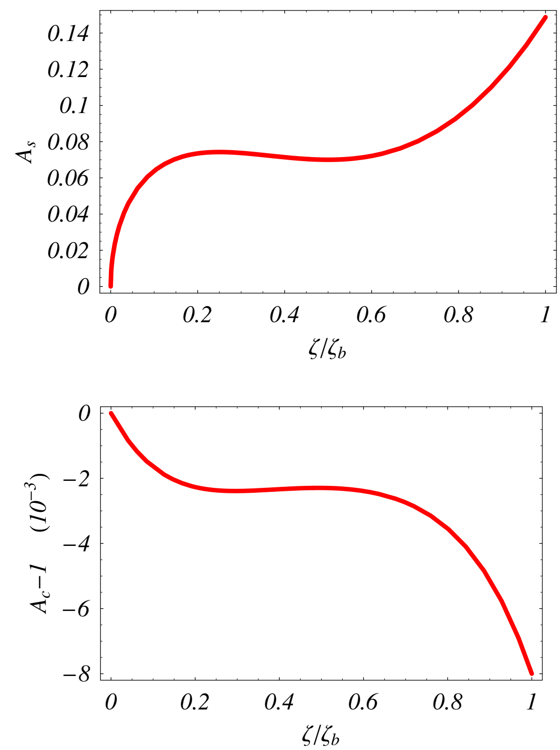

Fig. 4: Profile of an LCLS bunch assuming the charge distribution of Fig. 2.

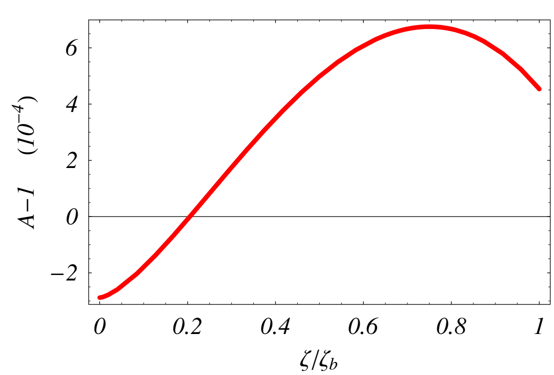

Fig. 5: Profile of an ILC bunch assuming the charge distribution of Fig. 1.

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