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Low and Medium β Superconducting Cavities and Accelerators

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SRF 2007

Beijing

October 2007

LOW AND MEDIUM β SUPERCONDUCTING CAVITIES AND ACCELERATORS

Jean Delayen

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Old Dominion University

Introduction

- There have been increased needs for reduced-beta ($\beta < 1$) SRF cavity especially in CW machine (or high duty pulsed machine; duty $> 10\%$)
 - Accelerator driven system (ADS)
Nuclear transmutation of long-lived radio active waste
Energy amplifier
Intense spallation neutron source
 - Nuclear physics
Radioactive ion acceleration
Muon/neutrino production
 - Military application
Tritium production
- SRF technology → **Critical path !!**

Introduction

- **Reduced beta Elliptical multi-cell SRF cavity**
 - for CW, prototyping by several R&D groups demonstrate as low as $\beta=0.47$
 - for pulsed, SNS $\beta=0.61$, 0.81 cavities & ESS (SNS; cryomodule test progressing, production cavities are coming now)
- **SRF cavity for CW application or long pulse application**
 - efforts for expanding their application regions down to $\beta\sim 0.1$,
- **Elliptical cavity has intrinsic problem as β goes down**
 - mechanical problem, multipacting, low RF efficiency
- **Spoke cavity; supposed to cover ranges $\beta=0.1\sim 0.5(6)$, $f=300\sim 900$ MHz**
 - design & prototype efforts in RIA, AAA, EURISOL, XADS, ESS, etc.

For proton $\beta=0.12$ corresponds ~ 7 MeV \rightarrow all the accelerating structures (except RFQ)

Low and Medium β Superconducting Accelerators

	High Current	Medium/Low Current
CW	Accelerator driven systems waste transmutation energy production	Production of radioactive ions Nuclear Structure
Pulsed	Pulsed spallation sources	

High-current cw accelerators

- Beam: p, H⁻, d
- Technical issues and challenges
 - Beam losses (~ 1 W/m)
 - Activation
 - High cw rf power
 - Higher order modes
 - Cryogenics losses
- Implications for SRF technology
 - Cavities with high acceptance
 - Development of high cw power couplers
 - Extraction of HOM power
 - Cavities with high shunt impedance

High-current pulsed accelerators

- Beam: p, H⁻
- Technical issues and challenges
 - Beam losses (~ 1 W/m)
 - Activation
 - Higher order modes
 - High peak rf power
 - Dynamic Lorentz detuning
- Implications for SRF technology
 - Cavities with high acceptance
 - Development of high peak power couplers
 - Extraction of HOM power
 - Development of active compensation of dynamic Lorentz detuning

Medium to low current cw accelerators

- Beam; p to U
- Technical issues and challenges
 - Microphonics, frequency control
 - Cryogenic losses
 - Wide charge to mass ratio
 - Multicharged state acceleration
 - Activation
- Implications for SRF technology
 - Cavities with low sensitivity to vibration
 - Development of microphonics compensation
 - Cavities with high shunt impedance
 - Cavities with large velocity acceptance (few cells)
 - Cavities with large beam acceptance (low frequency, small frequency transitions)

Common considerations (I)

- Intermediate velocity applications usually do not require (or cannot afford) very high gradients
- Operational and practical gradients are limited by
 - Cryogenics losses (cw applications)
 - Rf power to control microphonics (low current applications)
 - Rf power couplers (high-current applications)
- High shunt impedance is often more important
- To various degrees, beam losses and activation are a consideration

Common considerations (II)

- Superconducting accelerators in the medium velocity range are mostly used for the production of secondary species
 - Neutrons (spallation sources)
 - Exotic ions (radioactive beam facilities)
- Medium power (100s kW) to high power (~MW) primary impinging on a target
- Thermal properties and dynamics of the target are important considerations in the design of the accelerator (frequency, duration, recovery from beam trips)
- Some implications:
 - Operate cavities sufficiently far from the edge
 - Provide an ample frequency control window

Design considerations

- Low cryogenics losses
 - High $QR_s * R_{sh}/Q$
 - Low frequency
- High gradient
 - Low E_p/E_{acc}
 - Low B_p/E_{acc}
- Large velocity acceptance
 - Small number of cells
 - Low frequency
- Frequency control
 - Low sensitivity to microphonics
 - Low energy content
 - Low Lorentz coefficient
- Large beam acceptance
 - Large aperture (transverse acceptance)
 - Low frequency (longitudinal acceptance)

A Few Obvious Statements

Low and medium β

$$\beta < 1$$

Particle velocity will change

The lower the velocity of the particle or cavity β

The faster the velocity of the particle will change

The narrower the velocity range of a particular cavity

The smaller the number of cavities of that β

The more important it is that the particle achieve design velocity

Be conservative at lower β

Be more aggressive at higher β

A Few More Statements

Two main types of structure geometries

TEM class (QW, HW, Spoke)

TM class (elliptical)

Design issues of medium β elliptical cavities are similar to those of $\beta=1$

Most of the talk will be on TEM-class cavities

For TM-class cavities see:

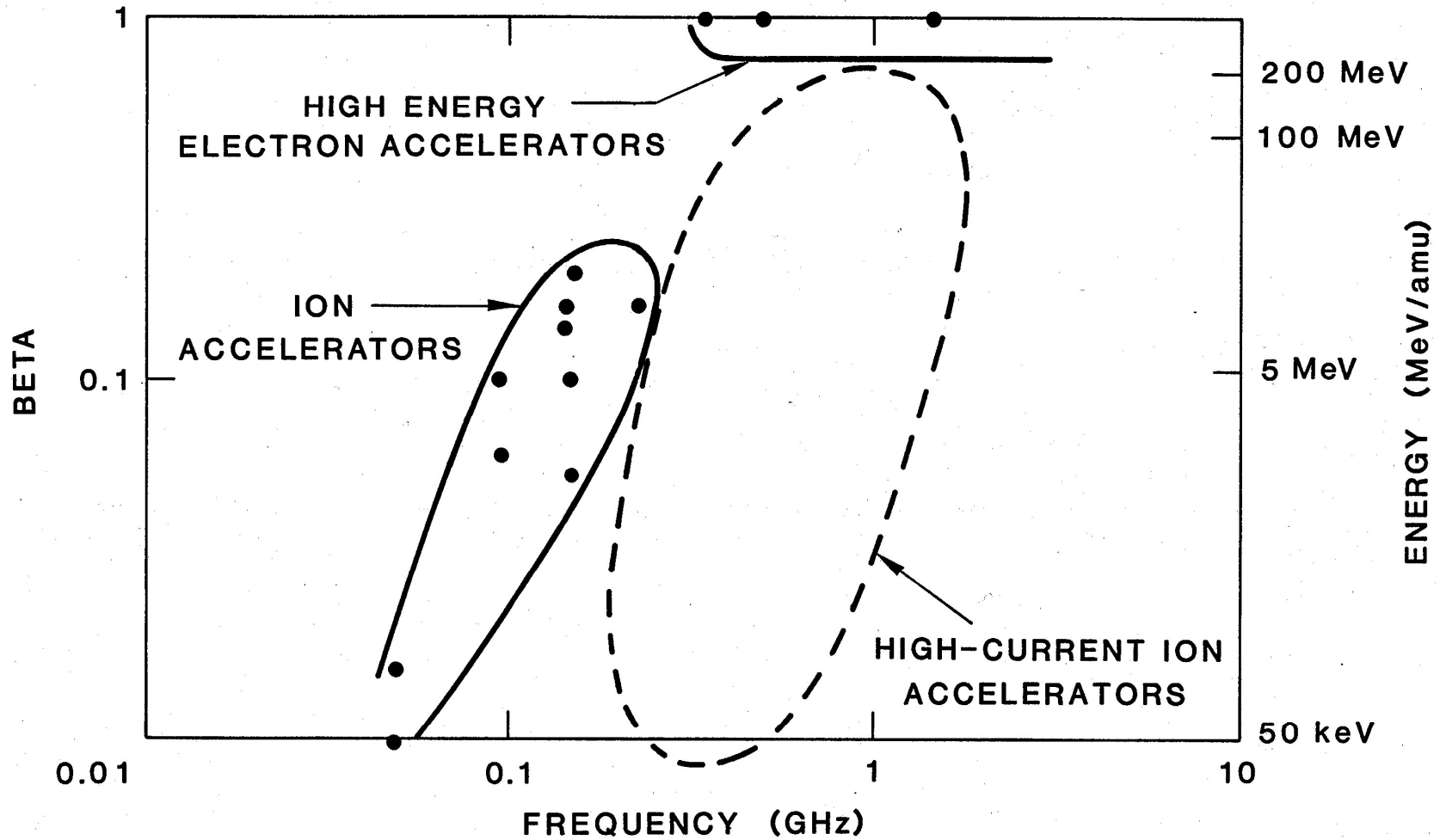
Design criteria for elliptical cavities

Pagani, Barni, Bosotti, Pierini, Ciovati, SRF 2001.

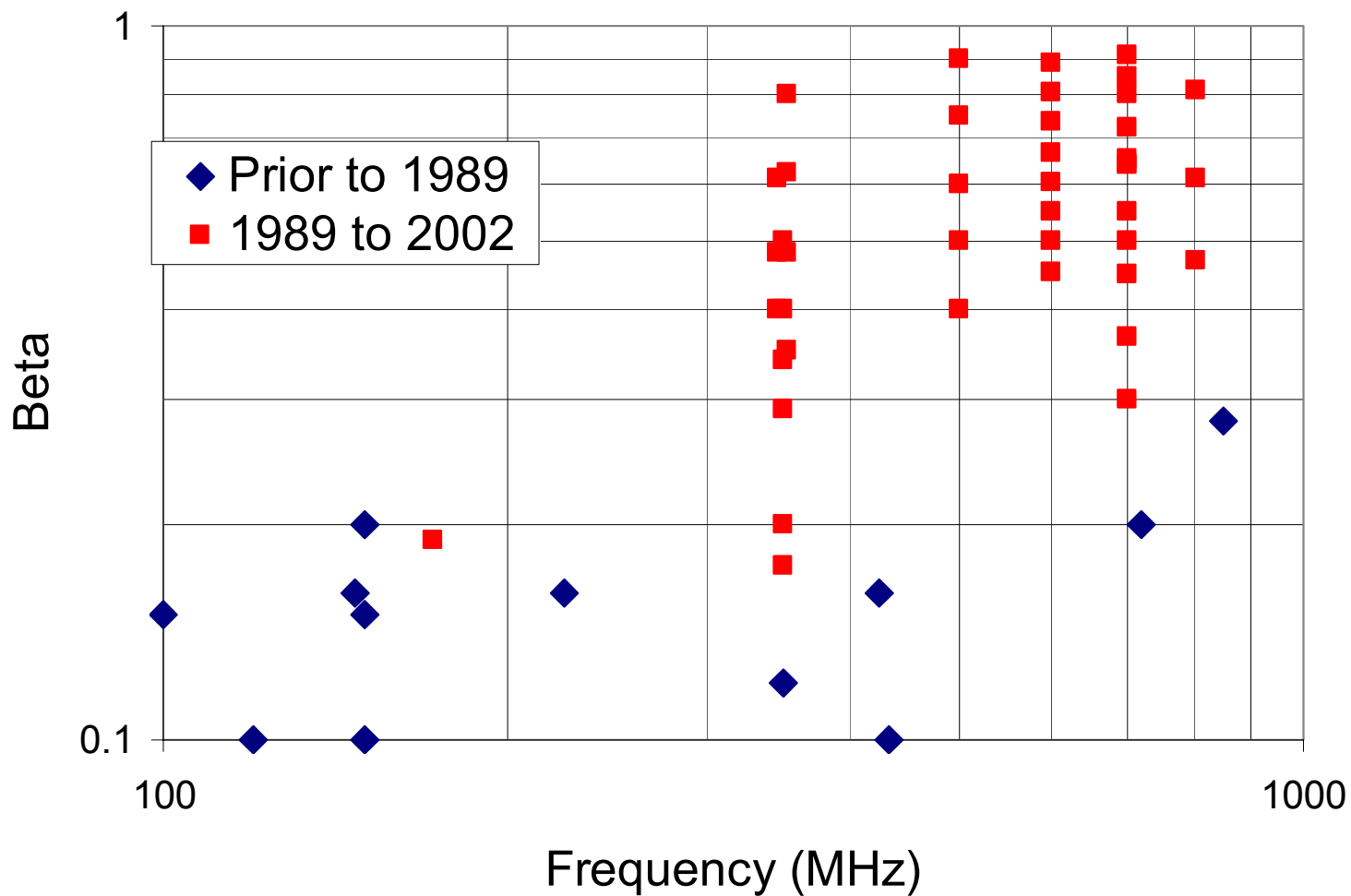
Challenges and the future of reduced beta srf cavity design

Sang-ho Kim, LINAC 2002.

Superconducting Structures – Circa 1987



$\beta < 1$ Superconducting Structures – 2002..



Basic Structure Geometries

Resonant Transmission Lines

- $\lambda/4$
 - Quarter-wave
 - Split-ring
 - Twin quarter-wave
 - Lollipop
- $\lambda/2$
 - Coaxial half-wave
 - Spoke
 - H-types

– TM

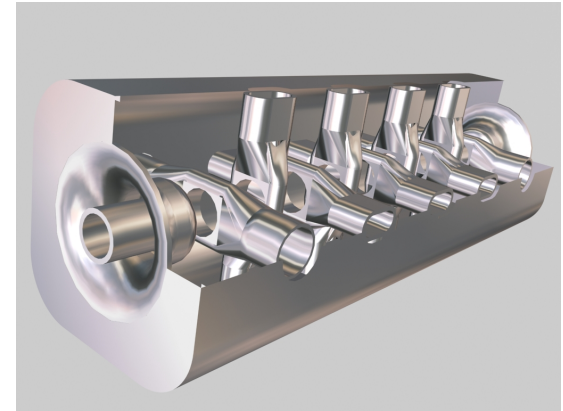
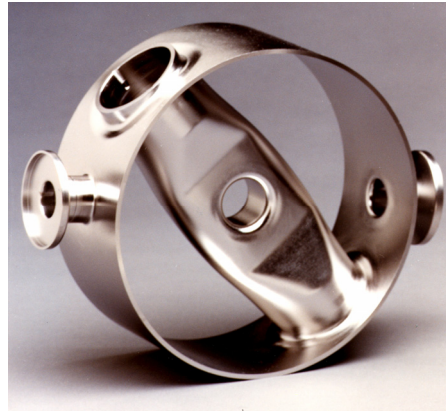
- Elliptical
- Reentrant

– Other

- Alvarez
- Slotted-iris

A Word on Design Tools

TEM-class cavities are essentially 3D geometries



3D electromagnetic software is available

MAFIA, Microwave Studio, HFSS, etc.

3D software is usually very good at calculating frequencies

Not quite as good at calculating surface fields

Use caution, vary mesh size

Remember Electromagnetism 101

Design Tradeoffs

Number of cells
Voltage gain
Velocity acceptance

Frequency

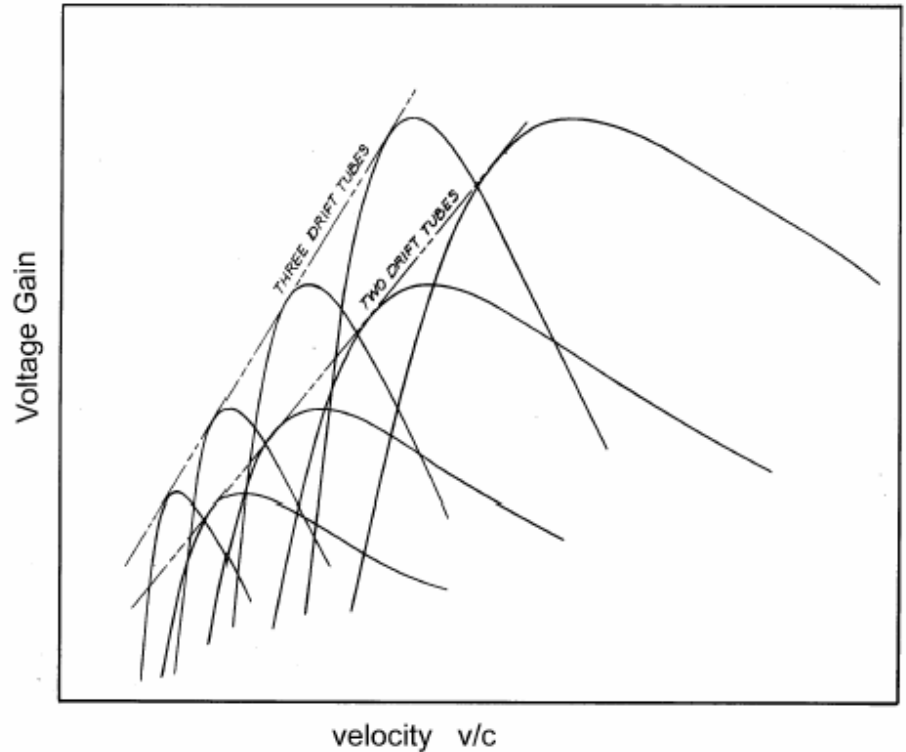
Size

Voltage gain

Rf losses

Energy content, microphonics, rf control

Acceptance, beam quality and losses



Energy Gain

Transit Time Factor - Velocity Acceptance

$$\Delta W = q \int_{-\infty}^{+\infty} E(z) \cos(\omega t + \phi) dz$$

- Assumption: constant velocity

$$\Delta W = q \cos \phi \Delta W_0 T(\beta)$$

$$\Delta W_0 = \Theta \int_{-\infty}^{+\infty} |E(z)| dz$$

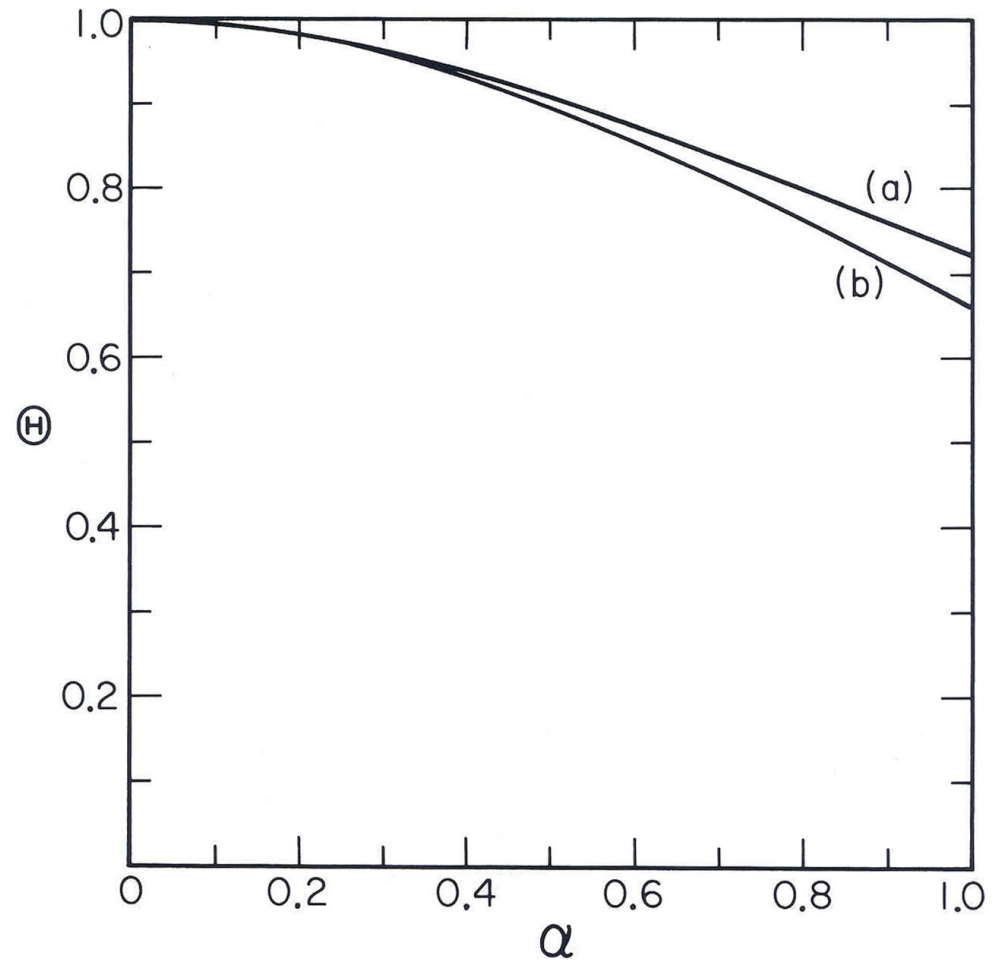
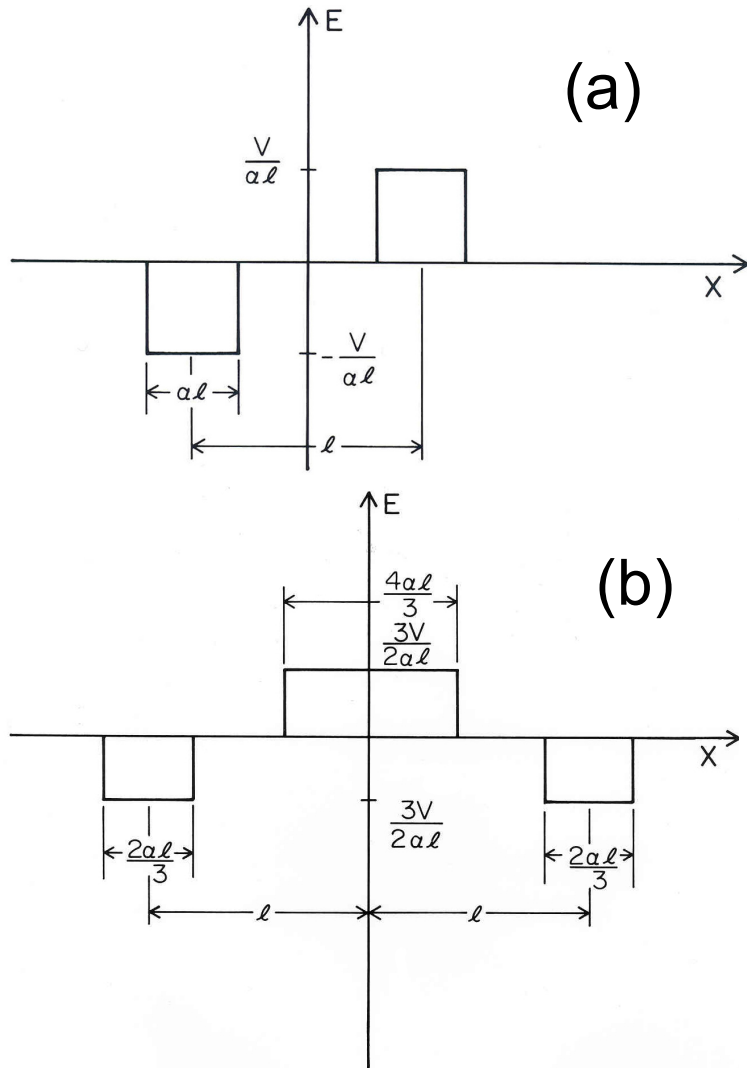
$$\Theta = \frac{\text{Max} \int_{-\infty}^{+\infty} E(z) \cos\left(\frac{\omega z}{\beta c}\right) dz}{\int_{-\infty}^{+\infty} |E(z)| dz}$$

Transit Time Factor

$$T(\beta) = \frac{\int_{-\infty}^{+\infty} E(z) \cos\left(\frac{\omega z}{\beta c}\right) dz}{\text{Max} \int_{-\infty}^{+\infty} E(z) \cos\left(\frac{\omega z}{\beta c}\right) dz}$$

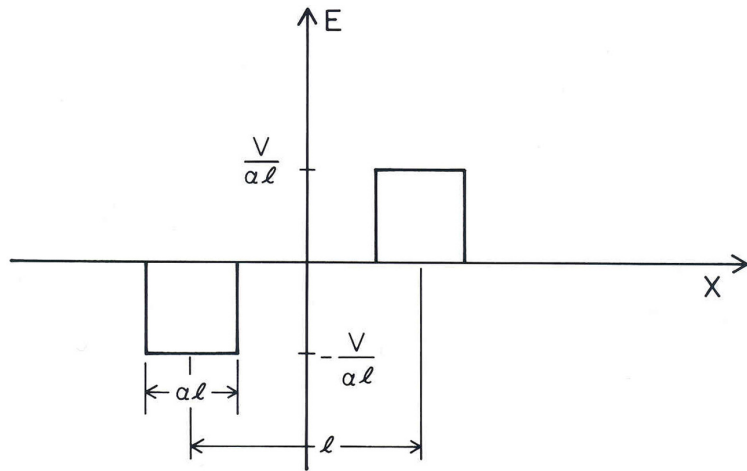
Velocity Acceptance

Transit Time Factor



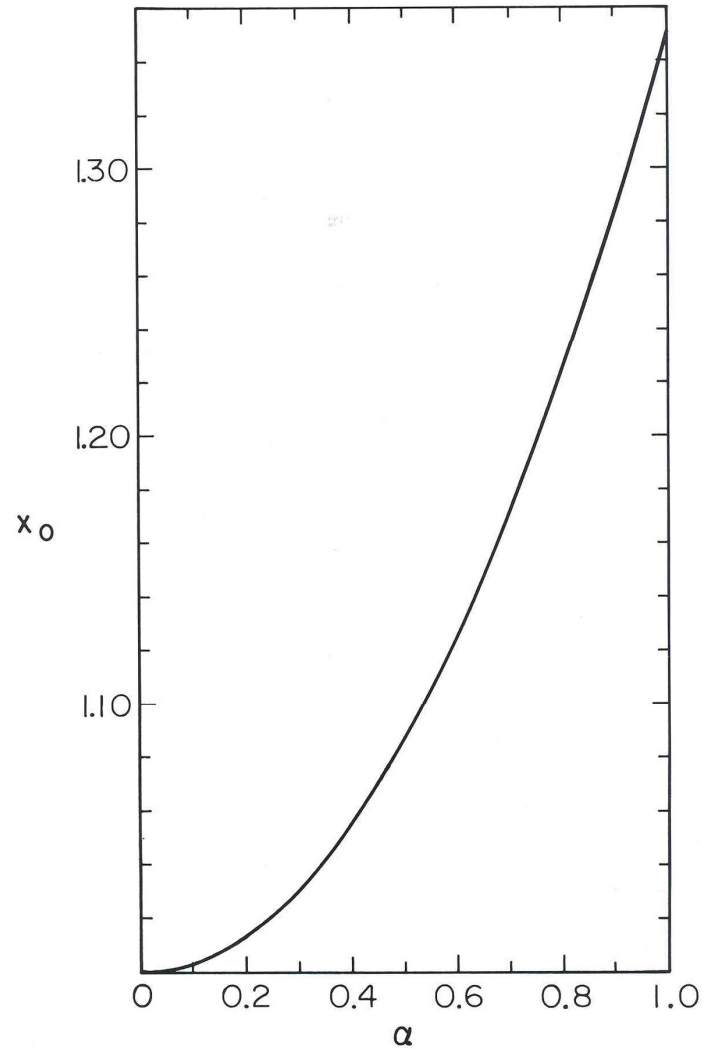
Velocity Acceptance for 2-Gap Structures

$$T(\beta) = \frac{\beta}{\beta_0} \frac{\sin\left(\frac{\pi\alpha\beta_0}{2x_0\beta}\right) \sin\left(\frac{\pi\beta_0}{2x_0\beta}\right)}{\sin\left(\frac{\pi\alpha}{2x_0}\right) \sin\left(\frac{\pi}{2x_0}\right)}$$



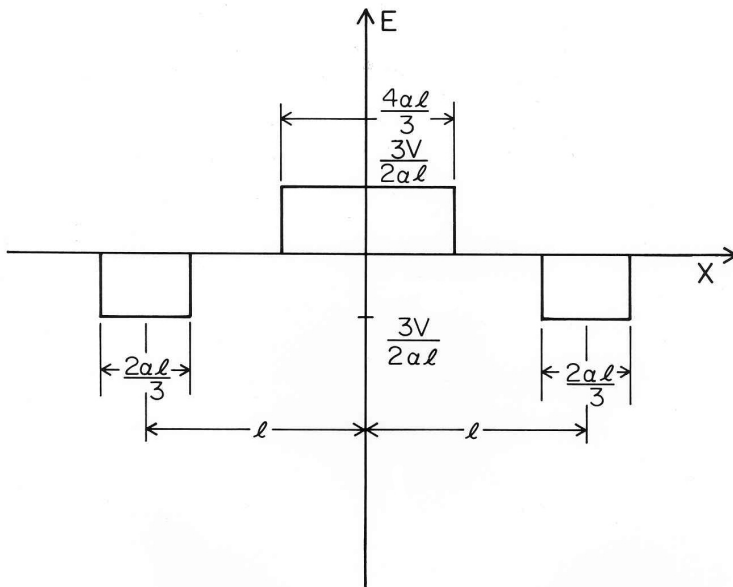
$$x_0 = \frac{\beta_0 \lambda}{2l}$$

$$l \approx \frac{\beta_0 \lambda}{2.2}$$

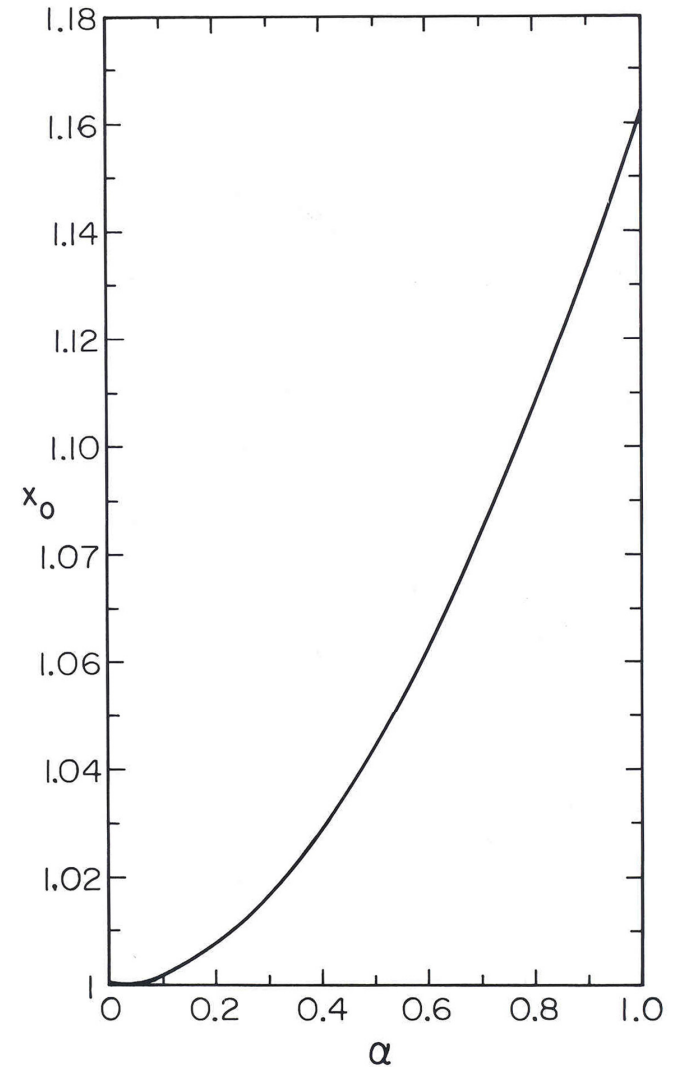


Velocity Acceptance for 3-Gap Structures

$$T(\beta) = \frac{\beta}{\beta_0} \frac{\sin\left(\frac{\pi\alpha\beta_0}{3x_0\beta}\right) \left[\cos\left(\frac{\pi\alpha\beta_0}{3x_0\beta}\right) - \cos\left(\frac{\pi\beta_0}{x_0\beta}\right) \right]}{\sin\left(\frac{\pi\alpha}{3x_0}\right) \left[\cos\left(\frac{\pi\alpha}{3x_0}\right) - \cos\left(\frac{\pi}{x_0}\right) \right]}$$



$$x_0 = \frac{\beta_0 \lambda}{2l}$$

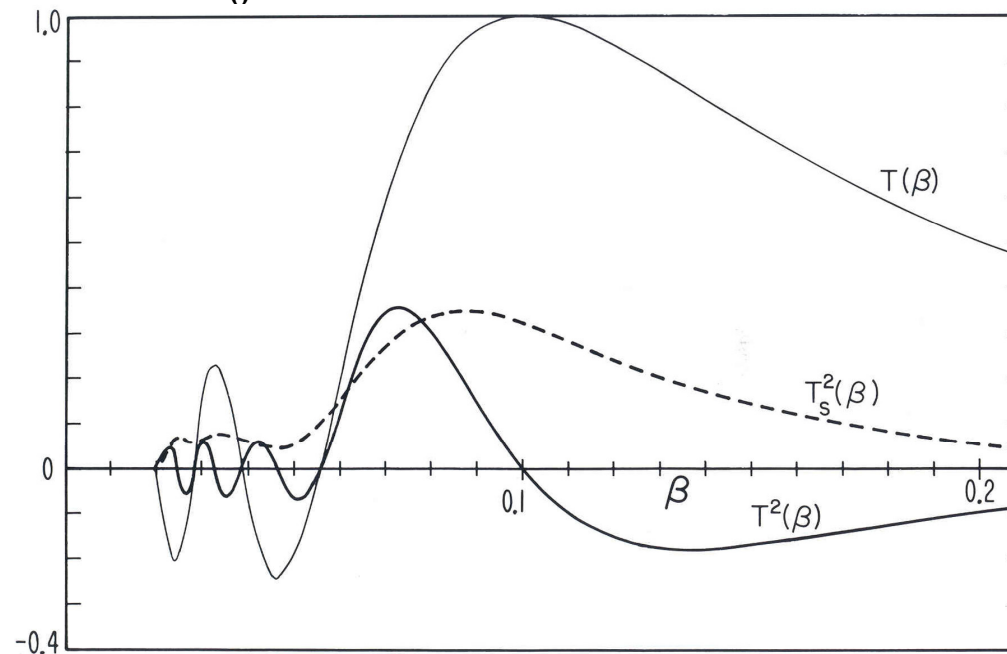


Higher-Order Effects

$$\Delta W = q \cos \phi \Delta W_0 T(\beta) + \frac{(q\Delta W_0)^2}{W} \left[T^{(2)}(\beta) + \sin 2\phi T_s^{(2)}(\beta) \right]$$

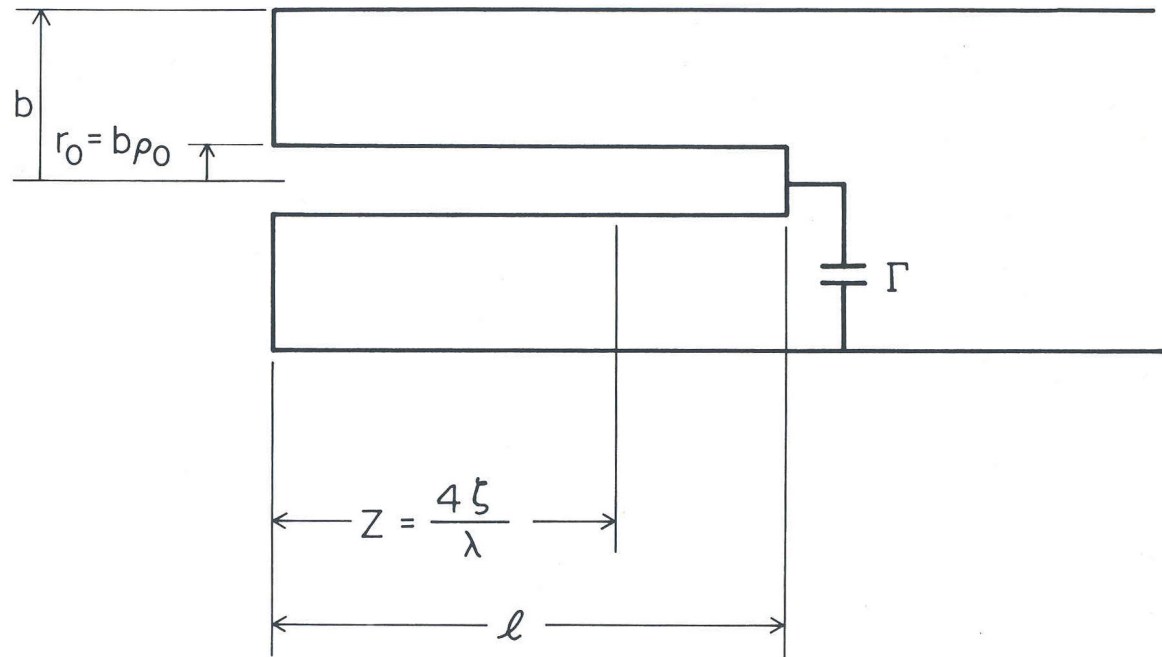
$$T^{(2)}(k) = -\frac{k}{4} T(k) \frac{d}{dk} T(k) \quad k = \omega / \beta c$$

$$T_s^{(2)}(k) = -\frac{k}{4\pi} \int_0^\infty \frac{T(k+k')T(k-k') - T(k)T(k)}{k'^2} dk'$$



A Simple Model: Loaded Quarter-wavelength Resonant Line

If characteristic length $\ll \lambda$ ($\beta < 0.5$), separate the problem in two parts:
Electrostatic model of high voltage region
Transmission line



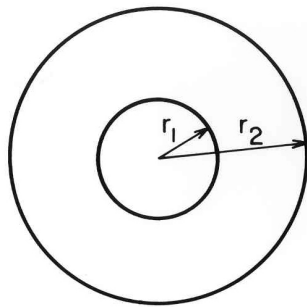
Basic Electrostatics

- a: concentric spheres
- b: sphere in cylinder
- c: sphere between 2 planes
- d: coaxial cylinders
- e: cylinder between 2 planes

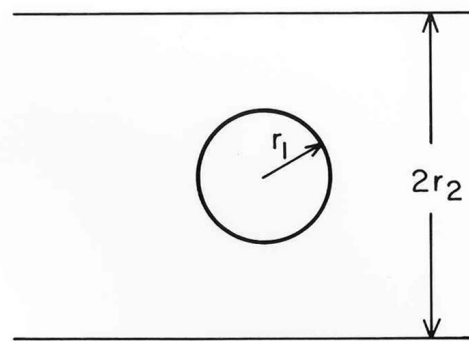
V_p : Voltage on center conductor

Outer conductor at ground

E_p : Peak field on center conductor

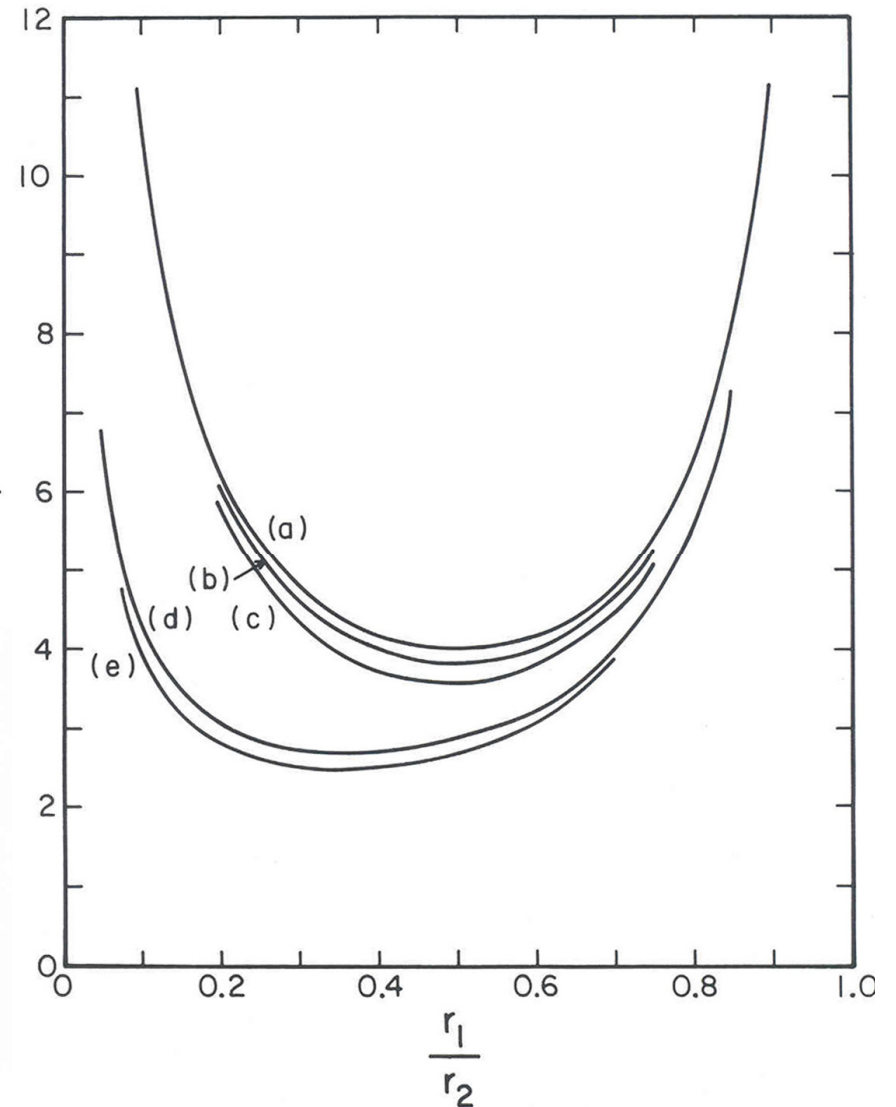


(a), (b), (d)



(c), (e)

$$E_p = \frac{r_2}{V}$$



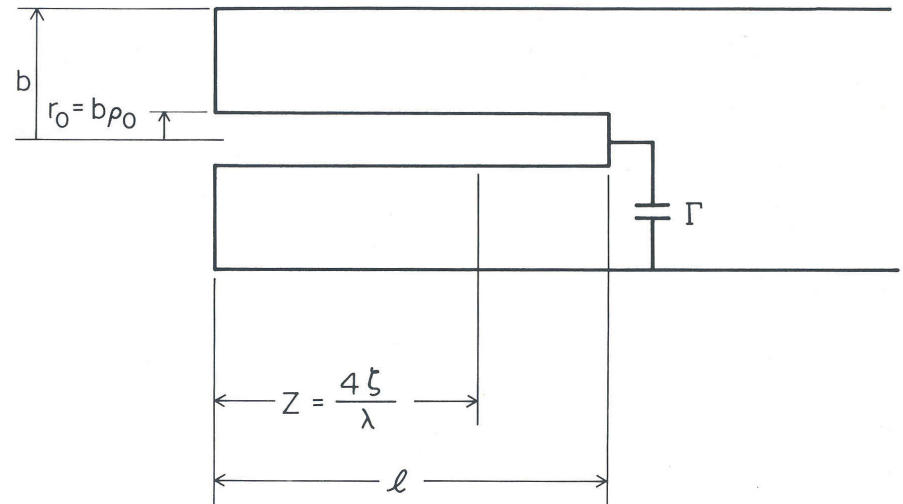
Loaded Quarter-wavelength Resonant Line

Capacitance per unit length

$$C = \frac{2\pi\epsilon_0}{\ln\left(\frac{b}{r_0}\right)} = \frac{2\pi\epsilon_0}{\ln\left(\frac{1}{\rho_0}\right)}$$

Inductance per unit length

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{r_0}\right) = \frac{\mu_0}{2\pi} \ln\left(\frac{1}{\rho_0}\right)$$



Loaded Quarter-wavelength Resonant Line

Center conductor voltage

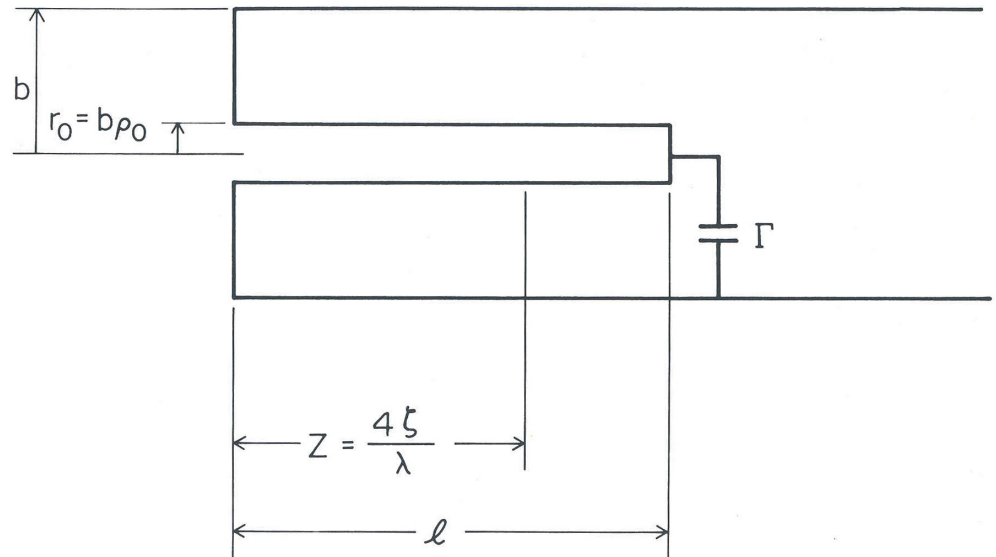
$$V(z) = V_0 \sin\left(\frac{2\pi}{\lambda} z\right)$$

Center conductor current

$$I(z) = I_0 \cos\left(\frac{2\pi}{\lambda} z\right)$$

Line impedance

$$Z_0 = \frac{V_0}{I_0} = \frac{\eta}{2\pi} \ln\left(\frac{1}{\rho_0}\right), \quad \eta = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\Omega$$

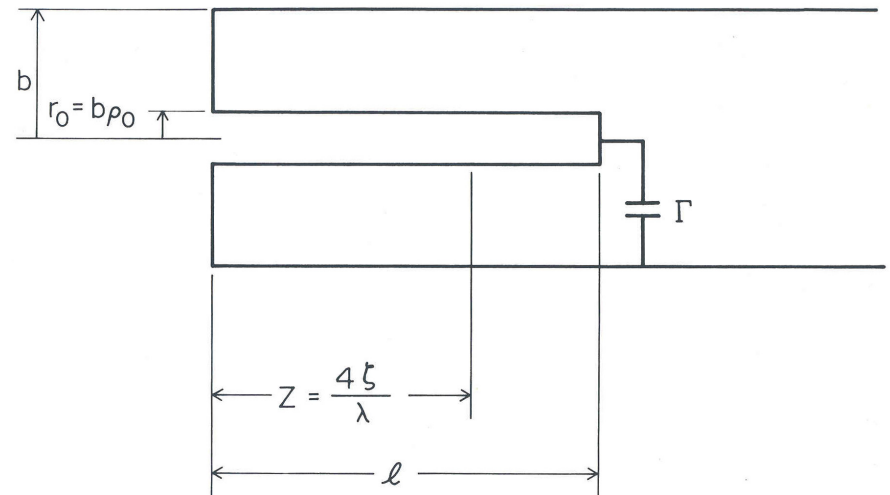


Loaded Quarter-wavelength Resonant Line

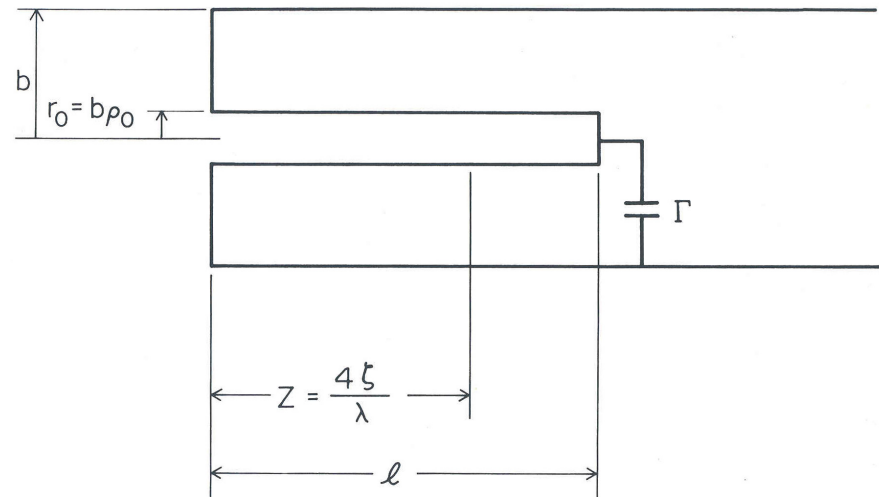
Loading capacitance

$$\Gamma(z) = \lambda \epsilon \frac{\cotan\left(\frac{2\pi}{\lambda} z\right)}{\ln(1/r_0)} = \lambda \epsilon \frac{\cotan\left(\frac{\pi}{2} \zeta\right)}{\ln(1/\rho_0)}$$

$$l = \frac{\lambda}{2\pi} \text{Arctan} \left[\frac{\lambda \epsilon}{\Gamma \ln(1/\rho_0)} \right]$$



Loaded Quarter-wavelength Resonant Line



Peak magnetic field

$$\frac{V_p}{b} = \begin{Bmatrix} \eta & H \\ c & B \\ 300 & B \end{Bmatrix} \rho_0 \ln\left(\frac{1}{\rho_0}\right) \sin\left(\frac{\pi}{2}\zeta\right) \quad \begin{Bmatrix} \text{m, A/m} \\ \text{m, T} \\ \text{cm, G} \end{Bmatrix}$$

V_p : Voltage across loading capacitance

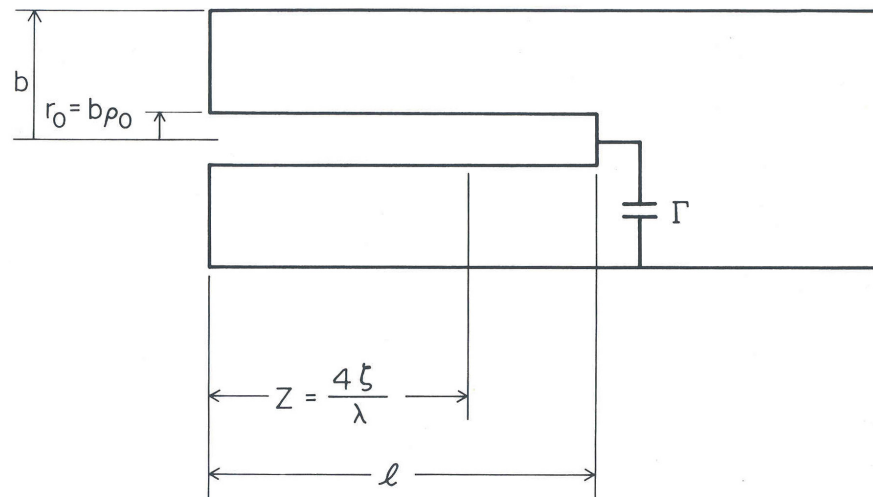
$B \approx 9 \text{ mT}$ at 1 MV/m

Loaded Quarter-wavelength Resonant Line

Power dissipation (ignore losses in the shorting plate)

$$P = V_p^2 \frac{8}{\pi} \frac{R_s}{\eta^2} \frac{\lambda}{b} \frac{1 + 1/\rho_0}{\ln^2 \rho_0} \frac{\zeta + \frac{1}{\pi} \sin \pi \zeta}{\sin^2 \frac{\pi}{2} \zeta}$$

$$P \propto \frac{R_s}{\eta^2} E^2 \beta \lambda^2$$

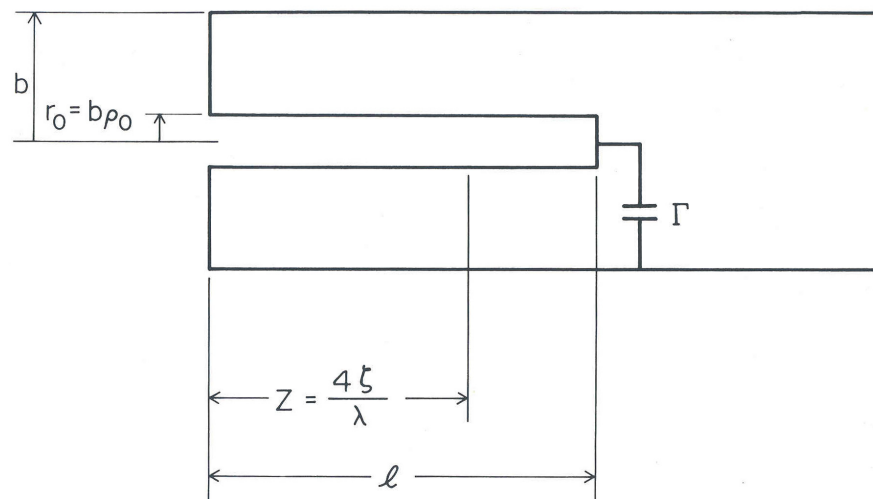


Loaded Quarter-wavelength Resonant Line

Energy content

$$U = V_p^2 \frac{\pi \epsilon_0}{8} \lambda \frac{1}{\ln(1/\rho_0)} \frac{\zeta + \frac{1}{\pi} \sin \pi \zeta}{\sin^2 \frac{\pi}{2} \zeta}$$

$$U \propto \epsilon_0 E^2 \beta^2 \lambda^3$$

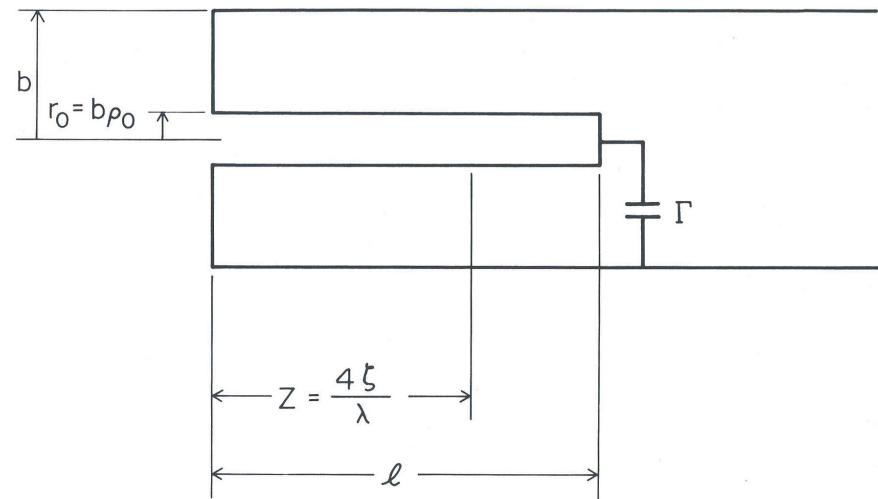


Loaded Quarter-wavelength Resonant Line

Geometrical factor

$$G = QR_s = 2\pi \eta \frac{b \ln(1/\rho_0)}{\lambda (1+1/\rho_0)}$$

$$G \propto \eta \beta$$

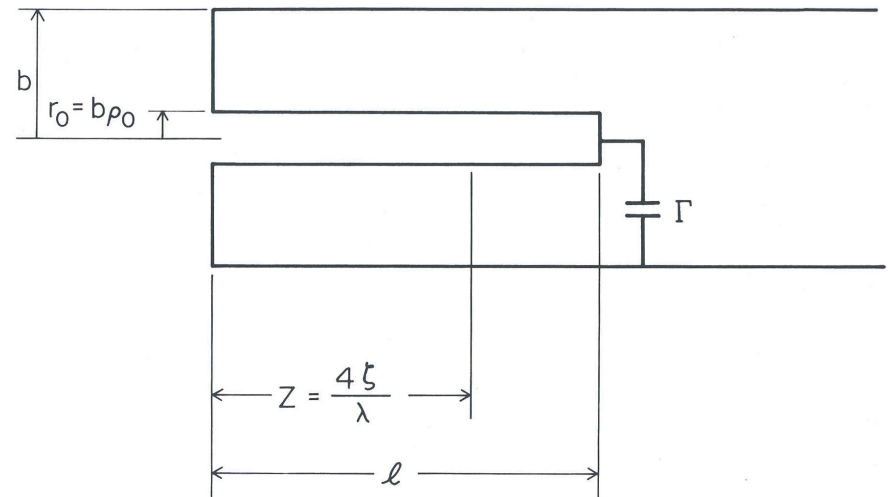


Loaded Quarter-wavelength Resonant Line

Shunt impedance $(4V_p^2 / P)$

$$R_{sh} = \frac{\eta^2}{R_s} \frac{32}{\pi} \frac{b}{\lambda} \frac{\ln^2 \rho_0}{1+1/\rho_0} \frac{\sin^2 \frac{\pi}{2} \zeta}{\zeta + \frac{1}{\pi} \sin \pi \zeta}$$

$$R_{sh} R_s \propto \eta^2 \beta$$

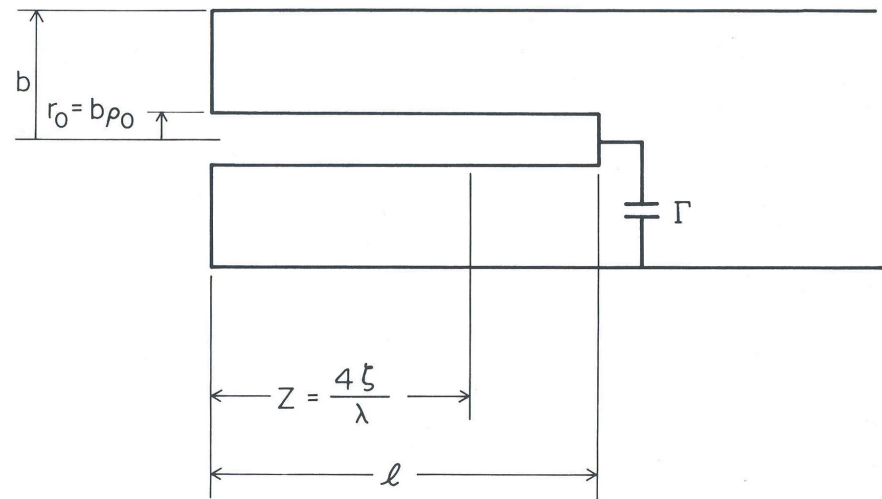


Loaded Quarter-wavelength Resonant Line

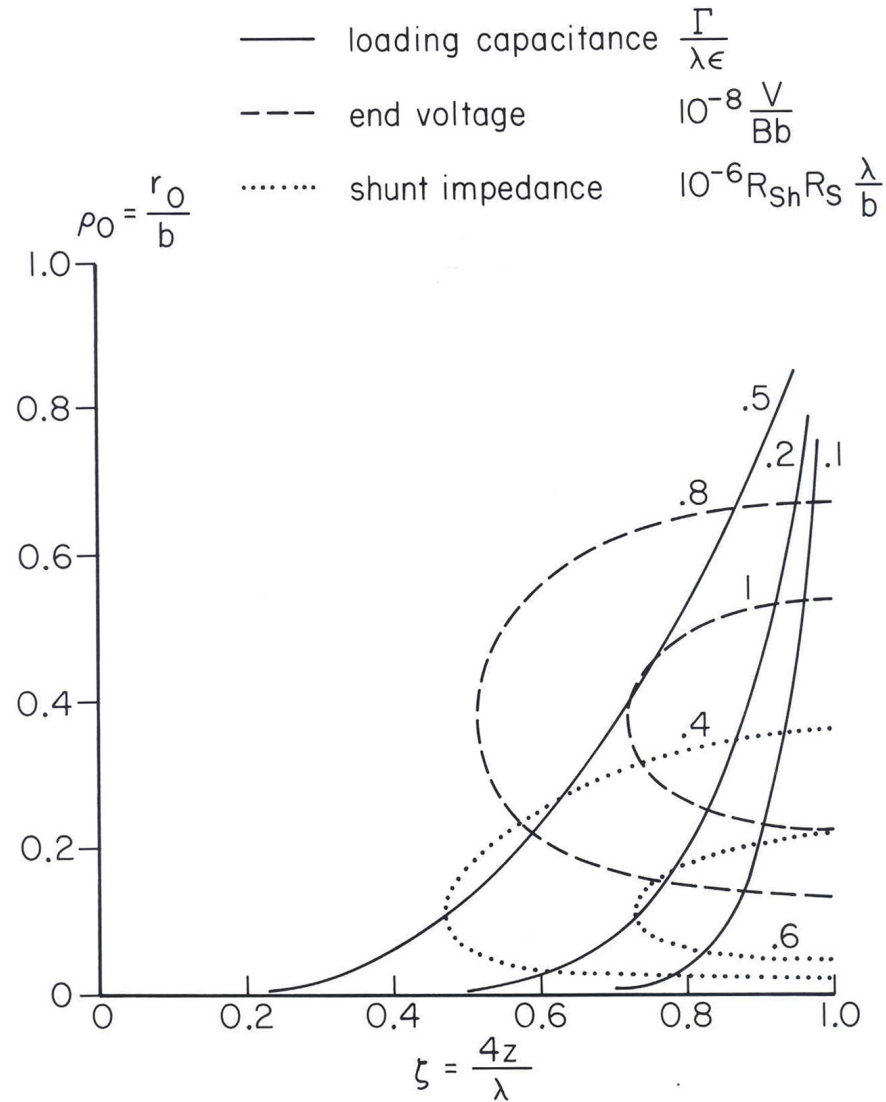
R/Q

$$\frac{R_{sh}}{Q} = \frac{16}{\pi^2} \eta \ln(1/\rho_0) \frac{\sin^2 \frac{\pi}{2} \zeta}{\zeta + \frac{1}{\pi} \sin \pi \zeta}$$

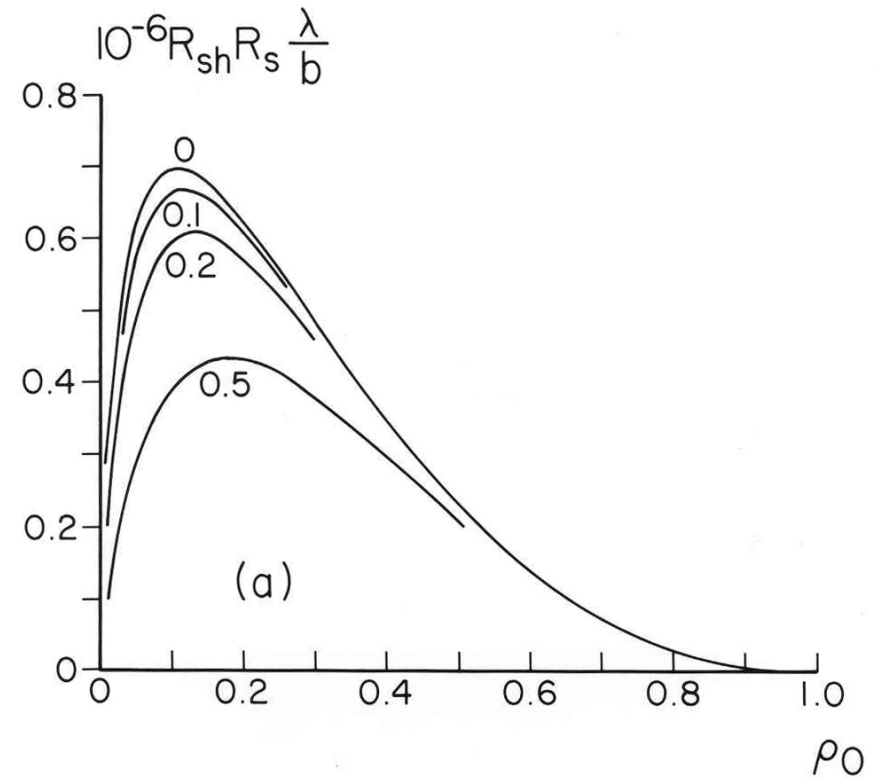
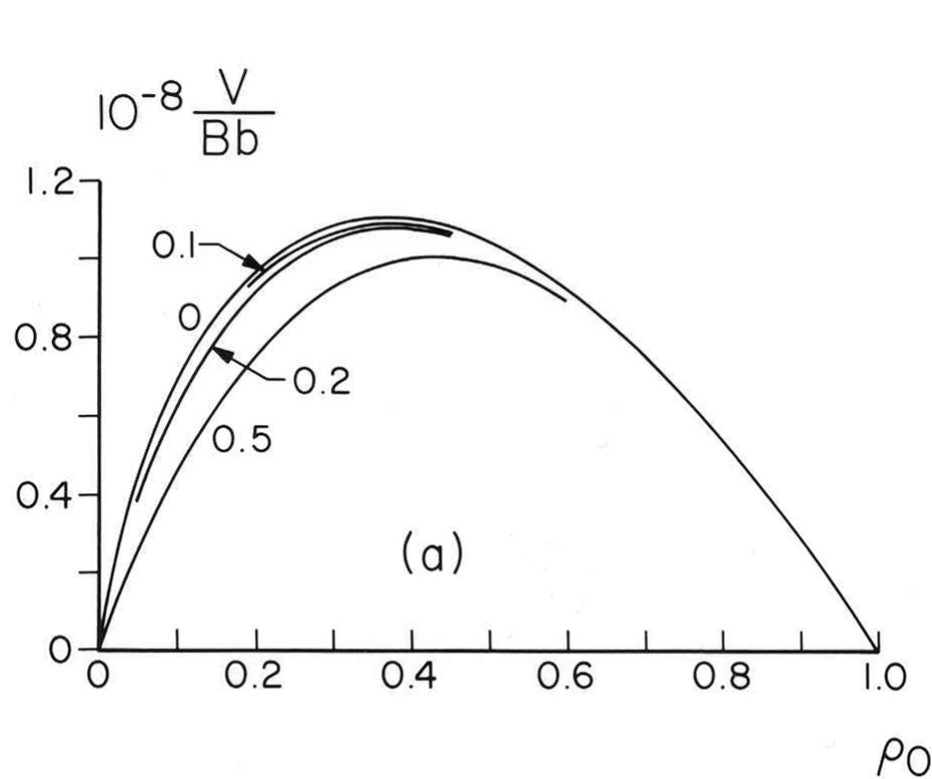
$$\frac{R_{sh}}{Q} \propto \eta$$



Loaded Quarter-wavelength Resonant Line

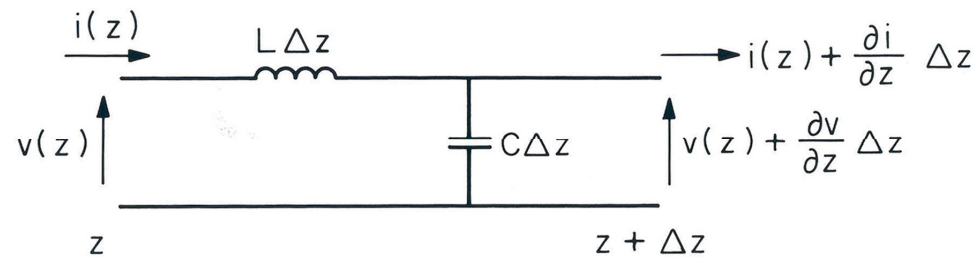


Loaded Quarter-wavelength Resonant Line



MKS units, lines of constant normalized loading capacitance $\Gamma/\lambda\epsilon_0$

More Complicated Center Conductor Geometries



$$\frac{d^2 v}{d\zeta^2} - \frac{1}{\rho \ln \rho} \frac{d\rho}{d\zeta} \frac{dv}{d\zeta} + \frac{\pi^2}{4} v = 0$$

$$\frac{d^2 i}{d\zeta^2} + \frac{1}{\rho \ln \rho} \frac{d\rho}{d\zeta} \frac{di}{d\zeta} + \frac{\pi^2}{4} i = 0$$

$$\Gamma(z) = -C(z) \frac{i(z)}{di/dz}$$

More Complicated Center Conductor Geometries

Constant logarithmic derivative of line capacitance

Good model for linear taper

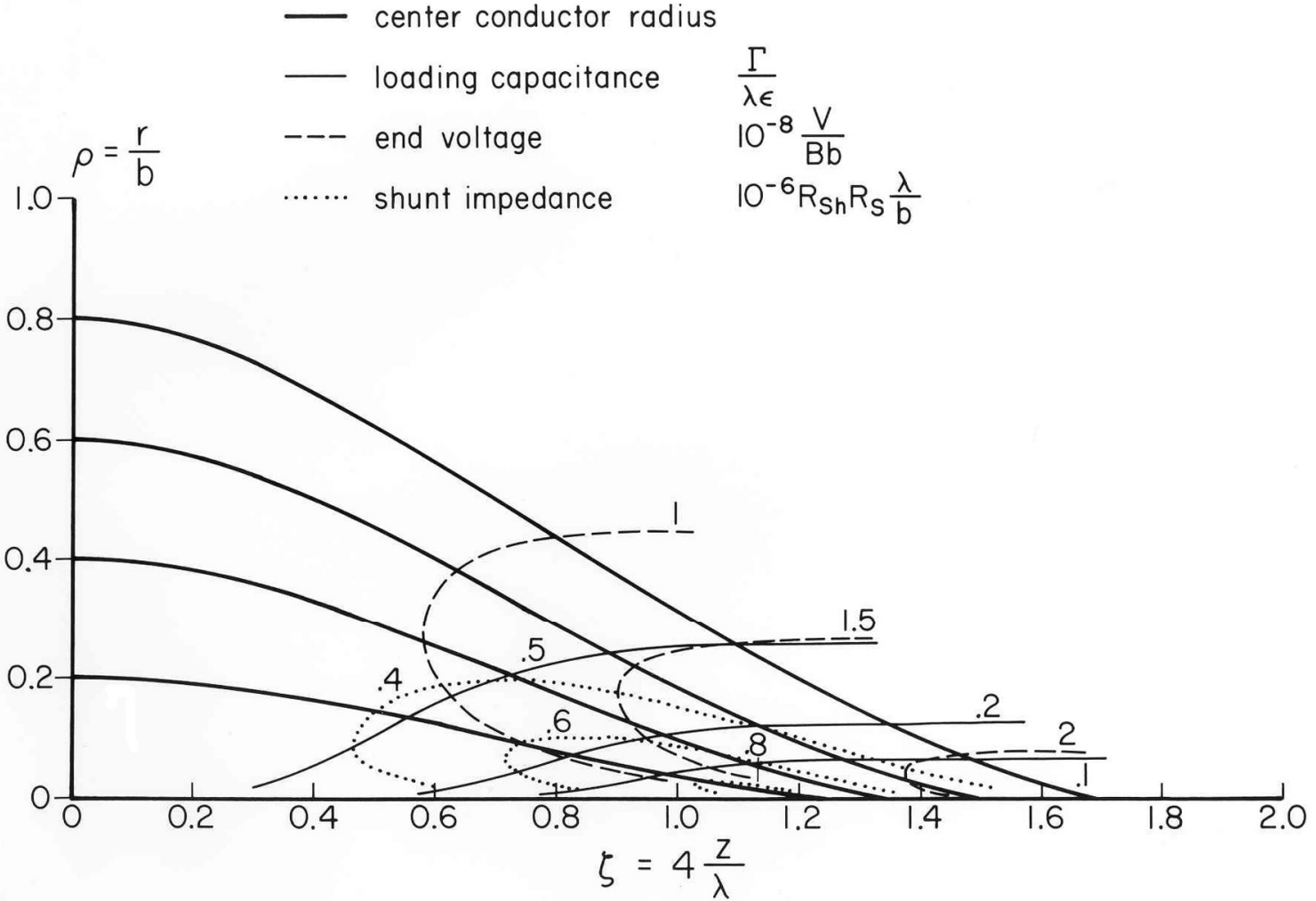
$$\frac{1}{C} \frac{dC}{dz} = -\frac{1}{d} \quad r(z) = b \left(\frac{r_0}{b} \right)^{\exp(z/d)}$$

Constant surface magnetic field

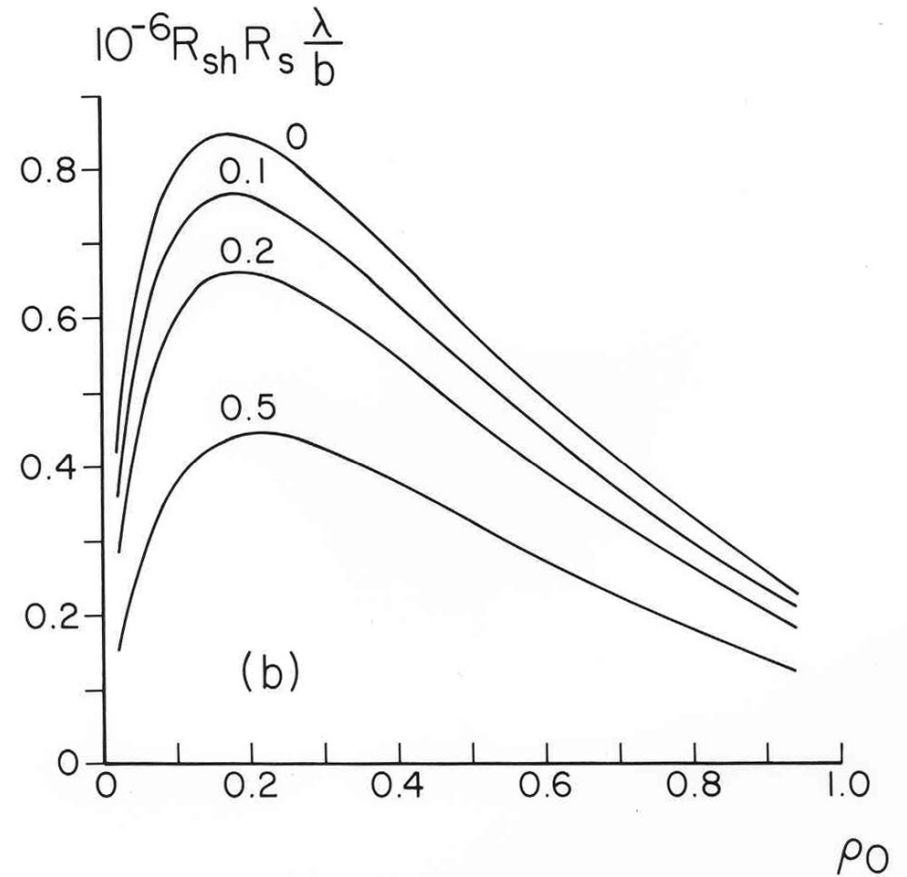
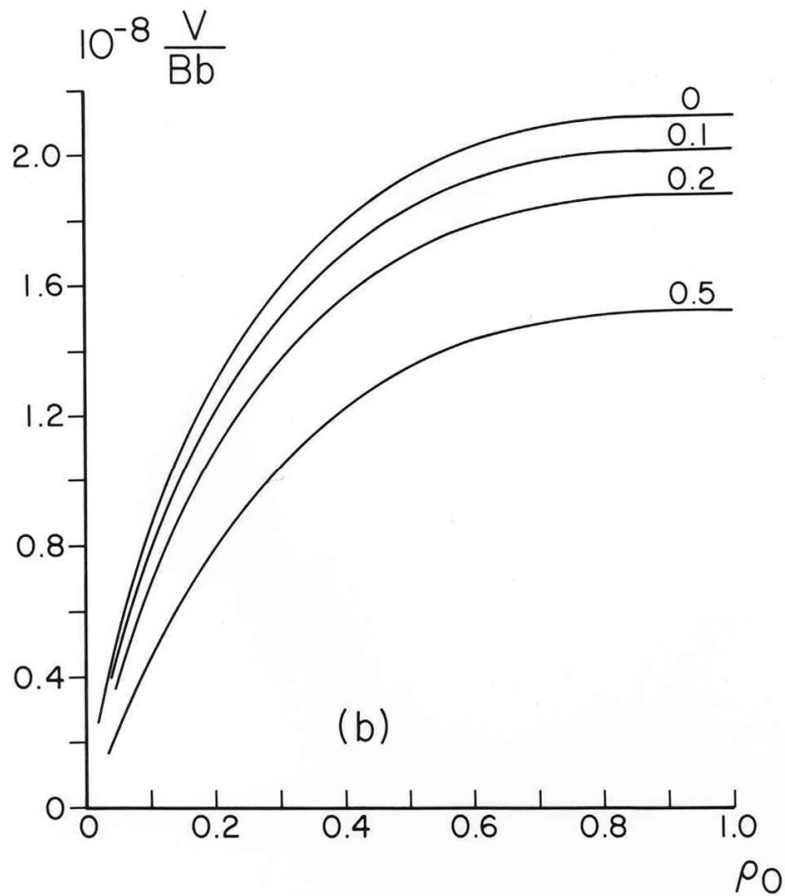
$$i(z) \propto r(z)$$

$$\frac{d^2 r}{dz^2} - \frac{1}{r \ln(b/r)} \left(\frac{dr}{dz} \right)^2 + \frac{4\pi^2}{\lambda^2} r = 0$$

Profile of Constant Surface Magnetic Field

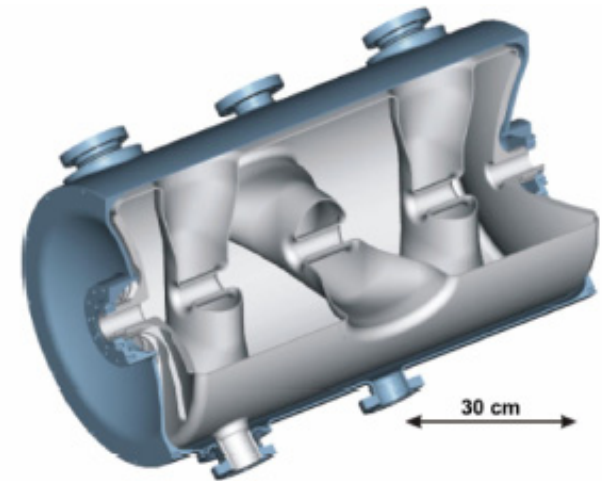
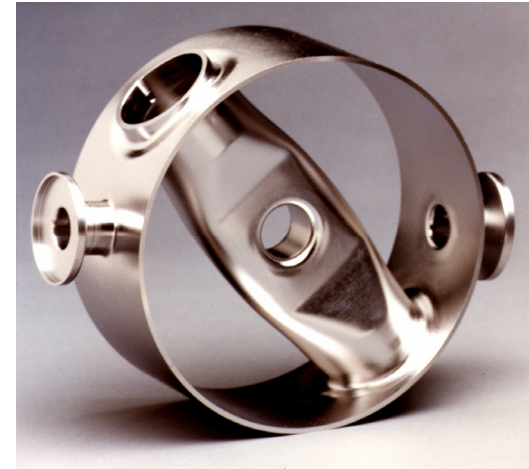
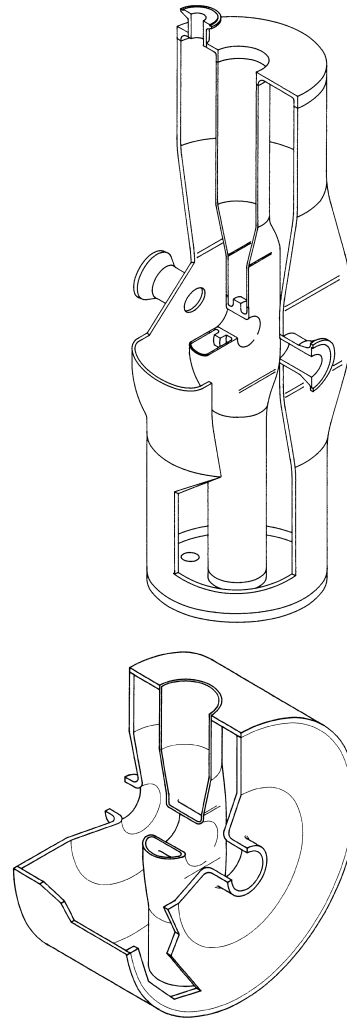
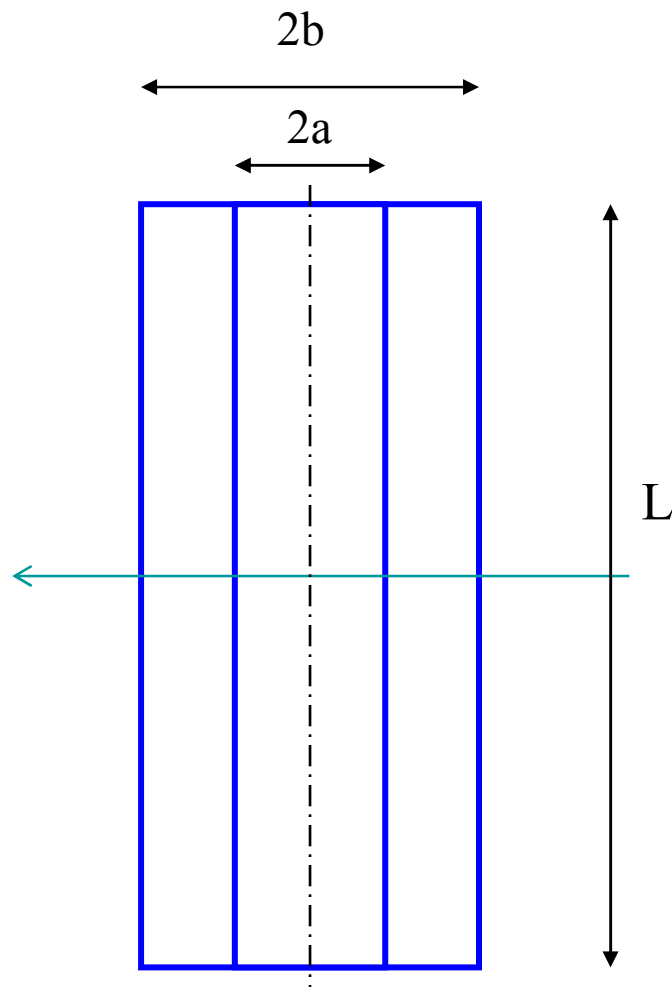


Profile of Constant Surface Magnetic Field



MKS units, lines of constant normalized loading capacitance $\Gamma/\lambda\epsilon_0$

Another Simple Model: Coaxial Half-wave Resonator



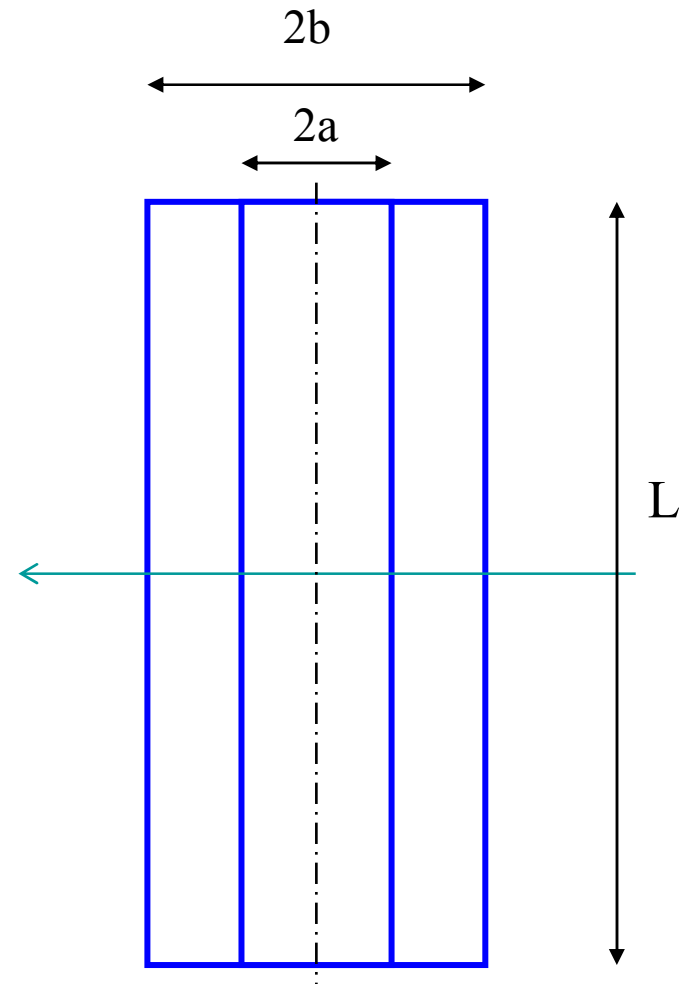
Coaxial Half-wave Resonator

Capacitance per unit length

$$C = \frac{2\pi\epsilon_0}{\ln\left(\frac{b}{a}\right)} = \frac{2\pi\epsilon_0}{\ln\left(\frac{1}{\rho_0}\right)}$$

Inductance per unit length

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{r_0}\right) = \frac{\mu_0}{2\pi} \ln\left(\frac{1}{\rho_0}\right)$$



Coaxial Half-wave Resonator

Center conductor voltage

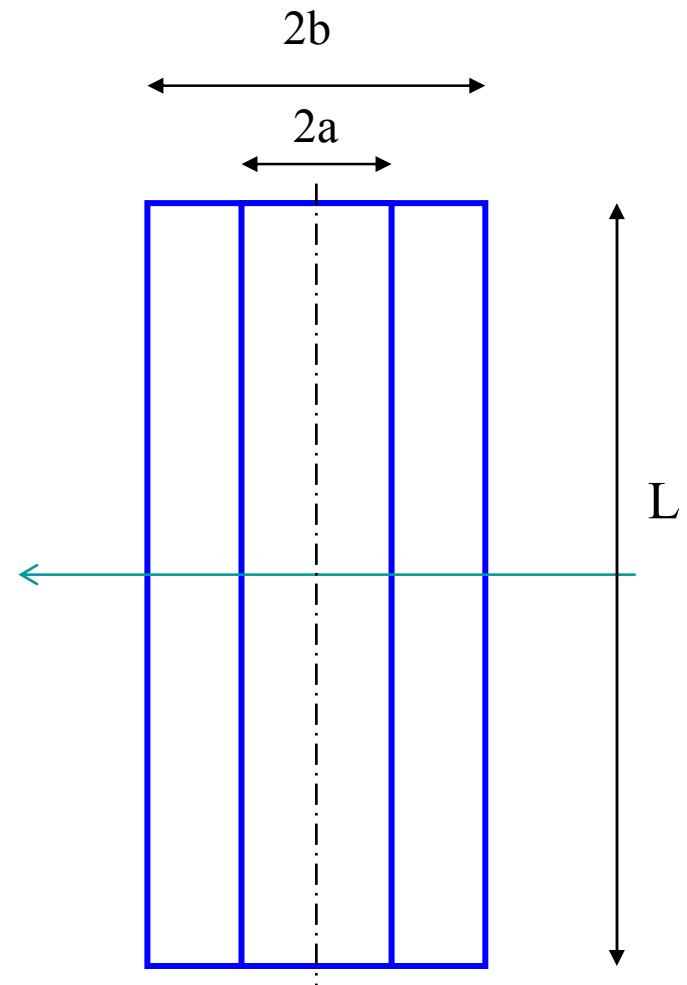
$$V(z) = V_0 \cos\left(\frac{2\pi}{\lambda} z\right)$$

Center conductor current

$$I(z) = I_0 \sin\left(\frac{2\pi}{\lambda} z\right)$$

Line impedance

$$Z_0 = \frac{V_0}{I_0} = \frac{\eta}{2\pi} \ln\left(\frac{1}{\rho_0}\right), \quad \eta = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\Omega$$



Coaxial Half-wave Resonator

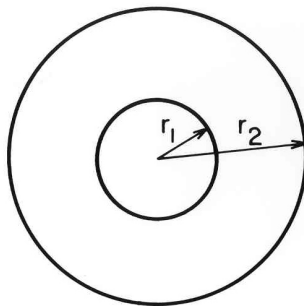
Peak Electric Field

d: coaxial cylinders

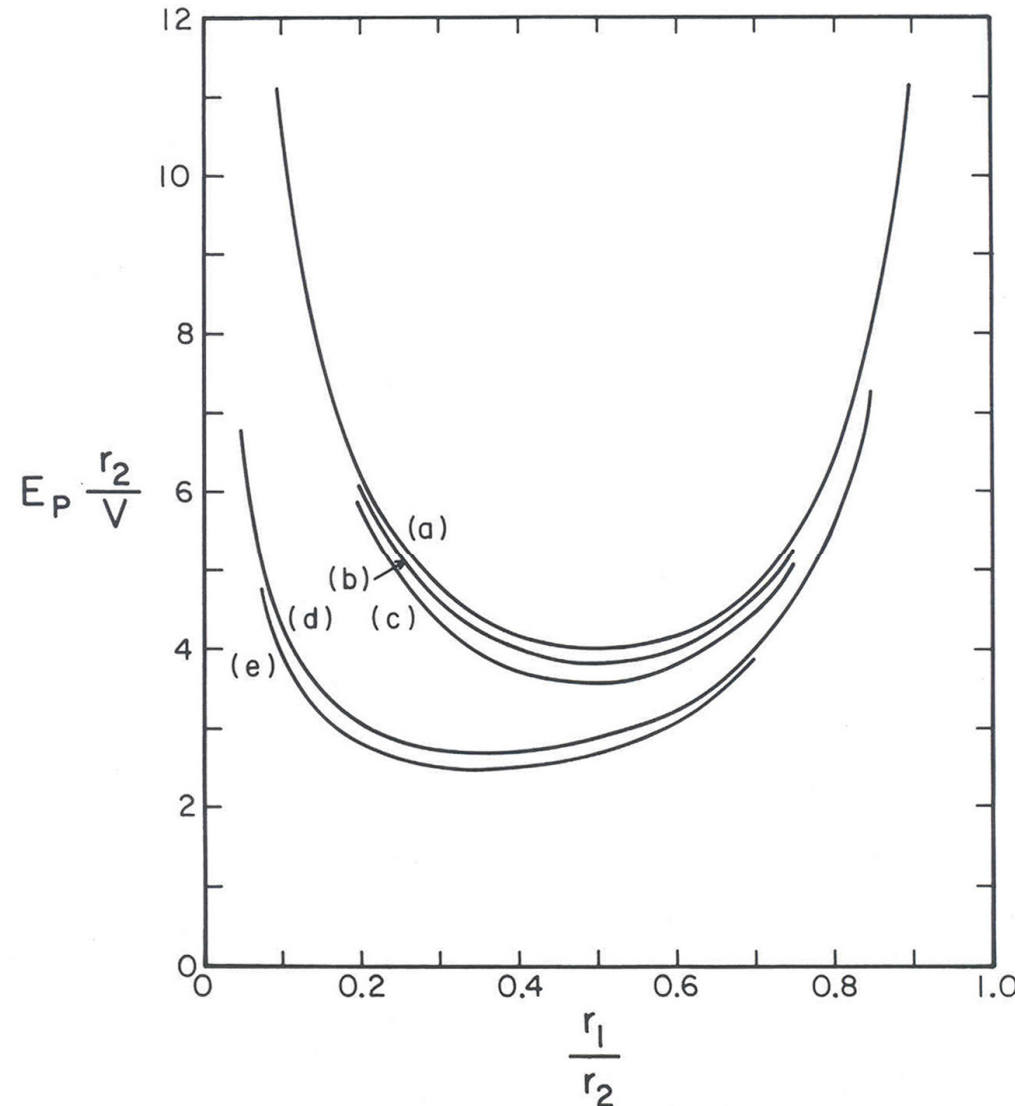
V_p : Voltage on center conductor

Outer conductor at ground

E_p : Peak field on center conductor



(a), (b), (d)



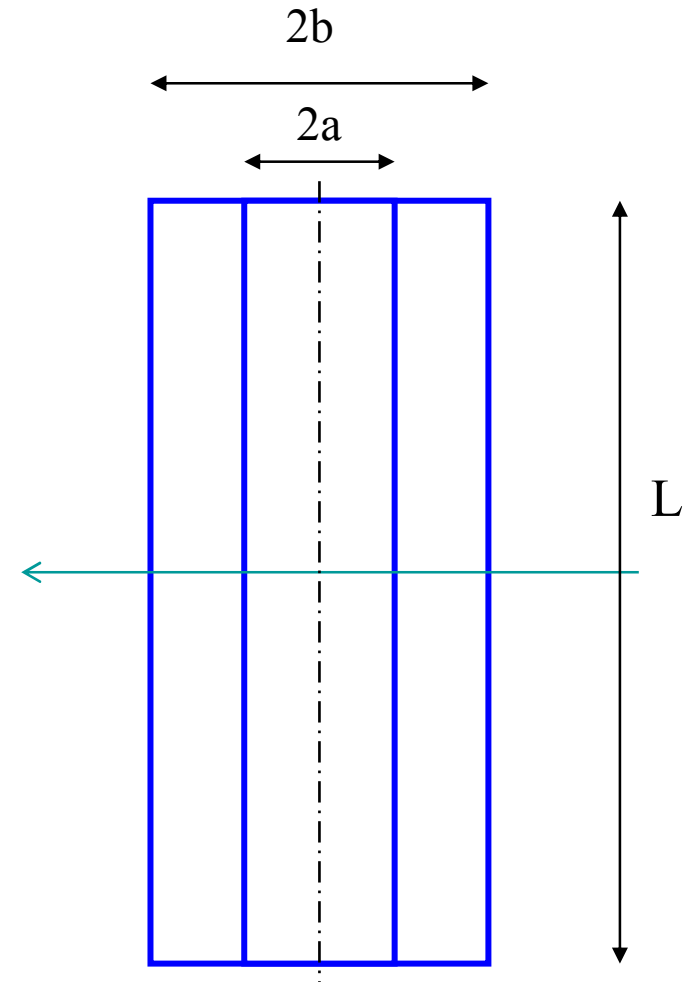
Coaxial Half-wave Resonator

Peak magnetic field

$$\frac{V_p}{b} = \begin{Bmatrix} \eta & H \\ c & B \\ 300 & B \end{Bmatrix} \rho_0 \ln\left(\frac{1}{\rho_0}\right) \quad \begin{Bmatrix} \text{m, A/m} \\ \text{m, T} \\ \text{cm, G} \end{Bmatrix}$$

V_p : Voltage across loading capacitance

$B \approx 9 \text{ mT}$ at 1 MV/m

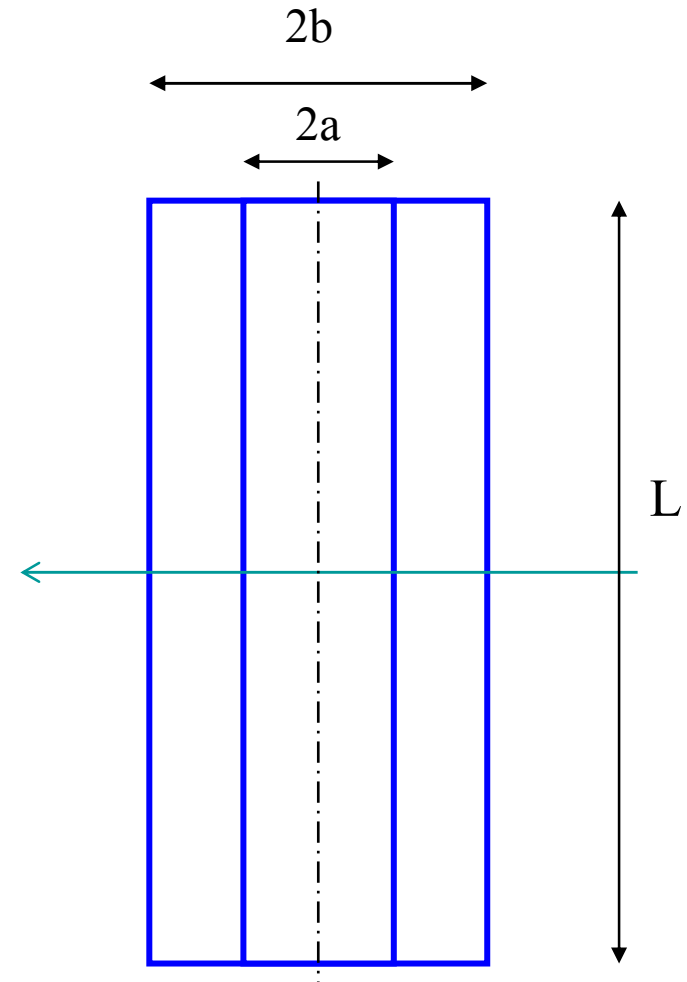


Coaxial Half-wave Resonator

Power dissipation (ignore losses in the shorting plate)

$$P = V_p^2 \frac{16}{\pi} \frac{R_s}{\eta^2} \frac{\lambda}{b} \frac{1 + 1/\rho_0}{\ln^2 \rho_0}$$

$$P \propto \frac{R_s}{\eta^2} E^2 \beta \lambda^2$$

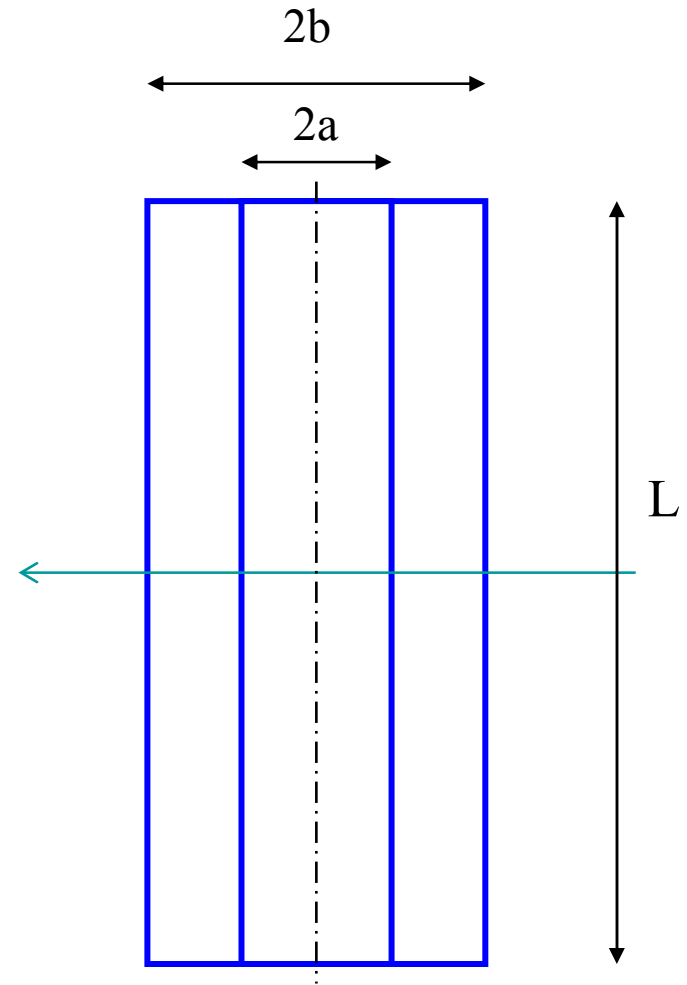


Coaxial Half-wave Resonator

Energy content

$$U = V_p^2 \frac{\pi \epsilon_0}{4} \lambda \frac{1}{\ln(1/\rho_0)}$$

$$U \propto \epsilon_0 E^2 \beta^2 \lambda^3$$

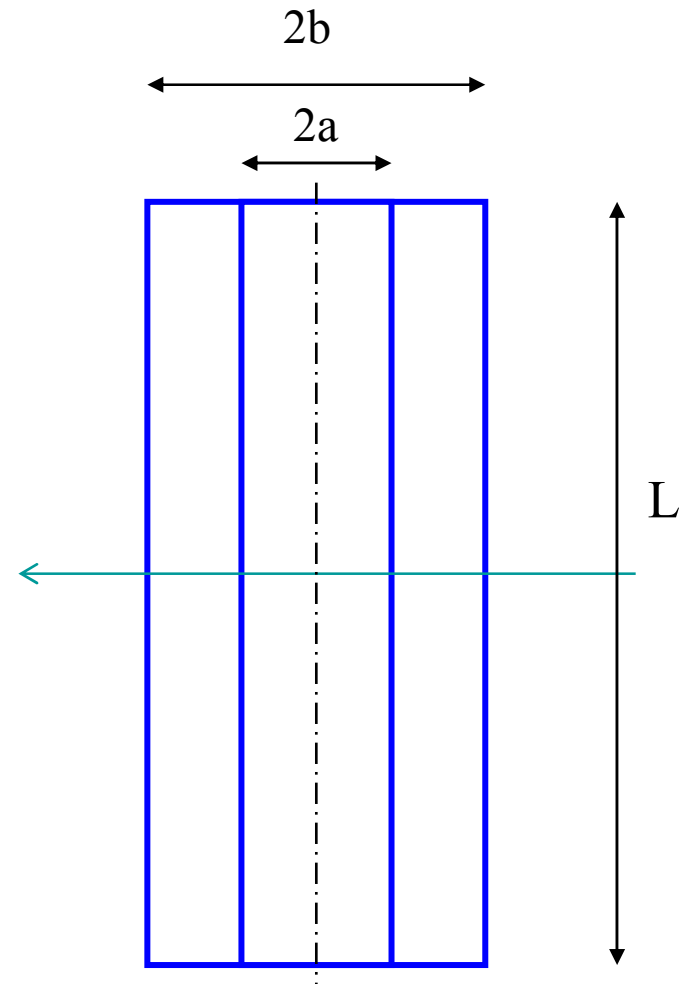


Coaxial Half-wave Resonator

Geometrical factor

$$G = QR_s = 2\pi \eta \frac{b \ln(1/\rho_0)}{\lambda (1+1/\rho_0)}$$

$$G \propto \eta \beta$$

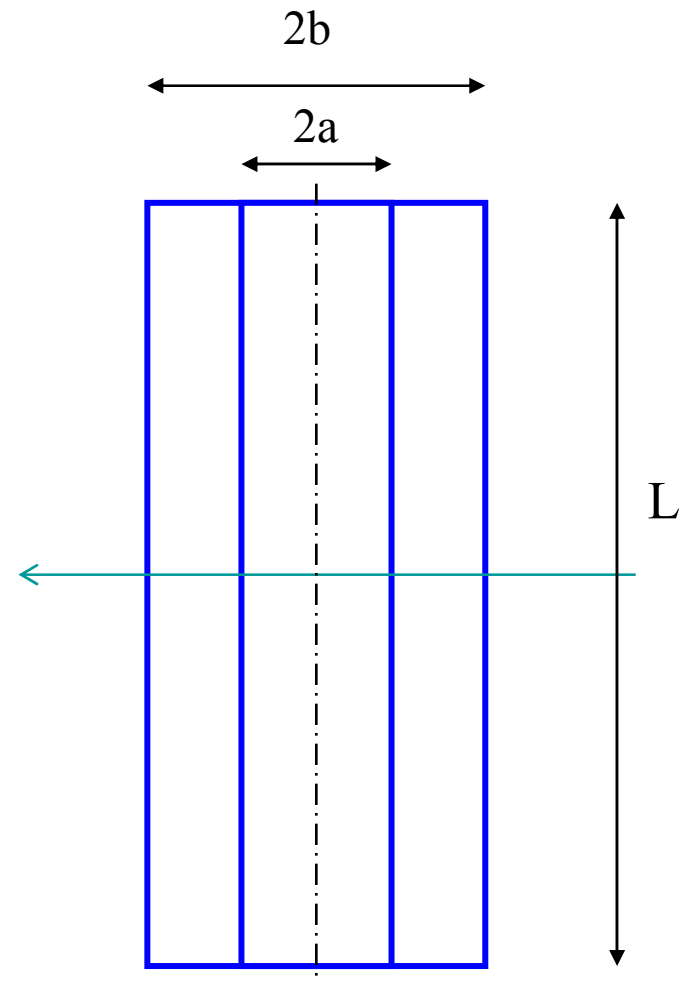


Coaxial Half-wave Resonator

Shunt impedance $(4V_p^2 / P)$

$$R_{sh} = \frac{\eta^2}{R_s} \frac{16}{\pi} \frac{b}{\lambda} \frac{\ln^2 \rho_0}{1 + 1/\rho_0}$$

$$R_{sh} R_s \propto \eta^2 \beta$$

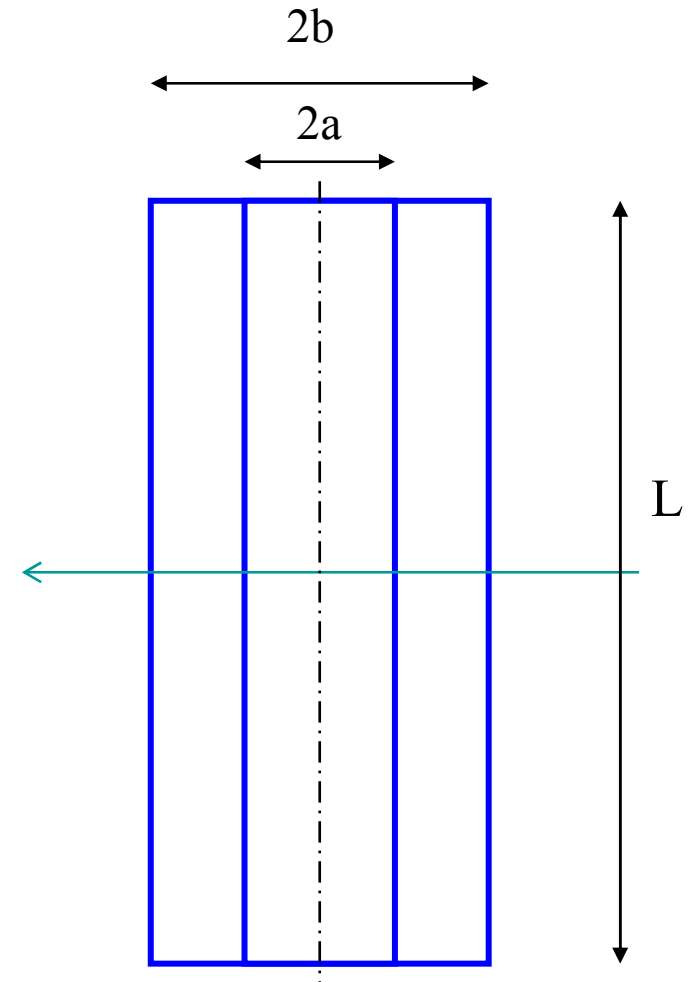


Coaxial Half-wave Resonator

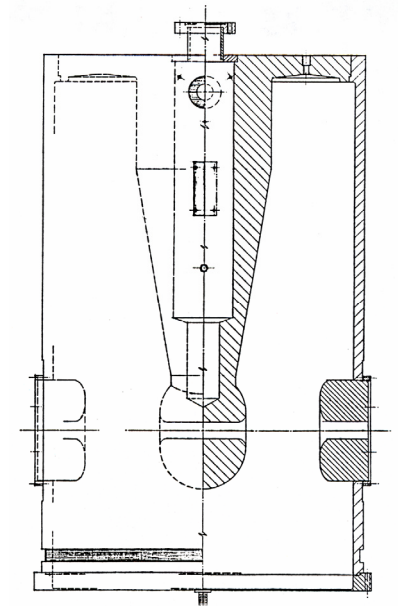
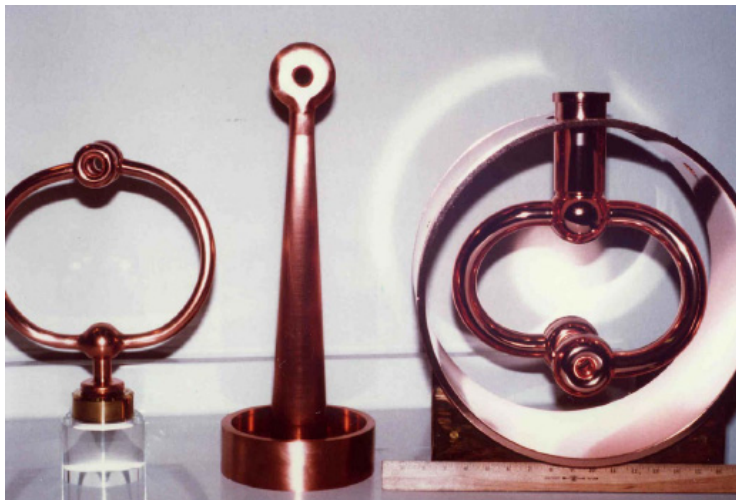
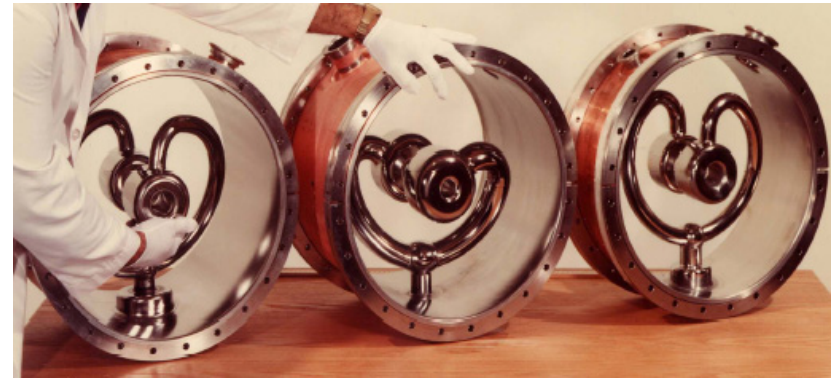
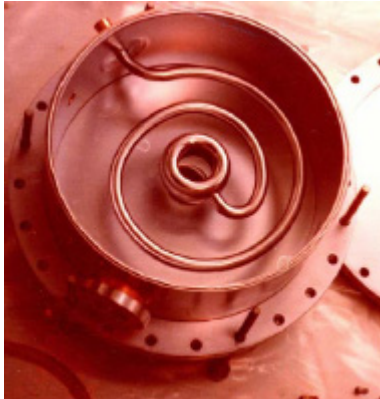
R/Q

$$\frac{R_{sh}}{Q} = \frac{8}{\pi^2} \eta \ln(1/\rho_0)$$

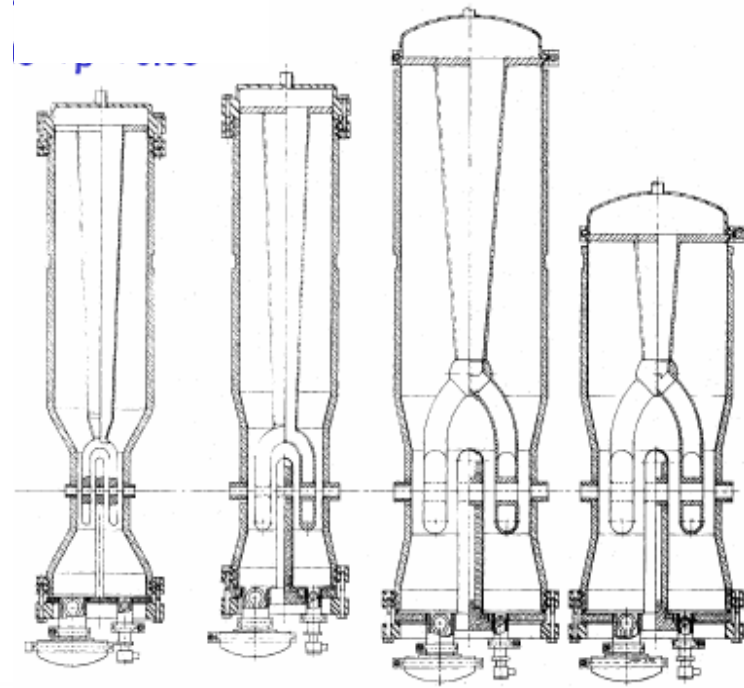
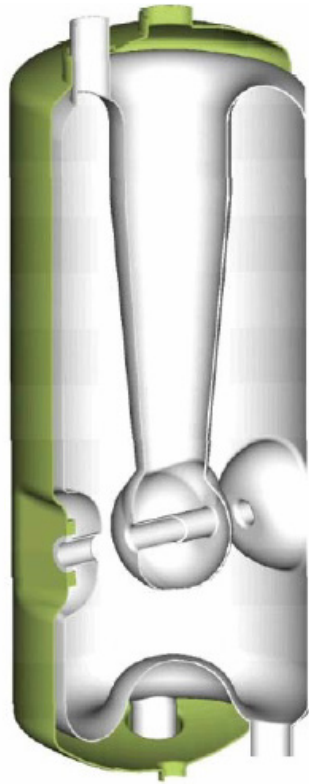
$$\frac{R_{sh}}{Q} \propto \eta$$



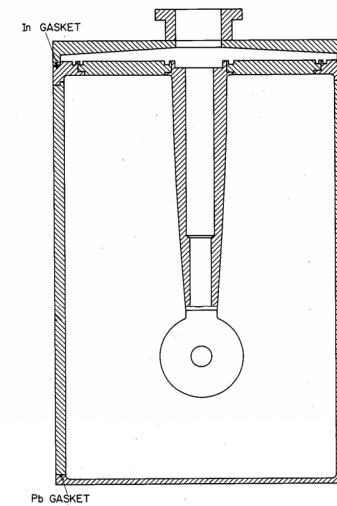
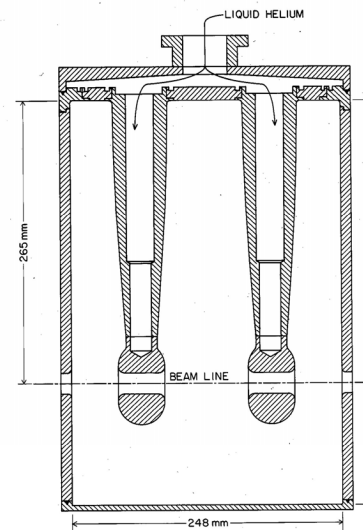
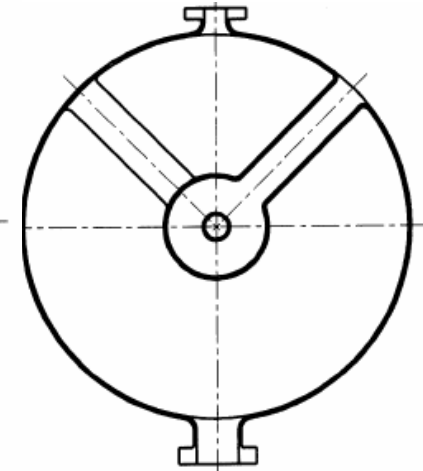
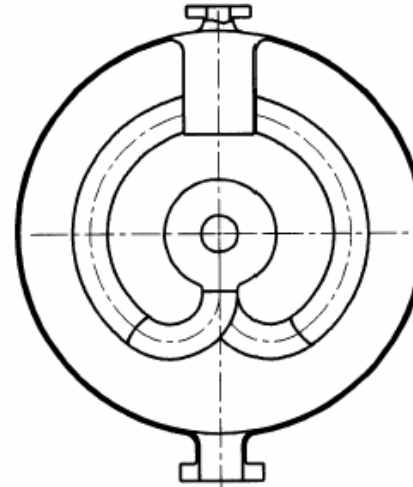
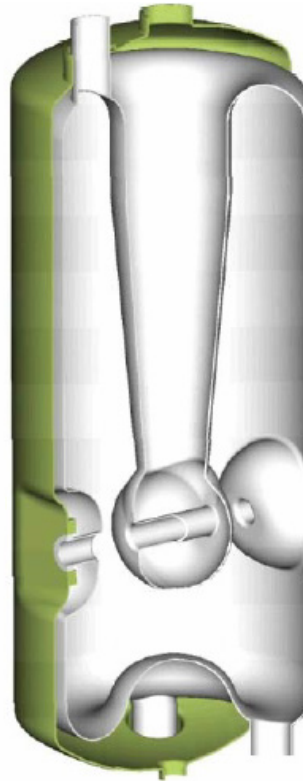
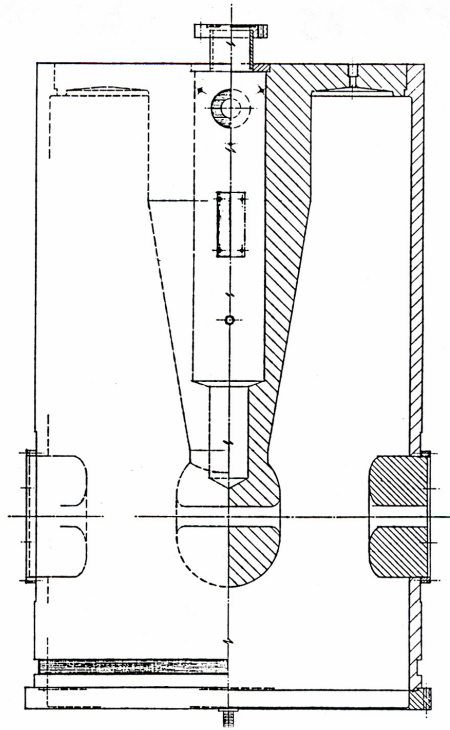
Some Real Geometries ($\lambda/4$)



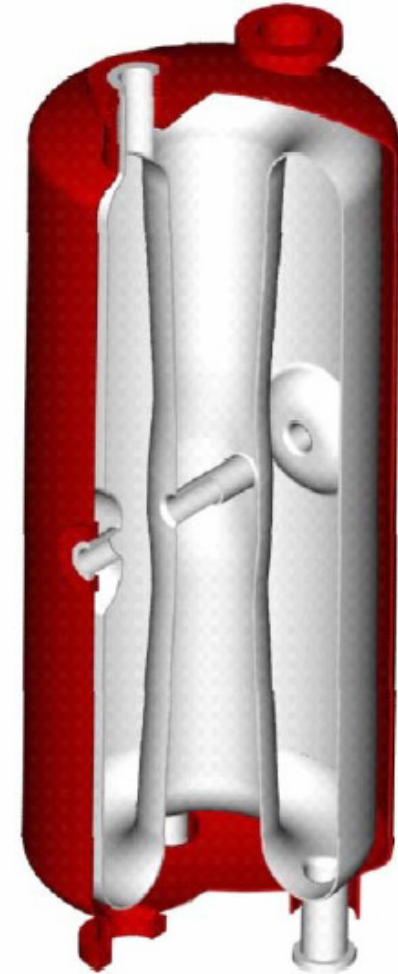
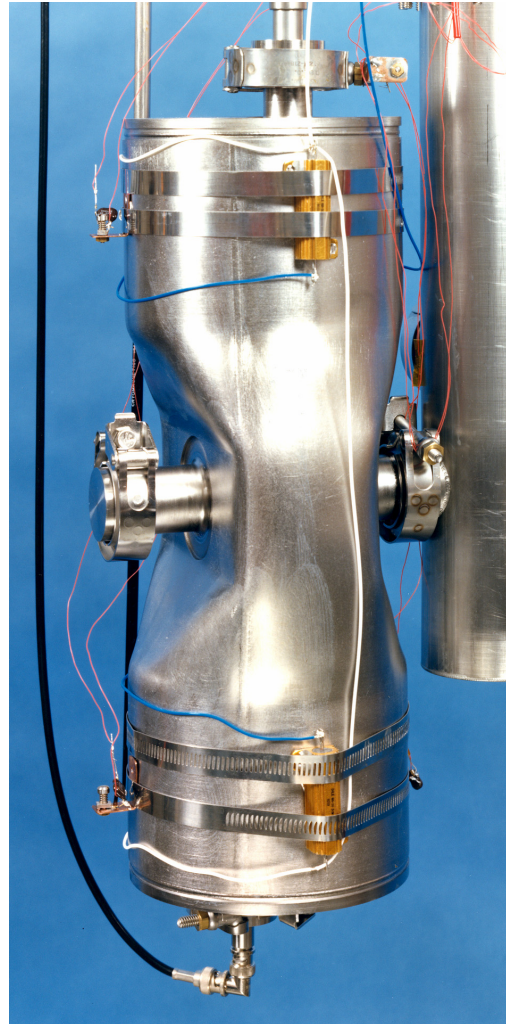
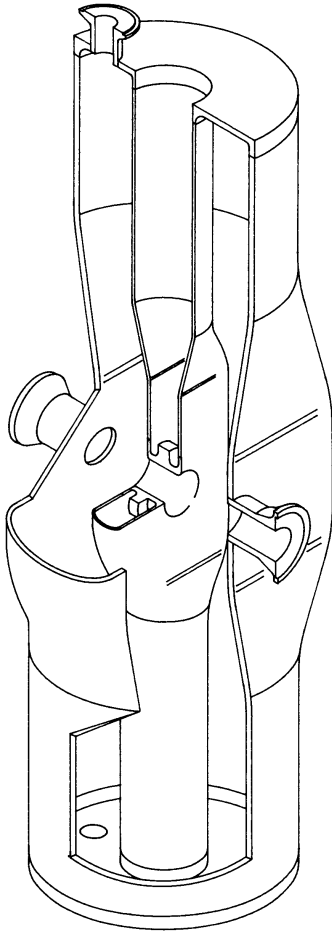
Some Real Geometries ($\lambda/4$)



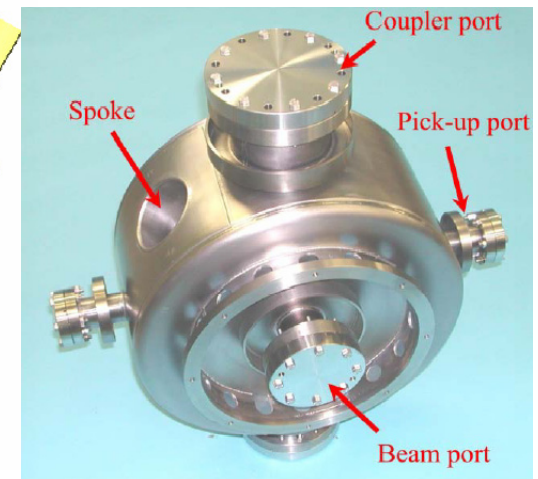
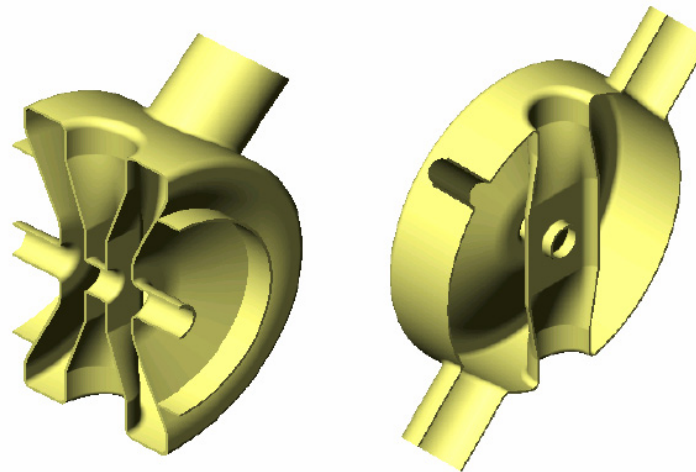
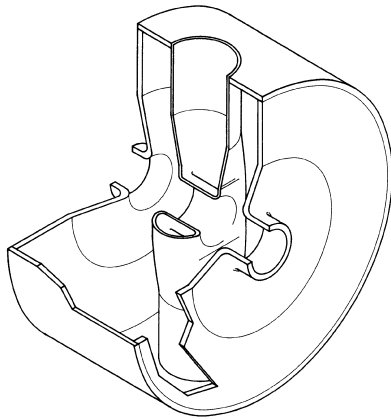
$\lambda/4$ Resonant Lines



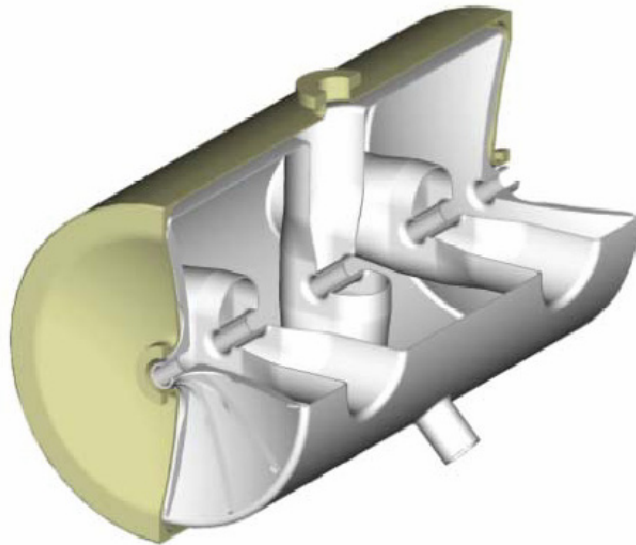
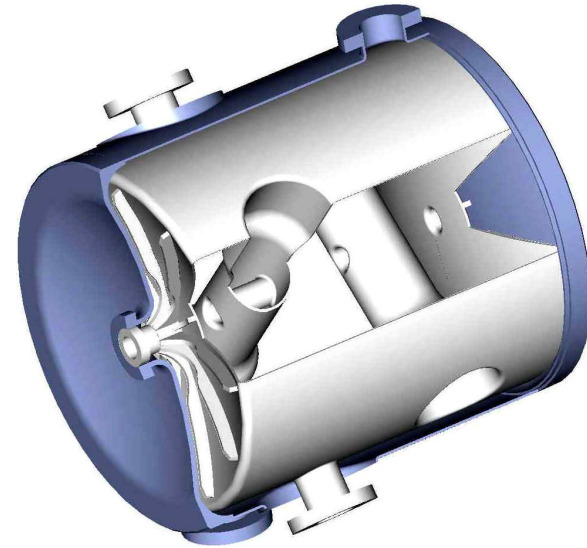
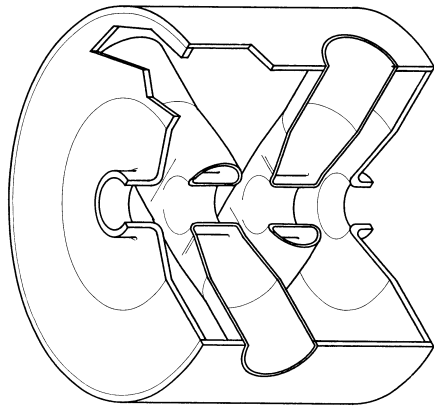
$\lambda/2$ Resonant Lines



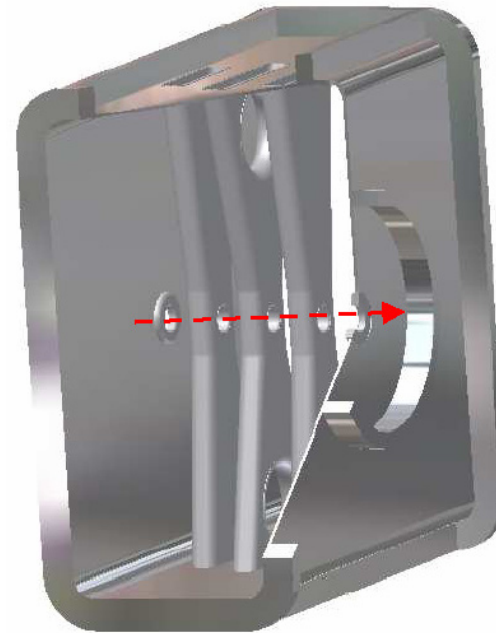
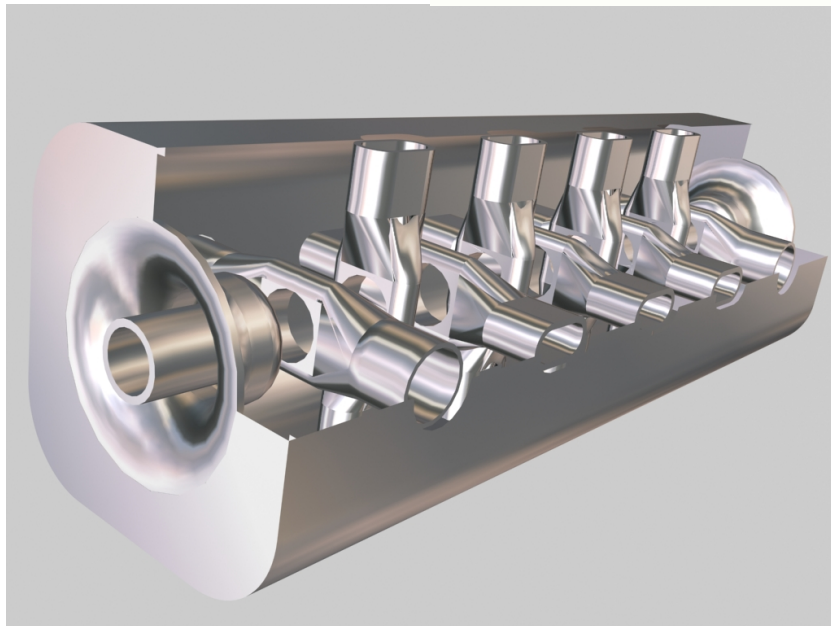
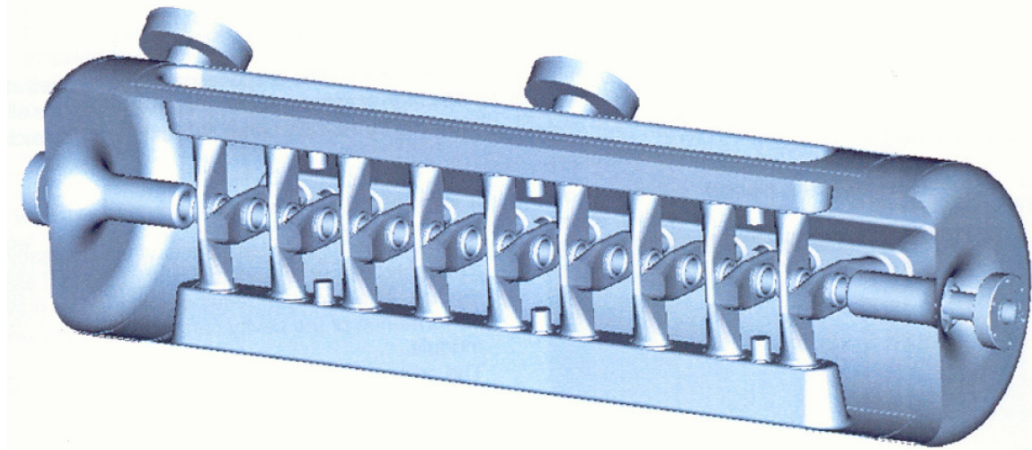
$\lambda/2$ Resonant Lines – Single-Spoke



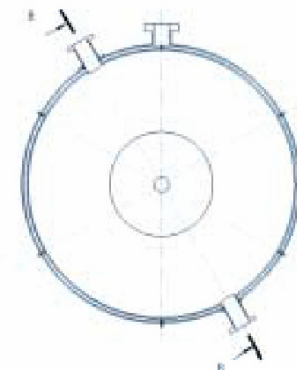
$\lambda/2$ Resonant Lines – Double and Triple-Spoke



$\lambda/2$ Resonant Lines – Multi-Spoke



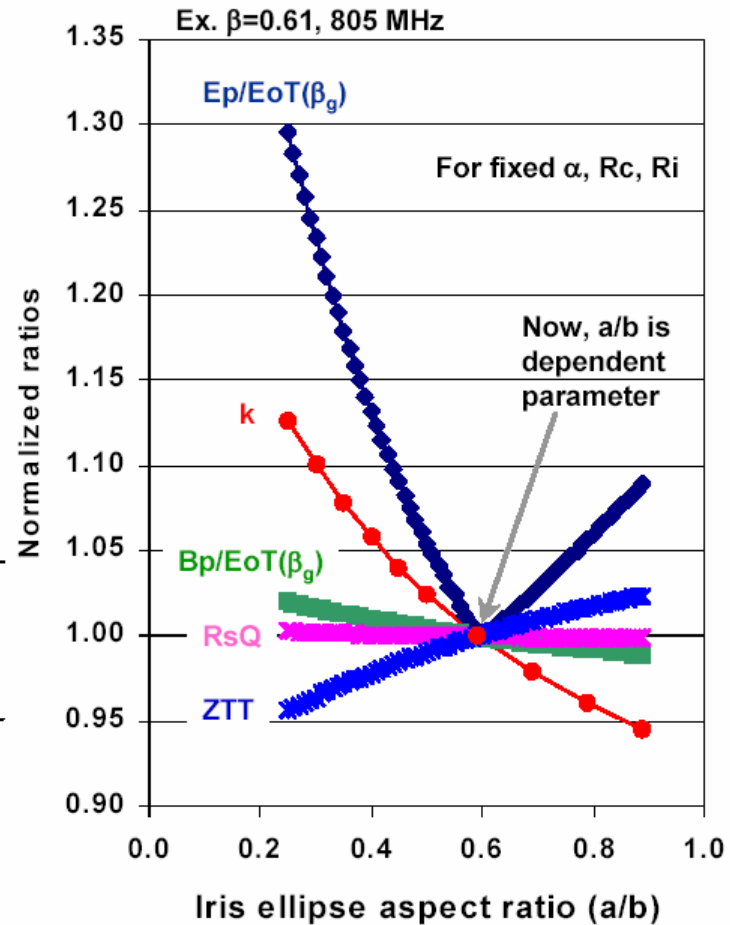
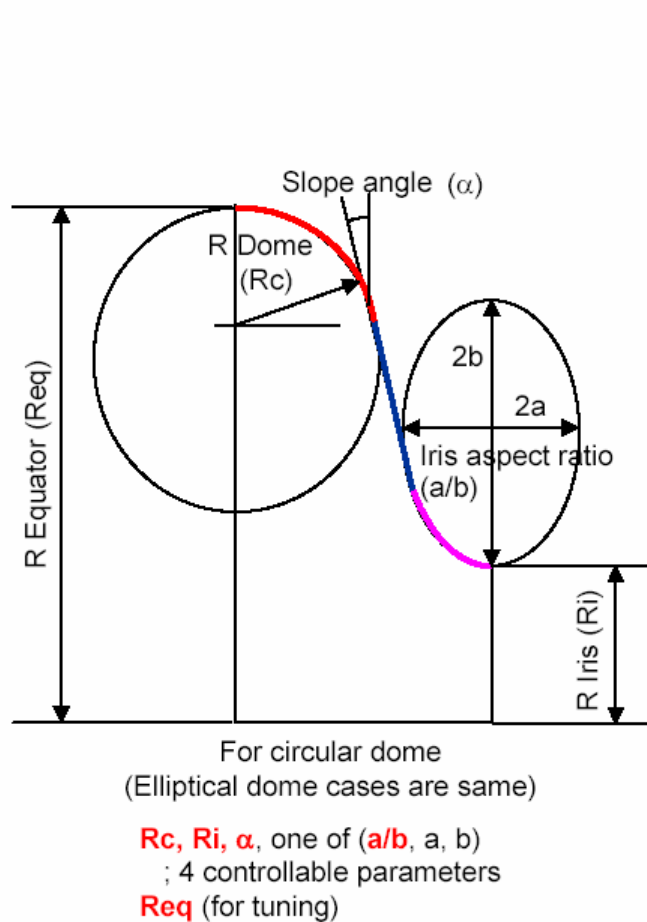
TM Modes



Design Considerations

- Minimize the **peak surface fields**
Bp; approaches to theoretical limit (190 mT)
← high RRR, defect control, better surface treatment (~170 mT)
Ep; fields exceed 80 MV/m ← improved surface cleaning tech.
- Reasonable **Inter-cell coupling** between cells in Elliptical cavity
- Spoke cavity intrinsically has big coupling constant
- Provide required **external Q**
- In CW, **higher shunt impedance** (mainly determined by the cavity type)
- Reasonable **mechanical stiffness**
common; reasonable tuning force, mechanical stability under vacuum pressure (test~2 atm), stable against microphonics pulsed; affordable dynamic Lorentz force detuning
- Safe from **Multipacting**
- Verify **HOM** and related issues
- **Coupled field problems** are common between RF, mechanical, thermal..
→ strong interfaces are needed

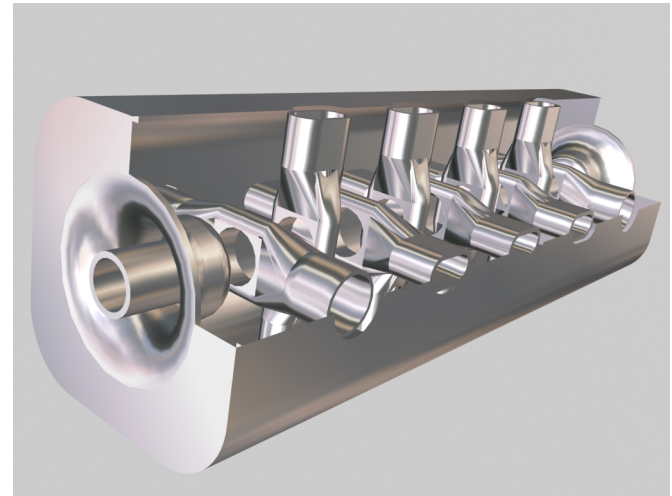
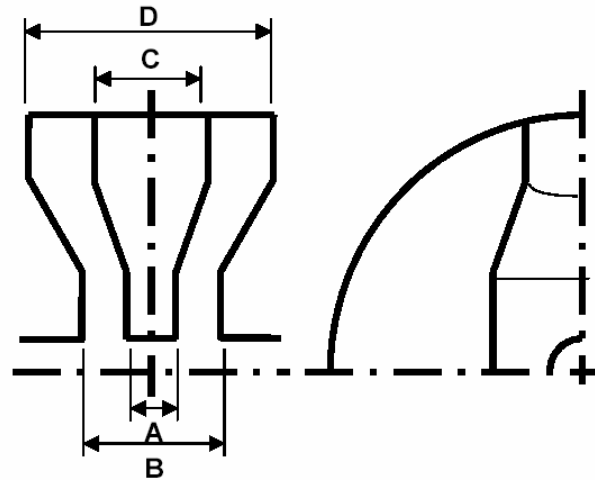
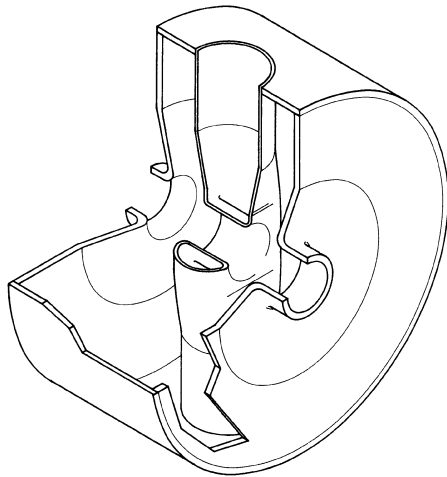
RF Geometry Optimization (elliptical cavity)



Elliptical cell geometry and dependencies of RF parameters on the ellipse aspect ratio (a/b) at the fixed slope angle, dome radius and bore radius.

RF Geometry Optimization (Spoke Cavity)

- There have been extensive efforts for design optimization especially to reduce the ratios of E_p/E_{acc} and B_p/E_{acc} .
 - Controlling A/B (E_p/E_{acc}) and C/D (B_p/E_{acc}) → **Shape optimization**
 - Flat contacting surface at spoke base will help in another minimization of B_p/E_{acc}
 - **For these cavities:**
 - Calculations agree well → $E_p/E_{acc} \sim 3$, $B_p/E_{acc} \sim (7 \sim 8)$ mT/(MV/m), though it is tricky to obtain precise surface field information from the 3D simulation.
 - Intrinsically have very strong RF coupling in multi-gap cavity.**
 - Have rigid nature against static and dynamic vibrations.**
 - Beta dependency is quite small.**
 - Diameter ~ half of elliptical cavity.**



Velocity Acceptance

- **Energy gain** $\Delta W = q V T(x) \Phi(x) \cos \varphi$

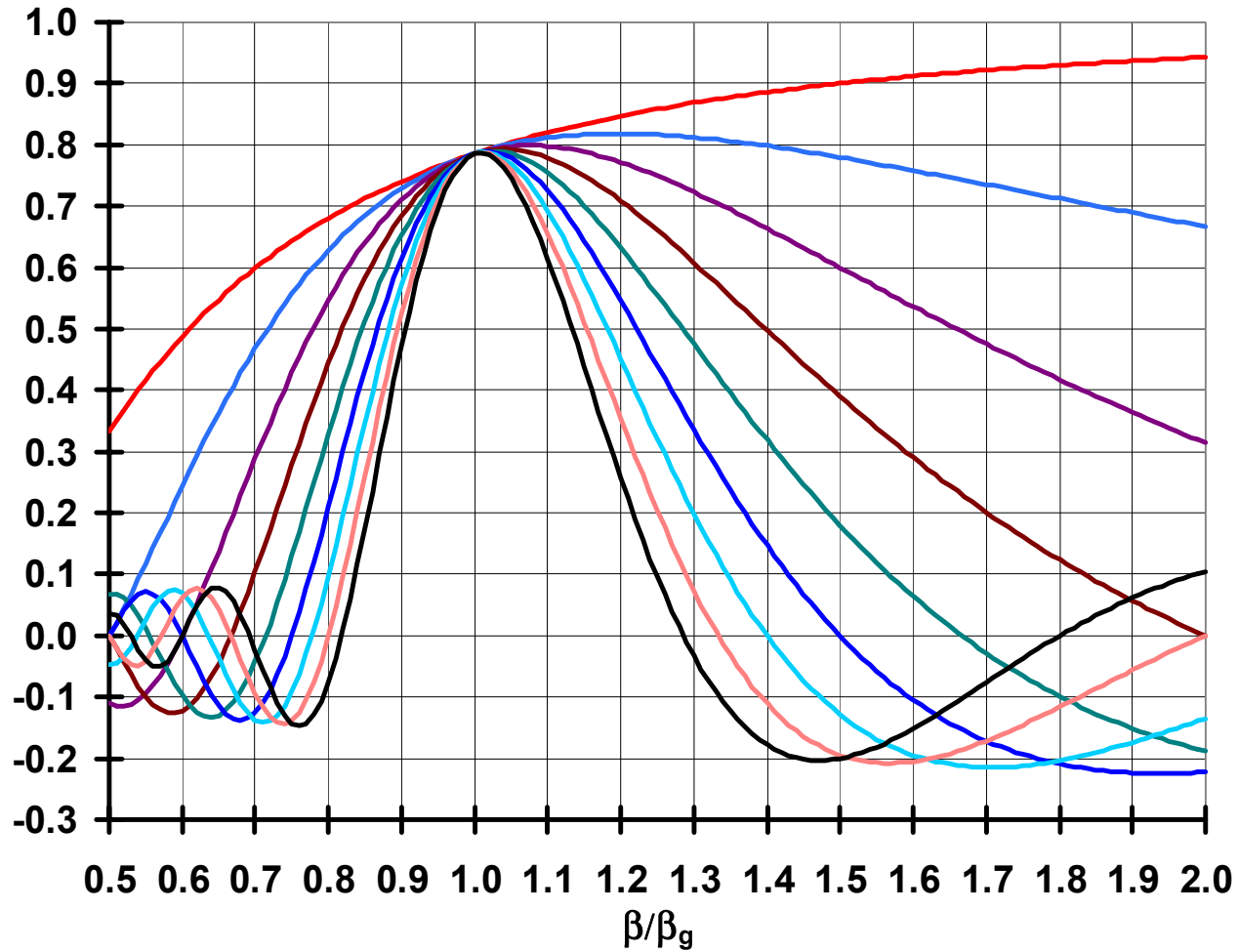
$$x = \frac{\beta \lambda}{2l}$$

$T(x)$ **Transit time factor for single cell**
Depends on field profile in cell

$\Phi(x)$ **Phasing factor in multicell cavities**
Depends on cell spacing and field amplitude in cells
Does not depend on field profile in cells (assumed to be identical)

Velocity Acceptance

Velocity Acceptance for Sinusoidal Field Profile



Voltage in Cells

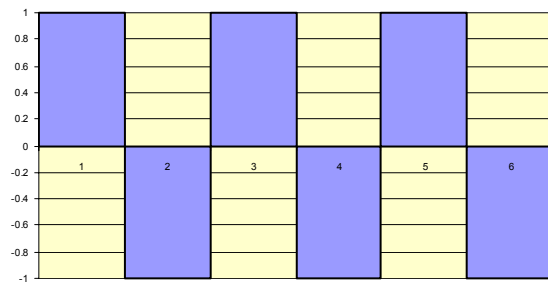
Voltage in j^{th} cell

$$V_j^M = \sin\left(\pi M \frac{(2j-1)}{2N}\right)$$

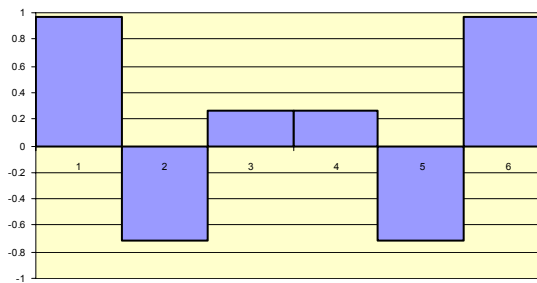
N: Number of cells,

M: Mode number

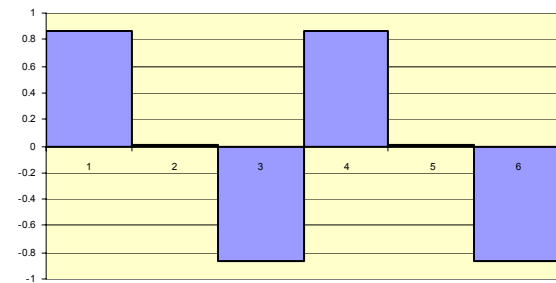
6 Cell, Mode 6



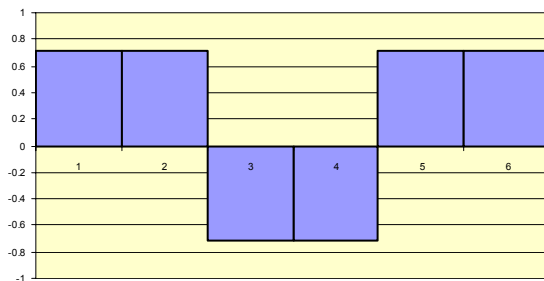
6 Cell, Mode 5



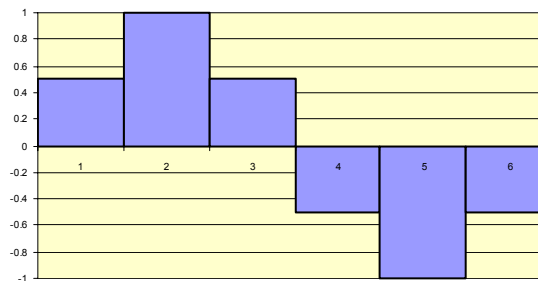
6 Cell, Mode 4



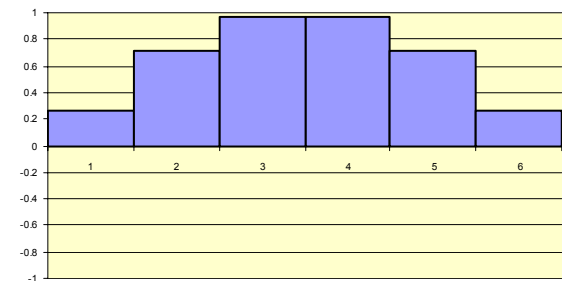
6 Cell, Mode 3



6 Cell, Mode 2



6 Cell, Mode 1



Phasing Factor

For fundamental (π) mode:
$$\Phi(x) = \frac{1}{\cos\left(\frac{\pi}{2x}\right)} \begin{cases} (-1)^{n+1} \sin\left(\frac{N\pi}{2x}\right), & N = 2n \\ (-1)^n \cos\left(\frac{N\pi}{2x}\right), & N = 2n + 1 \end{cases}$$

For all modes:

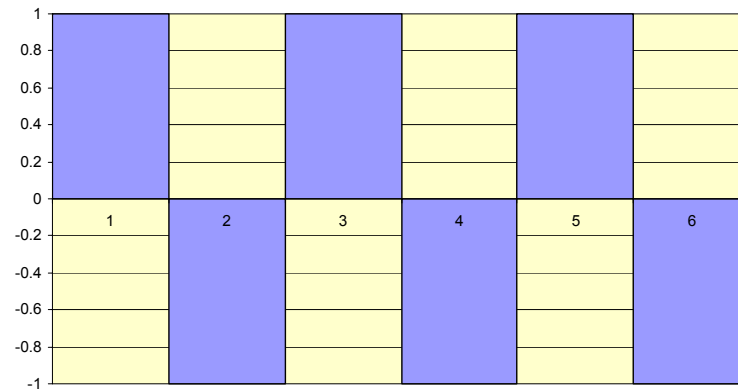
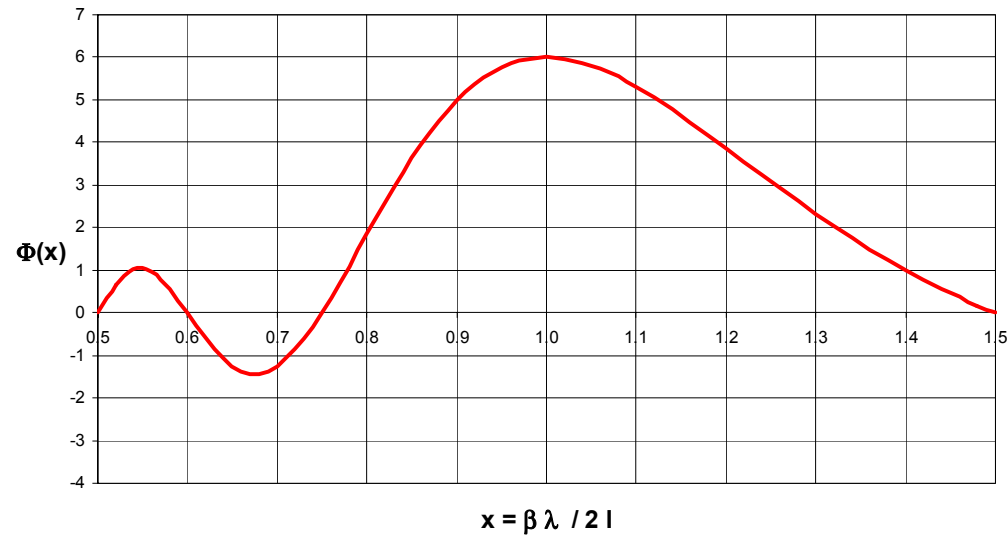
$$\Phi(x) = \frac{1}{2} \left(\frac{\sin\left[\frac{N\pi}{2}\left(\frac{M}{N} - \frac{1}{x}\right)\right]}{\sin\left[\frac{\pi}{2}\left(\frac{M}{N} - \frac{1}{x}\right)\right]} + (-1)^{M+1} \frac{\sin\left[\frac{N\pi}{2}\left(\frac{M}{N} + \frac{1}{x}\right)\right]}{\sin\left[\frac{\pi}{2}\left(\frac{M}{N} + \frac{1}{x}\right)\right]} \right)$$

If $M=N$, recover previous formula

If $x=1$
$$\Phi(x) = N\delta_{MN}$$

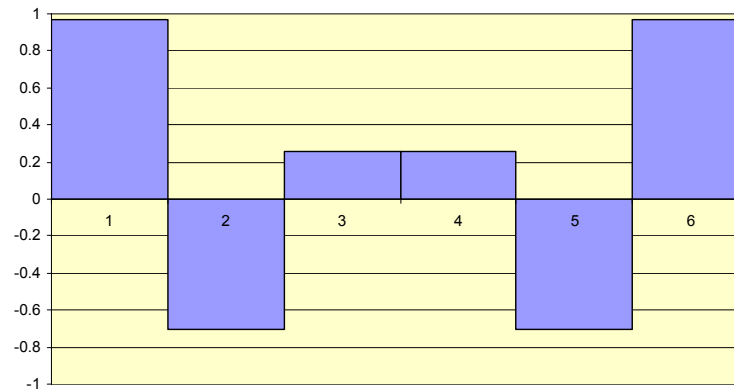
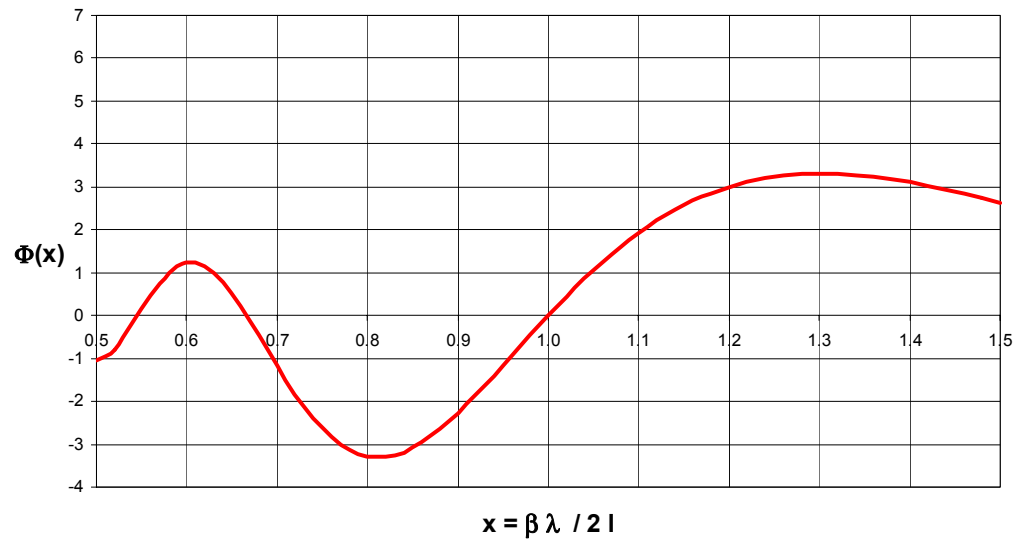
Phasing Factor

6 Cells, Mode 6



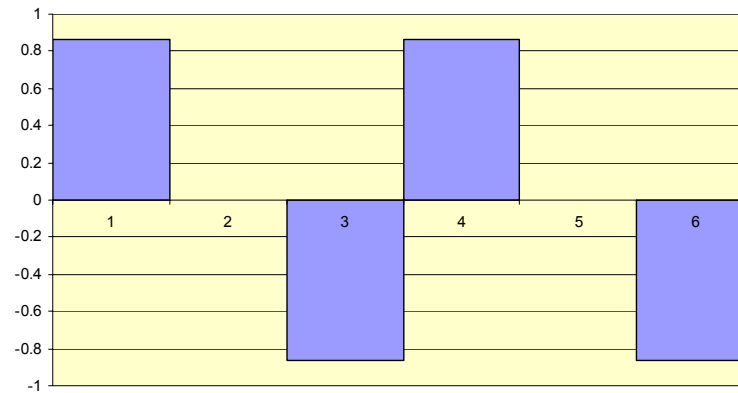
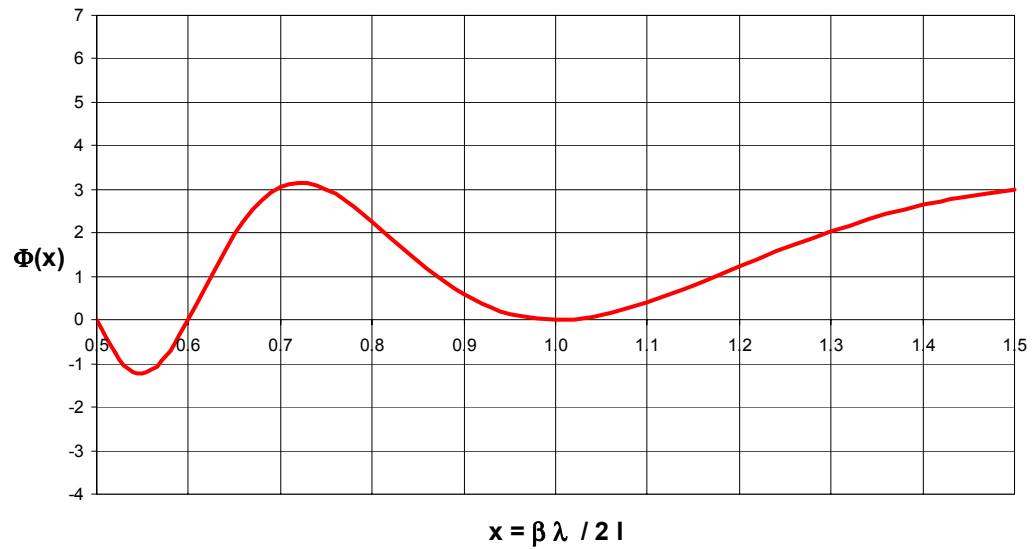
Phasing Factor

6 Cells, Mode 5



Phasing Factor

6 Cells, Mode 4



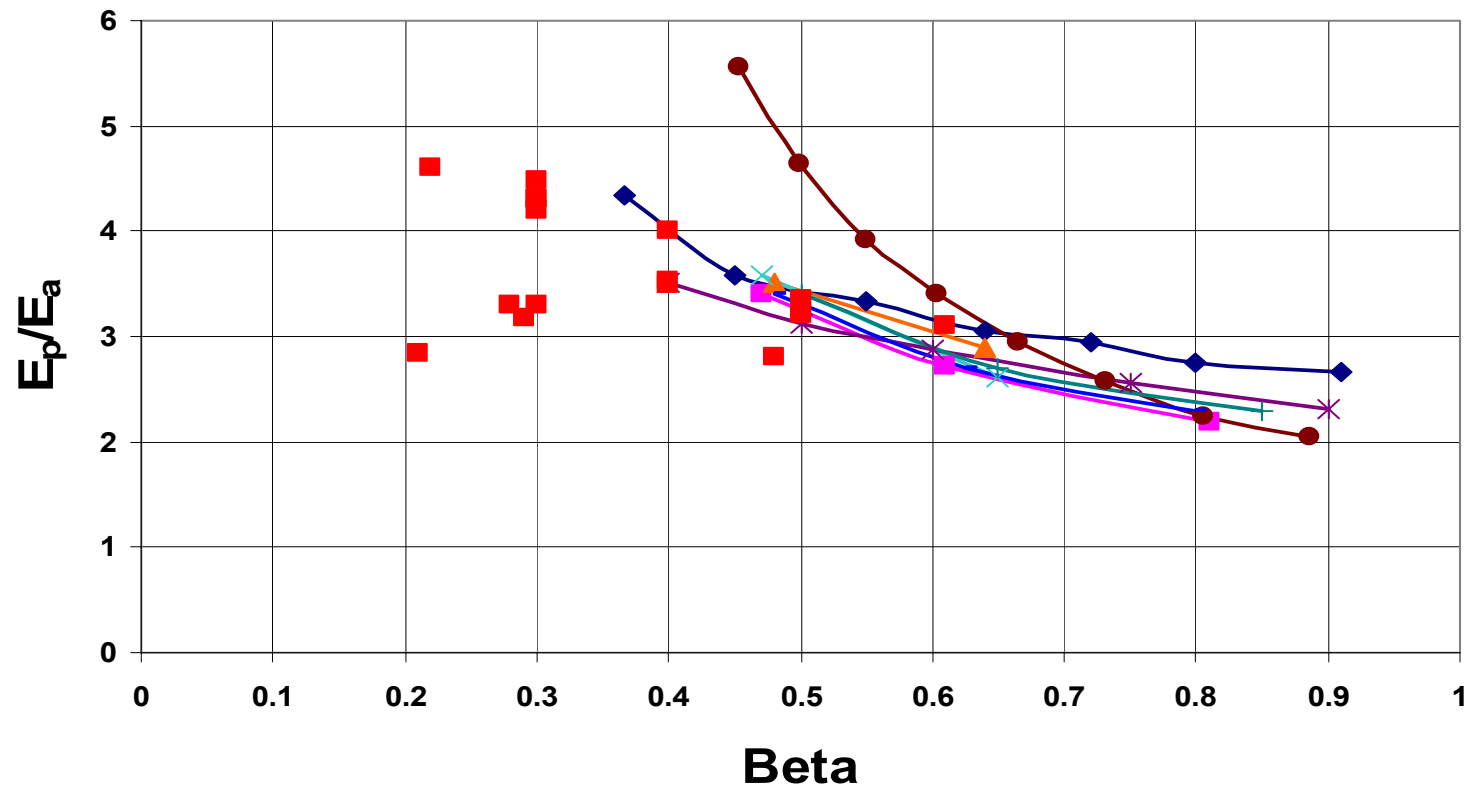
Surface Electric Field

- **TM₀₁₀ elliptical structures**
 - $E_p/E_a \sim 2$ for $\beta = 1$
 - Increases slowly as β decreases
- **$\lambda/2$ structures:**
 - Sensitive to geometrical design
 - Electrostatic model of an “shaped geometry” gives $E_p/E_a \sim 3.3$, independent of β

Surface Electric Field

- Lines: Elliptical

Squares: Spoke



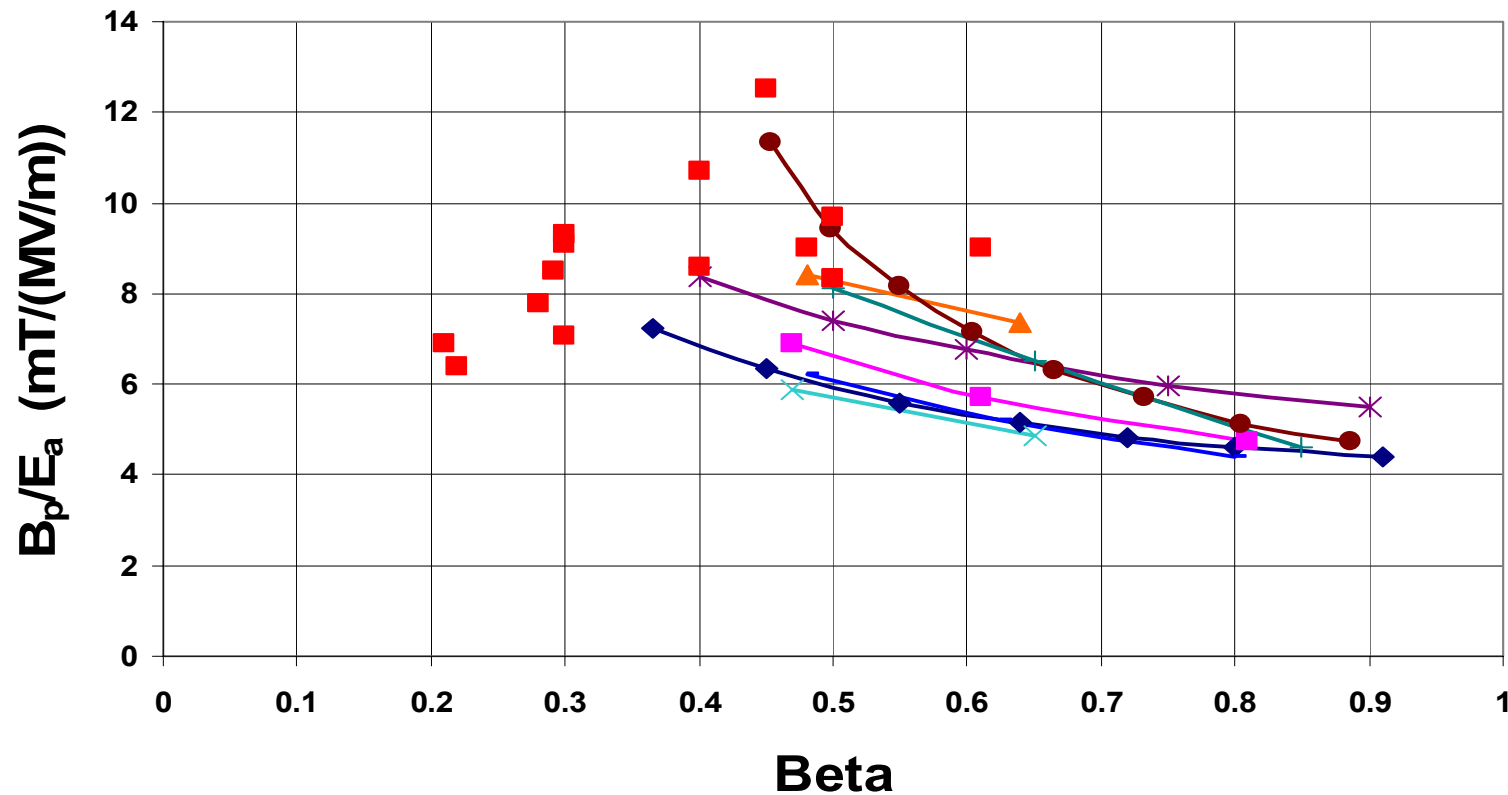
Surface Magnetic Field

- **TM₀₁₀ elliptical cavities:**
 - $B/E_a \sim 4 \text{ mT}/(\text{MV}/\text{m})$ for $\beta=1$
 - Increases slowly as β decreases
- **$\lambda/2$ structures:**
 - Sensitive to geometrical design
 - Transmission line model gives $B/E_a \sim 8 \text{ mT}/(\text{MV}/\text{m})$, independent of β

Surface Magnetic Field

- Lines: Elliptical

Squares: Spoke

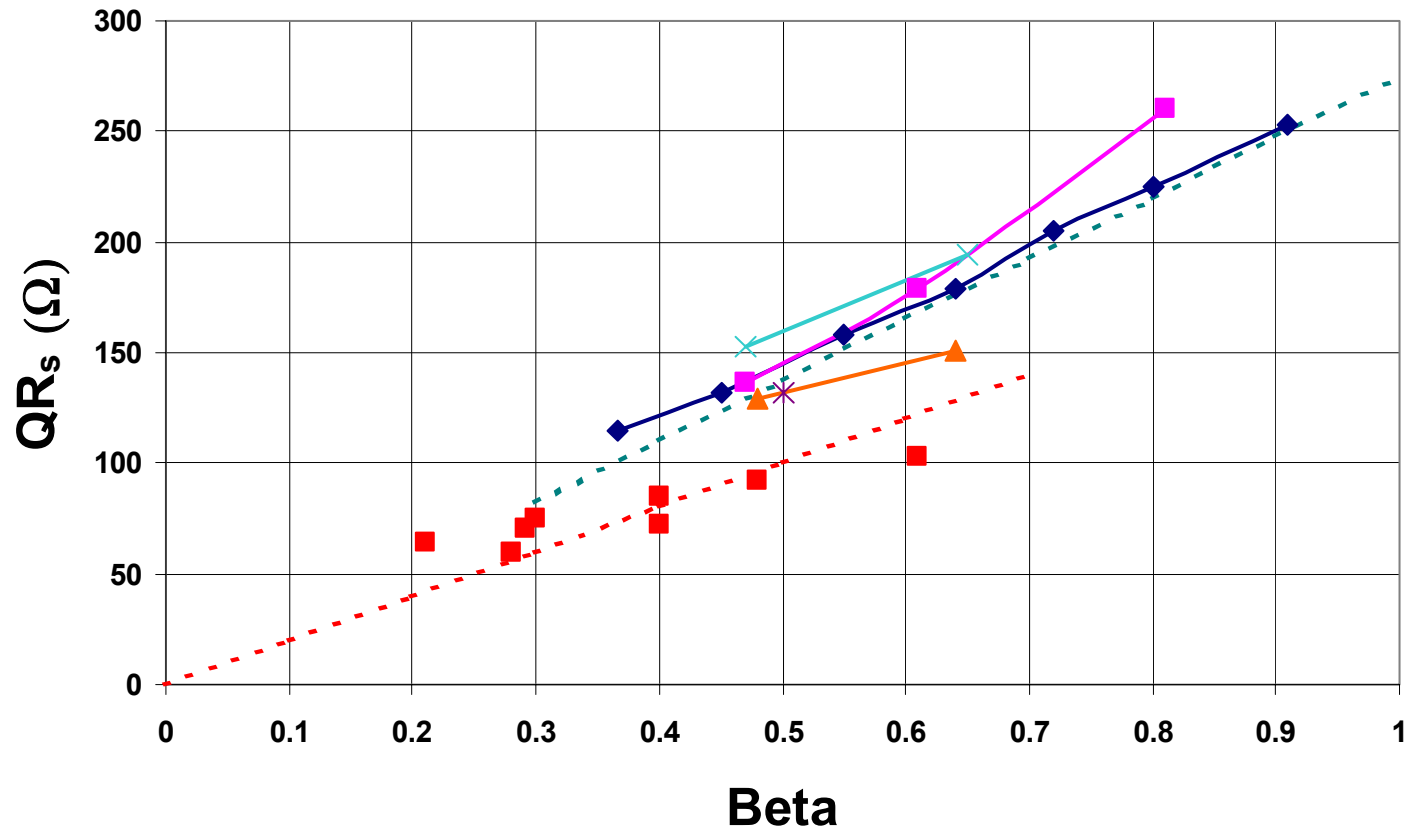


Geometrical Factor (QR_s)

- **TM_{010} elliptical cavities:**
 - Simple scaling: $QR_s \sim 275 \beta (\Omega)$
- **$\lambda/2$ structures:**
 - Transmission line model: $QR_s \sim 200 \beta (\Omega)$

Geometrical Factor (QR_s)

- Lines: Elliptical Squares: Spoke



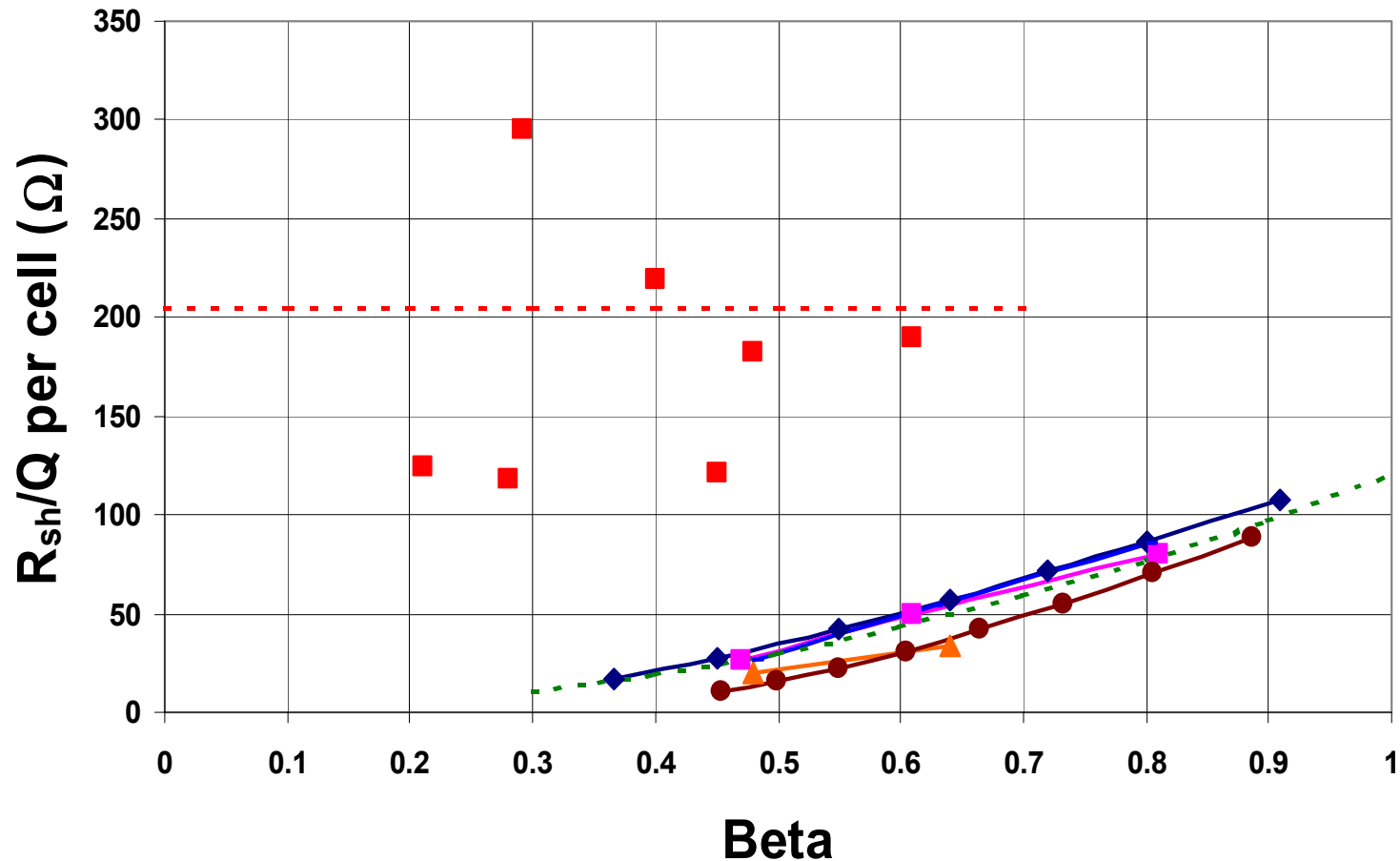
R_{sh}/Q per Cell or Loading Element

- $R_{sh} = V^2/P$
- TM_{010} elliptical cavities:
 - Simple-minded argument, ignoring effect of beam line aperture, gives: $R_{sh}/Q \propto \beta$
 - When cavity length becomes comparable to beam line aperture : $R_{sh}/Q \propto \beta^2$
 - $R_{sh}/Q \approx 120 \beta^2 \quad (\Omega)$
- $\lambda/2$ structures:
 - Transmission line model gives: $R_{sh}/Q \approx 205 \Omega$
 - Independent of β

R_{sh}/Q per Cell or Loading Element

Lines: Elliptical

Squares: Spoke



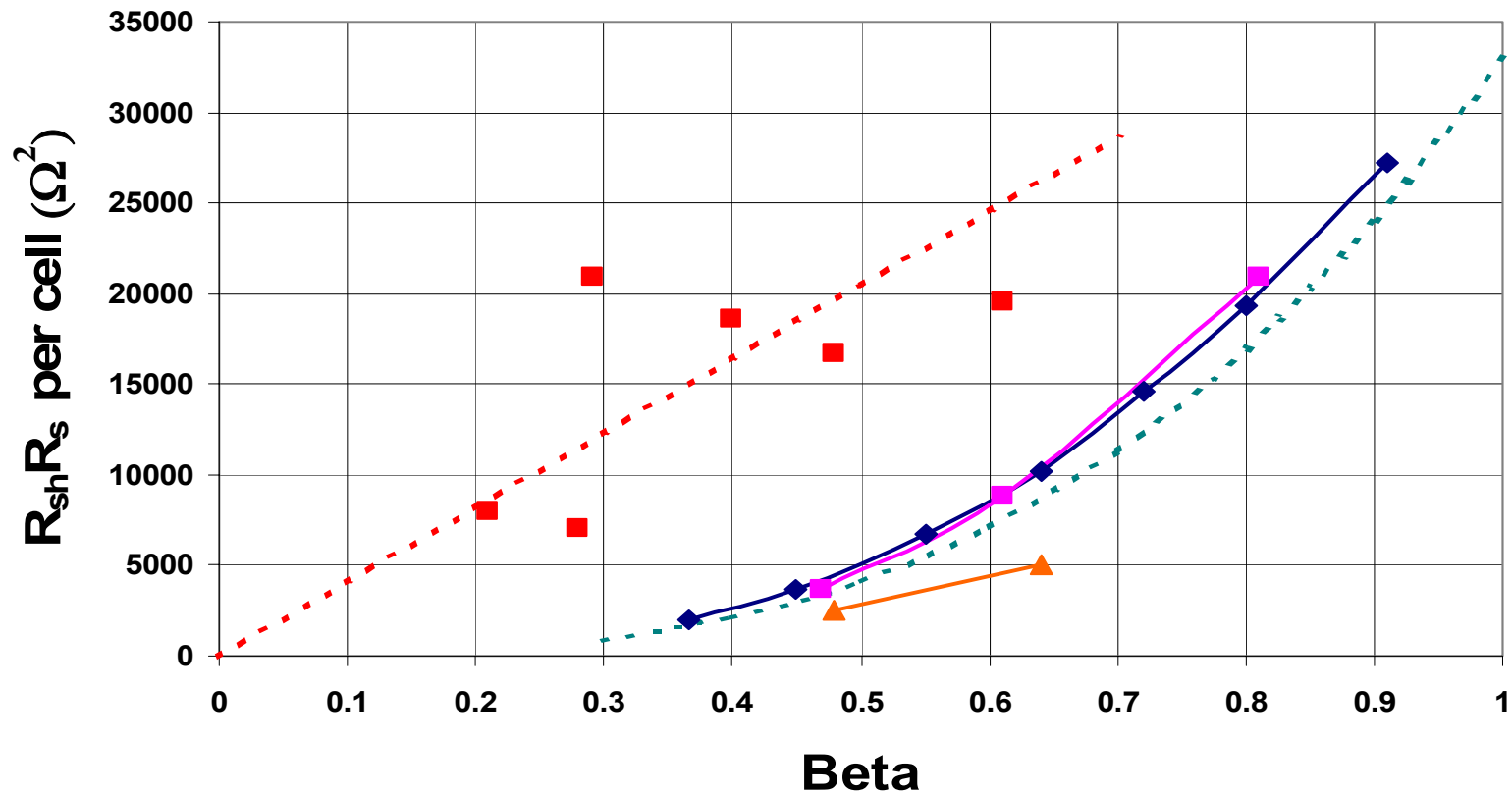
Shunt Impedance R_{sh}

($R_{sh}/Q \sim QR_s$ per Cell or Loading Element)

- **TM₀₁₀ elliptical cavities:**
 - $R_{sh} R_s \sim 33000 \beta^3 (\Omega^2)$
- **$\lambda/2$ structures:**
 - $R_{sh} R_s \sim 40000 \beta (\Omega^2)$

Shunt Impedance R_{sh} (R_{sh}/Q QR_s per cell or loading element)

- Lines: Elliptical
- Squares: Spoke



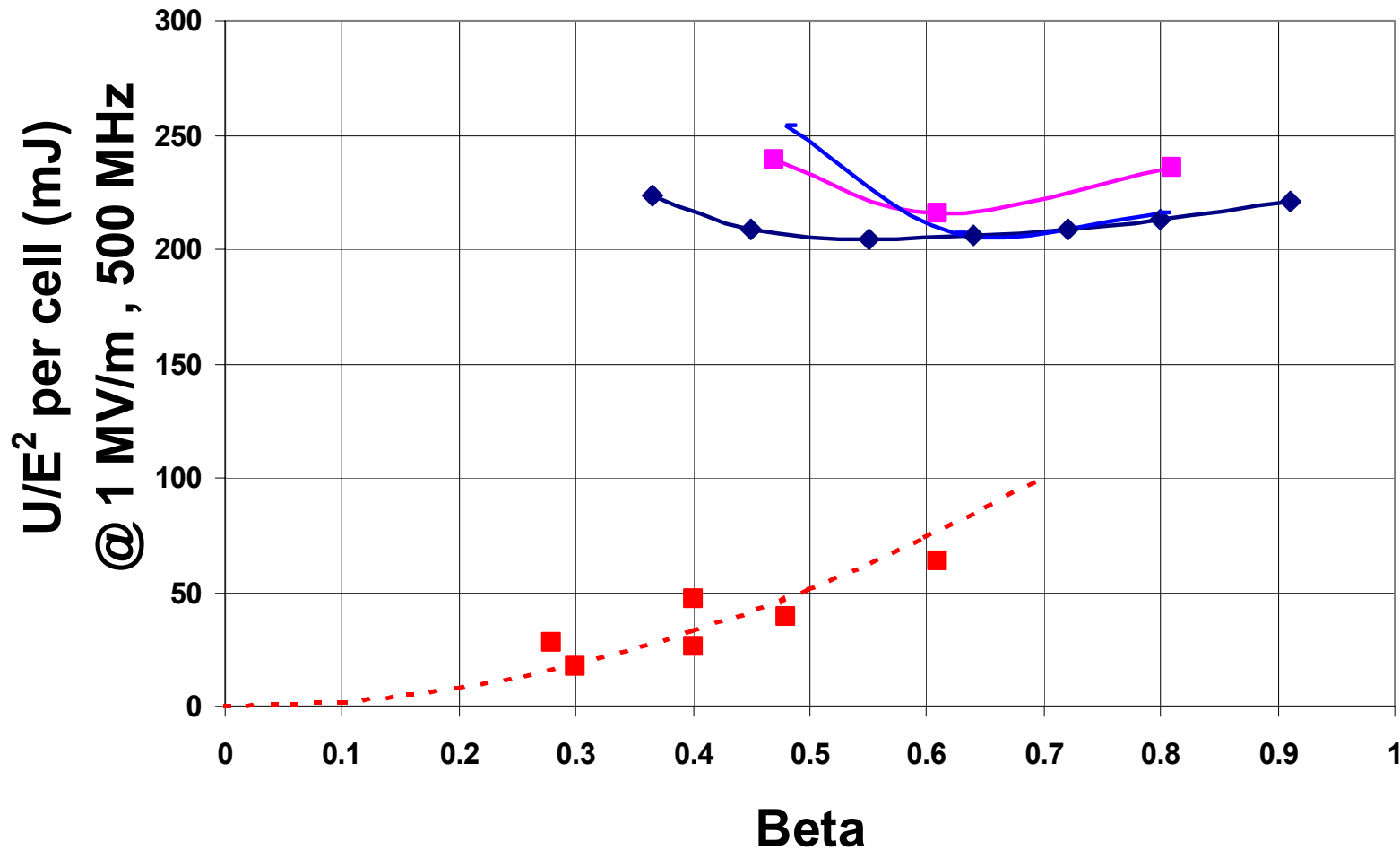
Energy Content per Cell or Loading Element

Proportional to $E^2\lambda^3$

At 1 MV/m, normalized to 500 MHz:

- **TM₀₁₀ elliptical cavities:**
 - Simple-minded model gives $U/E^2 \propto \beta$
 - In practice: $U/E^2 \sim 200\text{-}250$ mJ
 - Independent of β (seems to increase when $\beta < 0.5 - 0.6$)
- **$\lambda/2$ structures:**
 - Sensitive to geometrical design
 - Transmission line model gives $U/E^2 \sim 200 \beta^2$ (mJ)

Energy Content per Cell or Loading Element

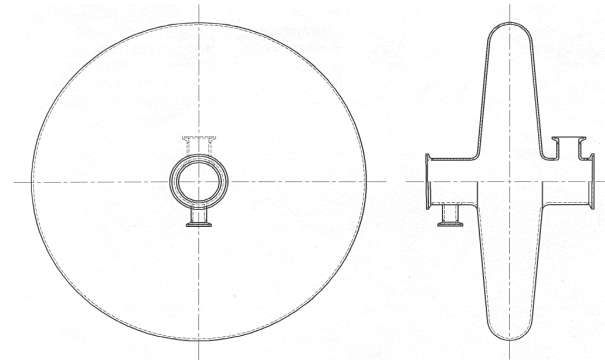


Size & Cell-to-Cell Coupling

TM₀₁₀ Structures

$$\varnothing \sim 0.88 - 0.92 \lambda$$

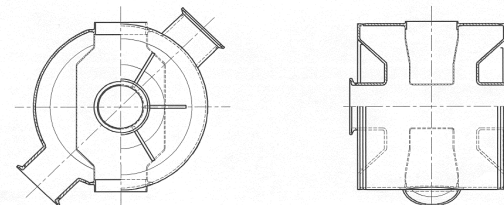
Coupling $\sim 2\%$



I/2 Structures

$$\varnothing \sim 0.46 - 0.51 \lambda$$

Coupling $\sim 20 - 30\%$



Example : 350 MHz, $\beta = 0.45$

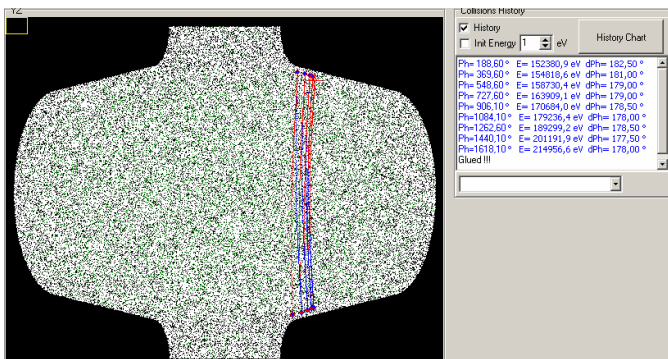
Multipacting

- **TM₀₁₀ elliptical structures**
 - Can reasonably be modeled and predicted/avoided
 - Modeling tools exist

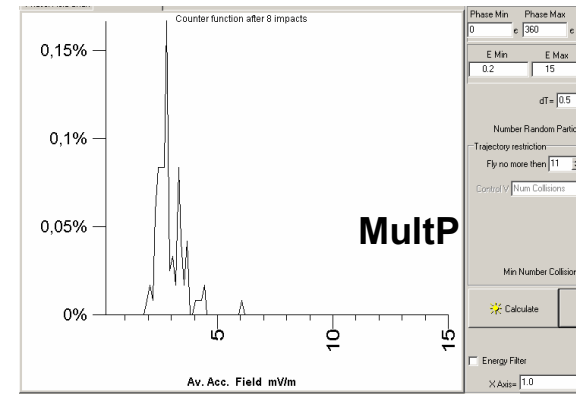
- **$\lambda/2$ Structures**
 - Much more difficult to model
 - Reliable modeling tools do not exist
 - Multipacting “always” occurs
 - “Never” a show stopper

Multipacting

- Resonant electron loading → strongly depends on geometry
 - Several 2D simulation codes are available;
- Multipacting condition
1. Closed trajectory
 2. insensitive to the initial energy
 3. $SEY(E) > 1$ (Physical surface condition) ← lowering thru He processing & better surface cleaning process
- A series validity test tells (Larger) circular dome will help to preventing MP.
 β Lower than $b \sim 0.45$ cavity is supposed to have more MP problem



Courtesy of L. Kravchuk (INR)

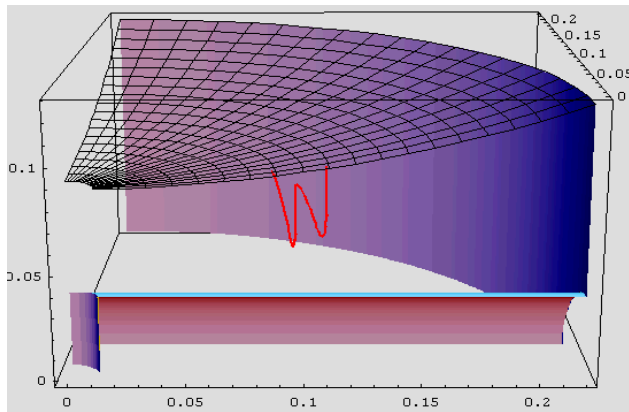


Ex. SNS 0.61 cavity, condition 1&2 are satisfied, but electron energy is too high $SEY \ll 1$

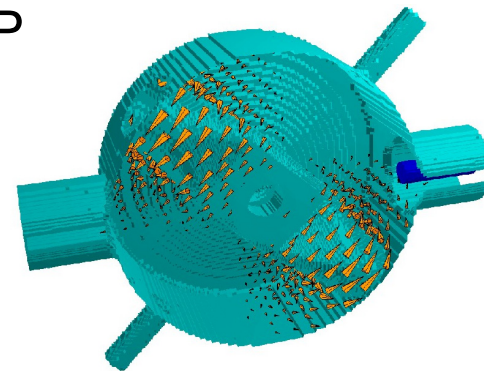
Multipacting

- Spoke cavity; **full 3D structure coupler** is connected to the cavity main body
→ should be checked very carefully
(will not be a showstopper)
- 3D MP codes are under development and/or available now.
- Benchmarking or comparison with experiments are strongly needed

RIA prototype 3D MP simulation with MultiP



(Courtesy of L. Kravchuk (INR))



AAA/LANL $\beta=0.175$, 350 MHz
Spoke cavity with
Power coupler (coupling concept)
Courtesy of F. Krawczyk (LANL)

TM Structures – Positive Features

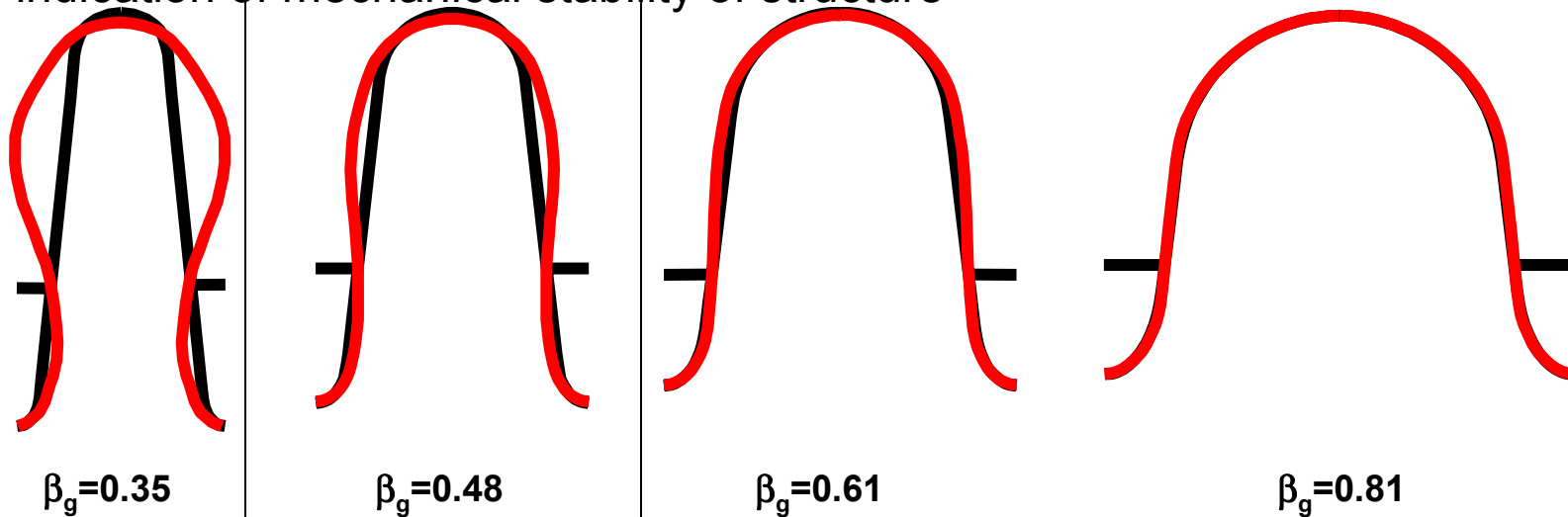
- **Geometrically simple**
- **Familiar**
- **Large knowledge base**
- **Good modeling tools**
- **Low surface fields at high β**
- **Small number of degrees of freedom**

$\lambda/2$ Structures – Positive Features

- **Compact, small size**
- **High shunt impedance**
- **Robust, stable field profile (high cell-to-cell coupling)**
- **Mechanically stable, rigid (low Lorentz coefficient, microphonics)**
- **Small energy content**
- **Low surface fields at low β**
- **Large number of degrees of freedom**

How Low Can We Go with β_g in TM Cavities ?

- Static Lorentz force detuning (LFD) at $E_0 T(\beta_g) = 10$ MV/m, 805 MHz (Magnification; 50,000)
- In CW application LFD is not an issue, but static LFD coeff. provides some indication of mechanical stability of structure



RF efficiency; x
Mechanical
Stability; x
Multipacting;
Strong possibility

Will work in CW
Pessimistic in
Pulsed application
Would be a
competing Region
with spoke cavity

Suitable for all CW & pulsed applications
Recent test results of SNS prototype cryomodule, $\beta_g = 0.61$;
quite positive; piezo compensation will work

How High Can We Go with β_g in Spoke Cavities?

- **What are their high-order modes properties?**
 - **Spectrum**
 - **Impedances**
 - **Beam stability issues**
- **Is there a place for spoke cavities in high- β high-current applications?**
 - **FELs, ERLs**
 - **Higher order modes extraction**

Parting Words

In the last 30+ years, the development of low and medium β superconducting cavities has been one of the richest and most imaginative area of srf

The field has been in perpetual evolution and progress

New geometries are constantly being developed

The final word has not been said

The parameter, tradeoff, and option space available to the designer is large

The design process is not, and probably will never be, reduced to a few simple rules or recipes

There will always be ample opportunities for imagination, originality, and common sense