3-29-2023

Mathematical modeling and inverse problems in applications

Thanh T. Nguyen
Rowan University, nguyent@rowan.edu

Mathematical modeling and inverse problems in applications

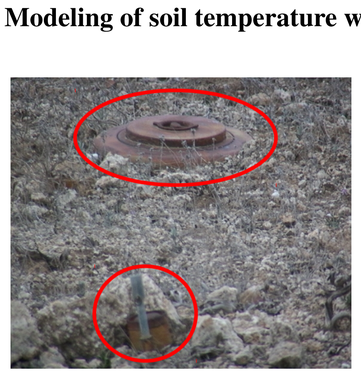
Thành T. Nguyen

Department of Mathematics, Rowan University (nguyent@rowan.edu)

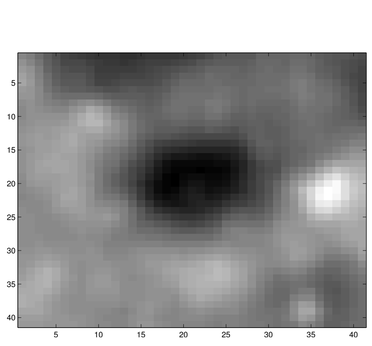
Examples of Applications of Mathematical Modeling

Mathematical models, based on ordinary or partial differential equations, are widely used to describe physical/chemical/biological processes and can be found in several applications: nondestructive testings, subsurface imaging, defense, medicine, environmental sciences, etc.

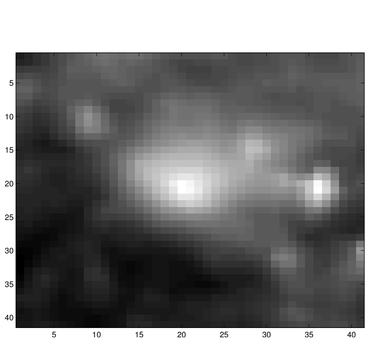
Modeling of soil temperature with landmines:



Landmines

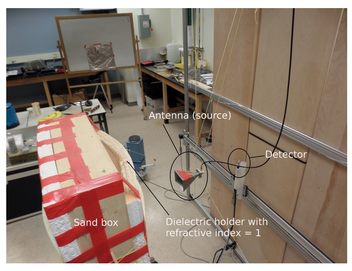


Infrared image at 12AM

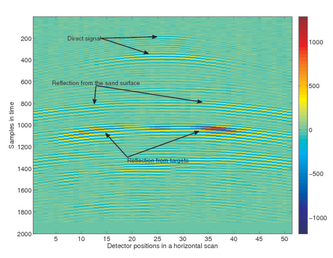


Infrared image at 12PM

Modeling of waves in heterogeneous media:

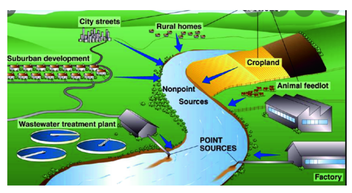


Radar for subsurface imaging



Measured radar signals

Modeling of water pollution in rivers/streams/lakes:



Modeling of water pollution

Inverse Problems

Forward model: A mathematical model can be described by a problem of ordinary/partial differential equations of the form

$$F(u, u_t, u_x, u_y, \ldots, \mathbf{p}) = 0,$$

where $\mathbf{p}$ represents coefficients of the model (parameters of the medium).

Data: Measurements of the state variable u (physical/chemical/biological process) are conducted.

Inverse Problems:

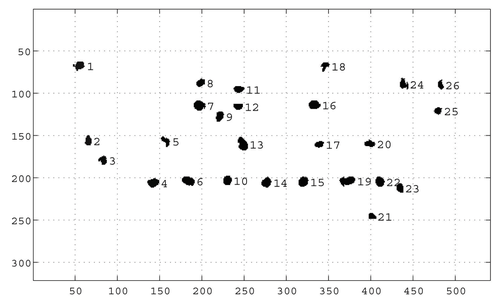
- Model identification problem: Find a model (equivalently, the parameter set $\mathbf{p}$) which best fits the measured data.
- Object detection/imaging: determine anomalies (objects) in a medium or reconstruct the physical properties of a medium (or an object).
- Source identification: Determine pollution sources.

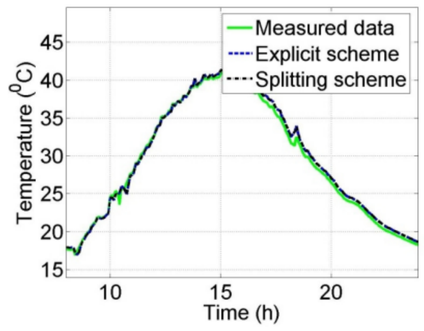
Methods for Solving Inverse Problems

- Numerical methods for differential equations
- Data processing: image processing, signal processing
- Optimization methods for parameter estimation problems

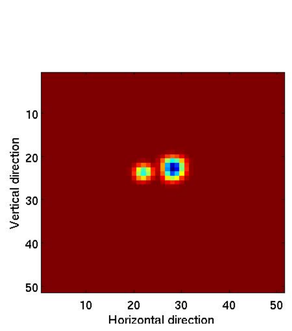
Sample results of Inverse Problems

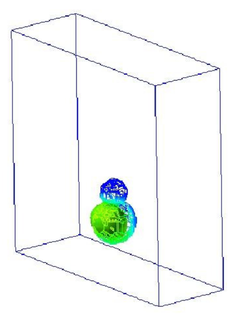
Detection of buried landmines using infrared cameras:

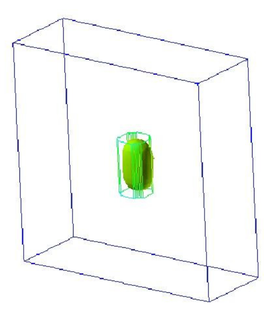




Detection of buried objects using ground penetrating radar:







Detection of water pollution sources: