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Frequency limited impulse response gramians based model reduction

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ABSTRACT

In order to simplify the analysis of complex electronic systems, they needsto be modeled accurately. Model reduction is further required to streamline the procedural and computational complexities. Further the instability caused by the model reduction techniques worstly effects the accuracy of a system. Therefore, we have proposed some improvements in the frequency limited impulse response Gramians based model order reduction techniques for discrete time systems. The propsed techniques assures the stability of the model after it get reduced. The proposed techniques provided better results than the stability preserving techniques.

1. Introduction

Analysis of large complex systems such as telecommunication and digital filters [1]-[6] is a challenging task. Model order reduction (MOR) approximates higher order models with reduced order models (ROMs). Balance truncation [7] is a common scheme to get stable ROM for stable original system. Ideally, it is important to have low frequency response error between original and ROM for all frequency ranges. In some cases, a particular frequency range is of interest.

Enns [8] has extended the balance truncation method into frequency weighted scenario. Enns technique may yield unstable ROMs for double sided weights. Wang and Zilouchian [9] proposed a frequency limited MOR technique without explicit weights. It may also yield unstable ROM. To overcome the problem of Wang and Zilouchian [9], some work (including Ghafoor and Sreeram [10], Imran and Ghafoor [11]) yields stable ROMs.

Impulse response Gramian (IRG) was proposed by Sreeram and Agathoklis [12] for MOR. IRG contains input/output behaviour of the original system and preserve markov properties of the original system. Sahlan et al [13] extended the work of Sreeram and Agathoklis [12] to frequency limited scenario. The proposed techniques yield stable ROM and less error as compared to existing frequency limited MOR technqiue.

2. Preliminaries

Consider a stable discrete time system

$$G(z) = D + C(zI - A)^{-1}B \tag{1}$$

where $\{A \in R^{n\times n}, B \in R^{n\times m}, C \in R^{p\times n}, D \in R^{p\times m}\}$. Let the frequency limited $\delta\omega \in [-\omega_1, -\omega_2] \cup$

$[\omega_1, \omega_2]$ controllability Gramian $W_c(\omega)$ and observability Gramian $W_o(\omega)$.

$$W_c = \frac{1}{2\pi}\int_{\delta\omega} (e^{j\omega}I - A)^{-1}BB^T(e^{-j\omega}I - A^T)^{-1}d\omega$$

$$W_o = \frac{1}{2\pi}\int_{\delta\omega} (e^{-j\omega} - A^T)^{-1}C^TC(e^{j\omega}I - A)^{-1}d\omega$$

satisfy

$$AW_cA^T - W_c + X = 0, \qquad (2)$$

$$A^TW_oA - W_o + Y = 0 \qquad (3)$$

$$X = BB^TF^* + FBB^T = USU^T = U\begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}U^T \qquad (4)$$

$$Y = C^TCF + F^*C^TC = VRV^T = V\begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}V^T \qquad (5)$$

$$F = -\tfrac{\omega_2-\omega_1}{4\pi}I + \tfrac{1}{2\pi}\int_{\delta\omega} (e^{j\omega}I - A)^{-1}d\omega \qquad (6)$$

$$S_1 = diag(s_1, s_2, s_3, \dots, s_l),$$

$$S_2 = diag(s_{l+1}, s_{l+2}, \dots, s_n),$$

$$R_1 = diag(r_1, r_2, r_3, \dots, r_k),$$

$$R_2 = diag(r_{k+1}, r_{k+2} \dots, r_n),$$

$l \le n$ and $k \le n$ are the number of positive eigenvalues of X and Y respectively.

Remark 1 *Let the original system in equation (1) be in the controllability or in the observability canonical form, then the solution of equation (2) can be written as*

$$W_c = W_o = W_h = \sum_{k=0}^{\infty}\begin{bmatrix} h_{k+1}h_{k+1} & \cdots & h_{k+1}h_{k+n} \\ \vdots & \ddots & \vdots \\ h_{k+n}h_{k+1} & \cdots & h_{k+n}h_{k+n} \end{bmatrix}$$

where W_h is the frequency limited IRG and $h_k = CA^{k-1}B$ is impulse response.

Remark 2 *As a consequence of Remark* 1, *only one Gramian (either W_c or W_o) is enough for MOR*

Since X and Y in equation (2) may be indefinite, which cause the possible unstable ROM. To address this issue Sahlan et al [13] modified X (to compute W_h with original system realization in controllability canonical form) and Y (to compute W_h with original system realization in observability canonical form) in frequency limited IRG case as

$$X_s = U\begin{bmatrix} S_1 & 0 \\ 0 & |S_2| \end{bmatrix}U^T, \qquad Y_s = V\begin{bmatrix} R_1 & 0 \\ 0 & |R_2| \end{bmatrix}V^T$$

3. Main Results

The proposed techniques also modify X and Y to ensure positive/semipositive definiteness of these matrices. These modifications yield stable ROM and low error. Let new synthetic IRG $W_{H_{M_i}}$ are computed as

$$AW_{H_{M_i}}A^T - W_{H_{M_i}} + B_{M_i}B_{M_i}^T = 0 \qquad (7)$$

where $B_{M_i} \in \{B_{M_1}, B_{M_2}, B_{M_3}\}$

$$B_{M_1} = U\begin{bmatrix} S_1^{1/2} & 0 \\ 0 & (S_2/s_n)^{1/2} \end{bmatrix} \quad \text{for} \quad s_n < 0$$

$$B_{M_2} = U\begin{bmatrix} S_1^{1/2} & 0 \\ 0 & (S_2 \cdot s_n)^{1/2} \end{bmatrix} \quad \text{for} \quad s_n < 0$$

$$B_{M_3} = U\begin{bmatrix} S_1^{1/2} & 0 \\ 0 & ((S_2 + s_nI)/s_n)^{1/2} \end{bmatrix} \quad \text{for} \quad s_n < 0$$

$$B_{M_i} = US^{1/2} \quad \text{for} \quad s_n > 0$$

Remark 3 *The Gramian $W_{H_{M_i}}$ can also be obtained when the original system is in observable canonical form as*

$$A^TW_{H_{M_i}}A - W_{H_{M_i}} + C_{M_i}^TC_{M_i} = 0 \qquad (8)$$

where $C_{M_i} \in \{C_{M_1}, C_{M_2}, C_{M_3}\}$

$$C_{M_1} = \begin{bmatrix} R_1^{1/2} & 0 \\ 0 & (R_2/r_n)^{1/2} \end{bmatrix}V^T \quad \text{for} \quad r_n < 0$$

$$C_{M_2} = \begin{bmatrix} R_1^{1/2} & 0 \\ 0 & (R_2 \cdot r_n)^{1/2} \end{bmatrix}V^T \quad \text{for} \quad r_n < 0$$

$$C_{M_3} = \begin{bmatrix} R_1^{1/2} & 0 \\ 0 & ((R_2 + r_nI)/r_n)^{1/2} \end{bmatrix}V^T \quad \text{for} \quad r_n < 0$$

$$C_{M_i} = R^{1/2}V^T \quad \text{for} \quad r_n > 0$$

Note that only equation (6) or (7) is enough for MOR.

Let similarity transformation matrix T_{M_i} is calculated as

$$T_{M_i}^TW_{H_{M_i}}T_{M_i} = diag\{\sigma_1, \sigma_2, \dots, \sigma_n\}$$

where $\sigma_j \ge \sigma_{j+1}$ and $\sigma_r \ge \sigma_{r+1}$, the ROMs $\{A_{11}, B_1, C_1, D\}$ are obtained by transforming and partitioning as

$$T_{M_i}^{-1}AT_{M_i} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},\ T_{M_i}^{-1}B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, CT_{M_i} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}^T,\ D$$

Remark 4 *Since* $B_{M_i}B_{M_i}^T \geq 0$, $C_{M_i}^T C_{M_i} \geq 0$. *Therefore, the realization* (A, B_{M_i}, C_{M_i}) *is minimal.*

Remark 5 *The ROM* $\{A_{11}, B_1, C_1, D\}$ *is asymptotically stable if and only if* $\{A, B_{M_i}\}$ *is stable and controllable. This follows similar to Theorem* 1 *of [13].*

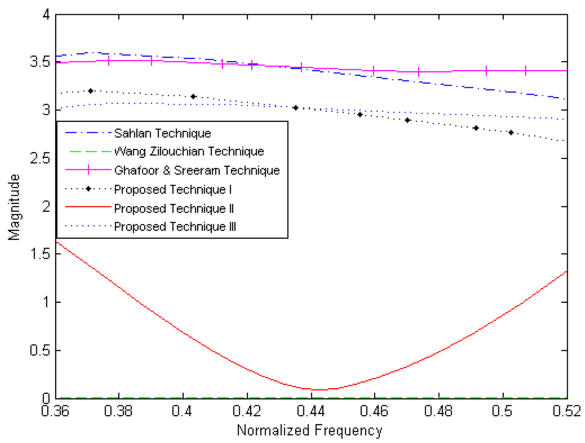

Fig. 1. Error comparison - zoom-in view

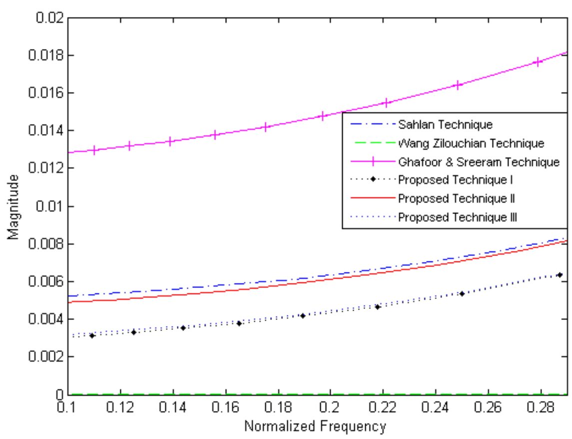

Fig. 2. Error comparison - zoom-in view

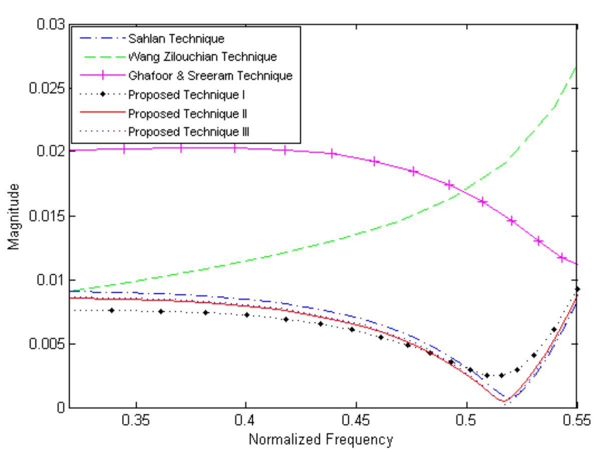

Fig. 3. Error comparison - zoom-in view

4. Numerical Simulations

Example 1: Consider a 6^{th} order discrete time system, having following transfer function with desired frequency interval $0.36\pi - 0.52\pi$.

$$G(z) = \frac{0.25z^5+1.25z^4+1.75z^3+2z^2+2.5z+0.25}{z^6+0.875z^5+0.75z^4+0.5z^3+0.3z^2+0.25z+0.1}$$

Example 2: Consider a 6^{th} order Elliptic band-pass $0.7\pi - 0.8\pi$ filter with 0.1 dB ripple in the pass band and 30 dB attenuation in the stop band, having following transfer function with desired frequency interval $0.1\pi - 0.29\pi$ and $0.32\pi - 0.55\pi$. Fig. 1, 2 and 3 represent zoom-in view of frequency response error $\sigma[G(z) - G_r(z)]$ plot, where $G_r(z)$ is $(4^{th}, 3^{rd}, 1^{th})$ order reduced order obtained using existing and proposed techniques respectively.

$$G(z) = \frac{0.03081z^6+0.07371z^5+0.06328z^4+1.711e^{-17}z^3 -0.06328z^2-0.07371z-0.03081}{z^6+3.827z^5+7.281z^4+8.275z^3+5.975z^2+2.572z+ 0.5518}$$

5. Conclusion

Some improved frequency limited IRG based MOR techniques are proposed for discrete time systems. Wang and Zilouchian's Technique performs better but sometimes it yield unstable ROM. The proposed techniques yield stable ROMs and error is also improved as comparable with other existing stability preserving frequency limited MOR techniques.

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