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Nonlinear vibrations and dynamic stability of viscoelastic anisotropic fiber reinforced plates

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Abstract. Fiber-reinforced plastic composites are one of the most widely used composite materials because they balance well between properties and cost. Despite their widespread use in the aviation and automotive industries, there is currently a lack of effective mathematical models for their calculation under various dynamic loads. The research object of this work is an anisotropic viscoelastic fiber-reinforced simply supported rectangular plate. Two dynamic problems are considered: vibrations of the plate under the influence of a uniformly distributed static load; stability of the plate compressed in one direction. Within the Kirchhoff-Love hypothesis framework, a mathematical model was built in a geometrically nonlinear formulation, taking into account the tangential forces of inertia. By the Bubnov-Galerkin method, based on a polynomial approximation of the deflection and displacement, the problem was reduced to solving systems of nonlinear ordinary integro-differential equations. With a weakly singular Koltunov-Rzhanitsyn kernel with variable coefficients, the resulting system was solved by a numerical method based on quadrature formulas. By using experimental studies, considering the directions of the fibers, the values of the rheological parameters of some plastic materials (KAST-V and EDF) were obtained. The plate's dynamical behavior was investigated depending on the plate's geometric parameters, viscoelastic and inhomogeneous material properties. Results show the importance of taking into account the viscoelastic properties of the material when solving dynamic problems of anisotropic reinforced plates made of composite materials. In particular, when studying the problem of dynamic stability of an anisotropic reinforced plate made of KAST-V, the results obtained in elastic and viscoelastic formulations in some cases differ from each other by more than 20 %.

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1. Introduction

Mechanics of composite materials has made great progress in recent years. The application of composite materials in engineering structures considerably improved structures' operational characteristics and, in several cases, create structures that could not be realized with traditional materials. Polymer composite materials are widely used in diverse engineering fields. A large class of engineering structures relies on the use of composite materials and ceramics (for example, operating under conditions of high temperatures, contact pressures, etc.). Therefore, developing efficient algorithms for solving nonlinear problems of dynamis stability of shells, panels, and plates made of composite materials is important in today's industry.

The development of high-modulus and high-strength boron and graphite fibers joined together in plastic or metal binders arouses great interest in structural elements made of layers, each of which has different mechanical properties. In most cases, these composite materials include many layers of fibers with specific stacking sequences. The dimensions of the structural elements are such that each layer can be adequately modeled as a homogeneous anisotropic layer. Due to the unique characteristics of modern composite materials, interest in the problems of studying the dynamic behavior of anisotropic reinforced layered structures has not weakened in recent years.

An overview analysis of the using various natural fibers to form composites with various matrix materials and the effect of fiber length, chemical processing, and compositions on the dynamic mechanical characteristics of composites is available in [1].

An overview of works related to the study of typical functional capabilities of composites reinforced with natural fibers, including thermal insulation properties, biodegradable properties, and vibration damping properties, are given in [2]. This article concludes that composite materials that fully utilize their functionality rather than mechanical characteristics are important for the future development of composites reinforced with natural fibers. The analysis of some published literature on vibration damping applied to composites reinforced with natural fibers by experimental and numerical methods was carried out here. The effect of reinforcement with natural fibers on the thermosetting and thermoplastic polymer matrix vibration response was considered.

The article [3] investigates the effect of transverse normal and shear deformations on fiber-reinforced viscoelastic beams resting on two-parameter (Pasternak's) elastic foundations. The results were obtained using the refined theory of the sinusoidal shear deformation beam theory and compared with the results obtained using the simple theory of the sinusoidal shear deformation beam theory, the Timoshenko first-order shear deformation beam theory, and the classical Euler-Bernoulli beam theory.

The characteristics of free vibration of rectangular plates reinforced with continuous grading fiber (CGFR) based on elastic bases were studied based on the three-dimensional linear theory of elasticity and the theory of small deformations [4]. The foundation was described by the Pasternak model or a two-parameter model. The CGFR plate was simply supported at the edges and was assumed to have an arbitrary change in the volume fraction of the fiber in the direction of thickness. Displacement functions that satisfy simply supported boundary conditions were used to reduce the equilibrium equations to a set of coupled ordinary differential equations with variable coefficients, which in turn have been solved by differential quadrature (DQM) to obtain natural frequencies. Studies have shown that this method has a high convergence rate, stable numerical performance, and accuracy. The results for a CGFR plate with an arbitrary change in the volume fraction of the fiber in the thickness of the plate were compared with a discrete layered composite plate. They also showed that the use of CGFR plates with graduated volume fractions of fibers has advantages over traditional discretely laminated plates.

The static response of an inhomogeneous fiber-reinforced viscoelastic multilayer plate was investigated in [5] by using the first-order shear deformation theory. Several types of sandwich plates were considered, taking into account the symmetry of the plate and the thickness of each layer. In addition, two cases were considered depending on the viscoelastic material that enters the core or the surface of multilayer plates. To solve equations determining the bending of simply supported inhomogeneous fiber-reinforced viscoelastic multilayer plates, the effective modules method and Ilyushin's approximation method were used. Numerical calculations were carried out to study the effect of the time parameter on deflections and stresses at different values of the aspect ratio, the ratio of sides to the thickness, and the constitutive parameter.

Amabili [6] investigated the dynamic stability of orthotropic thick plates subjected to periodic uniaxial stress and bending stress, considering the inertia of rotation and transverse stress. Based on Bolotin's method, the regions of dynamic instability of plates reinforced with graphite and fiberglass were estimated by solving eigenvalue problems. The influence of material properties and load parameters on the instability region and the index of dynamic instability of orthotropic plates was discussed.

The distorted modes in geometrically nonlinear forced oscillations of angular layered circular cylindrical shells were investigated using the highest order Amabili-Reddy's shear deformation theory [7]. Harmonic forces were applied in the radial direction and simply supported boundary conditions were assumed. The equations of motion were obtained using an energy approach based on Lagrange equations, which preserves dissipation. Numerical results were obtained using the method of pseudo-arc continuation and bifurcation analysis.

Anisotropic composite plates were evaluated with nanofiber-reinforced matrices (NFRM) [8]. Seven different loading conditions were considered: three for uniaxial loading, three for paired combined loading, and one with three combined loads. The anisotropy had a complete stiffness matrix of 6 by 6, which was satisfied and solved using the Galerkin algorithm.

The influence of some geometric and material parameters on the free vibrations of FML plates (fibrous metal laminates) was studied in [9]. For the analytical solution of the governing equations of the composite plate, the first-order shear deformation theory (FSDT) and the Fourier series method were used.

In combination with the elastic constitutive model and fiber direction, a plastic square plate with two materials (orthotropic and anisotropic) was simulated [10]. Anisotropic characteristics of the thermal field, pressure field, and vibration characteristics of fiber-reinforced PA66 were analyzed using simulation comparisons.

A new nonlinear model of vibrations of reinforced composite thin plates in a thermal environment was proposed [11] by introducing nonlinear thermal and amplitude fitting coefficients. Based on the classical laminated plate theory, the complex module approach, the power function, and the Ritz methods, dynamic governing equations in high-temperature environments for solving nonlinear natural frequencies, vibration characteristics, and damping parameters were obtained. Experimental tests of thin plates made of carbon-epoxy composite CF130 were carried out to test this model.

The article [12] presents a new nonlinear model of vibrations of composite plate structures with amplitude-dependent properties, taking into account the nonlinear characteristics of stiffness and damping by introducing the nonlinearity of the material into the dynamic modeling of fiber-reinforced composite thin plates. In this model, the elastic modulus and loss coefficients were expressed as a function of the strain energy density based on the nonlinear Jones-Nelson material model. These elastic moduli and loss coefficients were characterized as a function of the maximum dimensionless strain energy density using specific parameters at different excitation amplitudes. The power function fitting method was then used to determine the model's nonlinear stiffness and damping parameters. The nonlinear natural frequencies, vibration characteristics, and damping coefficients of the TC300 carbon-epoxy composite thin plate were calculated and measured in a case study.

A Fourier series method based on Mindlin theory and Hamilton's variational principle was proposed [13] for modeling and analyzing the vibration of composite multilayer plates with arbitrary boundary conditions, in which vibrational displacements were sought as a linear combination of a double series of Fourier cosines and auxiliary series functions. Three types of springs with constraints were introduced, and a vibration model was created by combining the Hamilton energy principle to create a general structural model of composite layered materials. The influence of boundary conditions, the laying angle, and the laying layer on the vibration characteristics were analyzed.

The nonlinear dynamic model in a thermal environment was established for modeling a reinforced composite structure made of thin plates [14]. This model uses Hamilton's principle in combination with the classical theory of layered plates, the complex modulus method, and the strain energy method. Nonlinear dependences between elasticity modules, Poisson coefficients, loss coefficients, and temperature change were expressed by the polynomial method. Dynamic equations in a high-temperature environment were derived to solve the inherent characteristics, dynamic characteristics, and damping parameters, considering temperature-dependent properties. The principle of identification of the corresponding fitting coefficients in the theoretical model was illustrated. An experimental test of a thin plate made of carbon-epoxy composite TC500 was carried out to substantiate the correctness of the obtained results. The results of the developed model and experimental tests showed good consistency.

The buckling of a multilayer composite reinforced plate, with porosity effects, was analyzed according to the theory of first shear deformation plates [15]. The properties of the plate material were considered orthotropic properties. Three different models were used considering the effect of porosity in a multilayer composite plate. The Navier procedure was used to solve the problem for a simply supported plate. The influence of porosity coefficients, porosity models, fiber orientation angles, and layer sequences on critical buckling loads was discussed.

Othman, Abouelregal, and Said [16] analytically studied the influence of variable thermal conductivity and initial stress on a reinforced transversely isotropic thick plate. A model of the problem of generalized thermoelasticity with phase delays in an isotropic elastic medium with temperature-dependent mechanical properties was developed. It was assumed that the upper surface of the plate was thermally insulated with a given surface load, and the lower surface of the plate rested on a rigid base. Normal mode analysis was used to obtain analytical expressions of displacement components, power voltage, and temperature distribution. The results obtained in the presence and absence of initial stress and reinforcement were compared.

The nonlinear Jones-Nelson theory combined with the classical theory of layered plates by Yadav, Amabili, et al. [17]. The polynomial fitting method, the strain energy method, and a damping model of a fiber-reinforced composite thin plate were used. The methods and model were created considering the amplitude-dependent properties. The elastic modulus was expressed as the strain energy density function in this model. The loss coefficients in the longitudinal, transverse, and shear directions were expressed as functions of the excitation amplitude. Three plates made of carbon-epoxy composite TC300 were taken as research objects for the case study. One of them was used to determine the amplitude-dependent coefficients of loss coefficients in fiber-reinforced composites by combining the least-squares method with the polynomial fitting method. The other two plates were used to verify the correctness of the theoretical model. The developed model results, taking into account the amplitude dependence and experimental verification, showed good consistency.

The strict polynomial of a higher order in the thickness coordinate was introduced to develop a theory for doubly curved multilayer composite shells, considering the thickness and shear deformations [18]. By applying the condition of zero transverse normal and tangential stresses on the upper and lower surfaces of the shells, the theory of thickness deformation and shear of the third order with six kinematic parameters was derived. At the same time, the nonlinear terms in all kinematic parameters were preserved. The accuracy of the proposed theory has been tested for static and dynamic reference cases. The results obtained based on the proposed theory were compared with the results obtained using a more complex and computationally time-consuming nine-parameter theory. Isotropic and cross-layered circular cylindrical shells under the action of radial forces and pressure and nonlinear forced oscillations of the cross-layered shell under harmonic radial excitation were considered.

The vibrations of circular cylindrical shells made of a carbon nanotube (CNT) fiber-reinforced composite (CNT-FRC) were studied in a geometrically nonlinear formulation [19]. Vibrations were created due to radial harmonic force, and the process itself was considered viscous structural damping. Elastic properties of randomly distributed CNTs in a polymer matrix (i.e., hybrid matrix) were calculated according to the Eshelby-Mori-Tanaka/Voigt scheme to account for the effect of CNT agglomeration in a hybrid matrix. The resulting hybrid matrix was reinforced with aligned fibers to prepare the CNT-FRC shell plate; its effective properties were evaluated using the Halpin Tsai homogenization approach. The CNT-FRC shell was modeled using von Karman geometric nonlinearity and first-order shear deformation theory (FSDT). According to the Hamilton principle, nonlinear control partial differential equations (PDEs) of CNT-FRC shells were derived. These PDEs were discretized into ordinary differential equations (ODES) using the Galerkin method. The ODES were solved by the incremental harmonic balance (IHB) method combined with the arc length extension method to obtain the frequency-amplitude characteristic of the shell. The influence of various types of CNT agglomeration models, CNT mass fraction, agglomeration parameters, and the sequence of laying layered materials on frequency-amplitude curves corresponding to forced and free nonlinear oscillations of the CNT-FRC shell was studied in detail.

The main purpose of the study [20] was to determine the effect of fiber orientation on the reduction of applied voltage for modeling the structure of a dielectric elastomer and improving its mechanical characteristics. Based on the nonlinear continuum mechanics and large inelastic deformations, the defining relations and equations determining the behavior of viscoelastic dielectric elastomers under harmonic electrical load were extracted and analyzed in various states. The obtained numerical results, such as phase diagrams, frequency amplitudes, and oscillations, illustrate the dynamic behavior of an anisotropic dielectric elastomer with different fiber orientations. Hyperelastic and rheological models were used to consider the viscoelastic properties of the material in combination with electrical coupling.

The analysis of many experimental and fundamental studies shows that most composite materials have clearly pronounced viscoelastic properties [21–23].

Most researchers who have attacked the class of problems mentioned above have considered the solution of problems with such a mathematical statement in an elastic case. In these works, only some properties of construction materials were taken into consideration: the problems were solved either in a linear context or in elastic wave propagation was not taken into account. Even if the problems were solved using a viscoelastic formulation, in many cases, the viscoelastic characteristics of the material were only taken into account in a restricted context. In these cases, viscoelastic properties were addressed by employing the Voight model or by using exponential relaxation kernels. However, mathematical models of problems of viscoelastic systems based on these assumptions cannot describe real processes occurring in shell constructions in early times [21, 23]. The choice of an exponential kernel in calculations was not incidental. The systems of integro-differential equations obtained from the calculations were, by way of differentiation, reduced to the solution of ordinary differential equations, which in most cases used to be solved by the well-known Runge-Kutta's numerical method. To the present day, none of the existing methods has allowed one to solve such problems with weakly singular kernels of the type of Koltunov's, Rzhanitsyn's, Abel's, Rabotnov's, and others [21, 23].

Badalov, Eshmatov, and Yusupov [24] developed the numerical method based on the use of quadrature rules, which makes it possible to solve a system of nonlinear integro-differential equations with weakly singular kernels of the type of Koltunov-Rzhanitsyn's, Abel's, and Rabotnov's. Verlan, Abdikarimov, and Eshmatov [25] modified this method. The method provides reasonably high accuracy of results, is universal, enables one to solve a wide class of dynamic problems in viscoelasticity, and is economical from computer time consideration. Based on this method, many numerical results have been obtained by Eshmatov, Mukherjee [26], Mirsaidov, Abdikarimov, Khodzhaev [27], and other researchers [28–31] that are in a general agreement with evaluable experiments' data.

This work aims to study the influences of the viscoelastic and anisotropic properties of a material on the dynamical behavior of the plate. For the first time, an integral model is used to consider the viscoelastic properties of materials of an anisotropic structure. The integral mode correctly describes the rheological processes occurring in the studied structure during the entire time. The presented mathematical model makes it possible to obtain sufficiently accurate solutions that are well combined with experimental results.

2. Materials and Methods

The classical Kirchhoff-Love theory is used to construct a mathematical model of the dynamic behavior of a plate made of a material having anisotropic viscoelastic properties under the influence of various external loads (Fig. 1).

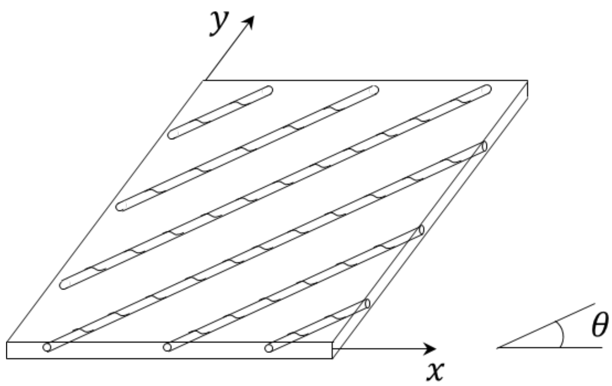

Figure 1. Anisotropic fiber-reinforced rectangular plate.

In this case, following [32–34], the stress and moment resultants have the form:

$$\begin{aligned}
N_x &= A_{11}^*\varepsilon_x + A_{12}^*\varepsilon_y + A_{16}^*\gamma_{xy} + B_{11}^*\chi_x + B_{12}^*\chi_y + B_{16}^*\chi_{xy},\\
N_y &= A_{12}^*\varepsilon_x + A_{22}^*\varepsilon_y + A_{26}^*\gamma_{xy} + B_{12}^*\chi_x + B_{22}^*\chi_y + B_{26}^*\chi_{xy},\\
N_{xy} &= A_{16}^*\varepsilon_x + A_{26}^*\varepsilon_y + A_{66}^*\gamma_{xy} + B_{16}^*\chi_x + B_{26}^*\chi_y + B_{66}^*\chi_{xy},\\
M_x &= B_{11}^*\varepsilon_x + B_{12}^*\varepsilon_y + B_{16}^*\gamma_{xy} + D_{11}^*\chi_x + D_{12}^*\chi_y + D_{16}^*\chi_{xy},\\
M_y &= B_{12}^*\varepsilon_x + B_{22}^*\varepsilon_y + B_{26}^*\gamma_{xy} + D_{12}^*\chi_x + D_{22}^*\chi_y + D_{26}^*\chi_{xy},\\
M_{xy} &= B_{16}^*\varepsilon_x + B_{26}^*\varepsilon_y + B_{66}^*\gamma_{xy} + D_{16}^*\chi_x + D_{26}^*\chi_y + D_{66}^*\chi_{xy},
\end{aligned} \tag{1}$$

where $A_{ij}^*,\ B_{ij}^*,\ D_{ij}^*,\ i,\ j=1,\ 2,\ 6$ are the operators having the following form:

$$A_{ij}^{*}\varphi=\sum_{k=1}^{K}\left(\bar{Q}_{ij}^{*}\varphi\right)_{k}\left(z_{k}-z_{k-1}\right),\quad B_{ij}^{*}\varphi=\frac{1}{2}\sum_{k=1}^{K}\left(\bar{Q}_{ij}^{*}\varphi\right)_{k}\left(z_{k}^{2}-z_{k-1}^{2}\right),$$

$$D_{ij}^{*}\varphi=\frac{1}{3}\sum_{k=1}^{K}\left(\bar{Q}_{ij}^{*}\varphi\right)_{k}\left(z_{k}^{3}-z_{k-1}^{3}\right),$$

$$\bar{Q}_{11}^{*}\varphi=\left[Q_{11}\cos^{4}\theta+\frac{1}{2}\left(Q_{12}+2Q_{66}\right)\sin^{2}2\theta+Q_{22}\sin^{4}\theta\right]\left(1-\Gamma^{*}\right)\varphi,$$

$$Q_{11}=\frac{E_{1}}{1-\mu_{12}\mu_{21}},$$

$$\bar{Q}_{12}^{*}\varphi=\left[\frac{1}{4}\left(Q_{11}+Q_{22}-4Q_{66}\right)\sin^{2}2\theta+Q_{12}\left(1-\frac{1}{2}\sin^{2}2\theta\right)\right]\left(1-\Gamma^{*}\right)\varphi,$$

$$Q_{22}=\frac{E_{2}}{1-\mu_{12}\mu_{21}},$$

$$\bar{Q}_{22}^{*}\varphi=\left[Q_{11}\sin^{4}\theta+\frac{1}{2}\left(Q_{12}+2Q_{66}\right)\sin^{2}2\theta+Q_{22}\cos^{4}\theta\right]\left(1-\Gamma^{*}\right)\varphi,$$

$$\bar{Q}_{16}^{*}\varphi=\left[Q_{11}\sin\theta\cos^{3}\theta-\frac{1}{4}\left(Q_{12}+2Q_{66}\right)\sin 4\theta-Q_{22}\sin^{3}\theta\cos\theta\right]\left(1-\Gamma^{*}\right)\varphi,$$

$$\bar{Q}_{26}^{*}\varphi=\left[Q_{11}\sin^{3}\theta\cos\theta+\frac{1}{4}\left(Q_{12}+2Q_{66}\right)\sin 4\theta-Q_{22}\sin\theta\cos^{3}\theta\right]\left(1-\Gamma^{*}\right)\varphi,$$

$$\bar{Q}_{66}^{*}\varphi=\left[\frac{1}{4}\left(Q_{11}-2Q_{12}+Q_{22}\right)\sin^{2}2\theta+Q_{66}\cos^{2}2\theta\right]\left(1-\Gamma^{*}\right)\varphi, \tag{2}$$

$$Q_{12}=\frac{E_{1}\mu_{21}}{1-\mu_{12}\mu_{21}}=\frac{E_{2}\mu_{12}}{1-\mu_{12}\mu_{21}},\ Q_{66}=G_{12},\ \Gamma^{*}\varphi=\int_{0}^{t}\Gamma\left(t-\tau\right)\varphi\left(\tau\right)d\tau.$$

Here K is the number of plate layers, E_1, E_2 are the elastic moduli, G_{12} is the shear modulus, μ_{12}, μ_{21} are the Poisson ratios, θ is the angle characterizing the direction of the fibers relative to the axis OX, Γ^{*} is the integral operator with the relaxation kernel $\Gamma(t)$.

The relationship between the strains in the median surface ε_x, ε_y, γ_{xy}, χ_x, χ_y, χ_{xy} and displacements u, v, w in directions x, y, z, written by considering the von Kármán type of geometric nonlinearity, in the form [32–35]:

$$\varepsilon_{x}=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^{2},\ \varepsilon_{y}=\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^{2},\ \gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y},$$
$$\chi_{x}=-\frac{\partial^{2}w}{\partial x^{2}},\ \chi_{y}=-\frac{\partial^{2}w}{\partial y^{2}},\ \chi_{xy}=-2\frac{\partial^{2}w}{\partial x\partial y}. \tag{3}$$

By substituting (1) and (3) into the equations of motion

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}+p_x=\rho\frac{\partial^2 u}{\partial t^2},\quad \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}+p_y=\rho\frac{\partial^2 v}{\partial t^2},$$
$$\frac{\partial^2 M_x}{\partial x^2}+\frac{\partial^2 M_y}{\partial y^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial}{\partial x}\left(N_x\frac{\partial w}{\partial x}+N_{xy}\frac{\partial w}{\partial y}\right)+ \tag{4}$$
$$+\frac{\partial}{\partial y}\left(N_{xy}\frac{\partial w}{\partial x}+N_y\frac{\partial w}{\partial y}\right)+q=\rho\frac{\partial^2 w}{\partial t^2},$$

the system of nonlinear integro-differential equations in partial derivatives that satisfies the boundary conditions of the problem (the edges are hinged) is obtained. The solution of this system is sought in the form:

$$u(x,y,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}u_{mn}(t)\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b},$$
$$v(x,y,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}v_{mn}(t)\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}, \tag{5}$$
$$w(x,y,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}w_{mn}(t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b},$$

where $u_{mn}(t)$, $v_{mn}(t)$ and $w_{mn}(t)$, $m,\ n=1,\ 2,\ 3,\ \ldots$ are the unknown functions of time. Note that eq.(5) gives just an approximation of the in-plane displacements u and v for the given boundary conditions since additional functions should be retained. Substituting the approximating functions (5) into the resulting system of equations and performing the procedure of the Bubnov-Galerkin method, a system of nonlinear ordinary integro-differential equations is obtained. Further, this system is integrated using the numerical method based on the use of quadrature formulas [24, 25].

3. Results and Discussion

The calculations use the simplest and, at the same time, quite common weakly-singular Koltunov-Rzhanitsyn kernel having the form $\Gamma(t)=Ae^{-\beta t}t^{\alpha-1}\left(0<\alpha<1\right)$ as the relaxation kernel, where A, α, β are the rheological viscosity parameters determined from the experiments [21, 23]. Experimental studies have shown that weak singular functions most accurately describe the rates of relaxation processes [21, 23]. One of the difficulties of using singular kernels in numerical calculations is their singularity at t = 0. A detailed description of the numerical method based on the use of quadrature formulas is given in [24, 25]. The method allows solving systems of nonlinear integro-differential equations, preliminarily transforming singular kernels into regular ones.

In the calculations, the following parameters for plastics: KAST-V (E_1 = 25.5 GPa, E_2 = 14.91 GPa, G_{12} = 4.41 GPa, μ_{12} = 0.2, ρ = 1900 kg/m³) and EDF (E_1 = 24.5 GPa, E_2 = 18.04 GPa, G_{12} = 4.1 GPa, μ_{12} = 0.18, ρ = 1830 kg/m³) have been used. To determine the rheological parameters of the viscosity of these materials for different directions of reinforced fibers, the results of experiments given in [36] have been used. In this paper, based on the experiments, the rheological parameters of the viscosity for some glass-reinforced plastics, such as Textolite, KAST-V, SVAM, and EDF have been obtained. The parameters given in the paper correspond to the samples with directions of reinforced fibers of 0, 45, and 90 degrees relative to the OX axis. The values of the rheological viscosity parameters for samples with an arbitrary fiber orientation were obtained in [37] from experimental results by using interpolation formulas.

The problem of buckling of the anisotropic fiber-reinforced rectangular plate of thickness h with the sides a and b, subjected to dynamic compression along with one of the sides by force $P(t)=P_0 t$ (P_0 is the loading rate) was considered as follows. A problem with a similar mathematical formuation (elastic problem) was considered in [29].

Hereinafter, the following parameters of the plate are used in the calculations (unless otherwise specified): $a = b = 0.5m,\ h = 0.5sm,\ \theta = 45^{\circ},\ P_0 = 2\,MPa/s.$

As a criterion determining the critical time, it is assumed that the sag of the deflection should not exceed a value equal to the thickness of the plate [35]. In shell structures, the greater the critical time, the more stable it is to dynamic loads.

The following graphs correspond to the results obtained for the midpoint of the simply supported plate. On the graphs, m (meter) is the dimension for the deflection, and s (second) is for time.

Fig. 2 shows the influence of the material's viscoelastic properties on the behavior of anisotropic reinforced plates made from KAST-V and EDF. In general, KAST-V and EDF are materials with different viscoelastic properties. It is evident from the figure that considering viscoelastic properties results in a decrease of the critical time. Especially, it is apparent for plates from KAST-V (A = 0.0208), which have more viscoelastic properties than EDF. The difference in critical times for plates from elastic and viscoelastic KAST-V reaches up to 21 %, whereas this difference is about 2.4 % for plates from EDF. This result shows that it is crucial to consider viscoelastic properties in solving this type of problem.

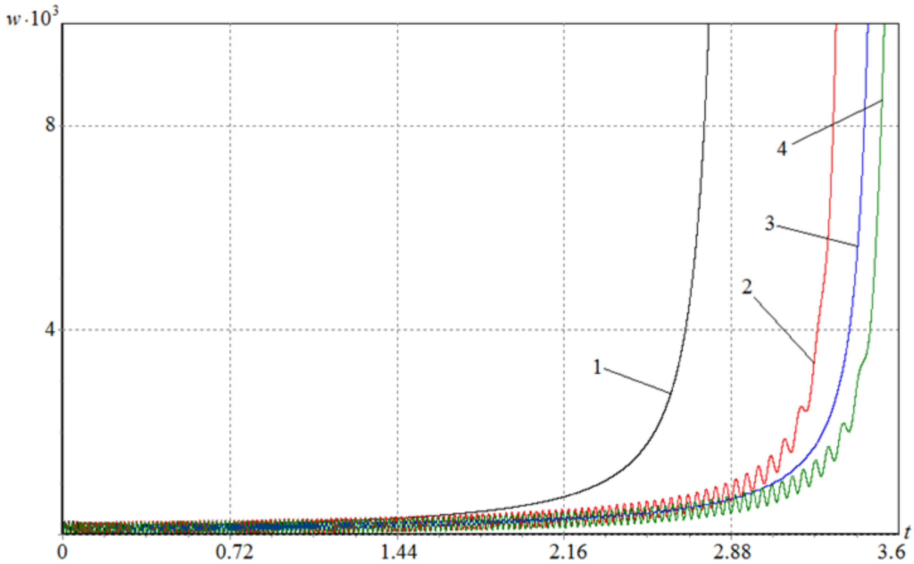

Figure 2. Effect of viscoelastic properties on the behavior of anisotropic reinforced plate from KAST-V (1 – viscoelastic, 2 – elastic) and EDF (3 – viscoelastic, 4 – elastic).

Fig. 3 shows a graph of the changes in the deflections of the midpoints of the plates of various thicknesses. The results show that an increase in plate rigidity due to increased plate thickness leads to a proportional increase in the critical time value.

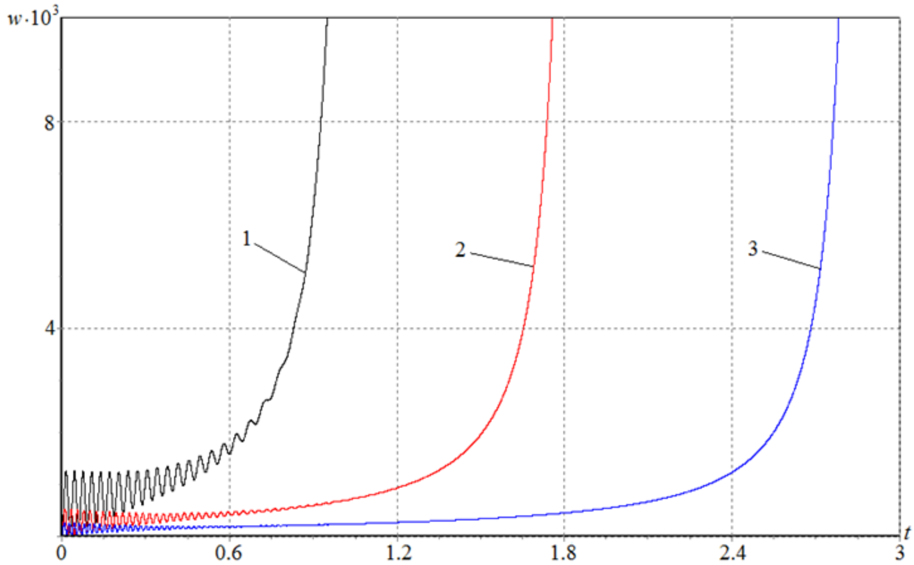

Figure 3. Dependence of the deflection on time for various values of the thicknesses of the plate: 1 – h = 0.3 *sm*; 2 – h = 0.4 *sm*; 3 – h = 0.5 *sm*.

Fig. 4 shows the analogous results for λ = 1; 1.2; 1.4. Here λ is the ratio of plate edges. Therefore, if λ = 1, then the plate is square-shaped. As it is clear from the graph, the increase in one of the edges of the plate increases the critical time.

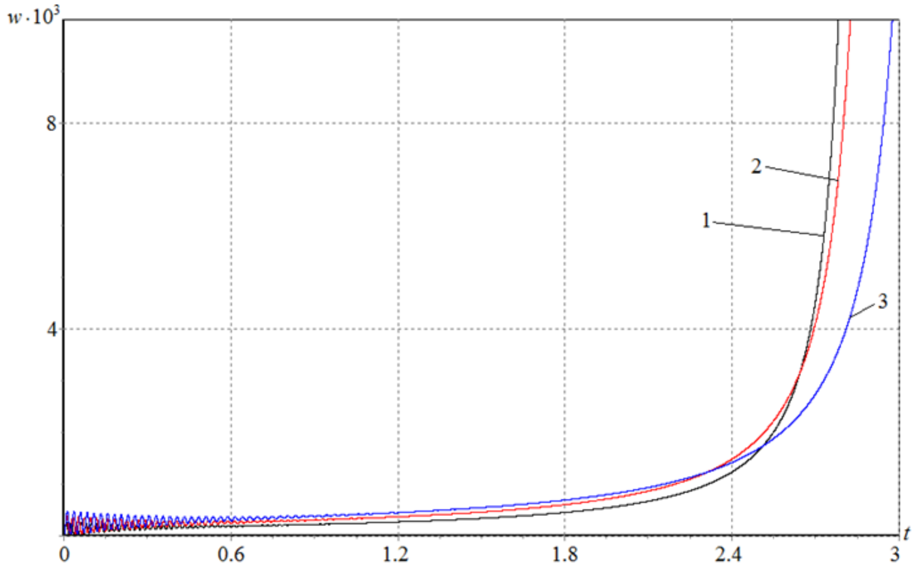

Figure 4. Dependence of the deflection on time for various values of the parameter λ (λ = *a/b*) 1 – λ = 1; 2 – λ = 1.2; 3 – λ = 1.4.

The various curves in Fig. 5 correspond to cases of changes in the deflections of the midpoint of a reinforced rectangular plate at different loading speeds. It should be noted here that in all cases, at the initial moments of time, the changes in the deflections are oscillations that are harmonic in shape, which begin to increase rapidly at certain points in time.

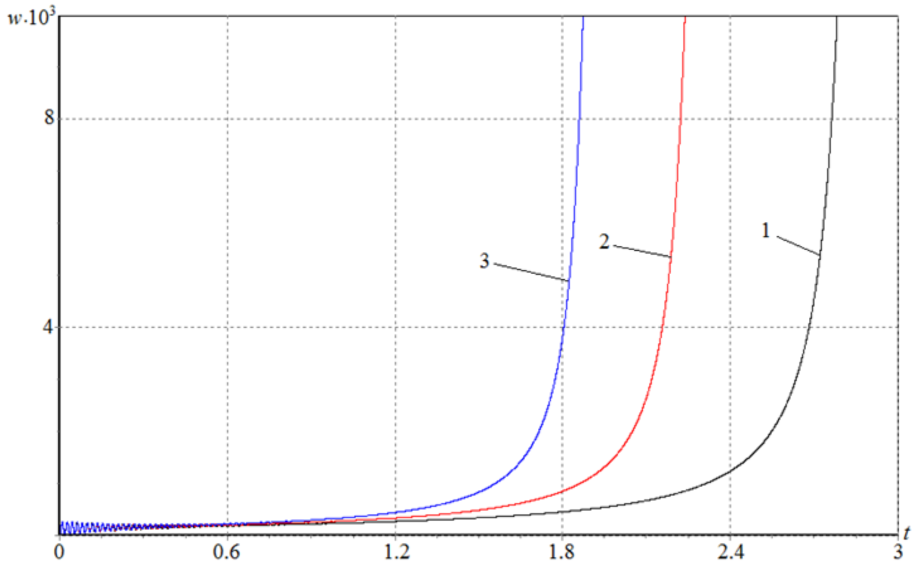

Figure 5. Dependence of the deflection on time for various values of the velocities of loading 1 – P_0 = 2 *MPa/s*; 2 – P_0 = 2.5 *MPa/s*; 3 – P_0 = 3 *MPa/s.*

Fig. 6 shows the influence of changes in the direction of the fibers of the reinforced plate on the dynamic process. As the angle of direction of the fibers increases from 0 to 45 degrees, an increase in the critical time is observed. The difference between the critical time values for single-layer plates with fiber directions of 0 and 45 degrees is 12 %.

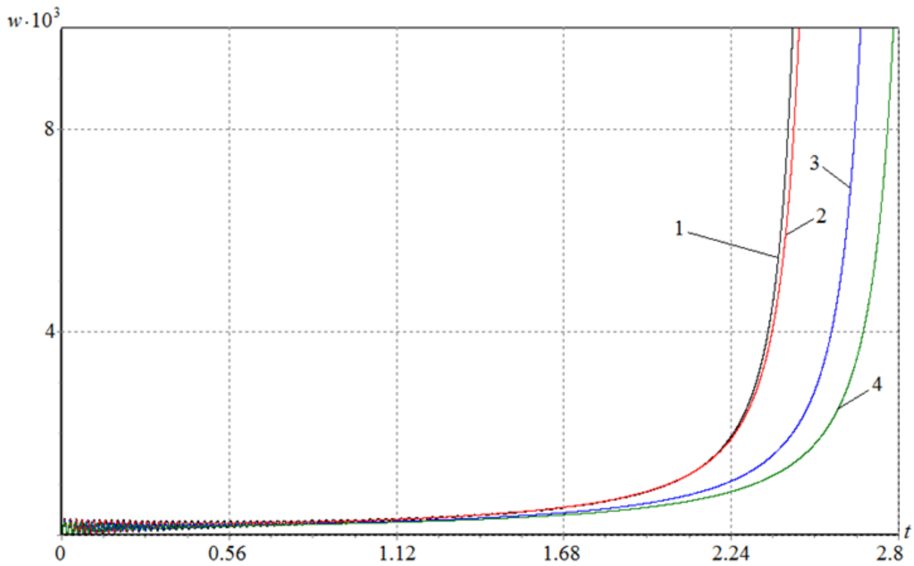

Figure 6. Dependence of the deflection on time for plates with different fiber orientations 1 – 0°; 2 − 15°; 3 − 30°; 4 – 45°.

As mentioned earlier, reinforced composites are a set (composition) of several reinforced layers, each with its own mechanical properties. Thus, by changing the composite structure, it is possible to create contractures, the behavior of which can be predicted in advance.

In this regard, the study of the behavior of laminated reinforced plates with different fiber orientations under the influence of axial compression is of particular interest. Fig. 7 shows the changes in the deflections of the midpoints of laminated reinforced plates made of KAST-V. Moreover, although all these plates have different fiber directions, their thickness is the same. The results show that for two-layer plates with fibers located at an angle of –45 degrees relative to the OX axis in one layer and 45 degrees in another, the critical time values are higher than the others. The layered fiber plate, parallel and perpendicular to the OX axis, has a lower critical time (i.e., it is less stable) than other plates with similar mechanical properties. The difference between the critical time values for the above two-layer plates is 13.3 %.

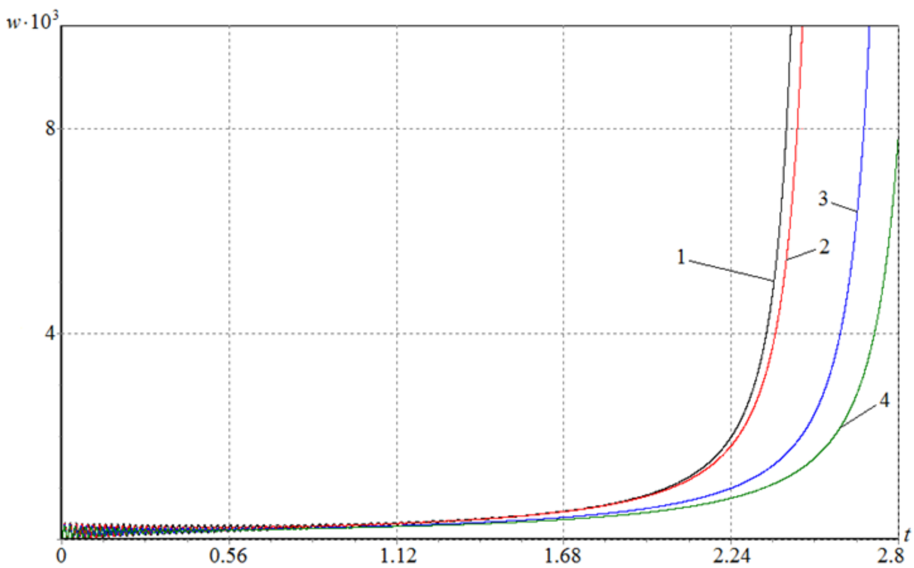

Figure 7. Dependence of the deflection on time for sandwich plates with different fiber-orientated layers 1 – 0°/90°; 2 – 15°/-15°; 3 – 30°/-30°; 4 – 45°/-45°.

The results of studies of the behavior of reinforced plates for a wide range of changes in their mechanical, physical, and geometric parameters under dynamic compression of one of their sides are shown in Table 1.

Table 1. Critical time values for various geometric and physical parameters of anisotropic fiber-reinforced plates from KAST-V.

№	Geometrical parameters of the plate			Physical parameters		Number of layers	Fiber orientations	Critical time values		
	a, *m*	b, *m*	h, *sm*	q, *Pa*	v, *MPa/s*			Elastic problem	Viscoelastic problem	differ. (in %)
1	0.5	0.5	0.5	100	2	1	45°	3.2798	2.7117	21
2	0.6	0.5	0.5	100	2	1	45°	3.3358	2.7338	22
3	0.7	0.5	0.5	100	2	1	45°	3.5262	2.8604	23.3
4	0.5	0.5	0.4	100	2	1	45°	2.0238	1.6522	22.5
5	0.5	0.5	0.3	100	2	1	45°	0.9192	0.7680	19.7
6	0.5	0.5	0.5	200	2	1	45°	3.2110	2.5986	23.6
7	0.5	0.5	0.5	300	2	1	45°	3.1422	2.4982	25.8
8	0.5	0.5	0.5	100	2.5	1	45°	2.6268	2.1806	20.5
9	0.5	0.5	0.5	100	3	1	45°	2.1858	1.8248	19.8
10	0.5	0.5	0.5	100	2	1	0°	2.5984	2.3884	8.8
11	0.5	0.5	0.5	100	2	1	15°	2.7640	2.4038	15
12	0.5	0.5	0.5	100	2	1	30°	3.1046	2.6044	19.2
13	0.5	0.5	0.5	100	2	2	0°/90°	2.5984	2.3830	9
14	0.5	0.5	0.5	100	2	2	15°/-15°	2.7860	2.4152	15.4
15	0.5	0.5	0.5	100	2	2	30°/-30°	3.1396	2.6344	19.2
16	0.5	0.5	0.5	100	2	2	45°/-45°	3.3242	2.7496	20.9
17	0.5	0.5	0.5	100	2	3	45°/-45°/45°	3.2900	2.7168	21.1

The numerical results in the table show that taking into account the viscoelastic properties of structural materials leads to a significant decrease in the critical time value. In some cases, the differences in the results obtained by solving elastic and viscoelastic problems differ from each other by more than 20 %. It is also shown that the direction of the fibers in the structures also significantly affects the value of the critical time. For example, in two-layer plates, the fibers of which have directions at an angle of 45 and –45 degrees, the critical time values are greater than in a single-layer plate of the same thickness, the fibers of which are located at an angle of 45 degrees. This means that by changing the structure of the composite material, it is possible to create structures more resistant to dynamic loads.

4. Conclusions

The study of the dynamic stability of viscoelastic anisotropic reinforced plates, subjected to dynamic compression along with one of their sides, shows:

- It is important to consider the viscoelastic properties of the construction material. The results indicate that the difference in the critical time of elastic and viscoelastic problems is 21 % for plates made from KAST-V.
- The critical time values mainly depend on the direction of the reinforced fibers in each layer. In single-layer and double-layer plates, the difference in the critical time values depending on the direction of the reinforced fibers in places is 12 % and 13.3 %, respectively. An analysis of the results shows that the most resistant to these loads are double-layer plates with fibers located at an angle of –45 degrees relative to the OX axis in one layer and 45 degrees in another.

The results obtained in the work and the conclusions drawn on their basis allow us to accurately predict the dynamic behavior of plates made of reinforced plastics.

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