Problem of the Week: A Student-Led Initiative to Bring Mathematics to a Broader Audience

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Abstract

Problem of the Week (POW!) is a weekly undergraduate mathematics competition hosted by two graduate students from the UNO Math Department. It started with the goal to showcase variety, creativity, and intrigue in math to those who normally feel math is dry, rote, and formulaic. Problems shine light on both hidden gems and popular recreational math, both math history and contemporary research, both iconic topics and nontraditional ones, both pure abstraction and real-world application. Now POW! aims to increase availability and visibility in Omaha and beyond. Select problems from Fall 2021 to Spring 2023 are highlighted here: these received noteworthy engagement and praise from participants and observers.

Goals

- Show math to students that they have never seen or heard of, and may never have otherwise.
- Evangelize various areas of math; inspire student interest.
- Provide a challenge for competitive personalities.
- Teach participants to research, experiment, and wonder, not just recall methods and follow directions.
- Minimize barriers to learning, maximize community outreach.
- Advertise tools and mathematical resources on campus.

Tiers

POW! has three difficulty tiers corresponding roughly to easy/medium/hard. The tiers are named after the classification of conic sections used widely throughout math and physics. Each week gets three problems, one per tier.

- Elliptic: Puzzles for non-math types.
- Parabolic: Exercises for math types.
- Hyperbolic: Challenges for math whizzes.

Tiers are decided by degree of background needed to understand and solve problems. By having participants compete only against others of similar mathematical maturity, participation is more rewarding for fresher minds. Each Spring the top nine scorers (three from each tier) receive prizes in a small ceremony.

Elliptic Problems

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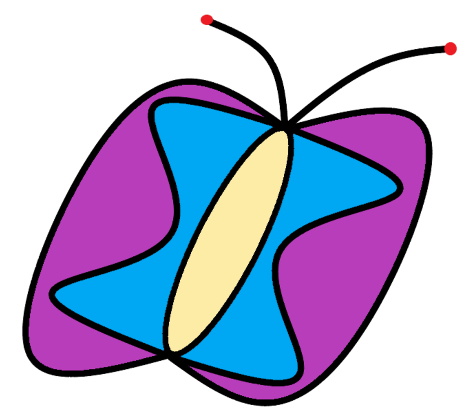


Problem: How many ways are there to trace this butterfly? You cannot lift your pen and must start from the left antenna.

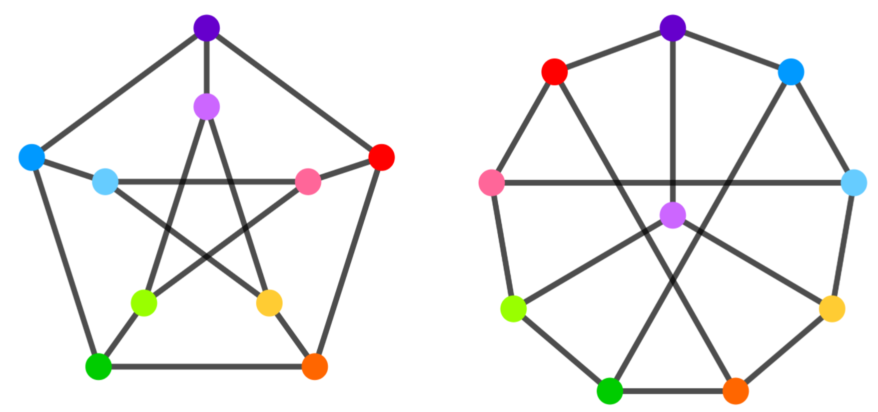
Differentiating Numbers

The arithmetic derivative D is a function satisfying the "product rule" $D(a \cdot b) = D(a) \cdot b + a \cdot D(b)$ for all integers a and b, and $D(p) = 1$ for prime numbers p. **Problem**. Find $D(168)$.

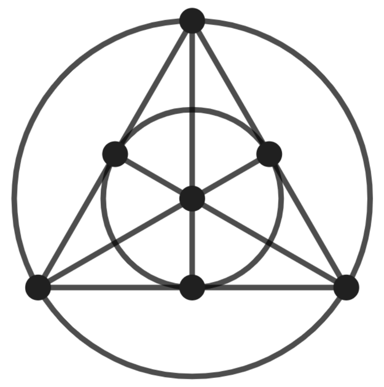
Parabolic Problems

Perspective Shift

A graph (vertices & edges) can be drawn many ways. Often it's impossible to illustrate all of a graph's symmetry in a single drawing. To compensate, we can use multiple different drawings to illustrate different kinds of symmetry of the same graph, e.g. the Peterson graph can be drawn to illustrate either fivefold or threefold symmetry as shown in this pair of color graphs:



Problem. Redraw the black graph to show *fourfold* symmetry:



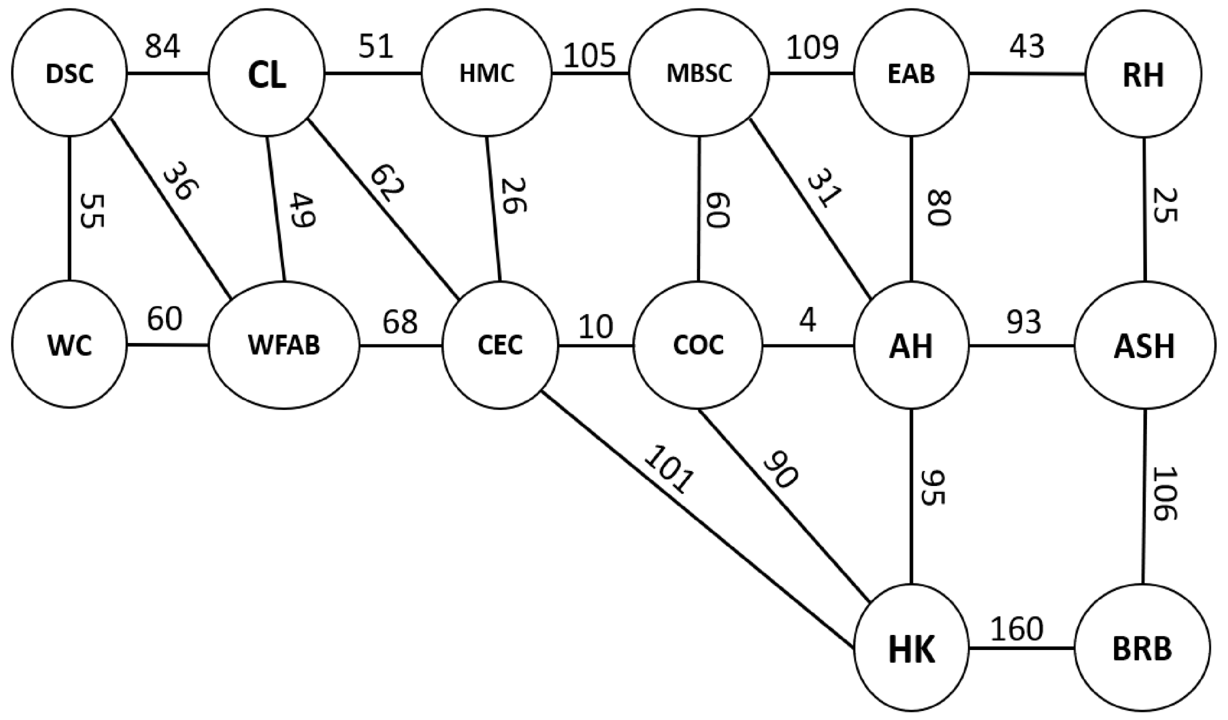
(Currently, of course, the graph illustrates threefold symmetry.)

Hint. Instead of drawing a simple 2D figure, lift some pieces off the page and imagine instead a 3D figure!

Campus Stroll

We're about to be late for our Graph Theory class in Durham Science Center (DSC), but right now we're all the way over in the Biomechanics Research Building (BRB).

This diagram records the time (in seconds) to walk between neighboring campus buildings

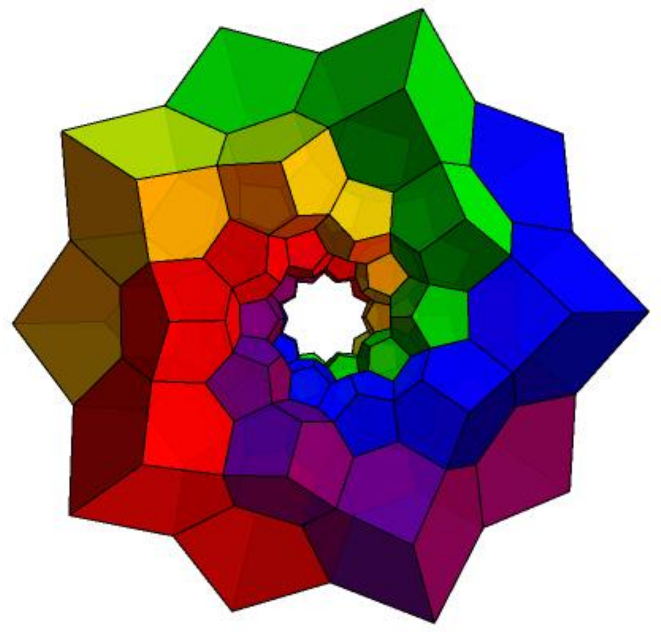


Problem: What path from BRB to DSC is quickest? Slowest?

Hyperbolic Problems

Good Fibrations

Three-dimensional space is filled by gluing 120 dodecahedra face-to-face in a particularly symmetric fashion. A **bundle** is a way of partitioning these dodecahedra into a dozen rings each consisting of ten dodecahedra. Every neighboring pair of dodecahedra can be extended to a unique ring, and every ring can be extended to a unique bundle.



Five rings of one bundle are shown above. (Inevitably, one or more dodecahedra will be "inside out" and infinitely large, but this will not be an issue for counting purposes.)

Problem. How many bundles are possible?

Gilded Radix

Our decimal number system uses base ten, where each digit represents a multiple of a power of ten, for example,

$$168 = 1 \cdot 10^2 + 6 \cdot 10^1 + 8 \cdot 10^0.$$

In **phinary**, all numbers are represented uniquely as sums of distinct, nonconsecutive powers of the golden ratio $\varphi = \frac{1+\sqrt{5}}{2}$

Problem. Express the nth Fibonacci number F_n in phinary.

Results

POW! has improved UNO participation in the Putnam Competition / Putnam Fridays and Honors Calculus problem-solving sessions and thus is settling alongside them as a critical part of our Math Department's pipeline for scouting and nurturing high-potential mathematics students. Upper-level courses and other departments here have talked about POW! and used its problems; a majority of top competitors are from local high schools; and even middle school teachers have reached out about kids with questions. Moreover, people have participated from out of state and even out of country.

Last Spring a 250+ page compendium of problems, solutions, and commentaries was published, with affordable pricing for physical copies and a free eBook version for download which is just starting to garner international attention. A second volume is anticipated. Next to POW!'s online fingerprint, its digital management has been made flexible and scalable for future hosts to smoothly inherit.

Bibliography

Goucher, A. (2021, April 24). *Good Fibrations*. Complex Projective 4-Space.

Horner, B. & Sahs, J. M. (2022). *UNOmaha DigitalCommons*. UNOmaha Problem of the Week (2021-2022 Edition).

Acknowledgements

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