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Lifting Work and Building Time at the 4th Dynasty Pyramids

Rolf KRAUSS

Earlier studies about how long it took to build the great stone pyramids focused on the cubic amounts of masonry which is not an adequate parameter for determining the achievement of pyramid builders. Rather, the appropriate parameter is lifting work measured in Joules – in brief, the product of mass multiplied by lifting height. Provided lifting of a block took on average the same time at the 4th dynasty pyramid construction sites, then there was a proportion between lifting work and building time. The upper limits of the resulting building times depend on the length of Kheops's reign.

Operaciones de levantamiento y tiempo de construcción en las pirámides de la Dinastía IV
Los estudios previos acerca del tiempo empleado para construir las grandes pirámides de piedra se han centrado en su volumen en metros cúbicos de la piedra utilizada. Sin embargo, este no es un parámetro adecuado para determinar el logro de los constructores de las pirámides. Por el contrario, el parámetro apropiado es la fuerza de elevación medida en julios, es decir, el producto de masa multiplicado por altura elevada. Dado que el levantamiento de un bloque tomaría, como promedio, el mismo tiempo en todas las localizaciones de construcción de pirámides, se puede deducir que hubo una relación proporcional entre trabajo de levantamiento y tiempo de construcción. Los límites resultantes más altos de tiempo de construcción dependen de la duración del reinado de Khufu.

Keywords: Red Pyramid, building time, microgravimetry, muography, epigraphy, Lepsius.
Palabras clave: Pirámide roja, tiempo de construcción, microgravimetría, muografía, epigrafía, Lepsius.

In the 1980s Egyptologists Dieter Arnold and Rainer Stadelmann searched for correlations between the reign lengths of Snofru, Kheops, and Khephren and their enormous pyramid building activities.[1] The Royal Turin Canon (TP) gives Snofru 24 regnal years equivalent to calendar years, 23 to Kheops, and 20+x to Khephren. If the year figures are accepted, then these kings or their working crews managed, respectively, as much as 428, 322, and 231 cubic meters of masonry on average per day. The regnal years in the TP may very well be vitiated by scribal copying mistakes over the centuries. Stadelmann suggested that Snofru actually reigned for 45 years, Kheops 30 to 32, and Khephren 26 to 28 years, implying average cubic amounts of masonry which he deemed to be achievable. For the purposes of comparison, he cited the

* I thank my friend and colleague Juan Antonio Belmonte for commenting on a first draft of the ms, and I acknowledge thankfully the muographic comments by Hiroyuki Tanaka. I am obliged to the peer reviewers who will find that I took care of the slips they pointed out, one by one. In particular I changed my loose use of the term density and added a paragraph about horizontal work.

1 Arnold 1981: 15–28; Stadelmann 1980: 437–449; Stadelmann 1983: 225–241; Stadelmann 1987: 229–240; Stadelmann 1990a: 226–227; Stadelmann 1990b: 258–261. For a volumometric statistic of Snofru's pyramids with remarks about building time, complementing the ones by Arnold and Stadelmann, see Monnier 2018: 16, Table 1.

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construction process at the first Aswan Dam and a figure of 300 cubic meters masonry per day as accomplished during its construction. I have shown in detail that the daily achievement surpassed 300 cubic meters of masonry by far, and that the building technique used at the first Aswan Dam is an inappropriate model for pyramid building.[2] In a reaction to Stadelmann's approach Jürgen von Beckerath maintained that pyramid building achievement should be estimated on the basis of what is known about each reign length.[3]

My own attempt to determine how long it took to build the Red Pyramid (Dahshur-North) of Snofru in particular was based on building dates found on blocks of the pyramid. These express the regnal years in terms of a census, expressed as *renpet zep n* and *renpet (m-) khet zep n* (*year of the nth time* and *year after the nth time*). According to traditional Egyptological interpretation the census took place biennially but also occasionally in successive years; this manner of counting years went out of use after the Old Kingdom. Recently John Nolan has suggested the alternative that the years *renpet m-khet zep* correlate to lunar intercalation years.[4] According to a paraphrase by Rita Gautschy *et alii* a *renpet m-khet zep* "was employed if an intercalary month was inserted into the lunar calendar at the end of the preceding year in order to keep it in line with the siderial and solar year. Nolan's hypothesis requires a reduction of the number of regnal years usually assigned to Old Kingdom Pharaohs by about 21% and hence a shortening of Old Kingdom chronology".[5] The procedure results, for example, in a shortening of the usually accepted 64 regnal years for Pepy II to 54 years. But there is another hypothesis which comprises a regular biennial census in the time of Pepy II and implies about 64 regnal years.[6] My argument about the building times of the Fourth Dynasty pyramids as presented below, refers in any case to calendar years which can be translated into any system of counting regnal years.

The dates which were available to me for computing the building time of the Red Pyramid were a *renpet-zep* 15 on the southwestern corner stone of the foundation (under the first course), in the following block A, further a *renpet zep* 15 and 16 respectively on two backing stones of the revetment, in the following blocks B and C (for details see further below). Casing blocks and backing stones of Tura limestone have been stripped off the four faces of the Red Pyramid above the first five courses or so; the core blocks of local limestone remained in place. The stripping off created rubble heaps on the surface and rubble on the faces of the pyramid.[7] In 1983 Stadelmann described the position of block B when found as *directly above course 12*, but in 1986 as *within course 12*; the find spot of block C he described in 1983 as *six courses above block B*, and later, in 1986, as *course 16/17*.[8] By deducing a deceleration

2 Krauss 2017a: 92–96.

3 Beckerath 1997: 158.

4 Nolan 2003: 75–98.

5 Gautschy *et alii* 2017: 69–108.

6 Krauss 2008: 377–385.

7 See Stadelmann 1982: 381, for remarks about removing rubble.

8 Stadelmann 1982: 235: "Unmittelbar über der 12. Steinlage fand sich ein abgesprengter *backing stone*, der aufgrund der Fundlage nicht sehr viel weiter von oben kommen kann, mit einem Datum des *15. Jahres, 2. Monats der šemu-Jahreszeit, Tag 14*. Sechs Steinschichten darüber lag ein weiteres Bruchstück mit einem Datum des *16. Jahres, 3. Monats der achet-Jahreszeit und letztem Monatstag*". Stadelmann 1987: 234: "Einer der Steine mit dem Datum des 15. Males wurde in der 12. Steinlage gefunden, der zweite mit einem Datum des 16. Mals in 16./17. Lage, in etwa 12 m Höhe …".

rate from the dates and find spots of blocks A, B, and C, I arrived at a building time between 10 and 11 years.[9] Not only did I make a computational error;[10] moreover the heights of the courses where blocks B and C were found, turned out to be incorrectly given by Stadelmann. He had counted the courses and computed their heights by using a mean value, rather than measuring the heights or using the measurements which are available in or deducible from the literature. The differences between computed and measured heights of courses 1 to 12, and 12 to 17 result in corrections of about 150% in the case of volumes and corrections of more than 200% in lifting work.[11] Finally in 2008 Stadelmann gave a description of the courses where blocks B and C were found which differed from those of the 1980s:[12] "… the 15th time [block B comes] from levels under the 12th layer, whereas the 16th time of counting [block C] was found on the back face of a backing stone still on the 16th layer".

Furthermore, Roman Gundacker argued for corrected readings of graffiti Maidum nos. 18 and 22 from the reign of Snofru, reading *renpet m-khet zep* 15 and 16 respectively, rather than *renpet zep* 15 and 16.[13] The correction of Maidum no. 18 would add another calendar year between the dates of blocks A and B, whereas I had presumed that blocks A and B were laid in one and the same calendar year. Although Gundacker's arguments based on epigraphy are sound, the possibility remains that the earlier readings were correct, since the epigraphic situation is difficult, allowing different interpretations. Furthermore, he points out the possibility that a dated building graffito on a backing stone does not indicate when the block was put in place; rather it notes the date when the block was stored after arriving from the quarry.[14] If so, the backing stone graffiti would inform about the progress of the construction though with delay. On the other hand, Stadelmann rejected early on the explanation of the building graffiti as referring to the transport from the quarry.[15] Later he cited the mastaba of Ptahshepses where "most certainly every stone of the casing had a date determining the day or week [of ten days], when it was put into its place in the building".[16] In other words, the chronological meaning of the building dates is open to interpretation; the blocks may have received dates not only on one occasion.

A revision of my attempt to deduce a deceleration rate on the basis of the building graffiti is not opportune at the moment, since there are unpublished building graffiti which should be considered.[17] Stadelmann mentioned:[18] "… not more than ca. 20 blocks,[19] graffiti with dates, mostly

9 Krauss 1996: 43–50; Krauss 1997: 1–14; Krauss 1998: 29–37.

10 Krauss 2017a: 89.

11 Krauss 2017a: 92.

12 Stadelmann 2008: 107.

13 Gundacker 2006: 42–43.

14 Gundacker 2006: 370.

15 Stadelmann 1982: 386 n. 18.

16 Stadelmann 2008: 104.

17 M. Lehner informed me in his email of 31 August 2020 that in Lehner 1997: 104, he made a mistake in citing Stadelmann on building graffiti.

18 Stadelmann 2008: 106–107.

19 Hemeda 2018: [9] asserts that "about every twentieth casing stone discovered had inscriptions on the back side". Stadelmann is not mentioned in the article, but the allusion to *every twentieth casing block* reads like a reflection of his *ca. 20 blocks* as cited above.

damaged and incomplete. The dates begin with a year of counting – *rnp.t sp* – 15, several times, and a year of counting 16, also several times …". In 2019 Stadelmann passed away, and since I shall follow him sooner rather than later I cannot wait until the eventual publication of the Dahshur building dates by those who inherited his excavation notes. Under these circumstances I risk another attempt at determining the time it took to build the Red Pyramid; with hindsight, it is at least possible to avoid falling for a second time into the same trap.

1 | Chronological constraints

Below I suggest a correlation between the building times of the Red Pyramid and the pyramids of Kheops and Khephren, presupposing that each king completed his pyramid during his own reign. The highest attested date of Kheops is *renpet khet zep* 13,[20] which would correspond to a 26th regnal year, provided the census was regularly biennial and a census coincided with his first full year after his accession. Actually attested are the *renpet zep* years 4 (?), 5, 8 10, and 12,[21] and *renpet m-khet zp* 12.[22] Note that the journal of Merer which records delivering Tura blocks for the pyramid of Kheops is not dated to a regnal year.[23]

A year 17 of Kheops which is mentioned in the 10th edition of William Flinders Petrie's *History of Egypt*,[24] is usually passed over in silence,[25] but it has been cited as possibly authentic,[26] and employed as chronologically valid though with a question mark.[27] Stadelmann referred to it as "ein 17. Mal der Zählung in einer der Entlastungskammern von Petrie beobachtet",[28] but he pointed also to Jean-Philippe Lauer:[29] "nous n'avons pu retrouver cette date sur aucune des inscriptions publiées par Perring ou par Lepsius. Petrie l'aura-t-il relevée lui-même sur place, mais sans la publier?". In Petrie's own publication of 1883 on the Giza pyramids he himself cited what Herodotus had to say about building the pyramid of Kheops within 20 years, but he said nothing about dated building graffiti in the pyramid in general nor in a relieving chamber in particular.[30] He described the relieving chambers in some detail,[31] although referring the reader to the accounts of Vyse as "most required for an account of … the chambers in the Great Pyramid over the King's Chamber …".[32] Finally I cite Anthony Spalinger who "suspect(s) that Petrie confused his data and mixed the evidence from Snefru at Meidum … with that of Cheops at Giza".[33] Evidently Petrie did not discover a dated graffito in 1880/81, but rather such a date crept somehow into the 10th edition of his *History*. The reader will find amusing information about Kheops and his year 17 in the article "Khufu" (Kheops) in Wikipedia.[34]

20 Kuhlmann 2005: 247–251.
21 Spalinger 1994: 283–285; Verner 2006: 124–128, 131–132; Verner 2008: 26–27.
22 Kuhlmann 2005: 245–246.
23 Tallet 2017.
24 Petrie 1923: 60.
25 Spalinger 1994: 285 n. 20, with literature.
26 Verner 2008: 26, n. 3.
27 Gautschy 2017: 85, 103.
28 Stadelmann 1987: 239, n. 1.
29 Lauer 1973: 134, n. 1.
30 Petrie 1883: 80–95.
31 Petrie 1883: 30–31.
32 Petrie 1883: XI.

For Khephren *renpet-zep* years 1, 5, 7, 10, 12, and 13 are attested, along with *renpet khet zep* 4 and 5.[35] On the basis of biographical information about *Nṯr(.j)-pw-nswt*,[36] Klaus Baer concluded a maximum reign of 25 years for Khephren. An inscription in *Nṯr(.j)-pw-nswt*'s tomb claims that he had been *nb jmꜣḫ* in the time of Djedefre, Khephren, Mycerinos, Shepseskaf, Userkaf, and Sahure: "Even assuming that Neṯeripunesut could have been *nb jmꜣḫ* as a child, this implies that he lived 65 years at least, longer if he was not a child under Djedefre. The reign of Khephren cannot, therefore, have been much longer than 25 years".[37]

The time it took overall to build the Red Pyramid is determined by a) laying three foundation courses;[38] b) *renpet-zep* 15 as the date on the southwestern corner block of the (upper course of the) foundation (block A); c) the construction of the crypts,[39] which Stadelmann tentatively dated to about *renpet zep* 13;[40] d) graffiti on backing stones citing *renpet zep* 15 and 16 from courses 12 to 17, middle of the east side (blocks B and C); e) a graffito on a casing or backing stone dated to *renpet zep* 16, seen by Carl Richard Lepsius in 1843 on the south side of the preserved casing and above the rubble (*LD* II 1f);[41] f) graffiti Dahshur-North nos. 7, 8 and 9 (Gundacker's numbering) as possible attestations of *renpet-zep* 24 which I discuss below in detail.

Stadelmann discovered graffiti nos. 8 and 9. As far as preserved, graffito no. 9 (fig. 1a) appears to read *renpet zep* 14; it was read as *renpet zep* 16 with a question mark by Hourig Sourouzian,[42] and as *renpet zep* 24 by Gundacker.[43] The latter rejected *renpet zep* 14 as a possible reading, citing block A of the foundation dated to *renpet zep* 15, implying that no casing block could date to an earlier year. Given his premise cited above, Gundacker should have considered the possibility that casing blocks arrived from the quarries and were stored in *renpet zep* 14 but laid in the following year. The fragment of a backing stone with graffito no. 9 was found in the area of the pyramid temple, and must have originated in a course above the preserved fifth course.[44]

33 Spalinger 1994: 285 n. 20.
34 <https://en.wikipedia.org/wiki/Khufu> accessed 15/05/2021.
35 Spalinger 1994: 286–288; Verner 2006: 133–134.
36 For the tomb of *Nṯr(.j)-pw-nswt* or Nesutpunüter, see Porter and Moss 1974: 278.
37 K. Baer, Unpublished Seminar papers Chicago: Old Kingdom Chronology 4 (a). The papers have been distributed widely among interested Egyptologists.
38 Stadelmann 1982: 382, mentions one layer of foundation stones. By contrast, Klemm and Klemm 2010: 59, describe the foundation as "three layers of large, fine [Tura] limestone blocks", confirmed by Rosemarie Klemm in an email of April 24, 2021.
39 Maragioglio and Rinaldi 1964: 136, observation no. 7.
40 Stadelmann 1987: 234.
41 Gundacker 2006: 53, 56. Lepsius reported another dated graffito without census indication (*LD* II 1f); for another dated graffito without census reported by Erbkam in 1843 see fig. 2 above.
42 Sourouzian 1982: 389.
43 Gundacker 2006: 55–56.
44 Stadelmann 1982: 386.

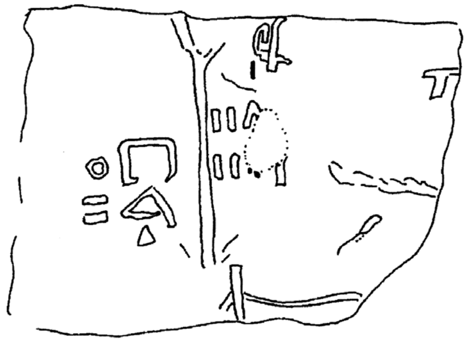

Figure 1a. Graffito no. 9.

Figure 1b. Graffito no. 8.

Stadelmann explained and argued: "All blocks with graffiti or the remains thereof which we discovered to date, originated in the lower 20 courses in the middle of the east face of the pyramid. There is little hope to find blocks with dates higher than year 15 and 16 in the rubble, since when the pyramid was stripped of its revetment, casing and backing stones got damaged in proportion to the height of their fall".[45]

Nevertheless, Gundacker suggests that the block with graffito no. 9 originated high up, and graffito no. 8 indeed confirms the possibility. Graffito no. 8 (fig. 1b) is written on a casing stone fragment found in the rubble on the ground.[46] Sourozian suggested reading *renpet zep* 24; Gundacker concurs.[47]

The fragment might be a leftover from the time when the pyramid was stripped of its uppermost casing stones. Stadelmann considered the find as *ausserordentlicher Glücksfall* (extraordinary happy coincidence). The vertical position of the numerals could indicate a census year, but also other numbers, except day numerals. The graffito shows neither a *renpet* nor a *zep* sign which can be combined with the numeral 24. Further, Gundacker notes the remains of other graffiti which possibly include a ship inter alia, "implying complications for the reading". Thus graffito no. 8 might refer to the 24th census but perhaps not.

Graffito no. 7 was reported by Georg Erbkam, architect of the Lepsius expedition, and published by Kurt Sethe and Ludwig Borchardt in Lepsius Denkmäler Text (LDT) 1.[48] Figure 2 presents the entry in Erbkam's Skizzenbuch. The publication has the graffito and three unconnected signs turned by 180° and it omits Erbkam's comment: "drawing with a red stylus in the northwest corner of the pyramid of Dashur".[49]

45 Stadelmann 1987: 237: "Je höher ein Verkleidungs- bzw. backing stone im Mauerwerk der Pyramide sass, desto stärker wurde er beim gewaltsamen Sturz beschädigt. Alle Steine mit Zeichenresten, die wir bisher gefunden haben, stammen aus den unteren Lagen der Mitte der Ostseite der Pyramide. Die Hoffnung, in den Schuttmassen Steine mit höheren Jahresangaben anzutreffen, ist daher gering".

46 Stadelmann 1987: 240, Abb. 4.

47 Gundacker 2006: 55.

48 Sethe and Borchardt 1897: 206. Borchardt, in his role as a specialist for architecture, assisted Sethe, see Erman, in Sethe and Borchardt 1899: IX; see also Krauss 2017b: 5.

49 "Rothstift(?)zeichnung in d(er) Nordwestecke d(er) Pyr(amide) v(on) Dashur". I thank Elke Freier for deciphering Erbkam's *Rothstiftzeichnung*, which will refer to the red ochre of the graffito.

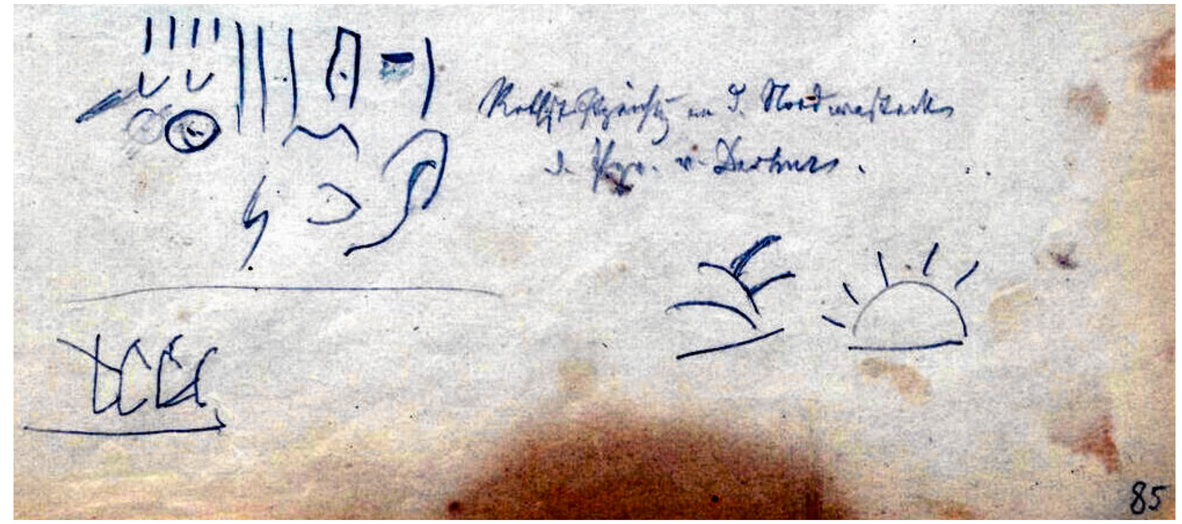

Figure 2. Erbkam's Skizzenbuch, p. 85. *SMB-PK Ägyptisches Museum und Papyrussammlung, Bucharchivalie Inv.-Nr. ÄM 98.*

In a later publication Borchardt read the two signs below Erbkam's remark as *kha*-hieroglyphs (SL N 28), meaning "*khawi*, the two (pyramids), (Snefru) appears".[50] Provided *kha*-hieroglyphs were meant, then not all of the sketch should be turned by 180°, rather the two hieratic lines only which implies that Erbkam saw and copied them upside down.

Sethe read the hieratic line as *renpet zep* 21 or 21 + x,[51] whereas Borchardt suggested 26.[52] Charles Maystre characterized both readings as *véritablement douteuse*;[53] he rendered Erbkam's copy as two vertical columns (fig. 3a).

Stadelmann accepted the 180° turn of Erbkam's copy as published in LDT 1. He postulated a *renpet* hieroglyph in front of the numeral 24, and he interpreted the three strokes which follow as numerals of III *akhet* (fig. 3b).[54] Gundacker concurs in principle, although correcting the reading in details. Since he cannot cite other examples for month numerals of the same relative length, he suggests halving the three strokes, thereby understanding the upper half as month numerals and the lower half as lotus buds (SL N8) with the meaning *akhe*t. He interprets as a sun disk what Stadelmann understood as *akhet*, to arrive at, finally, *renpet zep 24, III akhet sw* ...[55] By contrast, I accept Sethe's reading *month I shemu day* 24,[56] since it does not require any

50 Borchardt 1937: 16, n. 4.

51 Sethe 1905: 81, 119.

52 Borchardt 1937: 16, n. 4.

53 Maystre 1935: 96.

54 Stadelmann 1987: 235, Abb. 3.

55 Gundacker 2006: 54.

56 Sethe 1905: 119.

Figure 3a. After Maystre 1935, fig. 4.

Figure 3b. After Stadelmannn 1987, 235, Abb. 3.

emendations, and especially since the three long strokes are exemplary determinatives for *shemu*.

To sum up: graffito no. 8 perhaps attests *renpet zep* 24, but graffiti nos. 7 and 9 hardly can, if at all. In a draft of this paper, I argued on the basis of a *renpet zep* 24 as the presumed upper limit for the construction time of the Red Pyramid, but after renewed scrutiny I realized that *renpet zep* 24 is possibly attested only once, and definitely not for certain. It is clear that the lack of regnal years of Snofru after *renpet m-khet zep* 18 might well be coincidental, but it would be easier to presume the existence of a *renpet zep* 24, if a *renpet zep* 23 were unquestionably attested, which is not the case.[57]

The question of how the census operated in the time of Snofru remains. As is well known and often discussed, the Palermo Stone attests *renpet-zep* 8 of Snofru following on *renpet-zep* 7, both without an intervening *renpet khet zep* 7.[58] According to Pauline Posener-Kriéger's reading of the Maidum graffiti,[59] complemented by critical comments of Anthony Spalinger,[60] and Miroslav Verner,[61] and finally Gundacker's revised readings, the Maidum building graffiti contain the following regnal years of [Snofru]: *renpet (m-) khet zep* 10 (?), 13 (?), and 18 in a series of *renpet-zep* 6 (?), 7 (?), 8 (?), 9 (?), 12, 13, 14 (?), 15, 16, 17, 18, and 23 (?). As noted above, Gundacker argues in favor of a *renpet khet zep* 15 and a *renpet khet zep* 16 as census-free years. He opts for a regular census count in the reign of Snofru and explains the missing years *renpet khet zep* like Posener-Krieger did earlier,[62] viz. that "only rarely … a distinction was made between the years *renpet-zep* and *renpet khet zep*". While this is plausible, the length of Snofru's reign remains nevertheless unclear.

57 See Posener-Krieger 1991: 19, for the problematic reading of *renpet zep* 23.
58 Verner 2006: 128–131.
59 Posener-Krieger 1991: 17–21, pl. 7–12.
60 Spalinger 1994: 281–283.
61 Verner 2006: 128–131; Verner 2008: 23–43.
62 Posener-Kriéger 1991: 19.

At present, considering the readings of various authors as cited above, the building years of the Red Pyramid refer to attested or unattested census years and years after (*) of Snofru as shown in Table 1; census years which are attested or claimed either for Dahshur or Maidum are in bold characters. If building activity at the Red Pyramid itself covered the regnal years as indicated in Table 1, then it lasted 11 full calendar years and parts of *renpet-zep* 15 and *renpet-zep* 24, possibly full 12, but less than 13 full calendar years. Should the census have been regularly biennial, then building the Red Pyramid would have taken as many as 20 calendar years.

Census count of Snofru	**15**	***15**	**16**	***16**	**17**	**18**	***18**	?19	?20	?21	?22	**?23**	**?24**
Building years, Red Pyramid	1	2	3	4	5	6	7	8	9	10	11	12	13

Table 1. Building years of the Red Pyramid referred to regnal years of Snofru.

2 | Relevant pyramid measurements of all three pyramids as well as details of the Red Pyramid

Khephren pyramid:[63] Base length of 215.3 m,[64] tg 53.13° = 4/3 as slant of the faces and 143.5 m as original height are well established.[65] Hölscher made summary reference to a rock core which the builders utilized in the lower courses.[66] According to the more detailed account of Maragioglio and Rinaldi,[67] the core slopes upwards from south to north and to a lesser degree from east to west;[68] there is no core in the southeast section of the base. Note that the comments of Raynaud *et alii* on the rock core are not reliable. The authors cite Petrie 1883, without page number, as describing the rock "in the inner descent of Khephren at a height of 5 m above the reference level of the surrounding esplanade".[69] I do not find such an assertion in Petrie's book.[70]

Kheops pyramid:[71] Base length of 230.36 m,[72] tg 51.842° =14/11 as slant and 146.6 m as original height are well established.[73] The heights of each of the preserved courses have been measured

63 For the history of exploration see Maragioglio and Rinaldi 1966: 42.
64 Dorner 1981: 81.
65 Hölscher 1912: 61; Dorner 1981: 79.
66 Hölscher 1912: 61.
67 Maragioglio and Rinaldi 1966: 44–46, 50, 54, 58, Tav 5, fig. 2, sezione S–N.
68 From about 2 m in the south to about 8 m in the north; from about 1 m in the east to about 5 m in the west.
69 Raynaud 2008: 16.
70 *Cf.* Petrie 1883: 33, with a description of the rock core.
71 For the history of exploration see Maragioglio and Rinaldi 1965: 8–10.
72 Dorner 1981: 77.
73 For details see Maragioglio and Rinaldi 1965: 18.

on several occasions, last by Georges Goyon who arrived at a height of the preserved 201 courses of 138.745 m.[74] The missing 7.85 m are to be reconstructed as 9 to 10 layers and a pyramidion. In courses 1 to 7 the builders made use of a local stone core; for the size of the core I rely on the figures as determined by Michael Haase.[75] Note that the comments of Raynaud *et alii* on the rock core are unreliable. It seems they rely for the maximum height of the core on a drawing to scale in a publication by Max Eyth.[76] Eyth's book is a novel; the drawing is attributed by a character in the novel to Charles Piazzi Smyth,[77] but actually it is Eyth's own drawing, based on a publication of Smyth.[78]

Red Pyramid:[79] By contrast to the pyramids of Kheops and Khephren, the Red Pyramid has not been described in every detail, and in what follows I cite some pertinent observations compiled from different sources. For the original base length of 219.08 m, tg $45^\circ = 1$ as slant and 109.54 m as original height I rely on what Josef Dorner measured in February 1997 and later elaborated.[80] The available measurements of the courses allow a division of the original height into three sections: lower, middle, and upper part. The varying heights of the first 37 courses in their present state are indicated in a profile drawing to scale by Vito Maragioglio and Celeste Rinaldi;[81] presumably they measured these heights, amounting to a total of 34.79 m.[82] The respective mean height of 94 cm accords suspiciously well with 3 feet (0.944 m) as noted by Shae Perring as the height of the courses near the base.[83] According to Dorner, structural settlement has occurred; he argues for 37.55 m as the original cumulative height of the first 37 courses.[84]

There are no detailed measurements for the courses of the middle section. With regard to the overall heights of the courses, Maragioglio and Rinaldi stated:[85] "In line with the entrance, the height of the courses oscillates between 50 and 70 cm". This accords with 2 feet or 60.96 cm as stated by Perring.[86] For the purposes of the present article I count 115 courses of 60.6 cm each, adding up to 69.7 m for the middle section.[87] Samuel Birch rendered the observations of Perring about

74 Goyon 1978: 405–413.

75 Haase 1995: 20–27.

76 Raynaud *et alii* 2008: 18.

77 Eyth 1906: 400, 403, Blatt 1.

78 Smyth 1874: Frontispiece.

79 For the history of exploration see Maragioglio and Rinaldi 1964: 124; and Stadelmann 1982: 379–381.

80 Dorner 1998: 23–30.

81 Maragioglio and Rinaldi 1964: Tavole 18, fig. 3; the latter is the source for the values which I use.

82 It seems that I ascribed in Krauss 2017a: 91, mistakenly to Maragioglio and Rinaldi a compilation of these values *from various authors*.

83 Perring 1842: 15.

84 *Cf.* Krauss 2017a: 91.

85 Maragioglio and Rinaldi 1964: 126. There is a contradiction in the English translation: "The courses on the exterior are regularly horizontal and their height is rather constant, but variable from course to course". The contradiction stems from neglect of the words marked with italics in the original: "I corsi si presentano all'esterno, perfettamente orrizontali e sono di abbastanza costante *per tutta la loro estensione*: l'altezza è però variabile a corso a corso".

86 Perring 1842: 15.

87 To distribute the lifting work in the middle section among more or less than 115 courses does not change the total amount of the work.

the upper section as follows:[88] "The top of the pyramid was built entirely with Arabian stone.[89] The apex had been formed of one block [i.e. the pyramidion], and the course below it of four others, 4 feet 9 inches [1.448 m] thick". Maragioglio and Rinaldi commented on Perrings remarks:[90]

> Perring did not see, as often is said, the uppermost block (the pyramidion) in situ, which, Mariette affirms, no longer existed in his days.[91] The English author [Perring] says the uppermost course of the nucleus consisted in only one block, but this is a guess (however probable) because he did not put it in his drawings of the section nor in the perspective view of the monument. Instead, Perring states that the course under the uppermost block (which he thought to be the penultimate course) was formed by four blocks of *Arabic* [Tura] limestone, 1.47 m. thick.

They go on to cite "an interesting affirmation found in the notebooks of Lepsius … that pink mortar was used in the construction …". They overlooked another interesting remark of Lepsius as reported in LDT 1:[92] "The facing of Mokattam stone is nearly completely missing; at the top there is a cornerstone preserved and another stone of the facing". I compared original and published texts of Lepsius word by word without finding a remark about a cornerstone or another stone of the facing as preserved at the top.[93] The authors may have had Perring's observation of a preserved cornerstone in mind, although this leaves "another stone of the facing" unexplained. Adolf Erman asserted, that Sethe and Borchardt added references to publications other than Lepsius, "ohne dass dabei eine bibliographische Vollständigkeit angestrebt worden wäre".[94] The members of the Lepsius expedition climbed to the top of the Red Pyramid,[95] and were thus informed about the situation at top; still, the citation in question cannot be found in the archive material accessible to Sethe and Borchardt.

For the top section, I reckon a course 1.47 m high, as indicated by Perring, plus the Tura limestone pyramidion discovered by Stadelmann, about 75 cm in height.[96] Thus the overall height of the Red Pyramid may be distributed over a sequence of 37 + 115 + 2 = 154 courses. The slant of the pyramidion does not match the slant of the Red Pyramid, and therefore Corinna Rossi suggests that it might have been made for the second and abandoned phase of the Bent Pyramid.[97] The explanation is possible, but on the other hand, the pyramid builders were not obliged to construct a bulding which conformed in all details to a geometrically exact pyramid.

88 Perring 1842: 15.

89 Perring meant stone from Gebel Mokattam as part of the Arabian Mountain Range. The latter designation seems to be outdated among Westerners; it goes back to Herodotus II.8.

90 Maragioglio and Rinaldi 1966: 126, see also Tavole 18, fig. 6.

91 Mariette 1889: 573, wrote somewhat differently: "Je ne sais si Mr. Perring … a vu encore en place la pierre qui formait le sommet même de la pyramide; elle n'existe plus aujourd'hui, du moins à sa place primitive".

92 Sethe and Borchardt 1897: 206: "Die Bekleidung fehlt [fast] ganz; sie war von Mokattemstein; an der Spitze ist ein Eckstein und ein anderer Stein der Bekleidung erhalten". Sethe used the orthography *Mokattem*, not *Mokattam*, of the original text.

93 Notebooks and diaries of Lepsius are kept at the Berlin-Brandenburgische Akademie der Wissenschaften.

94 Erman, in: Sethe and Borchardt 1897: IX n. 1.

95 Lepsius, Abeken, Bonomi, and Max Weidenbach on February 26, 1843; Erbkam on April 15, 1843. Erbkam left a sketch book and a diary which are kept at the BBAW. For the diary transcribed by Elke Freier see <diary pom.bbaw.de/erbkam/index.html>. The draughtsman Max Weidenbach left a diary, now in the South Australian Museum in Adelaide; its publication is in preparation by Susanne Binder and Boyo Ockinga. I thank both for Weidenbach's respective diary entry.

96 Stadelmann 1983: 235–236, Tafel 75.

97 Rossi 1999: 219–222.

3 | Building materials

3.1 | Density: generalities

For the purposes of this article, density is the important property of the building materials, since it figures in the formula for lifting work (see below). Density is defined as proportion of mass to volume, measured in kg/m³; its symbol is ρ (rho). I would have liked to use this definition only, but in the sources cited below two other related terms are employed. One is *unit weight* or *specific weight* ρ*g (density times acceleration of gravity); the other is *specific gravity* or *relative density* $d = \rho/\rho_0$, where ρ_0 is the density of water at a specific temperature, usually 4° C, hence d is a unit-less property.

It seems that geologists are not interested in the density of stone because of its variability.[98] A prime example is the chapter on Giza and Tura limestone by K. J. Weber in J. D. de Haan's publication about engineering the Great Pyramid.[99] Weber discusses many geological and petrographic details, but not the density of any stone. De Haan chose ρ = 2500 kg/m³ as "density of the building material corresponding with the density of porous limestone", citing Arnold (Table 2),[100] who in his turn did not provide his source. Arnolds Table is headed by "Weight* (kg/l)"; the * explains *Weight* as *specific gravity*. The letter *l* in the symbol *kg/l* appears to mean *l(iter)*, indicating the source as a Table which included the densities of fluids.

Stuart Wier in his version of engineering the Great Pyramid chose ρ = 2700 kg/m³ as "average density of the pyramid";[102] no source is given. One would expect that those Egyptologists who write about the great *weight* of the building blocks of the pyramids know what they are talking about, but the *specific weight* of the respective limestone or the product of density times acceleration of gravity g seems to be unknown to them.

3.2 | Density: Kheops pyramid

According to Rosemarie and Dietrich Klemm, casing and backing stones of the pyramid of Kheops came from the quarries of Tura, Maasara, and Mokattam.[103] In 1865 Smyth brought fragments of casing stones taken from the pyramid of Kheops to Edinburgh,[104] where they are kept today in the National Museum of Scotland.[105]

Material	Specific gravity
Dense limestone	2.65 – 2.85
Porous limestone	1.7 – 2.6

Table 2. Specific gravity of limestone, after Arnold.[101]

98 As explained to me by Rosemarie Klemm in an email of August 18, 2020.
99 Weber 2010: 53–59.
100 Haan 2010: 15.
101 Arnold 1991: 28, Table 2.1.
102 Wier 1996: 150.
103 Klemm and Klemm 2010: 87–89.
104 Smyth 1874: 26.
105 See Brück and Brück 1988: 113, 128 (photo).

The museum houses also a large fragment of a casing stone found by Waynman Dixon in the early 1870s.[106] I directed an enquiry about the density of the small fragments to Dr. Margaret Maitland, Principal Curator of the Ancient Mediterranean, Department of World Cultures at NMS. When the restrictions effective in the museum due to the corona epidemic were eased I received the following information about T.1996.162, one of the fragments:[107]

> weight in air: 3651g; weight in water (t=0): 1940g; weight in water (t=30 min): 2025g;
> ρ = weight in air / (weight in air minus weight in water); dry density (air-filled porosity) = 2.1g/cm3.

In the meantime, I discovered that Smyth had himself determined the specific gravity of the stone material used in the pyramid of Kheops.[108] I could convert his value for the limestone of the casing into 2.086 g/cm³ which is identical to the result obtained for fragment T.1996.162.

The core blocks at Giza stem from quarries near the pyramids. The stone is characterized by fossils, especially nummulites which are so named because their form is reminiscent of small coins or Latin *nummulus*, hence nummulitic limestone. I could find three values for the density of Giza limestone other than the core material of the Kheops pyramid. In 1980 Denys Parsons (not an archaeologist) wrote in *New Scientist* that he determined ρ = 2.23 g/cm³ for a piece of limestone "from the Giza ridge".[109] He pointed to a remark of Petrie that an average block of the Great Pyramid has a volume of 50 x 50 x 28 inches [1.147 m³] "or [a mass of] 2½ tons each" which implies about ρ = 2200 kg/m³.[110] Since there was nothing at stake for Parsons I accept his assertion as trustworthy, whereas I presume that Petrie based his computation on the average density of limestone. Furthermore, S. M. Nakkhla and M. Abd el Kader reported in 2006 for the "rock south of the sphinx" the *specific gravity* of 2.28.[111]

In 1986, Lakshmanan, Bui *et alii*, a team of engineers specializing in microgravimetry, surveyed the pyramid of Kheops.[112] Microgravimetry is a form of gravimetry in which very small differences in the gravitational fields of closely spaced measurement points are determined. It is a geophysical technique primarily employed for detecting various underground cavities; as an applied science it yields reliable results.[113] Lakshmanan *et alii* searched in vain for unknown chambers in the pyramid, but they were able to determine density values for the pyramid itself, and to work out the density of its rocky underground:[114] "La densité moyenne du terrain sous et autour de la pyramide [de Kheops] est de l'ordre de 2.25 g/cm³, avec des valeurs un peu plus faible sous le centre de la pyramide".

106 Lightbody 2016: 39–56; see also Brück and Brück 1988: 113.
107 As determined by members of NMS staff: Dr Bob Gooday, Earth Systems Analyst, Dr Tacye Phillipson, Senior Curator of Science, and Julie Gibb, Assistant Curator of Science.
108 Smyth 1874: 240.
109 Parsons 1980: 669.
110 Petrie 1883: 83, note *.
111 Nakhla and Abd Elkader 2006: 213.
112 Lakshmanan and Montlucon 1987: 10–17; Bui 1988: 1063–1069.
113 I thank Mark Lehner for bringing the microgravity research and its importance to my attention.
114 Bui 1988: 1067; note that fig. 7 on the same page indicates 2.35 g/cm³ as density of the rock beneath the base of the pyramid.

Hemeda and Sonbol report that the core material of the pyramids of Kheops and Khephren "is not exposed"[115] – i.e., was inaccessible behind the backing stones. Nevertheless, Klemm and Klemm were able to analyse some samples of core material of both pyramids,[116] and there is no doubt about their reliability. Core material of the pyramid of Kheops should be accessible in the breach which Howard Vyse made on the south side of the pyramid.[117] Furthermore, Hemeda and Sonbol state that not only the core stones but also the backing stones originated in Giza in the so-called Kheops quarry,[118] saying nothing about Tura limestone. Remarks by Maragioglio and Rinaldi shed some light on the contradiction, since they observed that "at times, however, these [backing] blocks are not of white limestone but good local fossiliferous limestone. Thus it appears that the backing-stones were not always made with the best stone …".[119] Hemeda and Sonbol "selected a total of 45 samples of fallen fragments from different locations around the three [Giza] pyramids".[120] For three samples from the Kheops pyramid they list the values "2.03, 1.98, 2.01," under the heading "Unit weight (UW) (g/cm^3)".[121] They cite values only for ρ, i.e. for density; it seems that they use density and Unit weight interchangeably, as in colloquial language. The values cited are numerically very close to each other, and will refer therefore to one kind of stone – presumably backing stone material of Tura origin, if one considers their average of ρ = 2.00 g/cm^3 and compares ρ = 2.08 g/cm^3 as density of the casing fragments determined by Smyth. The latter's value for the nummulitic limestone of the core I convert into 2.35 g/cm^3.[122] Smyth alone refers without question to core material and I take his density value as binding.

Lakshman *et alii* computed the average bulk density of the pyramid in its present state as ρ = 2.05 g/cm^3;[123] furthermore, they determined zones of different bulk density within the pyramid.[124] Between 2015 and 2017 Kunihiro Morishima *et alii* undertook muographic measurements in the pyramid of Kheops. Muography is an applied science which yields reliable results. Muons induced by cosmic ray can be used to detect variations in the average density of various materials, in the present case the pyramid core consisting of blocks, mortar and interspaces. Morishima presupposed a density of ρ = 2.2 g/cm^3 for the limestone,[125] which corresponds to the value cited above after Bui although omitting the second decimal. Morishima did not determine the bulk density, i.e. the total of blocks, mortar, and empty spaces; rather he was searching for unknown cavities in the structure. I owe to the kindness of Hiroyuki Tanaka the following

115 Hemeda and Sonbol 2020: 13.

116 See Klemm and Klemm 2010: fig. 88 and 90 (Kheops), and Figs. 99–101 (Khephren).

117 Maragioglio and Rinaldi 1965: 14.

118 Hemeda and Sonbol 2020: 14.

119 Maragioglio and Rinaldi 1965: 106.

120 Hemeda and Sonbol 2020: 3.

121 Hemeda and Sonbol 2020: 22.

122 Smyth 1874: 240.

123 Bui 1988: 1066.

124 Bui 1988: 1066, fig. 4. See also Bui 2012: 44, fig. 3.15.

125 Morishima 2017: 391–392.

discussion which results in a correction of ρ = 2.2 g/cm^3 as assumed density of the respective limestone:[126]

> Muographic picturing of the density structure inside the Kheops pyramid shows that the simulated flux, as measured perpendicular under the King's chamber, was roughly 3.5 (see Extended Data Figure 2, in Morishima *et alii*, 2017: 394). In this region, the difference between the observed flux and the simulated flux was approximately 0.2. It follows that there is an excess in the flux of ~6%, by comparison to ρ = 2.2 g/cm^3 as the assumed density of the pyramid. Relative to the position of the detector in the Queen's chamber, the elevation angle that covers the King's chamber ranges between 15 and 30 degrees from zenith; therefore, the muons passing through this region arrive from a near-vertical direction. Since the rock thickness ranges between 180 and 740 meters water equivalent (m.w.e.), Eq. (7) in Tanaka and Ohshiro (2016) can be applied to estimate the bulk density of the region including the King's chamber. Then we obtain $N_0/N_1 = \langle X_0\rangle/\langle X_1\rangle^{-2.2} \sim 1.06$ (a), with N_0 and N_1 the measured, and theoretically expected flux, and $\langle X_0\rangle$ and $\langle X_1\rangle$ are the measured and theoretically expected muographically averaged densimetric thickness (MADT) of the target object, including backing stones.[127]
>
> Muography measures the density integrated over the path of muons, and since the Kheops pyramid is more or less completely stripped of its outer casing, it is difficult to determine directly the thickness of the backing stones. However, assuming that the thickness of the backing stone layer is much smaller than the core underneath, Eq. (a) roughly represents the ratio of the core bulk density; hence the application of (a) will not be essentially changed. As a consequence, the core bulk density can be estimated to be ~ 2.13 g/cm3, or approximately 3% less than the assumed density of 2.2 g/cm3.
>
> Another piece of structural information that we can infer from the muographic data is a variation in core density as a function of elevation. As shown in the Extended Data Figure 2; Morishima *et alii*, 2017: 394, the deviation of N_0 from N_1 is larger in the angular region that includes the King's chamber. The flux of the muons that arrived from this near-vertical angular region reflects the bulk density of the top region of the pyramid, indicating that the lower part of the pyramid's core is more tightly packed, probably for the purpose of increasing mechanical strength. Above the King's chamber the core density decreases by 3%, with the likely intention of lessening the weight on the lower part of the pyramid.

The corrected value of 2.13 g/cm^3 comes close to the average bulk density of 2.05 g/cm^3 as determined by Lakshaman *et alii*. I take 2.09 g/cm^3 as mean bulk density on the basis of the two techniques. Thus, one could compute the mass of the pyramid in its present state, as if it were made of tightly packed stones of about 89% of 2350 kg/m^3 as the density of the core material (or: relative bulk density of 89%) according to Smyth. Actually, the blocks are not packed tightly and the pyramid consists of solid blocks with 11% interspaces, either empty or filled with sand or mortar. Old Kingdom mortar is nowadays described as Nile mud- and sand mortar, clay-, gypsum-, gypsum- and lime mortar.[128] Before microgravimetry and muography arrived at Giza, Arnold estimated that a layer in one of the great stone pyramids amounts in general to about 90% of solid stone, the remaining 10% being mortar and empty spaces.[129] The new techniques modify Arnold's estimate at least for the pyramids of Kheops and Khephren.

126 E-mail of April 5, 2021.

127 Tanaka 2020.

128 Klemm and Klemm 1991: 445–454.

129 Arnold 1981: 26.

3.3 | Density: Khephren pyramid

Casing and backing stones of the pyramid of Khephren came from Tura and Maasara.[130] The material is described as "of a type greyer, harder and more fragile that that used in the pyramid of Kheops".[131] Lepsius took along large fragments from the casing of Khephren's pyramid.[132] The fragments still exist in the Berlin Egyptian Museum and I wish good luck to those who want to learn about their density.[133] Hemeda and Sonbol cite 2.31, 2.29 and 2.25 g/cm³, or on average 2.28 g/cm³ for limestone backing stone samples from Khephren's pyramid.[134] By comparison to the densities of core and casing material of the Kheops pyramid as determined by Smyth, and also to the values for Giza limestone in general, the average 2.28 g/cm³ appears to refer to core material; the other possibility is that the backing stone samples are of local Giza origin. Under these circumstances I employ 2.28 g/cm³ as density of core material in Khephren's pyramid. I am fully aware of the uncertainty without giving much weight to it, since there is an ample leeway for lifting work and resulting building time at the pyramid of Khephren as Tables 4 and 5 imply.

Whereas muographic measurements were carried out in both of the pyramids of Khephren and Kheops, no gravimetric measurements were undertaken in the pyramid of Khephren. In the late 1960s and early 1970s Luis Alvarez and Gerald Lynch recorded the cosmic muons arriving in Belzoni's chamber, the crypt of Khephren's pyramid.[135] Lynch determined a bulk density of $\rho = 1.9 \pm 0.2$ g/cm³,[136] taking into account that Lambert Dolphin had determined $\rho = 1.8$ g/cm³ from "a small piece of the Khephren pyramid". It may have been a fragment of a backing stone,[137] since "especially the remains of backing stone blocks allow sufficient sampling of the fine uniform white-grey limestone quality that was used for both casing and backing".[138] On the other hand, the density of Dolphin's fragment is markedly less than all other density values reported from Giza. Regardless, whether Lambert's fragment is accepted as relevant or not, the point is that Lynch determined the bulk density of the pyramid as if it were made of tightly packed blocks of $\rho = 1.9 \pm 0.2$ g/cm³. If the blocks are packed with ample interspaces, then the single stone blocks have a density which is higher than the bulk density. Note that the five lowest courses were much better constructed than the remainder.[139] Otherwise the masonry of the nucleus is very loose and without mortar, as can be seen in the breach made by early violators of the pyramid.[140]

130 Klemm and Klemm 2010: 82–89.

131 Maragioglio and Rinaldi 1966: 50.

132 Sethe and Borchardt 1897: 27: Egyptian Museum Berlin nos. 1335, 1339, 1341.

133 To determine the density of relatively voluminous pieces of stone is only possible with equipment normally not at hand in a museum.

134 Hemeda and Sonbol 2020: 22.

135 Alvarez *et alii* 1970: 832–839; Lynch 1973.

136 As reported by Alvarez in: Trower 1987: 184.

137 Trower 1987: 184. Dolphin will have picked up the sample in the 1970s when he investigated the Great Sphinx, see <www.ldolphin.org/egypt/sphinx.html> (accessed August 24, 2020)

138 Klemm and Klemm 2010: 95.

139 Maragioglio and Rinaldi 1966: 46.

140 Stadelmann 1990b: 178: "das Kernmauerwerk (ist) sehr lose und ohne bindenden Mörtel verlegt ... im Gegensatz zu der Bautechnik bei der Cheopspyramide".

It appears that mortar was used only with the outermost blocks of the pyramid of Khephren,[141] by contrast to the pyramid of Kheops.

In 2016 Hiroyuki Tanaka published a re-analysis of the Álvarez and Lynch measurements.[142] He assumed the same density for backing and core stones arriving at 1.89 ± 0.20 g/cm³ for the bulk density of the core. The figures correspond to a pyramid of tightly packed stones with a density of about 83% of 2280 kg/m³ (or: relative bulk density of 83%) as the supposed density of Giza limestone (or correspondingly loosely packed stones of the density of Giza limestone).[143]

3.4 | Density: Red Pyramid

Lepsius labeled the core stone of the Red Pyramid 'oyster stone' (German: Austerstein).[144] Maragioglio and Rinaldi described it in more detail as:[145] "coarse reddish limestone characterized by the presence of numerous big fossil shells". Citing M. Pawlikowski's findings,[146] Stadelmann reported that not only the casing but also the foundation blocks are of Tura limestone; the casing consists of an outer block and a backing stone behind it.[147] Klemm and Klemm describe the core material as coming from nearby quarries, consisting of a sandy limestone or calcareous sandstone containing many types of fossils, among them oyster shells of palm size.

Lepsius picked up two fragments of the Red Pyramid's casing;[148] but since the 1890s it has not been possible to locate them in the Berlin museum. According to the petrographic analysis of Klemm and Klemm the foundation and casing stones originated in the Tura and Maasara quarries.[149] They remark the difference between the casing material of the Giza pyramids and the Red Pyramid which latter they characterize as "of coarser-grained structure and the grinded [sic] surface appears more compact and less porous than the stones used later in the Gizeh pyramids".[150] Hemeda and Sonbol collected 48 samples from "surface and core layers", and list as Unit weight examples "2.03, 1.98, 2.01 [g/cm³]" or 2.0 [g/cm³] on average for the limestone of the casing, and "2.29, 2.30, 2.22 [g/cm³]" or 2.27 [g/cm³] on average for the core.[151] Both sets of figures compare well with Smyth's values for the core and casing material of the pyramid of Kheops. Maragioglio and Rinaldi commented with reference to the Red Pyramid:[152] "in the inferior [lower] courses of the nucleus we also saw much

141 See Maragioglio and Rinaldi 1966: 46, citing Hölscher 1912: 61: "Im Inneren ohne Mörtel gebaut. Nur die äussersten Steine sind durch Mörtel verbunden".

142 I thank Hiroyuki Tanaka for comments on his article.

143 Tanaka and Ohshiro 2016: 430–432.

144 Sethe and Borchardt 1897: 206.

145 Maragioglio and Rinaldi 1964: 126.

146 Stadelmann 1982: 382, n. 12.

147 Stadelmann 1983: 234.

148 Sethe and Borchardt 1897: 206: "2 kleine Stückchen Mokattemstein mit den Resten von roten Zeichen sind in Berlin Nr. 1343. 1344".

149 Klemm and Klemm 2010: 59, 65–68.

150 Klemm and Klemm 2010: 65–66.

151 Hemeda 2018: [16], Table 8. I noted one disturbing instance where the text cites the properties of limestone, although actually meaning sandstone: Hemeda 2018: [15] (7.1.1).

152 Maragioglio and Rinaldi 1964: 126.

yellowish mortar". Lepsius brought samples of mortar from the Red Pyramid to Berlin, which had disappeared in the 1890s, (whereas samples from the Khephren pyramid were preserved then).[153] Undated mortar samples from the Great Sphinx have a relative density or specific gravity d = 1.63 on average.[154]

4 | Horizontal and vertical work

Pyramid building logistics and techniques have been discussed in publications during the last 25 years, especially in the circumspect contributions by S. Wier,[155] H. de Haan,[156] and E. Graefe.[157] There is also the study of H. Tanaka which is accessible only to those who read Japanese.[158] Presumably logistics and techniques were perfected beginning with the second phase of the Bent Pyramid's construction to remain unchanged until the time of Khephren. I presume that the logistics functioned, the building processes were never interrupted, and the working schedules were the same at the construction of the Red Pyramid, and for the pyramids of Kheops and Khephren. For example, seasonally variable lengths of daylight would have affected all building projects which took years to accomplish and therefore can be disregarded in comparisons. The lengthening of daylight in summer would have been neutralized by high temperatures at midday, forcing the workmen to rest.[159]

In what follows all building activities which do not take place at the pyramid site – for example digging a branch channel from the Nile and building a harbor; constructing a mortuary temple; quarrying stone; transporting the blocks to the construction site, and so on – are disregarded. Such work can be dealt with separately from the piling up of blocks in the form of a pyramid. The latter process involves two kinds of work, namely vertical work – lifting – and horizontal work, for example hauling. Horizontal work W is defined as the product of the coefficient of friction μ, acceleration of gravity g, mass in kg, and horizontal distance in meters; the measuring unit is Joule.

There are various possibilities for the horizontal transport of blocks: dragging over a rough surface,[160] sliding on sledges over a rough or lubricated surface; moving on rollers. Transport on sledges is well attested in pharaonic Egypt;[161] the tribological and archaeological literature offers values for the coefficient μ. For example, Bowden and Tabor cite μ = 0.25 bis 0.5 for clean wood on wood, and μ = 0.2 for wet wood on wood.[162] Rolling friction is in general much less than sliding friction. This was first commented on in a pseudo-Aristotelian text.[163] In 1970 Henri Chevrier maintained that the Egyptians "n'ont donc jamais pu utiliser de rouleaux de bois".[164] By contrast,

153 Sethe and Borchardt 1897: 27, 206.
154 Nakhla and Abd Elkader 2006: 212, Table 4.
155 Wier 1996: 150–163.
156 Haan 2010.
157 Graefe 2003: 113–152.
158 Tanaka and Oshiro 2017.
159 At least as documented during the construction of the first Aswan Dam, see Fitzmaurice 1903: 97, 105.
160 Chevrier 1970: 20–21; Dowson 1979: 30–31.
161 For the Egyptians' use of sledges see Clarke and Engelbach 1930: 89; Arnold 1991: 275–280.
162 Bowden and Tabor 1954: 327.
163 Hett 1936: 367.
164 Chevrier 1970: 32, 39.

Arnold stated in 1991 that "the use of wooden rollers was quite common".[165] Back in 1930 Somers Clarke and Reginald Engelbach were able to cite excavated examples of rollers, describing them as "slightly thicker in the middle than at the ends which are rounded". Goyon reported rollers of the same type, though of bronze.[166] George A. Reisner found in Nuri Pyramid VIII (Aspelta) *two short thick rollers of granite* apparently used when moving a sarcophagus.[167] Clarke and Engelbach commented on certain disadvantages of rollers:[168]

> In the case of moderate sized blocks, and with sufficient men on the spot, the running of the sled over transversely laid sleepers is a considerably quicker process than the use of rollers, which need a good deal of attention to avoid jamming or running sideways.

Brian Cotterell and Johan Kamminga stated in 1990:[169] "To be really efficient rollers must be reasonably true and must run on a well-constructed track". Later, in 2011, experiments with a reconstructed archaic Japanese sledge yielded μ = 0.2 to 0.4 for moving with rollers on ground, but μ = 0.02 to 0.04 for moving with rollers on wooden tracks.[170] Note that neither Haan, nor Wier, considered the use of rollers for the transport of pyramid blocks.

Maragioglio and Rinaldi cite 1.12 m as height (thickness) of the blocks which make up the Red Pyramid's first course.[171] For the purpose of computation, 1 meter may be assumed as both length and depth of the blocks, corresponding to blocks with a volume of 1.120 m^3 and a mass of 2542 kg = 1.120 m^3 * 2270 kg/m^3. The presumed measurements determine the number of blocks and the length of transport distances in the first course; variable values of length and depth of blocks will nevertheless yield the same overall amount of horizontal work. If 1 m is the length of a block, then there are 219 blocks in a row of the Red Pyramid's base length. If brought in from one side, the sum Σ of transport distances for all blocks amounts to $\Sigma = ((219^2 + 219)/2)*219$ m. Table 3 presents the horizontal work for the first course with the coefficients of friction as cited above, and with ρ = 2270 kg/m^3, presuming that in the lowest courses the blocks were packed as tightly as possible for obvious reasons of stability. Note that the values for μ result in amounts of horizontal work which differ by a factor of about 20.

M	Work
0.4	$5.26 * 10^{10}$ J
0.25	$3.29 * 10^{10}$ J
0.04	$5.24 * 10^{9}$ J
0.02	$2.81 * 10^{9}$ J
0.177 (average)	$2.77 * 10^{10}$ J (average)

Table 3. Horizontal work for the first course of the Red Pyramid.

165 Arnold 1991: 273–275.
166 Goyon 1990: 58, n. 47.
167 Cotterell and Kamminga 1990: 224, cite Engelbach 1922: 38, who in turn cited from a letter he had received from Reisner. Peter Der Manuelian informed me (email of March 18, 2021) that the rollers are not mentioned in any publication and that the relevant archive material at the Museum of Fine Arts, Boston, is at present, due to Covid-19 restrictions, not accessible.
168 Clarke and Engelbach 1930: 89–91, here 90; cited by Dowson 1979: 34.
169 Cotterell and Kamminga 1990: 223.
170 Shimotsuma 2011: 176.
171 Maragioglio and Rinaldi 1964: Tav. 18.

Equation (1)

$$W = \rho g \frac{L^2 H}{3} \frac{H}{4} J = \rho g \frac{L^2 H^2}{12} Joules$$

Equation (2)

$$\frac{\text{lifting work Red Pyramid}}{\text{building time Red Pyramid}} = \frac{\text{lifting work Kheops Pyramid}}{\text{building time Kheops Pyramid}} = \frac{\text{lifting work Khephren Pyramid}}{\text{building time Khephren Pyramid}}$$

Equation (3)

$$\text{building time of Kheops Pyramid} = \frac{\text{lifting work Kheops Pyramid}}{\text{lifting work Red Pyramid}} \times \text{building time Red Pyramid}$$

In the case of the first course of the Red Pyramid one should reckon with horizontal work only, considering "il fatto che non occorreva solevare i blocchi, ma solo sportarli".[172] By comparison, horizontal work was of far less importance in the lower courses of the pyramids of Kheops and Khephren where the use of local stone cores saved work. In the higher courses of each pyramid horizontal work was to be done as soon as a block was lifted to the edge of the respective course. At the same time other blocks were lifted and thus horizontal work was done parallel to lifting work.[173] If I am not mistaken, the time for horizontal work was not an independent component of total building time, the first course excepted. In other words – as far as construction time is concerned it is by far mostly liftingwork that counts. Lifting work for a pyramid of base length L and height H can be computed in one step with equation (1) as the product of volume multiplied by ρ, g, and H/4 as height of its centre of gravity (centroid) where the mass can be thought of being concentrated.[174] Just for the fun of it, I cite the local values of g. The WELMEC formula yields for Giza g = 9.79305 m/s^2, and for Dahshur g = 9.7928 m/s^2, differing very slightly from the global value of g = 9.81m/s^2.

The lifting technique in pyramid building is not known. The golden rule of mechanics ensures that any device to lift a mass which saves force, results in a longer way of transport. Regardless which technique the pyramid builders used, the expenditure of energy remained the same. Without long discussions I postulate that constructing the great stone pyramids followed the same procedure and consider the result of the building activities as a total. In detail I presume that the technique of lifting

172 Maragioglio and Rinaldi 1965: 20–21: "it was not necessary to lift the blocks, but only to move them sideways" with reference to the Kheops pyramid, but applicable to the Red Pyramid and Khephren's pyramid.

173 I presume that the proportion between horizontal work and building time is the same as between lifting work and building time. The coefficient μ would be reduced in comparisons of horizontal work, but the uncertainty about bulk density would remain.

174 The zones of different bulk density in the pyramid of Kheops change with the position of the centroid, though reference to an average bulk density presumably neutralizes the change.

blocks was the same and to lift a block over a specific vertical distance took on average the same amount of time. These premises imply proportion (2) between lifting work and building times; the proportion allows to derive for example equation (3) for the building time of the Kheops pyramid.

Equations (2) and (3) do not do justice to the total piles of blocks in form of a pyramid, since the horizontal work for the first courses is not included. The omission can be corrected by replacing "lifting work" in equations (2) and (3) by the *sum of lifting work beginning at course 2 + horizontal work for course 1.* Lifting work at the Kheops pyramid, beginning with the second course,[175] amounted to $1.9933*10^{12}$ Joules, if ρ = 2350 kg/m^3. To be subtracted are $9.04*10^9$ Joules which were saved by making use of the rock core from course 2 to 7, leaving $1.9842*10^{12}$ Joules.[176] Horizontal work at the Kheops pyramid is negligible, since it is limited to a strip 5 m deep around the rock core; it amounts on average to $63*10^6$ Joules which would show in the 5th decimal of the lifting work. If we consider the relative bulk density of 89% we finally arrive at W = $1.7660*10^{12}$ Joules as the amount of work to be compared with the corresponding work for the Red Pyramid.

Lifting work at the pyramid of Khephren, beginning with the second course,[177] amounted to$1.7331*10^{12}$ Joules, if ρ = 2280 kg/m^3. It will have been slightly less, because using a rock core saved work in the lower courses. I estimate that about $7*10^9$ Joules were safed,[178] resulting in $1.7261*10^{12}$ Joules. If we consider the relative bulk density of 83% we finally arrive at W = $1.4327*10^{12}$ Joules as the amount of work to be compared with the corresponding work at the Red Pyramid.

The relative bulk density of the Red Pyramid is unknown. Circumstantial arguments speak in favor of planning and executing the Red Pyramid as a very solid building. I take the three foundation layers beneath the first course as indication that the architects aimed at a very stable construction which implies tight packing of the core blocks. (The three layers are a feature unique to this building, and therefore not to be considered in a comparison.) The architects decided to construct the Red Pyramid in a new technique with horizontal instead of inclined layers.[179] Surely the architects considered the possible consequences of the new technique. The stability of the pyramid called for tightly packed stones and as little space between them as possible. On the other hand, a bulk density which decreased with increasing height of the frustum would have shortened the overall building time, a definitely welcome side effect. The pyramid builders of the 4th dynasty will have learned by experience how much inter-space was possible; the low bulk density of Khephren's pyramid reflects their experiences. For these reasons, and pending further discussion, I assume that the relative bulk density of the Red Pyramid amounts to 90% which is slightly higher than the relative bulk density of 89% of the pyramid of Kheops; for pragmatic reasons I consider also a value of 95%. Lifting work at the Red Pyramid, beginning with the second course amounted to $1.0256*10^{12}$ Joules, if ρ = 2270 kg/m^3.[180] To be added are $2.77*10^{10}$ Joules

175 Measurements referring to the second course are: base length 228.46 m; height of the remaining pyramid 145.1 m; height of the course below (first course) 1.5 m.

176 Computations of lifting and horizontal work referring to the rock core are based on Haase 1995: 21–22.

177 Measurements referring to the second course are: base length 213.92 m; height of the remaining pyramid 142.6 m; height of the course below (first course) 0.92 m.

178 The estimate is based on the figures of Hölscher and Maragioglio and Rinaldi cited above, see notes 66–68.

179 Arnold 1991: 159.

180 Measurements referring to the second course are: base length 216.66 m; height of the remaining pyramid 108.33 m; height of the course below (first course) 1.12 m.

as average amount of horizontal work in the first course, resulting in $1.0533*10^{12}$ Joules. If relative bulk density is 95%, then W =$1.0007*10^{12}$ Joules, if 90%, W = $0.9480*10^{12}$ Joules. Table 4 presents the proportions between the work W for the pyramids of Kheops with 89%, or Khephren with 83% relative bulk density, and the work for the Red Pyramid with relative bulk density as cited.[181]

Red Pyramid bulk density	W (Kheops) / W (RedPyr)	W (Khephren) / W (RedPyr)
90%	1.764	1.431
95%	1.862	1.511

Table 4. Proportions of work on the basis of relative bulk densities.

The proportions in Table 4 are used in Table 5 to compute building times for the pyramids of Kheops and Khephren, assuming 10 to 20 years building time for the Red Pyramid. If, for example and according to the premises, it took 13 full years to build the Red Pyramid, then it took full 23 to less than 25 years to build the pyramid of Kheops. One has to consider that piling up the blocks of a pyramid cannot have begun on the first day of a reign. First a suitable location for the pyramid had to be found,[182] and preliminary work carried out. Thus, a building time of more than 13 full years for the Red Pyramid would result in a building time for the pyramid of Kheops longer than the attested reign of the king.

Proportionate building times in years				
Red Pyramid	Kheops		Khephren	
10 years	17.64	18.62	14.31	15.11
11	19.40	20.48	15.74	16.62
12	21.16	22.34	17.17	18.13
13	22.93	24.20	18.60	19.64
14	24.69	26.06	20.03	21.15
15	26.46	27.93	21.46	22.66
16	28.22	29.79	22.89	24.17
17	29.98	31.65	24.32	25.68
18	31.75	33.51	25.75	27.19
19	33.51	35.37	27.18	28.70
20	35.28	37.24	28.62	30.22
*	1.764	1.862	1.431	1.511
**	95%	90%	95%	90%

Table 5. Proportionate building times (* and **: proportions and relative bulk density of Table 4).

181 I computed for all three pyramids the work done as if core and casing were of different densities or of the same density. Since the resulting proportions corresponding to those in Table 4 differed only in the third decimal, I decided to refer here only to the possibility that core and casing had the same density.

182 Aigner 1982: 382.

5 | Final remarks

So far I considered the building times of the three pyramids as undifferentiated totals. Breaking down the totals into the building times of the successive courses may not be possible on the basis of available information. What is possible is to break down the total lifting work into the work for the successive courses. Figure 4 describes the lifting work for the successive courses of the Red Pyramid and the pyramid of Kheops; the values are computed as product of ρg, times the masses of the courses (each course in the form of a low frustum), times the heights of their centroids;[183] the low density of the upper courses is not considered, i.e. bulk density is taken as constant.

The variable heights of the layers cause irregular oscillations in the amounts of lifting work; the smooth part of the Red Pyramid's curve results from my choice of uniform course heights after course 37. A simple interpretation of fig. 4 is that lifting work first increased sharply, then reached a plateau, and decreased slowly thereafter. The dynamic results from the interplay between the increasing height of a pyramid frustum (= more lifting work), and the decreasing volumes/masses of the courses.

Knowing how much lifting work was achieved, course by course, does not inform us today about the building time of each course. By contrast, the ancient architects who had developed the lifting technique and watched it working ought to have known how long it took to lift a specific number of blocks to a specific height. It is feasible that after constructing the pyramids of Maidum and Dahshur the architects were able to extrapolate the time it would take to build a pyramid taller than the Red Pyramid. Thus I presume that the architects could provide Kheops with approximate building times for any projects of any dimensions the king may have had in mind.

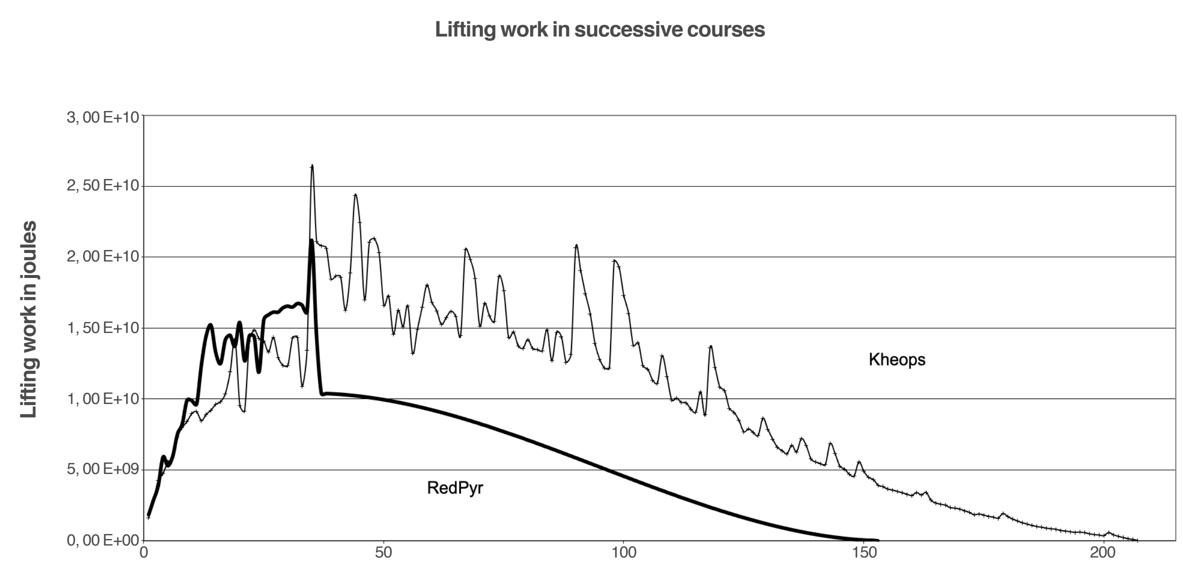

Figure 4. Lifting work in successive courses of the Red Pyramid and Kheops pyramid. (Notation on the Y-axis follows the conventions of excel: 5,00E+09 means 5*109).

183 The computation were to be modified if the inner cores "are stepped and built with accretion layers", for which possibility see Arnold 1991: 159–16, and Graefe 2003: 125.

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