



Research article

Nomography: A renewed pedagogical tool to sciences and engineering high-education studies

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HIGHLIGHTS

- Defining the alignment charts and the type of formulas that can be represented by nomograms.
- To present the open-source PyNomo software to construct complete, vector-based, and scalable nomograms.
- Nomograms for sciences and engineering-based formulas are examined to evaluate their renewed potentialities.

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ABSTRACT

Nomography is defined as a branch of mathematics in which methods of graphical representation of functional dependencies are studied. The resulting graphs are called nomograms, alignment charts, or abacs. Along the past century, this science branch experienced extensive development and use in many contexts to support scientists and engineers with accurate and fast calculations of complex formulas to a practical precision. However, nomography declined by the end of the twentieth century with the development and popularisation of more capable and powerful personal computers and handheld calculators. Despite this context, nomography remains attractive due to its potential for rapid and accurate graphical calculations contributing to a better understanding of complex formulas. Thus, this work defines the alignment charts and their capabilities to justify their importance as graphical tools in sciences and engineering studies. Also, PyNomo software is introduced to build vector-based and scalable nomograms. Then, this work emphasizes the importance of including nomography as a valuable and renewed computational tool, in conjunction with PyNomo software, in an academic context. In line with that, several nomograms, which have been obtained with PyNomo software, are examined to grasp the true importance of this science branch and its capabilities in an educational context.

1. Introduction

Nomography is the graphical representation of complex formulas and the inter-relationship among their variables employing nomograms which are calculating charts with scales that contain values of three or more mathematical variables, widely used in medicine, engineering, industry, and the biological and physical sciences (Encyclopedia Britannica, 2021). In fact, the word nomography is the result of the combination of two Greek words: “*nomos*” (meaning “law”), and “*gramme*” (meaning “line”), which could be translated as the science of representing the law of mathematics by graphs or nomograms (Glasser and Doerfler, 2019).

Nomography, “a forgotten art” as stated by Doerfler (2008), was comprehensibly introduced by Maurice d’Ocagne, in 1884, in France

(Adams, 1964). This area of practical and theoretical mathematics was used extensively for many years to support engineers with accurate and fast calculations of complex formulas to a practical precision (Doerfler, 2008). In fact, during many decades, especially through the twentieth century, there was an ever-increasing use of nomography, where many technical journals and books included nomograms that were time-saving in repetitive solutions of mathematical formulas (Evesham, 1982). In this way, it is quite impressive the variety of study fields in which nomography has been applied, such as statistics, electronics, ballistics, heat transfer, radioactivity, medicine, biomechanics, food technology, engineering, physical, and biological sciences, and business (Levens, 1959).

However, to the end of the twentieth century, with the development and popularisation of more capable and powerful personal computers

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and handheld calculators, the use of nomography significantly declined (Glasser and Doerfler, 2019). This usage decline has provoked that the scientific community, in general, be almost unaware of the actual capability of nomography. Thus, the number of nomography-based articles published in recent years has been quite scarce (Glasser and Doerfler, 2019; Toumès, 2012; Doerfler, 2008, 2009a). Among the most recent and relevant works in applied nomography are those focused on medicine (Chen et al., 2021; Potente et al., 2019; Iasonos et al., 2016; Marasco et al., 2011; Kattan and Marasco, 2010; Bevilacqua et al., 2007; Kattan et al., 1999), geothermal applications (Ekneligoda, 2015), forensics (Drábek, 2014; Henssge and Madea, 2004; Henssge, 1992), seismic hazards studies (Douglas, 2018, 2021; Douglas and Azarbakht, 2021; Douglas and Danciu, 2020; Gkimpraxis et al., 2019; Mahdyiar, 1987), engineering and mathematical education (Uzoigwe et al., 2017; Glasser and Doerfler, 2019). Moreover, the study of the practical application of nomography has been dismissed from most sciences and engineering universities syllabus. Undoubtedly, this absence of appropriate training in the basics of applied nomography contributes to certain illiteracy or difficulty among engineers, scientists, practitioners, etc., not only in the most elemental construction but also in the basic interpretation of nomograms. In spite of the fact of this short use of nomography to construct nomograms as a graphic aid, however, there is still a significant volume of technical documentation (Hydraulic Institute, 1990; Schlumberger's log interpretation charts, 1997), books (Serra, 2008; Michael et al., 2008; Liu et al., 2014), machinery specification leaflets (e.g.: IGT Testing Systems leaflet, 2022), manufacturers catalogs (e.g.: Xylem's industrial pumps catalog, 2016), etc., in which many mathematical relationships are presented under nomograms.

In this way, the actual importance of including nomography as a valuable computational tool in an academic context, such as sciences and engineering studies, could be justified by the next points: i) it can be attractive and provoke interest, potentially because nomograms are unusual, ii) it might provide the basic skills to design and interpret nomograms, avoiding that this centenary knowledge might become utterly extinct from universities, iii) it would contribute to a better understanding of complex formulas, and how are associated their variables as well as the sensitivity of the results to changes in those variables, iv) it may help those people who prefer pictures to equations understand the inputs and results of complex calculations, v) it could benefit of modern computing such as open-source codes and more advanced design-assistance software to produce in seconds outstanding and customized nomograms, which could be useful and reliable for repetitive tasks, and (vi) it could solve accurately and quickly struggling situations where the access to computers or handheld calculators might not be available (Douglas and Danciu, 2020; Glasser and Doerfler, 2019).

So, this work attempts to support the points listed above, emphasizing the capability of nomography, in terms of a pedagogical tool. Thus, to achieve this goal the following sections are going to be examined: i) the description of the main elements that define a nomogram and the different categories, ii) to put forward the PyNomo software, which is a very capable open source code to help design nomograms, and iii) to present and analyze some nomograms, obtained with the support of PyNomo, from sciences and engineering formulas of common use, to evaluate the actual potentialities of nomography into a learning environment of higher education in sciences and engineering.

2. The alignment charts: nomograms

In nomography the graphical representations are called nomograms, nomographs, alignment charts, and abacs (Doerfler, 2009a). Nomograms, in terms of a graphical tool, have the great capability to represent a diverse set of equations, from the simplest mathematical equation to the most complex one. Thus, the simplest alignment chart would consist of three parallel scales so graduated that a straight line which joins values on two of the scales will intersect the third scale at a value which satisfies the relation among the variables (Figure 1). In fact, a wide variety of

interesting nomograms can be constructed from specific derivations according to each equation form following certain representation rules that are suitably covered in Hewes and Seward (1923), Johnson (1952), Levens (1959), Davis (1962), Doerfler (2009a), and Glasser and Doerfler (2019). Also, a detailed description of the methodology behind the construction of nomograms is devoted in Evesham (1982), Allcock et al. (1963), Levens (1959), Mavis (1939), and Hewes and Seward (1923), in which the use of the elementary properties of determinants is essential.

Figure 1 shows the main elements of a three parallel scales nomogram, where it represents the simplest nomogram form. Moreover, this nomogram solves a simple addition of their reciprocal functions of $1/u_1 - 1/u_2 = 1/u_3$, where u_1 , u_2 , and u_3 are variables. Later, more elaborated nomograms will be discussed to justify the importance of this graphical tool as a relevant educational methodology in sciences and engineering with the support of the open-source code-based PyNomo software. In Figure 1 are represented other important elements such as the scales, which could be defined as the lines marked with graduations arranged in some systematic manner. In fact, on these scales, values of the variables (referred by means of labels) are conveniently plotted, permitting their easy and accurate reading. Those scale graduations can be linear, logarithmic, exponential, trigonometric, or projected (Mavis, 1939). It is worth noting that nonlinear scales generally need different tick levels for different parts of the scale to provide the greatest accuracy to the user. Another basic element of nomograms is the index line or isopleth (Figure 1), which helps understand how the nomogram works, solving the equation by means of deriving the values of the variables. So, the isopleth, generally, is a straight line drawn across the diagram intersecting the appropriate scales at values of the variables which satisfy the plotted equation or formula. Thus, in Figure 1 the isopleth provides that $1/50 - 1/133.3 = 1/80$.

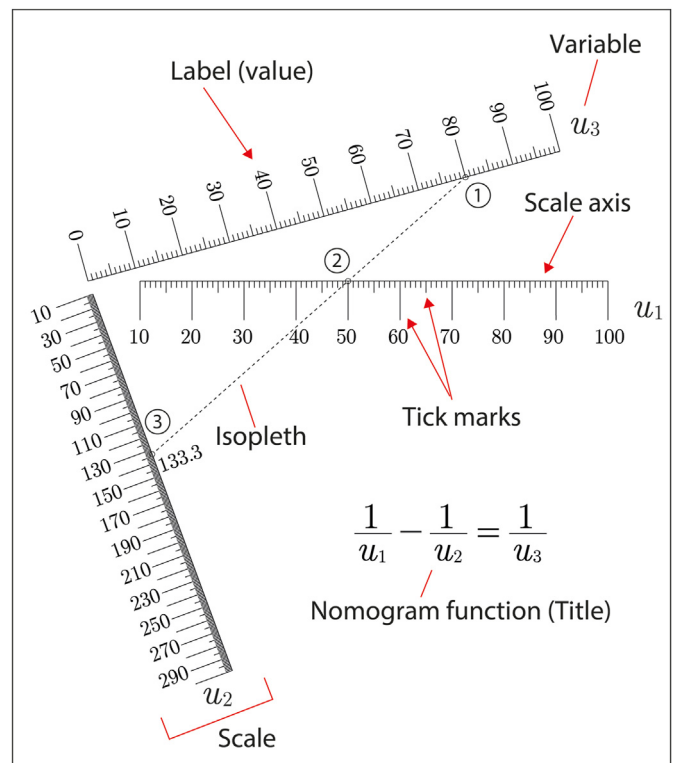


Figure 1. Main elements that constitutes a typical nomogram.

Figure 2 emphasizes the wide variety of formulas that can be represented by nomograms. It is worth recalling that almost any equation that could be taught in sciences or engineering studies, from the simplest formula to the most complex one, could be represented by an appropriate

Nomogram type	Form of equation	Representation
1	$f_1(u_1) + f_2(u_2) + f_3(u_3) = 0$	
2	$f_1(u_1) = f_2(u_2) * f_3(u_3);$ $f_1(u_1) = \frac{f_2(u_2)}{f_3(u_3)}$	
3	$f_1(u_1) + f_2(u_2) + \dots + f_n(u_n) = 0$	
4	$\frac{f_1(u_1)}{f_2(u_2)} = \frac{f_3(u_3)}{f_4(u_4)}$	
5	$f_1(v) = f_2(x, u)$	
6	$u = u$	
7	$\frac{1}{f_3(u_3)} = \frac{1}{f_1(u_1)} + \frac{1}{f_2(u_2)}$	
8	$y = f(u)$	
9	$\begin{vmatrix} f_1(u_1, v_1) & g_1(u_1, v_1) & h_1(u_1, v_1) \\ f_2(u_2, v_2) & g_2(u_2, v_2) & h_2(u_2, v_2) \\ f_3(u_3, v_3) & g_3(u_3, v_3) & h_3(u_3, v_3) \end{vmatrix}$	
10	$f_1(u_1) + f_2(u_2) * f_3(u_3) + f_4(u_3) = 0$	

Figure 2. Nomogram types according to equation forms and their graphical representation.

nomogram, and this is relevant from a pedagogical point of view as nomograms can be used easily to study the effects of different variables on the final formula results or conducting preliminary sensitivity studies before complex, numerical simulations are undertaken.

In Figure 2, $f_1, g_1, h_1, f_2, g_2, h_2, f_3, g_3, h_3$, etc., represents functions, and $x, u_1, v_1, u_2, v_2, u_3, v_3$, etc., represents the variables of the function. In Figure 2, the representation column indicates, for instance, that a type 1 nomogram would be a plot mainly composed of three parallel scales, however, a type 2 nomogram would be represented by two parallel and one oblique instead of three parallel scales. The type 3 nomogram is called “compound” nomogram because it uses more than three parallel scales, which are joined by auxiliary axes (Doerfler, 2009a). This type of

nomograms is ideal for equations of more than three variables (Hoelscher et al., 1952; Marshall, 1921; Peddle, 1910).

The type 4 nomogram is called “proportional” nomogram, which uses crossed isopleths within a boxed area delimited by the scales (Doerfler, 2009a). The type 5 is referred as “contour” nomogram, and it consists of an $x-u$ plot of a family of curves, where the u -value is found from a particular x -value by the curve for a particular value of a third variable (v). This is often seen as a graph with multiple curves (Glasser and Doerfler, 2019). The type 6 nomogram or “ladder” nomogram is simply a connection of two scales providing a transition between two scales that share a common variable (u) (e.g. a transition from linear to logarithmic scale). The type 7 nomogram is known as “reciprocal”, “concurrent”, or “angle” nomogram

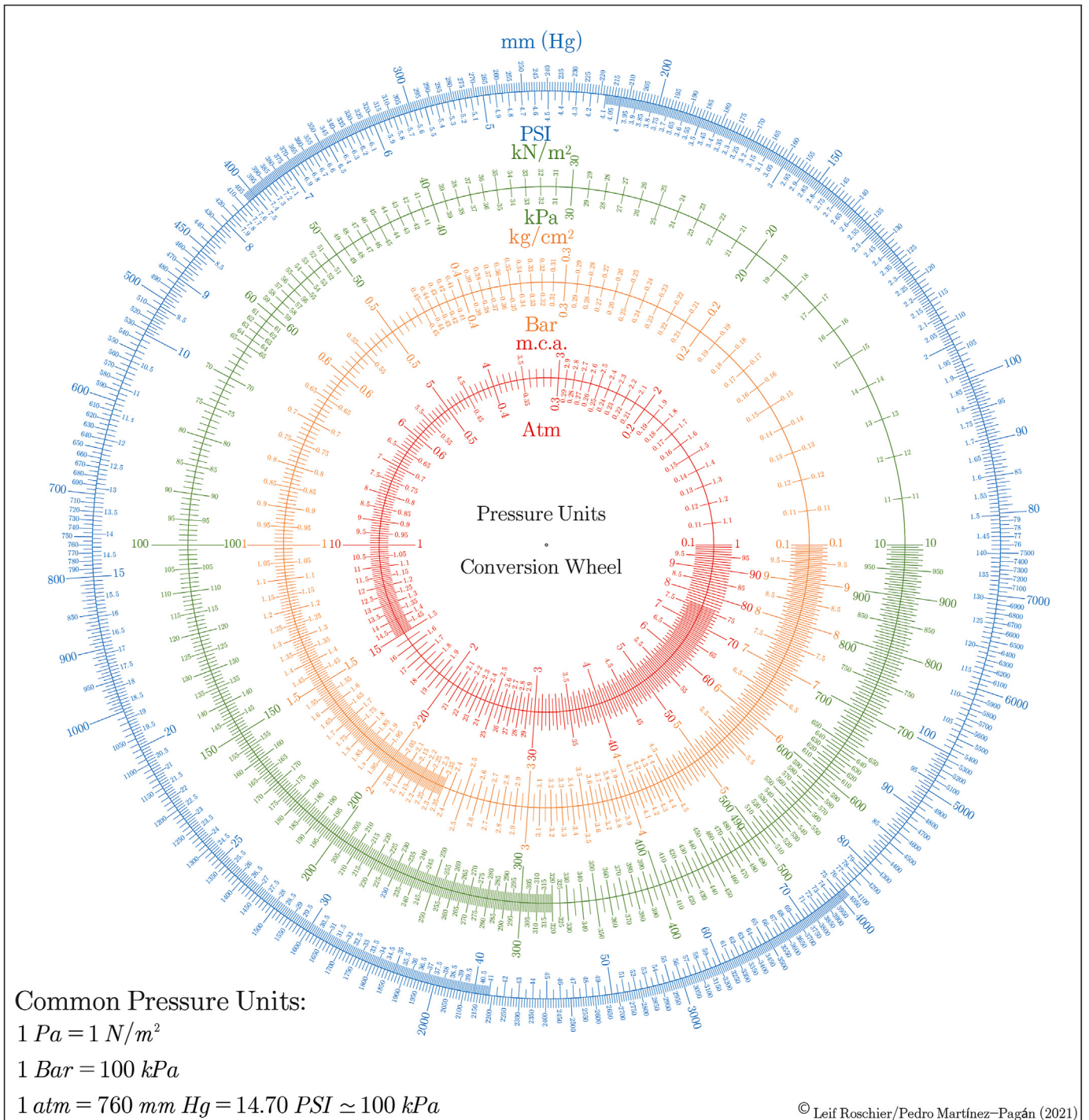


Figure 3. Circular nomogram to convert pressure units.

(Hoelscher et al., 1952). In this type of nomograms, the scales, having a common origin or vertex, are laid-out at different angles of 60° , 45° , etc. (Doerfler, 2009a). The type 8 nomogram, referred as “single” scale, and it is used to derive graphically the relationship of simple functions, which is very useful and illustrative to provide unit conversion of different variables (e.g. $^\circ F$ - $^\circ C$, m-feet, lb-kg, PSI-atm, etc.). The type 9 nomogram is a determinant-based nomogram, which is an elegant and sophisticated way of plotting curved or grid scales, which might be constituted with up to six variables (i.e. determinants from a 3×3 matrix). It is important to highlight that a wide range of engineering equations can be put in determinant form so they can be plotted using a type 9 nomogram (Adams,

1964). Finally, the type 10 nomogram would be a plot composed of two straight scales and one curved scale (Marshall, 1921).

3. PyNomo: a open-source software to create nomograms

In nomography, one of the most cumbersome stages is drawing the scales or grids. So, that inconvenient point likely has been one of the main reasons for the decline of nomogram construction, in conjunction with the advent of more capable computer-aided design software. However, the appearance of PyNomo software (Roschier, 2022) can help to turn this tendency since it has the amazing capability of drawing the

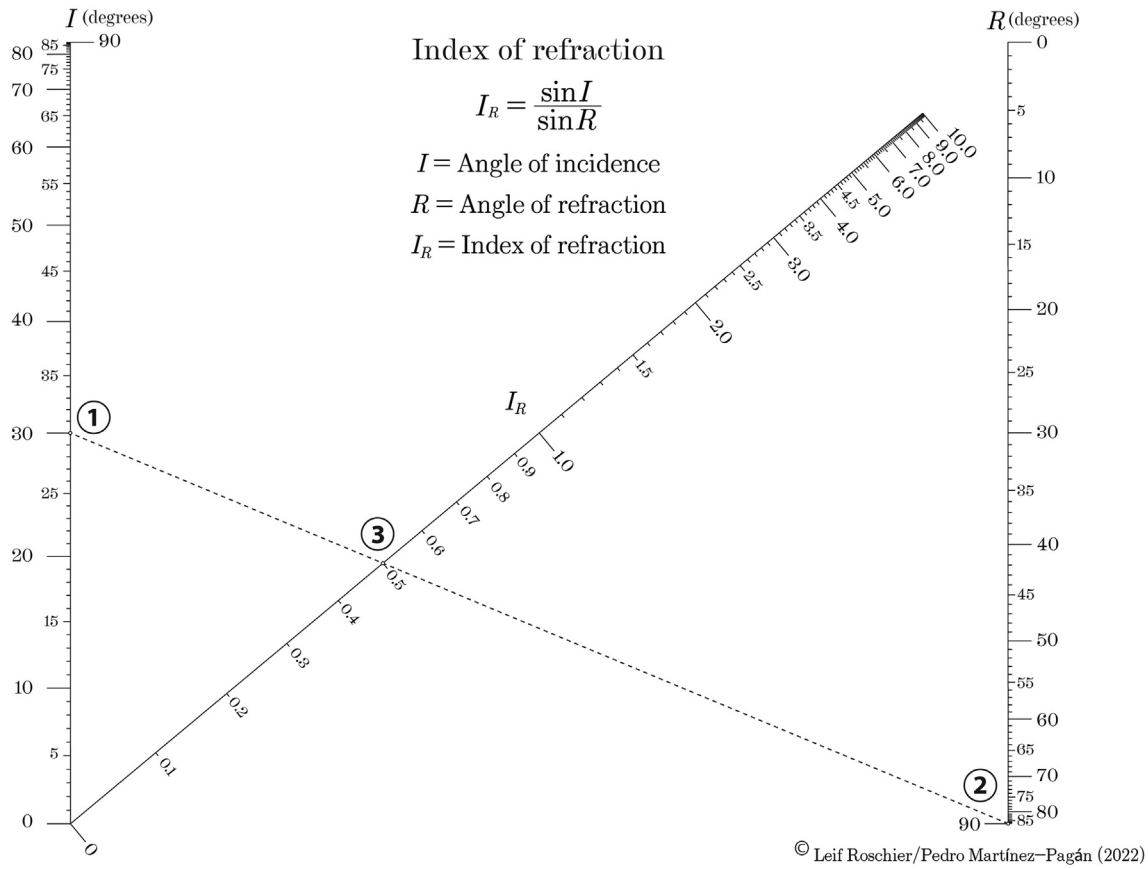


Figure 4. Nomogram for the index of refraction equation.

complete nomogram once the mathematical layout has been obtained (Doerfler, 2009b). PyNomo can be defined as a Python-based software to build nomograms under scalable pdf or eps format. This software was written by one of the authors (LR) and licenced under GPL3. More importantly, the nomograms created by PyNomo are in vector form so they can be edited for additional artwork in any graphic design software, which can support pdf and eps format, such as Adobe Illustrator, Microsoft Visio or LaTeX. Moreover, PyNomo supports nearly all types of nomograms (Boulet et al., 2021), which gives an idea of the key role of this software in introducing nomography in sciences and engineering studies.

At this point, it is highly recommendable to visit the web page http://pynomo.org/wiki/index.php/Pynomo_details and <http://lefakkomies.github.io/pynomo-doc/installation/installation.html> (Boulet et al., 2021) for software installation details, nomogram code examples, tutorials, basic documentation, etc. All of that sort of information and documentation will facilitate to grasp the basic knowledge on the use of PyNomo to construct easily, with a little self-training, the most fundamental nomograms which could solve a wide range of equations that mostly cover those used in sciences and engineering studies (civil, mining, mechanical, energy, hydraulics, medicine, biology, chemistry, physics, etc).

4. Examples of nomograms for sciences and engineering equations

In this section, some nomograms will be examined for sciences and engineering-based formulas to evaluate, in conjunction with PyNomo software, the aforementioned potentialities in an academic environment.

4.1. Sciences: pressure unit conversion wheel nomogram

Figure 3 illustrates a circular nomogram, which is very useful to easily convert pressure units that are of common use in the fields of hydraulics, geotechnics, physics, etc. This nomogram is the most basic nomogram type referred to as type 8 or single scale nomogram (Figure 2). This circular nomogram is constituted of a set of a circular set of scales assigned at each pressure unit, in which drawing a straight line, or isopleth, starting from its centre and crossing the whole nomogram, the conversion of the different pressure units are derived conveniently.

Its construction was made using the PyNomo open-source code provided by Doerfler (2009b) for the construction of circular slide rules. It should be pointed up that the circular nomogram is perfectly scalable, preserving its quality and precision since PyNomo outcomes are in vector form. Thus, it could be accommodated into any area without the need of worrying about final pixelated images, which gives a wide variety of possibilities; for instance, as Doerfler (2009b) states, it can be used as a dial face pasted behind a knob from which a clear plastic overlay is attached with a pointer line inscribed, or even printed on any object to easily have access to it (e.g. a customized mouse pad). So, the possibilities are endless.

This type of nomogram could suit properly for the conversion of other sciences and engineering factors, constants and symbols. In line with that, even though the International System (SI) of units is the standard metric system, however, Imperial units remain in use among engineers and scientists in the USA and a few other countries. So, this kind of nomograms could provide, in a very accessible way, the conversion of engineering and scientific units in those very repetitive tasks.

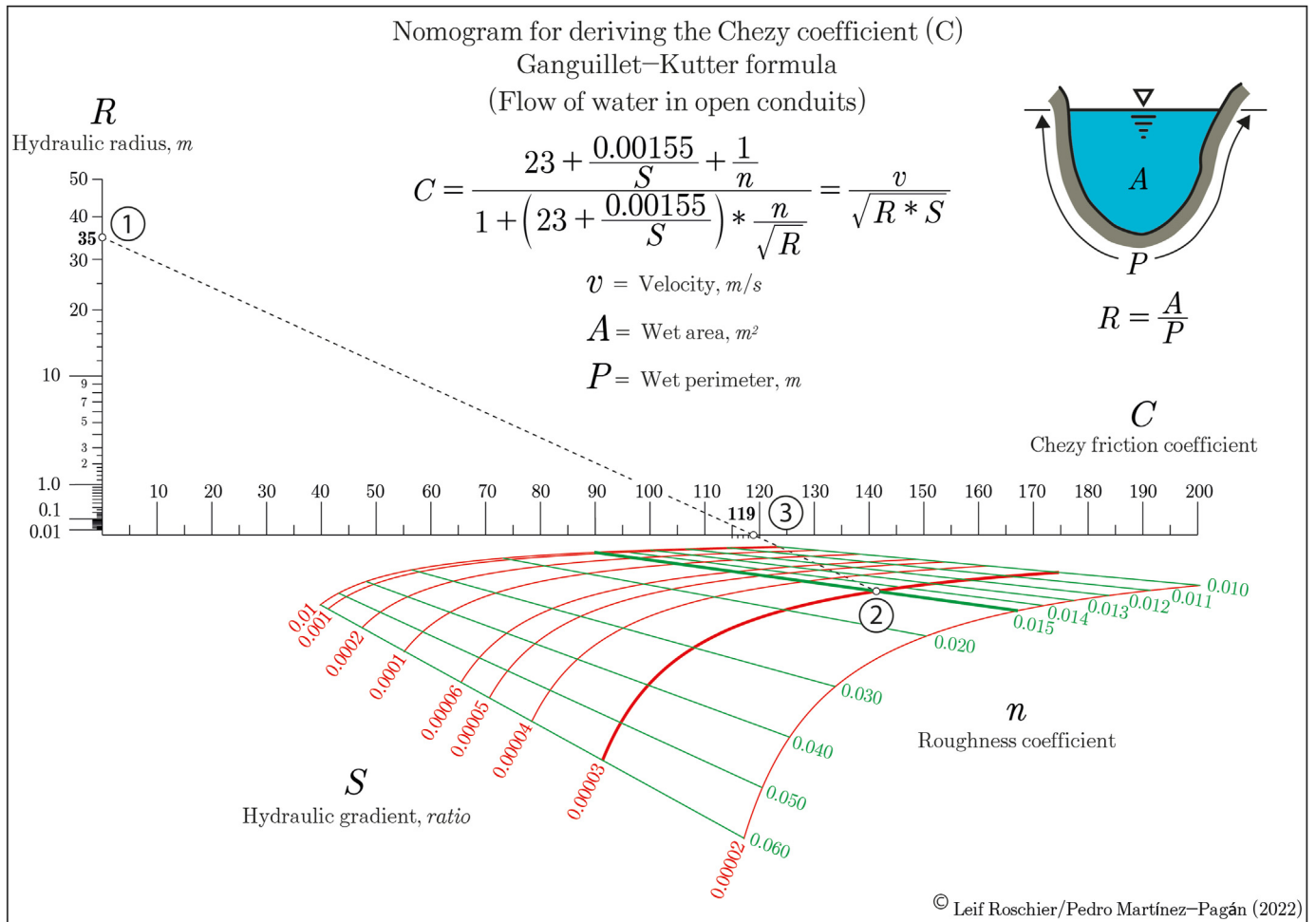


Figure 5. Nomogram for deriving the Chezy friction coefficient.

4.2. Sciences: the index of refraction formula

The index of refraction formula, or Snell's Law, is amply used in optics, seismic, etc. The index of refraction is obtained by the following equation (Robinson and Çoruh, 1988):

$$I_R = \frac{\sin I}{\sin R} \tag{1}$$

where I_R is the index of refraction, dimensionless; I is the angle of incidence, in degrees; and R is the angle of refraction, in degrees.

In seismic, this formula provides the way rays refract, compared to their incidence angle, when they enter a material with different seismic properties (e.g. wave transmission velocity, density, etc.). The index of refraction formula suits the general form of type 2 nomogram (Figure 2), so the graphical representation for this form is a Z chart (Levens, 1959), which is supported by PyNomo software.

Figure 4 illustrates the Z nomogram obtained through PyNomo for the index of refraction. The isopleth provides a value of 0.5 for the index of refraction when the angle of incidence corresponds to 30° and the angle of refraction is 90°. As a check, this value of 0.5 coincides with that obtained from Eq. (1) by a handheld calculator. It should be noted that this case, in which the angle of refraction is 90°, produces a critically refracted wave travelling along the boundary of the two materials (Robinson and Çoruh, 1988).

The final artwork of Figure 4 was done using Illustrator from Adobe. This artwork corresponds to the typing of the title, the formula, and the labels.

4.3. Sciences: The Chezy friction coefficient

Hydraulics is another science branch that has greatly benefited from nomography. In fact, presently, there are still many technical books in which many diagrams are provided to precisely solve hydraulics equations that are of common use. One of these examples is the graphical representation of the Ganguillet-Kutter coefficient, C , in the Chezy formula for the flow of water in conduits (Mavis, 1939):

$$C = \frac{23 + \frac{0.00155}{S} + \frac{1}{n}}{1 + \left(23 + \frac{0.00155}{S}\right) * \frac{n}{\sqrt{R}}} \tag{2}$$

where C is the Chezy friction coefficient; R is the flow velocity, in m/s; n is the roughness coefficient, dimensionless; S is the hydraulic gradient, dimensionless; and R is the hydraulic radius, in m.

The Chezy formula is of interest not only because it solves a complicated equation but also because it is one of the very early examples of nomographic charts for solving a function of four variables (Mavis, 1939).

To graphically represent four-variables formulas it is needed to put them under their equivalent determinant to construct a type 9 nomogram, which is supported by PyNomo software. In line with that, the associated determinant of Eq. (2) is:

$$\begin{vmatrix} C & 0 & 1 \\ 0 & \sqrt{R} & 1 \\ \frac{1}{n} + 23 + \frac{0.00155}{S} & -n * \left(23 + \frac{0.00155}{S}\right) & 1 \end{vmatrix} = 0 \tag{3}$$

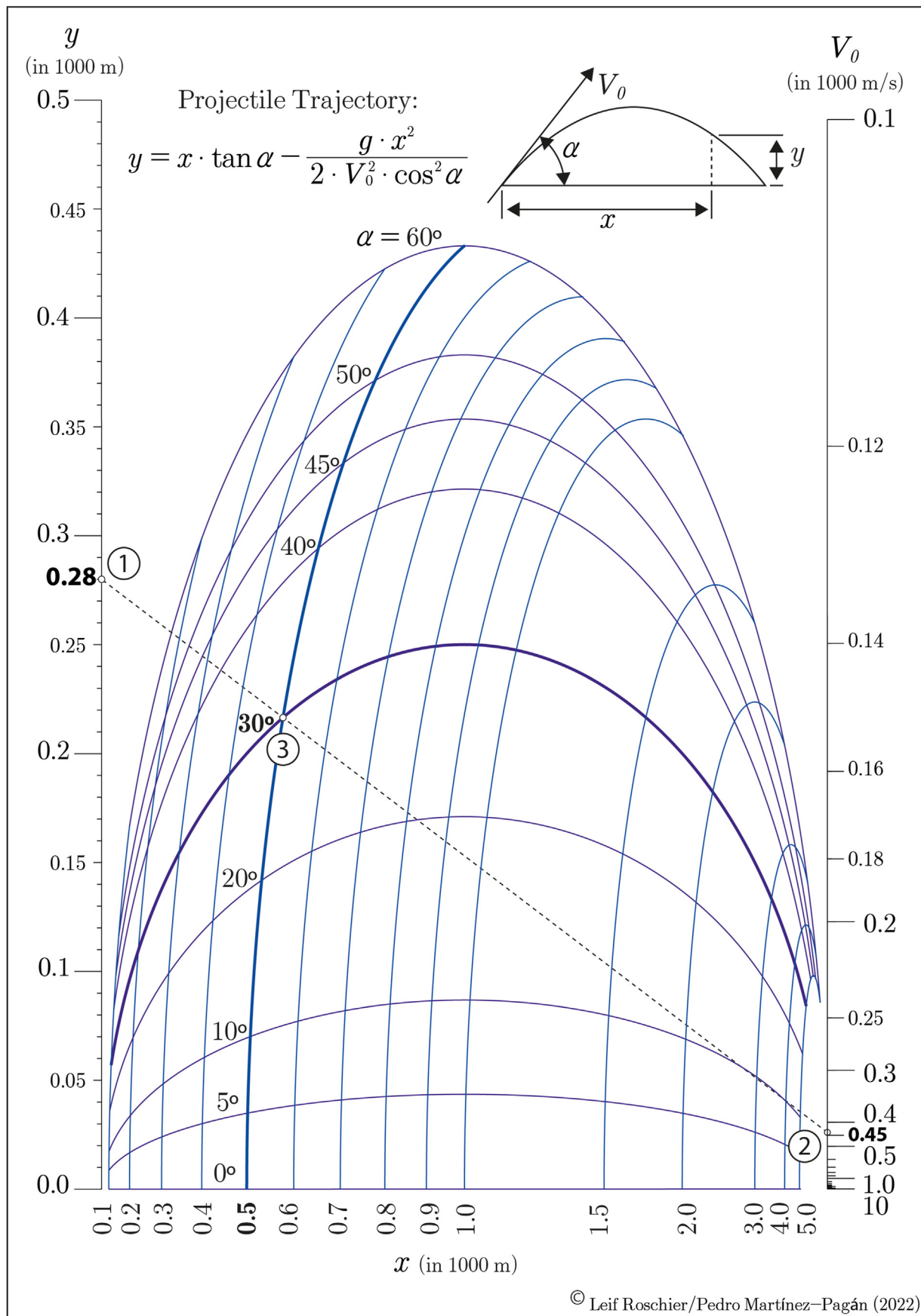


Figure 6. Nomogram for the projectile trajectory.

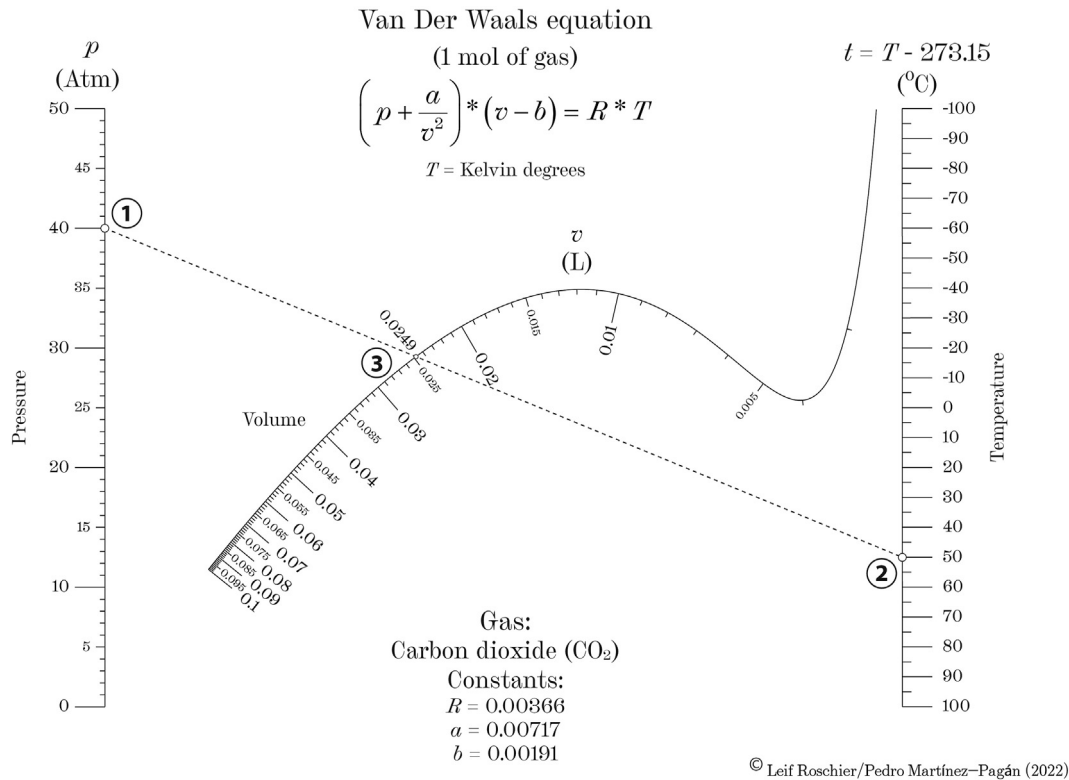


Figure 7. Nomogram for the Van Der Waals equation.

At this point, it should be pointed up that PyNomo solves grid nomograms for equations of up to six variables using the type 9 nomogram code if it is possible to find a way to express the determinant with no more than two variables per row and no variables used in more than one row. Each row having two variables is plotted as a grid rather than a curvilinear scale (Doerfler, 2009b). Eq. (3) satisfies this rule as long as in the first row there is only the variable C, in the second row there is the variable R, and in the third row there are two variables, n, and S.

Figure 5 shows the grid nomogram corresponding to Eq. (2) where the isopleth provides the value of 119 for the Chezy friction coefficient having a roughness coefficient of 0.015, an hydraulic gradient of 0.00003, and a hydraulic radius of 35 m. As a check, from Eq. (2), the Chezy friction coefficient obtained by a handheld calculator was 118.8.

4.4. Sciences: the projectile trajectory

Another important science branch in where nomography could be nicely applied is physics. A typical and common problem, that is covered in physics studies, is used as an example of an interesting and nice application of nomography. The equation for the projectile trajectory, neglecting air friction, is given by (Hoelscher et al., 1952):

$$y = x \cdot \tan \alpha - \frac{g \cdot x^2}{2 \cdot V_0^2 \cdot \cos^2 \alpha} \tag{4}$$

where α is the initial angle, in degrees; V_0 is the initial velocity, in m/s; and g is the acceleration due to gravity, in m/s²; and x and y are the horizontal and vertical position, respectively, of the projectile, in meters.

The projectile trajectory equation involves a curved-scale diagram, so the use of a determinant is required in the same way as it was discussed in the previous example.

A determinant that justifies Eq. (4) is:

$$\begin{vmatrix} \frac{g}{2 \cdot V_0^2} & 0 & 1 \\ y & 1 & 1 \\ \frac{x \cdot \tan \alpha \cdot \cos^2 \alpha}{x^2 + \cos^2 \alpha} & \frac{\cos^2 \alpha}{x^2 + \cos^2 \alpha} & 1 \end{vmatrix} = 0 \tag{5}$$

This determinant (Equation 5) agrees with the standard nomographic form for type 9 nomograms so it can be graphically represented with the support of PyNomo software. Figure 6 shows the grid nomogram corresponding to Eq. (4) where the isopleth provides the value of 280 m for variable y, 500 m for variable x, considering an initial angle of 30°, and assuming an initial velocity of 450 m/s. As a check, from Eq. (4), the vertical position (y) obtained by a handheld calculator was 280.6 m.

4.5. Sciences: Van Der Waals equation

Chemistry is another science branch that could benefit from the application of nomography in a learning environment. To support that a nomogram for the classical Van Der Waals equation is provided. The Van Der Waals equation is commonly used for gas law problems in chemistry. The Van Der Waals equation, considering one mol of gas, is written as (Parsegian, 2005):

$$\left(p + \frac{a}{v^2}\right) * (v - b) = R * T \tag{6}$$

Where p is the pressure, in Atm; v is the volumen in liters, L; R is a gas constant in L-Atm/mol-K; T is the absolute temperature in Kelvin, K; a and b are constants which depend on the type of gas considered. Eq. (6) can be expressed, in terms of centigrades degrees instead of Kelvin degrees, as:

$$\left(p + \frac{a}{v^2}\right) * (v - b) = R * (t + 273.15) \tag{7}$$

For the construction of the nomogram for the Van Der Waals equation, it is needed to consider a gas, which will provide its different

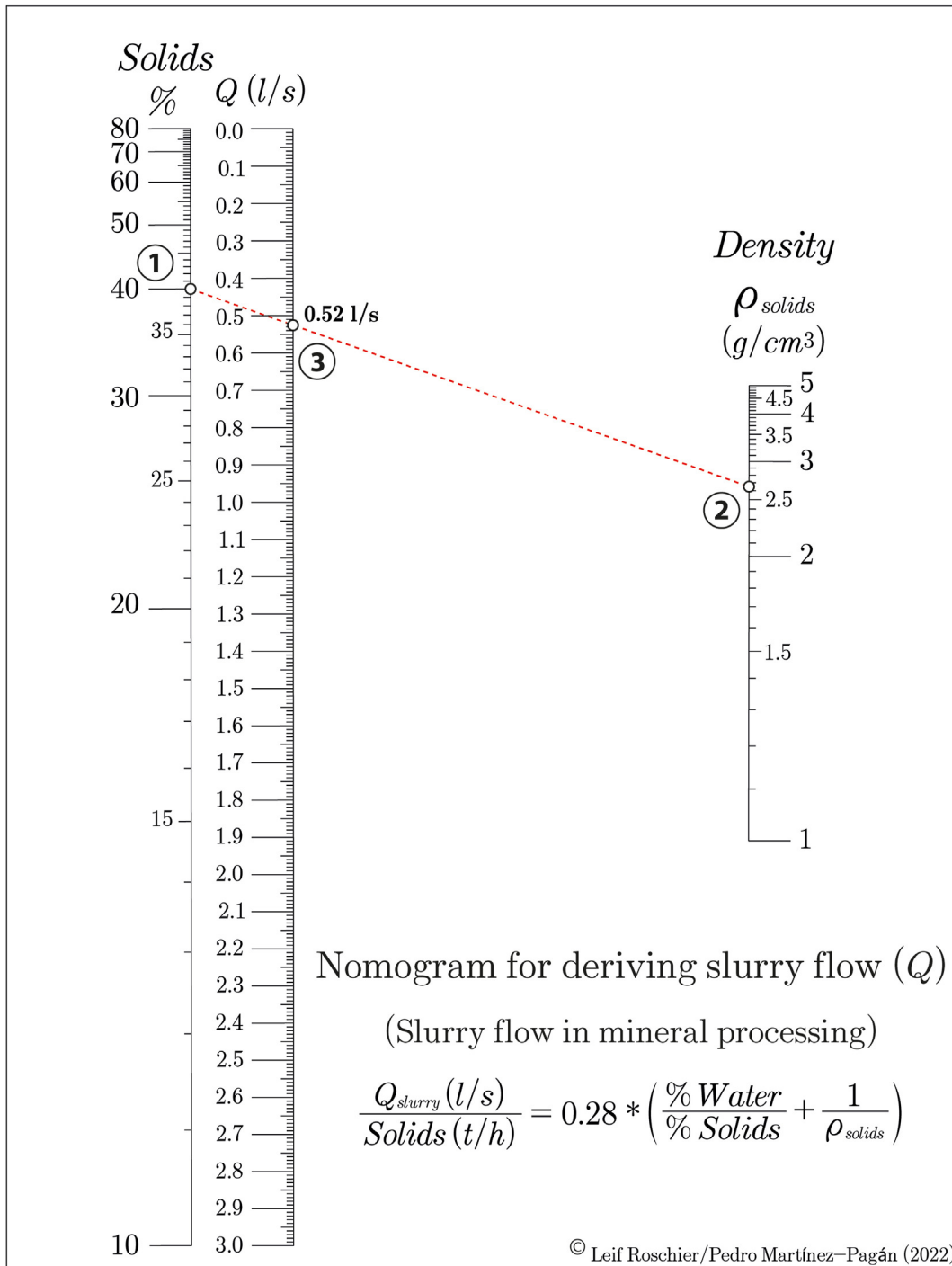


Figure 8. Nomogram for the slurry flow in pipes formula.

constants. For our example it was chosen the carbon dioxide (CO₂) gas which provided a value of 0.00366 for the constant R , a value of 0.00717 for the constant a , and a value of 0.00191 for the constant b .

If those values are substituted in Eq. (7), and then its terms are properly arranged, this new expression meets the type 10 nomogram (Eqs. (8) and (9)):

$$p - t * \frac{0.00366}{v - 0.00191} - \frac{273.15 * 0.00366}{v - 0.00191} + \frac{0.00717}{v^2} = 0 \tag{8}$$

Since:

$$f_1(p) - f_2(t) * f_3(v) + f_4(v) = 0 \tag{9}$$

Figure 7 shows the nomogram obtained of Van Der Waals equation for one mol of carbon dioxide in which the isopleth provides a value of 0.0249 L of volume when the conditions of temperature and pressure are 50 °C, and 40 Atms, respectively. As a check, from Eq. (7), the value obtained by a handheld calculator was 0.02485 L of volume.

4.6. Engineering: slurry flow in mineral processing

In engineering, there is a considerable number of equations that are suitable to be graphically represented by means of nomograms. Thus, the

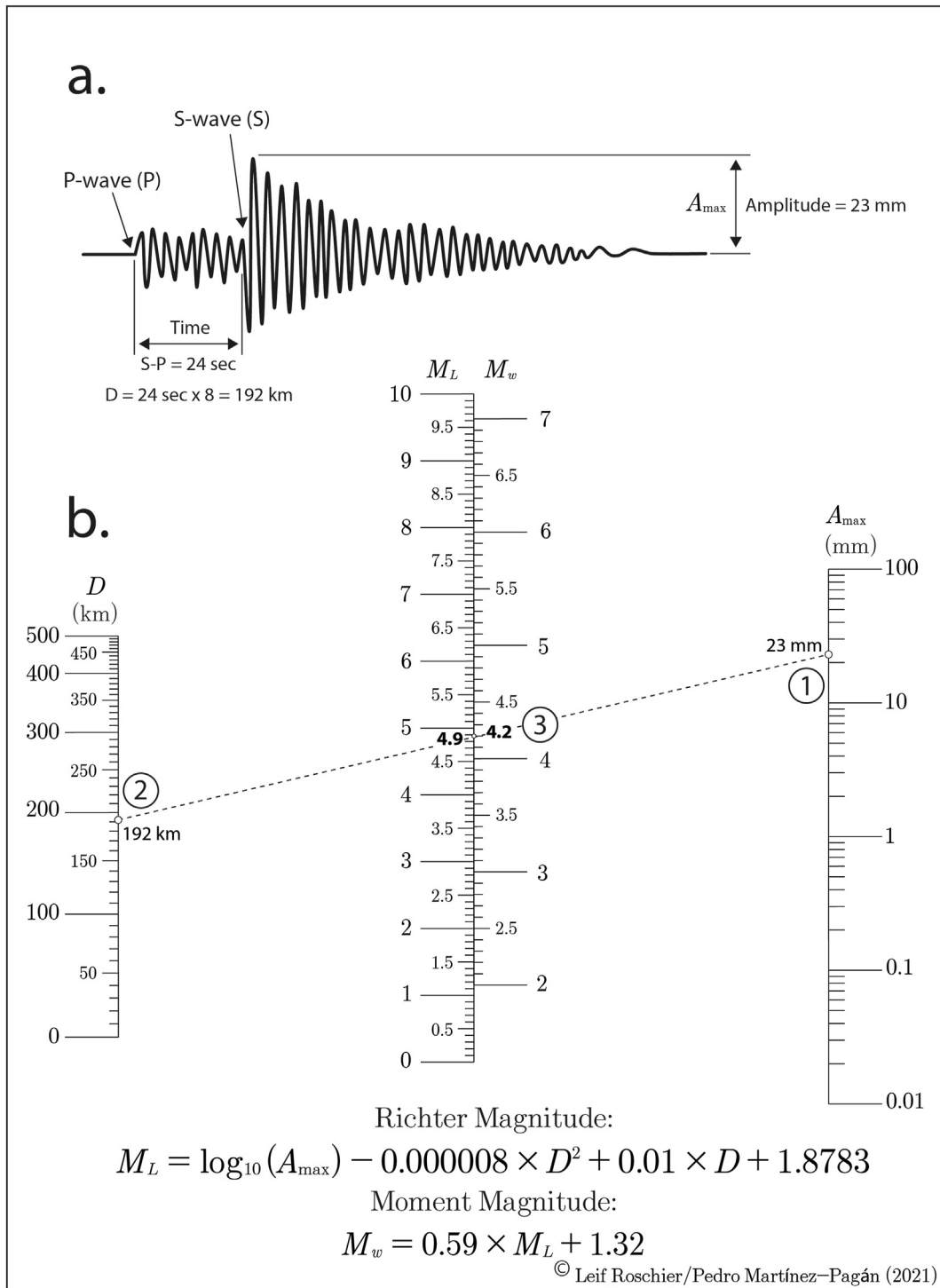


Figure 9. Nomogram for obtaining the earthquake magnitude.

expression to derive the slurry flow traveling in pipes in the mineral processing industry is one of these clear examples, which is represented by means of parallel and straight charts supporting that statement. The equation for the slurry flow per t/h of solids to be pumped through pipes is given by (López-Jimeno, 1994):

$$\frac{Q_{\text{slurry}} \text{ (l/s)}}{\text{Solids} \text{ (t/h)}} = 0.28 \times \left(\frac{\% \text{ Water}}{\% \text{ Solids}} + \frac{1}{\rho_{\text{solids}}} \right) \quad (10)$$

Where Q_{slurry} is the slurry flow per t/h of solids, in l/s; % Water refers to the percentage of water in the slurry, in weight; % Solids refers to the percentage of solids in the slurry, in weight; and ρ_{solids} is the specific density, in g/cm³. It must be pointed up that if the percentage of solids is 40%, then the percentage of water will be 60%, totalling both 100%.

The construction of this nomogram is based on type 1 form as Eq. (10) can be easily arranged as:

$$\frac{Q_{slurry} \text{ (l/s)}}{\text{Solids (t/h)}} - \frac{100 - \% \text{ Solids}}{\% \text{ Solids}} - \frac{1}{\rho_{solids}} = 0 \quad (11)$$

Now, Eq. (11) fulfills the expression for type 1 nomograms, which are made up of three straight parallel scales (Equation 12):

$$f_1(Q_{slurry}) - f_2(\% \text{ Solids}) - f_3(\rho_{solids}) = 0 \quad (12)$$

Figure 8 illustrates the alignment charts of Eq. (11) where the isopleth indicates a value of slurry flow per t/h of solids of 0.52 l/s for a solid specific density of 2.65 g/cm³, and a percentage of solids in the slurry of 40% (percentage of water in the slurry of 60% in the slurry). As a check, from Eq. (10), the value obtained by a handheld calculator was 0.53 l/s of slurry flow. At this point, it is important to highlight that PyNomo enables increase the scale subdivision is more precision is required thus obtaining values with one, two or more decimals.

4.7. Engineering: seismic magnitude of earthquakes

Another useful application of nomography in seismic engineering is obtaining the value of an earthquake magnitude immediately after its occurrence. For many years, the intensity of an earthquake occurrence has been provided in terms of the Richter magnitude (ML). The Richter magnitude scale was developed in 1935 by Charles F. Richter of the California Institute of Technology as a mathematical device to compare the size of earthquakes (Richter, 1958). The expression that provides the strength of an earthquake following the Richter scale is:

$$M_L = \log_{10}(A_{max}) - 0.000008 \times D^2 + 0.01 \times D + 1.8783 \quad (13)$$

Where M_L is the Richter magnitude, dimensionless; D is the distance between the point of earthquake occurrence and the seismic recording site, in km; A_{max} is the maximum amplitude, in mm. The distance, D , can be derived from the recorded seismogram knowing the time between the P-wave and the S-wave first recordings (Figure 9a).

Observing Eq. (13) it is seen that it fulfills again a type 1 nomogram of the form (Equation 14):

$$f_1(M_L) - f_2(A_{max}) - f_3(D) = 0 \quad (14)$$

Figure 9 illustrates the nomogram obtained which is characterized by three straight parallel scales. This nomogram is similar to that provided by Green and Hall (1994), which corroborates that the applied methodology has been appropriate and consistent. Another interesting and elegant implementation is adding to the Richter magnitude scale the moment magnitude (M_w) scale to obtain its conversion provided that the moment magnitude scale is of common use presently (Mereu, 2020). For that magnitudes conversion is adopted the expression of Mereu (2020):

$$M_w = 0.59 \times M_L + 1.32 \quad (15)$$

Eq. (15) is graphically implemented following a type 8 nomogram ($y = f(u)$) as Doerfler (2009b) describes. Figure 9b shows the final nomogram in which the isopleth indicates a Richter magnitude of 4.9 (moment magnitude of 4.2), for an earthquake that strikes at a distance of 192 km from the measurement point with a maximum amplitude of 23 mm. As a check, the values obtained by a handheld calculator for Richter and moment magnitude from Eqs. (13) and (15), respectively, were 4.87 and 4.19.

5. Conclusions

Nomography is a mathematical branch that, employing nomograms, easily derives the value of variables for mathematical expressions, no matter how complex they are. Along the past century, the employ of nomography, as well as its development, was considerable. However, its continuous and unstoppable decline started with the advent of handheld

calculators and computers, and as a consequence, nowadays is considered almost a forgotten science.

This work attempts to highlight the fundamental importance of nomography as a renewed academic tool to support sciences and engineering studies. This importance is because nomograms can solve very complicated formulas with amazing ease through PyNomo software. In fact, PyNomo software has come to solve the most troublesome and time-demanding point of nomography as it is scales construction. Immediately this aspect is facilitated, nomography can be introduced appropriately in the academic environment. It can play a relevant role in providing hands-on application of sciences and engineering formulation in an engaging, creative learning activity. Moreover, sciences and engineering students might benefit from the fact that nomograms can help study the effects of different variables of a formula. These nomograms are useful in conducting preliminary sensitivity studies and providing the skills to interpret those nomograms that are still in use.

Declarations

Author contribution statement

Leif Roschier: Conceived and designed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data.

Pedro Martínez-Pagán: Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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Data availability statement

Authors are working in making these data available as soon as possible through an open access repository.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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