

# On Ultimate Strength of an Inland Waterway Barge

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*Due to the sudden nature of hull girder collapse caused by extreme loadings, the ultimate strength of ships, i.e., ultimate capacity, has to be evaluated. Ultimate strength analysis procedures have already been provided within the rules of the classification societies for sea-going ships. However, rules for inland vessels are not fully addressing the issue. In addition, literature data on the ultimate strength of inland vessels are almost negligible, which is alarming, considering the frequency of grounding and overloading events in inland navigation. Moreover, inland vessels' structural elements are prone to buckling due to their slender plates. In order to evaluate ultimate strength, an inland waterway (IW) barge is chosen for progressive collapse analysis (PCA) employment. PCA has demonstrated that the buckling collapse of structural elements vastly governs a vessel's ultimate capacity. Results show the extent of the safety zone between the actual loss of the ultimate capacity and the linear-elastic behavior of the structure.*

**Keywords:** *ultimate strength, progressive collapse, inland vessel ultimate strength, inland vessel, inland barge, barge, ultimate bending moment, inland vessel collapse.*

## 1. INTRODUCTION

During the service, ships are subjected to loads and events that are generally regarded as predicted in the design phase. Thus, ship structures are almost exclusively designed according to design (service) loadings and allowable stress, which is generally taken as a share of the material's yield stress. This assumes that the structural response will always be within the linear-elastic structural behavior of the material.

However, extreme and accidental events can be experienced during the ship's lifetime and, therefore, can produce excessive bending moments. This may lead to catastrophic outcomes such as the complete loss of the ship. For instance, hull girder collapse or "brake in two" is the ultimate scenario of the ship's structural failure and can be a consequence of the grounding, overloading due to the large amount and distribution of weights on a ship, waves, etc. Taking into account the significance of the potential outcome, classification societies have already implemented procedures to evaluate the ship's ultimate capacity to carry loads, i.e., ultimate strength. One of the most notable methods for the ultimate strength assessments is a progressive collapse analysis (PCA), and its calculation procedure, based on the Smith method [1], is described in Common Structural Rules for Bulk Carriers and Oil Tankers (CSR), see [2]. To some extent, other classification societies incorporate ultimate strength procedures in their rules and regulations; see BV's [3] and LR's rules [4].

The main idea of the ultimate strength evaluation is

to provide a more realistic calculation of the margin of safety and to answer the question: until which level the ship's hull girder can be loaded before it experiences structural collapse or the loss of its capacity to carry loads?

In the past several years, numerous investigations have been performed regarding the ultimate strength of sea-going ships. In that sense, ultimate strength analysis has frequently been used in the preliminary design phase. More recent books dealing with the ship structural design are delivering their large portion to the ultimate strength analysis; see [5, 6, 7, 8]. Moreover, committee III.1 (Committee III.1 Ultimate Strength) of the International Ship and Offshore Structures Congress (ISSC) published their report on the ultimate strength of ship structures in [9], providing the benchmarking and guidance for the ultimate strength assessments.

In contrast, rules and regulations for the inland vessels of the classification societies and other shipping authorities still need to fully develop procedures for the ultimate strength assessment on the hull girder level. Moreover, literature data on ultimate strength evaluations for inland vessels barely exist. Nevertheless, more than 20 years ago, in order to address the structural collapse recordings and prevent future ones, a study [10] was published on the ultimate strength of inland tank barges, but the analysis was not performed using sophisticated methods nowadays. More recent investigations, for instance, included inland catamaran ultimate strength analysis [11]. In that regard, and according to the best of the authors' knowledge, traditional inland cargo vessels are still not investigated. However, some studies on their structures and service loadings can be found in [12] and [13, 14], respectively. This can be disturbing because inland vessels can experience the exceedance of their ultimate strength and, finally, hull girder collapse. The collapse can happen due to ground-*ing*, which is

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more frequent considering low water periods due to climate change effects, as pointed out in [15]. Moreover, overloading due to excessive loads and improper overhauling of an already grounded vessel can produce excessive bending moments. When coupled with elements diminution due to aging (inland vessels can be significantly older than sea-going ships), such bending moments may lead to the hull girder structural collapse.

Taking into account the actual lack of data on inland vessels' ultimate strength and on knowledge of their structural behaviour until the collapse, the paper's main goal is to evaluate the ultimate strength of the typical inland waterway (IW) barge using PCA. Such analysis delivers not only the measure of the ultimate bending moment but also provides sequences of structural members collapse until the collapse of the whole hull girder. This is expected to display a glimpse of understanding of the inland vessel collapse phenomenon. Thus, the following section 2 explains the nature of the ultimate strength and methods for its evaluation. Methods particularly explained in sections 2.2 and 2.3 are used for the case study presented in this paper. Section 3 introduces the benchmarking vessel, while sections 4 and 5 display obtained results and discussion, followed by the conclusion in section 6.

## 2. ULTIMATE STRENGTH

Bending in a vertical plane (symmetry plane) is the dominant loading for sea-going cargo ships and inland cargo vessels exceeding 60-65 m in length. When the maximum vertical bending moment (VBM) is reached, it is called the ultimate VBM, and the hull's corresponding (able to withstand it) strength is the ultimate strength. The vessel can reach ultimate strength due to VBM in sagging or hogging condition. In the sagging, elements above the neutral axis of the hull cross section are subjected to compression and may experience buckling if compressive loads exceed buckling critical loads. On the other side of the cross-section, elements located below the neutral axis will undergo tension which can lead to plastic deformations or yielding if the material's yield stress is reached. In the hogging condition, the vessel cross-section experiences the opposite structural behavior described in the sagging condition. Therefore, for each structural element, the stress-strain curve, both linear and nonlinear part, has to be presumed. Accordingly, the stress-strain curve has to be identified in compression and tension.

There are numerous methods to estimate the actual ultimate strength of the vessel. They are based on a set of assumptions and the level of complexity which involves computational time and inclusion of sophisticated local structural phenomena. Methods that use simple beam theory and presumed stress distribution along the height of the cross-section of the vessel are called presumed methods. Ones that model stress-strain linear and plastic structural behavior of each cross-section element in a more refined manner while employing the post-buckling behavior and large deformations are called progressive collapse methods.

### 2.1 Presumed methods

On the basic level, as traditionally used, one can assume that the vessel has reached its ultimate strength when the first (one) element achieves yield stress (first-yield VBM). This could be experienced by the bottom sagging or the deck (or hatch coaming) in hogging condition. Similarly, it can be assumed that the ultimate strength is attained when the first (one) element achieves buckling stress (first-collapse VBM), which is generally deck (or hatch coaming) in sagging and bottom in hogging conditions. Nevertheless, first-yield and first-collapse bending moments are not the ultimate ones. They could be assumed as ultimate VBM if the single element yielding or buckling phenomenon had been assumed to be an ultimate scenario for the cross-section [5, 7] during the structural assessments. Both approaches are based on the linear-elastic behavior of the structure. Note that the maximum VBM allowed by the rules of the classification societies is calculated according to the allowable yield stress criterion, which is the share of the yield stress of the material [2, 3, 4], set by the rules. This share acts as a safety factor. The actual ultimate VBM of cargo vessels is larger than the maximum allowable VBM and is generally larger than the first-yield or the first-collapse VBM, as the vessel should still be able to carry loads after the single element of the bottom or deck have yielded or collapsed.

Moreover, one can assume that the ultimate strength is reached when all elements experience yield stress (full-plastic VBM). Such value of the VBM is generally unrealistically high because the vessel will never experience a scenario in which all elements would undergo plastic deformation. Hence, full-plastic VBM can be used as an upper-level reference of VBM. Before that point, the vessel will reach its actual VBM due to multiple buckling. Full-plastic ultimate strength sequences are more related to the thick-plated structures with no buckling modes.

In order to adopt a more realistic ultimate state, Caldwell [16] proposed the assumption that the ship had reached ultimate strength when all the tensioned elements at one side of the neutral axis had experienced yield stress, and all the elements on the other side of the neutral axis had experienced buckling stress. Naturally, such VBM will be lower than full plastic because critical buckling stress is lower than yield stress for slender structures, usually found in ship structures. However, actual VBM will still be unreachable as ships would have lost their capacity to carry loads before all their elements experienced buckling or yielding. In both the full-plastic and Caldwell scenario, the position of the neutral axis changes to satisfy the force equilibrium of the cross-section, considering the updated stress distribution.

Paik and Mansour went a step further by developing two methods. In the first one, called Paik – Mansour [17], authors assumed that, whether in sagging or hogging conditions, several elements along the height of the compressed part of the cross-section experienced buckling stress, and just upmost distanced elements experienced yield stress, at the opposite side of the cross-section.

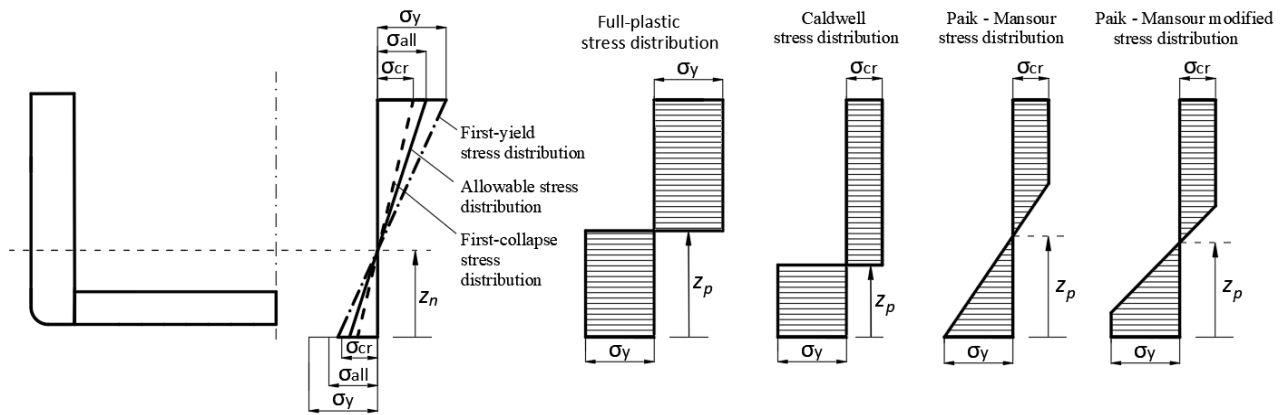


Figure 1. Presumed methods – stress distribution for the sagging condition

Table 1. Presumed methods – calculation of ultimate VBM

Methods	Maximum or ultimate VBM ( $M_U$ )
Allowable stress	$M_U = \min(SF \cdot \sigma_y \cdot Z_B, SF \cdot \sigma_y \cdot Z_D)$
First-yield	$M_U = \min(\sigma_y \cdot Z_B, \sigma_y \cdot Z_D)$
First-collapse	$M_U = \min(\sigma_{cr,B} \cdot Z_B, \sigma_{cr,D} \cdot Z_D)$
Full-plastic	Equilibrium of axial forces: $\int \sigma_x dA = 0$ New position of the neutral axis:
Caldwell	$z_p = \frac{\sum_{i=1}^n A_i z_i  \sigma_i }{\sum_{i=1}^n A_i  \sigma_i }$
Paik – Mansour	$\downarrow$
Paik – Mansour modified	$M_U = \sum_{i=1}^n A_i \sigma_i  z_i - z_p $

Furthermore, as the stress distribution changes, the position of the neutral axis also changes until the equilibrium of forces is reached for the cross-section. Later, Paik and Mansour proposed an improved version of the first method, called the Paik – Mansour modified method [18]. The method assumes that in both compressed and tensioned parts of the cross-section, along their respective heights, several elements undergo buckling on one side and yielding on another. Upon achieving the equilibrium of forces along the cross-section and neutral axis position changes; the reached VBM can be labeled as the ultimate VBM.

The simplistic illustration of the presumed methods' stress distribution for the sagging condition is given in Figure 1 (for hogging and vice versa), while their general calculation procedure is presented in Table 1.

## 2.2 Progressive collapse methods

More sophisticated methods model the progressive collapse of the hull. Some of them are The idealized Structural Unit Method (ISUM), Nonlinear Finite Element Method (NLFEM), and the Smith method.

ISUM simplifies the complex structural models. For instance, a stiffener plate is divided into segments that create an assembly. Each subdivision has a closed-form

expression that includes nonlinear behavior based on theoretical, numerical, and experimental studies. The method originated from [19]. It is less time-consuming than NLFEM. NLFEM considers interactions between the structural elements while employing nonlinear structural behavior. The method is time-consuming and requires modeling techniques described in [5, 7].

In this paper, analysis is based on the methodology for the Smith method, explained in the following, and modified Smith method, see section 2.3.

Smith method was first introduced in [1], and currently, it is used within IACS CSR [2]. Here, the method is labeled as CSR PCA and includes the following steps:

1. Hull cross-section is divided into the following segments, according to the procedure from [2]: stiffener with attached plating (SAP), plating (PL), and hard corner (HC).

2. For each segment, a stress-strain curve for both linear and the plastic domain is assumed. Considering compression and tension, see Figure 2 for mild steel used in this analysis, according to recommendations in [2]. Stress-strain curves consider buckling and post-buckling behavior, as well as imperfections, implicitly. Moreover, the stress-strain curve in tension is assumed to be elastic-perfectly plastic for all segments. In compression mode, the curve exhibits the compression behavior leading to the lowest buckling stress in all buckling modes, including beam-column buckling, torsional buckling, and local buckling of stiffener web (for SAP) and plate buckling (for PL). Contrary to SAP and PL, HC segments are assumed to experience elastic-perfectly plastic behavior in compression and tension, considering their larger rigidity. The method does not include interactions between the segments.

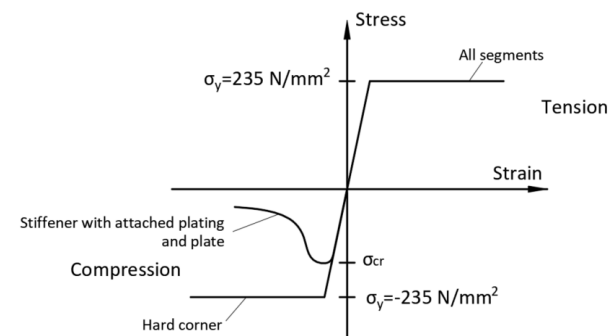
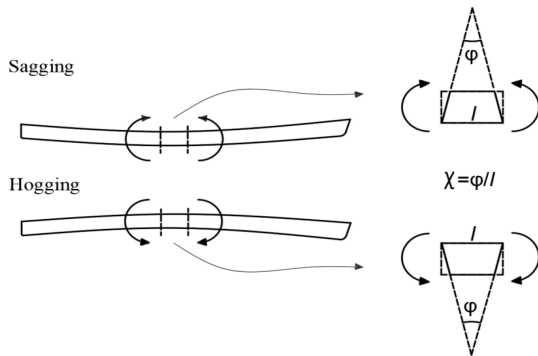


Figure 2. Stress-strain curves

3. The model is represented by the hull cross-section extended between web frames. The cross-section is subjected to the vertical bending curvature  $\chi$  (hogging or sagging), which is incrementally increased from  $\chi_U$  (curvature that induces stress equal to 1% of yield strength in strength deck) to the values leading the cross-section to the ultimate VBM, see Figure 3. In each step of the increment, the sum of all axial forces from all segments has to be equal to zero, and thus, the instantaneous new neutral axis has to be calculated, see (1):

$$\sum_{i=0}^n A_i \sigma_i = 0. \quad (1)$$



**Figure 3. VBM subjected to the mid hull cross-section creating curvature and rotation of the cross-section planes**

4. In each increment, the strain of segments ( $\varepsilon_i$ ) has to be calculated according to new curvature ( $\chi$ ), the position of the segments ( $z_i$ ), and the new position of the neutral axis above the baseline ( $z_p$ ), as in (2):

$$\varepsilon_i = \chi(z_i - z_p). \quad (2)$$

5. In each of the increment, VBM corresponding to the particular increment is calculated, according to (3):

$$M_U = \sum_{i=1}^n A_i \sigma_i |z_i - z_p|. \quad (3)$$

Ultimate VBM ( $M_U$ ) is calculated taking into account all segments and their stresses ( $\sigma_i$ ), which are determined based on a stress-strain curve and the actual strain from (2). Finally, the diagram VBM – curvature is to be designed. Maximum VBM on VBM – the curvature diagram, or the point from which the slope of the VBM-curvature diagram becomes negative, is called the ultimate VBM.

A complete procedure with equations for buckling in the case of the Smith method's progressive collapse analysis (CSR PCA) is described in [2] and will not be given here due to the complexity of the formulations.

### 2.3 Modified progressive collapse method

In this paper, IW barge ultimate strength will be analyzed using the CSR PCA method from [2] and the same CSR PCA but slightly modified in one aspect (here, the second approach is designated as – P-T PCA). Namely, formulations used in [2] for the buckling collapse of SAP are complex equations adapted for sea-going ships such as bulk carriers and oil tankers. Since the analyzed ship here is an inland vessel, we replaced those equations with

a more general and simpler one used for buckling steel structures. Such is called Paik – Thayamballi (P-T) empirical formulation for the steel plate-stiffener model. It is taken from [20] and presented here as:

$$\frac{\sigma_U}{\sigma_{Yeq}} = \frac{1}{\sqrt{0.995 + 0.936\lambda^2 + 0.17\beta^2 + 0.188\lambda^2\beta^2 - 0.067\lambda^4}} \quad (4)$$

According to a technical review of ultimate strength formulations in [21], the P-T formula delivers more realistic results than classical Euler and Johnson-Ostenfeld formulas used for SAP in CSR PCA [2]. Later ones tend to overestimate the buckling behavior of the stiffener-plate model, especially in cases with a more increased plate slenderness ratio, as studied in [21]. Note that inland vessels tend to have more slender plates. On the other hand, (4) is insensitive to lower column slenderness ratios due to the nature of the empirical formulations [21]. Equation (4) implicitly includes the effect of local buckling, lateral torsion buckling, initial deflections, and residual welding stresses considered at an average level [7, 21]. Some of these phenomena are also acknowledged in several formulations in CSR PCA [2] but in a more analytically complicated manner. Nonetheless, the disadvantage of (4) lays in the fact that the column ( $\lambda$ ) and plate slenderness ( $\beta$ ) are considered without the effect of the width of the attached plating, unlike in CSR PCA [2], where the previously described effect is applied. Moreover, the limit of equation (4) is that such cannot be used as larger than  $1/\lambda^2$ . The parameters for (4) are to be calculated according to standard equations for column and plate slenderness ratios [5, 7]:

$$\lambda = \frac{L}{\pi r} \sqrt{\frac{\sigma_{Yeq}}{E}} \quad (5)$$

$$\beta = \frac{b}{t} \sqrt{\frac{\sigma_Y}{E}}$$

In tension, for CSR PCA and P-T PCA, all segments are assumed to experience elastic-perfectly plastic structural behavior, see Figure 2. In compression, they differ, according to Table 2. In the case of CSR PCA, buckling formulations are given by IACS and generally explained in section 2.2 and thoroughly in [2]. In the basis of the plate buckling is the Frankland formula [22], and in the buckling of the stiffener with attached plating case – Euler and Johnson-Ostenfeld formulas. More on plate buckling can be found in [23].

**Table 2. CSR PCA vs. P-T PCA – buckling methods**

Segments	CSR PCA [1, 2]	P-T PCA
Stiffener with attached plating	- Beam-column buckling - Torsional buckling - Web local buckling of flanged profiles - Web local buckling of flat bars	Paik – Thayamballi equation for stiffener with attached plating, see (4) and [20]
Plate	Plate buckling (Euler and Johnson-Ostenfeld)	
Hard corner	Elastic-perfectly plastic stress-strain curve, see Figure 2	

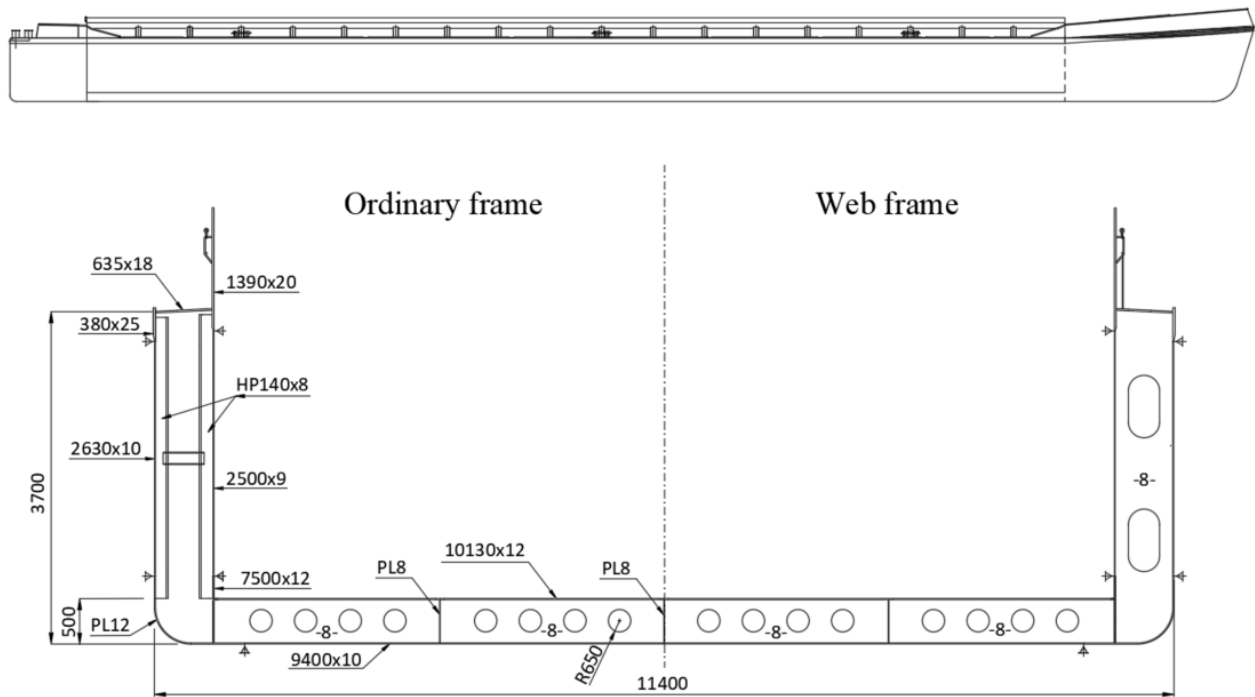


Figure 4. IW barge layout and cross section (dimensions are in mm)

### 3. THE VESSEL

A typical IW barge used in the analysis is designed mostly for navigation on European waterways, especially Rhine, due to its draught of 3.55 m. The barge is intended to carry containers. The vessel can reach the Danube if navigation conditions permit because its 11.45 m breadth allows passage through the 12 m wide lock connecting Rhine and Danube. The barge is transversely framed, which is typical for this type and lengths of inland vessels. The structure mostly consists of plates because it is transversely framed. Below the deck area, only ordinary frames are not the plates, but the bulb profiles - HP140x8. The general layout and the cross-section structure of ordinary and web frames at midship are given in Figure 4, while the main particulars are presented in Table 3. In Figure 5, the cross-section is divided into segments according to the procedure from [2]. Segment no. 10 has a specific stiffener which is modified to a T profile with the same geometrical characteristics for the purpose of the analysis and simpler assessment of cross-section properties. Material is the most commonly used one for inland vessels, which is a mild steel with the following properties:  $E = 206000 \text{ N/mm}^2$ ,  $\mu = 0.3$ ,  $\sigma_y = 235 \text{ N/mm}^2$ .

Table 3. Vessel particulars

$L_{OA}$	72.9 m
$B_{OA}$	11.40 m
$H$	3.70 m
$T$	3.55 m
TEU (maximum)	144
Spacings of normal frames	500 mm
Spacings of web frames	3000 mm

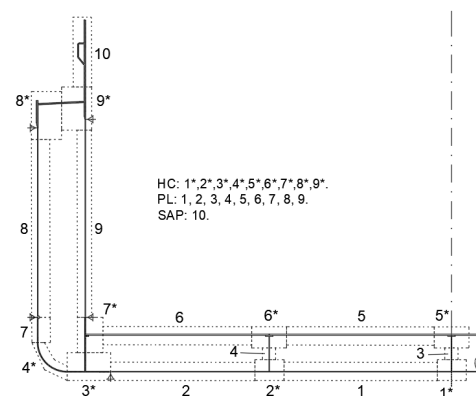


Figure 5. Cross-section subdivision into segments

### 4. RESULTS

The neutral axis (elastic) position is calculated at 1459.7 mm above the baseline. The calculated hull modulus of the bottom, deck, and hatch coaming (at its centroid of the segment) are  $Z_B = 0.86 \text{ m}^3$ ,  $Z_D = 0.56 \text{ m}^3$ , and  $Z_{HC} = 0.42 \text{ m}^3$ . First, buckling critical stresses are given for each segment in Table 4. Ultimate VBM is evaluated for the PCA methods: CSR PCA and P-T PCA. In addition, those results were compared to the maximum VBM calculated by the following presumed methods, considering Table 1 procedure: allowable stress, first yield, and first-collapse (see methodology in section 2.1).

The allowable stress criterion,  $190 \text{ N/mm}^2$ , is taken from BV rules in [24], for mild steel with a yield stress of  $235 \text{ N/mm}^2$ . Results were calculated for both sagging and hogging condition. The summary of results is presented in Table 5, while the VBM-curvature diagram is shown in Figure 6. Actual VBMs for service loading conditions were not assessed here because the authors of this paper had no data on the vessel's operational loading and unloading conditions (weight distribution). Nonetheless, in the case of the properly designed vessel,

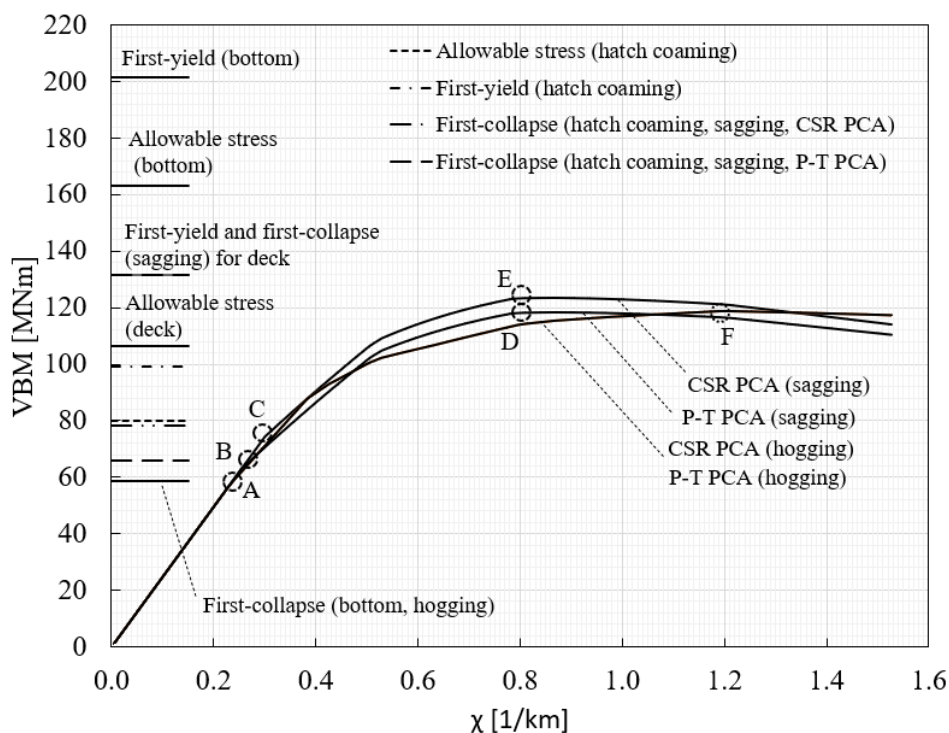


Figure 6. VBM – curvature

service VBM should not exceed limit values in any service scenario (allowable, first-yield, first-collapse), let alone the ultimate one.

Table 4. Buckling critical stress

Segments		$\sigma_{cr}$ [N/mm <sup>2</sup> ]	
No.	Type	CSR PCA	P-T PCA
1	PL	68.41	68.41
2	PL	72.17	72.17
3	PL	132.94	132.94
4	PL	132.94	132.94
5	PL	83.54	83.54
6	PL	82.78	82.78
7	PL	178.37	178.37
8	PL	54.03	54.03
9	PL	50.65	50.65
10	SAP	185.29	156.58
1*-9*	HC	235	235

Table 5. VBM

Method	Maximum VBM [MNm]	
	Sagging	Hogging
CSR PCA	123.64	118.85
P-T PCA	118.43	118.85
Allowable stress for by BV [24] (190 N/mm <sup>2</sup> , for mild steel with yield stress equal to 235 N/mm <sup>2</sup> )	163.1 (bottom) 106.3 (deck) 80.0 (hatch coaming)	
First – yield (Based on mild steel with yield stress equal to 235 N/mm <sup>2</sup> )	201.7 (bottom) 131.5 (deck) 99.0 (hatch coaming)	
First – collapse	58.7 (bottom, hogging) 131.5 (deck, for HC, sagging) 78.1 (hatch coaming, CSR PCA for SAP, sagging) 66.0 (hatch coaming, P-T PCA for SAP, sagging)	

## 5. DISCUSSION

Following the curvature increment, CSR PCA and P-T PCA in both conditions, sagging and hogging, behave linearly until point A because the structure is stressed below the proportional limit of the material.

In point A, during hogging, bottom plates (segments no. 1 and 2) experience buckling almost simultaneously (see their buckling stresses in Table 4). The neutral axis position above the baseline increases to meet the force equilibrium. The slope of the VBM- $\chi$  curve starts to decrease, the structural behavior becomes nonlinear, and the ultimate VBM is reached at around  $\chi = 1.2$  1/km, see point F. The buckling vastly governs the hogging collapse. Both methods (CSR PCA and P-T PCA) deliver the same curve, as their only difference is in the buckling stresses of the hatch coaming (segment no. 10), which is the only SAP here. In hogging, segment no. 10 will never experience buckling. Hence, the difference between the methods would be more visible if the framing system had been longitudinal, not transversal.

In the sagging condition, VBM- $\chi$  is linear until point B, when the hatch coaming starts to buckle according to the P-T CSR value for segment no. 10 (Table 4). From then on, the neutral axis decreases, VBM- $\chi$  curve slope rate starts to decline, and the ultimate VBM is reached (point D, at  $\chi = 0.8$  1/km) for the sagging condition by P-T PCA.

Regarding CSR PCA for sagging, the curve behaves linearly even further, until point C, due to around 18 % larger buckling stress of the hatch coaming (segment no. 10, Table 4). From that point onwards, the CSR PCA and P-T CSR are almost parallel. Their structural collapse is directed by the collapse of the segments for which they have identical formulations. Ultimate VBM curvature is set to be the same value for both methods.

However, CSR PCA has a 4.4% larger ultimate VBM due to the higher buckling stress of the hatch coaming.

The first-yield VBM for the bottom and deck is larger than the ultimate VBM in PCA methods. That means the hull will reach its ultimate strength before the yielding of the deck or bottom. Hatch coaming first-yield VBM is just 16-20 % smaller than the ultimate strength of the vessel (compared to both PCA methods); therefore, the safety margin between the first-yield (hatch coaming) and ultimate VBMs is low. The second segment (deck) will never yield because the structure will already collapse. Note that in the case of sea-going ships, the situation is different: the first yield will happen long before the ultimate strength.

In the presumed methods, allowable stress VBM gives a bit reduced values of VBM compared to the first yield, based only on the difference between the 190 N/mm<sup>2</sup> and 235 N/mm<sup>2</sup> stress. In that sense, only the bottom will reach its allowable stresses after the ultimate VBM is achieved. The difference between allowable stress and ultimate VBM provides a very low safety margin.

First-collapse methods deliver the lowest VBM, as expected, due to significantly lesser buckling stresses of segments than yielding ones. Only the deck collapse will never be reached as the ultimate strength of the hull will be experienced slightly before.

## 6. CONCLUSION

According to the best of the authors' knowledge, there are only a few literature data or regulations involving the ultimate strength of inland vessel hull girders. However, such structural failures are not quite rare, considering groundings due to increasingly prolonged low water levels on rivers, improper overhauling, overloading, etc. Therefore, the paper's main goal is to present one of the first assessments of such kind. The PCA showed the extent of the safety zone between the linear-elastic and ultimate VBM. In some scenarios, there are almost no such zones, contrary to the sea-going ships case. This makes the inland vessel extremely vulnerable after the VBM produces the single element's collapse (or yielding). Naturally, this is partly a consequence of the transverse framing of the vessel, which is a common framing solution for inland vessels with lengths up to 100 m. Therefore, based on the PCA assessment of the transversely framed IW barge, which is typical and a class of its own, the following remarks can be presented:

- more refined methods (PCA) can deliver the sequences and better understanding of the ultimate strength collapse of an IW barge,
- the buckling overwhelmingly governs transversely framed IW barge's ultimate strength,
- CSR PCA gives a higher ultimate VBM than the P-T PCA due to the difference between the IACS analytical and P-T empirical formulations.
- First-yield method should not be used to evaluate the maximum VBM of the presented IW barge type, as the vessel would probably experience the hull girder collapse much before yielding starts in almost any of the

elements, except hatch coaming, and hence, there is no significant safety margin,

- Allowable stress method for evaluation of the IW barge VBM, as a single criterion, should not be used as it is based on the yield stress, which, according to first-yield results - proved to be inadequate for hull girder analyses,

- However, the allowable stress method could be used for the IW barge; thus, in order to increase the safety zone between the ultimate strength and allowable stress VBM, a larger safety margin should be employed,

- First-collapse method delivers a much larger safety margin with respect to actual ultimate strength than other criteria, as expected, and should be used as a main criterion for the linear-elastic evaluations of the vessel hull girder, as it creates a measurable safety zone between the service and ultimate strength of the vessel.

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#### ABBREVIATIONS

BV	Bureau Veritas
CSR	Common Structural Rules for Bulk Carriers and Oil Tankers
HC	Hard corner segment

IACS	International Association of Classification Societies
ISSC	International Ship and Offshore Structures Congress
ISUM	Idealized structural unit method
IW	Inland waterway
LR	Lloyd's Register
NLFEM	Nonlinear finite element method
PCA	Progressive collapse analysis
P-T	Paik-Thayamballi method
PL	Plating segment
SAP	Stiffener with attached plating segment
TEU	Twenty-foot equivalent unit
VBM	Vertical bending moment

#### NOMENCLATURE

$A$	Area of the cross-section
$A_i$	Area of the $i$ -th segment
$b$	The breadth of the segment
$B_{OA}$	Breadth overall of the vessel
$E$	Modulus of elasticity
$H$	Height of the vessel
$l$	Extension of the cross-section along the length
$L_{OA}$	Length overall of the vessel
$M_U$	Ultimate vertical bending moment of the hull girder
$n$	Number of segments
$r$	Radius of gyration
$SF$	Safety factor
$t$	Thickness
$T$	Draught of the vessel
$Z_B$	Hull section modulus for bottom
$Z_D$	Hull section modulus for deck
$Z_{HC}$	Hull section modulus for hatch coaming
$z_i$	Neutral axis position of the $i$ -th segment above the baseline
$z_n$	Elastic neutral axis position of the cross-section above the baseline
$z_P$	Plastic neutral axis position of the cross-section above the baseline

#### Greek symbols

$\beta$	Plate slenderness ratio
$\varepsilon_i$	The strain of the $i$ -th segment
$\lambda$	Column slenderness ratio
$\mu$	Poisson's ratio
$\sigma_{all}$	Allowable stress
$\sigma_{cr}$	Buckling (critical) stress
$\sigma_{cr,B}$	Buckling (critical) stress for bottom
$\sigma_{cr,D}$	Buckling (critical) stress for the deck
$\sigma_i$	The normal stress of the $i$ -th segment
$\sigma_U$	Ultimate stress (buckling stress)
$\sigma_x$	Normal stress
$\sigma_y$	Yield stress
$\sigma_{yeq}$	Yield stress (equivalent)



$\varphi$  Cross section rotation  
 $\chi_1$  Initialize cross-section curvature

$\chi$  Cross section curvature

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## ГРАНИЧНА ЧВРСТОЋА РЕЧНЕ БАРЖЕ

**Н. Илић, Н. Момчиловић**

Услед изненадне природе колапса бродског трупа као еквивалентног носача изазваног екстремним оптерећењима, гранична чврстоћа, тј. гранична носивост бродова, требала би бити процењена. Процедуре за анализу граничне чврстоће већ су дате у правилима класификационих друштава за морске бродове. Међутим, правила за речне бродове не узимају у обзир овај проблем. Постојећа литература

о граничној чврстоћи речних бродова скоро и да не постоји, што је забрињавајуће узимајући у обзир честе случајеве насукана и преоптерећења бродова унутрашње пловидбе. Такође, услед виткости лимова, структура речних бродова је подложнија извијању. Гранична чврстоћа, у овом случају речне барже, израчуната је коришћењем анализе прогресивног колапса. Резултати приказују ниво зоне сигурности између губитка носивости и линеарно-еластичног одзива конструкције. Анализа прогресивног колапса је показала да је гранична чврстоћа (носивост) брода у највећој мери зависна од извијања структурних елемената.