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Endogenous capital stock and depreciation in the United States

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Abstract

There are several puzzles and unresolved problems in empirical economics that depend on the reliability of capital series. Productivity paradoxes, and certain recent trends in the US macroeconomic data, cannot be addressed correctly with the available standard measures of capital stock. Our paper contributes to the theory of capital by endogenizing capacity utilization and depreciation in an intertemporal optimization model with adjustment and maintenance costs. This model allows for corporate taxation and identifies the impact on the variables that shape the capital accumulation process. Depreciation is a control variable that is no longer assumed proportional to the capital stock. The model provides a system of equations that we run empirically with a data set of the US economy for the period 1960–2016. We obtain an empirical measure of the depreciation rate and the capital stock based on profitability and market values. They are economic estimations that consider the entire capital deterioration and obsolescence. Aggregate capital stock is a key variable in the description of the economy, and our results, which better fit the foundations of economic theory, can provide policymakers with a good understanding of the field in which specific public policy measures are to be implemented.

1 | INTRODUCTION

In recent years, there has been a revival of studies in the fields of macroeconomics and economic growth focused on productivity-related debates. First, the Solow paradox and paradox 2.0, which involve labor productivity and the total factor productivity (TFP) (Acemoglu et al., 2014; Byrne et al., 2016). Second, the overturning of certain stylized Kaldor facts and the emergence of new ones, which are related to capital productivity (Eggertsson et al., 2018). Empirical research in all these areas, ranging from productivity analyses to studies regarding the correct implementation of public policies, depends on the availability of a reliable series of capital stock. In this paper, we meet the challenge and provide a quantitative measure of the US capital stock that differs from that obtained through the Bureau of Economic Analysis (BEA) methodology.

The neoclassical capital theory has developed a powerful theory of investment,[1] but has failed to create a consensus around a theory of depreciation. Investment decisions are channeled toward the explicit acquisition of new capital goods that involve market observable transactions. However, capital depreciation is an umbrella under which a wide variety of heterogeneous concepts are sheltered. They range from the decline in productive capacity, through efficiency losses, to either retirements or scrapping. Moreover, decisions on the depreciation of capital are basically associated with market unobservable transactions, which make available records unreliable. The firm's depreciation decisions depend on several implicit costs and benefits related to deterioration and obsolescence. Deterioration, which may be both physical (output decay) and economic (input decay), is an inherent characteristic of capital goods associated with the aging, use, and maintenance of equipment. Obsolescence, which may be both technological and structural, is extrinsic to assets and comes from outside, and is associated with technical progress (mainly embodied but also disembodied), energy prices, patterns of international trade, regulatory programs, or changes in the output composition that affects relative prices.

The lack of a unified theory of capital depreciation has been compensated for years by the results of two main but separate branches of economic research (Bitros, 2010a, 2010b). The first uses the vintage capital model as a natural instrument and focuses on the study of obsolescence (Boucekkine et al., 2008; Johansen, 1959; Malcomson, 1975; Mukoyama, 2008). The second, through the proportionality theorem, has provided a framework for the study of physical deterioration, that is, wear and tear, which narrowly refers to depreciation caused by aging and the regular and constant use of capital (Jorgenson, 1963, 1974). These two views reveal a more fundamental controversy between those who treat the depreciation of capital as a simple technical requirement that is exogenously determined, and those who consider the depreciation as a complex phenomenon in which agents' decisions play an important role in determining endogenously its economic value.[2]

[1]For a review of the investment literature we recommend the works of Jorgenson (1967), Fazzari et al. (1988), Abel (1990), Chirinko (1993), and Caballero (1999).

[2]This was already highlighted by Feldstein and Rothschild (1974) and Nickell (1975), for which depreciation varies considerably under the influence of conventional economic forces. However, the variability does not necessarily mean that it is an endogenous and controlled variable. For example, in Benhabib and Rustichini (1991) technical progress and nonexponential depreciation rates are combined to account for obsolescence, but the different assumptions of schedules for variable depreciation are exogenously predetermined. More recently, Boucekkine et al. (2009) in a two-sector vintage capital model with neutral and investment-specific technical progress, assuming variable utilization of vintages, raised a study where depreciation is an endogenous phenomenon. Their economic rate of depreciation is the sum of a physical age-related depreciation rate, plus an economic use-related depreciation rate, and the scrapping or obsolescence rate.

The standard neoclassical model assumes capital malleability in the sense that old and new capital goods share the same marginal productivity. However, the heterogeneity of capital goods and investment-specific technical progress fit better in vintage capital models, as technology improvements only affect output through net investment or the replacement of old equipment. Nevertheless, after Solow (1960), capital stock can be represented by a single equivalent measure, the jelly capital. Solow assumes a putty–putty technology, where the average useful life of capital goods is constant. Under the aggregation properties of his model, it is possible to calculate both deterioration and obsolescence, but the latter is a constant fraction of the value of capital.

Beyond the theoretical controversies and conceptual debates, on the empirical side, we find that the more conventional measures of depreciation are incomplete, but the effort made by the statistical offices has been significant. For them, depreciation is the loss in value of the existing assets due to wear and tear, normal or expected obsolescence, accidental damage, and aging. In the US economy, BEA's capital stock measurement uses the perpetual inventory method (PIM) to estimate depreciation that reflects expected obsolescence and adjusts for quality change in assets.[3] Such expected obsolescence translates into the shortening of the economic service lives of assets, affecting the overall flow of its services. It is only the effects of unforeseen obsolescence that are not included in BEA's estimates of depreciation.

However, in practice, the BEA assumes that investment goods have age–price profiles that follow a strictly geometric pattern. In the absence of econometric estimates of the geometric depreciation rates, these are calculated using the declining balance method based on the information about average service lives and the appropriate values of the declining balance rate. Hence, capital stock estimates derived from the PIM are crucially dependent on service lives, and one important determinant of these lives is normal obsolescence. Service lives should reflect actual experiences as closely as possible. In other words, service lives should vary over time to account for changes in economic conditions and technology. However, barring a few exceptions, the assets' service lives used by the BEA are constant. In any case, although BEA's depreciation estimates are intended to include the effects of obsolescence, which cannot be separated from the effects of all other factors affecting asset prices, the results are far from ideal because of data limitations.

The purpose of our work is to address this challenge and obtain a comprehensive economic measure of the capital stock at the aggregate level. This measure is required because capital stock, or the flow of services it provides, is one of the basic macroeconomic aggregates that describe the main empirical facts of modern economies. However, it is also important to identify the optimal public policies to deal with the real problems of society. To achieve this goal, we need to measure depreciation correctly in economic terms, because any forgotten component or measurement error can lead researchers to misinterpret reality and make inaccurate economic predictions. The pursued measure of depreciation should include depreciation caused by age, use, maintenance, embodied technical progress, and so on.

An important question in public finance is whether taxing capital can lead to lower investment and lower capital stock, also affecting the economic growth rate (Kang & Ye, 2019; Lu & Chen, 2015; Piergallini, 2021; Renström & Spataro, 2021; Suzuki, 2021). In this paper we draw on the theoretical apparatus underlying the fundamental equation of capital theory. This equation states that, in equilibrium, the market prices of capital assets are determined by the discounted present value of what the purchasers of the assets expect to earn from its ownership. At this point, we

[3]A detailed methodological exposition of the "BEA Depreciation Estimates" can be found at https://apps.bea.gov/national/pdf/BEA_depreciation_rates.pdf

introduce a corporate income tax, which is a special tax on the profits of private firms. Corporate tax on profits is levied at an earlier stage than any other tax on capital income, such as the tax on distributed dividends. Consequently, it is a way of taxing profits that would otherwise avoid taxation by being kept inside corporations (Bastani & Waldenström, 2020). In any case, for the sake of simplicity and given that it empirically represents a small part of total taxes, we ignore the tax on dividends here. Therefore, there is no room for controversy regarding the double taxation of corporate income. In an integrated theoretical–empirical framework, we solve a dynamic optimization model that endogenizes the depreciation rate by adding it to the set of controls, together with gross investment, capacity utilization, and employment. Although in standard models, an aggregate capital stock is used at full capacity in the production process and depreciates at a constant exogenous rate, we know that firms do not always use all the installed capital, and the depreciation rate is subject to continuous changes.

The theoretical model suggests an alternative approach to the empirical series of the depreciation rate and the value of capital stock, yielding an economic estimation based on market values and profitability indexes, such as Tobin's q ratio (Escribá-Pérez et al., 2018, 2022). We do not consider capital vintages and, consequently, our model cannot produce an explicit scrapping rate. Additionally, our model does not differentiate between the consumption and capital sectors, and we do not make explicit the role of the embodied technological progress. Even so, we prove below that, according to our computations, the measured economic value of the endogenous variables encompasses depreciation because of obsolescence. Our empirical contribution in this paper is a measure of the US capital that corresponds to the market value of capital stock. We then compare our results with the series supplied by the BEA.

The structure of the paper is as follows. Section 2 describes and solves the theoretical model that supports the new economic measurement of capital stock and depreciation. Section 3 discusses the content of the economic depreciation rate and its relationship with obsolescence. Section 4 derives the computational procedure that is used to obtain quantitative results in an empirical application to the US economy. Finally, Section 5 summarizes.

2 | THEORY

Let us consider the supply side of an economy with a large number of identical firms. We shall present here a simple theoretical model that shows the optimizing behavior of the individual price-taking firm in a competitive environment. However, given the representative agent assumption and the absence of externalities, the variables of the model might also represent aggregate levels, and the problem could be read as if all firms made decisions jointly in a centralized economy. The optimization problem to be solved is an intertemporal maximization problem that generalizes the standard model, in which the employment $L(t)$ and the gross investment $I^G(t)$ are controlled to maximize the present discounted value of cash-flow. We add the rate of capital depreciation $\delta^*(t)$ and the rate of capital utilization $u(t)$ to the set of controls. These two endogenous variables are linked to each other due to their relationship to maintenance and repair expenditures. This relationship combines the depreciation-in-use hypothesis, which states that a higher level of economic activity leads to a higher rate of capital utilization and, hence, to a higher rate of depreciation, with the maintenance assumption according to which firms indirectly affect the depreciation rate by devoting resources to keep the equipment in good working conditions. We also include in the model the taxation of capital in the form of corporate taxes on profits, but we do not consider any tax on dividends.

The objective functional or cash-flow is determined by revenues that depend on the production function minus adjustment costs, maintenance costs, the wage bill, and investment spending. Adjustment and maintenance expenditures enter the cash-flow associated with their corresponding cost function because they are internal to the firm.[4] We assume that firms pay taxes at a rate $\tau(t)$ levied on revenues less current operating expenses, which do not include investment expenses. Firms take as given the tax rate decided by the government in each period. This tax structure excludes investment tax credits and fiscal depreciation allowances.[5] Consequently, the firm's problem is to maximize the discounted after-tax cash-flow stream. The model has a single state variable, the capital stock $K^*(t)$, so that the optimal control problem must include a dynamic constraint to express the corresponding accumulation process. Putting it all together, we can write

$$\begin{aligned} \max_{\{K^*, L, I^{\mathrm{G}}, \delta^*, u\}} V(t_0) \;=\; & \int_{t_0}^{+\infty} ((1-\tau(t))(G(p(t), A^*(t), K^*(t), L(t), I^{\mathrm{G}}(t), \delta^*(t), u(t)) \\ & - W(t)L(t)) - p^{\mathrm{k}}(t) I^{\mathrm{G}}(t)) e^{-\int_{t_0}^{t} R(s) ds} dt \\ & \text{s.t. } \dot{K}^*(t) = I^{\mathrm{G}}(t) - \delta^*(t) K^*(t), \\ & \quad K^*(t_0) = K_0^* > 0. \end{aligned} \tag{1}$$

In this economy output is produced according to the production function $Y = A^* F(L, uK^*)$. Here A^* is the current level of technology and $F(\cdot)$ is homogeneous of degree one in its two determinants: labor and the portion of the capital stock that is used in the productive activity.[6] This function satisfies Inada conditions. For the sake of simplicity we normalize the price of output, $p(t) = 1$. The price of labor $W(t)$, the market price of capital goods $p^{\mathrm{k}}(t)$ and the nominal interest rate $R(t)$ are given for the competitive firm. Function $G(\cdot)$ represents the value of net production after subtracting investment-related adjustment costs $C(I^{\mathrm{G}}, K^*)$ and maintenance expenditures $M(\delta^* K^*, uK^*)$. These two functions are assumed homogeneous of degree one in their corresponding pair of determinants.[7]

[4]In this optimal control problem depreciation is no longer a residual variable. Just like the rate of capital utilization, it is one of the instruments used by firms in setting their optimal plans. The key elements here are the costs of capital adjustment (Hayashi, 1982), the maintenance and repair of equipment (Agénor, 2009; Escribá-Pérez & Ruiz-Tamarit, 1996; McGrattan & Schmitz, 1999), the depreciation-in-use mechanism (Bischoff & Kokkelenberg, 1987; Boucekkine & Ruiz-Tamarit, 2003; Epstein & Denny, 1980; Motahar, 1992), and the technical progress (Boucekkine et al., 2009, 2010). An alternative stochastic general equilibrium model that also includes the above elements is developed in Albonico et al. (2014).

[5]A more complete corporate tax system is considered in Hall and Jorgenson (1967) and McGrattan and Schmitz (1999). The former considers both investment tax credits and tax depreciation allowances, but ignores adjustment and maintenance costs. The latter takes into account depreciation allowances and still omits adjustment costs. Both of them disregard the interaction with the capital utilization rate. For the sake of simplicity and in line with the theoretical purposes of this paper, the introduction of a uniform corporate tax rate seems an appropriate choice that does not undermine the subsequent analysis.

[6]Actually, the function could be written as $F(L, KU^*)$, where $KU^* = uK^*$, $F_L > 0$, $F_2 > 0$, $F_{LL} < 0$, and $F_{22} < 0$. Our homogeneity assumption involves the variables L and KU^* instead of L, u, and K^*, taken separately.

[7]The adjustment cost function $C(\cdot)$ has the usual properties: it is increasing in I^{G} and decreasing in K^*. The maintenance cost function originally could be written as $M(D^*, KU^*)$, where $D^* = \delta^* K^*$ represents the volume of total depreciation. It is assumed that maintenance expenditures decrease with depreciation D^*, but increase with the quantity used of capital stock KU^*. Our homogeneity assumption involves the variables D^* and KU^* instead of δ^*, u, and K^*, taken separately. All these assumptions are standard and have been considered as such in the literature.

Because of the linear homogeneity assumptions we can write $G(A^*, K^*, L, I^G, \delta^*, u) = A^*F(L, uK^*) - \phi\left(\frac{I^G}{K^*}\right)K^* - \varpi\left(\frac{\delta^*}{u}\right)uK^*$, and characterize this function by the sign of the first and second derivatives with respect to the controls and the state variable.[8] That is, $G_L = A^*F_L > 0$, $G_{LL} = A^*F_{LL} < 0$, $G_{I^G} = -C_{I^G} = -\phi' < 0$, $G_{I^GI^G} = -C_{I^GI^G} = -\frac{\phi''}{K^*} < 0$, $G_{\delta^*} = -\varpi' K^* > 0$, $G_{\delta^*\delta^*} = -\varpi''\frac{K^*}{u} < 0$, $G_u = A^*F_2K^* - \left(\varpi - \varpi'\frac{\delta^*}{u}\right)K^* \lesseqgtr 0$, $G_{uu} = A^*F_{22}K^{*2} - \frac{\varpi''\delta^{*2}K^*}{u^3} < 0$. Moreover, given that $\phi(\cdot)$ is strictly convex we get $C_{K^*} = \phi - \phi'\frac{I^G}{K^*} < 0$ and $C_{K^*K^*} = \phi''\frac{(I^G)^2}{(K^*)^3} > 0$, and then $G_{K^*K^*} = A^*F_{22}u^2 - C_{K^*K^*} < 0$. Finally, we assume that the net marginal productivity of capital before taxes is positive, $G_{K^*} = A^*F_2u - C_{K^*} - \varpi u > 0$.

The dynamic constraint can be added to the objective functional by introducing the multiplier μ as expression of the shadow price of capital. Then, we get the following Hamiltonian function written in current value

$$H^c = (1 - \tau(t))(G(A^*(t), K^*(t), L(t), I^G(t), \delta^*(t), u(t)) - W(t)L(t)) - p^k(t)I^G(t) + \mu(t)(I^G(t) - \delta^*(t)K^*(t)). \tag{2}$$

It is assumed that the discount rate for the cash-flow is given exogenously and is perceived as a constant R. This avoids the time-consistency problem associated with a nonconstant discount rate that makes preferences intertemporally dependent, and allows Pontryagin's maximum principle to be applied. We get the necessary conditions for the control variables[9]

$$H^c_L(\cdot) = 0 = (1 - \tau(t))(A^*(t)F_L(L(t), u(t)K^*(t)) - W(t)), \tag{3}$$

$$H^c_{I^G}(\cdot) = 0 = -(1 - \tau(t))\phi'\left(\frac{I^G(t)}{K^*(t)}\right) - p^k(t) + \mu(t), \tag{4}$$

$$H^c_{\delta^*}(\cdot) = 0 = -(1 - \tau(t))\varpi'\left(\frac{\delta^*(t)}{u(t)}\right)K^*(t) - \mu(t)K^*(t), \tag{5}$$

$$H^c_u(\cdot) = 0 = (1 - \tau(t))A^*(t)F_2(L(t), u(t)K^*(t))K^*(t) - (1 - \tau(t))\left(\varpi\left(\frac{\delta^*(t)}{u(t)}\right) - \varpi'\left(\frac{\delta^*(t)}{u(t)}\right)\frac{\delta^*(t)}{u(t)}\right)K^*(t), \tag{6}$$

the Euler equation

[8]The adjustment unit cost function $\phi(i)$ satisfies the properties $\lim_{i\to 0^+}\phi(i) = 0$, $\lim_{i\to+\infty}\phi(i) = +\infty$, $\phi'(i) > 0$, $\lim_{i\to 0^+}\phi'(i) = 0$, $\lim_{i\to+\infty}\phi'(i) = +\infty$, $\phi''(i) > 0$. The maintenance unit cost function $\varpi\left(\frac{\delta^*}{u}\right)$ satisfies the properties $\lim_{x\to 0^+}\varpi(x) = +\infty$, $\varpi\left(\frac{1}{u}\right) = 0$, $\lim_{x\to+\infty}\varpi(x) = 0$, $\varpi\left(\frac{\delta^*}{1}\right) \geqslant 0$, $\varpi'(x) < 0$, and $\varpi''(x) > 0$. It is easy to deduce the results $\varpi_{\delta^*} = \frac{\varpi'}{u} < 0$, $\varpi_{\delta^*\delta^*} = \frac{\varpi''}{u^2} > 0$, $\varpi_u = -\frac{\varpi'\delta^*}{u^2} > 0$, $\varpi_{uu} = \frac{\delta^*}{u^3}\left(2\varpi' + \varpi''\frac{\delta^*}{u}\right)$, and $\varpi_{\delta^*u} = -\frac{1}{u^2}\left(\varpi' + \varpi''\frac{\delta^*}{u}\right)$. Moreover, if we assume that $\varpi_{uu} > 0$, then we get $\varpi_{\delta^*u} < 0$.

[9]Although the model also includes the following control constraints $\forall\, t$: $L(t) \geq 0$, $I^G(t) \geq 0$, $0 \leq \delta^*(t) \leq 1$, and $0 \leq u(t) \leq 1$, for the sake of simplicity we do not make them explicit in the optimization problem. We are going to consider the case of interior solutions alone, which is guaranteed by our characterization of the involved functions.

$$\dot{\mu}(t) = R\mu(t) - H^{c}_{K^*}(A^*(t), K^*(t), L(t), I^{G}(t), \delta^*(t), u(t)), \tag{7}$$

where $H^{c}_{K^*}(\cdot) = (1-\tau)\left(A^*F_2u - \left(\phi - \phi'\frac{I^G}{K^*}\right) - \varpi u\right) - \mu\delta^*$, the dynamic constraint

$$\dot{K}^*(t) = I^{G}(t) - \delta^*(t)K^*(t), \tag{8}$$

the initial condition K_0^*, and the transversality condition

$$\lim_{t\to+\infty} \mu(t)K^*(t)e^{-R(t-t_0)} = 0. \tag{9}$$

We observe that the first-order conditions (3)–(6) could define a system of four control functions. However, after total differentiation we check that the implicit function theorem cannot be applied because the linear homogeneity assumed on $G(\cdot)$ makes the determinant of the Jacobian matrix equal to zero. Except for the investment equation that appears independently of the others, we define two new control variables as ratios between the remaining ones, and show the sign of the partial effects associated with the state, costate, and parameters.

$$\frac{L(t)}{u(t)} = N(\overset{+}{K^*}(t); \overset{+}{A^*}(t), \overset{-}{W}(t)), \tag{10}$$

$$I^{G}(t) = I^{G}(\overset{+}{K^*}(t), \overset{+}{\mu}(t); \overset{-}{p^{k}}(t), \overset{+}{\tau}(t)), \tag{11}$$

$$\frac{\delta^*(t)}{u(t)} = x(\bar{\mu}(t); \bar{\tau}(t)). \tag{12}$$

Equation (6), in turn, establishes a tight relationship between the two ratios

$$Z\left(\frac{L(t)}{u(t)}, \frac{\delta^*(t)}{u(t)}, K^*(t), A^*(t)\right) = 0. \tag{13}$$

It is clear from the above expressions that changes in the corporate tax rate applied to taxable income have a direct impact on investment and depreciation decisions. But there is also an induced effect on employment and capital utilization through changes in the capital stock and its shadow price. In any case, our study of the above first-order conditions should be taken as a first step toward a more general theory explaining simultaneously the behavior of investment and depreciation. In particular Equation (4) says that, ceteris paribus, an increase in taxes must be accompanied by an increase in the marginal adjustment cost, which is achieved by an increase in the investment-to-capital ratio. Equation (5) says, in turn, that a decrease in taxes should be coupled with an increase in the ratio of depreciation to capacity utilization. However, an exhaustive analysis of the question would require identifying on empirical grounds the particular functional forms, and then determining the exact solution of the dynamic systems (7)–(9) for K^* and μ. Then, with these solution trajectories we could go back to the control functions and obtain the true demand functions, where the endogenous variables depend only on the exogenous variables and parameters, an issue that we leave open because it is beyond the scope of the present paper.

Instead, it is easy to characterize the steady state in our model by the stationary level of gross investment $I_{ss}^{G} = \delta_{ss}^{*}K_{ss}^{*}$, and the user cost of capital at the profit maximization position

$$G_{K^*} = \left(\frac{p^{k}}{1-\tau} + \phi'\right)\left(R + \delta_{ss}^{*}\right). \tag{14}$$

The user cost is the pretax net marginal product of capital and is related to an equilibrium implicit rental price of capital, that is, the opportunity cost of capital (Creedy & Gemmell, 2017). The above expression shows that corporate taxation is associated with a higher user cost of capital, which also includes the marginal adjustment cost corresponding to the replacement investment rate. The latter is due to the fact that adjustment costs are defined on gross investment. In equilibrium we also have $\phi' = -\frac{p^{k}}{1-\tau} - \varpi'$.

Now, returning to the optimality conditions, we can integrate (7) forward solving for $\mu(t)$ under the nonexplosivity condition $\lim_{t_F \to +\infty} \mu(t_F)\exp\left\{-\int_t^{t_F}(R + \delta^*(s))\,ds\right\} = 0$. The result we get may be put in terms of the model-based definition of marginal q,

$$\begin{aligned} q^{M}(t) = \frac{\mu(t)}{p^{k}(t)} = \frac{1}{p^{k}(t)}\int_t^{+\infty} &(1-\tau(s))(A^*(s)F_2(L(s), u(s)K^*(s))u(s) \\ &-\left(\phi\left(\frac{I^{G}(s)}{K^*(s)}\right) - \frac{I^{G}(s)}{K^*(s)}\phi'\left(\frac{I^{G}(s)}{K^*(s)}\right)\right) - \varpi\left(\frac{\delta^*(s)}{u(s)}\right)u(s))e^{-\int_t^{s}(R+\delta^*(v))\,dv}\,ds. \end{aligned} \tag{15}$$

That is, the present value of the future stream of the after-tax net marginal productivity of capital, discounted by the sum of the constant discount rate plus the variable depreciation rate, and all that divided by the current market price of one unit of capital. In other words, the quotient between the shadow price of one unit of capital (the Hamiltonian multiplier μ) and its replacement cost (the market price of capital goods p^{k}). This variable directly explains the flows of investment and depreciation that determine the dynamics of capital stock.

On the other hand, the property of homogeneity assumed on the production and cost functions together with the first-order conditions of the dynamic optimization problem, allow us to set the linear ordinary differential equation in $X = \mu K^*$: $\dot{X} = RX - (1-\tau)(A^*F(L, uK^*) - \phi\left(\frac{I^{G}}{K^*}\right)K^* - \varpi\left(\frac{\delta^*}{u}\right)uK^* - WL) + p^{k}I^{G}$. This one may be integrated forward solving for the product $\mu(t)K^*(t)$ under the transversality condition (9). The result we get may be put in terms of the model-based definition of average q, the quotient between the market value of (all) the firm(s) and the economic value of the capital stock measured in nominal terms at its replacement cost,

$$\begin{aligned} q^{A}(t) &= \frac{\mu(t)K^*(t)}{p^{k}(t)K^*(t)} \\ &= \frac{\int_t^{+\infty}((1-\tau(s))(G(A^*(s), K^*(s), L(s), I^{G}(s), \delta^*(s), u(s)) - W(s)L(s)) - p^{k}(s)I^{G}(s))e^{-R(s-t)}\,ds}{p^{k}(t)K^*(t)}. \end{aligned} \tag{16}$$

Given (15) and (16) it is apparent the equality between marginal and average q.[10] That is, the two theoretical q ratios are equivalent and, according to the literature, they can be empirically approximated by the observable Tobin's q, which is the ratio of the stock market value of the firm to the current-cost book value of capital assets. The first two are endogenous variables that refer to economic or market values, although different authors may use other labels with the same meaning. For example, the valuation at replacement cost in nominal terms and the valuation at replacement cost at current prices, which means that capital goods are valued at the prices of the current period. Of course, this is opposed to valuation at historical prices, which means that assets are valued at the prices at which they were originally purchased. Instead of that, the empirical measures of Tobin's q are exogenous empirical measures that use financial data and adjust for inflation to compute the replacement cost of the assets and liabilities. According to Siegel (2008), available measures of Tobin's q ratio are based on financial market valuation of the corporate assets corresponding to the fundamentals of the firm, as well as on data obtained from balance sheets. Consequently, we shall use them as exogenous proxies for our model-based q ratios.

3 | DOES THE ENDOGENOUS RATE OF DEPRECIATION CAPTURE OBSOLESCENCE?

There is an important question related to the theory developed in Section 2 that needs an answer: Is the variable depreciation rate $\delta^*(t)$ representing total depreciation? That is, does it also account for the obsolescence of capital goods or just deterioration, both physical and economic? Our model endogenizes the depreciation rate, which is no longer a fixed proportion of the capital stock, assuming that economic agents choose it in an optimal way according to the costs and benefits of all activities related to capital goods. One central piece of the model is the maintenance cost function that connects the resources devoted to maintenance and repair with the intensity of capital utilization and the rate of depreciation. It is beyond question that, by combining the maintenance argument with the depreciation-in-use mechanism, the model accounts for total deterioration. However, when it comes to obsolescence, things are not so direct.

Traditionally, obsolescence has been analyzed in the context of vintage capital models studying the effect that the embodied technological change causes in the economic useful life of capital goods.[11] After Solow (1960) the issue of obsolescence began to play a role in the more standard neoclassical model in which it is assumed the existence of an aggregate stock of capital. And more recently, the literature has focused on the link between obsolescence, investment-specific technological progress, and economic growth. Greenwood et al. (1997) pointed out that the observed negative correlation between the series for the investment price index and the quantity of investment should be taken as evidence of technological change in the production of new units of capital. In what follows, we will use the term $\theta(t)$ to represent the state of the technology, or productivity level, in the production of new capital. Then, increases in $\theta(t)$ will be read as embodied quality improvements, or investment-specific technological change, that boost the productivity of the last vintage of capital goods. Embodiment means that it is necessary

[10]See Hayashi (1982), Blanchard et al. (1993), or Kalyvitis (2006).

[11]It is well known that improvements through neutral technical progress increase the profitability of all vintages, which lengthens their lifetime, whereas capital-embodied technical progress leads to shorter lifetimes.

to invest in new equipment to benefit from the advantages of innovation. But we must not forget that the embodied technical progress also has a counterpart in the obsolescence of the capital goods already installed.[12]

There are many reasons then to try to find out whether our strategy in modeling an endogenous rate of economic depreciation also captures obsolescence, even if only implicitly and indirectly. To address this important question we need to go to the conceptual advances developed by Solow (1960) and the methodological arguments exhibited in Greenwood et al. (1997), which significantly extends the model of the previous one. Solow was the first to show how to manage the investment-specific technological change in a model with aggregate capital stock. But it was Greenwood and his coauthors who, as a main novelty, proposed two equivalent representations of the underlying model. In Section 3 they provide a version of the model that includes both the level of neutral technical progress and the level of embodied technical progress, but the depreciation rate is assumed exogenous and constant. The latter represents a major shortcoming because under such an assumption only physical depreciation can be recorded. However, in Appendix B they recast the model "*so that it appears as a conventional model with neutral technological change.*" Hence, "*a key variable in the transformed model is the economic rate of depreciation. Investment-specific technological change can be measured by the spread between the economic and physical rates of depreciation*" (Greenwood et al., 1997, p. 353, footnote 12). The equivalence between these two specifications reveals an interesting feature: any change in the formal representation of technology will be compensated in the formal representation of depreciation and, hence, in the measure of capital stock.

Next, we will translate Greenwood's mapping to our framework. Remember that in the model of Section 2 $A^*(t)$ apparently takes the form of the standard neutral technological level, and $\delta^*(t)$ has been defined as the economic depreciation rate. Methodologically, we can establish a homomorphic parallelism showing, first, that depreciation caused by obsolescence is also included in $\delta^*(t)$ and, second, that the embodied technical progress is also taken into account in $A^*(t)$. However, Greenwood's is a general equilibrium approach, while ours is a partial equilibrium approach involving the supply side of the economy. Hence, we will reinterpret the representative firm of Section 2 as a representative consumer–producer agent. We also assume that there is a government that levies taxes only on profits, and keeps its current budget balanced by spending the revenues collected in the form of a lump-sum transfer to the consumer–producer. In this way, from an aggregate point of view output and the dynamics of capital are determined as follows:

$$Y(t) = A^*(t)F(L(t), u(t)K^*(t)), \tag{17}$$

$$\dot{K}^*(t) = I^{\mathrm{G}}(t) - \delta^*(t)K^*(t), \tag{18}$$

where K^* and δ^* represent the market value of capital stock and the depreciation rate, and A^* is the efficiency level in the production function. According to the agent's resource constraints,

[12]There is a well-known embodiment controversy concerning the relative contribution of investment-specific technological progress to output growth (Greenwood & Krusell, 2007; Hercowitz, 1998; Oulton, 2007). The debate was also between two approaches to the accounting practice: the traditional growth accounting (Hulten, 1992; Jorgenson, 1966) and the quantitative theory based on models (Greenwood et al., 1997; Solow, 1960). Nowadays, the discussion on what proportion of economic growth is due to embodiment is focussed on deciding which deflators for durable equipment and output should be used.

output may be allocated to consumption, investment, and adjustment and maintenance expenditures, $Y = C + \frac{p^k I^G}{p} + \phi\left(\frac{I^G}{K^*}\right)K^* + \varpi\left(\frac{\delta^*}{u}\right)uK^*$.

The specification of the above aggregate constraints can be recast, following the Solow–Greenwood (SG) arguments and notation, into these others

$$Y(t) = A_{SG}(t)F(L(t), u(t)K_{SG}(t)), \tag{19}$$

$$\dot{K}_{SG}(t) = \theta(t)I^G(t) - \delta_{SG}(t)K_{SG}(t). \tag{20}$$

Here, δ_{SG} is variable due to the presence of adjustment and maintenance costs, which makes it a vehicle for both physical and economic deterioration.[13] θ represents the current state of the technology for producing capital goods, but it is also the level of embodied technical progress. A_{SG} is a measure of TFP, and represents the exogenous level of neutral technical progress. K_{SG} is the value of capital stock measured in quality-adjusted units according to the PIM. In this case the corresponding output constraint would be $Y = C + \frac{p^k I^G}{p} + \phi\left(\frac{I^G}{K_{SG}}\right)K_{SG} + \varpi\left(\frac{\delta_{SG}}{u}\right)uK_{SG}$.

The above model specifications are connected by a critical relationship, which we take from Greenwood's appendix,

$$K^*(t) = \frac{K_{SG}(t)}{\theta(t)}. \tag{21}$$

Then, combining (18), (20), and (21) we get

$$\delta^*(t) = \delta_{SG}(t) + \frac{\dot{\theta}(t)}{\theta(t)}. \tag{22}$$

The economic rate of depreciation $\delta^*(t)$ is a key variable in our model, and Equation (22) shows that obsolescence is measured properly by this variable along with physical and economic deterioration. In fact, the above equation says that our endogenous $\delta^*(t)$ is the sum of the depreciation rate corresponding to deterioration plus the rate of embodied technical progress.[14]

Finally, from (17), (19), and (21) we get

[13]Neither Solow nor Greenwood included maintenance costs and variable capacity utilization in their studies of investment and technological change. In general, without adjustment and maintenance functions in the model, δ_{SG} would be a constant depreciation rate associated with physical deterioration, which would capture the effects of capital aging and its use at a constant rate (most probably at full capacity).

[14]In empirical studies, the hypothesis of the link between the investment price index and capital-embodied technological change is introduced by assuming $\theta(t) = p(t)/p^k(t)$, which is the reciprocal of the relative price index of investment goods with respect to output, that is, the amount of investment goods that can be purchased for one unit of output. The idea is simple, technological change makes new capital goods simultaneously less expensive and more productive than old ones. One unit of new capital is $\frac{\dot{\theta}(t)}{\theta(t)}$ times more productive than another unit one period older, but $\frac{\dot{\theta}(t)}{\theta(t)}$ is also the rate of decline in $\frac{p^k(t)}{p(t)}$. Consequently, the decline of the relative price index of new capital goods discloses an increasing level of investment-specific technology.

$$A^*(t) = A_{\mathrm{SG}}(t)\frac{F(L(t), u(t)\theta(t)K^*(t))}{F(L(t), u(t)K^*(t))}. \tag{23}$$

Given that $F(\cdot)$ is homogeneous of degree one we can write[15]

$$A^*(t) = A_{\mathrm{SG}}(t)\left(1 + (\theta(t) - 1)\left(1 - \frac{F_L L}{F}(t)\right)\right) = A_{\mathrm{SG}}(t)\cdot h(\theta(t), \beta(t)). \tag{24}$$

This expression says that the model-based measure of TFP $A^*(t)$ is something beyond the pure exogenous level of neutral technical progress associated with the measure of capital stock that arises from the PIM. The endogenous measure of TFP actually represents the level of the overall technical progress associated with the economic measure of capital stock and depreciation, including obsolescence. It encompasses the standard neutral technical progress, but also depends positively on the level of investment-specific technology. The contribution of the latter, that is, embodied technical progress, to overall technical progress is greater than the elasticity of output with respect to capital in the production function.

Having in mind all these new insights that affect obsolescence and TFP, we must conclude that the correct economic measurement of capital is important and will have major implications for empirical growth accounting exercises. To the extent that capital has economically depreciated but statistics assume that it is still active in providing its services, measured TFP growth will be less than it actually is.[16] Therefore, any attempt to explain old and new productivity slowdowns, the role of information and communication technologies in the productivity acceleration of the mid-1990s, the Solow paradox, or even some other macroeconomic puzzles, requires paying attention to all previous theoretical developments, but also to the empirical results presented in Section 4.

4 | EMPIRICAL METHODOLOGY AND MEASUREMENT

The theory of the firm that we have developed in the previous sections allows us to carry out an empirical exercise aimed at obtaining the economic values of capital and depreciation. To this end, one needs to rewrite the variables in discrete terms making the relevant expressions computationally operative. Furthermore, we will simplify the notation by calling the market value of the firm along the optimal equilibrium path as V_t^*. Then, from (15) and (16) we get

$$\frac{V_t^*}{p_t^{\mathrm{k}} K_t^*} = q_t. \tag{25}$$

Recall that the index of profitability can be conceptually differentiated between theoretical variables (the model-based q^{M} and q^{A}) and empirical measures (Tobin's q ratio). We have seen

[15]With a Cobb–Douglas specification for $F(\cdot)$ we get $1 - \frac{F_L L}{F} = \beta$ and $A^*(t) = A_{\mathrm{SG}}(t)\theta^{\beta}(t)$, which corresponds to the equation in Greenwood et al. (1997, p. 361).

[16]Studying the post-2004 slowdown in labor productivity and TFP in the United States, Byrne et al. (2016) examine the hypothesis of mismeasurement of capital. However, the authors focus on unmeasured quality improvements and unobserved intangible investments. They conclude, on the contrary, that true TFP grows slower, not faster, than measured.

that under some standard technical assumptions that characterize the functions of production, adjustment costs, and maintenance expenditures, marginal q^{M} equals average q^{A}. The first one is important because it explains the flows of investment and depreciation, which determine the dynamics of capital stock. The second can be expressed in terms of endogenously determined variables such as the market value of the firm in the numerator, and the economic value of the capital stock measured in nominal terms at its replacement cost in the denominator. Finally, from a practical standpoint, we introduce the observable Tobin's q ratio as an exogenous proxy for the theoretical index of profitability. Tobin's q ratio is an empirical measure taken from studies that manage the book value of capital assets, as well as financial and stock-exchange data of companies.[17]

The dynamics of the capital stock is determined at each moment according to the first-order difference equation

$$K_t^* = I_t^{\mathrm{G}} + (1 - \delta_t^*)K_{t-1}^*. \tag{26}$$

Given the capital stock of the previous period, by adding the flow of gross investment I_t^{G} and subtracting the depreciation flow $\delta_t^* K_{t-1}^*$, we obtain the stock of capital of the current period. Although the gross investment is a control variable in the model, from the point of view of our computations it will be considered as a forcing variable. Since transactions related to the acquisition of capital goods are observable transactions in the market, we shall consider these records as the outcome of optimal decisions, and as such we include them in our calculations. In this way, we can try to obtain a straightforward procedure to generate the values of depreciation and capital stock that are not directly observable in the market.

As usual, we consider the market value of the firm V_t^* given by the discounted present value of the infinite flow of distributed profits, B_t^*. Any profit distributed in the form of dividends is, by definition, a cash-flow net of adjustment and maintenance costs, but also net of corporate taxes and after deducting the amounts of fixed capital consumption. In addition, the revenues that allow dividends to be paid have been obtained with less than full capacity utilization. According to the financial theory of the firm, the best candidate for discounting profits is the required returns to capital or its average cost. But the empirical approach faces a problem of choice between different alternatives. In our case, we will use the long-term interest rate because we derive the measurement method from a set of hypotheses that include an infinite horizon (only distributed profits are relevant) and perfectly competitive capital markets (all measures of the returns to capital are equivalent).[18] Given that variables V_t^* and B_t^* are both expressed in nominal terms, we discount the stream of dividends with the nominal interest rate R_t,

$$V_t^* = \sum_{s=t}^{\infty} \frac{B_s^*}{\prod_{\tau=t+1}^{s}(1 + R_\tau)}. \tag{27}$$

[17]It is assumed that in no case does the empirical computation of Tobin's q ratio use data on capital stock at historical prices or acquisition costs.

[18]These assumptions place us in the realm of measurement with theory, but under an extreme position. However, our target here is mainly methodological. We are looking for a mechanism to measure the value of the aggregate capital stock at equilibrium, according to the purest requirements of the neoclassical capital theory. In this paper we focus on avoiding the shortcomings posed by the assumption of an exogenous constant depreciation rate.

In this intertemporal context without uncertainty, it is still necessary for our computations to specify how agents form their expectations regarding the future value of variables. We assume that, along the equilibrium path, economic agents expect that nominal profits increase with inflation. Moreover, under perfect competition, agents are price-takers; they consider the inflation rate and the nominal interest rate as exogenously given at the moment of making decisions. Although these two price variables change over time in accordance with the market forces of demand and supply, in each period the individual firm reacts by perceiving them as constant parameters. Consequently, we are assuming that they behave as if the observed current values of the rate of inflation and the nominal interest rate were to be repeatedly observed in the future; that is, static expectations. When we apply these assumptions to the terms of Equation (27) we find that, $\forall\ s, \tau \in [t, \infty[$, $B_s^* = B_t^*\left(1 + \pi_s^{\mathrm{k}}\right)^{s-t}$ being $\pi_s^{\mathrm{k}} = \pi_t^{\mathrm{k}}$ the inflation rate associated with the price index of capital goods p^{k}, and $\prod_{\tau=t+1}^{s}(1 + R_\tau) = (1 + R_\tau)^{s-t}$ with $R_\tau = R_t$. We define the real interest rate $r_t = R_t - \pi_t^{\mathrm{k}} > 0$ and approximate the term $\frac{1+\pi_t^{\mathrm{k}}}{1+R_t} = 1 + \pi_t^{\mathrm{k}} - R_t$, taking the product $r_t R_t$ as negligible. Then, we can write

$$V_t^* = B_t^* \sum_{s=t}^{\infty} \left(\frac{1 + \pi_t^{\mathrm{k}}}{1 + R_t}\right)^{s-t} = B_t^* \sum_{s=t}^{\infty} (1 - r_t)^{s-t} = \frac{B_t^*}{r_t}. \tag{28}$$

Substituting this result in (25) we get

$$q_t = \frac{B_t^*}{r_t\, p_t^{\mathrm{k}} K_t^*}. \tag{29}$$

Equations (29) and (26) give us a clear idea of how the process of capital accumulation is defined in economic terms. They also explain how markets evaluate this process globally in aggregate terms. In these equations there are different types of variables, some represent quantities and others are prices, some refer to observables in real markets and others do not account for any explicit market transactions, some are considered endogenous and others can be treated as exogenous. But in all cases they capture the economic or market value of the involved variable. On the one hand we have the stock of capital and the flows of distributed profits, gross investment, and depreciation. On the other the interest rate, the price of investment goods and the financial Tobin's q ratio.

Concerning the revenues of productive factors generated by firms and distributed through the market mechanisms, it must be remarked that the sum of the economic value of the distributed net profits, B_t^*, and the flow of economic depreciation in nominal terms is equal to the distributed gross profits expressed in nominal terms, $B_t^{\mathrm{G}} = B_t^* + \delta_t^* p_t^{\mathrm{k}} K_{t-1}^*$. Substituting in (29) we get

$$q_t r_t\, p_t^{\mathrm{k}} K_t^* = B_t^{\mathrm{G}} - \delta_t^* p_t^{\mathrm{k}} K_{t-1}^*. \tag{30}$$

Therefore, if we know the value of all the price variables and the value of the economic-accounting flows of gross investment and gross distributed profits, we can use sequentially Equations (26) and (30) to obtain the economic value of the unobservables: the depreciation rate δ_t^* and the capital stock K_t^*. This dynamic system of two first-order difference equations allows us to express both δ_t^* and K_t^* as a function of variables q_t, r_t, p_t^{k}, B_t^{G}, and I_t^{G}, given the predetermined value of K_{t-1}^*,

$$\delta_t^* = \frac{\frac{\left(B_t^{\mathrm{G}} / p_t^{\mathrm{k}}\right)}{q_t r_t} - K_{t-1}^* - I_t^{\mathrm{G}}}{\left(\frac{1}{q_t r_t} - 1\right) K_{t-1}^*}, \tag{31}$$

$$K_t^* = \frac{K_{t-1}^* + I_t^{\mathrm{G}} - \left(B_t^{\mathrm{G}} / p_t^{\mathrm{k}}\right)}{1 - q_t r_t}. \tag{32}$$

From these expressions it is easy to identify a number of correlations between the two endogenous variables and the set of independent variables. Ceteris paribus, an increase in investment expenditures will increase the capital stock but reduce the depreciation rate. Moreover, higher levels of distributed profits in real terms are associated with higher values of the depreciation rate and lower values of the capital stock. Finally, the financial Tobin's q ratio and the real interest rate are inversely (positively) correlated with the depreciation rate (capital stock).

4.1 | Quantitative results

Next, we run the above equations, which represent the new measurement method, with data from the nonfinancial business sector of the US economy over the period 1960–2016. This sector encompasses most of the activities in the economy but excludes the financial intermediation sector, real estate activities, and nonmarket services. From a known initial value K_0^* we can compute forward the above closed-form solutions to obtain the complete series for the capital stock and the depreciation rate. The data series used in this empirical exercise come from different databases: the Integrated Macroeconomic Accounts (IMA) and the National Income and Product Accounts (NIPA) from the BEA[19]; the Organisation for Economic Cooperation and Development (OECD) Statistics[20]; and Osborne and Retus (2017).

The sources for each variable in Equations (31) and (32) are the following: the data on gross distributed profits, B^{G}, calculated as the sum of Property Income (dividends and interest) and Consumption of Fixed Capital, are from the IMA[21]; the Gross Fixed Capital Formation at current prices and the capital stock in 1960 are also from the IMA[22]; to deflate, we use the NIPA

[19]The data can be accessed in https://www.bea.gov/data/economic-accounts and https://www.bea.gov/data/special-topics/integrated-macroeconomic-accounts

[20]The data can be accessed in https://stats.oecd.org/

[21]The BEA Account codes of Property Income are of both components, dividends (BEA Account codes: FA106121101 [received] and FA106121001 [paid]) and interest (BEA Account codes: FA106130101 [received] and FA106130001 [paid]). The codes of the series of Consumption of fixed capital are (BEA Account code: FA106300001) for nonfinancial corporate and noncorporate (BEA Account code: FA116300001). All of them correspond to Table s.4.a and Table s.5.a. Annual data from 1960 to 2016.

[22]The series of GFCF are in Table s.5.a BEA Account Code: FA105019085 in reference to nonfinancial corporate sector, and Table s.4.a BEA Account Code: FA115019085 for nonfinancial noncorporate sector. The fixed assets in 1960 correspond to the nonfinancial, corporate and noncorporate, business assets (BEA Account codes: LM102010005 and LM112010005) excluding real estate (LM105035005 and LM115035023).

private nonresidential investment price index[23]; the observable Tobin's q has been taken from Osborne and Retus (2017), specifically their $Q3$: the market value of outstanding equity plus market value of outstanding corporate bonds plus net liquid assets divided by the net stock of produced assets valued at current cost; finally, interest rates are taken from OECD.Stat, and refer to yields on government securities with outstanding maturities of 10 years.[24]

Once these economic measures are obtained, we can compare with the most common statistical measures of capital stock and depreciation. These standard measures are recorded by agencies, such as the BEA, based on the PIM. Therefore, they assume that the useful life of capital goods is determined exogenously by technological parameters, and depreciation is then calculated at a constant exponential rate. Figures 1 and 2 plot the series of the variables depreciation rate and capital stock, respectively. In Figure 1, the economic depreciation rate is compared with the statistical depreciation rate. Figure 2 shows the time profiles of economic and statistical capital stocks. Table 1 contains the numerical series of economic and statistical depreciation rates, as well as the corresponding capital stocks, over the period 1960–2016. Here it is assumed that the initial capital stocks are equal, $K^*_{1960} = K_{1960}$.

The complete series of variables needed to perform our calculations are shown in Table A1. The reader will also find Figures A1–A4, which help graphically identify the correlations between the endogenous rate of economic depreciation and the independent variables, as implicitly defined in Equations (31) and (32).

In Figure 1, we observe that the economic depreciation rate fluctuates around the statistical depreciation rate, but according to a low-frequency movement. This means that the economic rate is higher or lower than the statistical rate over long periods spanning many years. Throughout the sample, the gap between the two rates seems to obey an irregular but stationary process. During this long period, the statistical depreciation rate shows a smooth but constant upward trend, mainly explained by changes in the composition of capital, which turns toward goods with a shorter average useful life. As a result of the strong persistence of gaps between the two depreciation rates, we observe in Figure 2 that the economic measure of capital remains below or above the statistical measure of capital for several years. The cumulative nature of capital stock is the main reason for the observed profiles. In any case, the above features of results show that the dynamic process driven by our difference equations is nonexplosive.

The measurement of capital has been, for years, a real puzzle for economists. Given the lack of consensus to measure capital in economic terms with the support of economic theory, efforts have focused on obtaining good statistical measures. Any attempt at inventorying the economy's capital stock needs to add heterogeneous goods by using deflators for different capital goods. Even when economists measure the aggregate capital stock according to the PIM, it is also necessary to use the price indexes associated with the types of goods that make up the investment flow. Thus, the weaknesses detected in the elaboration or application of these deflators will be transferred to the statistical measure of capital stock. The economic measure of capital that we have just proposed is not exempt from this problem. Moreover, our algorithm uses price variables other than the market price of capital goods. Therefore, any mistake or shortcoming concerning the real interest rate or Tobin's q ratio will also have implications for

[23]The data can be found in Table 5.3.4, BEA Account Code: B008RG. Annual data from 1947 to 2017.

[24]Long-term interest rates, OECD.Stat. *Direct source*: Federal Reserve Board, US Data.

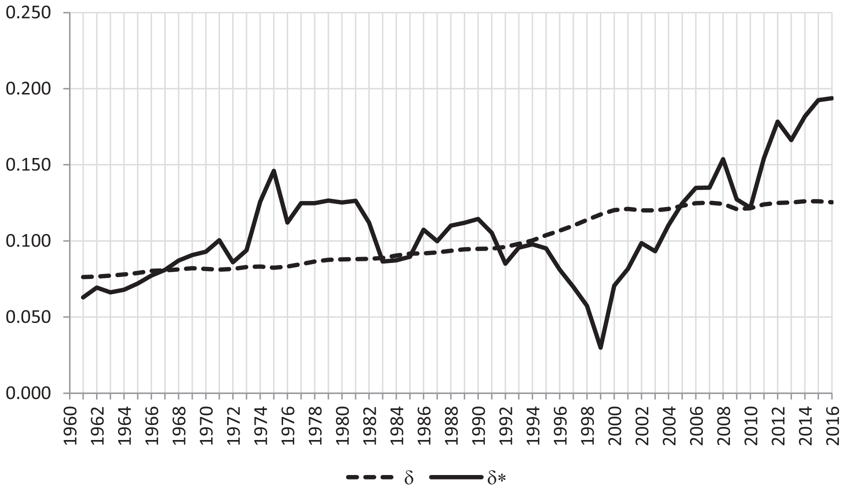

FIGURE 1 US economic and statistical depreciation rates. *Source*: Own elaboration and Bureau of Economic Analysis

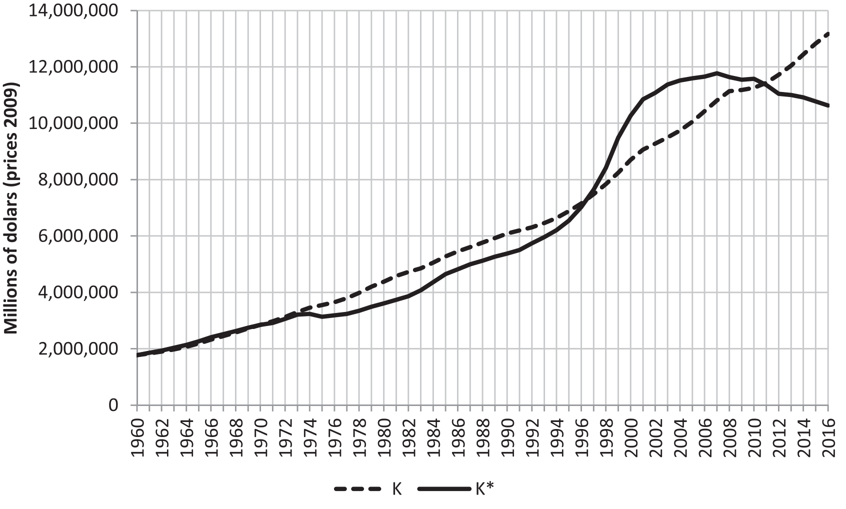

FIGURE 2 US economic and statistical capital stocks. *Source*: Own elaboration and Bureau of Economic Analysis

the estimated economic value of the capital stock. However, the main goal of this paper is methodological. On the basis of a theory of the optimizing firm, we obtain a new method that overcomes the limitations posed by the proportionality hypothesis on capital depreciation. Of course, the quality of our empirical results depends on the quality of the input data[25] and on some assumptions of the theory that could be modified.

[25]Following Hulten (2001, p. 41), if the available data on prices and quantities do not accurately reflect reality, or if the limits of the data set are set too narrow, attacking the measures of capital and TFP is like shooting the messenger for the message.

TABLE 1 Economic and statistical depreciation rates and capital stock (millions of dollars, prices 2009) in US nonfinancial business sector (1960–2016)

	Depreciation rate		Capital stock			Depreciation rate		Capital stock	
	Economic	Statistical	Economic	Statistical		Economic	Statistical	Economic	Statistical
Year	δ^*	δ	K^*	K	Year	δ^*	δ	K^*	K
1960			1,777,396	1,777,396	1989	0.1119	0.0944	5,264,742	5,931,358
1961	0.063	0.0762	1,856,425	1,832,817	1990	0.1144	0.0948	5,381,330	6,088,083
1962	0.0694	0.0766	1,936,615	1,901,354	1991	0.1052	0.0949	5,501,912	6,197,402
1963	0.0662	0.0772	2,033,257	1,979,326	1992	0.0851	0.0961	5,742,120	6,310,507
1964	0.0679	0.0779	2,142,278	2,072,220	1993	0.0956	0.0981	5,960,438	6,458,602
1965	0.0719	0.0789	2,269,426	2,189,960	1994	0.0977	0.1003	6,208,748	6,641,839
1966	0.0772	0.0804	2,407,093	2,326,821	1995	0.0952	0.1037	6,547,209	6,882,438
1967	0.081	0.0807	2,522,044	2,448,940	1996	0.0811	0.1066	7,024,398	7,156,831
1968	0.0871	0.0813	2,631,582	2,578,862	1997	0.0698	0.1101	7,646,737	7,481,428
1969	0.0907	0.082	2,748,465	2,723,027	1998	0.0574	0.1138	8,412,878	7,834,792
1970	0.0929	0.0816	2,844,895	2,852,714	1999	0.03	0.1173	9,485,802	8,240,705
1971	0.1004	0.0812	2,917,503	2,979,286	2000	0.0706	0.1202	10,269,508	8,702,977
1972	0.086	0.0815	3,061,622	3,131,358	2001	0.0815	0.121	10,846,158	9,063,624
1973	0.0939	0.0829	3,211,779	3,309,321	2002	0.0985	0.12	11,077,461	9,275,958
1974	0.1257	0.0832	3,234,407	3,460,335	2003	0.0932	0.1201	11,377,765	9,494,793
1975	0.1459	0.0823	3,138,094	3,551,243	2004	0.1103	0.1209	11,516,426	9,740,347
1976	0.1121	0.0832	3,185,738	3,655,362	2005	0.1246	0.123	11,597,194	10,058,262
1977	0.1248	0.0847	3,236,029	3,793,862	2006	0.1348	0.1248	11,654,840	10,423,731
1978	0.1248	0.0865	3,345,829	3,979,271	2007	0.135	0.1251	11,767,757	10,804,967

TABLE 1 (Continued)

	Depreciation rate		Capital stock			Depreciation rate		Capital stock	
	Economic	Statistical	Economic	Statistical		Economic	Statistical	Economic	Statistical
Year	δ^*	δ	K^*	K	**Year**	δ^*	δ	K^*	K
1979	0.1265	0.0876	3,490,569	4,198,444	2008	0.1537	0.1244	11,633,851	11,135,653
1980	0.1253	0.0878	3,607,214	4,383,697	2009	0.1273	0.1208	11,543,431	11,181,353
1981	0.1263	0.0881	3,733,961	4,579,735	2010	0.1217	0.1215	11,575,538	11,259,782
1982	0.1121	0.0881	3,863,625	4,724,484	2011	0.1545	0.124	11,356,972	11,433,336
1983	0.0865	0.0888	4,078,811	4,854,392	2012	0.1783	0.1249	11,047,042	11,720,277
1984	0.0872	0.0904	4,363,768	5,055,849	2013	0.1662	0.1253	10,998,597	12,039,430
1985	0.0896	0.0917	4,652,811	5,272,320	2014	0.1818	0.1261	10,919,466	12,442,020
1986	0.1073	0.0917	4,820,376	5,455,335	2015	0.1924	0.126	10,776,555	12,833,001
1987	0.0998	0.0924	4,997,138	5,609,385	2016	0.1937	0.1253	10,631,612	13,167,002
1988	0.11	0.0935	5,126,795	5,764,047					

Note: Own elaboration and Bureau of Economic Analysis.

Despite this, even if our empirical results concerning the measure of depreciation and capital stock can be improved, this does not disqualify our work. It is important to note that the results shown in Figures 1 and 2 are consistent with the facts and ideas commonly accepted about obsolescence and other issues related to the overall process of capital accumulation and substitution. In particular, they allow us to identify periods of greater or lesser destruction of capital, which are not recorded in official statistics. This is important because its repercussions on the study of many other macroeconomic topics. When the statistical measures underestimate depreciation, they overestimate capital growth, which in turn leads to an underestimation of TFP growth (Musso, 2004).

4.2 | Some key implications of our results

At the beginning of the paper, we referred to unsolved puzzles involving productivity paradoxes and slowdowns. The evolution of labor productivity growth in the United States, since World War II is usually divided into four periods: before 1973, it averaged 2.7% per year; it slowed down to 1.5% per year from 1974 to 1994; thereafter it rose to 2.8% average annual growth over 1995–2004; and, finally, a further slowdown lowered the average annual growth to 1.3% from 2005 to 2015. However, there is an ongoing debate about the causes of productivity slowdowns (Byrne et al., 2016; Syverson, 2017). On the one hand are those who do not recognize the decline in labor productivity growth as a real fact because of the mismeasurement of output. They consider that slowdowns are just a statistical fiction because the shift in economic activity toward the service sector and nonmarket activities make estimates of output unreliable (Varian, 2016). On the other hand are those who accept that the measured decline in productivity growth is meaningful and hypothesize alternative factors that could help explain them. The main candidates include: cyclical phenomena associated with supply and demand shocks, such as the rise in oil prices or the Great Recession linked to the real estate and financial collapses; the reduced dynamism of the economy due to corporate taxation and the increase in regulatory charges; and the waning of the exceptional rates of productivity growth associated with broad waves of technological breakthroughs such as electricity, internal combustion engines, new biochemical processes, or the surge in the manufacturing and utilization of information and communications technology (ICT) (Fernald, 2015).

Although there is evidence of output underestimation, it does not seem to explain the slowdowns because the biases have always been present and, also, it is not certain that the measurement has worsened. Moreover, as Fernald (2018) indicates, the first slowdown persisted when oil prices recorded a moderate increase in the mid-1980s, the second slowdown began before the onset of the Great Recession, and it is unclear whether the rising regulatory burdens have been the reason why productivity accelerations did not last. Therefore, further inspection of the reasons for productivity slowdowns and accelerations is required.

The above discussion suggests that the explanation we choose should be based on the hypothesis that changes in productivity growth are closely related to the diffusion of a new wave of technology, which usually follows a logistics-shaped curve. According to this, there is a significant delay between the beginning of the wave, the turning point at which growth takes off, and the end of the wave. Consequently, innovations characterized by creative destruction, in the sense of innovations leading to new capital goods that replace existing capital goods, for the economy as a whole will imply gradual replacement. This also means that obsolescence will be experienced with a certain delay, and most likely in overlapping layers corresponding to different technological waves.

Byrne et al. (2016) show that the time pattern of changes in labor productivity growth is similar to the pattern of changes in TFP growth, which was high before 1973, declined between 1974 and 1994, increased from 1995 to 2004, and decreased again from 2005 onward. According to a standard growth accounting methodology, the growth rate of labor productivity is the sum of the contribution of the growth rate of capital deepening and the growth rate of TFP itself. In the case that the measured capital is accurate and reliable, with such a strong parallelism in the dynamic patterns, we could be sure that the changes in the TFP growth rate have been the proximate drivers of the changes in labor productivity growth. However, as TFP is measured as a residual, any correction of capital will be translated to a correction of TFP for a given measured labor productivity. This is exactly what we do in this paper, we amend the official measure of the capital stock.

According to Aghion et al. (2021), there were important and innovative technological developments in the US during the 1970s and 1980s, but there were also many transformative ICT-related innovations during the second-half of the 1990s and the early 2000s, just as there are today. Musso (2006) suggests that the rapid pace of technological change, combined with the severity of economic recessions, induced a substantial acceleration in capital obsolescence since the early 1970s. Accelerated investment-specific technical progress accelerates the obsolescence of installed capital, especially computers and other conventional machine tools whose service lives became shorter. This confirms the idea that there is an overestimation of the capital stock in official statistics, which is tied to the underestimation of capital depreciation. Moreover, to the extent that traditional growth accounting computes TFP growth by assuming that the average useful life of capital goods is constant, the resulting estimates would be biased. In such a case, any explanation of labor productivity slowdowns and accelerations based on the relative importance of these distorted capital and TFP growth rates would be inaccurate.

We have studied and measured the full range of depreciation, from physical deterioration to obsolescence through economic deterioration. Instead, the BEA mainly measures physical deterioration and expected obsolescence. The two measures are represented as economic and statistical depreciation rates in Figure 1. Conceptually, the intersection between the two depreciation rates includes only part of the deterioration and part of the obsolescence, therefore it is difficult to infer what happens to economic deterioration and total obsolescence separately and independently by inspecting the difference between these rates. On the one hand, economic deterioration depends on the fluctuating economic forces of demand and supply, the varying intensity of capital utilization, and the expenditures incurred for maintenance and repair. On the other hand, unexpected obsolescence moves according to unforeseen accelerations and slowdowns in technical progress, shifts in energy prices, or changes in the output composition. Consequently, the measured economic depreciation may be higher or lower than the measured statistical depreciation. The results obtained for the relevant subperiods are as follows.

Before 1973, the average annual economic and statistical depreciation rates were almost equal to each other, that is, 7.9%. During the following two decades, between mid-70s and mid-90s, the economic depreciation rate was two points higher than the statistical rate, that is, 10.9% and 9%, respectively. However, this difference was reversed during the decade between 1995 and 2005, where the average economic depreciation rate was only 8.3%, whereas the average statistical depreciation rate reached 11.6%. Finally, from 2005 to 2016, the relationship between the economic and statistical average depreciation rates reversed again, the first one has been

15.8% and the second one 12.4%. During the entire period 1960–2016, the average annual rate of economic depreciation was 10.7%, which was only slightly higher than the 10% corresponding to the average annual rate of statistical depreciation.

Is this underestimation and overestimation of depreciation, along with the growth of capital stock, responsible for the productivity puzzles? A definitive answer to this question is both difficult to provide and beyond the scope of this paper. However, with our results, we believe we have provided relevant data for investigating this problem. These results strongly suggest that future efforts should be devoted to conducting comprehensive research that integrates the study of the different phases of labor productivity growth, the time pattern of waves of technological innovation, the evolution of economic measures of depreciation and capital stock, and the resulting phases of TFP growth that result from them.

5 | CONCLUSIONS

In this paper we have proposed a dynamic optimization model for a competitive firm in which capital utilization and depreciation are endogenous control variables. These variables, together with investment and labor demands, are determined by profit maximization at the microeconomic level. In the standard neoclassical model, both the capital utilization rate and the depreciation rate are unrealistically treated as exogenous constants. However, it is well known that firms do not always use all the installed capital, and the depreciation rate is subject to change. We refine the basic framework by lessening the objective functional of the representative firm with a maintenance cost function that depends on these two variables, but also assuming that firms pay taxes levied on revenues minus current operating expenses. Resolution under the usual homogeneity properties allows us to establish the decisive relationship of equality between the two model-based profitability indexes, the marginal q and the average q, which can be empirically approximated by the observable Tobin's q. Then, because markets have been assumed to be competitive, the planning horizon is infinite, and there is no uncertainty, the final solution can be arranged as a system of two first-order difference equations that enable endogenously calculation of the depreciation rate and the capital stock. This strategy yields an economic estimator for both variables based on profitability indicators.

The empirical purpose of our paper is to address the challenge of capital theory, which requires a comprehensive economic measure of the capital stock at the aggregate level. This measure is important because the flow of services provided by the capital stock is one of the basic macroeconomic aggregates that describe the main empirical facts of modern economies. To achieve this goal, we need to measure depreciation correctly in economic terms, because any forgotten component or measurement error can lead researchers to misinterpret reality and make inaccurate economic predictions. Our computation method differs, methodologically and empirically, from the standard procedure followed by official agencies that adhere to the PIM and assume a constant useful life for capital goods. These agencies provide a statistical measure of the depreciation rate that, at most, only revises the lifetime of a small number of assets once or twice, leaving the majority unchanged over time.

We tested the performance of our method in an empirical application focused on the US data. The results for the period 1960–2016 show that the endogenous economic depreciation rate fluctuates according to a low-frequency movement, being higher or lower than the exogenous statistical depreciation rate over long periods. Additionally, we obtain the series of both the economic and statistical capital stocks, whose difference is the consequence of a greater or lesser destruction of capital that is not recorded in the official statistics. Given the strong persistence of gaps between the two depreciation rates, the economic measure of capital remains below or above the statistical measure of capital for several years. With respect to the conceptual content of these measures, the BEA mainly computes physical deterioration and expected obsolescence. Instead, we also include economic deterioration, which is determined by market conditions such as the intensity of capital utilization and the maintenance and repair expenditures, and abnormal obsolescence associated with unexpected technological breakthroughs or changes in the relative prices of inputs.

Assuming a constant depreciation rate in the computations may cause a sizeable mismeasurement of capital stock. The misperception of the true amount of capital in the economy can lead policymakers to fail to choose the optimal public policy when trying to solve problems of the real economy. Our results are more realistic and more consistent with the fundamentals of capital theory and provide policymakers with a better understanding of the economic environment in which specific measures should be implemented. Additionally, it is well known that any distortion in the measurement of capital stock may cause a substantial bias in the measurement of TFP growth. If the economic value of depreciation is poorly measured, growth accounting will not correctly reflect the role played by the contribution of capital deepening and TFP growth in describing the observed pattern of labor productivity growth. In fact, since World War II, the US labor productivity has experienced two periods of exceptional growth, alternating with successive periods of slowdown. Turning points were detected in the mid-1970s, the mid-1990s, and the mid-2000s. In this paper, we find that the turning points of the phases corresponding to underestimation and overestimation of the true economic depreciation are identical. These temporal concordances are highly suggestive, but an in-depth study of their implications is left for future research. As interesting as it is, the relationship between the dynamics of productivity growth and the dynamics of the economic measures of depreciation and capital stock is beyond the scope of this paper.

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APPENDIX A

See Table A1

See Figure A1

See Figure A2

See Figure A3

See Figure A4

TABLE A1 US nonfinancial business sector 1960–2016

Year	I_t^G	p_t^k	R_t	q_t	B_t^G						
Year	[1]	[2]	[3]	[4]	[5]	Year	I_t^G	p_t^k	R_t	q_t	B_t^G
1960	186,793	0.297	0.041	0.75		1989	711,381	0.885	0.085	0.72	714,900
1961	190,930	0.296	0.039	0.87	53,600	1990	719,034	0.904	0.086	0.66	750,500
1962	208,987	0.296	0.039	0.83	56,500	1991	686,960	0.921	0.079	0.81	767,000
1963	224,803	0.296	0.040	0.89	59,300	1992	708,378	0.918	0.070	0.88	772,000
1964	247,158	0.298	0.042	1.01	64,000	1993	767,006	0.920	0.059	0.91	787,000
1965	281,251	0.302	0.043	1.09	70,100	1994	830,716	0.927	0.071	0.83	841,500
1966	312,919	0.306	0.049	0.9	76,100	1995	929,263	0.936	0.066	1	896,900
1967	309,919	0.314	0.051	1.06	83,200	1996	1,008,293	0.930	0.064	0.98	946,100
1968	329,115	0.325	0.056	1.14	91,600	1997	1,112,457	0.925	0.064	1.16	1,019,600
1969	355,687	0.339	0.067	0.9	101,400	1998	1,204,799	0.910	0.053	1.36	1,109,800
1970	351,847	0.355	0.073	0.82	111,800	1999	1,325,097	0.902	0.056	1.63	1,142,500
1971	358,172	0.371	0.062	0.87	119,900	2000	1,452,995	0.907	0.060	1.25	1,244,700
1972	395,021	0.384	0.062	0.98	128,500	2001	1,413,716	0.904	0.050	1.05	1,303,600
1973	437,525	0.400	0.068	0.71	140,400	2002	1,299,877	0.900	0.046	0.75	1,335,800
1974	426,352	0.438	0.076	0.39	165,000	2003	1,333,148	0.899	0.040	0.98	1,348,800
1975	375,723	0.496	0.080	0.54	190,600	2004	1,393,884	0.911	0.043	1.01	1,449,000
1976	399,473	0.523	0.076	0.6	205,200	2005	1,516,146	0.938	0.043	0.95	1,485,300
1977	447,932	0.558	0.074	0.5	229,100	2006	1,620,634	0.966	0.048	0.98	1,716,700

TABLE A1 (Continued)

Year [1]	I_t^G [1]	p_t^k [2]	R_t [3]	q_t [4]	B_t^G [5]	Year	I_t^G	p_t^k	R_t	q_t	B_t^G
1978	513,570	0.595	0.084	0.48	256,900	2007	1,685,739	0.986	0.046	0.99	1,842,600
1979	567,827	0.643	0.094	0.5	287,900	2008	1,674,557	1.003	0.037	0.6	1,946,100
1980	554,048	0.700	0.115	0.55	342,800	2009	1,390,700	1.000	0.033	0.74	1,788,000
1981	582,243	0.767	0.139	0.46	407,200	2010	1,437,468	0.991	0.032	0.84	1,791,500
1982	548,268	0.810	0.130	0.48	449,000	2011	1,570,143	1.005	0.028	0.82	1,919,900
1983	549,423	0.809	0.111	0.52	463,800	2012	1,714,507	1.022	0.018	0.92	2,083,900
1984	640,471	0.812	0.124	0.48	494,600	2013	1,787,368	1.030	0.024	1.17	2,098,300
1985	679,903	0.820	0.106	0.56	525,400	2014	1,920,516	1.044	0.025	1.27	2,262,100
1986	666,739	0.834	0.077	0.62	564,300	2015	1,958,522	1.051	0.021	1.15	2,402,500
1987	657,913	0.844	0.084	0.6	589,300	2016	1,942,552	1.048	0.018	1.2	2,469,200
1988	679,104	0.865	0.088	0.62	649,400						

Note: 1. The figures in column [1] represent the Gross Fixed Capital Formation and are expressed in millions of dollars, 2009 prices. *Source*: BEA, see footnote 22.

2. In column [2], the data correspond to the Investment price index. *Source*: BEA, see footnote 23.

3. In column [3], the figures refer to the long-term interest rates. *Source*: OECD, see footnote 24.

4. The figures in column [4] are the observable Tobin's q. *Source*: Osborne and Retus (2017).

5. Data in column [5] refer to the distributed gross profits and are in millions of current dollars. *Source*: BEA, see footnote 21.

Abbreviations: BEA, Bureau of Economic Analysis; OECD, Organisation for Economic Cooperation and Development.

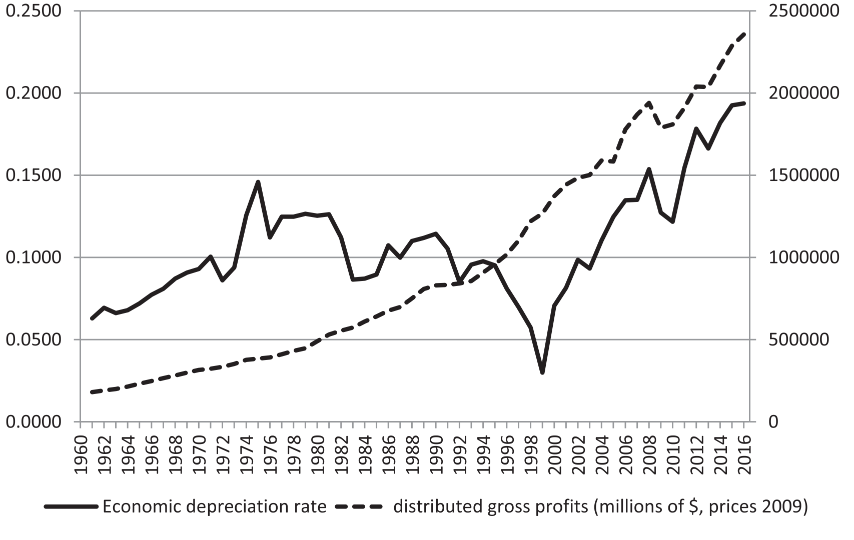

FIGURE A1 Economic depreciation rate and distributed gross profits: US nonfinancial business sector, 1960–2016. *Source*: Own elaboration and Bureau of Economic Analysis

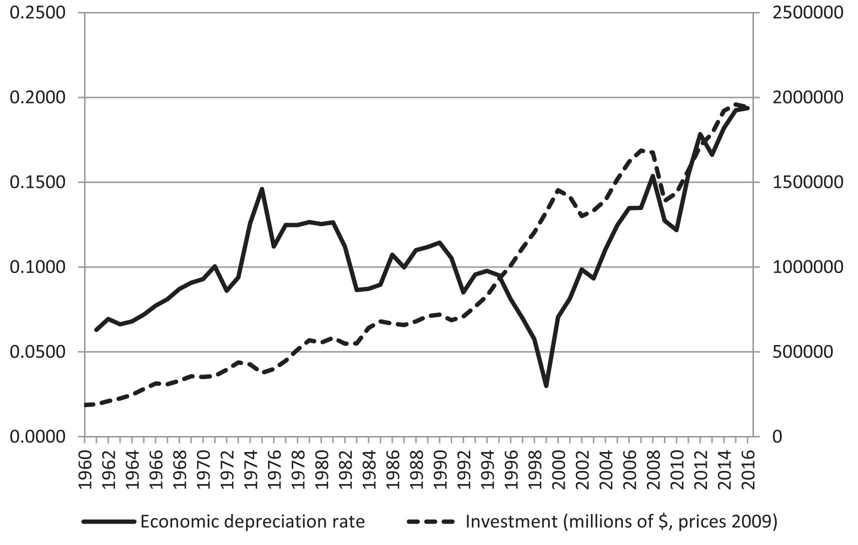

FIGURE A2 Economic depreciation rate and investment: US nonfinancial business sector, 1960–2016. *Source*: Own elaboration and Bureau of Economic Analysis

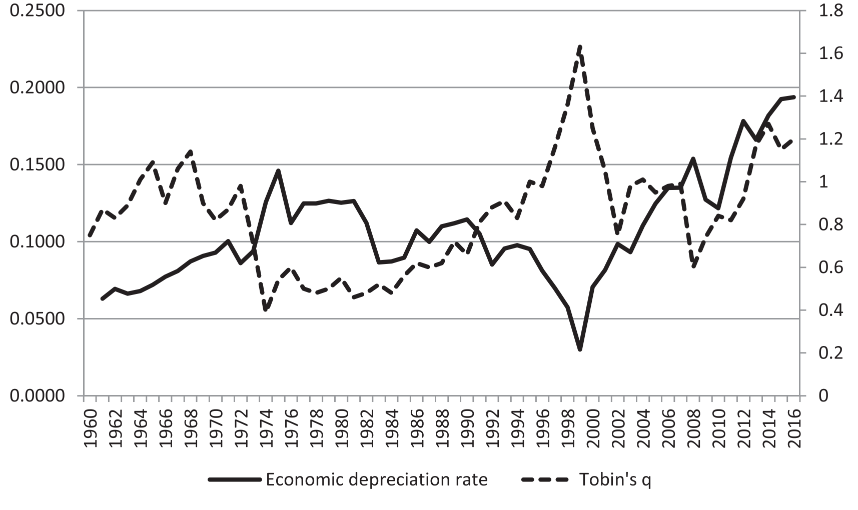

FIGURE A3 Economic depreciation rate and Tobin's q: US nonfinancial business sector, 1960–2016. *Source*: Own elaboration and Osborne and Retus (2017)

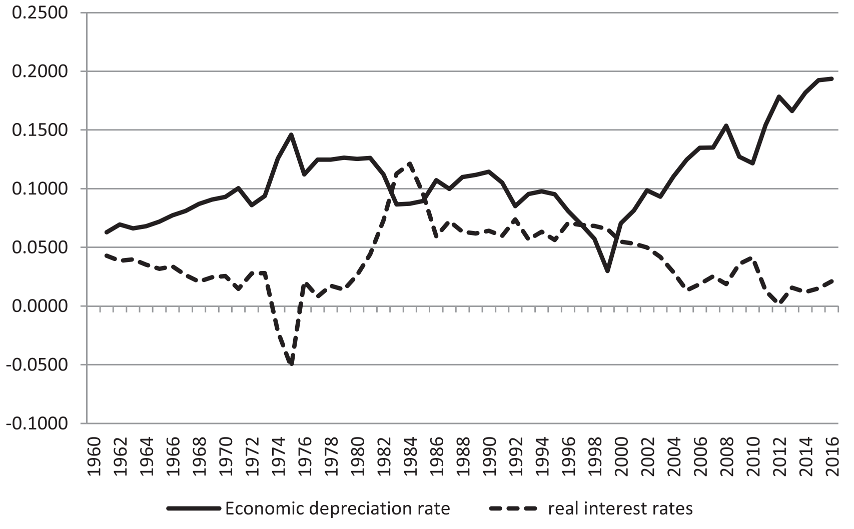

FIGURE A4 Economic depreciation rate and real interest rates: US nonfinancial business sector, 1960–2016. *Source*: Own elaboration and Organisation for Economic Cooperation and Development