Al-Mustansiriyah Journal of Science
ISSN: 1814-635X (print), ISSN:2521-3520 (online) Volume 34, Issue 1, 2023 DOI: http://doi.org/10.23851/mjs.v34i1.1215

Research Article Open Access

On α-Fuzzy Soft Irreducible Spaces

Majd Hamid Mahmood

Department of Mathematics, Collage of Education, Mustansiriyah University, Baghdad, IRAQ.

Contact: mgmg227@yahoo.com

Article Info

Received
26/08/2022

Accepted
04/12/2022

Published
30/03/2023

ABSTRACT

We define fuzzy soft irreducible sets, α-fuzzy soft irreducible sets in fuzzy soft topological spaces and study the properties including (fuzzy soft continuity; fuzzy soft homeomorphism and fuzzy soft topological properties) on α-fuzzy soft irreducible sets.

KEYWORDS: Fuzzy soft irreducible sets; fuzzy soft connected space; fuzzy soft continuous map.

الخلاصة

سوف نعرف المجاميع الضبابية الميسرة الغير قابلة للاختزال، المجاميع الضبابية الغير قابلة للاختزال- α في الفضاءات الضبابية الميسرة وسوف ندرس الخواص التي تتضمن (الاستمرارية الضبابية الميسرة، التكافؤ الضبابي الميسر والصفات التبولوجية الضبابية الميسرة) على المجاميع الضبابية الغير قابلة للاختزال- α.

INTRODUCTION

The fuzzy sets introduced in 1965 by Zadeh L. A. [12], Soft set introduced in 2001 by Molodtsov D. [8]. Fuzzy soft set introduced and studied in [1], [4], [7], [9] *(simply* $\mathcal{F}$*- set)*. α-fuzzy soft sets defined in [5] *(simply* α$\mathcal{F}$*- set)*. Soft topological space defined in [10]. Fuzzy soft topological space introduced and studied in [2],[6],[11].

In this research, we define fuzzy soft irreducible sets. α-fuzzy soft irreducible sets in fuzzy soft topological spaces and study the properties including: fuzzy soft continuity, fuzzy soft homeomorphism and fuzzy soft topological properties on α-fuzzy soft irreducible sets.

Definition 1.1. [7] *For a universal set* Ω*;* $\wp$ *set of parameters and soft set*$(f,\wp)$*. If each soft element* $\varkappa$ *in* $(f,\wp)$ *is associated with* $\eta\in[0,1]$*, then the resulting set is called a fuzzy soft set (*$\mathcal{F}$*- set).*

Definitions 1.2. [7]

For a soft set (F,E), F: E→P(Ω) where $F(e_i) \in P(\Omega), \forall e_i \in E$ the set of parameters, and for a family of $\mathcal{F}$*- sets* generated by the same soft set (F,E), { $(f,\wp)_\lambda : \lambda \in \Lambda$, where Λ is an infinite index set},

(1)the $\mathcal{F}$-union is defined by

$(h,\wp) = \tilde{\cup}_\lambda\ (f,\wp)_\lambda = \{x: x(e_i, F(e_i)^{\kappa i})\}$, where $\kappa i = \{\max \kappa i_\lambda : ki_\lambda$ are the memberships of each soft element in the soft set and $i\in\xi$, ξ is an infinite index set},

(2) the $\mathcal{F}$-intersection is defined by:

$(h,\wp) = \tilde{\cap}_\lambda\ (f,\wp)_\lambda = \{x : x = (e_i, F(e_i)^{\kappa i})\}$, where $\kappa i = \{\min \kappa i_\lambda : ki_\lambda$ are the memberships of each soft element in the soft set and $i\in\xi$, ξ is an infinite index set},

(3) $\tilde{A}$ is $\mathcal{F}$- subset of $\tilde{B}$, $\tilde{A} \tilde{\subseteq} \tilde{B}$ if each $\mathcal{F}$- element in $\tilde{A}$ is in $\tilde{B}$.

(4) $\tilde{\Phi}$ is the null $\mathcal{F}$- set *where each soft element associated to* $\eta=0$, $\tilde{\Omega}$ is the universal $\mathcal{F}$- set *where each soft element associated to* $\eta=1$.

Example 1.3.

For Ω = {a, b}, $\wp$ = {e}. Let

$\tilde{G}=(e,\{a^{0.2}, b^{0.3}\})$, $\tilde{L} = (e,\{a^{0.4}, b^{0.5}\})$,

$\tilde{\Phi}=(e,\{a^{0}, b^{0}\})$ and $\tilde{\Omega} = (e,\{a^{1}, b^{1}\})$

be $\mathcal{F}$- sets,

$\tilde{G}\ \tilde{\cup}\ \tilde{L}=(e,\{a^{0.2}, b^{0.3}\})\tilde{\cup}(e,\{a^{0.4}, b^{0.5}\})$

$= (e,\{a^{0.4}, b^{0.5}\}) = \tilde{L}$

$\tilde{G}\ \tilde{\cap}\ \tilde{L} =(e,\{a^{0.2}, b^{0.3}\})\tilde{\cap}(e,\{a^{0.4}\ b^{0.5}\})$

$= (e,\{a^{0.2}, b^{0.3}\}) = \tilde{G}$

$\tilde{\Omega}\ \tilde{\cap}\ \tilde{L} = \tilde{L}$, $\tilde{L}\ \tilde{\cup}\ \tilde{\Omega} = \tilde{\Omega}$

$\tilde{\Phi}\ \tilde{\cap}\ \tilde{G} = \tilde{\Phi}$, $\tilde{\Phi}\ \tilde{\cup}\ \tilde{L} = \tilde{L}$

$\tilde{\Phi}\ \tilde{\cap}\ \tilde{\Omega} = \tilde{\Phi}$, $\tilde{\Phi}\ \tilde{\cup}\ \tilde{\Omega} = \tilde{\Omega}$.

Definition 1.4.[11] For a non- empty universal set Ω, $\wp$ set of parameters, $\mathfrak{F}$ the collection of $\mathcal{F}$- sets

generated from the $\mathfrak{F}$ - set $\tilde{\Omega}$ (the non-null universal $\mathfrak{F}$-set), if $\mathfrak{F}$ satisfies the following axioms:
(a) $\tilde{\Phi}$, $\tilde{\Omega}$ are in $\mathfrak{F}$.
(b) The intersection of any two $\mathfrak{F}$ - set belongs to .
(c) The union of members of - sets is in $\mathfrak{F}$.
Then, is called ($\mathfrak{F}$ - topology).
A triple (Ω, $\wp$) is called $\mathfrak{F}$ - topological space over Ω (simply $\mathfrak{F}$ -Space),
the sets of $\mathfrak{F}$ are $\mathfrak{F}$- open sets denoted by $\mathfrak{F}$o - sets and their complements are called $\mathfrak{F}$c- sets.

Example 1.5.
For $\Omega = \{a, b\}$, $\wp = \{e\}$.
Let $\tilde{N} = (e,\{ a^{0.8}, b^{0.7}\})$,
$\tilde{D} = (e,\{ a^{0.5} , b^{0.4} \})$, $\tilde{K} = (e ,\{ a^{0.6} , b^{0.2} \})$
$\tilde{\Phi} = (e, \{a^0, b^0 \})$, $\tilde{\Omega} = (e, \{a^1, b^1 \})$
Let $\mathfrak{F}_1 = \{\tilde{\Phi}, \tilde{\Omega}, \tilde{N}, \tilde{D}\}$, $\mathfrak{F}_2 = \{\tilde{\Phi}, \tilde{\Omega}, \tilde{N}\}$
$\mathfrak{F}_3 = \{\tilde{\Phi}, \tilde{\Omega}\}$.
Then, $\mathfrak{F}_1$, $\mathfrak{F}_2$, $\mathfrak{F}_3$ are $\mathfrak{F}$ - topologies over Ω.
But $_4 = \{\tilde{\Phi}, \tilde{\Omega}, \tilde{D}, \tilde{K}\}$ is not a $\mathfrak{F}$ - topology on Ω since $\tilde{D} \tilde{\cap} \tilde{K} = (e,\{ a^{0.5} , b^{0.2} \}) \notin \mathfrak{F}_4$.

Definition 1.6. [11]
Let $\tilde{A}$ be a $\mathfrak{F}$ - set in $\mathfrak{F}$ - topological space (Ω, $\wp$). The $\mathfrak{F}$ - interior of $\tilde{A}$
(or $\mathfrak{F}$ - int ($\tilde{A}$)) is defined by
$\mathfrak{F}$ - in($\tilde{A}$) = $\tilde{\cup}$ {$\tilde{G}$: $\tilde{G}$ is $\mathfrak{F}$ o- set and $\tilde{G} \tilde{\subseteq} \tilde{A}$ } .

Definition 1.7. [11]
Let $\tilde{M}$ be a $\mathfrak{F}$ - set in $\mathfrak{F}$ - topological space (Ω ,, $\wp$) .The $\mathfrak{F}$ - closure of $\tilde{M}$
(or $\mathfrak{F}$- cl($\tilde{M}$)) is defined by
$\mathfrak{F}$ - cl($\tilde{M}$) = $\tilde{\cap}$ {$\tilde{C}$: $\tilde{C}$ is $\mathfrak{F}$c - set and $\tilde{M} \tilde{\subseteq} \tilde{C}$} .

Remarks 1.8.
1- $\mathfrak{F}$ - int ($\tilde{A}$)) is the largest $\mathfrak{F}$ - set contained in $\tilde{A}$.
2- $\mathfrak{F}$ - cl ($\tilde{A}$)) is the smallest $\mathfrak{F}$ - set containing $\tilde{A}$.

Examples 1.9.
1- For $\Omega = \{s, d\}$, $\wp = \{e\}$. Let
$\tilde{C} = (e,\{ s^1, d^0 \})$, $\tilde{D} = (e ,\{ s^0 , d^1 \})$
$\tilde{\Phi} = (e, \{s^0, d^0 \})$, $\tilde{\Omega} = (e, \{s^1, d^1 \})$,
$\mathfrak{F}_1 = \{\tilde{\Phi}, \tilde{\Omega}, \tilde{C}, \tilde{D}\}$
The sets $\tilde{\Phi}$, $\tilde{\Omega}$, $\tilde{C}$ and $\tilde{D}$ are $\mathfrak{F}$o - sets and $\mathfrak{F}$c - sets,
$\mathfrak{F}_1$- in($\tilde{C}$) = $\tilde{C}$, $\mathfrak{F}_1$ - in($\tilde{D}$) = $\tilde{D}$, $\mathfrak{F}_1$ - cl($\tilde{C}$) = $\tilde{C}$, $\mathfrak{F}_1$ - cl($\tilde{D}$) = $\tilde{D}$.
2- For $\Omega = \{s, d\}$, $\wp = \{e\}$. Let
$\tilde{B}$=(e,{$s^{0.2}$, $d^{0.3}$}) , $\tilde{E}$ = (e ,{ $s^{0.4}$, d^1 }) ,
$\tilde{F}$=(e,{$s^{0.5}$, d^1}) , $\tilde{\Phi}$ = (e , {s^0, d^0 }) ,
$\tilde{\Omega}$=(e, {s^1, d^1})
For $\mathfrak{F}_2 = \{\tilde{\Phi}, \tilde{\Omega} , \tilde{E} , \tilde{F}, \tilde{B}\}$
$\mathfrak{F}_2$-in($\tilde{F}$) = (e,{$s^{0.2}$, $d^{0.3}$}) $\tilde{\cup}$ (e,{ $s^{0.4}$, d^1 })$\tilde{\cup}$(e,{$s^{0.5}$, d^1 })=(e,{ $s^{0.5}$, d^1})=$\tilde{F}$
$\mathfrak{F}_2$-cl($\tilde{F}$) = (e,{$s^{0.5}$, d^1}) $\tilde{\cap}$ (e, {s^1, d^1})=$\tilde{F}$.

α$\mathfrak{F}$- IRREDUCIBLE SPACE

In this section we will define and study α$\mathfrak{F}$ - irreducible spaces, with examples.

Definition 2.1.
The $\mathfrak{F}$ - set $\tilde{M}$ in $\mathfrak{F}$ - topological space (Ω ,, $\wp$) is called (α$\mathfrak{F}$o-set) if:
$\tilde{M} \tilde{\subseteq} \mathfrak{F}$ - int [$\mathfrak{F}$ - cl (- int ($\tilde{M}$))].
And is called (α$\mathfrak{F}$c-set) if:
$\mathfrak{F}$ - cl [$\mathfrak{F}$ - int ($\mathfrak{F}$- cl($\tilde{M}$))] $\tilde{\subseteq} \tilde{M}$.

Remarks 2.2.
1- α$\mathfrak{F}$ - int ($\tilde{A}$)) is the largest $\mathfrak{F}$ - set contained in $\tilde{A}$.
2- α$\mathfrak{F}$ - cl ($\tilde{A}$)) is the smallest $\mathfrak{F}$- set containing $\tilde{A}$.
3- for α$\mathfrak{F}$-set $\tilde{M} = \{x_{ij} : x_{ij} = (ei , \{h_j^{kij}\})\}$, $\tilde{M}^c = \{x_{ij} : x_{ij} = (ei , \{h_j^{1-kij}\})\}$,
$\forall\ kij \in [0,1]$.
4- The complement of α$\mathfrak{F}$o-set (α$\mathfrak{F}$c-set) is α$\mathfrak{F}$c-set (α$\mathfrak{F}$o- set).

Remark 2.3.
Every $\mathfrak{F}$o- set ($\mathfrak{F}$c-set) is α$\mathfrak{F}$o-set (α$\mathfrak{F}$c-set) but the converse is not true in general.
The contra positive is true also, i.e. if the set is not α$\mathfrak{F}$o-set (α$\mathfrak{F}$c-set), then it is not $\mathfrak{F}$o-set ($\mathfrak{F}$c-set).

Examples 2.4.
For $\Omega = \{a, b\}$, $\wp = \{e\}$ and $\tilde{M}$, $\tilde{N}$, $\tilde{C}$, $\tilde{D}$ are $\mathfrak{F}$ - sets defined as follows:
$\tilde{M}$ = (e, {$a^{0.5}$, $b^{0.6}$ }), $\tilde{N}$ = (e,{ $a^{0.3}$, $b^{0.4}$})
$\tilde{C}$ = (e,{ $a^{0.8}$ $b^{0.7}$ }) , $\tilde{D}$ = (e ,{ $a^{0.5}$ $b^{0.4}$ })
$\tilde{\Phi}$ = (e, {a^0, b^0 }) , $\tilde{\Omega}$ = (e, {a^1, b^1 })
Let $\mathfrak{F} = \{\tilde{\Phi}, \tilde{\Omega}, \tilde{M}, \tilde{N}, \tilde{C}\}$ be $\mathfrak{F}$ - topology on Ω.
Since $\tilde{C} \in \mathfrak{F}$. Then, $\tilde{C}$ is $\mathfrak{F}$o - set so it is α$\mathfrak{F}$o-set,
α$\mathfrak{F}$ - int($\tilde{C}$) = $\tilde{C}$
[α$\mathfrak{F}$ - cl (α$\mathfrak{F}$-int($\tilde{C}$))] is the smallest α$\mathfrak{F}$c - set containing $\tilde{C}$ which is equal to $\tilde{C}$.
Then, α$\mathfrak{F}$ - int [α$\mathfrak{F}$ - cl (α$\mathfrak{F}$ - int($\tilde{C}$))] which is the largest α$\mathfrak{F}$o - set contains $\tilde{C}$ which is equal to $\tilde{C}$. So $\tilde{C}$ is α$\mathfrak{F}$o-set, similarly for $\tilde{\Phi}$, $\tilde{\Omega}$, $\tilde{M}$ and $\tilde{N}$ are α$\mathfrak{F}$o-set,
$\tilde{D}$ is ($\mathfrak{F}$c-set) since it's the complement is $\mathfrak{F}$o – set $\tilde{M}$, and $\tilde{D}$ is(α$\mathfrak{F}$c-set) since its complement is α$\mathfrak{F}$o - set $\tilde{M}$, $\tilde{D}$ is not $\mathfrak{F}$o - set since $\tilde{D} \notin \mathfrak{F}$ and $\tilde{D}$ is not α$\mathfrak{F}$o-set since by definition,
α$\mathfrak{F}$ - int ($\tilde{D}$)) = $\tilde{N}$
α$\mathfrak{F}$ - cl (α$\mathfrak{F}$ - int ($\tilde{D}$)) = α$\mathfrak{F}$ - cl ($\tilde{N}$) = $\tilde{D}$

Al-Mustansiriyah Journal of Science
ISSN: 1814-635X (print), ISSN:2521-3520 (online) Volume 34, Issue 1, 2023 DOI: http://doi.org/10.23851/mjs.v34i1.1215

$\alpha\mathfrak{F}$ - int [$\alpha\mathfrak{F}$ - cl (- int ($\tilde{D}$))] = $\alpha\mathfrak{F}$ - int [$\tilde{D}$]
= $\tilde{N}$
and $\tilde{D} \not\subseteq$ $\alpha\mathfrak{F}$ - int [$\alpha\mathfrak{F}$ - cl ($\alpha\mathfrak{F}$ - int ($\tilde{D}$))] = $\tilde{N}$.

Definition 2.5.

The $\mathfrak{F}$ - topological space (Ω ,, $\wp$) is called $\alpha\mathfrak{F}$ - irreducible if the intersection of any two non-null $\alpha\mathfrak{F}$o - sets is a non-null set otherwise it will be said to be $\alpha\mathfrak{F}$ - reducible.

Examples 2.6.

1- For $\Omega = \{a, b\}$, $\wp = \{e\}$ and $\tilde{M}$, $\tilde{N}$, $\tilde{C}$ are $\mathfrak{F}$ - sets defined as follows:
$\tilde{M}$=(e,{$a^{0.3}$, $b^{0.5}$ }) , $\tilde{N}$ =(e,{ $a^{0.2}$, $b^{0.1}$})
$\tilde{C}$=(e,{ $a^{0.02}$ b^{0} }) , $\tilde{\Phi}$ =(e , {a^0, b^0 }) ,
$\tilde{\Omega}$ = (e, {a^1, b^1 }) ,
let $\mathfrak{F} = \{\tilde{\Phi}, \tilde{\Omega}, \tilde{M}, \tilde{N}, \tilde{C}\}$ be $\mathfrak{F}$ - topology over Ω. Since $\tilde{\Phi}$ $\tilde{\Omega}$, $\tilde{M}$, $\tilde{N}$, $\tilde{C}$ are $\mathfrak{F}$o-sets, then $\tilde{\Omega}$, $\tilde{M}$, $\tilde{N}$, $\tilde{C}$ are $\alpha\mathfrak{F}$o-set. Since the intersection of any two non-null $\alpha\mathfrak{F}$o - sets is a non-null set.
Then, (Ω ,, $\wp$) is $\alpha\mathfrak{F}$ - irreducible space.
2- For $\Omega = \{p, m\}$, $\wp = \{e\}$. Let
3- $\tilde{U}$ =(e,{ p^1, m^0 }) , $\tilde{V}$ = (e ,{ p^0 , m^1})
$\tilde{\Phi}$ =(e, {p^0, m^0 }) , $\tilde{\Omega}$ =(e , {p^1, m^1 })
With $\mathfrak{F}$ - topology $\mathfrak{F} = \{\tilde{\Phi}, \tilde{\Omega}, \tilde{U}, \tilde{V}\}$
Since $\tilde{\Phi}$ $\tilde{\Omega}$, $\tilde{U}$, $\tilde{V}$ are $\mathfrak{F}$o-sets then they are $\alpha\mathfrak{F}$o-sets. Since $\tilde{U}$ $\tilde{\cap}$ $\tilde{V}$ =(e, {p^1, m^0})$\tilde{\cap}$(e,{p^0, m^1})=(e,{p^0, m^0})=$\tilde{\Phi}$ then the $\mathfrak{F}$-topological space be $\alpha\mathfrak{F}$ - reducible.

Definition 2.7.

In $\mathfrak{F}$ - topological space (Ω, $\wp$), the $\alpha\mathfrak{F}$ - set $\tilde{A}$ is $\alpha\mathfrak{F}$ - dense if $\tilde{A}$ intersect with any non-null $\alpha\mathfrak{F}$o - sets in $\mathfrak{F}$.

Example 2.8.

1- For $\Omega = \{m, n\}$, $\wp = \{c\}$ set of parameters and $\tilde{Z}$, $\tilde{D}$, $\tilde{K}$ are $\mathfrak{F}$ - sets are defined as follows:
$\tilde{Z}$ = (c,{$m^{0.2}$, $n^{0.7}$ }) ,$\tilde{D}$ = (c ,{ $m^{0.3}$, $n^{0.09}$})
$\tilde{K}$ = (c,{ $a^{0.4}$ b^1 }) , $\tilde{\Phi}$ = (c , {a^0, b^0 }) ,
$\tilde{\Omega}$ = (c, {a^1, b^1 })
Let $\mathfrak{F} = \{\tilde{\Phi}, \tilde{\Omega}, \tilde{Z}, \tilde{D}, \tilde{K}\}$ be a $\mathfrak{F}$ - topology over Ω.
Then, $\tilde{\Phi}$, $\tilde{\Omega}$, $\tilde{Z}$, $\tilde{D}$ and $\tilde{K}$ are $\alpha\mathfrak{F}$o-sets,
$\tilde{Z}$ is $\alpha\mathfrak{F}$ - dense set since $\tilde{Z} \tilde{\cap} \tilde{D} \neq \tilde{\Phi}$,
$\tilde{Z} \tilde{\cap} \tilde{K} \neq \tilde{\Phi}$ and $\tilde{Z} \tilde{\cap} \tilde{\Omega} \neq \tilde{\Phi}$.
Similarly, $\tilde{\Omega}$, is $\alpha\mathfrak{F}$ - dense set.
2- For $\Omega = \{m, n\}$, $\wp = \{e\}$,
Let $\tilde{R}$ = (e,{ m^1, n^0}) , $\tilde{S}$ = (e ,{ m^0, n^1 })
$\tilde{\Phi}$ = (e,{ m^0, n^0 }) , $\tilde{\Omega}$ = (e ,{ m^1, n^1})
With $\mathfrak{F}$ - topology $\mathfrak{F} = \{\tilde{\Phi}, \tilde{\Omega}, \tilde{R}, \tilde{S}\}$, the elements of $\mathfrak{F}$ are $\alpha\mathfrak{F}$o-sets.
Since $\tilde{R} \tilde{\cap} \tilde{S} = \tilde{\Phi}$, So $\tilde{R}, \tilde{S}$ are not $\alpha\mathfrak{F}$ - dense set.

Definition 2.9.

The $\mathfrak{F}$-topological space (Ω ,, $\wp$) is $\alpha\mathfrak{F}$ - connected if there are no proper, non-null $\alpha\mathfrak{F}$o - separated sets $\tilde{C}$, $\tilde{D}$ in $\tilde{\Omega}$ such that $\tilde{C} \tilde{\cup} \tilde{D} = \tilde{\Omega}$, if (Ω ,$\mathfrak{F}$, $\wp$) is not $\alpha\mathfrak{F}$ – connected, then it is said to be $\alpha\mathfrak{F}$ - disconnected space .

Examples 2.10.

1- For $\Omega = \{m, n, L\}$, $\wp = \{e\}$,
let $\tilde{R}$ = (e,{ m^1, n^0 , L^1 }) ,
$\tilde{T}$=(e,{ m^0, n^1 , L^0 }) ,
$\tilde{\Phi}$ =(e,{ m^0, n^0 , L^0 })
and $\tilde{\Omega}$=(e ,{ m^1, n^1 , L^1 }) .
With $\mathfrak{F}$ - topology $\mathfrak{F} = \{\tilde{\Phi}, \tilde{\Omega}, \tilde{R}, \tilde{T}\}$, the elements of $\mathfrak{F}$ are $\alpha\mathfrak{F}$o-set.
Since $\tilde{R} \tilde{\cap} \tilde{T} = \tilde{\Phi}$ so $\tilde{R}$, $\tilde{T}$ are non-null $\alpha\mathfrak{F}$o-separated sets. Then, the space (Ω,$\mathfrak{F}$, $\wp$) is $\alpha\mathfrak{F}$ - disconnected space.
2- For $\Omega = \{m, n\}$, $\wp = \{c\}$ and $\tilde{M}$, $\tilde{N}$, $\tilde{C}$ are $\mathfrak{F}$ - sets defined as follows:
$\tilde{M}$ = (c, {m^0, $n^{0.06}$ }) ,
$\tilde{N}$ = (c, { $m^{0.07}$, $n^{0.08}$})
$\tilde{C}$ = (c, { $a^{0.09}$ b^1 }) , $\tilde{\Phi}$ = (c, {a^0, b^0}) and
$\tilde{\Omega}$ = (c, {a^1, b^1 }) .
Let $\mathfrak{F} = \{\tilde{\Phi}, \tilde{\Omega}, \tilde{M}, \tilde{N}, \tilde{C}\}$ be a $\mathfrak{F}$ - topology on Ω.
Then, $\tilde{\Phi}$, $\tilde{\Omega}$, $\tilde{M}$, $\tilde{N}$ and $\tilde{C}$ are $\alpha\mathfrak{F}$o-sets,
since there are no proper, non-null $\alpha\mathfrak{F}$o - separated sets .Then, the $\mathfrak{F}$ - topological space
(Ω ,, $\wp$) is $\alpha\mathfrak{F}$ - connected.
In the next theorem we provide the equivalents of $\alpha\mathfrak{F}$ - irreducible space.

Theorem 2.11.

For (Ω,$\mathfrak{F}$, $\wp$) space, the next statements are equivalent :

1. The space (Ω ,, $\wp$) is $\alpha\mathfrak{F}$ - irreducible.
2. Any $\alpha\mathfrak{F}$o - set in (Ω ,, $\wp$) and non-null is $\alpha\mathfrak{F}$ - dense.
3. Any $\alpha\mathfrak{F}$o - set in (Ω ,, $\wp$) is $\alpha\mathfrak{F}$ - connected.

Proof.

(1) $\Leftrightarrow$ (2) Since the $\alpha\mathfrak{F}$ - set $\tilde{A}$ in a $\mathfrak{F}$ - topological space (Ω ,, $\wp$) is $\alpha\mathfrak{F}$ - dense if $\tilde{A}$ intersects with any non-null $\alpha\mathfrak{F}$o - sets so the condition (1) is equivalent to (2).
(1) $\Longrightarrow$ (3) Let $\tilde{A}$ be $\alpha\mathfrak{F}$o - set in $\mathfrak{F}$ and suppose $\tilde{A}$ is $\alpha\mathfrak{F}$ - disconnected so there exist two non - null $\alpha\mathfrak{F}$o - sets $\tilde{M}$, $\tilde{N}$ in $\mathfrak{F}$ such that $\tilde{A} = \tilde{M} \tilde{\cup} \tilde{N}$; $\tilde{M} \tilde{\cap} \tilde{N} = \tilde{\Phi}$, which is contradiction with (1).

(3) ⟹ (1) If $(\Omega, \mathfrak{F}, \wp)$ is α$\mathfrak{F}$ - reducible space, then the intersection of any two non-null α$\mathfrak{F}$o - sets is a null set, i.e. If $\tilde{M}, \tilde{N}$ are two non-null α$\mathfrak{F}$o – sets, then $\tilde{M} \tilde{\cap} \tilde{N} = \tilde{\Phi}$, so $\tilde{M} \tilde{\cup} \tilde{N}$ is α$\mathfrak{F}$-disconnected set which contradicts (3).

Definition 2.12.

Let $(\Omega, , \wp)$ be a $\mathfrak{F}$ - topological space and let $\tilde{A} \tilde{\subseteq} \tilde{\Omega}$. If the family $\mathfrak{F}_A=\{\tilde{M}^* : \tilde{M}^* = \tilde{A} \tilde{\cap} \tilde{M}, \tilde{M} \in \mathfrak{F}\}$ exists and it is $\mathfrak{F}$-topology on A. Then, $\mathfrak{F}_A$ called the relative $\mathfrak{F}$-topology on $\tilde{A}$ induced by the $\mathcal{F}$ - topology $\mathfrak{F}$ over Ω. Note that $(A, \mathfrak{F}_A, \wp)$ is called a $\mathcal{F}$ - subspace of
$(\Omega, \mathfrak{F}, \wp)$.

Example 2.13.

For $\Omega = \{m, n\}$, $\wp=\{c\}$ and $\tilde{M}$, $\tilde{N}$ are $\mathfrak{F}$-sets defined as follows:

$\tilde{M} = (c, \{m^1, n^{0.6}\})$, $\tilde{N}=(c,\{m^0, n^{0.4}\})$

$\tilde{\Phi} = (c, \{a^0, b^0\})$, $\tilde{\Omega}=(c,\{a^1, b^1\})$

Let $= \{\tilde{\Phi}, \tilde{\Omega}, \tilde{M}, \tilde{N}\}$ be a $\mathfrak{F}$ - topology over Ω.

$\tilde{A} \tilde{\subseteq} \tilde{\Omega}$, $\tilde{A} = (c, \{m^{0.1}, n^{0.6}\})$

$\tilde{M}_A= (c, \{m^1, n^{0.6}\}) \tilde{\cap} (c, \{m^{0.1}, n^{0.6}\})$

$= (c, \{m^{0.1}, n^{0.6}\}) = \tilde{A}$

$\tilde{N}_A= (c,\{m^0, n^{0.4}\}) \tilde{\cap} (c, \{m^{0.1}, n^{0.6}\})$

$= (c,\{m^0, n^{0.4}\}) = \tilde{N}$.

Then, $\mathfrak{F}_A=\{\tilde{\Phi}_A, \tilde{\Omega}_A, \tilde{M}_A, \tilde{N}_A\}$

$= \{\tilde{\Phi}_A, \tilde{A}, \tilde{N}_A\}$

is relative $\mathfrak{F}$ - topology on $\tilde{A}$.

Proposition 2.14.

Let $(\Omega, \wp)$ be a $\mathfrak{F}$ - topological space and $\tilde{A} \tilde{\subseteq} (\Omega, \mathfrak{F}, \wp)$. If $\tilde{G}$ α$\mathfrak{F}$ - dense in $(\Omega, \mathfrak{F}, \wp)$, then $\tilde{G}$ is α$\mathfrak{F}$ - dense in $\tilde{A}$.

Proof.

Let $\tilde{G}$ be α$\mathfrak{F}$ - dense in $(\Omega, \wp)$. To prove $\tilde{G}$ is α$\mathfrak{F}$ - dense in $\tilde{A}$,

i.e., to prove $\tilde{G} \tilde{\cap} \tilde{M} \neq \tilde{\Phi}$, $\forall \tilde{M} \tilde{\subseteq} \tilde{A}$.

Let $\tilde{M} \tilde{\subseteq} \tilde{A}$, $\tilde{M}$ is α$\mathfrak{F}$o - set in $\mathfrak{F}$, $\tilde{M} \neq \tilde{\Phi}$, then, $\tilde{M}^* = \tilde{A} \tilde{\cap} \tilde{M} = \tilde{M}$

since given that $\tilde{G}$ is α$\mathfrak{F}$ - dense in $(\Omega, \mathfrak{F}, \wp)$.

Thus, $\tilde{G} \tilde{\cap} \tilde{M} \neq \tilde{\Phi}$; $\forall \tilde{M} \neq \tilde{\Phi}$, for each $\tilde{M}$ in $\mathfrak{F}$, $\tilde{M}^*= \tilde{M}$.

Implying, $\tilde{G} \tilde{\cap} \tilde{M} \neq \tilde{\Phi}$; $\forall \tilde{M} \neq \tilde{\Phi}$, for each $\tilde{M}$ in $\mathfrak{F}_A$. So $\tilde{G}$ is α$\mathfrak{F}$ - dense in $\tilde{A}$.

Theorem 2.15.

If $(\Omega, , \wp)$ is α$\mathfrak{F}$ - irreducible space, then any non-null α$\mathfrak{F}$o - set is also α$\mathfrak{F}$ - irreducible.

Proof.

Let $(\Omega, \wp)$ be α$\mathfrak{F}$ - irreducible space, $\tilde{Y}$ be non-null α$\mathfrak{F}$o-set and $\tilde{G}$ be a non-null α$\mathfrak{F}$o - set in $(Y, \mathfrak{F}_y, \wp)$. Since $\tilde{Y}$ is α$\mathfrak{F}$o - in $(\Omega, , \wp)$ so $\tilde{G}$ is α$\mathfrak{F}$o - in $(\Omega, \mathfrak{F}, \wp)$ by theorem 2.8. and theorem 2.10., then $\tilde{G}$ is α$\mathfrak{F}$ - dense in $\tilde{Y}$ and $\tilde{Y}$ is α$\mathfrak{F}$ - irreducible set.

Definition 2.16.

For α$\mathfrak{F}$ - topological spaces $(\Omega, , \wp)$ and
$(U, \partial, \wp)$, we say a map
$J: (\Omega, \mathfrak{F}, \wp) \rightarrow (U, \partial, \wp)$ is α$\mathfrak{F}$ - continuous if the inverse image of any α$\mathfrak{F}$o - set in ∂ is
α$\mathfrak{F}$o - set in.

The next theorem shows that α$\mathfrak{F}$ - continuous image of α$\mathfrak{F}$-irreducible set in is α$\mathfrak{F}$ - irreducible set.

Theorem 2.17.

For $J: (\Omega, \mathfrak{F}, \wp) \rightarrow (U, \partial, \wp)$ be α$\mathfrak{F}$ - map from a $\mathfrak{F}$ - topological space $(\Omega, \mathfrak{F}, \wp)$ into $\mathfrak{F}$ - topological space $(U, \partial, \wp)$. Then, the α$\mathfrak{F}$ - continuous image of α$\mathfrak{F}$ - irreducible set in
$(\Omega, , \wp)$ is α$\mathfrak{F}$ - irreducible set in $(U, \partial, \wp)$.

Proof.

Let $\tilde{A}$ be α$\mathfrak{F}$ - irreducible set in $(\Omega, \mathfrak{F}, \wp)$, $\tilde{G}, \tilde{H}$ are two non -null α$\mathfrak{F}$o - sets in $(U, \partial, \wp)$ such that $\tilde{G} \tilde{\cap} J(\tilde{A}) \neq \tilde{\Phi}$, $\tilde{H} \tilde{\cap} J(\tilde{A}) \neq \tilde{\Phi}$, $J^{-1}(\tilde{G})$, $J^{-1}(\tilde{H})$ are two non-null α$\mathfrak{F}$o - sets in $(\Omega, \mathfrak{F}, \wp)$, since $\tilde{A}$ is α$\mathfrak{F}$ - irreducible set.

Then, $J^{-1}(\tilde{G}) \tilde{\cap} \tilde{A} \neq \tilde{\Phi}$, $J^{-1}(\tilde{H}) \tilde{\cap} \tilde{A} \neq \tilde{\Phi}$,

$(J^{-1}(\tilde{G}) \tilde{\cap} \tilde{A}) \tilde{\cap} (J^{-1}(\tilde{H}) \tilde{\cap} \tilde{A}) \neq \tilde{\Phi}$

$(J^{-1}(\tilde{G}) \tilde{\cap} J^{-1}(\tilde{H})) \tilde{\cap} \tilde{A} \neq \tilde{\Phi}$

$J^{-1}(\tilde{G} \tilde{\cap} \tilde{H}) \tilde{\cap} \tilde{A} \neq \tilde{\Phi}$ so

$(\tilde{G} \tilde{\cap} \tilde{H}) \tilde{\cap} J(\tilde{A}) \neq \tilde{\Phi}$

$(\tilde{G} \tilde{\cap} J(\tilde{A})) \tilde{\cap} (\tilde{H} \tilde{\cap} J(\tilde{A})) \neq \tilde{\Phi}$

where $(\tilde{G} \tilde{\cap} J(\tilde{A}))$, $(\tilde{H} \tilde{\cap} J(\tilde{A}))$ are non -null α$\mathfrak{F}$o - sets

so $J(\tilde{A})$ is α$\mathfrak{F}$ - irreducible set in $(U, \partial, \wp)$.

Definition 2.18.

For two α$\mathfrak{F}$ - topological spaces $(\Omega, \wp)$and $(U, \partial, \wp)$ a map $J : (\Omega, \mathfrak{F}, \wp) \rightarrow (U, \partial, \wp)$ is α$\mathfrak{F}$c - map if the image of any α$\mathfrak{F}$c - set in
$(\Omega, \wp)$ is α$\mathfrak{F}$c - set in $(U, \partial, \wp)$.

Theorem 2.19.

Let $J: (\Omega, \mathfrak{F}, \wp) \rightarrow (U, \partial, \wp)$ be an α$\mathfrak{F}$c - bijective map from a $\mathfrak{F}$ - topological space $(\Omega, \mathfrak{F}, \wp)$ into a $\mathfrak{F}$ - topological space $(U, \partial, \wp)$, if $(\Omega, \mathfrak{F}, \wp)$ is α$\mathfrak{F}$ - irreducible space, then
$(U, \partial, \wp)$ is α$\mathfrak{F}$ - irreducible space.

Proof.

Let $\tilde{B}$ be α$\mathfrak{F}$ - set in $(U, \partial, \wp)$ to prove
$(U, \partial, \wp)$ is α$\mathfrak{F}$ - irreducible
since J is bijective. Then, there exist $\tilde{A}$ in

(Ω, ℘) such that $J(\tilde{A}) = \tilde{B}$
since (Ω,𝔉, ℘) is α𝔉 - irreducible space ,
then α𝔉- cl($\tilde{A}$) = $\tilde{\Omega}$,
since J is bijective α𝔉c - map,
then J (α𝔉- cl ($\tilde{A}$)) = J ($\tilde{\Omega}$) = $\tilde{\mho}$,
J (α𝔉- cl ($\tilde{A}$))= α𝔉- cl (J ($\tilde{A}$))
α𝔉- cl (J ($\tilde{A}$))= $\tilde{\mho}$ ⟹ α𝔉- cl ($\tilde{B}$)= $\tilde{\mho}$
Thus $\tilde{B}$ α𝔉- dense set in (℧,∂, ℘)
By theorem 2.8. (℧,∂, ℘) is α𝔉 - irreducible space .

Theorem 2.20.
Let J: (Ω,𝔉, ℘) → (℧ ,∂, ℘) be an α𝔉- continuous bijective map from a 𝔉 - topological space (Ω ,, ℘) into a 𝔉 - topological space (℧,∂, ℘), if (Ω ,𝔉, ℘) is α𝔉 - reducible space ,then (℧ ,∂ , ℘) is α𝔉 - reducible space.

Proof.
Let $\tilde{B}$ be α𝔉 - set in (℧,∂, ℘) to prove that (℧,∂, ℘) is α𝔉 - reducible .
Since J is bijective, then there exist $\tilde{A}$ in (Ω ,, ℘) such that $J(\tilde{A}) = \tilde{B}$
since (Ω,𝔉, ℘) is α𝔉 - reducible space which provides (Ω ,𝔉, ℘) is an α𝔉 - disconnected space. Again, J is bijective α𝔉 - continuous map and so (℧,∂, ℘) is an α𝔉 - disconnected space, and therefore (℧ ,∂ , ℘) is α𝔉 - reducible space .

Definition 2.21.
The map f from α𝔉 - space (Ω ,ℑ, ℘) to α𝔉 - space (℧ ,∂ , ℘) satisfy :
(1) f is α𝔉 - continuous.
(2) f is α𝔉 - bijective.
(3) f^{-1} is α𝔉 – continuous (or f is α𝔉 - open).
Is called α𝔉 - homeomorphism.
The next corollary show that the α𝔉 - irreducible space is α𝔉 - topological property.

Corollary 2.22.
Let J be α𝔉-homeomorphism from α𝔉 - topological space (Ω,𝔉, ℘) onto α𝔉 - topological space (℧ ,∂ , ℘) .If (Ω ,𝔉, ℘) is α𝔉 - irreducible space ,then (℧ ,∂ , ℘) is α𝔉 - irreducible space .

Proof.
Directly from Definition 2.19. and Theorem 2.20.
The next corollary show that the α𝔉-reducible space is α𝔉 - topological property.

Corollary 2.23.
Let J be α𝔉 - homeomorphism from α𝔉 - topological space (Ω, ℘) onto α𝔉 - topological space (℧,∂, ℘) .Then, if (Ω, ℘) is α𝔉 - reducible space, then (℧,∂, ℘) is α𝔉 - reducible space

Proof. Directly from definition 2.21 and theorem 2.20.

Examples 2.24.
For Ω={m, n, L}, ℘={e} be the set of parameters. Let
$\tilde{O}$=(e,{ $m^{0.1}$, n^0 , $L^{0.3}$ }) ,
$\tilde{B}$=(e,{ m^0, n^1 , L^0 }) ,
$\tilde{\Phi}$ = (e,{ m^0, n^0 , L^0 }) ,
$\tilde{\Omega}$ = (e,{ m^1, n^1 , L^1 }) ,
with 𝔉 - topology 𝔉 = {$\tilde{\Phi}$, $\tilde{\Omega}$, $\tilde{O}$, $\tilde{B}$}.
The elements of are α𝔉o-sets.
Since $\tilde{O} \tilde{\cap} \tilde{B} = \tilde{\Phi}$ so $\tilde{O}$, $\tilde{B}$ are non-null α𝔉o-separated sets, then the space (Ω,𝔉, ℘) is α𝔉 - disconnected space and so (Ω , , ℘) is α𝔉-reducible space.
Let J: (Ω,𝔉, ℘)→(℧,∂, ℘) be α𝔉-homeomorphism ,
J (x_{ij}) = J (ei, {h_j^{kij}}) = (ei, {h_j^{1-kij}})},
∀ kij ∈ [0,1],
where $J(\tilde{O}) = J$((e,{$m^{0.1}$, n^0 , $L^{0.3}$ }))
= (e,{ $m^{0.9}$, n^1 , $L^{0.7}$ }),
$J(\tilde{B}) = J$((e,{ m^0, n^1 , L^0 }))
= (e,{ m^1, n^0 , L^1 }) and
$J(\tilde{\Phi}) = \tilde{\Omega}$, $J(\tilde{\Omega}) = \tilde{\Phi}$.
Since J is α𝔉 -open map, then the images are α𝔉o - sets in (℧,∂, ℘) .
Since $J(\tilde{O}) \tilde{\cap} J(\tilde{B}) = \tilde{\Phi}$, so $\tilde{O}$, $\tilde{B}$ are non-null α𝔉o - separated sets, therefore (℧,∂, ℘) is α𝔉-disconnected space and so is an α𝔉 - reducible space.

CONCLUSIONS

We define 𝔉 -irreducible sets, α𝔉- irreducible sets in 𝔉 - topological spaces, and provide equivalent definitions with examples. We also define α𝔉-continuous and prove that the α𝔉-continuous image of α𝔉-irreducible set is α𝔉- irreducible set. Define α𝔉-close map, α𝔉-homeomorphism and prove that α𝔉-irreducible (α𝔉-reducible) space is α𝔉-topological property.

Disclosure and conflict of interest: The authors declare that they have no conflicts of interest.

REFERENCES

[1] Abd Alrahman, Abd Allah S, Aslam M., Muhammad S.K.,"A study on fuzzy soft set and its operations", Annals of Fuzzy Mathematics and Informatics Volume 6, No. 2, (2013), pp.339-362.

[2] Aras G. C., Bayramov S., "Some results on fuzzy soft topological spaces", Hindawi Publishing Corporation Mathematical Problems in Engineering, Vol. (2013), pp.1-10. https://doi.org/10.1155/2013/835308

[3] Aygunoglu A., Vildan Cetkin , Halis Aygun , "An introduction to fuzzy soft topological spaces", Hacettepe Journal of Mathematics and Statistics, Vol.43 ,No. 2, (2014), pp.193 - 204 .

[4] Cagman N., Enginoglu S., Citak F.," Fuzzy soft set theory and its applications", IJFS, Vol.8, No.3, (2011), pp.137-147.

[5] Hussain S.," On Weak and Strong Forms of Fuzzy Soft Open Sets", Fuzzy Information and Engineering, Vol. 8, No. 4, (2016), pp.451-463. https://doi.org/10.1016/j.fiae.2017.01.005

[6] Mahanta J., Das P.K.,"Results on fuzzy soft topological spaces", Arunachal Pradesh, INDIA, arXiv:1203.0634v1, (2012), pp.791-109.

[7] Maji P.K., Roy A.R., Biswas R., "Fuzzy soft sets", J.Fuzzy Math., Vol.9, No.3, (2001), pp.589- 602.

[8] Molodtsov D.,"Soft set theory - first results", Computers and Mathematics with Applications, Vol. 37, (1999), pp.19-31. https://doi.org/10.1016/S0898-1221(99)00056-5

[9] Naz M.,Shabir M. ,"Fuzzy soft sets and their algebraic structures",World Applied Sciences Journal, Vol.22,(2013), pp.45-61.

[10] Shabir M., Naz M.," On soft topological spaces", Comput. Math. Appl. Vol. 61, (2011), pp.1786-1799. https://doi.org/10.1016/j.camwa.2011.02.006

[11] Tanya B., Kandemir M. B., " Topological structure of fuzzy soft sets", Computer and Mathematics with Applications Vol. 61, (2011), pp.2952 - 2957. https://doi.org/10.1016/j.camwa.2011.03.056

[12] Zadeh L. A., "Fuzzy sets", Inform. and Control, Vol. 8, (1965), pp.338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

How to Cite

M. H. Mahmood, " On α-Fuzzy Soft Irreducible Spaces", *Al-Mustansiriyah Journal of Science*, vol. 34, no. 1, pp. 65–70, Mar. 2023.