

A 4850 MHz survey of a part of the “GB region” and the combined counts of sources

J. Maslowski^{1,2}, I. I. K. Pauliny-Toth², A. Witzel², and H. Kühr^{2,3}

¹ Astronomical Observatory, Jagellonian University, “Fort Skala”, ul. Orła 171, PL-30-244 Cracow, Poland

² Max-Planck-Institut für Radioastronomie, Auf dem Hügel 69, D-5300 Bonn 1, Federal Republic of Germany

³ Steward Observatory, University of Arizona, Tucson, AZ 85721 USA

Received February 27, accepted May 21, 1984

Summary. The results of a survey of a part of the “GB region”, made at a frequency of 4850 MHz are presented. This region had been surveyed earlier at a frequency of 1400 MHz. The present survey includes 99 sources above a limiting flux density of 0.030 Jy in a solid angle of $9.18 \cdot 10^{-3}$ sterad. Two-point spectral indices for sources selected at 1400 and 4850 MHz are derived and discussed. The combined source counts of the present and other MPIfR-NRAO surveys near 5 GHz are presented, in tabular and graphical form. Power-law representations of the combined source counts are derived.

Key words: radio survey – source counts – spectra of sources – GB area

1. Introduction

This paper is the third in a series dealing with sensitive surveys of extragalactic sources at a frequency of 4850 MHz. In our previous two papers (Maslowski, Pauliny-Toth, Witzel and Kühr 1980, 1984; hereinafter referred to as MPWK₁ and MPWK₂, respectively) we have presented and discussed the results of two such surveys. These surveys overlap the 5C6 area (Pearson and Kus, 1978) and the 5C2 area (Pooley and Kenderdine, 1968) and are complete to flux density limits of 0.02 and 0.017 Jy respectively. They permitted a better determination of the number-flux density relation for weak sources at a high frequency and provided high-frequency spectral information for the sources detected.

In this paper, we report the results of a survey at 4850 MHz of a part of the GB area, to a limiting flux density of 0.030 Jy. The GB area, originally surveyed by Maslowski (1971, 1972) at 1400 MHz with the 91 m telescope of the National Radio Astronomy Observatory (NRAO), covered the range R.A. $7^{\text{h}}17^{\text{m}}$ to $16^{\text{h}}23^{\text{m}}$, Dec. $45^{\circ}8'$ to $51^{\circ}7'$ (1970.23). The aim of the present survey was to provide a complete sample of sources stronger than 0.030 Jy from this region, together with spectral information for this sample.

The 4850 MHz observations and their reduction are briefly described in Sect. 2. Section 3 presents the results of the survey and gives the error analysis. The two-point spectral index distributions for sources selected at either 1400 MHz or 4850 MHz are discussed in Sect. 4. Finally, in Sect. 5, we present the

combined source counts from a number of surveys made at frequencies near 4850 MHz.

2. Observations and data reduction

The survey was performed between 15 January and 4 February 1977 with the 100 m telescope of the Max-Planck-Institut für Radioastronomie (MPIfR), using the same equipment, observing techniques and procedures of data reduction as those described in detail by MPWK₁. It was made by means of scans in right ascension, each 805 arcmin long, covering a declination range from $47^{\circ}21'15''$ to $50^{\circ}59'25''$ (1950.0) at intervals of 85 arcsec, i.e. about half the half-power beamwidth of 2.8 arcmin. The scans began at right ascension $10^{\text{h}}45^{\text{m}}$ (1950.0). The whole area of $14.55 \cdot 10^{-3}$ sterad was surveyed twice in this way and gave two independent maps *A* and *B* of the area.

The data were calibrated by means of the same sources as in MPWK₁, namely 3C48, 3C147 and 3C295, for which the flux densities were taken to be 5.50, 8.37 and 6.72 Jy, respectively¹. The flux density scale for the present survey is thus a factor of 1.013 higher than that of Kellerman et al. (1969; “KPW” scale) and a factor of 1.020 higher than that of Baars et al. (1977; “BGPW” scale).

Proceeding further exactly as in MPWK₁, we obtained, *a*) by forming $(A+B)/2$, the final “source map” which contains the discrete sources, confusion and noise, and *b*) by forming $(A-B)/2$, the “source-free map” which contains *only* noise and the effects of possible receiver instabilities, atmospheric fluctuations, pointing errors and any imperfections in the analysis. The latter, characterised by a r.m.s. noise of 0.0056 Jy per point, was used for the Monte Carlo error analysis referred to in Sect. 3. As in MPWK₁ an automatic source-extracting programme was applied to the “source map” to determine the positions and flux densities of sources stronger than 0.018 Jy (about 3.2 r.m.s.).

3. Survey results and errors

The final “source map” of the whole area of the present survey is shown in Fig. 1. The survey, however, partly overlaps the 5C2

¹ Although there is evidence that 3C147 is variable (Kesteven et al. 1977), the scaling factors derived from the three sources agree with a standard deviation of 2.5%. The exclusion of 3C147 would change the flux density scale by 0.3%

Send offprint requests to: J. Maslowski

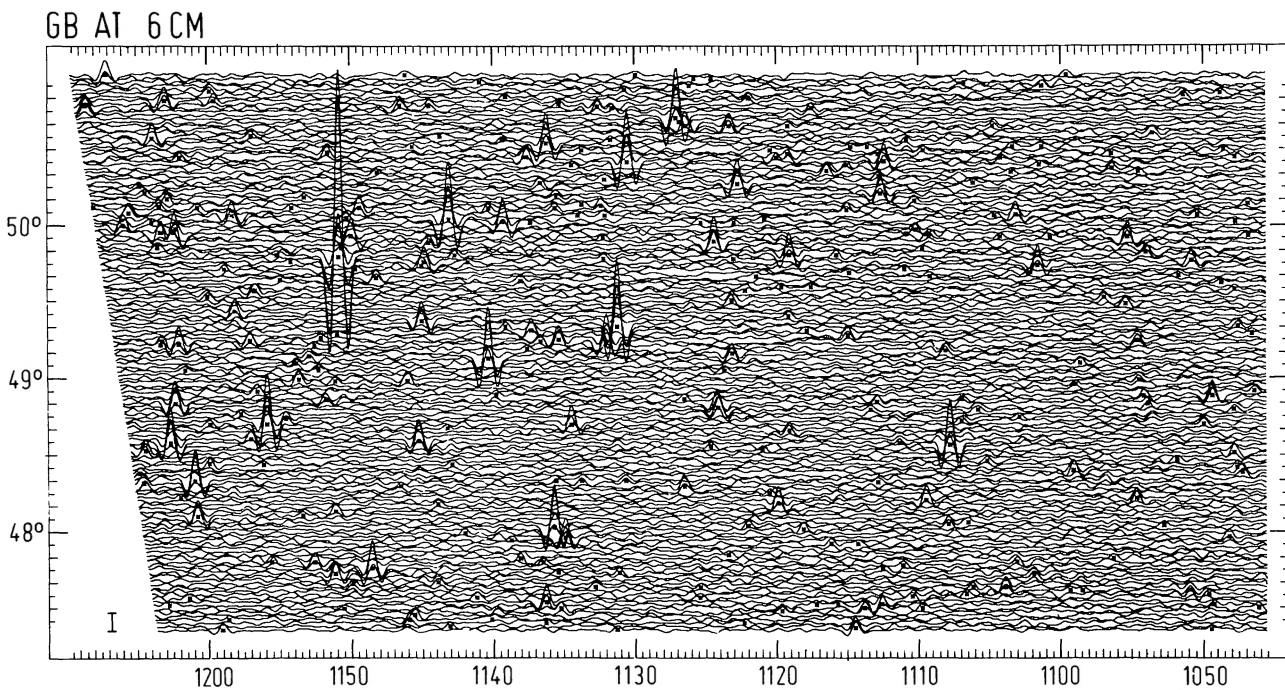


Fig. 1. Map of the area surveyed at 4850 MHz. The coordinates are right ascension (labelled hhmm) and declination, epoch 1950.0. The bar in the lower left corner represents a deflection of 0.1 Jy

area, which has already been surveyed by MPWK₂ to the lower limiting flux density of 17 mJy. The common area, of $5.37 \cdot 10^{-3}$ sterad (right ascension range $10^{\text{h}}45^{\text{m}}$ to $11^{\text{h}}15^{\text{m}}$ at declination $47^{\circ}29'45''$ and to $11^{\text{h}}18^{\text{m}}$ at declination $50^{\circ}59'25''$ (1950.0)) is not further considered here. The sources in this common area were only used to check the mutual consistency of the two surveys.

The Monte Carlo analysis, using artificial sources added to the “source-free” map in the manner described by MPWK₁, showed the present survey is 99% complete at the flux density level of $S(4850) = 0.030$ Jy. The comparison of sources in the area in common with the MPWK₂ survey was consistent with this value.

Table 1 lists the 99 sources having $S(4850) \geq 0.030$ Jy which lie in the remaining area of $9.18 \cdot 10^{-3}$ sterad of the present survey and is arranged as follows. Columns 1 to 3 give the source name, and the 1950.0 right ascension and declination. Columns 4 to 6 list the flux densities, in Jy, of the sources at 4850, 2695 and 1400 MHz. The 1400 MHz flux densities are taken from the GB catalogue (Maslowski, 1972), the 2695 MHz data are from measurements of GB sources stronger than 0.2 Jy (Maslowski, in preparation). In six cases, two neighbouring 4850 MHz sources were not resolved at 1400 and/or 2695 MHz; in these cases the sum of the 4850 MHz flux densities is quoted additionally in Col. 4, is indicated by an asterisk in Col. 1 and has been used for deriving spectral information. Column 7 gives the two-point spectral indices, α , based on the 4850 and 1400 MHz data and defined by $S \propto \text{frequency}^{\alpha}$.

The symbols *A*, *B* or *AB* given in Col. 8 identify the source as belonging to a sample selected at 4850 MHz, 1400 MHz or both frequencies and which is used for the spectral analysis (Sect. 4). Finally, Col. 9 lists other names of the 4850 MHz source, i.e. the GB designation (Maslowski, 1972), 3C (Bennett, 1962), BP (Bailey and Pooley, 1968), 4C/4CP (Gower et al. 1967; Caswell and Crowther, 1969). Peak flux densities are given in parentheses

for significantly extended sources at 4850, 2695 and 1400 MHz, respectively. Where available, flux densities at 408 MHz from the *BP* catalogue and at 14850 MHz from our own unpublished measurements are also given.

Table 2 presents the results of the error analysis, obtained from Monte Carlo experiments in the manner described by MPWK₁. The table is self-explanatory and we here emphasise only that

(i) the systematic flux density overestimation, ΔS , given in row 1 was taken into account in deriving the two-point spectral indices quoted in Table 1; the quoted 4850 MHz flux densities are, however, not corrected for this effect. The spectral indices given in Table 1 thus differ by $\Delta\alpha = -0.08, -0.06$ and -0.01 from those based on observed 4850 MHz flux densities of 0.03, 0.05 and 0.14 Jy, respectively.

(ii) the correction factors given in row 4 must be applied to the observed source counts if these are based on the 4850 MHz flux densities quoted in Col. 4 of Table 1.

4. Spectral properties of the sources

Two complete samples of sources, selected at 4850 MHz and 1400 MHz, and having two-point spectral indices $\alpha(\nu_1, \nu_2)^2$, can be obtained from the data given in Table 1.

² The notation $\alpha(\nu_1, \nu_2)$ is used here for the spectral index between frequencies ν_1 and ν_2 , for sources selected at frequency ν_1 .

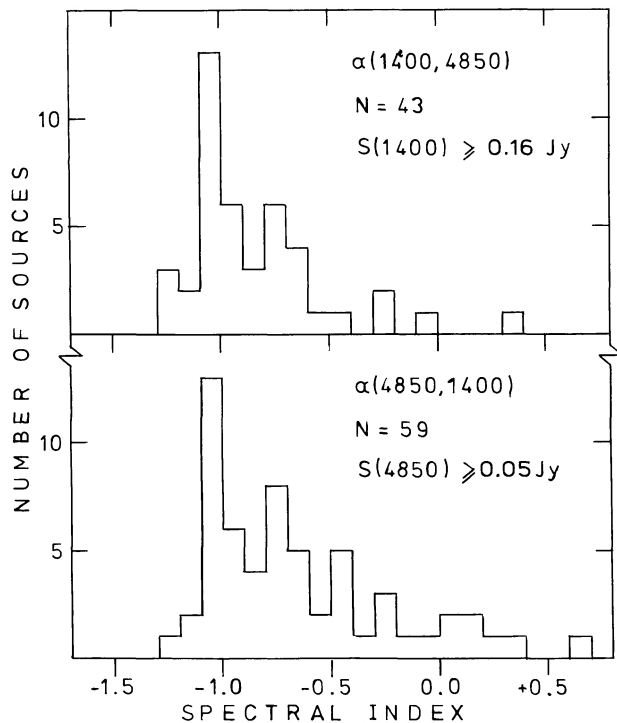
The spectral indices in Table 1 are based on the quoted flux densities. Because the flux density scales at 4850 MHz and 1400 MHz are 2.0% higher and 3.6% lower, respectively, than the BGPW scale, all spectra would be steeper, on the latter scale, by 0.04 in α_{4850}^{1400} .

Table 2. Errors, corrections and differential counts of sources from the present survey

Flux density range, S	≥ 600	<600 ≥ 140	<140 ≥ 50	<50 ≥ 30	mJy
Flux density over-estimation of point source, $\Delta S = S_{\text{Obs}} - S_{\text{Real}}$	0.0	0.9	3.6	2.8	mJy
Flux density error of point source, $\sigma_{\text{NCS}}^{\prime 1)}$		± 7.5	± 5.6	± 4.9	mJy
Position error, $\sigma_{\text{NCP}}^{\prime 2)}$		± 9.0	$\pm 15.$	$\pm 20.$	arcsec
Correction factor	1.000	1.018	0.957	0.871	
Observed source counts, n_{Obs}	1	12	46	40	
Corrected source counts, n_{Cor}	1.0	12.21	44.02	34.84	
Normalized source counts, $n_{\text{Cor}}/n_0^{\prime 3)}$	-	1.309 $\pm .375$	1.136 $\pm .171$	0.614 $\pm .104$	

- 1) Systematic calibration error not included (see text).
 2) Systematic pointing error of the telescope not included.
 3) The counts are normalized by: $n_0 = 90 \cdot S^{-2.5}$.
 All symbols are as in MPWK₁.

The sample selected at 1400 MHz (denoted by "B" in Col. 8 of Table 1) consists of 43 sources having $S(1400) \geq 0.16$ Jy (the completeness limit of the GB survey), all of which are detected by the present survey. The distribution of $\alpha(1400, 4850)$ is shown as a histogram in Fig. 2 (top) and is characterised by a median index $\bar{\alpha}(1400, 4850) = -0.96 \pm .033$, with a dispersion, $\sigma = 0.21$. In comparison, for sources from the 5C2 area, MPWK₂ found $\bar{\alpha}(1415, 4850) = -0.86 \pm .026$ and $\sigma = 0.21$. This suggests that the sources in the present survey have, on the average, a steeper spectrum in this frequency interval than those in the 5C2 area,

**Fig. 2.** The distribution of two-point spectral indices for two complete samples of sources selected at 1400 MHz (top) and at 4850 MHz (bottom)

by an amount, $\Delta\alpha = 0.10 \pm .043$. This effect, if real, is not due to systematic differences in the flux densities of the GB survey and the Westerbork 1415 MHz flux densities of sources in the 5C2 area since these have been found to be in good agreement (Gillespie, 1979; Katgert, 1975, 1977). It may reflect a dependence of the spectral index on the 1400 MHz flux density: for the 21 sources having $S(1400) < 0.27$ Jy, $\bar{\alpha} = -0.78 \pm .049$ whereas for the 22 sources with $S(1400) \geq 0.27$ Jy $\bar{\alpha} = -1.02 \pm .033$. In the sample of 68 5C2 sources selected at 1415 MHz, however, only 3 sources have $S(1415) \geq 0.27$ Jy. A similar flattening of the spectra with decreasing flux density has been found for even weaker sources in the 5C2 region (MPWK₂; Katgert, 1975 and 1977).

Although the present survey is complete to a flux density limit of 0.03 Jy, the 4850 MHz sample denoted by "A" in Col. 8 of Table 1 is confined to the 59 sources having $S(4850) \geq 0.05$ Jy for the purpose of spectral analysis. Even at this higher flux density limit, 11 sources in the sample have no counterparts in the GB catalogue and only lower limits to the spectral indices are given in Table 1. The distribution of the spectral indices $\alpha(4850, 1400)$ is shown in Fig. 2 (bottom) and is characterised by a median index $\bar{\alpha}(4850, 1400) = -0.77 \pm .052$, with a dispersion $\sigma = 0.40$. The percentage, F , of sources having $\alpha \geq -0.5$ is (31 ± 6) percent. These characteristics are in good agreement with those found by MPWK₂ for sources lying in the 5C2 area, for which $\bar{\alpha} = -0.74 \pm .044$, $\sigma = 0.36$ and $F = (32 \pm 5)\%$.

5. Combined source counts

Figure 3 shows (as a heavy line) the normalised, differential source counts near 5 GHz, obtained from a combination of the present survey and the following MPIfR-NRAO surveys: "S1" (Kellermann et al., 1968), "S2" (Pauliny-Toth et al., 1972), "S3" (Pauliny-Toth and Kellermann, 1972), "S4" (Pauliny-Toth et al., 1978a), "S5" (Kühr et al., 1981), "I" (Pauliny-Toth and Kellermann, 1972), "D" (Davis, 1971) and several deeper surveys (Pauliny-Toth et al., 1978b; Pauliny-Toth et al., 1980; Maslowski et al., 1981 and 1984). In addition, the 5 GHz source counts compiled from the Parkes surveys by Pauliny-Toth et al. (1978a) have been included. For all these surveys, sufficient information about systematic errors was available to enable us to apply corrections for incompleteness. In combining the data, sources in overlapping areas were excluded from the appropriate survey to ensure the independence of the final source counts. The normalisation has been made to $\Delta N_0 = 90 S^{-5/2}$. Figure 3 also shows the separate source counts from four surveys where the information was insufficient for us to make the proper corrections (Willis and Miley, 1979; Ledden et al., 1980; Wall et al., 1982a; Owen et al., 1983). For comparison, the separate source counts from the present survey are also shown.

The combined source counts in differential form are also tabulated in Table 3. They cover about 3 decades in flux density, divided into 24 intervals. The division was to some extent forced on us by the different flux density limits of the surveys and by the numbers of sources in the different intervals. Table 3 is arranged as follows:

- Col. 1 – the flux density range, $S_i < S_{i+1}$
- Col. 2 – Δm , the number of sources in $S_i \leq S < S_{i+1}$
- Col. 3 – $\Delta m'$, corrected number of sources
- Col. 4 – solid angle of the area containing these sources
- Col. 5 – $\Delta N'$, corrected source density (sterad^{-1})

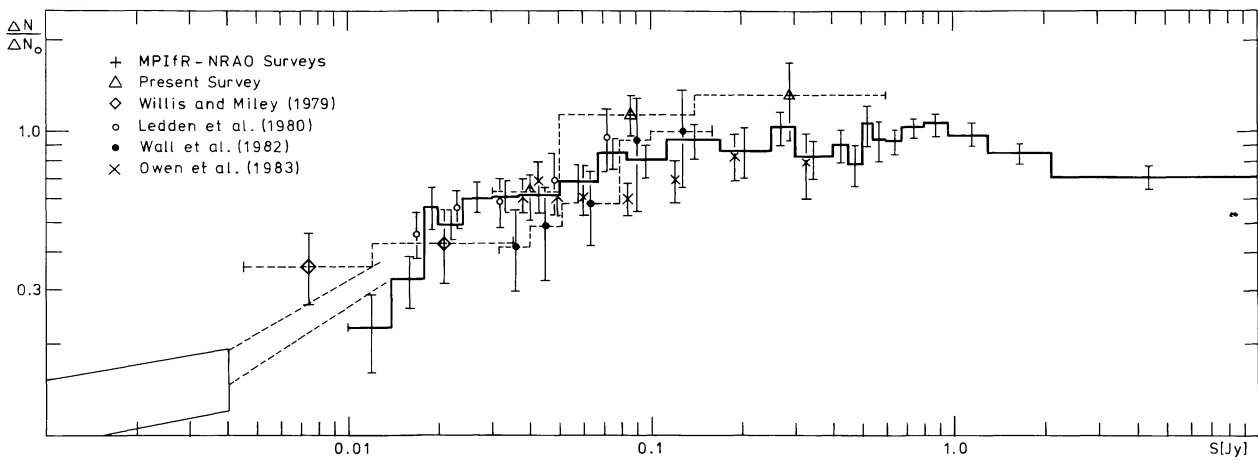


Fig. 3. Differential source counts near 5 GHz. The counts obtained by combining the present survey with other MPIfR-NRAO surveys are shown by the heavy line. The length of the error bars represents the $\pm 1\sigma$ statistical error $2(\Delta m)^{-1/2} \Delta N'/\Delta N_0$, where m is the number of sources in a given flux density interval of Table 3. The counts from four other published surveys are shown separately, and, for comparison, the counts from the present survey are also shown. All the counts are normalised to a differential count $\Delta N_0 = 90 S^{-5/2}$. The results of the $P(D)$ analysis of Wall et al. (1982b) are represented by the $\pm 1\sigma$ probability area outlined by a solid line

Col. 6 – $\Delta N'/\Delta N_0$, source count normalised by $\Delta N_0 = 90 S^{-5/2}$ and its statistical error $(\Delta m)^{-1/2} \Delta N'/\Delta N_0$.

Since the differential counts presented in Table 3 are well defined at least down to a flux density of 0.018 Jy, they can be

Table 3. Combined, differential counts of sources at 5 GHz

Flux density (Jy)	Δm	$\Delta m'$	Area (sterad)	$\Delta N'$	$\Delta N'/\Delta N_0$
0.010					
0.014	8	8.5	0.001582	5372.	0.226 ± 0.077
0.018	26	27.94	0.007555	3698.	0.325 ± 0.062
0.020	37	37.75	0.01834	2059.	0.567 ± 0.092
0.024	63	60.70	0.02482	2446.	0.482 ± 0.062
0.030	72	69.42	0.02482	2798.	0.610 ± 0.073
0.037	69	65.04	0.03400	1913.	0.614 ± 0.076
0.050	67	63.67	0.03400	1894.	0.618 ± 0.077
0.067	46	43.92	0.03400	1292.	0.678 ± 0.102
0.083	73	68.73	0.08606	799.	0.840 ± 0.101
0.113	66	63.36	0.08606	736.	0.791 ± 0.099
0.170	59	58.35	0.08606	678.	0.937 ± 0.123
0.250	28	28.00	0.08606	325.	0.865 ± 0.164
0.300	62	64.88	0.5476	118.	1.032 ± 0.128
0.400	56	58.02	0.5476	106.	0.828 ± 0.109
0.450	55	60.0	1.718	34.9	0.909 ± 0.117
0.500	37	39.0	1.718	22.7	0.781 ± 0.125
0.540	39	44.0	2.238	19.7	1.063 ± 0.160
0.600	44	46.0	2.238	20.6	0.930 ± 0.137
0.680	134	139.	6.807	20.4	0.924 ± 0.078
0.800	162	162.	6.807	23.8	1.028 ± 0.081
0.960	150	150.	7.081	21.2	1.055 ± 0.086
1.300	205	209.	9.301	22.5	0.964 ± 0.067
2.100	161	163.	9.301	17.5	0.844 ± 0.066
	130	130.	9.301	14.0	0.709 ± 0.062
	1849	1860.3		24451.3	

expressed in the form of power laws. For this purpose, we have applied the method of Refsdal (1969), which is statistically correct provided the number of sources in a given interval is not small (Crawford et al., 1970). This is true even though the method starts from the corrected integral counts ($N(>S) = KS^{-\beta}$), because it effectively works on uncorrelated data. The method, unlike other, statistically more sophisticated methods (Jauncey, 1967; Crawford et al., 1970), has the advantage of being easily adapted to the present case, where the sources observed in different flux density intervals come from different solid angles of sky.

Applying this method to the data in Table 3, we have estimated the exponent, γ , of the corresponding differential counts ($dN(S)/dS = -kS^{-\gamma}$, where $\gamma = \beta + 1$) for four flux density intervals in each of which the “goodness of fit” test showed that k and γ were constant. The resulting values are:

- 1) $S \geq 0.800$ Jy, $k = 90.1 \pm 2.4$, $\gamma = 2.75 \pm .069$
- 2) $0.800 > S \geq 0.067$ Jy, $k = 75.8 \pm 2.7$, $\gamma = 2.51 \pm .052$
- 3) $0.067 > S \geq 0.018$ Jy, $k = 131.3 \pm 4.3$, $\gamma = 2.24 \pm .067$
- 4) $0.018 > S \geq 0.010$ Jy, $k = 460 \pm 150$, $\gamma = 1.8 \pm .14$.

Below the level of the direct source counts (0.010 Jy), the population law has recently been estimated by Wall et al. (1982b) on the basis of the $P(D)$ analysis (Scheuer, 1957; Condon, 1974). The result of this analysis is shown in Fig. 3 by the $\pm 1\sigma$ probability area outlined by the solid line, the best-fit value being $(k, \gamma) = (127, 2.10)$. This result indicates less convergence in the counts below 0.004 Jy than suggested by the direct source counts in the range 0.010 to 0.018 Jy. Although the latter are based on only 34 sources, the further evidence of rapid convergence of the counts below 0.01 Jy came very recently from the VLA (Very Large Array) synthesis surveys of several small regions of the sky. Namely, Bennett et al. (1983) surveyed 13 small areas of the sky (each 12.8 arcmin square) and found 13 sources covering the flux density range $0.5 \leq S < 10$ mJy. Next, Fomalont et al. (1983) found 8 sources from their Deep Survey with $0.05 \leq S < 2$ mJy and 24 sources from the Intermediate Survey with $0.35 \leq S < 12$ mJy.

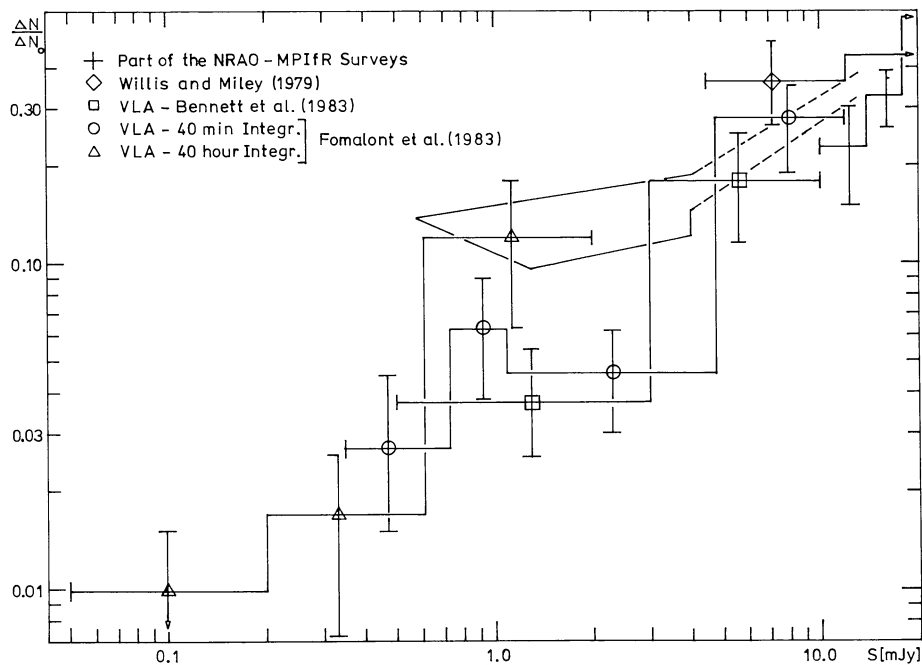


Fig. 4. VLA differential source counts near 5 GHz based on: 13 sources with $0.5 \leq S < 10$ mJy (Bennett et al., 1983), 8 sources with $0.05 \leq S < 2$ mJy and 24 sources with $0.35 \leq S < 12$ mJy (Fomalont et al., 1983). The estimation of the sky source density based on the $P(D)$ analysis (Wall et al., 1982b) is also shown by the probability area outlined by the solid line. Normalization - as in Fig. 3

These 45 sources allowed to extend the direct source counts below 10 mJy by about 2.5 orders in flux density as shown in Fig. 4. It is obvious that further VLA observations of sources in this very important flux density interval are required to improve the statistic for a quantitative determination of the slope of the counts and of the source density.

Acknowledgements. We wish to thank Dr. R. Porcas for valuable comments and critical remarks on the manuscript. We also wish to thank the telescope operators in Effelsberg, and the staff of the computer division at the MPIfR in Bonn and of the computer centre "CYFRONET" in Cracow for their help during data processing.

J. Maslowski wishes to thank Prof. R. Wielebinski for his invitation and hospitality during his stay at the MPIfR.

References

- Baars, J.W.M., Genzel, R., Pauliny-Toth, I.I.K., Witzel, A.: 1977, *Astron. Astrophys.* **61**, 99
- Bailey, J.A., Pooley, G.G.: 1968, *Monthly Notices Roy. Astron. Soc.* **138**, 51
- Bennett, A.: 1962, *Mem. Roy. Astron. Soc.* **68**, 163
- Bennett, C.L., Lawrence, C.R., Garcia-Barreto, J.A., Hewitt, J.N., Burke, B.F.: 1983, *Nature* **301**, 686
- Caswell, J.L., Crowther, J.H.: 1969, *Monthly Notices Roy. Astron. Soc.* **145**, 181
- Condon, J.J.: 1974, *Astrophys. J.* **188**, 279
- Crawford, D.F., Jauncey, D.L., Murdoch, H.S.: 1970, *Astrophys. J.* **162**, 405
- Davis, M.M.: 1971, *Astron. J.* **76**, 980
- Fomalont, E.B., Kellermann, K.I., Wall, J.V.: 1983, *Proc. IAU Symp.* **104**, p. 81
- Gillespie, A.R.: 1979, *Monthly Notices Roy. Astron. Soc.* **188**, 481
- Gower, J.F.R., Scott, P.F., Wills, D.: 1967, *Mem. Roy. Astron. Soc.* **71**, 49
- Jauncey, D.L.: 1967, *Nature* **216**, 877
- Katgert, P.: 1975, *Astron. Astrophys.* **38**, 87
- Katgert, P.: 1977, Ph.D. Thesis, Leiden University
- Kellermann, K.I., Pauliny-Toth, I.I.K., Davis, M.M.: 1968, *Astrophys. Letters* **2**, 105
- Kellermann, K.I., Pauliny-Toth, I.I.K., Williams, P.J.S.: 1969, *Astrophys. J.* **157**, 1
- Kesteven, M.J.L., Bridle, A.H., Brandie, G.W.: 1977, *Astron. J.* **82**, 541
- Kühr, H., Pauliny-Toth, I.I.K., Witzel, A., Schmidt, J.: 1981, *Astron. J.* **86**, 854
- Ledden, J.E., Broderick, J.J., Condon, J.J., Brown, R.L.: 1980, *Astron. J.* **85**, 780
- Maslowski, J.: 1971, *Astron. Astrophys.* **14**, 215
- Maslowski, J.: 1972, *Acta Astron.* **22**, 227
- Maslowski, J., Pauliny-Toth, I.I.K., Witzel, A., Kühr, H.: 1981, *Astron. Astrophys.* **95**, 285
- Maslowski, J., Pauliny-Toth, I.I.K., Witzel, A., Kühr, H.: 1984, *Astron. Astrophys.* **139**, 85.
- Owen, F.N., Condon, J.J., Ledden, J.E.: 1983, *Astron. J.* **88**, 1
- Pauliny-Toth, I.I.K., Kellermann, K.I.: 1972, *Astron. J.* **77**, 797
- Pauliny-Toth, I.I.K., Kellermann, K.I., Davis, M.M., Fomalont, E.B., Shaffer, D.B.: 1972, *Astron. J.* **77**, 265
- Pauliny-Toth, I.I.K., Steppe, H., Witzel, A.: 1980, *Astron. Astrophys.* **85**, 329
- Pauliny-Toth, I.I.K., Witzel, A., Preuss, E., Kühr, H., Kellermann, K.I., Fomalont, E.B., Davis, M.M.: 1978a, *Astron. J.* **83**, 451
- Pauliny-Toth, I.I.K., Witzel, A., Preuss, E., Baldwin, J.E., Hills, R.E.: 1978b, *Astron. Astrophys. Suppl.* **34**, 253
- Pearson, T.J., Kus, A.J.: 1978, *Monthly Notices Roy. Astron. Soc.* **182**, 273

Pooley, G.G., Kenderdine, S.: 1968, *Monthly Notices Roy. Astron. Soc.* **139**, 529
Refsdal, S.: 1969, *Astrophys. J.* **155**, 373
Scheuer, P.A.G.: 1957, *Proc. Camb. Phil. Soc.* **53**, 764
Wall, J.V., Savage, A., Wright, A.E., Bolton, J.G.: 1982a, *Monthly*

Notices Roy. Astron. Soc. **200**, 1123
Wall, J.V., Scheuer, P.A.G., Pauliny-Toth, I.I.K., Witzel, A.:
1982b, *Monthly Notices Roy. Astron. Soc.* **198**, 221
Willis, A.G., Miley, G.K.: 1979, *Astron. Astrophys.* **76**, 65