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A viscous fluid in a thin domain satisfying the slip condition on a slightly rough boundary

Fluide visqueux dans un domaine de faible épaisseur vérifiant la condition de glissement sur une frontière lègèrement rugueuse

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ABSTRACT

We consider a viscous fluid of small height ε on a periodic rough bottom Γ_ε of period r_ε and amplitude δ_ε, $\delta_\varepsilon \ll r_\varepsilon \ll \varepsilon$, where we impose the slip boundary condition. When ε tends to zero we obtain a Reynolds system depending on the limit λ of $(\delta_\varepsilon\sqrt{\varepsilon})/(r_\varepsilon\sqrt{r_\varepsilon})$. If $\lambda = +\infty$, the fluid behaves as if we would impose the adherence condition on Γ_ε. This justifies why this is the usual boundary condition for viscous fluids. If $\lambda = 0$ the fluid behaves as if Γ_ε was plane. Finally, for $\lambda \in (0, +\infty)$ it behaves as if Γ_ε was flat but with a higher friction coefficient.

RÉSUMÉ

On considère un fluide visqueux de faible épaisseur ε sur un fond rugueux Γ_ε, périodique de période r_ε et amplitude δ_ε, $\delta_\varepsilon \ll r_\varepsilon \ll \varepsilon$, où on impose la condition de glissement. Quand ε converge vers zéro on obtient un système de type Reynolds qui dépend de la limite λ de $(\delta_\varepsilon\sqrt{\varepsilon})/(r_\varepsilon\sqrt{r_\varepsilon})$. Si $\lambda = +\infty$, le fluide se comporte comme si on aurait imposé la condition d'adhérence sur Γ_ε. Ceci justifie la condition usuelle pour un fluide visqueux. Si $\lambda = 0$ le fluide se comporte comme si Γ_ε était plate. Enfin, pour $\lambda \in (0, +\infty)$, tout se passe comme si Γ_ε était plate, mais avec un coefficient de frottement plus élevé.

Version française abrégée

Pour un fluide visqueux dans un ouvert de $\mathbb{R}^3$ à frontière rugueuse, on sait que la condition de glissement et la condition d'adhérence sont asymptotiquement équivalentes. Ceci donne une justification mathématique de l'imposition usuelle de la condition d'adhérence pour les fluides visqueux. L'équivalence entre la condition de glissement et la condition d'adhérence a été montrée dans [10] dans le cas d'une frontière rugueuse de période ε et amplitude ε. Une extension au cas de frontières non périodiques a été obtenue dans [8]. Dans [11], on a considéré le cas d'une rugosité faible, plus exactement, la frontière est décrite par une fonction périodique de periode ε mais d'amplitude δ_ε, où δ_ε tend vers zéro. Alors, on a établie que si $\delta_\varepsilon/\varepsilon^{3/2}$ tend vers l'infini, l'équivalence entre la condition de glissement et la condition d'adhérence est maintenue, mais si $\delta_\varepsilon/\varepsilon^{3/2}$ converge vers zéro le fluide se comporte comme si la frontière était plate. Dans le cas où $\delta_\varepsilon \sim \varepsilon^{3/2}$ la rugosité

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n'est pas assez grande pour impliquer la condition d'adhérence, mais elle est assez grande pour augmenter le coefficient de frottement. Un rèsultat général sur la forme de la limite du système de Navier–Stokes avec des conditions de glissement sur une frontière non nécessairement périodique a été obtenue dans [7].

Dans cette Note, on généralise les résultats obtenus dans [11] au cas d'un domaine de hauteur ε. Plus exactement, pour un ouvert borné lipschitzien $\omega \subset \mathbb{R}^2$ et une fonction $\Psi \in W^{2,\infty}_{loc}(\mathbb{R}^2)$, périodique de période $Z' = (-1/2, 1/2)^2$, on définit Ω_ε par

$$\Omega_\varepsilon = \left\{ x = (x', x_3) \in \omega \times \mathbb{R}\colon -\delta_\varepsilon \Psi\left(\frac{x'}{r_\varepsilon}\right) < x_3 < \varepsilon \right\},$$

où les paramètres r_ε, δ_ε vérifient $\lim_{\varepsilon\to 0} \frac{r_\varepsilon}{\varepsilon} = 0$, $\lim_{\varepsilon\to 0} \frac{\delta_\varepsilon}{r_\varepsilon} = 0$. On considère alors un fluide satisfaisant le système de Stokes dans Ω_ε et la condition de Navier (ou glissement) $u_\varepsilon \cdot \nu = 0$, $\frac{\partial u_\varepsilon}{\partial \nu}$ parallel to ν sur la frontière rugueuse,

$$\Gamma_\varepsilon = \left\{ x = (x', x_3) \in \omega \times \mathbb{R}\colon x_3 = -\delta_\varepsilon \Psi\left(\frac{x'}{r_\varepsilon}\right) \right\},$$

où $u_\varepsilon = (u'_\varepsilon, u_{\varepsilon,3})$ est la vitesse et ν la normal extérieure à Ω_ε sur Γ_ε. Pour simplifier on impose aussi la condition d'adhérence $u_\varepsilon = 0$ sur $\partial\Omega_\varepsilon \setminus \Gamma_\varepsilon$. Notre but est d'étudier le comportement asymptotique de ce système pour ε tendant vers zéro. On obtient à la limite un système de type Reynolds qui dépend de $\lambda = \lim_{\varepsilon\to 0} \frac{\delta_\varepsilon \sqrt{\varepsilon}}{r_\varepsilon \sqrt{r_\varepsilon}}$. De là on déduit :

Si $\lambda = +\infty$, le fluide se comporte comme si on supposait $\Gamma_\varepsilon = \{x_3 = 0\}$ avec la condition à la frontière sur Γ_ε, $u_\varepsilon \in W^\perp \times \{0\}$, $\partial_3 u'_\varepsilon \in W$ avec $W = \{\nabla_{z'}\Psi(z') \in \mathbb{R}^2\colon z' \in Z'\}$. En particulier, si W est de dimension 2, le fluide se comporte comme si on aurait imposé la condition d'adhérence dans Γ_ε.

Si $\lambda = 0$ on voit que la rugosité n'a pas d'effet à la limite.

Si $\lambda \in (0, +\infty)$, le fluide se comporte comme si on supposait $\Gamma_\varepsilon = \{x_3 = 0\}$ avec la condition à la frontière sur Γ_ε, $u_{\varepsilon,3} = 0$, $-\mu\partial_3 u'_\varepsilon + \lambda^2 R u'_\varepsilon = 0$, où μ est la viscosité du fluide et R une matrix symétrique carrée non negative de dimension 2 qui est définie positive sur l'espace W. On observe que le nouveau terme $\lambda^2 R u'_\varepsilon$ est un terme de frottement. Cette condition à la frontière peut être considerée comme la condition générale puisque quand λ tend vers zéro ou $+\infty$, elle donne les résultats antérieurs.

Ce résultat est semblable à celui obtenu dans [11] pour un fluide de hauteur fixe, mais la taille critique est différente de $\delta_\varepsilon \sim r_\varepsilon^{3/2}$ qui serait la taille correspondante à [11]. Ceci vient du fait que loin de la frontière rugueuse le comportement du fluide est différent. Dans notre cas on montre que la vitesse est d'ordre ε^2 et la pression est d'ordre 1 et ne dépend pas de la profondeur (en une prémière approximation).

Pour finir on trouve dans les références [1,2,4–6,13], sur l'étude du comportement des fluides visqueux dans des domaines à frontière rugueuse, avec des conditions aux limites différentes de celles considérées dans cette Note.

1. Introduction

For a viscous fluid in an open set of $\mathbb{R}^3$ with a rugous boundary, it is known that if the normal velocity vanishes on the boundary (slip condition), then the fluid behaves as if the whole velocity vector vanishes on the boundary (adherence condition). This gives a mathematical explanation of why it is usual for a viscous fluid to impose the adherence condition. The equivalence between the slip and adherence conditions was proved in [10] for a periodic rough boundary of small period ε and amplitude ε. An extension to non-periodic boundaries was obtained in [8]. In [11] it was considered the case of a weak roughness, namely the boundary was described by a periodic function of small period ε and amplitude δ_ε, with $\delta_\varepsilon/\varepsilon$ converging to zero. It was proved that if $\delta_\varepsilon/\varepsilon^{3/2}$ tends to infinity, then the adherence and the slip conditions are still equivalent, while if $\delta_\varepsilon/\varepsilon^{3/2}$ tends to zero the fluid behaves as if the boundary was plane. In the critical case $\delta_\varepsilon \sim \varepsilon^{3/2}$ the roughness is not so large to imply the adherence condition but it is enough to increase the friction coefficient. A general result about the form of the limit equation for the Navier–Stokes system satisfying the slip condition on a (non-necessarily periodic) rough boundary has been obtained in [7].

Our aim in the present Note is to generalize the results in [11] to the case of a domain of small height ε. Namely, for a Lipschitz bounded open set $\omega \subset \mathbb{R}^2$ and a function Ψ in $W^{2,\infty}_{loc}(\mathbb{R}^2)$, periodic of period $Z' = (-1/2, 1/2)^2$, we define Ω_ε by

$$\Omega_\varepsilon = \left\{ x = (x', x_3) \in \omega \times \mathbb{R}\colon -\delta_\varepsilon \Psi\left(\frac{x'}{r_\varepsilon}\right) < x_3 < \varepsilon \right\}, \tag{1}$$

where the parameters r_ε, δ_ε are chosen non-negative and satisfying $\lim_{\varepsilon\to 0} \frac{r_\varepsilon}{\varepsilon} = 0$, $\lim_{\varepsilon\to 0} \frac{\delta_\varepsilon}{r_\varepsilon} = 0$. We consider a fluid satisfying the Stokes system in Ω_ε, the Navier (or slip condition) on the rough boundary

$$\Gamma_\varepsilon = \left\{ x = (x', x_3) \in \omega \times \mathbb{R}\colon x_3 = -\delta_\varepsilon \Psi\left(\frac{x'}{r_\varepsilon}\right) \right\}, \tag{2}$$

and (to simplify) the adherence condition on the rest of the boundary $\partial\Omega_\varepsilon \setminus \Gamma_\varepsilon$. Our purpose is to study the asymptotic behavior of this system when ε tends to zero. We show that it depends on

$$\lambda=\lim_{\varepsilon\to 0}\frac{\delta_\varepsilon}{r_\varepsilon}\sqrt{\frac{\varepsilon}{r_\varepsilon}}\in[0,+\infty]. \tag{3}$$

If $\lambda=+\infty$ and the space $W=\{\nabla_{z'}\Psi(z')\in\mathbb{R}^2\colon\ z'\in Z'\}$ agrees with $\mathbb{R}^2$, then the fluid behaves as if we impose the adherence condition on the whole $\partial\Omega_\varepsilon$.

If $\lambda=0$, then the fluid behaves as if Γ_ε agrees with the plane boundary $\{x_3=0\}$.

If $\lambda\in(0,+\infty)$ the fluid behaves as if we had considered a plane boundary and added a friction coefficient to the Navier condition (see Theorem 2.1 and Remark 2).

This is analogous to the result proved in [11] for a fluid with fixed height, but the critical size is not $\delta_\varepsilon\sim r_\varepsilon^{3/2}$ which would be the expected size from [11]. This is due to the fact that far of the rugous boundary the behavior of the fluid is different from the corresponding one in [11]. Here one can show that the velocity is of order ε^2 while the pressure is of order 1 and does not depend on the depth (in a first approximation).

To finish this introduction we refer to [1,2,4–6,13], for the study of viscous fluids in rugous domains satisfying different boundary conditions from the ones considered in the present paper.

2. Main results and some comments

Along this section, the points x of $\mathbb{R}^3$ are supposed to be decomposed as $x=(x',x_3)$ with $x'\in\mathbb{R}^2$, $x_3\in\mathbb{R}$. We also use the notation x' to denote a generic vector of $\mathbb{R}^2$.

Given a bounded connected Lipschitz open set $\omega\subset\mathbb{R}^2$ and $\Psi\in W^{2,\infty}_{loc}(\mathbb{R}^2)$, periodic of period Z', we define Ω_ε by (1) and Γ_ε by (2). Then, for $f=(f',f_3)\in L^2(\omega)^3$, we consider the Stokes system in Ω_ε

$$\begin{cases}-\mu\Delta u_\varepsilon+\nabla p_\varepsilon=f \quad \text{in } \Omega_\varepsilon, \qquad \operatorname{div} u_\varepsilon=0 \quad \text{in } \Omega_\varepsilon,\\ u_\varepsilon=0 \quad \text{on } \partial\Omega_\varepsilon\setminus\Gamma_\varepsilon, \qquad u_\varepsilon\cdot\nu=0 \quad \text{on } \Gamma_\varepsilon, \qquad \dfrac{\partial u_\varepsilon}{\partial\nu} \text{ parallel to } \nu \quad \text{on } \Gamma_\varepsilon.\end{cases} \tag{4}$$

Here ν denotes the unitary outside normal vector to Ω_ε in Γ_ε and $\mu>0$ corresponds to the viscosity of the fluid. It is well known that (4) has a unique solution $(u_\varepsilon,p_\varepsilon)\in H^1(\Omega_\varepsilon)^3\times L^2_0(\Omega_\varepsilon)$ ($L^2_0(\Omega_\varepsilon)$ denotes the space of functions in $L^2(\Omega_\varepsilon)$ whose integral in Ω_ε is zero). Moreover, we can show the following estimates:

$$\fint_{\Omega_\varepsilon}|u_\varepsilon|^2\,dx\leqslant C\varepsilon^4, \qquad \fint_{\Omega_\varepsilon}|Du_\varepsilon|^2\,dx\leqslant C\varepsilon^2, \qquad \fint_{\Omega_\varepsilon}|p_\varepsilon|^2\,dx\leqslant C. \tag{5}$$

Our aim is to study the asymptotic behavior of u_ε and p_ε when ε tends to zero. For this purpose, as usual, we use a dilatation in the variable x_3 in order to have the functions defined in an open set of fixed height. Namely, we take $\Omega=\omega\times(0,1)$ and we define $\tilde u_\varepsilon\in H^1(\Omega)^3$, $\tilde p_\varepsilon\in L^2_0(\Omega)$ by

$$\tilde u_\varepsilon(y)=u_\varepsilon\big(y',\varepsilon y_3\big), \qquad \tilde p_\varepsilon(y)=p_\varepsilon\big(y',\varepsilon y_3\big), \qquad \text{a.e. } y\in\Omega. \tag{6}$$

Then, our problem is to describe the asymptotic behavior of these sequences $\tilde u_\varepsilon$, $\tilde p_\varepsilon$. This is given by the following theorem which is the main result of the present Note:

Theorem 2.1. *Let $(u_\varepsilon,p_\varepsilon)\in H^1(\Omega_\varepsilon)^3\times L^2_0(\Omega_\varepsilon)$ be the solution of the Stokes system* (4) *and let $\tilde u_\varepsilon$, $\tilde p_\varepsilon$ be defined by* (6)*. Then, there exist $v\in H^1(0,1;L^2(\omega))^2$, $w\in H^2(0,1;H^{-1}(\omega))$ and $p\in L^2_0(\Omega)$, where p does not depend on y_3, such that, up to a subsequence,*

$$\frac{\tilde u_\varepsilon}{\varepsilon}\rightharpoonup 0 \quad \text{in } H^1(\Omega)^3, \qquad \frac{\tilde u_\varepsilon}{\varepsilon^2}\rightharpoonup(v,0) \quad \text{in } H^1\big(0,1;L^2(\omega)\big)^3, \qquad \frac{\tilde u_{\varepsilon,3}}{\varepsilon^3}\rightharpoonup w \quad \text{in } H^2\big(0,1;H^{-1}(\omega)\big), \tag{7}$$

$$\tilde p_\varepsilon\rightharpoonup p \quad \text{in } L^2(\Omega), \qquad \frac{\partial_{y_3}\tilde p_\varepsilon}{\varepsilon}\rightharpoonup f_3 \quad \text{in } H^{-1}(\Omega). \tag{8}$$

According to the value of λ defined by (3)*, the functions v, w and p are given by*:

(i) *If $\lambda=+\infty$, then denoting by $P_{W^\perp}$ the orthogonal projection from $\mathbb{R}^2$ to the orthogonal of the space $W=\{\nabla_{z'}\Psi(z')\in\mathbb{R}^2\colon\ z'\in Z'\}$, we have that v and p are given by*

$$v(y)=\frac{(y_3-1)}{2\mu}(y_3I+P_{W^\perp})\big(\nabla_{y'}p\big(y'\big)-f'\big(y'\big)\big), \quad \text{a.e. } y\in\Omega,$$

$$-\operatorname{div}_{y'}\left(\left(\frac{1}{3}I+P_{W^\perp}\right)\big(\nabla_{y'}p-f'\big)\right)=0 \quad \text{in } \omega, \qquad \left(\frac{1}{3}I+P_{W^\perp}\right)\big(\nabla_{y'}p-f'\big)\cdot\nu=0 \quad \text{on } \partial\omega.$$

Moreover, the distribution w is given by

$$w(y) = -\int_0^{y_3} \operatorname{div}_{y'} v\big(y', s\big)\, ds \quad in\ \Omega. \tag{9}$$

(ii) *If $\lambda \in (0, +\infty)$, then defining $(\hat{\phi}^i, \hat{q}^i)$, $i = 1, 2$, as solutions of the Stokes systems*

$$\begin{cases} -\mu \Delta_z \hat{\phi}^i + \nabla_z \hat{q}^i = 0 \quad in\ \mathbb{R}^2 \times (0, +\infty), \qquad \operatorname{div}_z \hat{\phi}^i = 0 \quad in\ \mathbb{R}^2 \times (0, +\infty), \\ \hat{\phi}^i_3\big(z', 0\big) + \partial_{z_i} \Psi\big(z'\big) = 0, \qquad \partial_{z_3} (\hat{\phi}^i)'\big(z', 0\big) = 0, \qquad \hat{\phi}^i(., z_3), \hat{q}^i(., z_3)\ periodic\ of\ period\ Z', \\ D_z \hat{\phi}^i \in L^2\big(Z' \times (0, +\infty)\big)^{3\times 3}, \qquad \hat{q}^i \in L^2\big(Z' \times (0, +\infty)\big), \end{cases}$$

and $R \in \mathbb{R}^{2\times 2}$ by

$$R_{ij} = \mu \int_{Z' \times (0,+\infty)} D_z \hat{\phi}^i : D_z \hat{\phi}^j\, dz, \quad \forall i, j \in \{1, 2\},$$

we have

$$v(y) = \frac{(y_3 - 1)}{2\mu} \left(y_3 I + \left(I + \frac{\lambda^2}{\mu} R \right)^{-1} \right) \big(\nabla_{y'} p\big(y'\big) - f'\big(y'\big)\big), \quad a.e.\ y \in \Omega,$$

where p satisfies

$$\begin{cases} -\operatorname{div}_{y'} \left(\left(\frac{1}{3} I + \left(I + \frac{\lambda^2}{\mu} R \right)^{-1} \right) \big(\nabla_{y'} p - f'\big) \right) = 0 \quad in\ \omega, \\ \left(\frac{1}{3} I + \left(I + \frac{\lambda^2}{\mu} R \right)^{-1} \right) \big(\nabla_{y'} p - f'\big) \cdot \nu = 0 \quad on\ \partial\omega. \end{cases}$$

Moreover, the distribution w is given by (9).

(iii) *If $\lambda = 0$, then*

$$v(y) = \frac{(y_3^2 - 1)}{2\mu} \big(\nabla_{y'} p\big(y'\big) - f'\big(y'\big)\big), \quad a.e.\ y \in \Omega,$$

where p satisfies

$$-\Delta_{y'} p = -\operatorname{div}_{y'} f' \quad in\ \omega, \qquad \frac{\partial p}{\partial \nu} = f' \cdot \nu \quad on\ \partial\omega.$$

Moreover, the distribution w is zero.

Remark 1. An analogous result to Theorem 2.1 is proved in [11] where it is studied the Stokes and Navier–Stokes systems with the slip condition on a rough boundary for an open set of $\mathbb{R}^3$ of fixed height. The functions $(\hat{\phi}^i, \hat{q}^i)$ are the same functions which appear in [11] to describe the behavior of the velocity and the pressure near the rough boundary. Moreover, it is proved there that $D_z \hat{\phi}^i$, $\hat{q}^i$ belong to $L^r(Z' \times (0, +\infty))^{3\times 3}$ and $L^r(Z' \times (0, +\infty))$ respectively for every $r \geqslant 1$ and have exponential decay at infinity.

Remark 2. For $\lambda = +\infty$, Theorem 2.1 shows that u_ε, p_ε behave as if in (4) we had assumed that Γ_ε was the plane boundary $\{x_3 = 0\}$ and that the boundary condition on Γ_ε was:

$$u_\varepsilon \in W^\perp \times \{0\} \quad \text{on } \Gamma_\varepsilon, \qquad \partial_3 u'_\varepsilon \in W. \tag{10}$$

In particular, if W agrees with $\mathbb{R}^2$ (which is true except if $\Psi(z_1, z_2)$ does not depend on z_1 and/or z_2) we deduce that the slip condition in (4) is equivalent to the adherence condition $u_\varepsilon = 0$ on $\{x_3 = 0\}$.

For $\lambda \in (0, +\infty)$, Theorem 2.1 shows that the asymptotic behavior of u_ε and p_ε is the same that if Γ_ε was the plane boundary $\{x_3 = 0\}$ and the boundary condition on Γ_ε was

$$u_{\varepsilon,3} = 0 \quad \text{on } \Gamma_\varepsilon, \qquad -\mu \partial_3 u'_\varepsilon + \lambda^2 R u'_\varepsilon = 0 \quad \text{on } \Gamma_\varepsilon, \tag{11}$$

i.e. although the roughness is not strong enough to deduce that the slip condition on Γ_ε is equivalent to (10), it is sufficient to provide the friction coefficient $\lambda^2 R u'_\varepsilon$ in (11).

For $\lambda = 0$, the roughness is so weak that u_ε and p_ε behave as if Γ_ε was plane.

The critical size $\lambda \in (0, +\infty)$ can be considered as the general one. In fact, the cases $\lambda = 0$ and $\lambda = +\infty$ can be obtained from this one by taking the limit when λ tends to zero and infinity respectively.

Remark 3. In the cases $\lambda = 0$ or $+\infty$, we can prove that the convergences in (7)–(8) are strong. In fact, assuming ω smooth enough (for example C^2), we can show that defining $\bar{u}_\varepsilon$, $\bar{p}_\varepsilon$ by:

$$\bar{u}_\varepsilon(x) = \left(\varepsilon^2 v\left(x', \frac{x_3}{\varepsilon}\right), 0\right), \qquad \bar{p}_\varepsilon(x) = p(x') \quad \text{a.e. } x \in \Omega_\varepsilon,$$

we have

$$\frac{1}{\varepsilon^4} \fint_{\Omega_\varepsilon} |u_\varepsilon - \bar{u}_\varepsilon|^2 \, dx \to 0, \qquad \frac{1}{\varepsilon^2} \fint_{\Omega_\varepsilon} \left|D(u_\varepsilon - \bar{u}_\varepsilon)\right|^2 dx \to 0, \qquad \fint_{\Omega_\varepsilon} |p_\varepsilon - \bar{p}_\varepsilon|^2 \, dx \to 0.$$

In the critical case $\lambda \in (0, +\infty)$, the above assertion still holds replacing $\bar{u}_\varepsilon$ by:

$$\bar{u}_\varepsilon(x) = \left(\varepsilon^2 v\left(x', \frac{x_3}{\varepsilon}\right), 0\right) + \lambda\varepsilon\sqrt{\varepsilon r_\varepsilon}\left(v_1(x', 0)\hat{\phi}^1\left(\frac{x}{r_\varepsilon}\right) + v_2(x', 0)\hat{\phi}^2\left(\frac{x}{r_\varepsilon}\right)\right).$$

Remark 4. The proof of Theorem 2.1 is based on the unfolding method [3,9,12]. For a.e. $x' \in \mathbb{R}^2$, we define $\kappa(x') \in \mathbb{Z}^2$ by $x' \in \kappa(x') + Z'$. Then, to study the behavior of $(u_\varepsilon, p_\varepsilon)$ near Γ_ε, the idea is to study the behavior of the sequences $\hat{u}_\varepsilon$, $\hat{p}_\varepsilon$ defined as

$$\hat{u}_\varepsilon(x', z) = u_\varepsilon\left(r_\varepsilon \kappa\left(\frac{x'}{r_\varepsilon}\right) + r_\varepsilon z', r_\varepsilon z_3\right), \qquad \hat{p}_\varepsilon(x', z) = p_\varepsilon\left(r_\varepsilon \kappa\left(\frac{x'}{r_\varepsilon}\right) + r_\varepsilon z', r_\varepsilon z_3\right),$$

for a.e. $x' \in \omega$, $z' \in Z'$, $-(\delta_\varepsilon/r_\varepsilon)\Psi(z') < z_3 < 1/r_\varepsilon$. This is similar to the idea used in [11], but here it is necessary to combine this change of variables with (6).

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