Model-based PI design for irrigation canals with faulty communication networks

T. Arauz[1], J. M. Maestre[1], A. Cetinkaya[2], E. F. Camacho[1]

***Abstract*— A PI design method for faulty networks is provided based on Linear Matrix Inequalities (LMIs). Feedback controllers for irrigation canals are designed based on LMIs, but sparsity constraints are also imposed to make zero the feedback control law elements not corresponding to the tuning PI parameters. Therefore, the design method is halfway between a PI controller and an optimal feedback control law, also providing stability guarantees up to a maximum probability of packet losses. The objective of the downstream controller is to maintain the water levels upstream from each downstream check structure of each canal pool, while gravity-offtake gates satisfy downstream water demands. The proposed approach is tested using the irrigation system of ASCE Test Canal 1 and compared with other tuning methods via simulation. Our results show that the design method can be a useful tool when dealing with control systems under faulty networks.**

I. Introduction

Irrigation systems are Cyber-Physical Systems (CPS), i.e., heterogeneous systems composed of computing devices that interact with physical processes with high societal relevance [14]. At the beginning of the 21st century, 2.7 million km^2 of land (18% of the total cropland area) were irrigated, using nearly 85% of the whole amount of human used water [10]. However, the global distribution of these percentages is geographically unbalanced with the USA and Southeast Asia having the majority of irrigation areas [8]. Many authors refer to the lack of canal performance in some regions, e.g., South and Southeast Asia [33], and the challenges in arid and semi-arid areas, such as Iran [30].

As a response to irrigation concerns, researchers have been working to develop effective control algorithms [2], [9], [19], [29]. However, most commonly employed control algorithms in canals are based on PI controllers, and different methods for PI tuning in this type of application can be found in the literature [11], [15], [32]. Furthermore, many approaches are based on Linear Matrix Inequalities (LMIs) to design feedback controllers for irrigation canals, by imposing some specific constraints that allow the tuning of PI parameters, e.g., [20]. The introduction of constraints in the design of feedback controllers has also been studied previously in other works [17], [18].

During the last years, cybersecurity threats have become a relevant research topic for CPS. Different strategies have been developed to guarantee properties such as system stability when CPS cybersecurity is endangered [16], [24]. Regarding water systems under faulty networks, tragic consequences may be caused when the system is under cyber-attacks, and some of those methods can also be helpful [13]. One cybersecurity incident is packets loss, which can be caused either by malicious attacks or unreliable transmissions [31]. Different controllers have been developed to guarantee system stability in spite of packet losses [22], [23].

This work proposes a new method to tune the coefficients of PI controllers for irrigation canals control under packet losses. Our approach is based on LMIs, which are used to design feedback controllers for irrigation canals. The rationale is to identify the elements of the feedback control law that correspond to the tuning parameters of classical controllers such as P and PI, and to impose sparsity constraints on the controller design so that the rest of the elements in the feedback matrix become zero. The proposed method is based on [1] but including packet losses. Therefore, a PI controller is designed that guarantees system stability up to a maximum probability of packet losses. Also, constraints definition is based on [3]. To study the performance of the method developed, the ASCE Test Canal 1 [4] is selected as a case study, and its linear model is built in MATLAB to implement the controller.

The rest of the paper is organized as follows. First, the problem setting is given in Section 2. Then, the LMI based controller design method is presented in Section 3. Section 4 introduces the ASCE Test Canal 1, which is the model used for tests, the proposed controller parameters along with other controllers for comparison, and the performance indicators considered for the assessment. Simulation results are provided in Section 5. Finally, concluding remarks are given in Section 6.

II. Problem formulation

An irrigation canal is formed by a set of sections where water flows, which are separated by gates to control the water volume of each one. From a mathematical viewpoint, we assume that the overall system is composed of a set of $\mathcal{N} = \{1, 2, \ldots, N\}$ discrete linear time-invariant subsystems. The dynamics of subsystem $i \in \mathcal{N}$ are given by

$$x_i(k+1) = A_{ii}x_i(k) + B_{ii}u_i(k) + d_i(k), \qquad (1)$$

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where $x_i \in \mathbb{R}^{q_i}$ and $u_i \in \mathbb{R}^{r_i}$ are the states and inputs, respectively, which in this context are typically water level errors and flows. The number of states and controls of each subsystem are respectively denoted as q_i and r_i, and in the overall system q and r. Here, d_i represents the disturbances due to the influence of neighbors' states and inputs in the update of x_i, which becomes

$$d_i(k) = \sum_{j \neq i} A_{ij} x_j(k) + \sum_{j \neq i} B_{ij} u_j(k), \qquad (2)$$

where matrices A_{ij} and B_{ij} map respectively the state and inputs of a subsystem $j \in \mathcal{N}$ into the state of subsystem i. External disturbances are omitted for simplicity and without loss of generality regarding the design method proposed.

Each subsystem described by (1) has its own goal: to minimize a stage cost defined as

$$\ell_i(k) = x_i^{\mathrm{T}}(k) Q_i x_i(k) + u_i^{\mathrm{T}}(k) R_i u_i(k), \qquad (3)$$

where $Q_i \in \mathbb{R}^{q_i \times q_i}$ and $R_i \in \mathbb{R}^{r_i \times r_i}$ are respectively positive semi-definite and definite constant weighting matrices. Physically, the cost penalizes quadratically water deviations and changes in gates and flows with respect to the operation point, which is assumed to be the origin for simplicity.

Finally, the overall system dynamics and stage cost can be obtained aggregating all the subsystem ones, so that the global model and goal can be described as

$$x_{\mathcal{N}}(k+1) = A_{\mathcal{N}} x_{\mathcal{N}}(k) + B_{\mathcal{N}} u_{\mathcal{N}}(k), \qquad (4)$$

$$\ell_{\mathcal{N}}(k) = x_{\mathcal{N}}^{\mathrm{T}}(k) Q_{\mathcal{N}} x_{\mathcal{N}}(k) + u_{\mathcal{N}}^{\mathrm{T}}(k) R_{\mathcal{N}} u_{\mathcal{N}}(k), \qquad (5)$$

where the subscript $\mathcal{N}$ stresses that all system vectors and matrices come from the aggregation of local subsystems, i.e., $x_{\mathcal{N}} = [x_i]_{i \in \mathcal{N}}$, $u_{\mathcal{N}} = [u_i]_{i \in \mathcal{N}}$, $A_{\mathcal{N}} = [A_{ij}]_{i,j \in \mathcal{N}}$, and $B_{\mathcal{N}} = [B_{ij}]_{i,j \in \mathcal{N}}$; $Q_{\mathcal{N}} = \mathrm{diag}(Q_i)_{i \in \mathcal{N}}$ and $R_{\mathcal{N}} = \mathrm{diag}(R_i)_{i \in \mathcal{N}}$. Finally, note that there is no disturbance term in (4) because all mutual interactions are embedded in the overall dynamics.

A. Control strategy

The main goal for the overall system defined by (4) is to minimize the total cost function (5) so that the water level is steered towards the operation point. To this end, a sparse linear feedback controller will be designed, i.e., not all the states can be used to calculate the control actions. For example, the error in the first reach is not used to calculate the actions of the last one. Therefore, there is a mapping from system states to control actions that must be considered in the design to obtain the desired sparsity pattern in the feedback controller, thus implementing constraints in the information flows.

The mapping, denoted hereafter as Λ, connects the set of state variables, $\mathcal{X}$, and $\mathcal{U}$, which is the set control actions, i.e., $\Lambda : \mathcal{X} \rightarrow \mathcal{U}$. This mapping is clearly observed in the feedback matrix K, which sets that control action u_i can receive information from state x_j only if $K_{i,j} \neq 0$. Superscripts will be added in the control law to stress this fact: $u_{\mathcal{N}} = K^{\Lambda} x_{\mathcal{N}}$, with $\Lambda \in \mathcal{M}$, where $\mathcal{M}$ is the set of all possible information mappings.

This way, the resulting controller receives actual physical states through the state vector $x_{\mathcal{N}}(k)$ (water-level errors with respect their target levels) to generate structure flow adjustments. Moreover, since the gain matrix K^{Λ} has to include both proportional and integral constants of a PI controller, not only error values have to be considered, but also the integral error values. Therefore, the state vector $x_{\mathcal{N}}(k)$ is extended with the water level integral errors of all states. Finally, the system is considered to be initially in the steady-state, so all the system variables are referred to this point, which becomes the origin for the linear model (4).

B. Packet losses

Packet losses are modelled by using a Bernoulli process with parameter $\rho \in [0, 1]$. Here, ρ represents the probability (and hence the long-run average ratio) of packet exchange failures between the system and its controllers, which can be caused due to the unreliability of transmissions or the malicious interference of malicious attackers. A binary-valued process $l_i(k) \in \{0, 1\}$ for each subsystem $i \in \mathcal{N}$ is defined to characterize success or failure of packet exchange attempts. In this way, the control input applied to subsystem i is given by

$$u_i(k) = (1 - l_i(k)) K_i x_i(k) \qquad (6)$$

following the packet dropout compensation of [25]: $u_i(k) = K_i x_i(k)$ when there is a success of packet reception ($l_i(k) = 0$), and $u_i(k) = 0$ when there is a failure ($l_i(k) = 1$), whatever the cause of the packet loss is.

III. LMI based Controller Design

First, we will introduce two theorems that will provide us with useful theoretical results for the design of the controller. Their proofs are not included, but can be found in the corresponding references.

Theorem 1. [1] *Consider the system (4) with system and input matrices $A_{\mathcal{N}}$ and $B_{\mathcal{N}}$ together with the stage cost (5) defined with $Q_{\mathcal{N}}$ and $R_{\mathcal{N}}$. Also, let constraints in the communication flows between states and controls define a mapping be denoted as Λ. If there exist a positive-definite matrix $W = W^{\mathrm{T}} = \mathrm{diag}(W_i)_{i \in \mathcal{N}}$, where $W_i \in \mathbb{R}^{q_i \times q_i}$, and matrix $Y \in \mathbb{R}^{r \times q}$ such that the following constraint is satisfied*

$$\begin{bmatrix} W & W A_{\mathcal{N}}^{\mathrm{T}} + Y_{\Lambda}^{\mathrm{T}} B_{\mathcal{N}}^{\mathrm{T}} & W Q_{\mathcal{N}}^{1/2} & Y_{\Lambda}^{\mathrm{T}} R_{\mathcal{N}}^{1/2} \\ A_{\mathcal{N}} W + B_{\mathcal{N}} Y_{\Lambda} & W & 0 & 0 \\ Q_{\mathcal{N}}^{1/2} W & 0 & I & 0 \\ R_{\mathcal{N}}^{1/2} Y_{\Lambda} & 0 & 0 & I \end{bmatrix} > 0, \qquad (7)$$

for the specific mapping Λ, with $Y_{\Lambda,ij} = Y_{ij}$ if the information flow from x_j to calculate u_i is allowed, i.e., if $\{x_j, u_i\} \in \Lambda$, and $Y_{\Lambda,ij} = 0$ otherwise; then there exists a feedback that provides a stabilizing control law $K^{\Lambda} = Y_{\Lambda} W^{-1}$ where $K_{ij}^{\Lambda} = 0$ if $Y_{\Lambda,ij} = 0$. Also, a Lyapunov function $f(x_{\mathcal{N}}(k)) = x_{\mathcal{N}}^{\mathrm{T}}(k) P x_{\mathcal{N}}(k)$ that provides a bound on the cost-to-go of the closed-loop system is obtained, with $P = W^{-1}$.

Next, we present a result for the case with centralized packet losses, i.e., $l_i(k) = l_j(k),\ i, j \in \mathcal{N}$.

Theorem 2. [3] *Consider the system (4) with input matrices $A_{\mathcal{N}}$ and $B_{\mathcal{N}}$. Also, let $\rho \in [0, 1]$ represent the probability packet losses. If there exist a positive-definite matrix $W = W^{\mathrm{T}} = \mathrm{diag}(W_i)_{i\in\mathcal{N}}$, where $W_i \in \mathbb{R}^{q_i\times q_i}$, and matrix $Y \in \mathbb{R}^{r\times q}$, and scalars $\beta \in (0, 1)$ and $\varphi \in [1, \inf)$ such that the following constraints are satisfied*

$$\begin{bmatrix} \beta W & W A_{\mathcal{N}}^{\mathrm{T}} + Y_{\Lambda}^{\mathrm{T}} B_{\mathcal{N}}^{\mathrm{T}} \\ A_{\mathcal{N}} W + B_{\mathcal{N}} Y_{\Lambda} & W \end{bmatrix} > 0, \tag{8}$$

$$\begin{bmatrix} \varphi W & W A_{\mathcal{N}}^{\mathrm{T}} \\ A_{\mathcal{N}} W & W \end{bmatrix} > 0, \tag{9}$$

$$(1 - \rho) \ln \beta + \rho \ln \varphi < 0 \tag{10}$$

then the control law (6) with $P = W^{-1}$ and $K^{\Lambda} = Y_{\Lambda} W^{-1}$ guarantees almost sure asymptotic stability of the zero solution of the closed-loop system dynamics.

A. Design method

To design the controller, we find a possible solution that complies with (7) – (10) for the specific mapping Λ and a specific packet loss probability ρ. Then, it is enough to take $K^{\Lambda} = Y_{\Lambda} W^{-1}$.

Note that these LMIs can be solved via any LMI toolbox, but computation is not direct since β and φ have to be initially set. In this way, the problem has to be solved by iterating over a set of values for $\beta \in (0, 1)$ and $\varphi \in [1, \inf)$. However, it is sufficient to check for larger values of β and φ that are close to the boundary range of (10). To this end, a small positive real number $\Delta > 0$ can be set to iterate over a set of values that fulfill with [3]:

$$(1 - \rho) \ln \beta + \rho \ln \varphi = -\Delta. \tag{11}$$

Therefore, the algorithm followed to solve the LMIs is presented in Algorithm 1. Note that there is an upper bound for the packet loss probability, $\rho_{\max}$, for which the LMIs (7), (8), (9) can be solved, i.e., the stability guarantees hold. The maximum probability of packet exchange failures $\rho_{\max}$ can be found by iterating ρ using Algorithm 1 until the LMIs in Step 2 become unfeasible.

Algorithm 1 Iterative process to solve the Optimization problem

Set probability of packet losses: ρ value
Initialization: $\beta \leftarrow 0.999$; $\Delta\beta \leftarrow 0.001$; $\Delta \leftarrow 0.00001$
Iterative process:
  Step 1: Update value $\varphi \leftarrow e^{-\frac{(1-\rho)\ln\beta - \Delta}{\rho}}$
  Step 2: Find a solution for LMIs (7), (8), (9)
  Step 3: Suitability test
    If solution infeasible **then**
      Update value $\beta \leftarrow \beta - \Delta\beta$
      GO TO Step 1
    else
      $P \leftarrow W^{-1}$
      $K^{\Lambda} \leftarrow Y_{\Lambda} W^{-1}$
    end if

IV. Case Study

The Test Canal 1 is described in [4] and [5]. The ASCE Test Canal 1 is based on an actual operating canal, concretely the lateral canal WM within the Maricopa Stanfield Irrigation and Drainage District in central Arizona, about 50 km south of Phoenix, which receives water from Colorado River and irrigates a district area of about 35,000 ha.

The ASCE test canal 1 is 9.5 km long and drops 40 m in elevation [5]. The whole canal is divided into eight pools by a series of controllable orifices gates. These gates are vertical sluice gates and, for the first canal, all gates are assumed to be always unsubmerged to avoid the problems associated with the transition from free to submerged flow. Gate movement is restricted by a minimum value of 0.5% of the gate height per control time step, and it can be modeled with suitable equations for the simulation software. Each of the eight pools starts in a different descending height to enable the water gravity movement. The canal also includes some gravity offtakes, located 5 m from the downstream end of each pool, but it has no flow at the downstream end. The control of the system is possible thanks to the motorized gates placed at the end of each canal, including some ultrasonic flowmeters located in the 11 offtakes placed next to the gates. The regulation time steep has to be around 5 min, depending on the selected control algorithm.

A. The Linear Canal Model

A simple discrete-time linear model for canal pool response was proposed by [28]: the integrator delay (ID) model, which is based on the method presented in [26] where an approximation model for an open channel with backwater effects was derived using the linearized Saint Venant equations. A canal pool for the ID model is considered to be formed by the first portion of normal depth and the other portion, closed to the downstream end of the pool, under backwater, named reservoir. The ID model is

$$\Delta h(k) = \frac{T_s}{A_s}[q_u(k - \tau_R) - q_d(k)] \tag{12}$$

where $\Delta h(k)$ represents the water level change; k, the integer time step number; T_s, the duration of the time step; A_s, the backwater surface area; q_u and $q_d(k)$, the upstream and the downstream flow rates, respectively; and τ_R the closest integer representing the delay time (τ/Ts).

Therefore, the model equation relates water level changes at the downstream end of the pool to flow changes through gates at the upstream and downstream end of the pool. The delay time is the main reason for the difference between the response of the upstream and downstream gates results. However, although the ID model has been verified as a good approximation for controller design purposes [27], the fact that the ID model is a simplification of the real canal response has to be also taken into account [6]. Finally, note that (12) can be easily written in the form of (4).

B. Proposed controller

The objective of a downstream feedback canal controller is to maintain the water levels upstream from each check structure, which is the downstream end of each canal pool, with the assumption that gravity-offtake gates are set at the set-point level satisfying downstream water demands. The water level at the downstream end of each canal pool is controlled by a gate at the upstream end of the same pool, bringing about a significant delay between the gate action and the water-level response to that action [7].

Furthermore, because of the delay time for water to travel across canals, the gate control changes of upstream ends may not be translated in water level changes of downstream end by the next control time. Thus, to account for this effect in a discrete-time controller, some additional terms related to these previous control actions have to be added in the control law. This addition turns out to be another extra extension of the state vector $x_{\mathcal{N}}(k)$ including these lagged flow measurements. The proper number of intermediate measurements depends on the real physical characteristics of each pool. Note that the added elements in the gain matrix corresponding to the extra lagged flow measurements will be forced to be zero since they cannot be controlled by the designed PI controller.

Regarding control inputs, the controller needs to be designed to keep them fixed when there is a packet loss. Therefore, the model has to be redefined to get incremental control signals. This way, the input signal $u_{\mathcal{N}}$ of (4) corresponds to

$$u_{\mathcal{N}}(k) = u_{\mathcal{N}}(k+1) + \Delta u_{\mathcal{N}}(k), \tag{13}$$

where $\Delta u_{\mathcal{N}}(k) = K^{\Lambda} x_{\mathcal{N}}(k)$. Consequently, the state vector is enlarged to include also the previous input signals as well.

The linear feedback controller was designed to minimize the total cost function defined by (5). This expression contains two weighting matrices: $Q_{\mathcal{N}}$ for deviations from the setpoint and $R_{\mathcal{N}}$ for control effort. Their assigned values have a significant influence on the tuning of controller parameters and system performance. In this work, the matrix $R_{\mathcal{N}}$ is set as $R_{\mathcal{N}} = \mathrm{I} \cdot 10^{-3}$, where I represents the corresponding identity matrix. The matrix $Q_{\mathcal{N}}$ is set as the corresponding identity matrix except for the diagonal elements corresponding to water level errors and their integrals, which are $[2.5189\ 1.5314\ 1.9881\ 0.6536\ 0.6196\ 0.5000\ 0.8058] \cdot 10^9$ for the error elements, and the same values but $[\cdot] \cdot 10^6$ for the error integrals. These elements are set depending on some coefficients related to the backwater surface area of each canal, giving higher weights to water level errors than water level error integrals. Furthermore, the diagonal elements of $Q_{\mathcal{N}}$ that are related to the previous input signals are set equal to the corresponding value of matrix $R_{\mathcal{N}}$.

Regarding packet losses, their probability of occurrence for controller design is set as $\rho = 0.6$, i.e., 60% of the packets are lost along the way almost surely. To design the corresponding PI controller, the LMIs of Section III-A are

TABLE I
PROPORTIONAL AND INTEGRAL PARAMETERS OF PI CONTROLLERS.

No. Pool	Designed PI controller for 60% of packet losses			PI controller without losses [1]		PIF controller [6]	
	K_p	K_i	K_u	K_p	K_i	K_p	K_i
1	0.5651	0.0194	-1.0005	1.4104	0.0428	1.4274	0.0151
2	0.6607	0.0214	-1.0002	2.5172	0.0727	1.4626	0.0201
3	0.6843	0.0233	-1.0006	2.1411	0.0659	1.4942	0.0175
4	0.6647	0.0208	-1.0003	6.1181	0.1715	1.7483	0.0255
5	1.0588	0.0307	-1.0006	6.8971	0.1939	1.4000	0.0303
6	0.7607	0.0214	-1.0001	8.5357	0.2331	1.3129	0.0349
7	1.0065	0.0312	-1.0020	5.3329	0.1621	1.2043	0.0467

simultaneously solved with $\beta = 0.999$ and $\varphi = 1.0007$. The solution found satisfies all requirements at the limit and the resulting PI coefficients are presented in Table I. Note that there is an extra element K_u that corresponds to the elements added in the state vector for the previous control signals.

Remark: The controller is designed in a centralized way with all canals considered a single system regarding packet losses, i.e., $l_i(k) = l_j(k),\ i, j \in \mathcal{N}$. However, note the model is almost decentralized due to system matrices A and B, and so is matrix P, making the controller suitable in the case where each canal manages packet losses independently.

C. Alternative tuning methods

The proposed method is compared with the same controller obtained with the same design method but without considering the possibility of packet losses. That is, the controller is design by following the method of Section III-A but only subject to the LMI constraint (7). Thus, this is the method proposed in [1].

Another tuning method taken from [6] is used to assess the proposed controllers: the fourth Filtered Proportional-Integral (PIF) controller. This method presents the best performance of all methods presented in [6] and consists of a PIF controller for each pool with the resonance frequency determined based on the maximum cross-over frequency and integral constants adjusted based on downstream resonance.

The PI coefficients used for both controllers are also presented in Table I.

D. Key Performance indicators (KPIs)

Some performance indicators commonly used in canal control and recommended by [4] are the *maximum absolute error* (MAE), the *integral of absolute magnitude of error* (IAE) and the *integrated absolute discharge change* (IAQ).

Besides those indicators, there are some other performance indicators considered that are usually applied in engineering studies: the *mean of mean absolute error* (MMAE), the *mean of standard deviations* (MSTD), and the *sum of standard deviations* (SSTD).

Finally, regarding water resources management, there are two indicators commonly used to compare the performances of different water resource systems: *resilience*, which is a metric defining how quickly a system is likely to recover or bounce back from failure once a failure has occurred, and *vulnerability*, which refers to the likely magnitude of failure

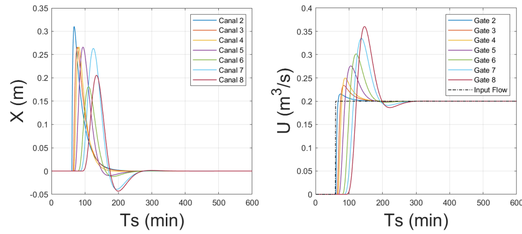

Fig. 1. Simulation results for step change of 0.2 m^3/s: Scenario 1 with the designed PI controller.

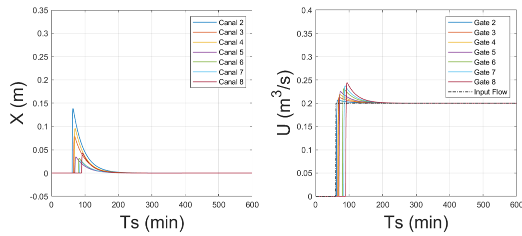

Fig. 2. Simulation results for step change of 0.2 m^3/s: Scenario 1 with the PI controller of [1].

if it occurs [12], [21]. Note that failure is considered to be the occurrence of unsatisfactory performance. If the system is in a steady state, any input flow or offtakes changes are considered as a failure.

V. Simulation Results and discussion

All the methods considered were evaluated using the same scenario. The system starts in a steady state of input flows and offtakes, and a step of 0.2 m^3/s in input flow is given at hour 4.

Besides, two different scenarios have been simulated:

- **Scenario 1**: no packet losses.
- **Scenario 2**: packet losses with a probability of occurrence of 30%.
- **Scenario 3**: packet losses with a probability of occurrence of 60%.

For Scenarios 2 and 3, in case of packet loss in a reach, it maintains the input signal from the previous time step.

The results obtained from Scenario 1 are presented in Figs. 1, 2 and 3 for the proposed PI controller, the PI controller of [1] and the PIR controller of [6], respectively. Likewise, Figs. 4, 5 and 6 represent the results for the three methods from Scenario 2, when packet losses are considered with a probability of occurrence of 30%. And Figs. 7, 8 and 9 represent the results for the three methods from Scenario 3, where packet loss probability is 60%. For each method the water level errors, X (m), and the flow rates through check gates, U (m^3/s), are depicted at each time sample (T_s = 4min).

Moreover, the KPIs indicated in the previous section have been calculated for each method and simulation, and they are presented in Tables II–V for the three scenarios.

To summarize all the obtained results, a broad comparison

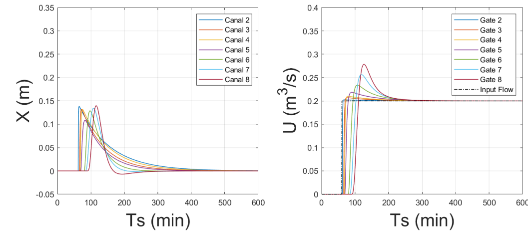

Fig. 3. Simulation results for step change of 0.2 m^3/s: Scenario 1 with the PIF controller of [6].

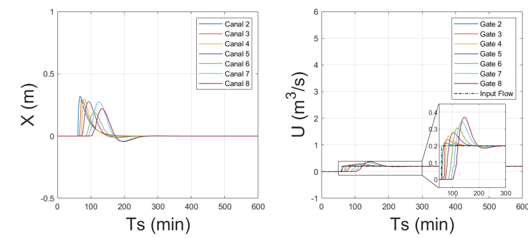

Fig. 4. Simulation results for step change of 0.2 m^3/s: Scenario 2 with the designed PI controller.

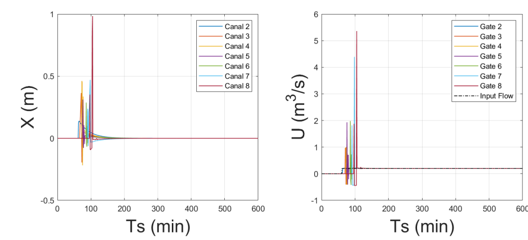

Fig. 5. Simulation results for step change of 0.2 m^3/s: Scenario 2 with the PI controller of [1].

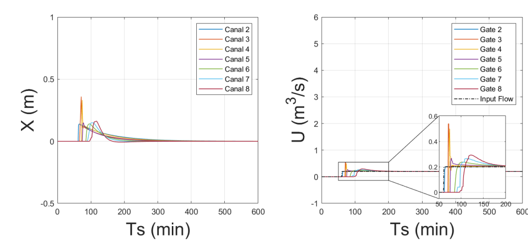

Fig. 6. Simulation results for step change of 0.2 m^3/s: Scenario 2 with the PIF controller of [6].

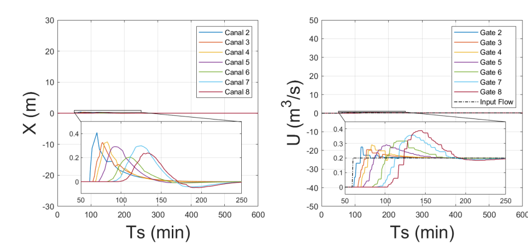

Fig. 7. Simulation results for step change of 0.2 m^3/s: Scenario 3 with the designed PI controller.

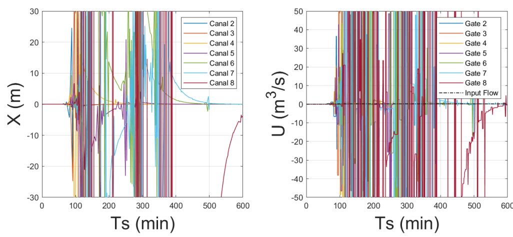

Fig. 8. Simulation results for step change of 0.2 m^3/s: Scenario 3 with the PI controller of [1].

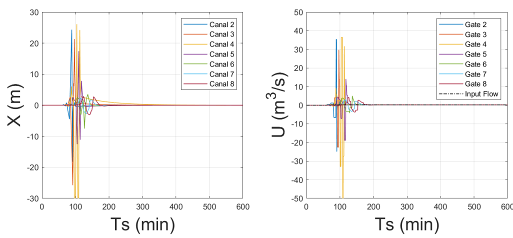

Fig. 9. Simulation results for step change of 0.2 m^3/s: Scenario 3 with the PIF controller of [6].

TABLE II

MAE, IAE AND IAQ.

Methods	MAE (10^{-2})		IAE(10^3)		IAQ (10^{-2})	
	Max	Mean	Max	Mean	Max	Mean
Scenario 1: no packet losses						
Proposed PI controller	31.07	25.15	3.20	2.43	34.87	14.95
PI controller of [1]	13.87	6.45	1.12	0.51	8.86	4.44
PIF controller of [6]	13.98	13.07	3.17	2.08	15.69	5.17
Scenario 2: packet losses with a probability of occurrence of 30%						
Proposed PI controller	31.80	26.76	3.24	2.43	36.86	15.96
PI controller of [1]	98.46	43.12	1.12	0.71	1524.49	604.71
PIF controller of [6]	35.91	20.42	3.17	2.08	71.26	24.28
Scenario 3: packet losses with a probability of occurrence of 60%						
Proposed PI controller	40.89	30.04	3.23	2.43	41.03	22.50
PI controller of [1]	108297218.57	15568047.12	1956151.31	281569.27	3596254605.61	454470210.53
PIF controller of [6]	5978.08	2009.44	167.87	54.14	41248.86	10389.44

TABLE III

MMAE, MSTD AND SSTD.

Methods	MMAE (10^{-3})	MSTD (10^{-2})	SSTD (10^{-2})
Scenario 1: no packet losses			
Proposed PI controller	16.82	4.99	34.95
PI controller of [1]	3.51	1.04	7.30
PIF controller of [6]	14.39	2.90	20.27
Scenario 2: packet losses with a probability of occurrence of 30%			
Proposed PI controller	16.85	5.14	35.97
PI controller of [1]	4.93	2.85	19.98
PIF controller of [6]	14.40	3.09	21.62
Scenario 3: packet losses with a probability of occurrence of 60%			
Proposed PI controller	16.84	5.23	36.62
PI controller of [1]	1952088.64	1217410.94	8521876.58
PIF controller of [6]	375.35	171.03	1197.22

TABLE IV

RESILIENCE.

RESILIENCE (10^{-3} min^{-1})								
Methods	Canal 1	Canal 2	Canal 3	Canal 4	Canal 5	Canal 6	Canal 7	Mean
Scenario 1: no packet losses								
Proposed PI controller	6.45	7.25	9.43	12.50	12.50	12.05	12.82	10.43
PI controller of [1]	6.37	7.04	7.52	8.93	9.43	10.00	10.20	8.50
PIF controller of [6]	2.17	2.91	2.50	3.41	5.75	8.70	13.51	5.56
Scenario 2: packet losses with a probability of occurrence of 30%								
Resilience $10^(- 3) : ProposedPIcontroller$	6.45	7.35	9.35	12.50	13.16	12.66	13.33	10.69
PI controller of [1]	6.37	7.41	8.00	9.52	9.90	7.19	50.00	14.06
PIF controller of [6]	2.17	2.95	2.52	3.46	5.88	9.26	14.93	5.88
Scenario 3: packet losses with a probability of occurrence of 60%								
Proposed PI controller	6.49	7.09	9.09	12.82	13.16	12.82	5.92	9.63
PI controller of [1]	4.33	4.29	2.85	2.42	1.96	1.96	1.98	2.83
PIF controller of [6]	1.86	2.02	1.89	2.01	2.84	5.92	5.81	3.19

TABLE V

VULNERABILITY.

VULNERABILITY (10^{-2} m)								
Methods	Canal 1	Canal 2	Canal 3	Canal 4	Canal 5	Canal 6	Canal 7	Mean
Scenario 1: no packet losses								
Proposed PI controller	31.07	26.64	26.66	26.64	18.15	26.34	20.58	25.15
PI controller of [1]	13.87	7.99	9.65	3.48	3.18	2.64	4.32	6.45
PIF controller of [6]	13.87	13.23	13.20	10.84	12.89	13.44	13.98	13.07
Scenario 2: packet losses with a probability of occurrence of 30%								
Proposed PI controller	31.80	29.15	30.03	27.78	19.07	27.48	21.99	26.76
PI controller of [1]	13.87	36.42	46.17	31.15	28.74	47.02	98.46	43.12
PIF controller of [6]	13.87	35.91	33.11	14.90	14.01	14.90	16.23	20.42
Scenario 3: packet losses with a probability of occurrence of 60%								
Proposed PI controller	40.89	32.76	33.16	29.19	20.47	29.92	23.93	30.04
PI controller of [1]	2459.91	4629.26	18176.27	61891.34	132780.88	459173.61	108297218.57	15568047.12
PIF controller of [6]	2434.81	2577.15	5978.08	1740.62	753.20	273.02	309.18	2009.44

between all methods is provided for each scenario independently:

- Scenario 1: the proposed PI controller reaches higher maximum error values than the other two controllers, as Figs. 1 to 3 and MAE values of Table II show. On the other hand, the designed PI is the softest of the three controllers, since it has been designed considering packet losses. This is reflected by the input flow rate graphs of Figs. 1 to 3, and also, by the IAQ values of Table II. Therefore, the designed PI performance is not as good as the performance of the PI designed without considering packet losses [1], which has the best KPIs values.
- Scenario 2: in case of packet losses with a low probability of occurrence, all controllers get worse performances and all KPIs achieve higher values than in the previous scenario (Tables II–V). The worst KPI values are achieved by the second method considered, PI [1], since it is the fastest controller and in case of loss, the performance worsening becomes greater. Its MAE and IAQ values of Table II are the ones that increase further if comparing with Scenario 1. However, the PIF controller [6] successfully faces data losses and system stability is still held. Its KPIs values are still kept in the same range as the PI designed.
- Scenario 3: in case of packet losses with a higher probability of occurrence, the PI controller of [1] provides very poor results as Fig. 7 shows, and thus, all KPIs values suffer a large increase. Furthermore, the PIF controller [6] presents a much worse performance than before (Fig. 9). However, the proposed PI is characterised by the lowest IAQ values (Table II), since it is the softest controller, with the corresponding advantage for the preservation of gates. Its reached error values become greater than in previous scenarios, but the increase is not as much as the one of the other two methods, as it is reflected by MAE values (Table II). Therefore, the designed PI is the only method that efficiently manages packet losses, without getting hard system performance worsening.

In conclusion, the designed PI is the controller that faces better the presence of packet losses between the system and the controller, especially when their probability of occurrence increases. Therefore, the proposed controller is the only one that provides the certainty of stability guarantees when dealing with faulty communication networks.

VI. Conclusion

PID controllers require tuning parameters that create a map between functions of the error and control actions, possibly considering several performance indicators along the way. Here, we tune stage cost weighting parameters, which are related to water level errors and control effort, and then the proposed LMI generates the PI controller mapping seeking for the minimization of the cost-to-go of the closed-loop system. Hence, the designer is much closer to system performance. For this reason, our proposal is halfway between a PI controller and an optimal feedback control law. The designer focuses on tuning parameters of a cost function and obtains PI controllers to accomplish that goal. Also, it is worth mentioning that the design method is centralized, hence exploiting synergies between local controllers and avoiding undesired interactions between PIs, as happens with other design methods.

Finally, the fact that the problem is cast as an LMI also allows including additional requirements into the design problem with simplicity. In this case, the controller is designed as robust against communication failures. This way, the proposed controller keeps nearly the same performance features despite the presence of packet losses. Moreover, this controller is the only one that guarantees system stability when dealing with packet losses. Hence the proposed PI controller represents a useful tool for irrigation canal control.

Further work will include the extension towards infinite dimensional control approaches which take into account the time delay problem among others.

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