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Jakovljević, Ž., Marković, V., Miladinović, M.,

RECOGNITION OF ELLIPTICAL SEGMENTS IN SCANNED LINES

Abstract: *Cylindrical surfaces, as one of the most frequent surfaces in mechanical engineering, are represented by elliptical (circular) or linear segments in scanned lines within structured point cloud. Having this in mind, segmentation and fitting of elliptical regions is a very important issue in the recognition of cylindrical surfaces. This paper presents the research in the recognition of elliptical (or circular) segments in scanned lines. Segments connected with G1 (or higher) continuity are considered. Presented method is based on seed independent region growing using direct least squares fitting of ellipses. The method is tested in the case studies considering synthesized as well as real world (scanned lines) examples.*
Key words: *Reverse engineering, segmentation, ellipse*

1. INTRODUCTION

The state of the art 3D scanning devices are characterized by high speed, resolution and accuracy and after scanning generate very dense point clouds with more than sufficient data [1]. The efficient algorithms for raw data preprocessing (registration, integration and meshing) are available [2] and creation of 3D triangular mesh from point cloud is a standard feature of contemporary CAD software. However, the efficient algorithms for recognition (segmentation, classification and fitting) of geometric primitives from point cloud still remain a challenge. The most critical element of recognition process is the segmentation of elementary regions (surfaces) from point cloud. Once the point cloud is segmented, there is a number of efficient surface fitting and classification techniques [3, 4] that are usually based on surface parameter estimation.

The majority of parts in mechanical engineering are bounded by planar surfaces and quadrics. Therefore, besides planes, the recognition of quadrics in point clouds is of a crucial importance.

There are two basic groups of strategies for segmentation of quadrics from point cloud [5]: 1) strategies based on edge detection, and 2) strategies based on regions.

In edge based techniques, the edges (boundaries) of surfaces are first detected and the segmentation is carried out using obtained faces' boundaries. This approach is convenient for G0 continuous surfaces where there is an abrupt change between adjacent faces. However, for G1 continuous surfaces, it is very difficult to detect edges. Note that in mechanical engineering there are a large number of parts with G1 continuous quadrics, usually as a result of cutting tool radius.

Region based segmentation is carried out by split and merge approach or by region growing. In these methods, the points belonging to homogenous regions are classified/clustered using different criteria usually based on differential parameters. For example in [6] region growing is carried out using local surface normals, in [7] curvature tensor and principle curvature in vertices were utilized as features, while in [8] region growing is carried out based on average curvature in the point and specific geometric constraints for selected quadrics. The results of segmentation by region growing are highly dependant on seed point which is a rule selected manually [8]. Alternative approach is random seed selection, or selection of seed point near estimated edges [7].

An interesting approach for segmentation of G0 continuous quadric regions from point cloud based on numerically stable least square fitting of ellipse is presented in [9].

The most of scanning devices as a low level output give structured point cloud which consists of a sequence of scanned lines. This structure is sometimes deteriorated by further processing during registration and integration of multiple views, and in some devices scattered point cloud is generated as a high level output. However, it is possible to restore this structure using certain techniques.

Of all quadrics (ellipsoid, sphere, hyperboloid, cone…), the cylinder is most frequently met in mechanical elements. In scanned lines, cylindrical surfaces are represented by elliptical (circular) or linear segments. Consequently, segmentation and fitting of elliptical regions in scanned lines is a very important issue in the recognition of cylindrical surfaces.

This paper presents the research in the field of recognition of elliptical segments in scanned lines. Segments connected with G1 continuity are considered. Presented segmentation method is based on direct least squares fitting of ellipses [4] and, in its essence, it is region growing method. The method is tested using a number of synthesized and real world examples.

The rest of the paper is organized as follows. In Section 2 we provide basic background regarding direct least squares fitting of ellipses [4]. Section 3 presents the methodology for segmentation of ellipses from scanned lines. In Section 4 the method is tested using a real world example, while in Section 5 we give concluding remarks.

2. LEAST SQUARES FITTING OF ELLIPSE

Conic section, in general, can be represented by the following equation:

$$a_1x^2 + a_2xy + a_3y^2 + a_4x + a_5y + a_6 = 0 \quad (1)$$

For ellipse, the coefficients a_i must satisfy the relation:

$$a_2^2 - a_1a_3 < 0 \quad (2)$$

Eq. (1) can be given in the vector form:

$$\mathbf{x} \cdot \mathbf{a} = 0 \quad (3)$$

where $\mathbf{a}=[a_1\ a_2\ a_3\ a_4\ a_5\ a_6]^T$ denotes the coefficients and $\mathbf{x}=[x^2\ xy\ y^2\ x\ y\ 1]$.

The problem of ellipse fitting can be formulated as estimating parameters **a**, given the observations $\mathbf{x_n}$, n=1,…N. Using the least squares method, the coefficients **a** can be obtained by the following minimization:

$$\min_{\mathbf{a}} \|\mathbf{Da}\|^2 \quad (4)$$

where **D** represents the design matrix in the form:

$$\mathbf{D} = \begin{bmatrix} x_1^2 & x_1y_1 & y_1^2 & x_1 & y_1 & 1 \\ x_2^2 & x_2y_2 & y_2^2 & x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_N^2 & x_Ny_N & y_N^2 & x_N & y_N & 1 \end{bmatrix} \quad (5)$$

In order to customize the least squares method for direct least squares fitting of ellipses, and make the parameter estimation more efficient Fitzgibbon et al. [4] introduced the following constraint:

$$a_2^2 \quad a_1a_3 = 1 \quad (6)$$

to the minimization problem (4). Constraint (6) can be rewritten in the matrix form:

$$\mathbf{a}^T\mathbf{Ca} = 1$$

with **C**(3, 2)=2, **C**(2, 2)=-1 **C**(3, 1)=2 and **C**(i, j)=0 otherwise. The minimization problem can be represented by the system of equations:

$$\mathbf{Sa} = \lambda\mathbf{Ca} \\ \mathbf{a}^T\mathbf{Ca} = 1 \quad (7)$$

where $\mathbf{S}=\mathbf{D}^T\mathbf{D}$ denotes the scatter matrix.

Fitzgibbon et al [4] have shown that the system of equations (7) has a single positive eigenvalue, and that this value gives the solution for **a**.

When all points $\mathbf{x_n}$ are sampled from an ellipse without noise, the scatter matrix **S** is close to singular matrix, and it is impossible to estimate the parameters using presented procedure.

3. SEGMENTATION OF ELLIPTICAL REGIONS FROM SCANNED LINES

The methodology for segmentation of G1 continuous elliptical regions from scan lines that we propose in this paper is based on the singularity of scatter matrix in the case of points sampled from exact ellipse. When points are sampled from approximate ellipse (e.g. scanned ellipse) matrix **S** will be close to singular matrix, and two conditions will be fulfilled: 1) matrix reciprocal condition number will be close to zero; 2) the eigenvalue can be close to zero.

If the scanned profile is examined by growing point by point starting from the first point on the line, as long as all points belong to one elliptical segment, scatter matrix will be close to singular. When during region growing, points sampled from a different ellipse are taken into consideration, i.e. on the transition from the preceding to the next elliptical (or other) region, scatter matrix will not be close to singular any more. The transition point can be detected by simple tresholding of reciprocal condition number and eigenvalue of

INPUT: N points on scan line, coordinates **x1, y1**

```
for i=1:N
   x=x1(1 : i);
   y=y1(1 : i);
   D=[x^2 x*y y^2 x y ones(i)]
   S=D'*D
   if rcond(S)>threshold_1
      coef=i; break
   else
      [svek, svr]=eig(inv(S)*C)
      sv=abs(svr(find(svr>0)))
      if sv<threshold_2
         coef=i; break; end; end
end;
```

OUTPUT: coef-point on the segment's boundary

Fig. 1. Segmentation of elliptical regions using scatter matrix

scatter matrix **S**. By subsequent application of proposed methodology until all points on the scan line are exhausted, all points on region boundaries can be detected.

The pseudo code for application of the presented methodology is given in Figure 1. The thresholds for reciprocal condition number and eigenvalue should be tuned depending on the noise level in the scanned line.

To verify the presented methodology, we have synthesized a line which is composed of 5 C1 continuous elliptical/circular segments. Segments' parameters and region boundary points are given in Table 1. Note, that the points are not uniformly sampled along x axis. Proposed algorithm recognized boundary points presented in Table 2 .

To test the performances of the algorithm in the presence of noise, we have added white Gaussian noise with signal to noise ratio (SNR) of 100dB to the synthesized signal. During testing, we have noticed that the original algorithm is prone to over-segmentation. However, since in the presence of noise, sampled points do not belong to exact ellipse, it is possible to estimate the elliptical

Seg no.	Coefficients [a1 a2 a3 a4 a5 a6]	Sample no. Start	End
1	[5.88 -8.73 10.92 -341.47 -312.47 10^4]	1	1617
2	[-1.04 0 -1.04 237.30 -275.52 10^4]	1618	1747
3	[0.99 0 1.76 -190.86 -129.68 10^4]	1748	2653
4	[-1.98 0 -1.98 263.83 -92.37 10^4]	2654	2847
5	[0.68 0 0.30 -165.69 -16.28 10^4]	2848	3903

Table 1. Parameters of synthesized C1 continuous segments

INPUT: **x**, **y** – coordinates of points on scan line
coef – vector of **N** points on boundaries

```
l=1; k=0;
for i=l:N-1
   x1=x(coef(l):coef(i-1)); y1=y(coef(l):coef(i-1));
   estimate a1
   calculate estimated y1_e
   x2=x(coef(l) : coef(i)); y2=y(coef(l) : coef(i));
   estimate a2
   calculate estimated y2_e
   if abs(sum(abs(y1_e-y1))/length(y1)-
      abs(sum(abs(y2_e-y2))/length(y2)>threshold
       k=k+1; l=i-1;
      coef1(k)=coef(i-1); break
end; end;
```

OUTPUT: coef1 - point on the merged segment boundary

Fig. 2. Merging of over-segmented elliptical regions using parameters' estimate

Seg. no.	Sample number: Case 1 Start of segment	Case 1 End of segment	Case 2 Start of segment	Case 2 End of segment
1	1	1617	1	1624
2	1618	1747	1625	1763
3	1748	2657	1763	2660
4	2658	2852	2661	2831
5	2853	3903	2832	3903

Table 2. Recognized elliptical regions: case 1 – signal without noise; case 2 – signal with noise

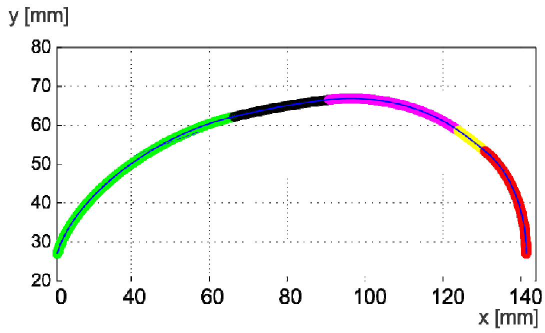

Fig. 3. Merging of over-segmented elliptical regions using parameters' estimate

Table 3. Estimated segments' parameters for synthesized signal with noise

Seg. no.	$a_1\times10^{-4}$	$a_2\times10^{-4}$	$a_3\times10^{-4}$	$a_4\times10^{-4}$	$a_5\times10^{-4}$	a_6
1	5.311	-7.312	10.176	-366.7	-293.5	1
2	0.156	-1.444	4.054	66.7	-399.4	1
3	1.123	-0.077	1.818	-211.4	-115.0	1
4	0.253	0.648	0.421	-100.6	-129.4	1
5	0.686	-0.004	0.306	-166.1	-15.9	1

segments' parameters **a**. These estimated parameters can be further exploited for merging of adjacent over-segmented regions using algorithm presented in Figure 2. After segmentation and merging of over-segmented regions, the boundary points presented in Table 2 (case 2) were recognized. Figure 3 shows synthesized signal along with segmented elliptical regions. Finally, estimated parameters of elliptical segments are presented in Table 3.

From Tables 1 and 2, it can be observed that the proposed algorithm adequately segmented synthesized elliptical regions. In the case of signal without noise, the tolerance between actual and detected region boundary was in the range from 0 to 5 samples, while in the case of signal with noise, as expected, this tolerance was higher - in the range from 7 to 16 samples. Maximal relative error between estimated (Table 3) and actual (Table 1) values of elliptical segments' parameters is 12%.

4. EXPERIMENTAL VERIFICATION

For experimental validation of proposed algorithm we have used a scan profile of a part presented in Fig. 4. This part represents a benchmark test for algorithms in reverse engineering tasks [5]. Part was scanned using ATOS Compact Scan 3D scanner, and unstructured 3D point cloud was generated. To extract scan lines from 3D triangular mesh, we have employed modified z buffer algorithm with representation of mesh triangles in barycentric coordinates. The direction of z axis was arbitrarily selected, and the scan lines on the surface of interest are generated from point cloud. One of the scan lines, along with the results of application of proposed segmentation algorithm is presented in Figure 5.

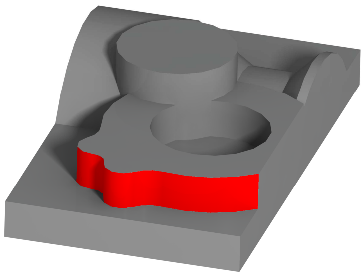

Fig. 4. Scanned part and surface of interest

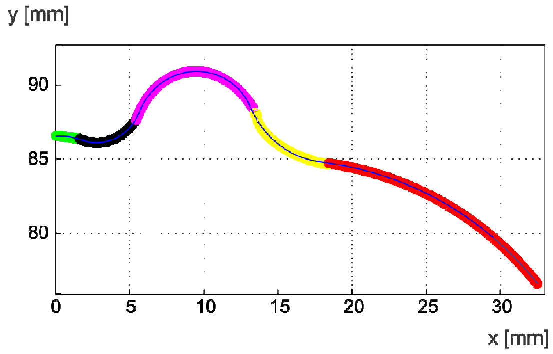

Fig. 5. Segmented elliptical regions on the scanned line

From Figure 5 it can be observed that the algorithm was able to adequately segment elliptical regions in the scan line. These regions correspond to cylinders on the scanned part.

5. CONCLUSION

In this paper we have presented a method for recognition (segmentation and fitting) of elliptical regions from scanned lines. The method is based on region growing using direct least squares fitting of ellipses and it is independent of seed selection. Although the objective of the research was segmentation of ellipses, the estimation of ellipses' parameters (fitting) was carried out as an additional effect of the proposed algorithm.

Very important characteristic of the method is that it is applicable for effective segmentation of regions with G1 (or higher) continuity where there is no abrupt change in the form of visible edge between adjacent segments. The algorithm has shown excellent performances on the synthesized as well as on the real world signal.

The future work will address the extension of the proposed method to segmentation of quadrics.

6. REFERENCES

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Authors: **Assist. Prof. Dr. Živana Jakovljević, M.Sc. Veljko Marković, M.Sc. Momčilo Miladinović,** University of Belgrade, Faculty of Mechanical Engineering, Department for Production Engineering, Kraljice Marije 16, 11000 Belgrade, Serbia, Phone.: +381 11 3302404, Fax: +381 11 3370364.
E-mail: zjakovljevic@mas.bg.ac.rs
vmarkovic@mas.bg.ac.rs
mmiladinovic@mas.bg.ac.rs