Chapter

Performance Comparison of the Ball and Beam System using Linear Quadratic Regulator Controller

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Abstract

This paper proposes the performance comparison of a linear quadratic regulator (LQR) controller for the ball and beam system (BBS). The BBS is a standard benchmark control system, which has two degree-of-freedom (2 DOF). It is an open loop and a highly nonlinear unstable system. This makes its parameter difficult to be estimated accurately, hence designing a controller for it is a challenging task. MatheThe BBS was modelled using Euler–Lagrange modeling technique, while the LQR controller was used for the stabilization of the ball on the beam. Simulation was done in MATLAB/Simulink 2022b environment, and the results simulated showed that for the two weighting matrices (Q *and* R), the state weighting matrix had a higher penalty on the ball displacement, ball velocity, beam angle, and beam angular velocity at lower values of Q. For the state weighting matrix had a better effect of penalty performance on the BBS with lower values. Also, as the diagonal element of the state weighting matrix Q increases from 0.1 to 20, the values of the optimal controller K increase, the reduced Ricatti matrix P increases, and the estimated eigenvalues E reduce. This implies that the ball displacement, ball velocity, beam angle, and beam angular velocity are better at lower values of Q.

Keywords: ball and beam system (BBS), linear quadratic regulator (LQR), weighting matrices, optimal controller

1. Introduction

Nonlinear systems play an important role in the field of control engineering. This is because suitable control techniques are used to improve the system performance [1]. The ball and beam system (BBS) is a highly nonlinear benchmark control problem in the field of control engineering. This is similar to practical control problems like balance control, position control, and tracking control problems [2]. BBS consists of a rigid beam that rotates freely in a vertical plane around the axis, while the ball rolls along the beam. The system can be categorized into two configurations, the first configuration is the ball and beam balancer, in which the beam is supported in the middle, and it rotates against its central axis. The second configuration is built with the beam supported by two-level arms on both sides. One of the level arms acts as the pivot, while the other is coupled to the motor output gear [3, 4]. The purpose of the BBS is to hold the ball in a desired position on the beam while controlling the ball position by adjusting the angle of the beam [5, 6].

The BBS is used to implement and analyze the results of different modern control algorithms [7]. The control structure of this system is used for many different schemes in practical applications. It is used for demonstrating control applications like aircraft roll-yaw applications [8]. It is widely used due to its nonlinearity, simplicity, and open-loop instability. The control objective is the stabilization of the ball on the beam while tracking the reference trajectory [1, 9].

The BBS comprises of the base, the ball, a beam, support block, gear, and motor. This is shown in **Figure 1**.

The beam consists of two parts; the first part of the beam consists of a rigid shaft, while the other part rotates up and down on which the ball moves freely on it [10].

However, there are some research done on the system in applying different control algorithms to stabilize and perform trajectory tracking of the ball on the beam. Rahmat et al. [11] investigated the performance of some control techniques that consist of a proportional-integral-derivative (PID) controller, linear quadratic regulator (LQR) controller, and neural network (NN) designed in terms of stabilization and trajectory tracking. It showed that the LQR controller had a better satisfactory result. In Ref. [12], the particle swarm optimization (PSO) algorithm was used to tune the gains of the PID controller for the BBS. The optimized PSO-PID controller was compared with fuzzy logic controller (FLC) and integral of time multiplied by absolute error (ITAE), which the optimized PID outperformed the other two techniques. Kazemi et al. [13] designed cascade PD and fuzzy cascade controllers for stabilization of the BBS. The gains of the PD controller were optimized using the asexual reproduction optimization (ARO) algorithm. The results of PD-optimized ARO were compared with the fuzzy-cascaded controller in which the tuned PD-ARO outperformed the fuzzy-cascaded PD. Also, Ezzabi et al. [14] demonstrated a nonlinear backstepping technique for controlling the ball position of the BBS. The results were compared with LQR controller, it showed that the nonlinear technique required less input magnitude to achieve a better performance than the LQR controller. In Ref. [15], control strategies that were based on optimal control synthesis were presented. LQR and H_2 controllers were used to control the ball on the beam. The control systems were implemented on a real BBS with a data acquisition card of DSP F28335. A new control strategy was proposed by Ref. [16] to control the stabilization of the BBS by the use of

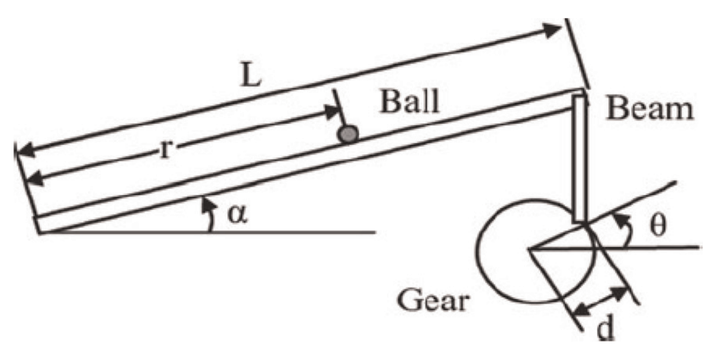

Figure 1.
Ball and beam system [10].

active disturbance rejection control (ADRC) on the system. The simulated results were compared with the proportional integration differentiation controller in which ADRC had a better performance than the integration differentiation controller. While Howimanporn et al. [17] developed a nonlinear discrete optimal control technique for the regulation of all the state variables in the discrete mode of the BBS. The proposed controller showed passivity, stability, and optimality properties during closed-loop operation. In Ref. [18], the BBS was designed using pole placement and LQR. The ball was able to be stabilized on the beam, and the results showed that LQR performed better than the pole-placement method. While an adaptive control was implemented in Ref. [19] for the BBS. Linear state-feedback model reference adaptive control (MRAC) was used in synchronizing the states of the BBS with the given reference model. Results showed that the error convergence was improved for different sets of the sinusoidal reference signal for the MRAC with modified feedback gains.

The main contribution of this article is the investigation of the performance effect of LQR controller on the BBS. This will be done by varying the Q and R matrix on the system, and observing which of the weighting matrix has a penalty effect on the minimization of the performance index of the LQR controller. However, the simulation was implemented in MATLAB/Simulink 2022b environment by adopting Lagrange's equation for modeling the system.

The rest of the paper is organized as follows: Section one introduces the BBS, while section two presents the mathematical model of the system. Section three discusses the controller design of the system and section four gives the simulated results. Finally, the conclusion is given in section five.

2. Mathematical model of the ball and beam system

The BBS is a two degree-of-freedom (DOF) system, the lateral movement of the ball is represented by its position on the horizontal axis, while the vertical movement of the beam is represented by the angle with the horizontal axis [16, 18]. The ball position is given by a sensor allocated at one end of the beam. The angle of the beam is adjusted by a toque provided by an actuator placed at the other end, where there is a connected axis [20]. The BBS can be simplified and linearized using the following assumptions [21]:

i. The ball rolls on the beam without slippage.

ii. The link connected to the beam is solid.

iii. The is no friction on the ball and beam surface, gears, and motor.

iv. The beam angle of rotation has no effect on the behavior of the system.

The equation which describes the dynamics of the system is obtained by using Euler Lagrange equation based on the energy balance of the system as follows [22, 23]:

$$\left(\frac{I_b}{R^2} + m\right)\ddot{r} + mg\sin\theta - mr\dot{\theta}^2 = 0 \tag{1}$$

$$\left(mr^2 + I + I_b\right)\ddot{\theta} + 2mr\dot{r}\dot{\theta} + mgr\cos\theta = u \tag{2}$$

where m is the mass of the ball, g is the acceleration due to gravity, I is the beam moment of inertia, I_b is the ball moment of inertia, r is the position of the ball, R is the radius of the ball, θ is the angle of the beam, and u is the torque applied to the beam.

The model of the system can be described by the following state variables as x_1 represents the ball position along the beam, x_2 is the velocity of the ball, x_3 is the beam angle, and x_4 is the beam angular velocity. The generalized coordinate is given as [24]:

$$[x_1 \;\; x_2 \;\; x_3 \;\; x_4] = \begin{bmatrix} r \\ \dot{r} \\ \theta \\ \dot{\theta} \end{bmatrix} \tag{3}$$

The state space representation of the system is given by [25]:

$$\dot{x}_1 = x_2 \tag{4}$$

$$\dot{x}_2 = -\alpha x_3 \tag{5}$$

$$\dot{x}_3 = x_4 \tag{6}$$

$$x_4 = -\beta x_3 + \gamma x_4 \tag{7}$$

where

$$\alpha = \frac{M_g}{\frac{J_b}{R_b{}^2} + M} \tag{8}$$

$$\beta = \frac{M_g}{J + J_b} \tag{9}$$

$$\gamma = \frac{1}{J + J_b} \tag{10}$$

The parameters of the BBS used for this article are given in **Table 1**.

Parameters	Value
M	$0.05kg$
R_b	$0.01m$
g	$9.81m/s^2$
l	$40cm$
d	$4cm$
J	$2.0 \times 10^{-2}kgm^2$
J_b	$2.0 \times 10^{-6}kgm^2$

Table 1.
Ball and beam system parameters.

3. Linear quadratic regulator (LQR) controller

The LQR controller takes the state equation of the system as feedback and generates a feedback error signal, as shown in **Figure 2**.

Given the system dynamics, the optimization procedure is to find an optimal control law that minimizes the performance index J, which is given as [26]:

$$J = \int_0^{t_1} \left(x^T Q x + u^T R u\right) dt \tag{11}$$

The optimal control law and the optimal controller are given as [26]:

$$u_{opt} = -R^{-1} B^T P x \tag{12}$$

$$K = R^{-1} B^T P \tag{13}$$

Substituting the value of the Eq. (13), into (12), the optimal control law is given as [23]:

$$u_{opt} = -Kx \tag{14}$$

From Eq. (11), the P matrix must satisfy the reduced matrix equation, which is given as [26]:

$$A^T P + PA - PBR^{-1} B^T P + Q = 0 \tag{15}$$

3.1 Selection of Q and Rmatrices

Using Eq. (11), matrices Q and R penalizes the performance of the states, which control the response of the system and the cost of the energy consumed by the system. To improve on the system performance, the Q matrix is been considered, while to improve on the cost, R matrix is been focused on. The Q and R matrices are chosen to be a diagonal matrix while considering the effect of increasing or decreasing their values.

The Q and R matrices used for this research are:

$$Q_0 = diag([\,0.1 \quad 0.1 \quad 0.1 \quad 0.1\,]) \tag{16}$$

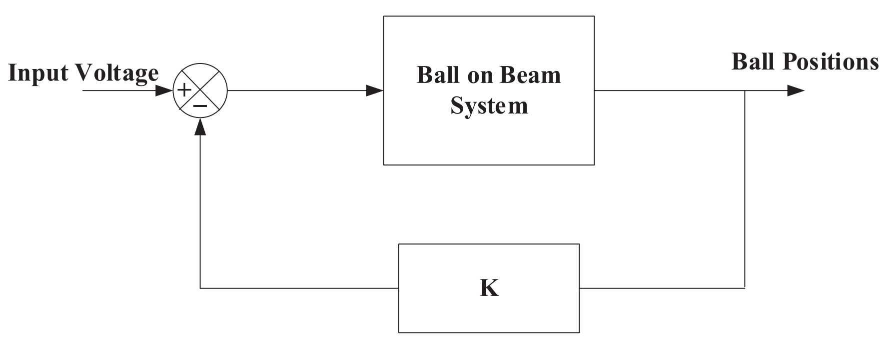

Figure 2.
LQR control structure.

$$Q_1 = diag([1 \quad 1 \quad 1 \quad 1]) \tag{17}$$

$$Q_2 = diag([10 \quad 10 \quad 10 \quad 10]) \tag{18}$$

$$Q_3 = diag([20 \quad 20 \quad 20 \quad 20]) \tag{19}$$

$$R_0 = diag([1]) \tag{20}$$

4. Results and discussion

The results of the BBS were generated from MATAB 2022b environment. The system has been proven to be controllable and observable, which gives way to further perform analysis of stabilizing the ball on the beam. Various values of the states weighting matrices Q were simulated against the control weighting matrix R, and the various values of the optimal controllers K, reduced Ricatti matrix P, and the estimated eigenvalues E were extracted.

For Q_0, R_0, the values of the K_0, P_0, E_0 extracted are:

$$K_0 = [-1.0741 \quad -0.6445 \quad 1.8936 \quad 0.4192] \tag{21}$$

$$P_0 = \begin{bmatrix} 0.3761 & 0.1577 & -0.2447 & -0.0215 \\ 0.1577 & 0.1393 & -0.2487 & -0.0129 \\ -0.2447 & -0.2487 & 0.7035 & 0.0379 \\ -0.0215 & -0.0129 & 0.0379 & 0.0084 \end{bmatrix} \tag{22}$$

$$E_0 = \begin{bmatrix} -1.6935 + 0.0000i \\ -1.7190 + 2.1616i \\ -1.7190 - 2.1616i \\ -15.8277 + 0.0000i \end{bmatrix} \tag{23}$$

The effect of the ball displacement, ball velocity, beam angle, and beam angular velocity is shown in **Figure 3**.

From **Figure 3**, it is seen that the ball stabilizes at about 2.8 secs on the beam on the x-axis, while the ball's velocity was stabilized at about 5.05 secs. Also, the beam angle was stabilized at about 4.7 secs, while the beam angular velocity was stabilized at about 5.1 secs. This shows that the state weighting matrix Q has a penalty on the ball position and also on the beam angle.

Also, for Q_1, R_0, the values of the K_1, P_1, E_1 extracted are:

$$K_1 = [-1.6043 \quad -1.6123 \quad 5.0314 \quad 1.0960] \tag{24}$$

$$P_1 = \begin{bmatrix} 1.7958 & 0.7998 & -1.2208 & -0.0321 \\ 0.7998 & 0.9835 & -1.7350 & -0.0322 \\ -1.2208 & -1.7350 & 5.2886 & 0.1006 \\ -0.0321 & -0.0322 & 0.1006 & 0.0219 \end{bmatrix} \tag{25}$$

$$E_1 = \begin{bmatrix} -1.1087 + 0.0000i \\ -1.8503 + 1.9018i \\ -1.8503 - 1.9018i \\ -49.9866 + 0.0000i \end{bmatrix} \tag{26}$$

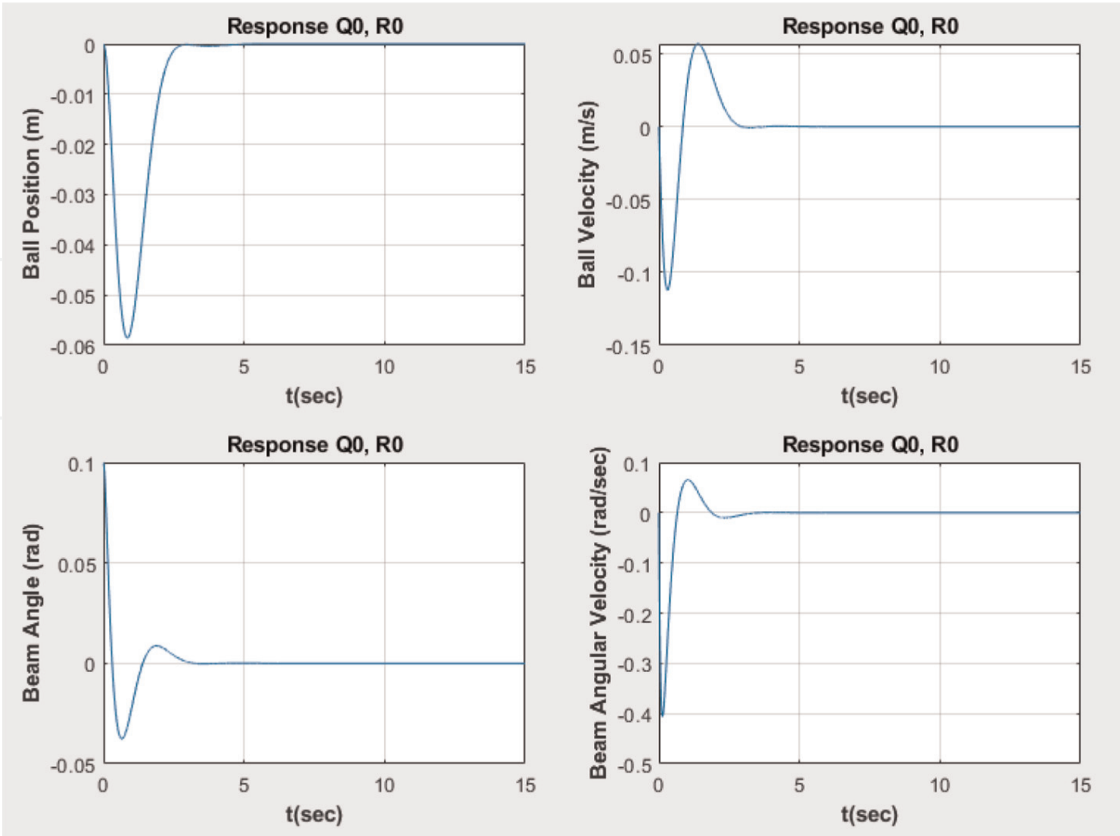

Figure 3.
Q_o, R_o weighting matrices.

The effect of the ball displacement, ball velocity, beam angle, and beam angular velocity is shown in **Figure 4**.

From **Figure 4**, it is seen that the ball stabilizes at about 5.5 secs on the beam on the x-axis, while the ball's velocity was stabilized at about 7.5 secs. Also, the beam angle was stabilized at about 5.2 secs, while the beam angular velocity was stabilized at about 5.8 secs. This shows that the state weighting matrix Q has a penalty on the ball position and also on the beam angle.

Also, for Q_2, R_0, the values of the K_2, P_2, E_2 extracted are:

$$K_2 = [-3.6906 \quad -4.8907 \quad 15.2386 \quad 3.2572] \tag{27}$$

$$P_2 = \begin{bmatrix} 15.6506 & 6.9593 & -10.4235 & -0.0738 \\ 6.9593 & 9.1483 & -15.8562 & -0.0978 \\ -10.4235 & -15.8562 & 48.9502 & 0.3048 \\ -0.0738 & -0.0987 & 0.3048 & 0.0652 \end{bmatrix} \tag{28}$$

$$E_2 = 10^2 \begin{bmatrix} -0.0101 + 0.0000i \\ -0.0187 + 0.0187i \\ -0.0187 - 0.0187i \\ -1.5809 + 0.0000i \end{bmatrix} \tag{29}$$

The effect of the ball displacement, ball velocity, beam angle, and beam angular velocity is shown in **Figure 5**.

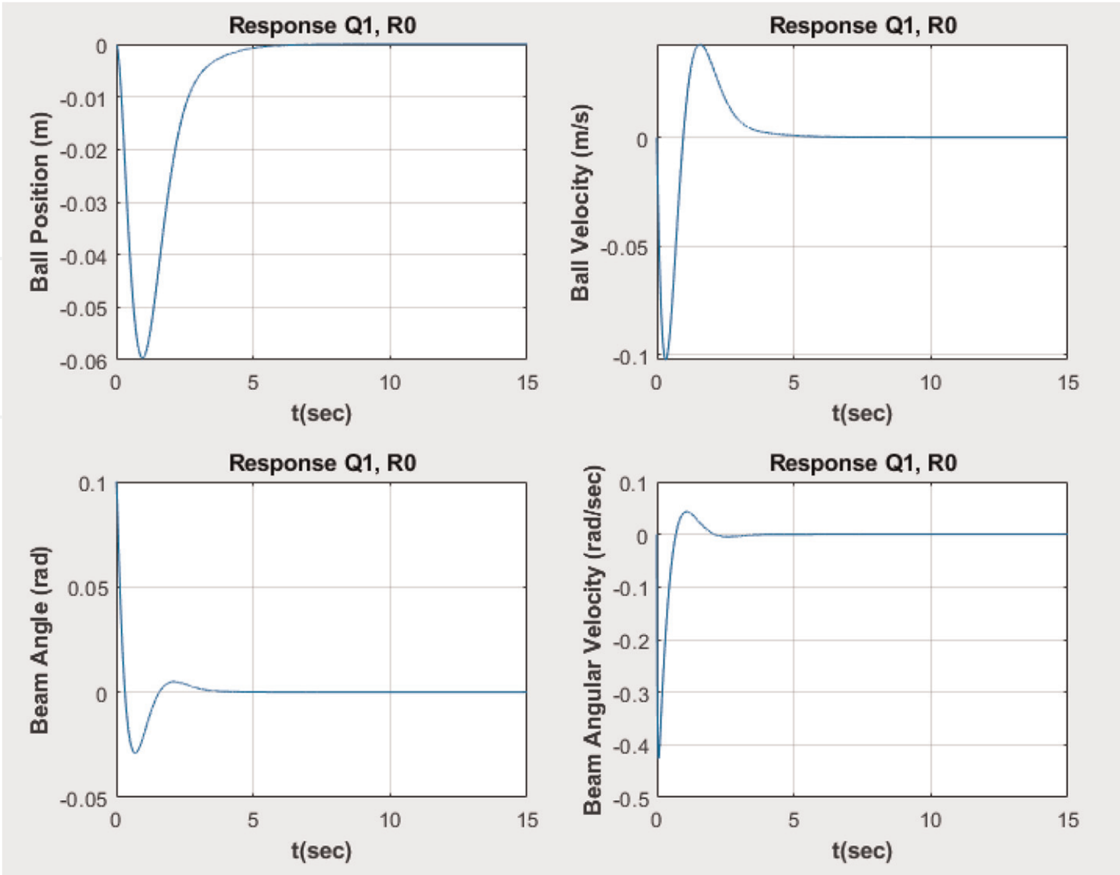

Figure 4.
Q_1, R_0 weighting matrices.

From **Figure 5**, it is seen that the ball stabilizes at about 7.1 secs on the beam on the x-axis, while the ball's velocity was stabilized at about 8.2 secs. Also, the beam angle was stabilized at about 6.1 secs, while the beam angular velocity was stabilized at about 6.6 secs. This shows that the state weighting matrix Q has a penalty on the ball position and also on the beam angle.

However, for Q_3, R_0, the values of the K_3, P_3, E_3 extracted are:

$$K_3 = [-4.9895 \quad -6.8948 \quad 21.4451 \quad 4.5670] \tag{30}$$

$$P_3 = \begin{bmatrix} 31.0192 & 13.7689 & -20.5469 & -0.0998 \\ 13.7689 & 18.1689 & -31.3889 & -0.1379 \\ -20.5469 & -31.3889 & 96.9744 & 0.4289 \\ -0.0998 & -0.1379 & 0.4289 & 0.0914 \end{bmatrix} \tag{31}$$

$$E_3 = 10^2 \begin{bmatrix} -0.0101 + 0.0000i \\ -0.0187 + 0.0187i \\ -0.0187 - 0.0187i \\ -2.2358 + 0.0000i \end{bmatrix} \tag{32}$$

The effect of the ball displacement, ball velocity, beam angle, and beam angular velocity is shown in **Figure 6**.

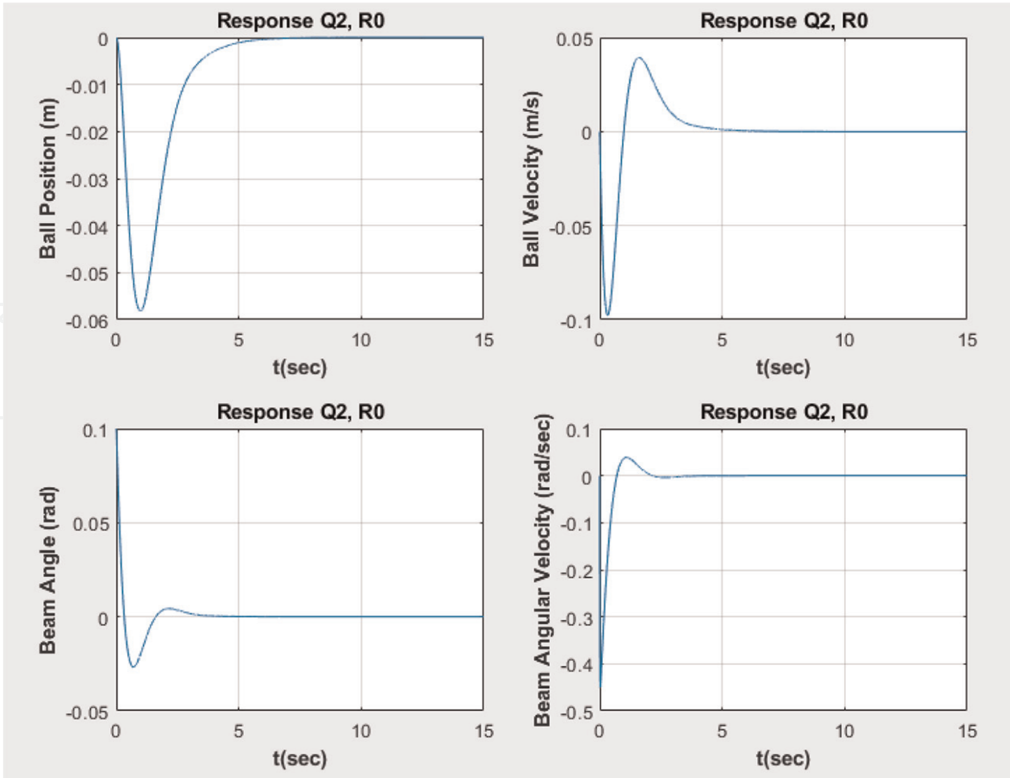

Figure 5.
Q_2, R_0 *weighting matrices.*

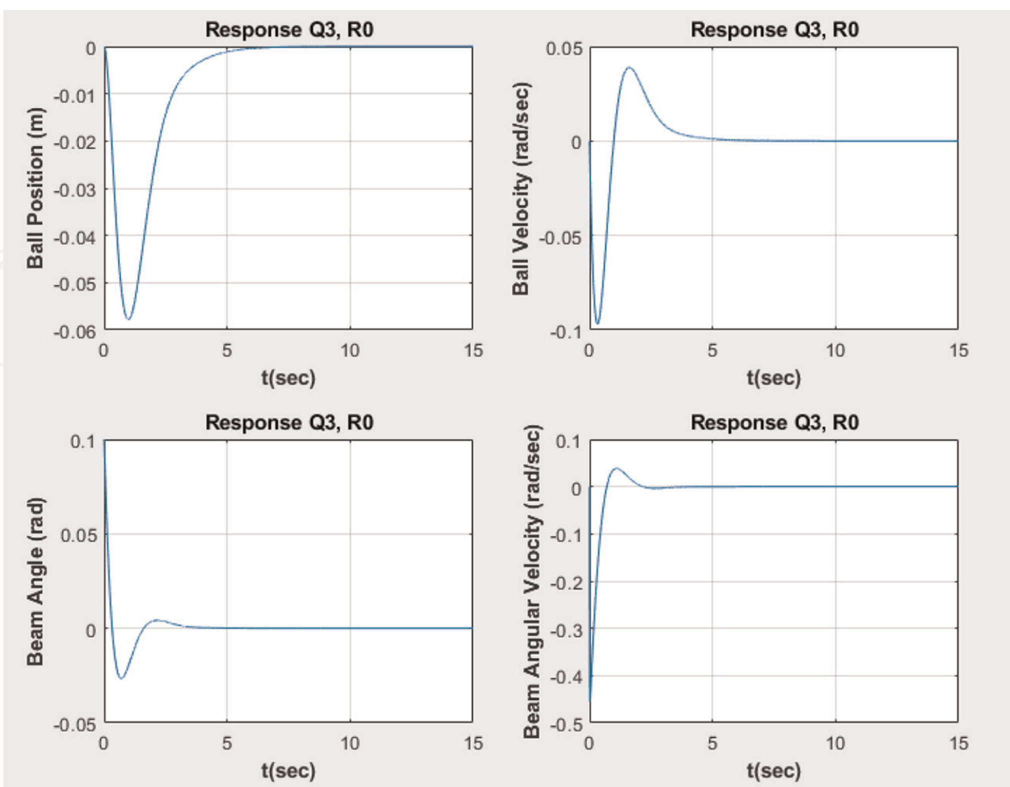

Figure 6.
Q_3, R_0 *weighting matrices.*

From **Figure 6**, it is seen that the ball stabilizes at about 7.6 secs on the beam on the x-axis, while the ball's velocity was stabilized at about 8.7 secs. Also, the beam angle was stabilized at about 6.5 secs, while the beam angular velocity was stabilized at about 7.2 secs. This shows that the state weighting matrix Q has a penalty on the ball position and also on the beam angle.

From **Figures 3–6**, it can be deduced that as the leading element of the state weighting matrix Q increases from 0.1 to 20, the values of optimal controller K increases, the reduced Ricatti matrix P also increases, and the estimated eigenvalues also reduces. This implies that the ball displacement, ball velocity, beam angle, and beam angular velocity are better at lower values of Q. This shows that the state weighting matrix has an effect on penalty performance on the BBS within lower values of Q.

5. Conclusion

An analysis on the effect of the state and control weighting matrices $(Q\, and\, R)$ on a benchmark control problem, the ball and beam system (BBS) has been studied. The system is a 2 DOF, an open loop, and a highly nonlinear unstable system, which makes estimating its parameters difficult, hence designing a controller was a difficult task. The BBS is an underactuated system with multiple input multiple output characteristics. Lagrange modeling technique was used for modeling the system, and LQR controller was used for the stabilization of the ball on the beam. A simulation was done in MATLAB/Simulink 2022b environment, and it showed that the state weighting matrix had a higher penalty on the ball displacement, ball velocity, beam angle, and beam angular velocity at lower values of Q. Also, it showed that as Q increases from 0.1 to 20, the values of the optimal controller K increase, the reduced Ricatti matrix P increases, and the estimated eigenvalues E also reduce. This implies that the ball displacement, ball velocity, beam angle, and beam angular velocity are better at lower values of Q. Future research will consider the effect of varying the control weighting matrix R over some range of values on the BBS system, which can be further applied to the field of autonomous vehicles.

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