# Efficient query processing over uncertain road networks 

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# EFFICIENT QUERY PROCESSING OVER UNCERTAIN ROAD NETWORKS 

## A Thesis

by

## BAMIKOLE A OGUNDELE

Submitted to the Graduate School of<br>The University of Texas- Pan American<br>In partial fulfillment of the requirements for the degree of MASTER OF SCIENCE

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Major Subject: Computer Science

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May 2015

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#### Abstract

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One of the fundamental problems on spatial road networks has been the shortest traveling time query, with applications such as location-based services (LBS) and trip planning. Algorithms have been made for the shortest time queries in deterministic road networks, in which vertices and edges are known with certainty. Emerging technologies are available and make it easier to acquire information about the traffic. In this paper, we consider uncertain road networks, in which speeds of vehicles are imprecise and probabilistic. We will focus on one important query type, continuous probabilistic shortest traveling time query (CPSTTQ), which retrieves sets of objects that have the smallest traveling time to a moving query point $q$ from point $s$ to point $e$ on road networks with high confidences. We propose effective pruning methods to prune the search space of our CPSTTQ query, and design an efficient query procedure to answer CPSTTQ via an index structure.


## DEDICATION

This thesis is dedicated to my sweetheart and loving wife, Adekumbi, and my children, Precious, Cherish, and Emmanuel. My mother-in-law, Yetunde and my mother Abigael. The completion of my master program would not have been possible without the unfailing love, sacrifice and support of my family. I give my deepest expression of love and appreciation for the encouragement that you gave and the sacrifices you made during this graduate program. Thank you for the support and company during late nights of typing and endurance when away from home.

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## CHAPTER I

## INTRODUCTION

It has been a major problem determining the shortest traveling time from one location to the other due to some variation in traffic. Finding objects like a gas station, hotel, or coffee shop with the shortest traveling time on a path from a particular source point $s$ to a destination point $e$ has been a challenge over the decades. One of the fundamental problems is the shortest traveling time query, which has many significant applications, such as Web-based mapping services and network routing. As a result, several methods have been developed for shortest time queries in deterministic graphs, in which vertices and edges (associated with edge weights) are known with certainty.

However, in some applications, the road networks may contain some uncertainties. For example, a road network can be modeled as a graph, in which each edge represents a highway, but in reality some highways are likely to have traffic jams and thus be associated with uncertain vehicle speeds.

Therefore, route planning on such road networks should consider uncertain traveling time on roads and provide reasonable paths for users in any circumstances. However, recent GPSenabled path services ignore the uncertainty inherently contained in traffic, and this uncertainty leads to probabilistic graphs in which edges are labeled with uncertain velocity values (samples) associated with existence probabilities. Moreover, the uncertainty could be as a result of noisy measurements from sensors/experiments, the existence of unstable communication links, or
unknown factors such as traffic accidents, and weather conditions (e.g. snow or thunderstorms). The existence probability of an edge can be predicted by some machine learning algorithm, or inferred from historical traffic data.

Figure 1 below shows an example of a road network (graph), which contains 14 nodes that are intersection points of roads $\mathrm{n}_{1}, \mathrm{n}_{2}, \ldots, \mathrm{n}_{14}$ denoted as circles, and edges that are road segments, $e_{i, j}$ may not exist between two nodes $n_{i}$ and $n_{j}($ for $1 \leq i, j \leq 14)$. On road segments (e.g., $e_{3,4}$ ), there are several facilities like the gas station and the rest area denoted with a square shape (e.g., $o_{1}$ and $o_{8}$, respectively).


Figure 1: Uncertain Road Network Model

In many real-world applications such as route planning, some people may want to travel from one location to another with the best alternate road. But the shortest traveling time from one point (hotel) to another point (airport) is challenging. Many analysis has shown that the shortest
path from one location to another does not necessarily guarantee that it will have the shortest traveling time. There are many factors (bad road, road traffic, accident, stop lights and signs) that could contribute to the fact that the shortest path may take longer traveling time.

In this paper, we will investigate an important type of query, namely continuous probabilistic shortest traveling time queries (CPSTTQ), over uncertain road networks, which retrieves points of interest (POIs) or facilities on an uncertain road networks with the shortest traveling time and high confidence, while a given query point $q$ is moving from the source point $s$ to the destination point $e$ on road networks. In order to efficiently process CPSTTQ queries, we propose effective pruning methods, time bound pruning and probabilistic bound pruning, to reduce the CPSTTQ search space. Even though many works have been proposed for various queries on road networks, most of the prior works did not consider the uncertainty of the traveling time but rather focused on certain shortest traveling path (with the shortest traveling distance). Thus, we cannot borrow previous techniques to tackle our CPSTTQ query.

## CHAPTER II

## PROBLEM DEFINITION

Uncertainty Model. In this paper, we will analyze the data model of traffic network over an uncertain road network with continuous probabilistic nearest neighbor queries. Several applications have been developed over the years to track the location of an object and calculate the traveling time. For example, GPS Tracking is installed on most phones which use state-of-the-art GPS technology quickly and accurately locate the people, providing real-time location updates using GPS navigation. We will consider the shortest traveling time to reach a nearest neighbor over continuous line segment.

Figure 1 illustrates that there are intersection nodes $\left(n_{1} \sim n_{14}\right)$ and POI objects ( $o_{1} \sim o_{8}$ ) in a road network. The nodes are the intersection points of roads, which are denoted with $n_{1}, n_{2}$, $\ldots, n_{14}$ on the road network, whereas POI objects are facilities, which are denoted with $o_{1}, o_{2}, \ldots$, , $o_{8}$ corresponding to a gas station, hotel, airport, coffee shop, hospital, restaurant, or rest area on the road network. Previous analyses have always considered the shortest traveling distance from one location to the other based on the belief that the shorter the distance, the faster approach to the destination. However, in reality, the road network may be associated with uncertain traffic information such as uncertain vehicle speeds, which can thus be modeled as an uncertain road network.

In particular, each edge in uncertain road networks is associated with an uncertain speed variable, which follows some probabilistic distribution (pdf) (represented by vehicle speed
samples). Table 1 shows an example of nodes in uncertain road networks in Figure 1, and Table 2 depicts the edge information of uncertain road network in Figure 1. For example, edge $e_{1,2}$ has 3 possible speed samples, capturing the uncertainty of the vehicle speed on this edge.

| Node | Location $(\boldsymbol{x}, \boldsymbol{y})$ | Node | Location $\boldsymbol{x}, \boldsymbol{y})$ |
| :---: | :---: | :---: | :---: |
| $n_{1}$ | $(6,7)$ | $n_{8}$ | $(5,3)$ |
| $n_{2}$ | $(5,8)$ | $n_{9}$ | $(7,4)$ |
| $n_{3}$ | $(4,8)$ | $n_{10}$ | $(10,4)$ |
| $n_{4}$ | $(1,8)$ | $n_{11}$ | $(10,6)$ |
| $n_{5}$ | $(1,6)$ | $n_{12}$ | $(10,8)$ |
| $n_{6}$ | $(2,5)$ | $n_{13}$ | $(8,8)$ |
| $n_{7}$ | $(4,4)$ | $n_{14}$ | $(8,5)$ |

Table 1: The Nodes of Uncertain Road Networks

| Edge | Distance <br> dist. $\left(\boldsymbol{n}_{\boldsymbol{i}}, \boldsymbol{n}_{\boldsymbol{j}}\right)$ | Velocity <br> $\left(\boldsymbol{e}_{\boldsymbol{i}, \boldsymbol{j}}\right)$ |
| :---: | :---: | :---: |
| $e_{i, 2}$ | $\sqrt{2}$ | $(35,0.3)(36,0.3)(37,0.4)$ |
| $e_{2,3}$ | $\sqrt{1}$ | $(35,0.4)(45,0.2)(50,0.4)$ |
| $e_{3,4}$ | 3 | $(55,0.3)(58,0.3)(60,0.4)$ |
| $e_{3,6}$ | $\sqrt{13}$ | $(30,0.6)(32,0.3)(35,0.1)$ |
| $e_{4,5}$ | 2 | $(40,0.7)(42,0.2)(45,0.1)$ |
| $e_{5,6}$ | $\sqrt{2}$ | $(36,0.2)(40,0.4)(46,0.4)$ |


| $e_{6,7}$ | $\sqrt{5}$ | $(55,0.3)(58,0.3)(60,0.4)$ |
| :---: | :---: | :---: |
| $e_{7,8}$ | $\sqrt{2}$ | $(42,0.3)(45,0.3)(50,0.4)$ |
| $e_{8,9}$ | $\sqrt{5}$ | $(30,0.5)(35,0.2)(45,0.3)$ |
| $e_{9,10}$ | 3 | $(50,0.3)(55,0.3)(60,0.4)$ |
| $e_{10,11}$ | 2 | $(45,0.6)(48,0.1)(55,0.3)$ |
| $e_{11,12}$ | 2 | $(28,0.5)(38,0.3)(48,0.2)$ |
| $e_{11,14}$ | $\sqrt{5}$ | $(25,0.2)(30,0.1)(35,0.7)$ |
| $e_{12,13}$ | 2 | $(40,0.6)(42,0.2)(47,0.2)$ |
| $e_{13,14}$ | 3 | $(50,0.3)(55,0.4)(60,0.3)$ |
| $e_{13,2}$ | 3 | $(32,0.7)(42,0.1)(52,0.2)$ |
| $e_{14,1}$ | $\sqrt{8}$ |  |

## Table 2: The Edges of Uncertain Road Networks

DEFINITION 1. (Uncertain Road Network). An uncertain road network is define as a Graph $G=(V, E, T)$ where vertices $V$ is a set $n_{1}, n_{2}, n_{3}, \ldots, n_{|s|}$, each $n_{i}(1 \leq i \leq|S|)$ residing at a $2 D$ location $\left\{x\left(n_{i}\right), y\left(n_{i}\right)\right\}, E$ is a set of edges, where each $e_{i, j}$ in $E$ associated with a velocity variable $Z\left(e_{i, j}\right)$, and $T$ is a mapping from $V \times V$ to $E$ with edges $E$ that shows the traveling time along the edge $e$.

In Definition 1, the variable of a velocity $Z\left(e_{i, j}\right)$ on each edge $e_{i, j}$ of an uncertain road network follows a probabilistic distribution, represented by discrete velocity samples. In particular, each sample $S_{i, j}$ of random variable $Z\left(e_{i, j}\right)$ is associated with an appearance probability $S_{i, j} \cdot p\left(\sum \forall s_{i, j} S_{i, j} \cdot p=1\right)$.

Example 1. (Uncertain Road Network) In an uncertain road network from location $S$ to $E$ there are three possible path to arrive at location $E$ from the starting point of location $S$. The traveling time from all the three paths are different even though they are starting and arriving at the same location, but due to some uncertain traffic network on the road the traveling time varies.

From the example of Figure 1, nodes $n_{1}, n_{2}, \ldots, n_{14}$ are the intersection points on road segments of a continuous road network. Table 1 illustrates all the locations of each node $n_{i}(1 \leq$ $i \leq 14$ ), which is described by 2 D coordinates. Moreover, $e_{i, j}$ is an edge from node ( $n_{i}, n_{j}$ ) whose distance is denoted by $\operatorname{dist}\left(n_{i}, n_{j}\right)$.

Table 2 shows traveling distances and uncertain velocities on each edge, where each edge $e_{i, j}$ is associated with a probability distribution function, $\operatorname{pdf}\left(e_{i, j}\right)$ of the velocity variable.

| Facility | Position $(\boldsymbol{x}, \boldsymbol{y})$ | Edge $\left(\boldsymbol{e}_{i, j}\right)$ |
| :---: | :---: | :---: |
| $\mathrm{O}_{1}$ (Gas Station) | $(2,8)$ | $(3,4)$ |
| $\mathrm{O}_{2}$ (Hotel) | $(3,6.5)$ | $(3,6)$ |
| $\mathrm{O}_{3}$ (Airport) | $(6.5,8)$ | $(2,13)$ |
| $\mathrm{O}_{4}$ (Coffee Shop) | $(8,4)$ | $(9,10)$ |
| $\mathrm{O}_{5}$ (Hospital) | $(3,4.5)$ | $(6,7)$ |
| $\mathrm{O}_{6}$ (Gas Station) | $(9,5.5)$ | $(11,14)$ |
| $\mathrm{O}_{7}$ (Restaurant) | $(6,3.5)$ | $(4,5)$ |
| $\mathrm{O}_{8}$ (Rest Area) | $(1,7)$ | $\left(\begin{array}{l}\text { ( }) \\ \hline\end{array}\right.$ |

## Table 3: Facilities on Uncertain Road Networks

There are many paths from one point, $u$, to another destination location, $v$, on the road network. Following, we define the best path to a destination location.

DEFINITION 2 (The Best Path to a Destination) In a graph $G=(S, T, Y)$, two points $u, v \in V(G)$, and a probability threshold $\alpha \in(0,1)$, a probabilistic path query $(u, v)$ finds all paths $l \in \operatorname{path}(u, v)$, such that $\operatorname{Pr}(l) \geq \alpha$.

There can be many paths between two vertices in a large graph. Often, a user is interested in only the "best" paths. In this paper, we will consider the best alternative path to a location that provides the shortest traveling time. Given a path, $l$, from vertex $u$ to $v$ in an uncertain graph $G, l$ is called the shortest path over $G$ if and only if $l$ is the shortest path in at least one instance of $G$. The probability that $l$ is the shortest path from $u$ to $v$ in $G$ is denoted as $\operatorname{Pr}\{l\}$.

Example 2. Table 3 illustrates all the facilities points on the road network in Figure 1, $o_{1}$ (Gas Station), $o_{2}$ (Hotel), $o_{3}$ (Airport), $o_{4}$ (Coffee Shop), $o_{5}$ (Hospital), $o_{6}$ (Gas Station), $o_{7}$ (Restaurant), and $o_{8}$ (Rest Area). Each facility $o_{i}(1 \leq i \leq 8)$ has its own location on the road network and its corresponding edge, e.g., the facility Gas Station $o_{1}$ is at position $(2,8)$ on road segment $e_{3,4}$, Hotel $o_{2}$ is at position $(3,6.5)$ on road segment $e_{3,6}$, and so on.

In the example, there are different paths to get to a location on the road network. For example, traveling on this road network from edge $e_{1}$ to a facility Hotel $o_{2}$, there are different paths that lead to the facility. You may travel on the road network $(R N)$ from $e_{1} \rightarrow e_{2} \rightarrow e_{3} \rightarrow\left(o_{2}\right)$, or from $e_{1} \rightarrow e_{2} \rightarrow e_{3} \rightarrow e_{4} \rightarrow e_{5} \rightarrow e_{6} \rightarrow\left(o_{2}\right)$, or from $e_{1} \rightarrow e_{8} \rightarrow e_{7} \rightarrow e_{6 \rightarrow}\left(o_{2}\right)$. We will discuss the best path to get to each location on the road network.

Definition 3 (Probabilistic Shortest Traveling Time Queries, PSTTQ). Consider a road network $R N$ with a set of facilities $o_{1}, o_{2}, \ldots, o_{n}$ on $R N$, a static query point $q$ with a probabilistic threshold $\alpha$, a PSTTQ returns a set of objects $o_{i}$ that have the shortest traveling time to the static query point $q$ with probability, $\operatorname{Pr}_{P S T T Q}\left(q, o_{i}\right), \geq \alpha$, that is,

$$
\begin{equation*}
\operatorname{Pr}_{P S T T Q}\left(q, o_{i}\right)=\sum\left(\operatorname{Pr}\left\{t\left(q, o_{i}\right)=T\right\} \Pi\left(1-\operatorname{Pr}\left\{t\left(q, o_{j}\right) \leq T\right\}\right)\right) \geq \alpha \tag{1}
\end{equation*}
$$

In Definition 3, we calculate the probability of the traveling time, $t\left(q, o_{i}\right)$ from $q$ to $o_{i}$, for each possible value $T$ of $t\left(q, o_{i}\right)$ which will allow us to find the query result with the shortest time to a moving query point $q$ and with high confidence.

The condition below determines the probability that $o_{i}$ is the PSTTQ, i.e., The probability that the time cost $t\left(q, o_{i}\right)$ equals $T$ and the time costs $t\left(q, o_{i}\right)$ of any other objects are higher than $T$ should be greater than or equal threshold $\alpha$. Assuming independent velocities on connecting edges, the PSTTQ probability $\operatorname{Pr}_{P S T T Q}\left(q, o_{i}\right)$, is given by summing up the probabilities that $t\left(q, o_{i}\right)=T$ and $t\left(q, o_{i}\right)>T$ for other objects $o_{j}$ over all possible values of $T$.

Example 3. Given a query point $q$ from location sto $e$ on a road network with a probabilistic threshold $\geq \alpha$, the PSTTQ query retrieves those objects (e.g. gas stations) from $S$ ( $o_{1}, o_{6}$ ) that have the smallest traveling times with probabilities $\geq \alpha$. We compute the traveling time $t$ and its probability $p$ from point $q$ to $o_{1}$ and $o_{6}$ the traveling time and its probability on each edge are shown in the Table 4.

| Path | $\boldsymbol{t}\left(\boldsymbol{q}, \boldsymbol{n}_{\mathbf{1}}\right)$ | $\mathbf{p}$ | $\boldsymbol{t}\left(\boldsymbol{n}_{\mathbf{1}}, \boldsymbol{n}_{\mathbf{2}}\right)$ | $\mathbf{p}$ | $\boldsymbol{t}\left(\boldsymbol{n}_{\mathbf{2}}, \boldsymbol{n}_{\mathbf{3}}\right)$ | $\mathbf{p}$ | $\boldsymbol{t}\left(\boldsymbol{n}_{\mathbf{3}}, \boldsymbol{o}_{\mathbf{1}}\right)$ | $\mathbf{p}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{Path}\left(q, o_{1}\right)$ | 0.024 | 0.1 | 0.031 | 0.3 | 0.050 | 0.3 | 0.19 | 0.2 |
|  | 0.022 | 0.2 | 0.046 | 0.5 | 0.048 | 0.3 | 0.17 | 0.1 |
|  | 0.020 | 0.3 | 0.056 | 0.1 | 0.044 | 0.2 | 0.16 | 0.3 |
|  | 0.019 | 0.4 | 0.053 | 0.1 | 0.041 | 0.2 | 0.14 | 0.4 |
| $\operatorname{Path}\left(q, o_{6}\right)$ | 0.054 | 0.6 | 0.56 | 0.5 |  |  |  |  |
|  | 0.049 | 0.4 | 0.54 | 0.5 |  |  |  |  |

Table 4: Possible traveling times of road segments on path $\left(\boldsymbol{q}, \boldsymbol{o}_{\boldsymbol{i}}\right)$ from $\boldsymbol{q}$ to gas station

In Table $4, t\left(q, n_{1}\right)$ records four possible values of the traveling time in minutes on the first edge $e_{q: n_{1}}$ of the path from $q$ to $n_{1}$, and $\mathbf{p}$ represents their probabilities. Based on Eq. (1) we have $\operatorname{Pr}_{P S T T Q}\left(q, o_{1}\right)=0, \operatorname{Pr}_{P S T T Q}\left(q, o_{6}\right)=89.663 \%$. Object $o_{6}$ has a high probability of having the shortest time to query object $q$, among all traveling times of the three objects. Thus, object $o_{6}$ is one of our PSTTQ query answers.

## Definition 4 (Continuous Probabilistic Shortest Traveling Time Queries, CPSTTQ).

Consider a road network $R N$ with a set of facilities $o_{1}, o_{2}, \ldots, o_{n}$ on $R N$, a moving query point $q$ (moving from source point s to destination point e), and a probabilistic threshold $\alpha$, a continuous probabilistic shortest traveling time query (CPSTTQ) returns sub-paths of moving query point $q$ between s and e on the road network, each associated with a set of objects $o_{i}$ that have the shortest traveling time to query point $q$ within this sub-path with probability, $\operatorname{Pr}_{P S T T Q}\left(q, o_{i}\right) \geq \alpha$, where $\operatorname{Pr}_{\text {PSTTQ }}\left(q, o_{i}\right)$ is given by Eq. (1).

As an example, in Figure 2, we have a query point q moving along three connected subpaths, $p_{1}, p_{2}$, and $p_{3}$, with a starting location $s(5,7.5)$, through nodes $n_{1}$ and $n_{14}$, and ending at location $e(10,6)$. The CPSTTQ retrieves these three sub-paths because the query point moves from point $s$ to $e$, and each sub-path is associated with an object set such that any object $o_{i}$ in the set has the shortest traveling time $t\left(q, o_{i}\right)$ to the moving query point $q$ in this sub-path with probability greater than $\alpha$.

From Eq. (1) in Definition 3 we can define the condition of the CPSTTQ answer by: $\forall q$ in the sub-path of path(s,e),

$$
\operatorname{Pr} \operatorname{CPSTTQ}\left(q, o_{i}\right)=\sum\left(\operatorname{Pr}\left\{t\left(q, o_{i}\right)=T\right\} \Pi\left(1-\operatorname{Pr}\left\{t\left(q, o_{j}\right) \leq T\right\}\right) t\left(s, o_{i}\right), t\left(e, o_{i}\right)\right) \geq \alpha
$$

To find the query result with the shortest traveling time to point $q$, we calculate the traveling time $t\left(q, o_{i}\right)$ from $q$ to $o_{i}, t\left(s, o_{i}\right)$ from $s$ to $o_{i}$ and $t\left(e, o_{i}\right)$ from $e$ to $o_{i}$ and its probability for each possible value of $T$ of the traveling time. The shortest traveling time to a moving query point $q$ is then determined within each sub-path on the road network $R N$ and each sub-path is associated with an object set which is the answer to that query point $q$.


Figure 2: Uncertain Road Network Model with Query Point

Example 4. In the example of Figure 2, consider a moving query point $q$ on a road networks with a starting point $s(5,7.5)$ through a path $n_{1}, n_{14}$ and ending point $e(10,6)$ location. There are three sub-paths $p_{1}, p_{2}$, and $p_{3}$ of the path, path $(s, e)$, from the starting point $s$ to the ending point $e$. Each sub-path $p_{i}$ is associated with a set of objects such that the object set is the query answer to query point $q$ on this sub-path. That is, no matter where the query point $q$ is on the sub-path $p_{i}$, the object set within that sub-path is the PSTTQ answer set of $q$.

Figure 2 illustrates an example on a road network with a moving query point $q$ from starting point $s$ to the ending point $e$. As the query point $q$ moves on a road network in any of the subpaths $p_{1}, p_{2}$, and $p_{3}$, the object set contains objects with the shortest traveling time to the query point $q$ with probability $\geq \alpha$.

## CHAPTER III

## PRUNING METHODS

The straightforward method to answer the CPSTTQ is to compute the PSTTQ answers for any static query point $q$ on the path from $s$ to $e$, which is quite inefficient. Therefore, in this section, we propose effective pruning methods to reduce the search space of the CPSTTQ problem. In particular, our goal is to retrieve a small candidate set of CPSTTQ answers for any particular query sub-path (on path $(s, e)$ ), by filtering out bad candidates.

### 3.1 Time Bound Pruning

In this subsection, we propose a time bound pruning mechanism which eliminates objects that are not CPSTTQ query answers. We consider the intervals of two traveling times on road networks $t\left(q, o_{i}\right)$ and $t\left(q, o_{j}\right)$. The basic idea of the time bound pruning is as follows. This time bound pruning method utilizes the lower/upper bounds of the traveling time to enable pruning. Specifically, if the upper bound of the traveling time $t\left(q, o_{j}\right)$ is less than the lower bound of the traveling time $t\left(q, o_{i}\right)$, then we can conclude that $o_{i}$ is not a query answer to the CPSTTQ due to the existence of $o_{j}$. This way, we can eliminate objects from a facility set, $M=\left(o_{1}, o_{2}, \ldots, o_{n}\right)$ by using this pruning method.

LEMMA 1. (Time Bound Pruning). Assume that lower bound $l b_{-} t(x, y)$ and upper bound $u b_{-} t(x, y)$ are the bounds of the traveling time from location $x$ to location $y$ on road networks, respectively. Given a candidate object $o_{i}$, we can safely prune
$o_{i}$ from $M($ set of facilities $)$, if $l b_{-} t\left(q, o_{i}\right)>u b_{-} t\left(q, o_{j}\right)$ assuming that
$t\left(q, o_{i}\right) \epsilon\left[l b_{-} t\left(q, o_{i}\right), u b_{-} t\left(q, o_{j}\right)\right]$ and $t\left(q, o_{j}\right) \in\left[l b_{-} t\left(q, o_{i}\right), u b_{-} t\left(q, o_{j}\right)\right]$ for $o_{i}, o_{j} \in M$.

## PROOF.



## Figure 3: Illustration of Time Bound Pruning

We prove that we are able to prune object $o_{i}$ from the facility set $M$ as follows:

Assume that: $t\left(q, o_{i}\right) \in\left[l b_{-} t\left(q, o_{i}\right), u b_{-} t\left(q, o_{i}\right)\right]$ and $t\left(q, o_{j}\right) \in\left[l b_{-} t\left(q, o_{j}\right), u b_{-} t\left(q, o_{j}\right)\right]$, where $\left[l b_{-} t\left(q, o_{i}\right), u b_{-} t\left(q, o_{i}\right)\right]$ represent the traveling time interval of $t\left(q, o_{i}\right)$ from $q$ to $o_{i}$ and $\left[l b_{-} t\left(q, o_{j}\right), u b_{-} t\left(q, o_{j}\right)\right]$ is the traveling time interval of $t\left(q, o_{j}\right)$ from $q$ to $o_{j}$. By using the transitivity of the inequality, we have: $t\left(q, o_{i}\right) \leq u b_{-} t\left(q, o_{i}\right)$ and $t\left(q, o_{j}\right) \leq u b_{-} t\left(q, o_{j}\right)$

Since the condition that $t\left(q, o_{i}\right) \geq l b_{-} t\left(q, o_{i}\right)>u b_{-} t\left(q, o_{j}\right) \geq t\left(q, o_{j}\right)$ holds, therefore, we can obtain $t\left(q, o_{i}\right)>t\left(q, o_{j}\right)$. This result shows that $o_{i}$ has higher traveling time from $q$ than $o_{j}$, which implies that it is not a CPSTTQ answer. Hence, object $o_{i}$ can be safely pruned from the facility set $M$.

Lemma 1 allows us to use a time bound pruning method to filter out bad candidates via lower and upper bounds of the traveling time on a road network. Intuitively, we use Lemma 1 to prune object $o_{i}$ from the set of objects from the facilities $M$.

The Computation of Lower/Upper Bounds for the Traveling Time. To use the time bound pruning method, we need to compute the lower and upper bounds of the traveling time. For instance, consider the road network containing a path $s \rightarrow n_{1} \rightarrow n_{2} \rightarrow \ldots \rightarrow n_{k}$, where $s$ is the starting point and $n_{k}$ is an ending point of a segment on the road network. This segment on the road network has different sub-paths with nodes $n_{f}, n_{g}$ which belong to path $\left(q, o_{i}\right)$. Therefore, the velocity of the vehicle in this segment $e_{f, g}$ on road networks is given by $v \in\left[v^{-}\left(n_{f}\right.\right.$, $\left.\left.n_{g}\right), v^{+}\left(n_{f}, n_{g}\right)\right]$. Thus, we have the lower and upper bounds of the traveling time from $n_{f}$ to $n_{g}$ on edge $e_{f, g}$, denoted as $l b_{-} t\left(n_{f}, n_{g}\right)$ and $u b_{-} t\left(n_{f}, n_{g}\right)$, which are given by $l b_{-} t\left(n_{f}, n_{g}\right)=\operatorname{dist}\left(n_{f}\right.$, $\left.n_{g}\right) / v^{+}\left(n_{f}, n_{g}\right)$ and $u b_{-} t\left(n_{f}, n_{g}\right)=\operatorname{dist}\left(n_{f}, n_{g}\right) / v^{-}\left(n_{f}, n_{g}\right)$, respectively. Thus, the summation of $l b_{-} t\left(n_{f}, n_{g}\right)$ for all edges $e_{f, g}$ on path $\left(q, o_{i}\right)$ is the lower bound of the time cost $l b_{-} t\left(n_{f}, n_{g}\right)$ of path from $q$ to $o_{i}$, whereas the summation of $u b_{-} t\left(n_{f}, n_{g}\right)$ for all edges $e_{f, g}$ on path $\left(q, o_{i}\right)$ is the upper bound of the time cost $u b_{-} t\left(n_{f}, n_{g}\right)$ of path from $q$ to $o_{i}$.

LEMMA 2. (Lower and Upper Bounds of the Traveling Time). The lower bound of the traveling time from a moving query point $q$ on any subpath of a road network from a location st to location ed is given by:

$$
\begin{equation*}
l b_{-} t\left(q, o_{i}\right)=\min \left\{l b_{-} t\left(s t, o_{i}\right), l b_{-} t\left(e d, o_{i}\right)\right\} \tag{3}
\end{equation*}
$$

The upper bound of the traveling time of a moving query point $q$ on the subpath of a road network from location s to location $e$ is given by:

$$
\begin{equation*}
u b_{-} t\left(q, o_{i}\right)=\max \left\{u b_{-} t(s t, e d)+u b_{-} t\left(s t, o_{i}\right), u b_{-} t(s t, e d)+u b_{-} t\left(e d, o_{i}\right)\right\} \tag{4}
\end{equation*}
$$



Figure 4: A Query point $q$ on a sub-path of a Road Network

## PROOF.

We want to prove that Eq. 3 gives the minimum possible traveling time from $q$ to $o_{i}$. In particular, we consider two possible paths from $q$ to $o_{i}$. Assume that query point $q$, is on a road segment from node $s t$ to node $e d$. Then, the path from $q$ to $o_{i}$ may go through either node $s t$ (i.e., $q \rightarrow s t \rightarrow \ldots \rightarrow o_{i}$ ) or node ed (i.e., $q \rightarrow e d \rightarrow \ldots \rightarrow o_{i}$ ). We have the following inequality with respect to the traveling time, $t\left(q, o_{i}\right)$, from $q$ to $o_{i}$.

$$
t\left(q, o_{i}\right) \geq \min \left\{\left(l b_{-} t(q, s t)+l b_{-} t\left(s t, o_{i}\right)\right),\left(l b_{-} t(q, e d)+l b_{-} t\left(e d, o_{i}\right)\right)\right\} .
$$

Since we have:

$$
l b_{-} t(q, s t) \geq 0 \text { and } l b_{-} t(q, e d) \geq 0
$$

by the inequality transition, we can obtain

$$
t\left(q, o_{i}\right) \geq \min \left\{l b_{-} t\left(s t, o_{i}\right), l b_{-} t\left(e d, o_{i}\right)\right\}
$$

which is exactly equal to the RHS of Eq. 3 (i.e., $l b_{-} t\left(q, o_{i}\right)$ ). Thus, $\min \left\{l b_{-} t\left(s t, o_{i}\right), l b_{-} t\left(e d, o_{i}\right)\right\}$ is the lower bound of the traveling time $t\left(q, o_{i}\right)$.

Similarly, we can also prove the correctness of the upper bound, $u b_{-} t\left(q, o_{i}\right)$, of the traveling time from $q$ to $o_{i}$ in Eq. 4. Specifically, similar to the proof of the lower bound, we consider two paths through st and ed, respectively, on path $\operatorname{path}\left(q, o_{i}\right)$, and derive the following inequality w.r.t. time upper bounds.

$$
t\left(q, o_{i}\right) \leq \max \left\{u b_{-} t(q, s t)+u b_{-} t\left(s t, o_{i}\right), u b_{-} t(q, e d)+u b_{-} t\left(e d, o_{i}\right)\right\} .
$$

Next, since we have:

$$
u b_{-} t(q, s t) \leq u b_{-} t(s t, e d) \text { and } u b_{-} t(q, e d) \leq u b_{-} t(e s t, e d)
$$

by the inequality transition, we can obtain:

$$
t\left(q, o_{i}\right) \leq \max \left\{u b_{-} t(s t, e d)+u b_{-} t\left(s t, o_{i}\right), u b_{-} t(s t, e d)+u b_{-} t\left(e d, o_{i}\right)\right\}
$$

which is exactly equal to the RHS of Eq. 4 (i.e., $u b_{-} t\left(q, o_{i}\right)$ ).

Thus, $\max \left\{u b_{-} t(s t, e d)+u b_{-} t\left(s t, o_{i}\right), u b_{-} t(s t, e d)+u b_{-} t\left(e d, o_{i}\right)\right\}$ is the upper bound of the traveling time $t\left(q, o_{i}\right)$.

Intuitively, to compute traveling time bounds from $q$ to $o_{i}$, we consider two possible paths. The first path is $q \rightarrow s t \rightarrow \ldots \rightarrow o_{i}$ and the other way is $q \rightarrow e d \rightarrow \ldots \rightarrow o_{i}$. The traveling time from $q$ to $o_{i}$ through the path $s t$ is greater than that from st to $o_{i}$. Similarly, the traveling time from $q$ to $o_{i}$ through the path ed is greater than that from ed to $o_{i}$. Therefore, the traveling time from $q$ to $o_{i}$ can be lower bounded by the minimum between the traveling time from $e d$ to $o_{i}$ and that from st to $o_{i}$.

Similarly, to compute the upper bound of the traveling time from $q$ to $o_{i}$, we also consider two possible paths. The traveling time from $q$ to $o_{i}$ through the path st is smaller than that from ed to $o_{i}$ (through $s t$ ). The traveling time from $q$ to $o_{i}$ through the path $e d$ is greater than that
from st to $o_{i}$ (through $e d$ ). Therefore, the traveling time from $q$ to $o_{i}$ can be upper bounded by the maximum between the traveling time from $s t \rightarrow e d \rightarrow \ldots \rightarrow o_{i}$ and that from $e d \rightarrow s t \rightarrow \ldots$ $\rightarrow o_{i}$.

### 3.2 Probabilistic Bound Pruning

In this subsection, we will consider probabilistic information and propose an effective probabilistic bound pruning method to filter out false alarms. For continuous queries, we will take into account all possible locations within the sub-paths as the $q$ moves along the road network from s to $e$. Specifically, we mainly focus on the shortest traveling time $t\left(q, o_{i}\right)$ on a sub-path, path $\left(q_{1}, q_{2}\right)$, within a road network as the query point $q$ moves from one point $q_{1}$ to another one $q_{2}$. We introduce a way to prune those false alarms $o_{i}$ that have low CPSTTQ probabilities (as given in Eq. (2)).

DEFINITION 5 ( $\boldsymbol{\beta}$-Upper-Bound). Let the traveling time $t\left(q, o_{i}\right) \epsilon\left[l b_{-} t\left(q, o_{i}\right), u b_{-} t\left(q, o_{j}\right)\right]$ for $o_{i} \in M($ set of facilities $)$, a $\beta$-upper-bound is denoted as $\left(u b_{-} t\left(q, o_{i}\right) . \beta\right)$, which satisfies $\operatorname{Pr}\left\{t\left(q, o_{i}\right) \in\left[l b_{-} t\left(q, o_{i}\right), u b_{-} t\left(q, o_{i}\right) \cdot \beta\right]\right\}=\beta$.

The $\beta$-upper-bound, $u b_{-} t\left(q, o_{i}\right) \cdot \beta$, is given by an upper bound of the traveling time $u b_{-} t\left(q, o_{i}\right)$ such that $t\left(q, o_{i}\right)$ is the shortest traveling time within the interval $\left[l b_{t\left(q, o_{i}\right)}, u b_{-} t\left(q, o_{j}\right) \cdot \beta\right]$ with probability $\beta$.

LEMMA 3. (Probabilistic Bound Pruning). Assuming that $t\left(q, o_{i}\right) \in\left[l b_{-} t\left(q, o_{i}\right)\right.$, $\left.u b_{-} t\left(q, o_{i}\right)\right]$ and $t\left(q, o_{j}\right) \in\left[l b_{-} t\left(q, o_{j}\right), u b_{-} t\left(q, o_{j}\right)\right]$ for $o_{i}, o_{j} \in M$, if it holds that $\beta>1-\alpha$ and $l b_{-} t\left(q, o_{i}\right)>u b_{-} t\left(q, o_{j}\right) . \beta$, then object $o_{i}$ can be safely pruned from $M$ (set of facilities).

PROOF. We want to prove the correctness of our probabilistic bound pruning, by using the diagram in Figure 5. Specifically, for $o_{i}, o_{j} \in M, t\left(q, o_{i}\right)$ and $t\left(q, o_{j}\right)$ are within the intervals of two traveling times, $l b_{-} t\left(q, o_{i}\right), u b_{-} t\left(q, o_{i}\right)$ and $l b_{-} t\left(q, o_{j}\right), u b_{-} t\left(q, o_{j}\right)$ respectively.

Assume that, $u b_{-} t\left(q, o_{j}\right) \cdot \beta$ is a $\beta$-upper-bound of $t\left(q, o_{j}\right)$ which satisfies $\operatorname{Pr}\left\{t\left(q, o_{j}\right) \in\right.$ $\left.\left[l b_{-} t\left(q, o_{j}\right), u b_{-} t\left(q, o_{j}\right) \cdot \beta\right]\right\}=\beta$. The $u b_{-} t\left(q, o_{j}\right) \cdot \beta$ from Definition 5 indicates that the $\operatorname{Pr}\left(q, o_{j}\right)$ of an object is $\geq 1$. Since the probability of $o_{i}$ has the shortest traveling time to $q$, therefore:

$$
\begin{gathered}
\operatorname{Pr}_{C P S T T Q}\left(q, o_{i}\right) \leq \Sigma\left(\operatorname{Pr}\left\{t\left(q, o_{i}\right)=T\right\} \cdot \Pi\left(1-\operatorname{Pr}\left\{t\left(q, o_{j}\right) \leq T\right\}\right)\right. \\
\leq \Sigma \operatorname{Pr}\left\{t\left(q, o_{i}\right)=T\right\} \cdot\left(\left(1-\operatorname{Pr}\left\{t\left(q, o_{j}\right) \leq T \mid t\left(q, o_{j}\right) \in\left[l b_{-} t\left(q, o_{j}\right), u b_{-} t\left(q, o_{j}\right) . \beta\right]\right\}\right)\right. \\
\cdot \operatorname{Pr}\left\{t\left(q, o_{j}\right) \in\left[l b_{-} t\left(q, o_{j}\right), u b_{-} t\left(q, o_{j}\right) \cdot \beta\right]\right\} \\
+\left(1-\operatorname{Pr}\left\{t\left(q, o_{j}\right) \leq T \mid t\left(q, o_{j}\right) \in\left[u b_{-} t\left(q, o_{j}\right) \cdot \beta, u b_{-} t\left(q, o_{j}\right)\right]\right\}\right) \\
\left.\cdot \operatorname{Pr}\left\{t\left(q, o_{j}\right) \in\left[u b_{-} t\left(q, o_{j}\right) \cdot \beta, u b_{-} t\left(q, o_{j}\right)\right]\right\}\right) \\
=\Sigma \operatorname{Pr}\left\{t\left(q, o_{i}\right)=T\right\} \cdot\left(\left(1-\operatorname{Pr}\left\{t\left(q, o_{j}\right) \leq T \mid t\left(q, o_{j}\right) \in\left[l b_{-} t\left(q, o_{j}\right), u b_{-} t\left(q, o_{j}\right) \cdot \beta\right]\right\}\right) \cdot \beta\right. \\
\left.+\left(1-\operatorname{Pr}\left\{t\left(q, o_{j}\right) \leq T \mid t\left(q, o_{j}\right) \in\left[u b_{-} t\left(q, o_{j}\right) \cdot \beta, u b_{-} t\left(q, o_{j}\right)\right]\right\}\right) \cdot(1-\beta)\right)
\end{gathered}
$$

Based on the lemma assumption that $l b_{-} t\left(q, o_{i}\right)>u b_{-} t\left(q, o_{j}\right) \cdot \beta$ and the definition of $\beta$ upper bound, we have:

$$
T=t\left(q, o_{i}\right) \geq l b_{-} t\left(q, o_{i}\right)>u b_{-} t\left(q, o_{j}\right) . \beta .
$$

Then, in the formula above, we infer that:

$$
\operatorname{Pr}\left\{t\left(q, o_{j}\right) \leq T \mid t\left(q, o_{j}\right) \in\left[l b_{-} t\left(q, o_{j}\right), u b_{-} t\left(q, o_{j}\right) . \beta\right]\right\}=1 .
$$

Moreover, since probability is non-negative, we have:

$$
\operatorname{Pr}\left\{t\left(q, o_{j}\right) \leq T \mid t\left(q, o_{j}\right) \in\left[u b_{-} t\left(q, o_{j}\right) . \beta, u b_{-} t\left(q, o_{j}\right)\right]\right\} \geq 0 .
$$

As a result, we can obtain:

$$
\operatorname{Pr} \operatorname{CPSTTQ}\left(q, o_{i}\right) \leq \Sigma \operatorname{Pr}\left\{t\left(q, o_{i}\right)=T\right\} \cdot(1-\beta)=1-\beta
$$

We can thus conclude that $(1-\beta)$ is an upper bound of $\operatorname{Pr} \operatorname{CPSTTQ}\left(q, o_{i}\right)$. If $\beta>1-\alpha$, that is, $1-\beta<\alpha$, by the inequality transition, we can obtain $\operatorname{Pr}_{C P S T T Q}\left(q, o_{i}\right)<\alpha$. In other words, the probability $\operatorname{Pr}_{C P S T T Q}\left(q, o_{i}\right)$ that object $o_{i}$ has the shortest traveling time to $q$ on the sub-path must be less than $\alpha$, and object $o_{i}$ cannot be the CPSTTQ answer. Hence, we can safely prune object $o_{i}$ from $M$.

How to compute the $\boldsymbol{\beta}$-Upper-Bound. To enable the probabilistic bound pruning, we need to compute the $\beta$-upper-bound. Assuming that $t\left(q, o_{i}\right) \in\left[l b_{-} t\left(q, o_{i}\right), u b_{-} t\left(q, o_{j}\right)\right]$ and $t\left(q, o_{j}\right) \in$ $\left[l b_{-} t\left(q, o_{i}\right), u b_{-} t\left(q, o_{j}\right)\right]$, then $t\left(q, o_{i}\right)$, the traveling time on any subpath $\left(p_{1}, p_{2}, p_{3}\right)$ within the road segment such that we are able to satisfy $\operatorname{Pr}\left\{t\left(q, o_{i}\right) \in\left(l b_{-} t\left(q, o_{i}\right), u b_{-} t\left(q, o_{j}\right) \cdot \beta\right)\right\}=\beta$.

Let us assume that there are $e$ edges on any subpath within the road network which is upper bounded by the interval $\left[l b_{-} t_{e}\left(q, o_{i}\right), u b_{-} t_{e}\left(q, o_{j}\right)\right]$ such that the traveling time is within this probability.

Greedy Algorithm to Compute the $\boldsymbol{\beta}$-Upper-Bound. Here, we are getting a very small $\beta$-upperbound of the traveling time on each sub-path on a road network segment by not considering all the sample combinations but very specific with few sample combinations to be considered. We vary the length and progressively extend our greedy algorithm paths, $u$, from 1 to $l$, and will then compute $M\left(u b_{-} t_{\beta}(l), \beta\right)$ with path $\left(q, o_{j}\right)$.

Initial Case. $P_{1}$ is the first subpath from location $s$ to $e$ on the road network segment with length $u=1$ and edge of path with path $\left(q, o_{j}\right)$ that we first consider. Assuming $\operatorname{arr_{-}} t_{d}$ is a sorted array of size for $c_{1}$, which contains $c_{1}$ discrete time samples in ascending order of the traveling time on $P_{1}$. For every time sample $\operatorname{arr}_{-} t_{d}[i]\left(1 \leq i \leq c_{1}\right)$, which is associated with probability $\operatorname{arr_{-}} t_{d}[i] . p$. If the $u b_{-} t_{\beta}(1)=u b=a r r_{-} t_{d}[i]$ holds, since the corresponding $\beta$ value is given as $z_{1}=\sum_{j=1}^{i} \operatorname{arr} r_{-} t_{d}[j] . p$ the probability that the traveling time on $P_{1}$ is not greater than $\operatorname{arr_{-}} t_{d}[j]$. From $c_{1}$ traveling time samples from array $\operatorname{arr_{-}} t_{d}$, randomly select $M$ and denote it as $t_{d}^{(a)}, t_{d}^{(b)}, \ldots, t_{d}^{(n)}$ therefore $t_{d}^{(a)} \leq t_{d}^{(b)}$ and so on.

Progressive Computation. For $P_{k}$, a subpath of $\operatorname{path}\left(q, o_{j}\right)$ has length $k(i . e ., u=k)$, we can obtain two arrays, $a r r_{-} u b_{k}$ and $\operatorname{arr_{-}} \beta_{k}$, which contain $M \beta$-upper-bounds and $M$ corresponding $\beta$ values for subpath $P_{k}$ (in ascending order), respectively. When we consider the subpath, $P_{k}+1$, of path $\operatorname{path}\left(q, o_{j}\right)(u=k+1)$, we combine the $\beta$-upper bound on subpath $P_{k}$ with that of the $(k+1)$-th edge on path $\left(q, o_{j}\right)$.

Therefore we obtain combinations of $M \beta$-upper-bound $\operatorname{arr}_{-} u b_{k}[i]$, for subpath $P_{k}$, and $C_{k}+1$ traveling time samples, $t_{k+1}^{j}$ on the $(k+1)$-th edge of the path. Each combination results in $\beta$ -upper-bounds, $\operatorname{arr}_{-} u b_{k}[i]+\operatorname{arr_{-}} t_{k+1}[j]$, and its $\beta$ value $\operatorname{arr} \beta_{k}[i] \cdot z_{k+1}^{(j)}$, where $z_{k+1}^{(j)}=$ $\sum_{r=1}^{j} t_{k+1}^{(r)} \cdot p$, for $1 \leq i \leq M$ and $1 \leq j \leq c_{k+1} . M$ out of $\left(m . c_{k+1}\right)$ combinations where $\geq 1-\alpha$ . The $\beta$-upper-bounds and their values in arrays $\operatorname{arr}_{-} u b_{k+1}$ and arr_$\beta_{k+1}$ could be store pairs, therefore we are able to derive arrays, $\operatorname{arr}_{-} u b_{i}$ and $\operatorname{arr}_{\_} \beta_{i}$, for path $\operatorname{path}\left(q, o_{j}\right)$ of length $l$. $u b_{-} t_{\beta}\left(q, o_{j}\right)$ smallest value $\operatorname{arr}_{-} u b_{i}[i]$, as the $\beta$-upper-bound, where $\operatorname{arr}_{-} \beta_{i,}[i] \geq 1-\alpha$, for $1 \leq$ $i \leq l$.

```
Algorithm Compute_Beta_UB\{
Input: arrays arr_ \(t_{u}(1 \leqslant u \leqslant l)\)
Output: arrays arr \({ }_{\text {ubl }}\) and arr \(\beta_{i}\)
1) randomly select \(M\) samples, from \(C_{1}\) traveling time samples from array, arr \(_{t i}\)
(2) let the array \(\operatorname{arr}_{\text {_ubl }}(i)=t_{i}\) for \(M \geqslant I \geqslant 1\)
(3) let the array \(\operatorname{arr}_{\beta i}(i)=z\) for \(M \geqslant I \geqslant 1\)
(4) for \(k=1\) to \(l-1\)
(5) for each of the \(\left(M . P_{(k+1)}\right)\) where \(i \in[1, M]\) and \(j \in\left[1, P_{(k+1)}\right]\)
(6) compute the \(\beta\)-upper-bounds \(\left(a r r_{-u b k}(i)+a r r_{-} t_{k+1}(j)\right)\)
(7) randomly select \(M\) out of \(\left(M . P_{(k+1)}\right)\)
(8) store \(M\) pairs to arrays arr \(\operatorname{cubl}_{k+1}\) and \(\operatorname{arr}_{\beta k+1}\) )
(9) return arrys arr_ubi and arr_ßi
\}
```

Figure 6-Algorithm for computing $\boldsymbol{\beta}$-upper-bound

Algorithm of Computing $\boldsymbol{\beta}$-Upper-Bound. Figure 6 demonstrates the pseudo-code for computing the $M \beta$-upper-bound of the traveling time on path $\operatorname{path}\left(q, o_{j}\right)$ where $\beta \geq 1-\alpha$. Since we have subpaths $P_{1}, P_{2}, P_{3}$ we begin from the first edge $P_{1}$ on path path $\left(q, o_{j}\right)$ and randomly selected $M$ out of $C_{1}$ traveling time samples from array $\operatorname{arr}_{-} t_{i}$ and updates $\beta$ arr_ $\beta_{1}$ array and $\beta$-upper-bound arr_ub each time. The $P_{k}$ subpath is extended by adding the $(k+$ 1)-th edge on path path $\left(q, o_{j}\right)$ by considering $m . c_{k+1}$ combinations of $m \beta$-upper-bounds of possible traveling times on path $P_{k}$ and $c_{k+1}$ time samples on the $(k+1)$ - th edge, then calculate their corresponding pairs of $\beta$-upper-bounds and $\beta$ values for the path $P_{k+1}$. We also randomly select $M$ out of $m . c_{k+1}$ combinations satisfying $\beta \geq 1-\alpha$, and update the arrays $a r r_{-} u b_{k+1}$ and $a r r_{-} \beta_{k+1}$ with $\beta$ - upper-bounds and $\beta$ values for path $P_{k+1}$. The iterations $(l-1)$, for path $P_{l}$ therefore, we return the $M \beta$-upper-bounds in $\operatorname{arr}_{-} u b_{l}$ and $\beta$ values in $\operatorname{arr} r_{-} \beta_{l}$.

## CHAPTER IV

## CPSTTQ PROCESSING

In this section, we consider building an index over offline pre-computed data, in terms of Minimum Bounding Rectangle (MBR), from each object $o_{i}$ which could lead to improved efficiency of CPSTTQ processing.

### 4.1 Index Construction

Data Structure of the Index. Firstly, we consider constructing a spatial index R-tree over the Minimum Bounding Rectangles (MBRs) of objects $o_{i}$ and their pre-computed data, which can be used to retrieve CPSTTQ answers. We use the example in Figure 6 to illustrate our index structure over 8 objects, $o_{1}, o_{2}, \ldots ., o_{8}$. Specifically, we first pre-compute the neighbors (breadth-first search) of each object in the road networks, and then use an MBR to bound this object and its neighbors. Thus, we can obtain 8 MBRs for 8 objects, respectively. Figure 6 illustrates the R-tree index over these 8 MBRs, which are inserted into the R-tree with the normal insertion method of the R-tree code. The R-tree index is a tree structure, which contains both leaf and non-leaf nodes.

Leaf Nodes. Each of the leaf nodes has entry $A_{i}$ that stores octuplet with objects $o_{1}$ to $o_{8}$. MBRs of objects are stored in each leaf node, each of which tightly bounds objects $o_{i}$ and their corresponding neighbors in the road network. Here, the reason that we use MBRs to bound objects and their neighbors is that we can utilize these MBRs to retrieve CPSTTQ candidates.

Intuitively, if a path, path $(s, e)$, intersects with MBRs (including objects and their neighbors on road networks), then these MBRs might be CPSTTQ candidates (i.e., PSTTQ answers of query point) for query point moving from $s$ to $e$.

In the example of Figure 6, there are 4 leaf nodes, $A_{4} \sim A_{7}$, each of which contains MBRs of two objects. For instance, leaf node $A_{4}$ contains MBRs of two objects, $o_{2}$ and $o_{5}$. In particular, the neighbors of object $o_{2}$ are $o_{5}$ and $o_{7}$. Thus, the MBR of $o_{2}$ tightly bounds objects $o_{2}, o_{5}$ and $o_{7}$.

Non-Leaf Nodes. For non-leaf nodes, we recursively use MBRs to bound MBR of their child nodes on a lower level. In the example of Figure 6 , we have non-leaf nodes $A_{2}$ and $A_{3}$, which contain pointers, pointing to child nodes $A_{4}, A_{5}$ and $A_{6}, A_{7}$, respectively. In particular, the MBR of non-leaf node $A_{2}$ tightly bounds child entries $A_{4}$ and $A_{5}$ under $A_{2}$. The case of $A_{3}$ is similar. Furthermore, the root of the tree, $A_{1}$, recursively bounds MBRs of $A_{2}$ and $A_{3}$.

The Construction of the Tree Structure. We are interested in showing our pre-computed minimum value from object $\left(o_{i}, o_{j}\right)$ to its nearest neighbor by inserting the Minimum Bounding Rectangles (MBR) in the R-tree. Figure 7 illustrates how we determine the minimum values to objects $o_{i}$ and give more explanation on the construction of the tree index. We are able to insert Minimum Bounding Rectangles (MBR) of the object $o_{i}$ into the R-tree tightly bounded to its nearest neighbor through the standard insertion method to be able to locate the nearest neighbor and we considered all the nearest neighbor and all points associated within our Minimum Bounding Rectangles (MBR). To insert an object, the tree is traversed recursively from the root node. At each step, all rectangles in the current directory node are examined, and a candidate is chosen by choosing the rectangle which requires least enlargement. The search then descends into
this page, until reaching a leaf node. If the leaf node is full, it must be split before the insertion is made


Figure 7- The Index Structure


Figure 8- Minimum Bounding Rectangles (MBR)


Figure 9-Offline pre-computation

### 4.2 Query Procedure

Pruning with the Index. We are able to use the pruning method with the R-tree index to reduce the search space, and retrieve the CPSTTQ candidates by traversing the tree index. We use the example in Figure 6 to illustrate our basic idea. From the root node $A_{1}$ in the index tree, we begin to check the MBR of each child node (i.e., $A_{2} \sim A_{7}$ ) and the path, $\operatorname{path}(s, e)$, of moving query point $q$ from $s$ to $e$. When the query path, path(s,e), is not intersecting with node $A_{i}$, then we can safely prune all objects $o$ under this node $A_{i}$. This is because the CPSTTQ answers can only be those neighbor objects, $N N\left(o_{i}\right)$, of each object $o$ under $A_{i}$, but not object $o_{j}$ (i.e., the traveling time from $q$ on path $\operatorname{path}(s, e)$ to neighbor $N N\left(o_{i}\right)$ is always greater than or equal to that from $q$ to $o_{j}$, since $q$ is not in the MBR of object $o_{j}$ ). Thus, we are able to safely prune objects $o_{i}$ under $A_{i}$. On the other hand, if an index node $A_{i}$ intersects with the query path $\operatorname{path}(s, e)$, then we cannot prune this node
$A_{i}$, and we need to further check its children to retrieve CPSTTQ candidates through the index traversal.

LEMMA 4. (Index Pruning). Given a query point q moving along a path, path(s,e), from location s to location e, and Minimum Bounding Rectangle (MBR) of a node $A_{i}$, we can safely prune $A_{i}$, if the MBR,MBR $\left(A_{i}\right)$, of node $A_{i}$ intersects with path $(s, e)$.

CPSTTQ Processing Algorithms. In the sequel, we will discuss how to use the index to answer the CPSTTQ query. Given a path, path $(s, e)$, of a moving query point $q$ from $s$ to $e$. First, calculate the shortest path from point $s$ to $e$ for query point $q$, and then check if this shortest path, path $(s, e)$, intersect the Minimum Bounding Rectangles (MBR), $M B R\left(A_{i}\right)$, and then traverse the index $I$ by maintaining a queue $Q$.

For each non-leaf node, check each child node whether the shortest path, path $(s, e)$, is intersecting with the corresponding MBR of $A_{i}$ (lines 7-9) in algorithm CPSTTQ processing. If they intersect with each other, we will add it to the queue (line 10). But if not, we will ignore it, because it cannot contain any CPSTTQ candidates within the MBR.

For the leaf node that contains objects $o_{i}$, we will check one by one if we are able to apply our proposed pruning methods (lines 12-13). Then, we will do the pruning for each object $o_{i}$. We first use the index pruning to see whether or not $\operatorname{path}(s, e)$ intersects with $\operatorname{MBR}\left(o_{i}\right)$ (line 13). If the answer is yes, then $o_{i}$ is a potential CPSTTQ answer. We will further check whether $o_{i}$ can be pruned by the time bound pruning or probabilistic bound pruning. If $o_{i}$ survives, then we will add it to the candidate set $S_{\text {cand }}$ (lines 14-15). Finally, we will refine the candidates in the set $S_{\text {cand }}$ by computing actual CPSTTQ answers (line 16).

[^0]Input: index I, a query path, path(s,e), from sto $e$, and a probability threshold $\alpha$
Output: CPSTTQ Answer
(1) initialize a queue $Q$ accepting the tree nodes
(2) $S_{\text {cand }}=\emptyset ; r=r_{\beta}=+\infty$
(3) calculate the shortest path from $s$ to $e$
(4) insert root, into queue $Q$
(5) while $Q$ is not empty
(6) node $A=d e$-queue $Q$
(7) if A is a non-leaf node
(8) for each entry $A_{i}$ of $A$
(9) if path( $s, e$ ) intersects with $A_{i} . M B R$
(10) insert $A_{i}$ into queиe $Q$
(11) else
(12) $\quad$ for each object $o_{i}$ in leaf node $A_{i}$
(13) if path(s, e) intersects with o o $o_{i} M B R$
(14) if $o_{i}$ cannot be pruned by time bound pruning and probabilistic bound pruning (15) add to candidate set to $S_{\text {cand }}$
(16) refine candidates in $S_{\text {cand }}$ and return the actual CPSTTQ answers.

Figure 10: CPSTTQ Query Processing Algorithm

## CHAPTER V

## RELATED WORK

The study from this paper is related to some previous works on probabilistic shortest time queries over uncertain databases, continuous nearest neighbor, path queries on probabilistic graphs and on uncertain road networks.

Uncertain Databases and Continuous Nearest Neighbor. Probabilistic databases [CP1987] can store types of information that cannot be represented using the relational model. Probabilistic databases may also be viewed as generalizations of relational databases; any relational database can be represented without loss of information by a probabilistic database. Cavallo and Pittarelli [CP1987] defined a new project-join mapping for relational databases that is based on transforming a relational to a probabilistic database. Probabilistic databases [Z1997], are formalized by means of logic theories based on a probabilistic first-order language proposed by Halpem.

Probabilistic shortest time queries considers the uncertainty of traveling times in uncertain road networks, it considers the probabilistic path problem of traveling time from query point q to a facility, with the shortest traveling time. A probabilistic database [DS2007] comprises of a list of x-tuples and each x-tuple has one or more exclusive alternatives in which each of these alternatives is associated with an existence probability. [CKP2003] studied probabilistic query evaluation based upon uncertain data. They proved that classification of queries is made based upon the nature of the result set and thereby they develop algorithms for
computing probabilistic answers. They also address the important issue of measuring the quality of the answers to the queries, and provide algorithms for efficiently pulling data from relevant sensors or moving objects in order to improve the quality of the executing queries. [CKP2003] considers all the possible semantic world over the uncertain road networks, due to the fact that each possible world is a deterministic road network which could eventually appear in the real world.

Many times probabilistic database query processing mostly looks into the avenue of semantics. [DS2007] in their previous works describe an optimization algorithm that can compute efficiently most queries. They show the data complexity of some queries is \#P-complete, which implies that these queries do not admit any efficient evaluation methods. There are an exponential number of possible worlds in a probabilistic database and many previous works including nearest neighbor queries [CKP2003], skyline queries [PJLY2007], and top-k queries. [LSD2009] have address the issue. Frank [F1969] simulates a system which generates a set of random branch-length vectors according to given probability distributions and thereby apply a shortest path algorithm to compute the value for this simulation.
[YPS2002] studied how a continuous nearest neighbor query retrieves the nearest neighbor (NN) of every point on a line segment. In their result they have a set of (point, interval) tuples, such that point is the nearest neighbor of all points in the corresponding interval. [RKV1995] studied the efficient branch-and-bound R-tree traversal algorithm to find the nearest neighbor object to a point, and then generalize it to finding the k nearest neighbors. They furthermore discussed the metrics for an optimistic and a pessimistic search ordering strategy as well as for pruning and propose a depth-first method that starts from the root of the tree to the leaf node in which the process is repeated recursively until the first nearest neighbor is detected.
[ZR2001], [BJKS2002] in their previous work they addressed the problem of finding $k$ nearest neighbors for moving query point ( $k$-NNMP) and in dealing with the problem they assume that the query point is not static, as in $k$-nearest neighbor problem, but varies its position over time.

Probabilistic Graphs Path Queries. Some of the works that have been previously studied under the probabilistic graph and on uncertain road networks include a study by Rasteiro and Anjo [RA2004] who examine the issue of optimal paths in directed random networks in which this path is the path that maximizes utility function expected values. Ming and Pei [MP2010] in their model consider arbitrary weight distributions and correlations between the weights of adjacent edges. Also in tackling the problem of traffic uncertainty they proposed probabilistic path queries and two new types of top-k path queries.
[HL2009], [JLDW2011], [PBGK2010] are some of the other previous works that review the RDF graph with containing edges associated with existence probabilities. Also previous works from [WMGH2008], [F2005], [LC2011] use graphical models to model the probabilistic graph, in which the labels of vertices connecting through edges are dependent and also they are represented by the conditional probability tables. [CA2012] considers a new network reachability problem where their goal is to find the most reliable path between two nodes in a network, represented as a directed acyclic graph. They computed a path between two end nodes when each edge has a failure probability.
[HP2010] studied how to take traffic uncertainty into account in answering path queries in road networks. They proposed to capture the uncertainty in traffic such as the travel time between two vertices and the weight of an edge is modeled as a random variable and is approximated by a set of samples. They also proposed three novel types of probabilistic path queries using basic probability principles and reviewed uncertain traffic information by studying the path queries on
road networks of which they consider the paths between two vertices having a total weight less than a threshold with high probability.
[L1983] studied the proposed simulation method to approximate the probability distribution of the shortest path and defined the utility function that specifies the preferences among the paths. Moreover, [RA2004] studied the problem of optimal paths in directed random networks, whereby the cost of each arc is a real-valued random variable whereas the optimal path is regarded as the path that maximizes the expected values of a utility function. They consider the linear, quadratic, and exponential cases, presenting a theoretical formulation based on multicriteria models as well as the resulting algorithms and computational tests.

## CHAPTER VI

## CONCLUSIONS

In this paper, we studied the uncertain road network by modelling the road network traffic and considered many factors which may cause speeds of vehicles to be imprecise and probabilistic. We therefore focus more on continuous probabilistic shortest traveling time query (CPSTTQ), which retrieves sets of objects that have the smallest traveling time to a moving query point $q$ from one location to the other on road networks with high confidences. We propose effective pruning methods and time bound pruning to prune the search space of our CPSTTQ query and filter out CPSTTQ false alarms. Moreover, we designed an efficient query procedure to answer CPSTTQ via an index structure and are thereby able to retrieve the CPSTTQ answer efficiently.

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## BIOGRAPHICAL SKETCH

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[^0]:    Algorithm CPSTTQ_Processing

