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Effect of instruction and textbook adoption procedures on kindergarten students' learning of the concept of rectangle

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EFFECT OF INSTRUCTION AND TEXTBOOK ADOPTION PROCEDURES ON KINDERGARTEN STUDENTS' LEARNING OF THE CONCEPT OF RECTANGLE

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ABSTRACT

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The purpose of this study was to produce scientific knowledge about the ability of kindergarten children to learn the correct mathematical concept of rectangle. The subjects of this study were kindergarten children in a public school district whose population had a large proportion of Hispanic and economically disadvantaged children. A Solomon Four-Group Design as described by Campbell and Stanley (1963) was used. Both the control and experimental groups received instruction using a district adopted curriculum. A description of how the district's curriculum addressed the concept of rectangle was included in the results. The experimental group received additional instruction using a curriculum selected for its compatibility with the development of the correct concept of rectangle. The students' knowledge of the concept of rectangle was measured using a test designed specifically for that purpose with methods consistent to developmentally appropriate practice for kindergarten children. Test results were analyzed using a 2 by 2 analysis of variance design described by Campbell and Stanley (1963) and a significant result was found, $F(1, 63)=6.54$, $p<.05$, between the mean scores of the experimental group and the control group. Thus, this study provides evidence that young children can learn a correct concept of rectangle. There was not a significant result, $F(1, 63) = 1.34$, $p<.05$, between the group of students who were pretested and the group of students who were not pretested. Thus, this study provided no evidence that pretesting children impacted their posttest results. There was not a significant cell effect between treatment and testing, $F(1,63)=0.091$, $p<.05$. Thus, this study provided no evidence of an interaction effect between testing and treatment. A survey administered to kindergarten teachers uncovered that more than half of the teachers surveyed seemed to have a misconception about rectangles. This study is significant to educational leaders and educators who are involved in the development and implementation of policy and rules regarding how errors, such as the one addressed in this study, are corrected through the textbook adoption process. Additionally, the results document the need for communication between districts and teacher preparations programs, continued teacher in-service training in districts, and formative evaluation of teachers.

DEDICATION

To my wife, Gina, and my daughter, Ashley, thank you for your patience, support and encouragement as I completed this work. To my brother Tony, thank you for being a sounding board for my thoughts. And to all my family members and friends who always listened to my stories about rectangles, thank you for listening.

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CHAPTER I
INTRODUCTION

Public school student performance on mathematics tests has raised concerns about the quality of public school mathematics curricula for decades (Madaus, Clarke, & O'Leary, 2003). A common error has existed for decades in kindergarten mathematics textbooks (Fuys, Geddes, Lovett & Tischler, 1988). This study seeks to produce knowledge that can be used to correct that error.

Seeley (2003) described the process by which public school textbooks are adopted. Seeley stated that in the late 20th century, the textbook adoption process increasingly included requirements that textbooks match state curricula; however, the amount of time allotted by state governments for textbook publishers to edit their textbooks was too short, in some cases making it difficult for textbook companies to significantly change their textbooks. Texas established a statewide curriculum that included mathematics in 1984 titled the Essential Elements followed by Texas Essential Knowledge and Skills (TEKS) in 1997. By 1992, Texas required that textbooks address the Texas standards, as did many other states with their own curricula, a practice that may have led to the inclusion of many topics at a superficial level in textbooks (Seeley 2003).

Statewide textbook adoption procedures have not prevented a common error from persisting in kindergarten textbooks for decades. Fuys, Geddes, Lovett and Tischler (1988) found that two out of three major publishers whom they studied in the early 1980's published elementary mathematics textbooks which contained lessons on rectangles that could cause students to learn an incorrect definition of rectangle. Approximately two decades after Fuys, et al. identified this problem with public school mathematics textbooks, researchers found that students were still exhibiting a common misconception about rectangles (Clements & Sarama, 2007). This study sought to produce data that could be used to change how the concept of rectangle is taught in kindergarten with the ultimate goal of eliminating a problem that has existed for decades; that is the production of written materials that teach the misconception that squares are not rectangles.

Statement of the Problem

Young children have often displayed the misconception that squares are not rectangles (Kay, 1987; Fuys, Geddes, Lovett & Tischler, 1988; Clements & Sarama, 2000). Kay suggested that the misconception was born from instructional practices rather than being inherent to human cognitive development. Many materials produced for the teaching of the concept of rectangle have incorporated design flaws such as showing only oblongs as examples of rectangles (Fuys et al., 1988; Monaghan, 2000). It has been hypothesized by some researchers that seeing only limited examples of rectangles has caused children to form an incorrect prototype for the concept of rectangle (Fuys et al., 1988). Clements and Sarama (2007) noted that many books and materials for young children which were available to parents included limited examples of shapes providing opportunities for the creation of misconceptions. The National Council of Teachers of Mathematics (2000) in its Standards document comments that children in grades 3 to 5 easily recognize oblongs as rectangles but find a need for discussion as to why a square is a rectangle. In summary, the problem addressed in this study was that many young children have developed a misconception about rectangles that may be caused by the instructional materials and practices being used by early childhood educators and parents.

Need for the Study

This study was needed to generate data regarding the impact of curriculum materials on the development of the mathematical concept of rectangle among kindergarten children. Data exists that shows that young children have developed the misconception that squares are not rectangles (Clements & Sarama, 2007); however, a search of the literature yielded no data indicating that a particular curriculum or instructional practice caused the misconception. Kay (1987) did provide a treatment, a curriculum, which produced data that seemed to indicate that first grade children could learn the correct conception of rectangle; however, that study did not include a control group as part of the research design. There could have been factors other than the treatment which caused the students to demonstrate mastery of the concept of rectangle.

Purpose of the Study

The purpose of this study was to determine the effect of instruction on kindergarten children's mastery of the concept of rectangle. The treatment will consist of instruction using a curriculum which was designed to teach the correct concept of rectangle. The control group will not be treated with the experimental curriculum. The study will show whether kindergarten children can learn the correct concept of rectangle given instruction designed to have them do so.

Significance of the Study

Data gathered in this study will increase the body of knowledge that teachers, textbook publishers and toymakers can use to make informed decisions regarding early childhood mathematics instructional practices, curricula and educational toys with respect to the development of the concept of rectangle. It may inform educational leaders responsible for producing or delivering kindergarten mathematics curricula, and it may possibly guide lawmakers in our states and nation as they develop textbook adoption regulations particularly with respect to the process for the correction of errors.

Many materials and toys in existence supported teaching young children that squares were not rectangles by providing examples of only oblongs and often explicitly stating that squares are not rectangles or requiring students to sort squares and rectangles into separate categories (Clements & Sarama, 2007). These practices and resources built, through induction, a prototypical concept of rectangle that was incorrect. This study produced knowledge that may influence the behavior of teachers, textbook publishers and educational toymakers by showing them that it is possible for young children to learn the correct concept of rectangle if the instructional materials and practices are designed appropriately.

Research Questions

The main research question addressed in this study is what is the difference in mathematics performance as measured by a researcher-made test of students' ability to identify rectangles between kindergarten students who receive an intervention that includes squares as rectangles and kindergarten students in a control group who do not receive that instruction?

Additionally, the potential interaction of testing with the treatment will be examined, as well as the math performance difference of those children who were pretested versus those who were not pretested. By examining these relationships, the researcher will provide evidence of factors other than the treatment which may have confounded the results. The research questions were developed into hypothetical constructs that were presented in the methodology section of this dissertation.

Limitations of the Study

This study was limited to the population of two particular schools in a particular district in south Texas. The site of the study was a predominantly Hispanic community where Spanish was the primary language in over 40% of the homes and more than 90% of the students were economically disadvantaged.

The experimental treatment was a period of instruction lasting approximately 1 hour provided by two kindergarten teachers. As described through the learning trajectory theory by Simon (1995) and Clements and Sarama (2004), the instruction was shaped by the particular interactions between the teacher and the students. Therefore, there was a certain amount of variation in the instruction provided which was consistent with learning trajectory theory. The particular teachers' effectiveness was not being controlled in this study. The delivery of the instruction was observed by the researcher who noted, in a qualitative descriptive fashion, factors which may have influenced the results of the experiment such as delivery of the lesson, classroom environment and student behavior. One such factor was the effectiveness of instruction when conducted at the Van Hiele Level 0 as opposed to Level 1. The students demonstrated more success when the teachers led them in Level 0 learning activities. This was noticed by the researcher during the treatment protocols which included reteaching activities so that most students in the experimental group were given extra instruction, reteaching, using Level 0 activities. There were 11 out of the 34 students in the experimental group who did not receive the additional Level 0 reteaching activities because the posttest had already been administered to them by the time the researcher realized that Level 0 activities were more effective.

Delimitations of the Study

There are several questions that this study did not address but would be interesting for future studies. First, why do some educators and some of those who produce educational materials continue to support the teaching of the incorrect concept of rectangle?

Another question this study did not address has to do with the impact of learning logically inconsistent knowledge. That is, what impact on children's ability to think logically about mathematics does accepting contradictory statements about rectangles have? This question was spawned from the practice of teaching children two contradictory ideas: (1) that rectangles have two long sides and two short sides and (2) that squares are rectangles. This confusing practice was found in materials dating back several decades (Merton & May 1966). Merton and May wrote a book designed to provide guidance for teachers of the *new* math. In that book, the authors first led students to discover that “the opposite sides only are the same length” (p. 173) in a rectangle. Then they proceed to guide students in understanding that “All squares are also rectangles…” (p. 173). Does having to accept such contradictory statements cause children to stop trying to make sense of mathematics and simply strive to replicate whatever the teacher presents no matter how illogical? Related questions are what was the impact on the students’ anxiety about mathematics, and does having to accept contradictory statements about rectangles lead to anxiety about learning mathematics?

Also not addressed in this study was the rate at which the misconception was corrected as students progressed through their education. Was there a particular grade level in which the correct concept of rectangle was finally taught and learned? For example, when students were taught about the hierarchical classification of polygons as quadrilaterals, parallelograms, rectangles, and squares, did they experience confusion due to previously learned incorrect concepts about rectangles?

This study did not attempt to determine when or how children developed the misconception that squares were not rectangles. That is, the pretest and posttest results may have indicated that the children involved in the study had the misconception, but the study did not produce knowledge of whether the children developed the misconception without any instruction, whether the misconception was the result of parental attempts to teach the child about shapes using commercially available materials, or whether the children were taught the misconception in their formal education. In other words, no attempt was made to determine whether the misconception was an inherent trait due to human psychology.

Definition of Terms

In this study, when the word misconception was used in reference to kindergarten students' concept of rectangle, the judgment as to whether the concept of rectangle was correct was based on what was conventionally accepted by mathematicians as part of Euclidean geometry. An argument was made in this study that a particular definition of rectangle was the conventionally accepted definition. Any conception of rectangle that differed in a mathematically relevant way from the conventional definition of rectangle in Euclidean geometry was deemed a misconception.

Chapter Summary

In the 1980s, Fuys, et al. (1988) and Kay (1987) found that children exhibited a common misconception that squares were not rectangles and that textbook publishers produced materials that may have been designed in such a way as to contribute to the formation of the misconception. Two decades later, Clements and Sarama (2007) found that elementary students continued to exhibit that misconception and that many of the materials available for children to learn about the concept of rectangle were designed in such a way that they might produce the misconception that squares are not rectangles. This study seeks to produce knowledge about whether kindergarten children are able to learn the correct concept of rectangle, and to provide guidance about how to correct the problem with instructional materials. The next chapter will provide a review of relevant literature to include theories of learning related to geometry, instruction that is appropriate for kindergarten children, and some history and procedures related to the correction of errors in textbooks used in public schools.

CHAPTER II

REVIEW OF LITERATURE

Introduction

The literature found in this section came from searches of books available at the University of Texas – Pan American (UTPA) library, the UTPA College of Education library, the Academic Search Complete, the Education Full Text (H. W. Wilson) as well as the Dissertations and Theses databases available through the UTPA Curriculum Resource Center. The most significant sources of literature related to the teaching of the concept of rectangle to young children were the dissertation by Kay (1987) who conducted a study similar to the one described in the current study and the Monograph Number 3 by Fuys, Geddes, Lovett and Tischler (1988) which documented the existence of the problem and described the Van Hiele learning theory. Clements and Sarama (2000, 2004, & 2007) published several books and articles which provide additional evidence that the problem addressed in this study existed. The kindergarten textbooks adopted in 2009 were obtained from the UTPA College of Education Curriculum Resource Center as well as from access to textbooks given by publishers to public school districts. Since the problem identified in this study was that an incorrect concept of rectangle was being learned, the first part of the review establishes the conventional definition of rectangle. The review also encompasses relevant learning theory such as the Van Hiele theory of geometric thought and the concept of developmentally appropriate practice for young children to establish that the experimental treatment being proposed was aligned to effective and appropriate practices for teaching kindergarten children. The review includes literature about the textbook adoption policies and procedures with respect to impact on the correction of errors in instructional materials in order to address how the errors in the current textbooks can be corrected.

Textbook Adoption Policy and Procedures for Error Correction

According to Stein, Stuen, Carnine and Long (2001), relatively little research has been conducted about the textbook adoption process. Stein et al. found that the literature that did exist was consistent with the practice of textbook adoption at the time of their research. Stein et al. found that Texas and California had significant influence on textbook development and publication attributed to the large share of the market that those two states represented. The textbook adoption processes described by Stein et al. were either two-tiered, state and local, or just local. In the two-tiered process, a statewide committee first developed a list of approved textbooks before the local committees examined the textbooks. Stein et al. identified several weaknesses in the adoption process: (1) the educators involved in the adoption process were not well trained to conduct the evaluation of the textbooks, (2) the amount of time allocated to the evaluation of textbooks was not sufficient, (3) allowing all teachers to vote for a textbook limited the validity of the adoption process, and (4) there was a lack of research-based criteria for the evaluation and selection of textbooks. Stein et al. gave recommendations for the improvement of the adoption process which included providing additional time for educators to review the textbooks and providing screening instruments along with training on how to use them.

Farr and Tulley (1989) found that states that used a two-tier system and those that used only local committees adopted similar textbooks with similar costs. The major difference was that states with only local committees allowed local committees to view a larger number of different textbooks. Farr and Tully attributed the similarity in textbooks between those states with statewide adoption procedures and those without to the influence that the states with statewide adoptions had on the development of textbooks. Farr and Tulley recommended that we eliminate statewide adoption procedures so that local committees have a wider range of choices and to reduce the influence of testing programs on the content of textbooks.

Crane (1975) describes the evolution of textbook adoption procedures in California where there was a shift from state printers printing adopted textbooks to publishers changing their nationally available editions to meet the needs of California. Crane also describes how corrections to the textbooks were done. In making decisions about printing and corrections, publishers considered the economic impact of those decisions.

Sewall and Emberling (1998) described the adoption of history textbooks and listed three states as having great impact on the adoption process: California, Texas and Florida. Sewall and Emberling described how trying to meet the needs of local adoption committees was costly to publishers thus reducing the likelihood that corrections to textbook content would be made.

Cloud-Silva and Sadoski (1987) surveyed teachers in Texas about the statewide reading adoption process. They found that teachers felt that they were not well represented in the local adoption committees and that they needed more time to review the textbooks. Teachers also wanted to be able to review textbooks which the state had not included in the list of allowable textbooks.

Jongsma (1992) suggested that educators involved in the selection of reading textbooks in Texas should read material in preparation for the textbook selection process. Jongsma indicated that the materials suggested were not easy to read. A possible reason for the material not being easy to read is the quantity of material suggested as well as the topics which could challenge the readers to think and leave the readers questioning their views about the influence of textbooks on students.

The Charles A. Dana Center (2007) published a set of documents intended to provide guidance to Texas educators engaged in the selection of instructional materials, textbooks, for mathematics. The documents included an overview of a four phase process and a detailed rubric for evaluating how textbooks addressed each learning objective in the State curriculum for mathematics. In Phase 3 of the document for kindergarten, the learning objective which directly indicated that students were expected to learn the concept of rectangle in kindergarten was listed, "The student was expected to…describe, identify, and compare circles, triangles, rectangles, and squares (a special type of rectangle.)" (p. 16).

Johnston (2011) describes the trend towards electronic media as opposed to printed textbooks. According to Johnston, it was likely that e-texts would replace printed textbooks as the dominant instructional material in higher education in the near future. Johnston stated that over the next five years, it was predicted that somewhere between 12% to 30% of the textbook market would transition to electronic media.

The State of Texas has passed laws regarding textbook adoption. Texas Education Code (TEC) Chapter 31 and Texas Administrative Code (TAC) Chapter 66 were about textbook adoption. TEC section 31.151(a) required textbook publishers to certify that their textbooks were free of factual errors when contracts for textbook adoption were signed. TAC section 66.54 required that publishers submit corrected versions of their textbooks to the commissioner of education by a certain date and that the publisher sign an affidavit that all corrections required by the commissioner and the State Board of Education had been made. TEC chapter 31 defined the term textbook to include electronic textbooks which may include software, online services and any other means of conveying information electronically.

The Texas Education Agency (2012) published a draft of procedures, Proclamation 2014 Draft, for the adoption of mathematics instructional materials in grades kindergarten to grade 8. Those procedures contained a timeline of events for materials to be selected by school districts during the 2013-2014 school year and implemented in the classrooms during the 2014-2015 school year. As part of the procedures, the public had an opportunity to review the materials and submit a request for corrections to errors to the Texas Education Agency (TEA). The procedures required regional education service centers located throughout Texas to publish the date, time and location for the public to review the materials. The deadline for the regional service centers to publish the arrangements for the public review of the materials was February 8, 2013 and those materials were made available to the public by May 17, 2013. The deadline for Texas residents to submit a list of alleged errors to be corrected or written comments about the textbooks was September 6, 2013 by 5:00 PM CDT. Typically, public school districts in Texas do not begin analyzing textbooks until the fall semester, or later, of the adoption year after the deadline for corrections has past. Therefore, publishers may not be required to correct errors discovered by school districts during the adoption process. The Texas State Board of Education was scheduled to consider these procedures for approval during April 2012. The textbook adoption procedures published by the Texas Education Agency (2012) regarding the correction of errors in public school textbooks were designed to ensure that textbooks were free of factual errors. TEA's procedures also required a committee which was nominated from throughout Texas to review the materials.

A question that may have impeded the correction of the kindergarten curriculum was whether teaching the incorrect concept of rectangle would be considered a factual error by the Texas Education Agency. The Texas Administrative Code, Title 19, Ch 66, Rule 66.10, *February 22, 2010, 35 TexReg 1454* (2010) defines a factual error in instructional material as follows, "A factual error shall be defined as a verified error of fact or any error that would interfere with student learning. The context, including the intended student audience and grade level appropriateness, shall be considered." It is possible that the State review panel did not know or did not agree that an error in the kindergarten curriculum existed.

Definition of Rectangle

Sinclair (2008) stated that the geometry curriculum taught in American public schools had its roots in Euclid's Elements. A timeline given by Sinclair showed that, in 1844, geometry became a college entrance requirement and Euclidean geometry was the curriculum. While the geometry curriculum evolved over the years as mathematicians made discoveries of different non-Euclidean geometries, Euclidean geometry remained the foundational piece of the public school geometry curriculum. Sinclair stated that the Committee of Ten, a national committee working to improve public school curricula, made a recommendation in 1892 that geometry be taught in elementary schools. The Committee of Ten also recommended that the topics of Euclidean geometry be in the high school curriculum. In 1912, another similar committee, The Committee of Fifteen, continued to promote Euclidean geometry although it was found that the work of mathematician Adrien-Marie Legendre was having great influence in public school geometry textbooks of the time. Legendre made improvements upon the material produced by Euclid and introduced measurement into the geometry curriculum. Still, public school geometry continued to be greatly influenced by Euclidean geometry through the 1950s. In the 1960s, the New Math movement in the United States brought great changes to the way geometry was taught in the public schools; nevertheless, even in the New Math era, the geometry taught in the public schools was a transformed version of geometry still based on Euclidean geometry (Sinclair, 2008). Burger et al. (2007) wrote a popular high school geometry textbook adopted for use in Texas in 2007 in which they state, "Euclidean geometry was based on figures in a plane" (p. 726). There are topics in the textbook by Burger et al. which addressed non-Euclidean geometry; however, most of the text addressed figures on a plane. Therefore, throughout the history of geometry education in the United States until the present day, Euclidean geometry has been the driving force of what was taught in the public schools.

Blau (2003) lists 23 definitions by the famous ancient mathematician Euclid with the following definition of square and oblong, "Of quadrilateral figures, a square was that which was both equilateral and right-angled; an oblong was that which was right-angled but not equilateral..." (p. 264). In Byrne's (1847) translation of some of Euclid's work into English with colored diagrams and symbols, the definition of square, oblong and rectangle are given. The definition of rectangle which Byrne attributes to Euclid was consistent with the definition found in many sources, "A rectangle or a right angled parallelogram is said to be contained by any two of its adjacent or conterminous sides." Sinclair (2008) described the translation of Euclid's work by Heath in 1926 as "definitive" (p. 6). Heath (1956) in a second unabridged edition of his 1926 work included this in his translation of Euclid's work, "…an oblong that which was right-angled but not equilateral…" to define an oblong (p. 154). Thus was established the definition of oblong as a rectangle with unequal sides.

A different translation of Euclid's definition of rectangle was found in Fitzpatrick's (2008) translation which states, "a rectangle that which was right-angled but not equilateral…" (p. 7). According to Fitzpatrick, R. (personal communication, November 1, 2010), his use of the word rectangle rather than oblong was based on the fact that the word which he was translating, "orthogwnion", meant right-angled which was equivalent to rectangle. In that same paragraph, Fitzpatrick, R. (personal communication, November 1, 2010), stated that Euclid seemed to classify a square as a rectangle contradicting his definition of rectangle. Heath (1956) and Byrne (1847) both defined rectangle in their translations of Euclid in a manner that would be consistent with the conventionally accepted definition of rectangle while Fitzpatrick (2008) defined rectangle in his translation of Euclid in a manner which would lead to a contradiction.

Three common dictionaries (Soukhanov & Ellis, 1984; Guralnik, 1987, Berube, et al., 1985) define a rectangle with definitions which are equivalent to stating that rectangles have four sides and four right angles. These definitions lead to the logical conclusion that all squares are rectangles. The State of Texas, in the curriculum standards for kindergarten mathematics, states that squares are a special kind of rectangle (Texas Education Agency, 2006). Schwartz (2005) lists the attributes of rectangles as “4 right angles, 4 sides” in a textbook intended to provide guidance regarding the mathematics instruction of young children. Welchons and Krickenberger (1940) defined a rectangle as “…a parallelogram having one right angle…” (p. 156). Of course, if a parallelogram has one right angle, all of its angles will be right. Adams (1958) also defined a rectangle as “a parallelogram having a right angle” (p. 56), and a square as “a rectangle having two adjacent sides equal” (p. 56). Rich (1963) defined a rectangle as “an equiangular parallelogram” and a square as “an equilateral and equiangular parallelogram” (p. 79). Kay (2001) defined a rectangle in two equivalent ways. First, “If a parallelogram has a right angle, it was a rectangle” (p. inside cover). Second, “A parallelogram was a rectangle if its diagonals are congruent” (p. 225). Aarts (2008) provided a more sophisticated definition.

> The quadrilateral ABCD was called a parallelogram if AB//DC and AD//BC; if, moreover, two intersecting sidelines of a parallelogram, for example AD and AB, are perpendicular to each other, we call the quadrilateral ABCD a rectangle. (p. 21)

The definition of rectangle as a four-sided figure with ninety degree angles was well established. A rectangle was a quadrilateral with four right angles. Another way to state the same definition was that a rectangle must have four sides and four square corners. In these widely accepted and equivalent definitions, there was no reference to the length of the sides of the rectangle. It does not matter whether the sides are of different lengths or the same length in determining whether a shape was a rectangle. Therefore, all squares are rectangles.

Textbooks from Major Publishers and the Concept of Rectangle

During the 2007-2008 school year, Texas adopted new textbooks for elementary mathematics to be used starting in the 2008-2009 school year. Five major publishers whose textbooks were adopted by the Texas State Board of Education included Houghton Mifflin, SRA/McGraw-Hill, Harcourt School Publishers, Macmillan McGraw-Hill, and Pearson Scott Foresman – Addison Wesley. The following paragraphs will provide a listing of the authors and descriptions of how rectangles are addressed in the kindergarten mathematics textbooks published by publishers listed above.

Stiff et al. (2009) wrote a kindergarten mathematics textbook published by Houghton Mifflin titled *Texas Math* in which the lesson on rectangles was designed to help kindergarten students learn the correct concept of rectangle. The teacher was to draw a square and a rectangle and explain to the students that a square was a special kind of rectangle. Students were to create shape rubbings of squares and rectangles using crayons. The teacher was to emphasize that squares were special types of rectangles in which all sides are equal. The teacher was directed to encourage students to discuss whether squares were rectangles with the correct answer being that they were because squares have four straight sides and four equal corners.

Another activity had the students use attribute blocks to examine both square and oblongs and discuss why not all rectangles were squares. The student page of the textbook was labeled "Squares and Other Rectangles" (Stiff et al., p. 109A). It consisted of an aerial view of a cartoon town where the building tops were in the shapes of squares, rectangles, circles, triangles and other non-rectangles. The directions were for students to circle the rectangles and color rectangles that were also squares. The answer key correctly showed all rectangles circled, including the squares. Only the square rectangles were colored. The material was designed to teach the correct concept; however, the material had errors. There were two aspects of the material which might produce or reinforce the misconception that rectangles must have two sides longer than the others. First, the assessment portion of the lesson included this question, "How are these figures alike? different?" (Stiff et al., p. 109B) potentially causing students to conclude that rectangles and squares were distinct objects. Second, the glossary in the textbook shows only an oblong as an example of a rectangle potentially leading students to form a limited prototype of rectangle that includes only the oblong shape. Overall, however, this textbook was designed to teach the correct concept of rectangle.

Willoughby, Bereiter, Hilton, and Rubinstein (2009) wrote a kindergarten mathematics textbook, published by SRA/McGraw-Hill, in which a correct definition of rectangle was given to the teacher in the teacher's edition, "a parallelogram having four right angles" (p. 115A). While the written definition of rectangle was correct, the visual images of rectangles presented in the textbook consisted exclusively of oblongs.

Maletsky, et al. (2009) authored a kindergarten mathematics textbook published by Harcourt which states that "The sides of a rectangle are two different lengths" (p. 145A). Students were asked to explain how a square and a rectangle were different. Students were expected to respond that rectangles have sides of different lengths. The idea that rectangles have two sides longer than the other two was reinforced through a hands-on activity using craft sticks.

Altieri, et al. (2009) wrote a kindergarten mathematics textbook published by Macmillan/McGraw-Hill which included the following statement as a sample answer to a question asking students to state the difference between squares and rectangles, "…a rectangle has four sides with two sides that are longer" (p. 263). The textbook also warned teachers about a common error in which students may not understand that a square was a rectangle. In that same warning, the textbook directs teachers to explain to students "…that a rectangle has two sides that are longer than the other two sides" (p. 263).

Charles, et al. (2009) wrote a kindergarten mathematics textbook published by Pearson Scott Foresman in which a correct definition of rectangle was clearly stated, "a quadrilateral with four right angles" (p. 217A). However, in the learning activity described by the authors, the examples given are all oblong and it was emphasized that in rectangles “opposite sides are the same” (p. 217). In an activity designed for students who did not master the concepts of rectangle and square, the textbook used connecting cubes and posterboard shapes to have students measure the side lengths showing that in a rectangle, “only the opposite sides are the same length” (p. 220C). A review problem in a later section of the textbook shows a collági of rectangles including squares. The script accompanying the problem asks, “How are squares and rectangles alike? How are they different?” (p. 221A).

Decades ago, Fuys et al. (1988) found that two out of the three major textbook publishers of elementary mathematics textbooks published in 1984 provided materials that were designed such that they taught the wrong definition of rectangle. Those textbooks taught that rectangles had two long sides and two short sides. One textbook series did this by omitting squares and thus showing only oblong examples of rectangles. Another example of activities which taught the wrong idea was requiring students to color rectangles a different color than squares implying that squares were not rectangles. A final example was asking students to identify rectangles and included squares among the wrong answers.

In summary, out of five major elementary mathematics textbook series adopted by the State of Texas and reviewed here, four of them included learning activities that would lead young children to form the misconception that rectangles must have two sides longer than the others and therefore concluding that squares are not rectangles.

When asked to justify the development of rectangle prototypes which have two long sides and two short sides, one publisher gave two reasons for decisions to develop an incorrect concept of rectangles: (1) that the practice was based on research and (2) that it would not be appropriate, too complicated, to teach young children the correct concept of rectangle. The particular research that was quoted by the publisher was produced by Leushina (1991); however, an examination of that research revealed that the Leushina did not justify the use of the incorrect definition of rectangle with young children. Leushina (1991) addressed the psychological development of young children with respect to mathematics concepts such as sets, number, counting, size, shape and mass. Leushina provided guidance on how mathematics should be taught to young children. In particular, Leushina gave examples of how the concept of rectangle could be taught. In that guidance, Leushina included the self-contradictory statement, "…not every rectangle was a square, since in a square all sides are equal, but in a rectangle only the opposite sides are equal" (Leushina 1991, p. 115). The examples given by Leushina illustrating how the concept of rectangle should be taught led to the contradiction that rectangles must have two sides longer than the other two while at the same time asserting that squares are rectangles. The quote shown above produces that contradiction within a single statement.

In this section, I presented information regarding the content of public school textbooks regarding the concept of rectangle. The next sections explore whether it would be appropriate to teach the correct concept of rectangle to kindergarten-age children.

Cognitive Development of Children

One potential rationale for the practice of teaching the incorrect concept of rectangle was that the children were too young to be able to learn the correct concept of rectangle, that the practice of teaching the incorrect concept was appropriate for children of that age. Perhaps this belief stemmed from the part of Jean Piaget's theory which holds that children's cognitive development was constrained by their developmental stage (Brainerd, 2003). Brainerd (2003) gave the following analogy, "No matter how much practice it receives, a butterfly cannot learn to fly while it was still a caterpillar...." (p. 260). However, Piaget's theories relating to how young children learn geometric concepts were not confirmed (Brainerd, 2003; Clements & Sarama, 2007; Pressley & McCormick, 2007). Brainerd (2003), Clements and Sarama (2007) and

Pressley and McCormick (2007) asserted that the cognitive development stages theory was not supported by later research showing that younger children can do more than what Piaget claimed they could do and adults often are less developed than Piaget claimed they would be. Small (1990) reported that Piaget posited that the ability to recognize hierarchical relationships developed with the onset of the concrete operations around 7 years of age; however, Small noted that this assertion was not supported by replication studies which found that some children develop the ability earlier while others struggle with it even in later years.

Kay (1987) studied the ability of young children to learn at higher levels. Kay suggested that first grade students learn geometric concepts in two distinct ways: inductively or deductively. In the inductive process, students experience examples of a concept and develop a prototype or schema for that concept. For example, a student who was exposed to a variety of triangles would come to understand that triangles must have three sides. The deductive process would consist of students first learning a set of characteristics or properties such as quadrilaterals have four sides. Once students learned that quadrilaterals had four sides, they would be able to identify quadrilaterals using deductive reasoning. Kay hypothesized that students would be inclined to use deductive methods when the concept being learned was easier to describe with a rule than with prototypes or templates. The example given by Kay was that the concept of quadrilateral has an easy rule. Quadrilaterals have four sides. Therefore, students are more prone to succeed with deductive thinking when identifying quadrilaterals. On the other hand, the concept of square requires a more restrictive set of rules. A square must have four sides, four right angles, and the sides must be of equal length. While the rules for identifying a square are more complicated than those for a quadrilateral, the prototype or template of a square was simple. All squares are similar in both the common and mathematical senses. Rectangles have only two prototypes: an oblong and a square. Kay considered the definitions of square and rectangle to be “relatively complex” (Kay, 1987 p. 152) therefore, Kay hypothesized that students would prefer to use inductive reasoning when understanding rectangles. In other words, students would find more success learning the concept of rectangle by seeing examples rather than by definition.

Prototype Development

Prototype development was acknowledged by many researchers (Monaghan 2000, Clements & Sarama 2007, Fuys & Liebov 1993) as a source of misconceptions. If all the examples of rectangles that a child experiences have two long sides and two short sides, the child was likely to learn that rectangles must have two long sides and two short sides. In order to prevent or dispel these misconceptions, children need to see as many variations of a concept as possible. Pressley and McCormick (2007) described *symbolic schemes* as cognitive structures defined by Piaget which allow preoperational age children to represent objects by means of symbols such as language, mental images, and gestures.

Monaghan (2000) uses the phrase “one-to-one object-word match” (p. 187) to describe the prototypical conceptualization that many children form between pictures or drawings of oblongs and the word rectangle. Monaghan found that most of textbook materials found in London in 1997 contained depictions of rectangles that were oblong. Fuys et al. (1988) also noted that prototype formation was complicated for the concept of rectangle because in order to gain a correct concept of rectangle, students needed to be exposed to many variations of rectangles including squares.

Clements and Sarama (2007) reported that many of the books available at public book stores for helping young children learn shapes give only a single example of a rectangle and a single example of a square. Clements and Sarama (2000) ask, “As we continually contrast squares and rectangles, do we convince children of their separateness?” (p. 485).

Van Hiele Theory of Geometric Thought

Another theory relevant to whether teaching young children the correct concept of rectangle was appropriate is the theory developed by Pierre Van Hiele and his wife Dina Van Hiele. During the 1950s, the Van Hieles conducted research which has been influential among researchers of mathematics education (Fuys et al., 1988). The Van Hieles devised a theory of geometric thought that included five levels of which the first three are of interest in this study and the first, lowest, level will be directly implemented in this study.

The first level, the lowest level, of geometric thought was visual. Students operating at this level are able to identify shapes and sort different shapes. Fuys et al. (1988) shared an example of thinking in this first Van Hiele level when a student identified squares as kites simply because the student had seen a drawing of a collection of kites and some of them were squares. The student learned that squares were kites not because squares had the properties of kites, but because when the student was introduced to the concept of kite, squares were included in the examples of kites given to the student.

The second level in the Van Hiele Theory was characterized by the learner's demonstrated recognition of properties of shapes (Fuys et al., 1988). In this level, the student discovers and operates with the properties of figures. Given a set of quadrilaterals, the student generates a rule stating that quadrilaterals have four sides. Once that the understanding of the properties of quadrilaterals has been developed students operating at the second Van Hiele level are able to sort shapes based on properties and discuss the properties of the shapes.

The third level in the Van Hiele theory occurs when students are able to use the properties of shapes to order classes of shapes (Fuys et al., 1988). At this level students would be able to justify the inclusion or non-inclusion of one class of shapes within another class of shapes. “The square was recognized as being a rectangle because at this level definitions of figures come into play” (Fuys et al., 1988, p. 75).

Students operating at the fourth Van Hiele level are able to prove the truth of mathematical relationships within a given mathematical system (Fuys et al., 1988). Students at this level understand the need for definitions and the necessary and sufficient conditions that make definitions equivalent.

Students operating at the fifth Van Hiele level are able to compare mathematical systems, for example comparing Euclidean with non-Euclidean geometries (Fuys et al., 1988). At this level, students use rigorous mathematical arguments to prove the truthfulness of the axioms which make up the mathematical systems. Students at this level explore how changes to axioms affect the system and invent general solutions to problems.

The first three Van Hiele levels are most relevant to this study. The Van Hieles theorized that people have to go through the levels moving up from the lowest level to the highest. So it would be expected that young children would start at the lowest level of geometric thought (Fuys et al., 1988). As illustrated in Figure 1, the elementary mathematics curriculum approved by the State of Texas seems to be aligned to the Van Hiele levels progressing from the lowest level in kindergarten to the third level of geometric thought in grade 5.

Kay (1987) questioned how the Van Hiele Theory was applied to the concepts of quadrilaterals, rectangles and squares. The common practice was to leave the classification of shapes for later years, but Kay believed that young children could learn to think at the third Van Hiele level in the first grade. Kay conducted an experiment with a class of 16 first grade students where the students went through a series of lessons whose objective was for the children to learn the concepts of quadrilateral, rectangle, and square as well as the relationship among

Figure 1

Alignment of the Rectangle Concept in the Texas Essential Knowledge and Skills (TEKS) and the Van Hiele Model of Geometric Thought

Grade Level	•**Key Words from Texas Curriculum** •**Van Hiele Level**
Kindergarten	•identify...rectangles •Van Hiele Level 0, developing Level 1
1st Grade	•identify...rectangles...and describe attibutes •Van Hiele Level 1
2nd Grade	•classify ...polygons...acccording to attributes •Van Hiele Level 1
3rd Grade	•use attributes to recognize...rectangles...as...quadrilaterals •Van Hiele Level 1
4th Grade	•classify two-dimensional figures based on...angles •Van Hiele Level 1, developing Level 2
5th Grade	•classify two-dimensional figures in a hierarchy of sets...based on their attributes •Van Hiele Level 2

these classes of shapes. Kay found that before the students were exposed to the instruction that developed the correct concept of rectangle, eleven of them already had formed the misconception that rectangles had two long sides and two short sides. After the experimental instruction, only one student persisted in stating that rectangles had two long sides and two short sides. With regards to the higher level of thought or the 3^{rd} Van Hiele level where learners classify shapes by using hierarchical relationships, only one student performed this task before the experimental instruction while 8 did so after instruction at level 3 occurred. Kay concluded that instruction designed to develop higher level thinking does impact students' cognitive development. It seemed that she guided the students to learn the correct concept of rectangle with 10 of 15 students tested identifying squares as rectangles.

Van de Walle (1994) suggested several activities as appropriate for students operating at the first Van Hiele level, Level 0. Students should use physical models that can be manipulated by the students. Students should be presented with a variety of examples of shapes so that irrelevant features are not misidentified as being a defining characteristic of the shape. Students should experience many sorting, identifying and describing activities. Students should also experience building, composing and decomposing shapes. Fuys et al. (1988, p. 159) indicate that "…a crucial aspect of the progression from level 0 to level 1 was the recognition of properties as characteristics of classes of figures…", a progression they claim was encouraged by exposing children to many different examples, not just one example of the figure.

Hierarchic Interactionalism and Learning Trajectories

While Clements and Sarama (2007) did not support all of Piaget's theories, they did support the parts of Piaget's theories related to constructivism and interactionalism. With regard to interactionalism, Clements and Sarama suggested a new perspective that joined verbal descriptions and what Pierre Van Hiele (1986) called the visual level of geometric thought. Syncretic was what Clements and Sarama called the merger of the visual with the verbal. The syncretic level was defined as a synthesis or interaction of imagistic and verbal knowledge (Clements & Sarama, 2007).

Clements and Sarama (2007) built the Theory of Hierarchic Interactionalism most directly from the Van Hiele Theory of geometric thought, a theory they characterize as constructivist. Both theories propose that learning progresses through levels of thinking, but Clements and Sarama added a level that precedes the Van Hiele levels to account for children's thinking at the earliest stage. One of the tenets of hierarchical interactionalism, the hypothetical learning trajectory, was particularly relevant to this study because it provides a theoretical basis for allowing flexibility in the instruction provided to students.

The term *hypothetical learning trajectory*, shortened to learning trajectory, was introduced by Martin A. Simon in 1995 (Clements & Sarama, 2004). The definition of a learning trajectory given by Clements & Sarama (2004) was as follows:

> …we conceptualize learning trajectories as descriptions of children's thinking and learning in a specific mathematical domain and a related, conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesized to move children through a developmental progression of levels of thinking, created with the intent of supporting children's achievement of specific goals in that mathematical domain. (p. 83)

Simon (1995) had a simpler definition for learning trajectory that consisted of the following: (1) a learning goal, (2) learning activities, and (3) "the thinking and learning in which the students might engage" (p. 133). The learning trajectory was imbedded in what Simon called the Mathematical Learning Cycle. The Cycle was a tool for planning constructivist learning activities for the mathematics classroom. Simon believed that teachers could plan constructivist lessons but that interaction with students during the execution of the plan would usually lead to modifications of the plan. The need to be open to modifications during execution was emphasized as part of the constructivist philosophy.

Developmentally Appropriate Practice

Teachers not only need to model correct behavior, they also need to provide scaffolding, based on the theories of Vygotsky, to support students as they learn (Pressley & McCormick 2007). Fields, Groth, and Spangler (2008) describe scaffolding actions taken by teachers that help students experience success while exploring challenging thinking. Scaffolding was intended to help students master new skills or move on to higher levels of thinking without experiencing frustration that diminishes motivation to learn. Another of Vygotsky's influential ideas was the zone of proximal development or ZPD. Tasks are in the ZPD when they are between the most difficult task a child can accomplish independently and the most difficult task a child can accomplish with assistance (Pressley & McCormick 2007). Through the application of scaffolding techniques, teachers help children become proficient in more tasks and bring more challenging tasks into their ZPD. Scaffolding was part of creating a positive learning environment where children are emotionally supported to continue trying to master tasks which are challenging to them (Otto, 2008).

Otto (2008) and Fields et al. (2008) define Developmentally Appropriate Practice (DAP) as an optimal learning environment for young children where the learning activities are matched to the developmental level of each child. They discuss using relevant authentic tasks that engage students' interest. Play should be included in the curriculum as it is an important learning activity for young children. Therefore, a developmentally appropriate activity for learning about rectangles may include a center that involves rectangles in the real world such as a construction center where children learn about different geometric shapes while playing.

The definition of rectangle is arbitrary, defined by man, not nature. It can neither be derived from the physical laws of nature nor from logical thought. This kind of knowledge was labeled as social/arbitrary knowledge by Fields et al. (2008). This knowledge must be transmitted from the teacher to the student; however, the teacher can design activities which are exploratory such as sorting physical models of rectangles and non-examples of rectangles. Through such experiences with physical models, children can inductively establish mental schemes or prototypes of rectangles that capture the correct definition of rectangle.

In conclusion, the literature on DAP for young children does not preclude challenging students to learn the correct definition of rectangle at a young age. Fuys et al. (1988) in their analysis of elementary school textbooks stated, "…if squares are omitted from all examples of rectangles, it was entirely natural for student to form the incorrect definition of rectangle observed so often…." (p. 172). Early childhood educators should note the correct definition of a rectangle and critically analyze the curriculum material they use to ensure that the learning activities do not create the misconception that squares are not rectangles.

Chapter Summary

This chapter presented definitions of rectangle that were consistent with a rectangle having 4 sides and 4 right angles with no reference to the length of the sides so that squares are rectangles (Welchons and Krickenberger, 1940, Heath, 1956, Rich, 1963, Guralnik 1987, Schwartz, 2005, and Aarts, 2008). Also presented in this chapter was evidence that many materials produced for the purpose of teaching young children the concept of rectangle were designed with flaws such as not including a square as an example of a rectangle (Clements and Sarama, 2007). Those types of errors have been difficult to fix possibly due to state policies in place with regard to publishers making corrections in textbooks used by public schools (Seeley, 2003). Studies showed that young children have exhibited the misconception that squares are not rectangles (Fuys, et al., 1988, Clements and Sarama, 2000). The learning theories about geometric thinking produced by the Van Hieles supported the idea that young children can progress to higher levels of geometric thinking if given the correct experiences (Fuys et al., 1988). The theories of Developmentally Appropriate Practice and prototype development were taken into account when designing the experimental treatment for the present study (Otto, 2008, Fields et al., 2008). Prototype development theory indicated that students need to be exposed to as many variations of a shape as possible in order for them to develop a correct concept of the shape (Fuys et al., 1988). Hierarchic interactionalism and learning trajectory theory indicated that different paths could be taken by students in learning about rectangles depending upon the interaction between the students and the teacher (Clements and Sarama, 2007). In the following chapter, an experiment will be described where a lesson designed with the above learning theories in mind was delivered to a group of kindergarten students in an effort to determine whether they could learn the correct concept of rectangle.

CHAPTER III

METHODOLOGY

Research Design

The Solomon Four-Group Design described by Campbell and Stanley (1963) was used in the present study. The null hypotheses for the present study were tested with an F distribution at the 0.05 level of significance. There were four groups with randomly selected subjects. Two of the groups were pretested: one experimental and one control group. Table 1 shows a symbolic representation of the design. The R symbolizes the random assignment of subjects to each group. Each O was a test and each X was an experimental treatment.

Table 1

Solomon Four-Group Research Design

Group	Assignment	Pretest	Treatment	Posttest
1	R	O_1	X	O_2
2	R	O_3		O_4
3	R		X	O_5
4	R			O_6

This design was most appropriate for this experiment because it will control for several possible rival hypotheses (Campbell & Stanley, 1963). To begin with, history was a factor since the subjects were actively learning mathematics in their respective classrooms. It was possible that during this experiment, the subjects' teachers may have provided instruction that directly impacted the dependent variable in this experiment, knowledge of rectangles as measured by the

Rectangle Identification Test in Appendix A. This design controlled for history by including control groups who did not experience the treatment. Therefore, any impact of history was measured by comparing the performance of the groups who experienced the treatment to those who did not.

Another rival hypothesis was that exposure to the test during the pretest period influenced the subjects' performance on the posttest. This design attempted to control for that by including both control and experimental groups who were exposed to pretesting as well as control and experimental groups who were not exposed to pretesting. The interaction of the testing with the treatment was measured by comparing the performance of pretested groups to non-pretested groups. Still, since the teachers of the students became aware of the topic being tested, they may have provided additional instruction in that topic to all or some of the students.

To minimize the impact of history and testing, the time between pretesting, treatment and posttesting was kept to a minimum. The pretest, treatment and posttest were all administered within a two-week period. By minimizing the time between pretest, treatment and posttest, the opportunity for students to gain other experiences aside from the treatment which might influence their performance on the posttest was minimized.

Population and Sample Size

The students in the population of this experiment were children in schools in south Texas. The population consisted of about 180 kindergarten students in two schools, and all students except those who have significant cognitive disabilities were eligible for random selection to participate as subjects in this study. The student population was comprised of a large proportion of Hispanic and economically disadvantaged students which are characteristic of the overall population in the community where the study was conducted. The results of the study were generalizable to the kindergarten population on the campuses where the study was conducted.

Gay and Airasian (2003) and Ravin (2015) suggested that a minimum of 30 subjects per group be included in a study such as the current study, and Brainerd (2003) was critical of studies with less than 30 students in a group. Best and Kahn (2006) stated that a sample size of 30 was considered a large sample. Kay (1987) used a group size of 16 in her study of children's ability to learn about quadrilaterals. This study replicated some of Kay's work with randomly selected subjects assigned to the four groups of at least 16 subjects per group. In the current study, two of the groups, 32 subjects, received the experimental treatment and two groups, 35 subjects, did not receive the experimental treatment.

If pretesting indicated that the selected population of subjects already had the correct concept of rectangle, then a different school would have been selected for the study. It was necessary for the purposes of this study to find a population of kindergarten children who either possessed the misconception described earlier or lacked knowledge about the concept of rectangle in order to determine the effect of the experimental treatment. In this study, it was not necessary to select a different school since all the randomly selected subjects exhibited the misconception about rectangles or had no knowledge of rectangles.

Materials

The treatment in this experiment consisted of instruction provided by a qualified teacher, as assessed by the researcher, using a lesson designed to teach the correct concept of rectangle. The pre- and posttest were the same test developed by the researcher, the Rectangle Identification Test found in Appendix A. Other materials used in the treatment are described in the Intervention Protocol in Appendix B.

The validity of the Rectangle Identification Test was derived by obtaining feedback from university and public school educators with expertise and experience in the testing of young children. To further establish the validity and reliability of the test instrument, a process of comparing interview results to test results was used. Students were interviewed to determine their understanding of the concept of rectangle. The written test instrument was administered to the students. The design of the test instrument was adjusted until the results of the administration of the written test instrument matched at least ninety percent of the time with the results of the face-to-face interviews.

Dependent Variable

The dependent variable in this study was kindergarten students' mastery of the concept of rectangle as measured by the Rectangle Identification Test that required students to identify shapes that were rectangles. The tests were administered to small groups of students with students from both the experimental and control groups tested together.

As a further control contributing to the validity of the test results, the testing was conducted by an observer who did not know whether the students were part of the control group or the experimental group. Also, the test administration was observed by the researcher and qualitative notes were taken describing the test administration and any perceived confounding factors such as unexpected discipline issues. A qualitative summary of those notes is included in the section in Chapter 4 titled Summary of Researcher's Observations of Treatment and Posttesting.

Independent Variable

The independent variable, the treatment, in this study was mathematics instruction about the concept of rectangle. The instruction was provided by a teacher certified to teach elementary children in small groups of 6 to 11 students each. The length of the instruction was about one hour. The instruction followed a lesson designed to teach the correct concept of rectangle. The teacher was allowed to modify the lesson as was required by practical limitations that existed in the school while the key feature of including squares as rectangles was preserved in the lesson at every opportunity.

The instruction was observed by the researcher and qualitative notes were taken describing the instruction. A qualitative summary of those notes was included in the results of the study.

Research and Null Hypotheses - Experimental

$H_{1.1}$: There was a difference in mathematics achievement of kindergarten students instructed with a curriculum that includes squares as rectangles and the mathematics achievement of a control group as measured by a test of students' ability to identify rectangles.

$H_{0.1}$: There was no difference between the mathematics achievement of kindergarten students instructed with a curriculum that includes squares as rectangles and the mathematics achievement of kindergarten students instructed with other curricula as measured by a test of students' ability to identify rectangles.

$H_{1.2}$: There was a difference between the mathematics achievement of kindergarten students who were pretested and the mathematics achievement of kindergarten students who were not pretested as measured by a test of students' ability to identify rectangles.

$H_{0.2}$: There was no difference between the mathematics achievement of kindergarten students who were pretested and the mathematics achievement of kindergarten students who were not pretested as measured by a test of students' ability to identify rectangles.

$H_{1.3}$: There was an interaction between testing and treatment as indicated by the mathematics achievement of kindergarten students as measured by a test of students' ability to identify rectangles.

$H_{0.3}$: There was no interaction between testing and treatment as indicated by the mathematics achievement of kindergarten students as measured by a test of students' ability to identify rectangles.

Data Collection Procedures

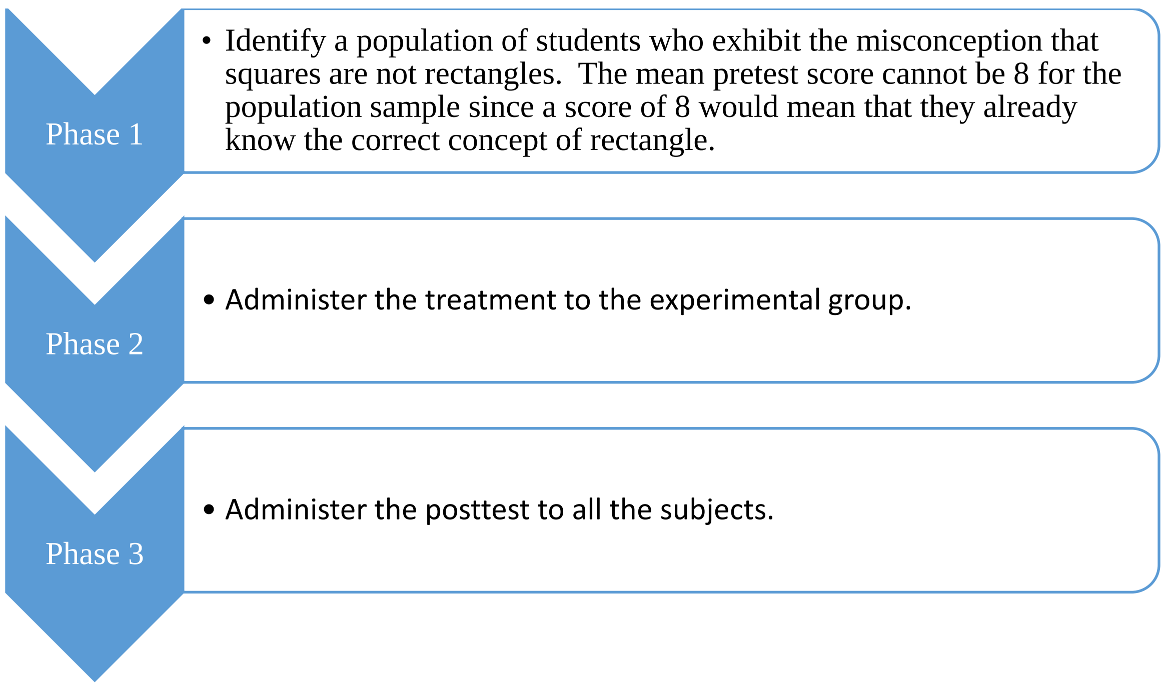
Following approval by the Institutional Review Board at the University of Texas Pan American and the school district involved in this study and the collection of signed consent and assent forms as described above, a district employee administered the Rectangle Identification Test which was developed by the researcher to the subjects as described in the experimental design and procedures. Only Groups 1 and 2 were pretested. All four groups were posttested. The researcher collected the pre- and post-tests after they were administered by a district employee and scored them using the Rectangle Identification Test Scoring Rubric found in Appendix A. The researcher observed the administration of the instructional treatment and posttests and made qualitative notes describing the events.

Experimental Procedure

Following approval by the Institutional Review Board at the University of Texas Pan American and the school district involved in this study, the researcher obtained the necessary signed consent forms from the participating campus principal, teachers and parents of student subjects. Since the subjects were kindergarten children, signed informed consent forms were obtained from the parents of the subjects. The subjects were informed about the experiment and given the choice to participate, assent, or to decline to participate. Developmentally appropriate, signed assent forms were obtained from the subjects. See Appendix C for a blank copy of the consent and assent forms. Subjects were selected at random from the population of kindergarten students at the campus excluding students with significant cognitive disabilities for whom the teacher does not believe the learning task was appropriate. The study did not use the existing classroom groupings. The experimental treatment was administered through arrangement with the campus principals during the regular school day. This study was conducted in three phases depicted in Figure 2 below.

Figure 2

Three Phases of Experimental Procedure



Phase 1 of the study: Identification of prospective subjects and pretesting procedures. To establish the extent to which students have already mastered the concept of rectangle, the Rectangle Identification Test was administered to the students in Groups 1 and 2 before the treatment was given to the experimental Groups 1 and 3. This phase served to establish that students either had no concept of rectangle, had the correct concept of rectangle, or had a misconception about rectangles. If this phase had indicated that the majority of the students had a well-formed and correct concept of rectangle, then a different sample from a different school would have been selected. This study was deemed appropriate because a population sample with the misconception about rectangles existed.

The researcher recruited a non-biased district employee to administer the pretest. The pretest administrator was a district employee who did not work at the particular campus where this study was conducted. The pretest administrator administered the pretest to the subjects in Groups 1 and 2 in small groups typically of 3 students. The pretest administrator was compensated for her preparation time which occurred after regular school hours, paid a stipend, by the researcher. The same pretest administrator was used for all students who were pretested whether part of the control group or the experimental group.

Based on the design of the Rectangle Identification Test, a subject who has the misconception that a square was not a rectangle was expected to score a 4 on the portion of the test designed to measure that construct. Students who had the correct concept of rectangle were expected to score an 8 on that test. Therefore, a One-Sample case test on SPSS was used to determine whether the mean score was different from 4 and different from 8.

If the mean pretest score had not been different from 8, then the subjects would have exhibited the correct concept of rectangle. In that case, a different sample from a different school would have been used. A new random sample of subjects would have been selected. This did not occur since almost all the subjects scored a 4 on the pretest. The pretest was administered and analyzed as described above, and the sample exhibited the misconception about rectangles.

If the mean pretest score was 4, then the subjects would have exhibited the misconception about rectangles. If the mean pretest score was different from both 4 and 8, then the subjects would have exhibited neither the misconception nor the correct concept of rectangle. In either of these cases, the researcher would have proceeded to Phase 2 of the study.

Phase 2 of the study: Treatment procedures. The researcher made arrangements with the campus principal for the delivery of the experimental treatment, instruction on the correct concept of rectangles. The treatment occurred during the school day. Two district staff members who delivered the experimental instructional treatment were compensated by the researcher for preparation time that occurred after regular school hours.

As treatment in this study, instruction on the concept of rectangle was provided for one period of about 60 minutes to students in groups of 6 to 11. More than one treatment period was necessary to keep the treatment group sizes small. Also, as was the practice in schools, a session was scheduled for students who do not master the concept during the initial treatment. The re-teaching was limited to about 30 minutes. After the treatment, the students were administered the Rectangle Identification Test to gather information about their understanding of the concept of rectangle.

The researcher recruited two district employees to deliver the instruction which was the treatment in this study. The employees were selected based on certain characteristics. They were certified to teach kindergarten students and had a mastery of the correct concept of rectangle. They were not teachers at the schools where the students were enrolled. The employees philosophically espoused the practice of teaching the correct concept of rectangle to kindergarten children. These characteristics were measured with the Teacher Rectangle Survey in Appendix E. The employees were compensated, paid a stipend, by the researcher for the time spent preparing for the delivery of the treatment. The employees did not know which subjects received pretesting.

Phase 3 of the study: Posttesting procedures. The researcher recruited a posttest administrator to administer the posttests. The posttest administrator did not know which subjects participated in the experimental treatment and did not know which subjects participated in the control group. The posttest administrator was trained on how to administer the Rectangle Identification Test. The posttest administrator was a district employee who did not work at the particular campus where this study was conducted. The posttest administrator administered the posttest to all the subjects in this study. The posttest administrator was compensated for her preparation time, paid a stipend, by the researcher.

Data Analysis Procedure

The quantitative data collected was in the form of items correct on the Rectangle Identification Test. The analysis featured a comparison of the mean number of items correct for the treatment group versus the control group. There was also an analysis of the effects of pretesting and the interactions of testing with the treatment.

The analysis was organized as shown in Table 2 below. As suggested by Campbell and Stanley (1963), an analysis of the column means were used to estimate the effect of the experimental treatment and an analysis of the row means were used to estimate the effect of pretesting. A 2 by 2 analysis of variance was used as indicated in Table 2 below. There was also an analysis of the interaction of testing with X using the cell means. All statistical calculations were done using a computer program, SPSS version 22.

Table 2

2 X 2 Analysis of Variance Design Suggested by Campbell and Stanley (1963)

	Independent Variable 2	
Independent Variable 1	No X	X
Pretested	O_4	O_2
Unpretested	O_6	O_5

In the analysis, if the mean for column X was significantly higher than the mean for column No X, then the conclusion would have been that the treatment caused the increase. The statistical calculations were done through a comparison of means using Two-Way Analysis of Variance (ANOVA). The data was coded into two groups: one group was the data for the No X column and the other group was the data for the X column. Each of those groups was further coded into the four groups of the Solomon Four-Group experimental design.

If there had been a significant difference between the row means, Pretested and Unpretested, then the conclusion would have been that pretesting effected student achievement. If the cell mean for cell Pretested with X, cell O_2 in Table 2, was significantly different from the cell means for the other cells, then the conclusion would have been that there was an interaction between testing and treatment.

Table 3 shows how the data was coded so that it could be analyzed with SPSS. There were 67 entries in the data table. The General Linear Model (GLM) Univariate procedure from SPSS 22 was used to produce the Two-Way ANOVA. The Dependent Variable was the posttest scores, PostScores. The Fixed Factors were the variables Treated and Pretested. This produced the required F test of significance at an alpha level of 0.05 for testing the null hypotheses for the effect of treatment and pretesting. Since the treatment produced a significant result, an estimate of the effect size was produced using the Options in the Univariate analysis on SPSS 22.

Table 3

Data Coding For 2 x 2 ANOVA Input into SPSS

SPSS Variables			
Treated	Pretested	Group	Scores
YES	YES	1	16 subjects
NO	YES	2	17 subjects
YES	NO	3	16 subjects
NO	NO	4	18 subjects

The qualitative notes taken by the researcher during the treatment and posttesting described any anomalies which may have affected the validity of the quantitative results.

Chapter Summary

The design for this study included a quantitative component intended to produce data regarding whether kindergarten students could demonstrate that they learned the correct concept of rectangle as measured through the Rectangle Identification Test after having experienced learning activities designed to teach the correct concept of rectangle. A Solomon Four-Group Design described by Campbell and Stanley (1963) was used to control for confounding factors. A 2-way ANOVA, as suggested by Campbell and Stanley, was used to determine whether the performance of the students was different between the control group and the experimental group. The sample was pretested to determine whether the sample of students exhibited a problem with the understanding of the concept of rectangle. This was determined through a one-sample t test. Qualitative notes were taken by the researcher during the treatment and posttesting. A survey was administered to kindergarten teachers in the geographical area where the study took place to gather data regarding their knowledge about the concept of rectangle and how to teach it. The next chapter presents the results of the experiment, a summary of the qualitative observations made by the researcher and the results of the teacher survey.

CHAPTER IV

RESULTS

Introduction

This chapter consists of teacher survey results and exploratory as well as confirmatory results for the hypotheses tests. A total of 67 kindergarten students participated in this study and 39 responses to a teacher survey were submitted.

Teacher Rectangle Survey Results

Table 4 below shows the results of the teacher survey found in Appendix F.

Table 4

Results of Teacher Rectangle Survey

	Teacher Responses		
Item	True	False	Correct Answer
1	36	3	True
2	26	13	False
3	26	13	True
4	38	1	True
5	21	17	False
6	25	13	True

Twelve out of the 39 teachers who responded to the survey instrument shown in Appendix F wrote a definition of rectangle in response to survey item 7 that defined the concept of rectangle in a manner that was correct and sufficient. Nine of the teachers left the response for item 7 blank and therefore did not define the concept of rectangle. Ten of the teachers indicated that rectangles need to have two sides longer than the other two exhibiting the misconception which is at the center of the problem addressed by this study.

Of the 39 teachers who responded, 12 had responses that were not logically consistent, 12 had responses that were logically consistent and 15 left some of the items blank. Seven of the teachers wrote responses to items 7 and 8 that were logically inconsistent stating that rectangles had to have 2 sides longer than the other two yet picking Activity 2 in Item 7 which classifies squares as rectangles. Nine of the teachers indicated that Items 5 and 6 were both true which is a logical inconsistency stating at the same time that rectangles must have two sides longer than the other two and that all squares are rectangles.

Student Rectangle Identification Test Results for the Pretest

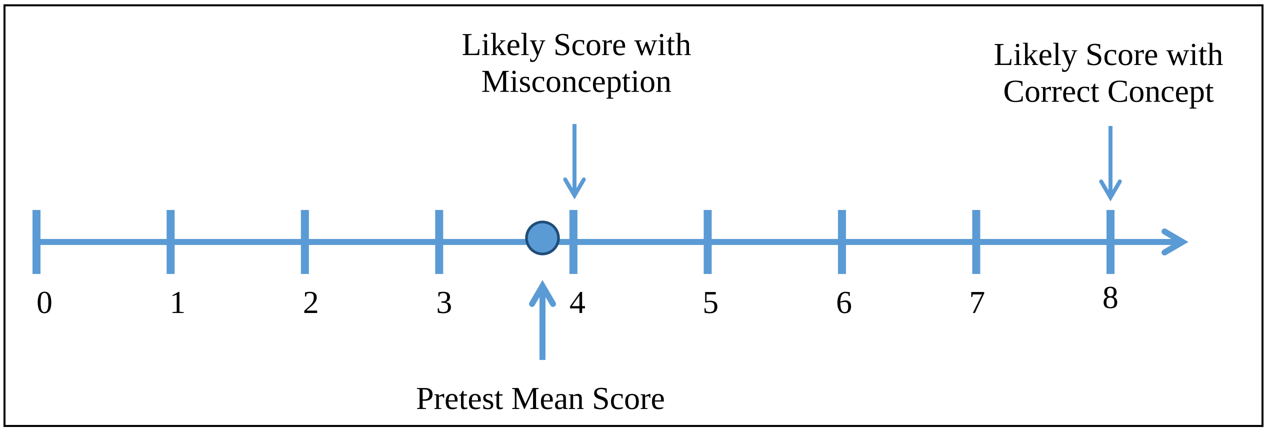
The pretest was administered after the students had experienced learning activities about the concept of rectangle using the State approved district adopted curriculum written by Charles, et al. (2015). The material was the second edition of the textbook and it showed improvements with regard to the concept of rectangle from the first edition. As described in the review of literature, the first edition by Charles, et al. (2009) included a design flaw where all the examples of rectangles were oblong. That was corrected in the second edition by Charles, et al. where squares were included as examples of rectangles in the learning activities for the students. While the written curriculum adopted by the district has improved, the actual learning experiences which occurred prior to the pretest were not observed so there is no data in this study regarding the actual instruction provided prior to the pretest administration.

A One-Sample T-Test was run on the pretest results of the Rectangle Identification Test found in Appendix A with two different Test Values, 4 and 8. The sample size for the pretest was 33 kindergarten students. The mean pretest score was 3.58 with a standard deviation of

0.792 and a standard error mean of 0.138. Tests of normality for the pretest data indicated that the data were not normally distributed. The Shapiro-Wilk statistic for 33 pretest scores was 0.603, df(33), $p<.05$. Thus, the null hypothesis of the Shapiro-Wilk test of normality was rejected indicating that the data were distributed differently than normal. Still, as per Ravid (2015), the one-sample *t*-test can be used even when the assumption of normality of distribution of the data is not met. The mean difference from the score of 8 was -4.424 while the mean difference from 4 was -0.424. While there was a statistically significant difference between the pretest scores and both 8 and 4 according to the one-sample t-test, the difference was ten times larger from 8 than from 4 as can be seen graphically in Figure 3 below.

Figure 3

Graphical Representation of Pretest Scores



Student Rectangle Identification Test Results for the Posttest

The posttest result for one kindergarten student was invalidated because the student circled all figures on the posttest. Even in the practice item on the posttest where the student was supposed to circle only the stars, the student circled all shapes. Therefore, that particular student's results were not a valid indication of whether the student understood the concept of rectangle. That subject's results were not included when calculating means or other statistics.

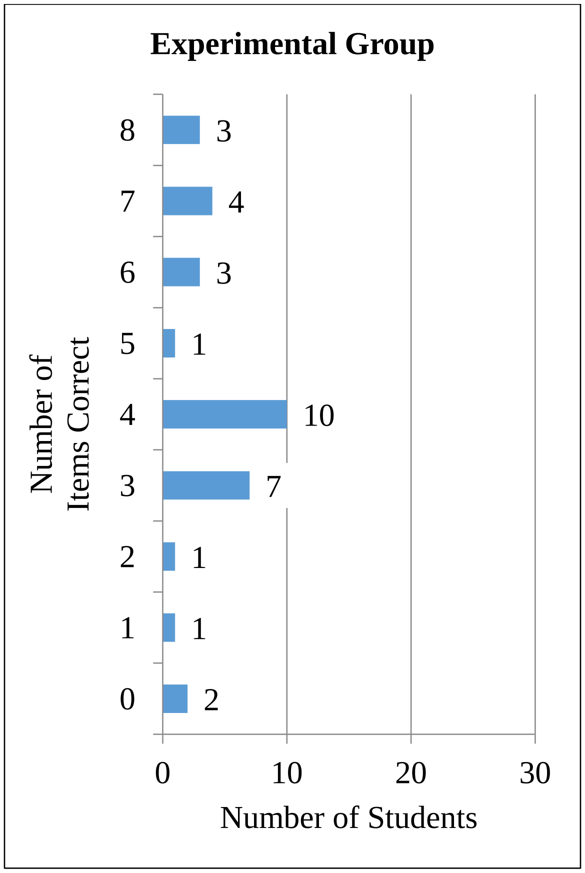
The groups in this study were organized into four groups in accordance with the Solomon Four-Group Design presented by Campbell and Stanley (1963). Groups 1 and 2 consisting of a total of 33 students were pretested, and Groups 1 and 3 consisting of a total of 32 students received the treatment. Campbell and Stanley recommended that 2 x 2 analysis of variance design be used to analyze the results in studies using the Solomon Four-Group design. Ravid (2015), stated that a Two-Way Analysis of Variance (ANOVA) is designed to study the relationship between two or more independent variables and a dependent variable. In this study the independent variables were treatment and pretesting. The dependent variable was posttest scores. Therefore, a Two-way ANOVA was used as the statistical method to analyze the posttest scores for the present study using SPSS 22.

As per Ravid (2015), the following assumptions exist for ANOVA: (1) the groups are independent of each other, (2) the dependent variable is measured on an interval or ratio scale, (3) the dependent variable is normally distributed, (4) the scores are random samples from their respective populations, and (5) the variances of the populations from which the samples were drawn are equal, homogeneity of variances. According to Ravid, the first two assumptions must be met and the second two may be violated to some extent.

There were 67 students posttested. Thirty five students were in the control group with no treatment and 32 students received the treatment. The Wilks-Shapiro test for normality of distribution indicated that neither the control group nor the experimental group had results that were normally distributed. Figure 4 shows a histogram of the results of the student posttest, and Table 5 shows a summary of the results of the Two-Way ANOVA produced with SPSS 22.

Figure 4

Results of Student Posttest



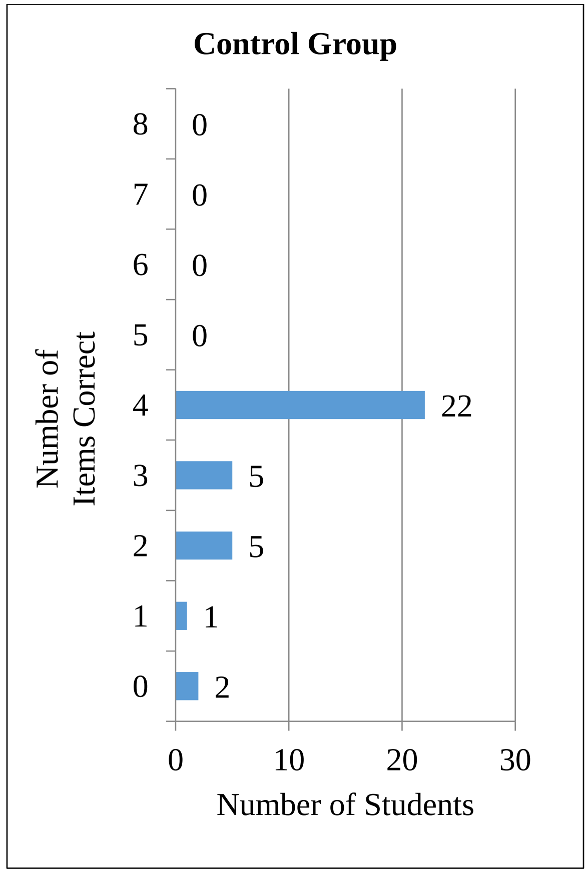


Table 5

Summary of a Two-Way Factorial ANOVA (2X2) using a Solomon Four-Group Design

Source	Sums of Squares	df	Mean Square	F	Significance	Effect Squared
Bet Treatment Conditions	19.515	1	19.515	6.544	0.013*	0.094
Bet Pretest	3.992	1	3.992	1.339	0.252	0.021
Cell Effect Bet Treat and Pretest	0.091	1	0.091	0.030	0.862	0.000
Error	187.867	63	2.982			
Corrected Total	211.642	66				

* $p < .05$

As shown in Table 5, the null hypothesis $H_{0.1}$ was rejected at the alpha level of 0.05. Therefore, there was a significant difference between the mathematics achievement of kindergarten students instructed with a curriculum that includes squares as rectangles and the mathematics achievement of kindergarten students instructed with other curricula as measured by the Rectangle Identification Test. The mean score on the Rectangle Identification Test for the students who received the treatment was 4.34 while the mean score for students who did not receive the treatment was 3.26. The effect size using the Partial Eta Squared from SPSS indicates that about 9% of the variance in the posttest scores was due to the treatment.

The second null hypothesis of this study, $H_{0.2}$, was not rejected at an alpha level of 0.05. Therefore, the data seem to indicate that there was no difference between the mathematics achievement of kindergarten students who were pretested and those who were not as measured by the Rectangle Identification Test.

The third null hypothesis, $H_{0.3}$, was not rejected at the alpha level of 0.05. Therefore, the data seem to indicate that there was no interaction between testing and treatment as indicated by the mathematics achievement of kindergarten students as measured by the Rectangle Identification Test.

Summary of Researcher's Observations of Treatment and Posttesting

Treatment was a lesson about rectangles administered to small groups of kindergarten students. After taking notes, the researcher found a theme regarding at which Van Hiele level the learning activities were. Learning activities were identified as being at the lowest Van Hiele level, level 0 or the visual level, if the activities involved simply identifying squares and oblongs as rectangles and not dwelling on the characteristics such as number of sides or whether the shape had square corners. The learning activities were identified as being at the second Van

Hiele level, Level 1, if the activities focused on having students master the characteristics of rectangles and squares.

The first group was a group of 6 students. There was one Level 0 activity identified and four Level 1 activities. The Level 0 activity consisted of the teacher simply placing plastic shapes for the students to see under the document camera and identifying the shapes. The shapes included rectangles, squares, triangles, circles, a trapezoid and a rhombus. The Level 0 activity was very brief. The first Level 1 activity consisted of the teacher modeling how a square had corners that fit perfectly onto the square corner of a paper. The students mimicked the teachers actions using their own set of manipulative materials at their tables. The second Level 1 activity consisted of the teacher leading the class in exploring the whether the oblong had square corners. The third Level 1 had the teacher placing different shapes under the document camera and comparing the corners of the shapes to the square corners of a sheet of pink paper by laying the shape on the paper. The teacher would ask the students whether the corners were square and follow-up with "Why?". The teacher asked several "why" questions. "Why is it a rectangle?" "Why is it not a rectangle?" In the fourth Level 1 activity, the teacher spent a lot of time asking "why" questions and classifying rectangles as either oblong or square. The students were given the Rectangle Worksheet in the format of a formative assessment. The use of the phrase "not oblong" rather than "square" on the Rectangle Worksheet seemed to confuse the students. The lesson lasted approximately 1 hour. Student performance on the Rectangle Worksheet indicated that they did not grasp the concept that squares were rectangles.

The second group of students consisted of 6 students. Four Level 0 activities and four Level 1 activities were identified. The first Level 1 activity consisted of the teacher explaining to the students the difference between a rhombus and a square, essentially teaching an incorrect concept in the similar fashion as when teachers present rectangles and squares as if they are distinct sets of quadrilaterals. The first Level 0 activity followed with the teacher telling the students that oblongs and squares were both rectangles. The teacher had the students repeat that statement in chorus. Teacher had students sort plastic shapes into rectangles and not rectangles on a mat while the teacher modeled under the document camera. The teacher then transitioned into a Level 1 activity by demonstrating how the corners of the square fit onto the corners of the mat. The teacher then explained that the orientation of a square on the mat does not change the fact that the square is still a square. In another Level 1 activity, the teacher demonstrated how a non-rectangular parallelogram did not have square corners. In a Level 1 activity, the teacher placed the plastic shapes under the document camera and removed them one at-a-time asking the students, "Why is it not a rectangle?". The teacher had 2 squares and 2 oblongs left under the document camera and asked the students, "How many squares?". Most students responded in chorale, "Two". One student said, "Four".   Teacher held up a plastic shape and asked whether it was a rectangle. If the answer was "yes", then the teacher put it on a mat under the document camera; otherwise, the teacher placed it to the side off of the mat. The teacher asked "why" was the oblong a rectangle, and a student responded that it was a rectangle because it was long. The teacher had the students complete the Rectangle Worksheet and decided to reteach since the students did not seem to understand that squares were rectangles. During the reteach activity, the teacher focused on using the words square and oblong telling the students that there were two types of rectangles, square rectangles and oblong rectangles. During the lesson, there was distracting noise that could be heard from groups of students moving up and down the stairs outside of the classroom.

Prior to beginning the treatment for the third group, the researcher gave the teachers feedback that much of the instruction seemed to be at Level 1 rather than Level 0. The researcher directed the teachers to spend less time on the vocabulary of oblong and square and to spend more time on Level 0 activities identifying rectangles.

The third group to receive the treatment had 11 students in it. During this observation, the researcher noted five Level 0 learning activities and three Level 1 learning activities. At Level 0, the teacher had the students pull out all the rectangles including the squares from the plastic bags where the shapes were stored. The teacher then moved to Level 1 by asking what was special about squares. A student pointed out that there were examples of oblongs in the room so the teacher proceeded to lead the class in discussing the different examples of oblongs that could be seen in the room. At Level 1, the teacher had the students put the corner of a square onto a mat and asked whether it covered the entire corner. The teacher repeated that with an oblong, trapezoid and parallelogram in an attempt to have the students understand the concept of square corners. Going back to Level 0, the teacher asked the students to count the number of rectangles they had. The teacher then had the students name the shapes that were rectangles and those that were not rectangles. The teacher attempted to check for understanding by displaying 2 squares and 2 oblongs under the document camera and asking the students to state name of the shapes. The students gave different responses but did not call them rectangles. The teacher then demonstrated each shape, one-by-one, and asked whether it was a rectangle. The teacher had the students sort the rectangles into squares and oblongs. Then, the teacher had the students sort the shapes into rectangles and non-rectangles. The teacher then put the 2 squares and 2 oblongs back under the document camera and asked the students to state how many rectangles were there. The students successfully responded that there were 4 rectangles. The teacher then had the students complete the Rectangle Worksheet.

The students on one campus where treatment was finished were posttested except for two students who were absent. The students were tested in groups of 3 except for the last two groups. On group consisted of 4 students and the very last group consisted of 2 students. The posttest administrator was a teacher certified to teach kindergarten with several years of experience teaching kindergarten. The teacher explained the directions to the students in English or Spanish depending upon which language the students understood. The teacher used dividers to prevent the students from seeing each other's responses. The teacher obtained assent from students who had not been pretested or received treatment. One student did not assent to posttesting, so that student was excluded from the analysis. The students seemed to follow directions. No factors that might have affected the validity of the posttest results were noted.

The fourth group to receive the treatment consisted of 11 students. The first learning activity was at Level 1 with the teacher telling students to pick up a large square and then asking them about the characteristics. The teacher asked whether the square had 4 sides and square corners. The teacher proceeded to demonstrate under the document camera that the corners of the square covered the corner of the green mat completely so that no green could be seen in the corner. The teacher placed a trapezoid with a non-square corner on the green mat to show the students that they could still see part of the green mat that was not covered by the corner of the trapezoid. The teacher stated to the students that the trapezoid was not a rectangle because the corners of the trapezoid were not square. The teacher then demonstrated how the small square was a rectangle because it had four sides and the corners were all square. Several students were not paying attention to the demonstration being conducted by the teacher. The teacher began working with one student in particular and other students began playing and not working on understanding the concept of rectangle. The teacher then spent some time helping the students differentiate between squares and oblongs. In another Level 1 activity, the teacher told the students that rectangles had two names, a first name and a last name. The first name was either square or oblong and the last name was rectangle. The teacher attempted to elicit the characteristics of rectangles from the students, but the students were having difficulty stating that the lengths of the sides were different in an oblong. The teacher then spent some time helping the students understand that a rotated square is still a square even if it is also called a diamond. The teacher then attempted to check for understanding by putting a square under the document camera and asking whether it was a rectangle. The students responded that the square was not a rectangle. The teacher proceeded with a Level 0 activity asking the students to pull out all the rectangles. The students pulled out only the oblongs and the teacher directed them to pull out the squares also. The teacher had the students complete the Rectangle Worksheet and it was clear that the students had not mastered the concept of rectangle. The teacher reviewed the oblong with the students and then spent time addressing one student in particular. Other students began playing rather than learning. The teacher directed the students to get back to work. The teacher then placed the 2 squares and the 2 oblongs under the document camera and stated that all were rectangles. The teacher had the students put an X on all the shapes that were not rectangles on the Rectangle Worksheet. The teacher then reviewed one more time using the document camera and a student volunteer to demonstrate that squares as well as oblongs were rectangles, a Level 0 activity.

After the fourth group was administered the treatment, the researched discussed with the teacher whether re-teaching was needed since the researcher noticed that students responded more successfully to Level 0 activities rather than to Level 1 activities. The researcher and the teacher agreed that re-teaching was needed with an emphasis on Level 0 activities. Re-teaching was scheduled for the following day. Only with the treatment groups on one campus since all except two students on the other campus had already been posttested.

The first re-teaching lesson consisted of four Level 0 activities. It was a group of 11 students. The first learning activity consisted of the teacher placing each shape under the document camera and asking whether it was a rectangle. The teacher gave corrective feedback as needed. One student spontaneously stated that a square was a special kind of rectangle. In the second learning activity, the teacher had each student pick out a shape from a bag containing platic shapes without looking. The student was then required to state to the class whether the shape was a rectangle or not. The teacher made sure the student got the correct answer, but most students were correct. As each student identified the shape as a rectangle or as not a rectangle, the student was told to move to one side of the room or the other. On one side of the room stood all the students who had rectangles in their hands, including squares. On the other side of the room stood students who had shapes that were not rectangles such as circles, triangles, trapezoids and non-rectangular parallelograms. The teacher had all the students on one side hold up their shapes and say whether it was a rectangle or not. The teacher then repeated that with students on the other side of the room. The students had a high success rate and seemed to enjoy the activity. For the third learning activity, the teacher had the students select one shape from their bag of shapes and trace it onto the back of the Rectangle Worksheet. The students were then required to write "Yes" if the shape was a rectangle and "No" if the shape was not a rectangle. The teacher then had the students complete the Rectangle Worksheet. The students showed a high level of success in correctly identifying all the rectangles including the squares on the Rectangle Worksheet. Those who did not were given corrective feedback.

With the second reteach group, the teacher repeated the Level 0 activities conducted with the first reteach group. The teacher put shapes under the document camera and had the students declare whether they were rectangles or not. This group also demonstrated high levels of success in identifying rectangles including squares initially, but when teacher had students pick out the shapes from a bag, the students did not successfully identify the square as a rectangle. The teacher had the go to opposite sides of the room, one side for students holding rectangles and one side for shapes that were not rectangles. The teacher had the students pick a shape from their own bag of shapes and draw it on the back of the Rectangle Worksheet. Then the students wrote "Yes" next to the shape if it was a rectangle and "No" if it was not a rectangle. The teacher picked up one shape at-a-time and asked whether it was a rectangle. The students replied in chorale. The teacher added an activity to demonstrate to students that a rotated square was still a square.

Chapter Summary

In this chapter, the researcher described the results obtained through the Teacher Rectangle Survey, Rectangle Identification Test, and qualitative observation notes. The Teacher Rectangle Survey results indicated that over 50% of the teachers had a misconception about rectangles. The pretest results of the Rectangle Identification Test indicated that the students had the misconception that two sides of a rectangle had to be longer than the other two. The posttest results of the Rectangle Identification Test indicated that the treatment had a statistically significant effect on the performance of students on the Rectangle Identification Test. In the next chapter, the researcher will present discussion, recommendations and limitations of the study.

CHAPTER V

DISCUSSION, RECOMMENDATIONS FOR RESEARCH, RECOMMENDATIONS FOR PRACTITIONERS, AND LIMITATIONS OF THE STUDY

Discussion Regarding the Teacher Rectangle Survey

The results of the Teacher Rectangle Survey indicate that many kindergarten teachers in the population studied have the same misconception that is the focus of this study. Almost all of the teachers identified the oblong as a rectangle; however, only 2/3 of the teachers identified a square as a rectangle. Furthermore, 21 out of 39 teachers indicated that rectangles had to have two sides longer than the other two. Nine of the 39 teachers had logically inconsistent responses such as indicating that rectangles had to have two sides longer than the other two and at the same time indicating that all squares are rectangles. If teachers have these misconceptions, then it is possible that the reason students learn the misconceptions is because teachers arrange learning activities that are aligned to their own misconceptions. In order to improve learning of the concept of rectangle for kindergarten students, the researcher recommends that kindergarten teachers be given the opportunity to learn the correct concept for themselves through staff development activities. Further research could be conducted to determine the effectiveness of staff development activities regarding teaching of the concept of rectangle.

Care needs to be taken when designing staff development activities for teachers so that the misconception that rectangles must be oblong is not inadvertently promoted. For example, Sousa (2010) and Tucker, Singleton and Weaver (2013) provide activities that may be used to teach the concept of rectangle, but both recent publications have the potential to create the misconception that rectangles must be oblong. Sousa includes activities where squares and rectangles are treated as separate shapes. Tucker, Singleton and Weaver include a warning to the reader that students often develop the misconception that squares and rectangles are different shapes; however, several activities are given where the only examples of rectangles are oblong. Sarama and Clements (2009) suggest that the best approach is to present many examples of squares and rectangles with varying sizes and orientations, and to include squares as examples of rectangles.

Discussion Regarding Errors in the Curriculum

As part of the present study, the researcher analyzed the curriculum which was adopted by the schools who participated in the study. The researcher found that the newly adopted curriculum has an improved treatment of the concept of rectangle. The definition of rectangle provided in the material is correct. Squares are clearly identified as rectangles. There are some learning activities where it is possible that students may become confused if the material is presented by the teacher in an incorrect manner. That is, sometimes students are expected to identify rectangles and squares as if they are distinct sets.

Errors in textbooks used by public schools for teaching the concept of rectangle have been well-documented for decades. While there have been improvements in the treatment of the concept of rectangle, some errors still need to be corrected. To report an error in a public school textbook in the State of Texas, individuals may send an email to textbookerrors@tea.texas.gov with the following information: (1) title of instructional materials, (2) publisher name, (3) itme type, (4) ISBN, (5) media type, (6) number of the page on which the error is found, (7) location on page, (8) description of error, (9) name of person reporting the error, (10) teacher name if applicable, (11) school and school district if applicable (Reporting Errors, 2015).

Discussion of Student Posttest Results

The posttest results indicated that kindergarten students were able to learn that the shapes which are called rectangles include both oblong rectangles and square rectangles. The study was not designed to determine whether students actually learned the word oblong. It was designed to gather evidence that the students learned that squares are rectangles.

Smith (2013) made reference to the Van Hiele levels describing Level 0 as if students were not able to identify a square as a rectangle because it did not look like a rectangle. That characterization of Level 0 of Van Hiele was not supported by the results of the current study. The data produced by the current study seem to indicate that students in kindergarten can learn that squares are rectangles simply by including squares as examples of rectangles. Smith went on to state that precise language was needed when discussing squares and rectangles among other shapes. That statement by Smith is indicative of the misconception that squares are a distinct class of shapes apart from rectangles. The results of the current study do support the suggestion from Sarama and Clements (2009) that multiple examples of squares as rectangles would produce the best mastery of the concept of rectangle by kindergarten students.

The qualitative data produced by this study was supportive of the Van Hiele theory which indicates that students begin their geometric thinking at Level 0. When the teachers presented Level 1 learning activities, the students exhibited confusion, lack of understanding and discipline problems. When the teachers presented Level 0 learning activities, the students seemed to enjoy the activities and exhibited high rates of success in mastering the concept that squares are rectangles.

Recommendations for Further Research

One question which this study did not address is why some educators and some of those who produce educational materials continue to support the teaching of the incorrect concept of rectangle. By finding the answer to that question, we might be able to understand how we can change the curriculum material that is being produced with poor learning activities regarding the concept of rectangle. Characteristics of poorly written material include materials that treat square and rectangle as distinct classes of shapes and materials that provide only oblong rectangles as examples of rectangles.

Another area of possible research is to explore the impact on students' ability to think logically when they experience logically inconsistent learning activities. This happens when teachers conduct learning activities that teach students that rectangles must have two sides longer than the other two. In other words, rectangles are oblong. Then, the same teacher, sometimes in the same lesson, teaches the students that all squares are rectangles. When students experience these two logically incompatible experiences, how does that impact their thinking about shapes and mathematics? Do students learn that mathematics does not always make sense? Do they learn to stop trying to make sense out of mathematics?

Another possible area for research is to determine the grade level at which the concept of rectangle gets corrected for students who learned it incorrectly in kindergarten. In Texas, students are expected to use characteristics to identify rectangles by third grade (19 TAC Chapter 111, 2012). How does learning the concept of rectangle incorrectly in kindergarten at Van Hiele Level 0 impact students' ability to master the classification of shapes as rectangles based on the shapes' characteristics in grade 3 at the Van Hiele Level 2?

Recommendations to Practitioners

Seeley (2003) suggested that the time allotted for reviewing instructional materials was not adequate for ensuring the quality of the materials. While that may still be true, in Texas, the Texas Education Agency (TEA) has provided a method by which interested parties may submit information to TEA about errors in curricula by sending information to textbookerrors@tea.texas.gov. The researcher recommends that parties who are interested review textbooks topics, in particular, review the way the concept of rectangles is taught in the lower elementary grades. If errors are found, communicate those errors to state education agencies and publishers so that the errors can be corrected. As early as 1988, Fuys, Geddes, Lovett and Tischler found that errors existed that were the topic of the present study in 2015. Perhaps through the use of modern communication technology we can correct these pernicious errors that have persisted for decades.

The results of this study indicate that students at the kindergarten level are able to learn the correct concept of rectangle in the sense that squares are rectangles. The researcher recommends that kindergarten teachers examine their knowledge and practices regarding how they teach the concept of rectangle. They should collaborate with 1st, 2nd and 3rd grade teachers who are responsible for further developing the students' concept of rectangle to make sure that the learning activities they use in kindergarten help students develop the correct concept of rectangle. The correct concept of rectangle, in language appropriate for kindergarten, is that rectangles have four sides and four right angles. There are many ways to convey the same concept, but in all cases, squares are rectangles. This concept of rectangle has been confirmed by the researcher to be consistent with many sources such as Heath (1956) and Schwartz (2005).

From the qualitative aspect of this study, the data indicate that the kindergarten students involved in the study were more successful in learning about the correct concept of rectangle when the teacher provided learning activities aligned to the lowest Van Hiele level. A recommendation for practitioners is that kindergarten teachers should be trained to understand the Van Hiele levels so that teachers are able to adjust the learning activities to match the level at which the students are able to learn.

Limitations of This Study

One limitation of this study was that it was conducted within two schools in South Texas with a population that was over 99% Hispanic, about 90% economically disadvantaged and about 60% English language learners whose primary language at home was Spanish. The students who participated in the study were those whose parents consented. Thus, the results of this study may be generalized, at most, to the kindergarten population of the two schools involved.

Another limitation and possible confounding factor was the quality of instruction provided by the two teachers who delivered the treatment. Both were certified to work with elementary children and had more than 5 years of experience doing so. Both were administered the Teacher Rectangle Survey and gave correct answers throughout the survey. Still, they had subtle differences in their styles of presentation which may have produced differing effects. Through the qualitative observations which the researcher conducted of the treatments, it seemed there was more variation in student success due to the level of instruction, Van Hiele Level 0 versus Van Hiele Level 1, than there was due to the different instructors. Students with either teacher demonstrated lack of mastery when the teacher focused on the properties of rectangles, Level 1 of Van Hiele. The kindergarten students experienced more success when the teachers simply showed them correct examples of rectangles rather than trying to explain how to use the properties of rectangles to identify rectangles.

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APPENDIX A

APPENDIX A

RECTANGLE IDENTIFICATION TEST

This test instrument was designed for kindergarten students. The instrument was designed to gather information regarding the student's knowledge about the mathematical concept of rectangle. It also gathers information about a common misconception about the concept of square.

Rectangle Identification Test

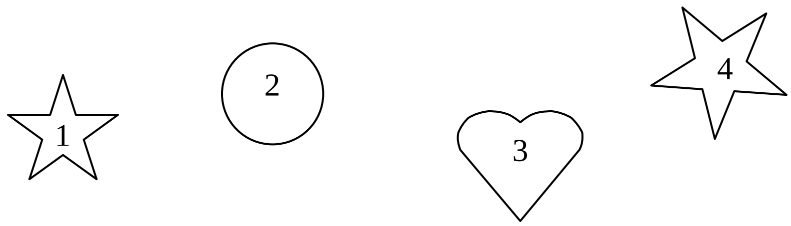
<u>Texas Essential Knowledge and Skills (TEKS)</u>
The student was expected to describe, identify, and compare circles, triangles, rectangles, and squares (a special type of rectangle).

<u>Directions to Pretest Administrator</u>

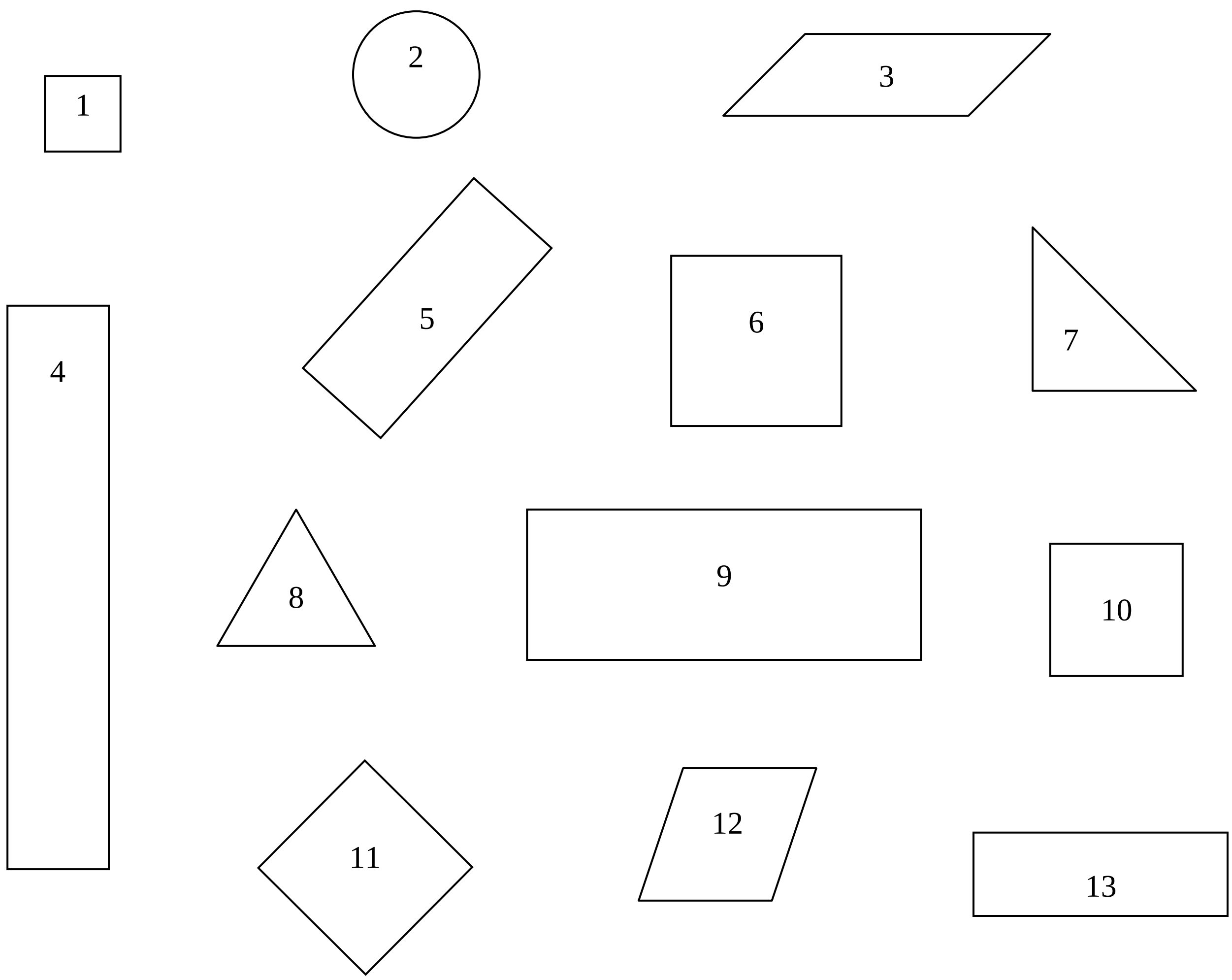
a. Administer this test individually or in small groups.
b. Provide each examinee with a copy of the Rectangle Identification Test and a pencil.
c. Say to the examinees, "**You will take a test today to see whether you know your shapes. First, write your name on the test at the top. At the top of the page, circle all the stars. Do not circle the shapes that are not stars**." The teacher may elaborate on these instructions to make sure students understand what to do.
d. Say, "**Now we will start the test. It was important that you do your own work. Do not share you answers with other students.**"
e. Say, "**Circle all the rectangles. Do not circle the shapes that are not rectangles.**" Elaborate as needed to ensure that the students understand what to do.
f. Give the examinees time to circle the rectangles. Redirect behavior as needed without giving the students the answers.
g. After the examinees are done circling, collect the test and submit it for scoring.

Name:____

Sample Items: Circle all the stars. Do not circle the shapes that are not stars.



Test Items: Circle all the rectangles. Do not circle the shapes that are not rectangles.



Rectangle Identification Test – Spanish Version

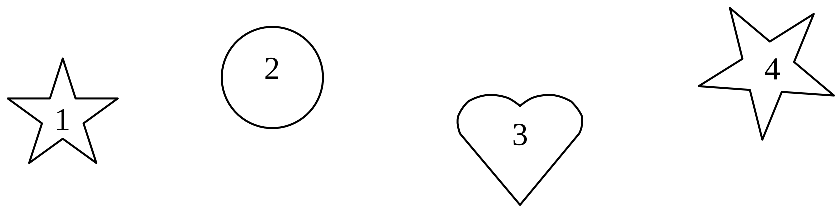
<u>Texas Essential Knowledge and Skills (TEKS)</u>
The student was expected to describe, identify, and compare circles, triangles, rectangles, and squares (a special type of rectangle).

<u>Directions to Pretest Administrator</u>

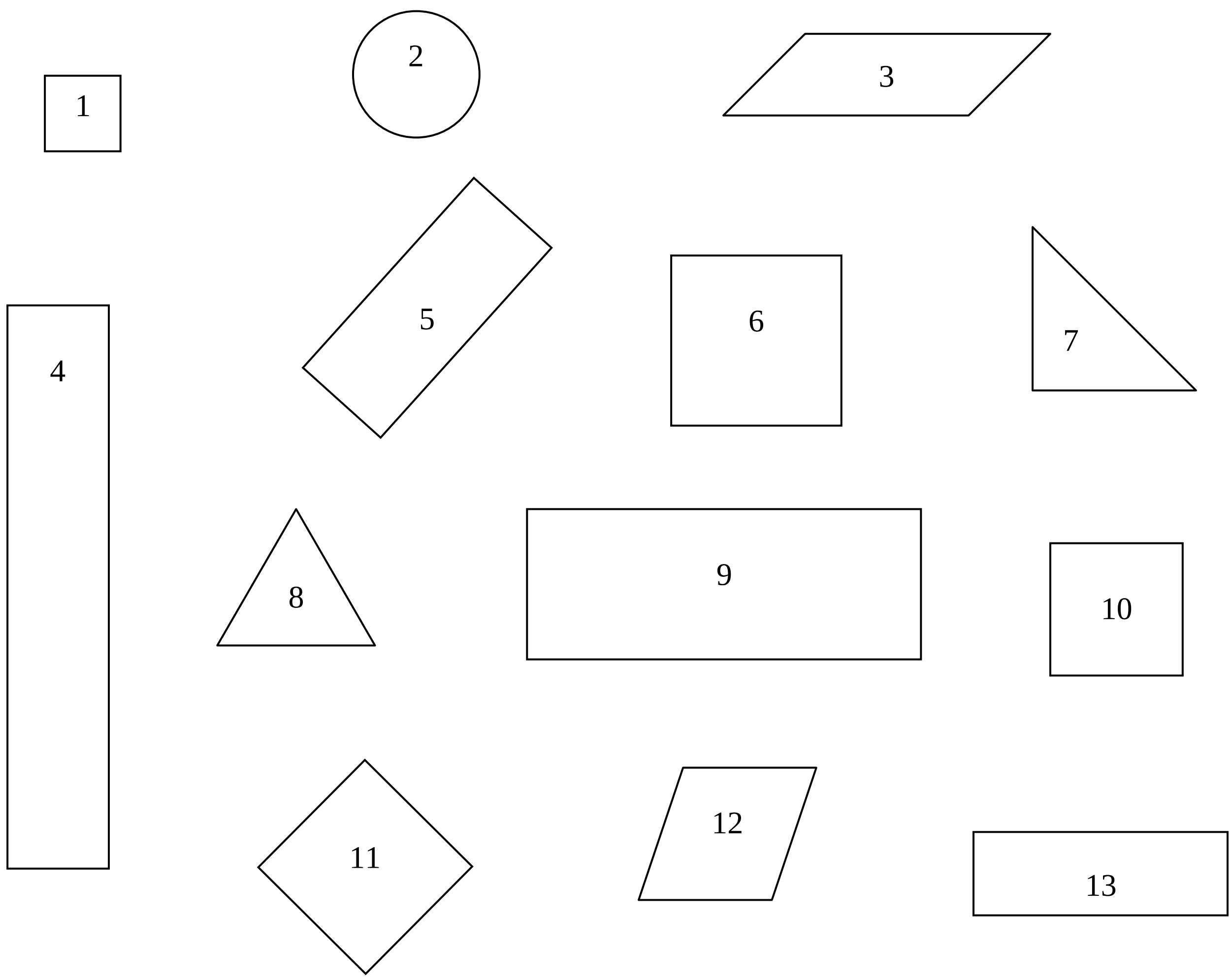
a. Administer this test individually or in small groups.
b. Use the Spanish version of the test if the students usually are instructed and tested in Spanish.
c. Provide each examinee with a copy of the Rectangle Identification Test and a pencil.
d. Say to the examinees, "**Ustedes tomarán una prueba hoy para ver si ustedes saben sus formas. Primero, escriban su nombre en la prueba. Circulen todas las estrellas. No rodee las formas que no son estrellas.**" The teacher may elaborate on these instructions to make sure students understand what to do.
e. Say, "**Ahora vamos a comenzar la prueba. Es importante que hagan su propio trabajo. No compartan sus respuestas con otros estudiantes.**"
f. Say, "**Circulen todos los rectángulos. No rodee las formas que no son rectángulos.**" Elaborate as needed to ensure that the students understand what to do.
g. Give the examinees time to circle the rectangles. Redirect behavior as needed without giving the students the answers.
h. After the examinees are done circling, collect the test and submit it for scoring.

Name:____

Artículos de Muestra: Rodea todas las estrellas. No rodee las formas que no son estrellas.



Artículos de Prueba: Rodea todos los rectángulos. No rodee las formas que no son rectángulos.



Scoring Rubric for the Rectangle Identification Test

Instructions: For each item listed below, if the shape associated with that item number was circled by the examinee, award one point.

Squares		Oblongs	
Item #	Points	Item #	Points
1		4	
6		5	
10		9	
11		13	
Total Points for Squares:		Total Points for Oblongs:	

Total Points for Squares		Total Points for Oblongs		Total Score
	+		=	

APPENDIX B

APPENDIX B

INTERVENTION PROTOCOL – LESSON PLAN

The intervention consists of a lesson designed for use in a real-world classroom. The lesson plan included here was formatted as are real-world lesson plans in a large deep south Texas public school district.

Lesson Title: Rectangles
Duration: 1 Hour
Materials: Attribute Blocks – 1 set per student of 2 triangles, 2 squares, 2 oblongs, and 2 circles
Tangrams – 1 non-rectangular parallelogram per student
Pattern Blocks – 1 trapezoid per student
Crayon – 1 red and 1 blue per student
Index Card – 1 per student
Rectangle Worksheet
Rectangle Sorting Mat A – Rectangles and Not Rectangles
Rectangle Sorting Mat B – Oblongs and Squares

Activities: Initial Learning: Hold up a shape and say its name: oblong, square, triangle and circle. Hold up the oblong and the square and tell the students that those are both rectangles. Hold up each of the shapes and ask the students whether it was a rectangle. Hold up the non-rectangular parallelogram and the trapezoid and show the students that the corner angles are not right angles using the index card. Tell them that the corners need to be square in order for the shapes to be rectangles.

Hands-on Learning: Distribute the attribute blocks rectangle sorting mat A, one set per student. Put one set of attribute blocks under the document camera so that all students can see your set. Use Rectangle Sorting Mat A to demonstrate to the students how to sort the set into Rectangles and Not Rectangles. Have the students mimic your actions at their tables. Monitor the students' work and give them corrective feedback. As you sort the shapes think out loud so that students hear your thoughts about the properties of each shape that cause you to put the shape in one group or the other. See the answer key for Rectangle Sorting Mat A to see how the shapes could be sorted. Point out to the students that the orientation of the shapes on the mat does not matter as long as they are in the correct area. Use Rectangle Sorting Mat B to sort the rectangles into oblongs and squares. Think out loud and have the students follow along with their own attribute blocks. Monitor their actions and give them corrective feedback.

Drill: Hold up a shape and have the students call out the name of the shape. Use choral response to help students feel safe. Repeat until there was a high level of success. Students may identify oblongs and squares as rectangles. That was OK. In either case, ask them what the other name for the shape is.

Independent Practice: Give the students the Rectangle Worksheet and have them circle all the rectangles and color the oblongs red and the squares blue. Send the students who successfully complete the assignment back to their regular classes escorted by your assistant.

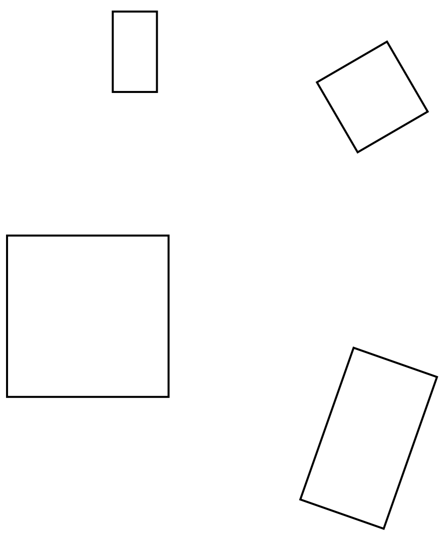
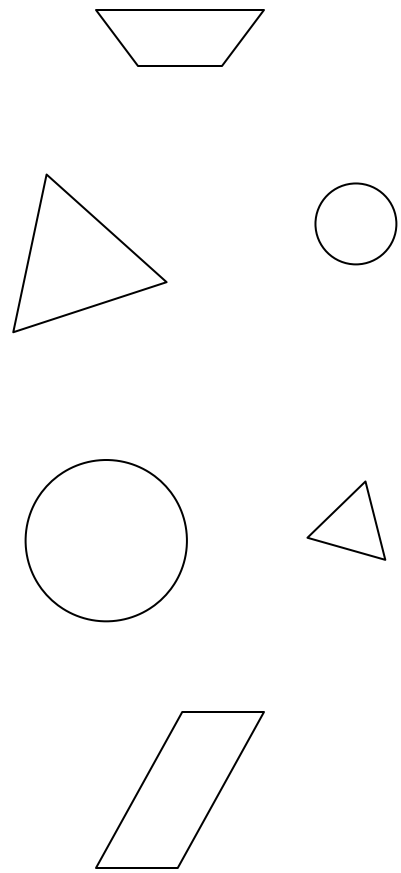
Reteach: For students who did not successfully complete the Rectangle Worksheet, review the shapes with them again and give them an opportunity to redo the Rectangle Worksheet.

Rectangle Sorting Mat A

Rectangles	Not Rectangles

Rectangle Sorting Mat A

Sample Answer Key

Rectangles	Not Rectangles
	

Name:____

Rectangle Sorting Mat B

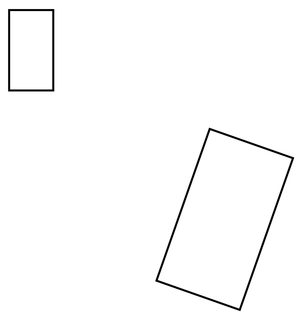
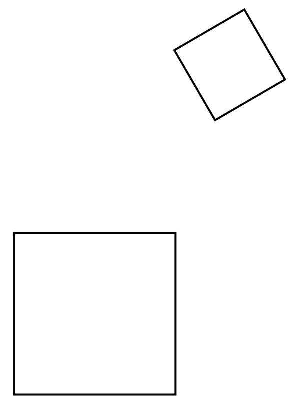
Rectangles

Oblong Rectangles	Not Oblong Rectanlges

Rectangle Sorting Mat B

Sample Answer Key

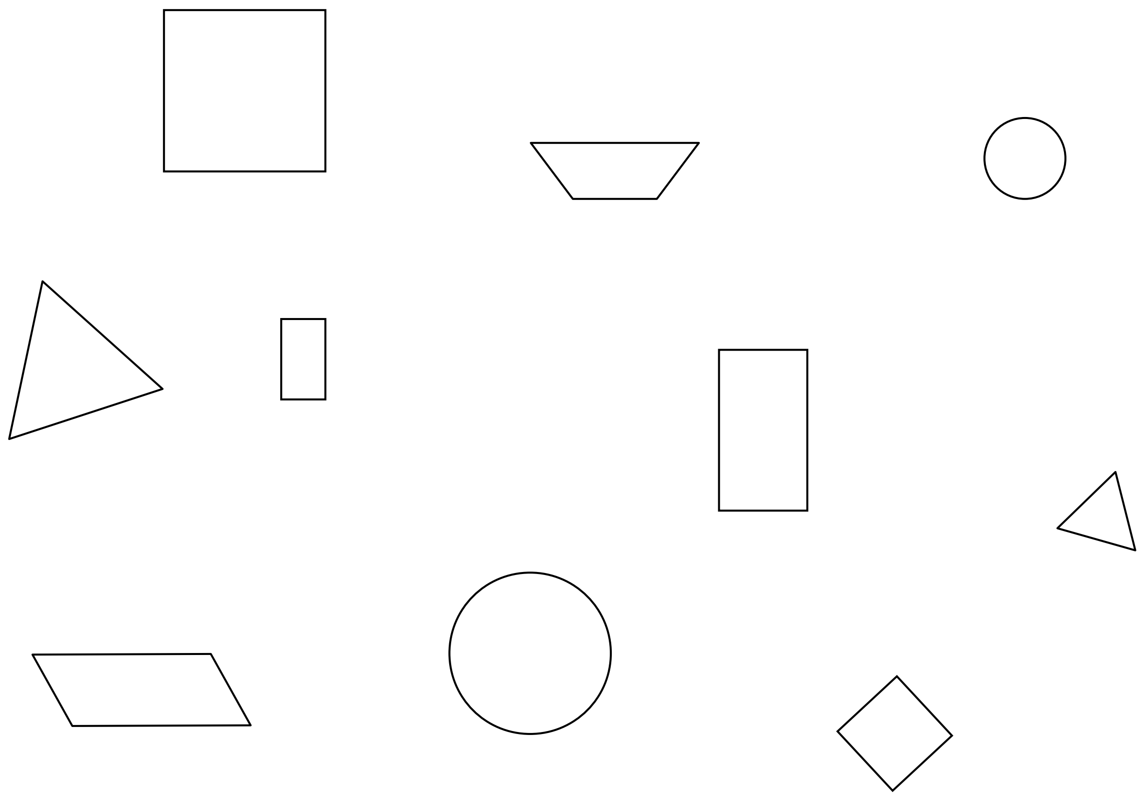
Rectangles

Oblong Rectangles	Not Oblong Rectangles
	

Name:____

Rectangle Worksheet

Circle all the rectangles. Color the oblong rectangles red. Color the rectangles that are not oblong blue. Cross out the shapes that are not rectangles.



APPENDIX C

APPENDIX C

PARENT CONSENT FORM

Date:

Dear Parent:

Your child, ________________________________, has been randomly selected for an opportunity to participate in a study being conducted by me, Francisco Rivera, doctoral student at The University of Texas – Pan American. The purpose of the study was to gain knowledge about how kindergarten children learn the mathematical concept of rectangle.

If you consent to have your child participate, your child may be selected to take a pretest or posttest. The purpose of the tests will be to determine whether your child has mastered the concept of rectangle. Your child may be selected to experience a lesson on rectangles designed to teach the correct concept. While learning the new lesson, your child may experience confusion especially if your child learned the concept incorrectly and his/her concept of rectangle has to be corrected. Your child will experience the lesson along with a small group of children from your child's school who were also selected randomly. The lesson will be done in one or two one-hour sessions at the school which your child already attends, and it will be presented by a district employee certified to teach kindergarten children.

Participating in this study could benefit your child by identifying a misconception that your child may have about rectangles and attempting to correct it; however, participation in the study does not guarantee that any misconception will be corrected. After the study was complete, the results will be shared with teachers to improve instruction.

The identities of the participating school, school district and the children will be kept confidential; however, it was possible that the participants may reveal their own participation or that of others in their group. I will maintain a written record of who the participants were. Materials that I publish will not identify the participants but only describe their demographics (eg. grade level, age, gender, race, etc.).

If you have questions about this study, contact me, Francisco Rivera, at (956) 453-8904.

Your signature on this form means that you understand the information presented, and that you want your child to participate in the study. You understand that participation was voluntary, and you may withdraw your child from the study at any time.

____

Parent Signature Date

Thank you for considering this request.

Sincerely

Francisco Rivera
Doctoral Student, The University of Texas – Pan American
Email: F.RIVERA2005@SBCLGOBAL.NET
Phone: (956) 453-8904

Fecha:_________________________

Querido Padre:

Su hijo, ______________________________, ha sido seleccionado para una oportunidad de participar en un estudio conducido por mí, Francisco Rivera, estudiante doctoral en la Universidad de Texas – Pan American. El objetivo del estudio es ganar el conocimiento sobre como los niños de jardín de infancia aprenden el concepto matemático del rectángulo.

Si usted consiente que su hijo participe, su hijo puede ser seleccionado para tomar una prueba antes o después de enseñanza. El objetivo de las pruebas será determinar si su hijo ha ganado maestría del concepto de rectángulo. Su hijo puede ser seleccionado para tomar una lección en rectángulos diseñados para enseñar el concepto correcto. Aprendiendo la nueva lección, su hijo puede tener confusión especialmente si en el pasado su hijo ha aprendido el concepto incorrectamente y su concepto del rectángulo tiene que ser corregido. Su hijo tomara la lección junto con un pequeño grupo de niños de la misma escuela donde su hijo ya asiste. La lección será hecha en una sesión o dos de una hora en la escuela a la cual su hijo ya asiste, y será presentada por un empleado del distrito certificado para enseñar a niños de jardín de infancia.

La participación en este estudio podría beneficiar a su hijo con identificando una idea falsa que usted hijo puede tener sobre rectángulos y intentando a corregir esa idea; sin embargo, la participación en el estudio no garantiza que cualquier idea falsa será corregida. Después de que el estudio es completo, los resultados serán compartidos con profesores para mejorar la instrucción.

Las identidades de los participantes, la escuela, el distrito escolar y los niños serán guardadas confidenciales; sin embargo, es posible que los participantes puedan revelar su propia participación o esto de otros en su grupo. Yo mantendré un registro escrito de quién fueron participantes. Los materiales que publico no identificarán a los participantes pero sólo describirán sus datos demográficos (por ejemplo: nivel de grado, edad, género, raza, etc.).

Si usted tiene preguntas sobre este estudio, llámeme al (956) 453-8904.

Su firma en esta forma significa que usted entiende la información presentada, y que usted quiere que su hijo participe en el estudio. Usted entiende que la participación es voluntaria, y usted puede retirar a su hijo del estudio en cualquier momento.

____ _________________________
Firma de Padre Fecha

Gracias por considerar esta petición.

Sinceramente,

Francisco Rivera
Estudiante Doctoral, La Universidad de Texas – Pan American
Correo electrónico: F.RIVERA2005@SBCLGOBAL.NET
Teléfono: (956) 453-8904

Fecha:________________________

Querido Padre:

Su hija, _____________________________, ha sido seleccionada para una oportunidad de participar en un estudio conducido por mí, Francisco Rivera, estudiante doctoral en la Universidad de Texas – Pan American. El objetivo del estudio es ganar el conocimiento sobre como los niños de jardín de infancia aprenden el concepto matemático del rectángulo.

Si usted consiente que su hija participe, su hija puede ser seleccionada para tomar una prueba antes o después de enseñanza. El objetivo de las pruebas será determinar si su hija ha ganado maestría del concepto de rectángulo. Su hija puede ser seleccionada para tomar una lección en rectángulos diseñados para enseñar el concepto correcto. Aprendiendo la nueva lección, su hijo puede tener confusión especialmente si en el pasado su hija ha aprendido el concepto incorrectamente y su concepto del rectángulo tiene que ser corregido. Su hija tomara la lección junta con un pequeño grupo de niños de la misma escuela donde su hija ya asiste. La lección será hecha en una sesión de una hora en la escuela a la cual su hija ya asiste, y será presentada por un empleado del distrito certificado para enseñar a niños de jardín de infancia.

La participación en este estudio podría beneficiar a su hija con identificando una idea falsa que usted hija puede tener sobre rectángulos y intentando a corregir esa idea; sin embargo, la participación en el estudio no garantiza que cualquier idea falsa será corregida. Después de que el estudio es completo, los resultados serán compartidos con profesores para mejorar la instrucción.

Las identidades de los participantes, la escuela, el distrito escolar y los niños serán guardadas confidenciales; sin embargo, es posible que los participantes puedan revelar su propia participación o esto de otros en su grupo. Yo mantendré un registro escrito de quién fueron participantes. Los materiales que publico no identificarán a los participantes pero sólo describirán sus datos demográficos (por ejemplo: nivel de grado, edad, género, raza, etc.).

Si usted tiene preguntas sobre este estudio, llámeme al (956) 453-8904.

Su firma en esta forma significa que usted entiende la información presentada, y que usted quiere que su hija participe en el estudio. Usted entiende que la participación es voluntaria, y usted puede retirar a su hija del estudio en cualquier momento.

_____________________________________ ________________________

Firma de Padre Fecha

Gracias por considerar esta petición.

Sinceramente,

Francisco Rivera
Estudiante Doctoral, La Universidad de Texas – Pan American
Correo electrónico: F.RIVERA2005@SBCLGOBAL.NET
Teléfono: (956) 453-8904

APPENDIX D

APPENDIX D

STUDENT ASSENT FORM

Do you want to learn about shapes? Circle your answer.

Yes No

____

Name

____

Date

APPENDIX E

APPENDIX E

TEACHER CONSENT FORM AND SURVEY

Teacher Consent Form and Survey

Date:

Dear Teacher:

You have been selected for an opportunity to participate in a study being conducted by me, Francisco Rivera, doctoral student at The University of Texas – Pan American. The purpose of the study was to gain knowledge about how kindergarten children learn the mathematical concept of rectangle. If you consent to participate, you may be selected to complete an online survey about the mathematical concept of rectangle and how to teach it.

Participating in this study could benefit you by identifying a misconception that you may have about rectangles and attempting to correct it. After the study was complete, the results will be shared with teachers to improve instruction.

There was minimal risk for you in participating in this study. The identities of the participating school, school district and the children will be kept confidential; however, it was possible that the participants may reveal their own participation or that of others in their group. I will maintain a written record of which schools participated. Materials that I publish will not identify the participants or schools but only describe their demographics (eg. grade level, age, gender, race, etc.).

If you have questions about this study, contact me, Francisco Rivera, at (956) 453-8904.

Your submission of the attached survey to my office means that you understand the information presented, and that you consent to participate in the study. By submitting the attached survey, you also indicate that you understand that participation was voluntary, and you may withdraw your consent at any time. Finally, by submitting the attached survey, you attest that you are 18 years of age or older.

Thank you for considering this request.

Sincerely

Francisco Rivera
Doctoral Student, The University of Texas – Pan American
Email: F.RIVERA2005@SBCLGOBAL.NET
Phone: (956) 453-8904

APPENDIX F

APPENDIX F

TEACHER RECTANGLE SURVEY

This survey was designed for inservice kindergarten teachers. It was designed to gather information regarding the teacher's knowledge concerning the mathematical concept of the rectangle and methods for teaching the concept of the rectangle.

This instrument was related to the Texas Essential Knowledge and Skills (TEKS): Kindergarten (9)(C) – The student was expected to describe, identify, and compare circles, triangles, rectangles, and squares (a special type of rectangle).

School:____________________________Date:____________________

Teacher Rectangle Survey

True or False Section

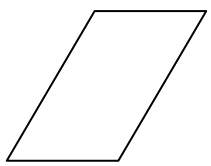
Directions: For items 1 through 6, circle either the word *True* or the word *False* in front of each statement to indicate whether the statement was true or false.

1. True or False

This is a rectangle.

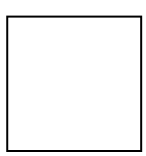
2. True or False

This is a rectangle.



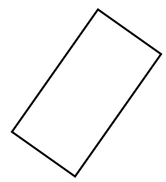
3. True or False

This is a rectangle.



4. True or False

This is a rectangle.



5. True or False

Rectangles must have two sides longer than the other two.

6. True or False

All squares are rectangles.

Restricted Response Section

Directions: Answer question 7 in the space provided below.

7. What is a rectangle?

____

____

____

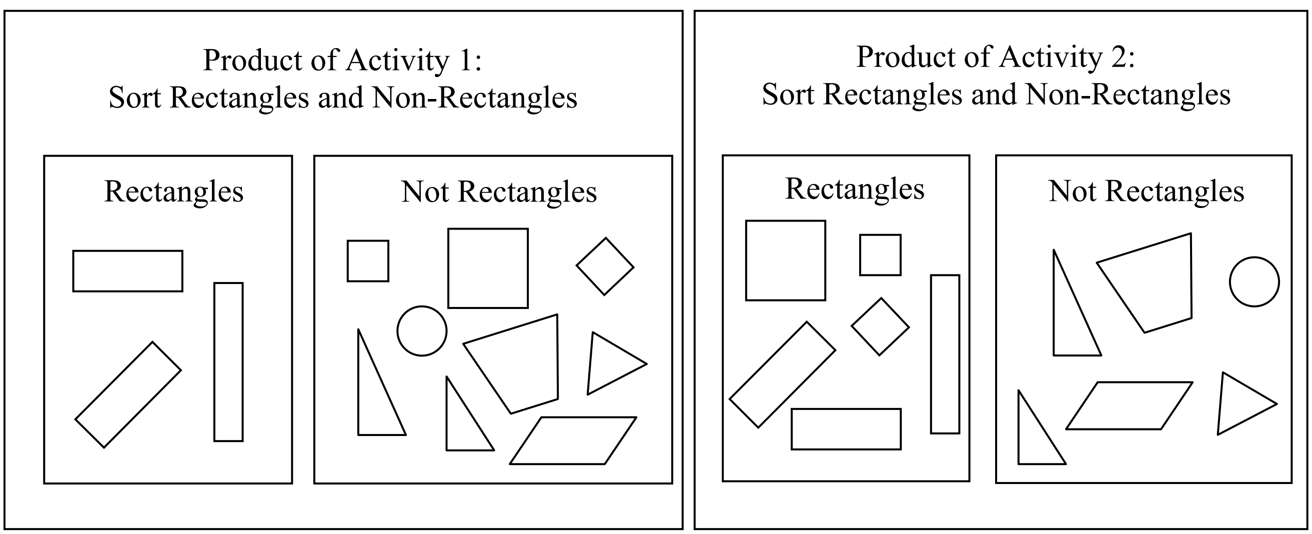
____

Extended-Response Section

Directions: For question 8, write your answer in the space below. Add pages if needed.

8. Which activity best supports the learning of the correct concept of the rectangle? Why?

In Activity 1, students are expected not to include squares as rectangles. In Activity 2, students are expected to include squares as rectangles.



____

____

____

(Attach additional pages if needed.)

APPENDIX G

APPENDIX G

PRETEST AND POSTTEST DESCRIPTIVE STATISTICS FROM SPSS 22

```
FREQUENCIES VARIABLES=PreScores PostScores
  /STATISTICS=STDDEV VARIANCE RANGE MINIMUM MAXIMUM MEAN MEDIAN MODE SKEWNESS
SESKEW KURTOSIS SEKURT
  /HISTOGRAM NORMAL
  /ORDER=ANALYSIS.
```

Frequencies

Notes

Output Created		30-MAR-2015 20:16:13
Comments		
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	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	67
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on all cases with valid data.
Syntax		FREQUENCIES VARIABLES=PreScores PostScores /STATISTICS=STDDEV VARIANCE RANGE MINIMUM MAXIMUM MEAN MEDIAN MODE SKEWNESS SESKEW KURTOSIS SEKURT /HISTOGRAM NORMAL...
Resources	Processor Time	00:00:01.34
	Elapsed Time	00:00:01.16

Page 1

Statistics

		PreScores	PostScores
N	Valid	33	67
	Missing	34	0
Mean		3.58	3.78
Median		4.00	4.00
Mode		4	4
Std. Deviation		.792	1.791
Variance		.627	3.207
Skewness		-1.873	.266
Std. Error of Skewness		.409	.293
Kurtosis		2.821	.827
Std. Error of Kurtosis		.798	.578
Range		3	8
Minimum		1	0
Maximum		4	8

Frequency Table

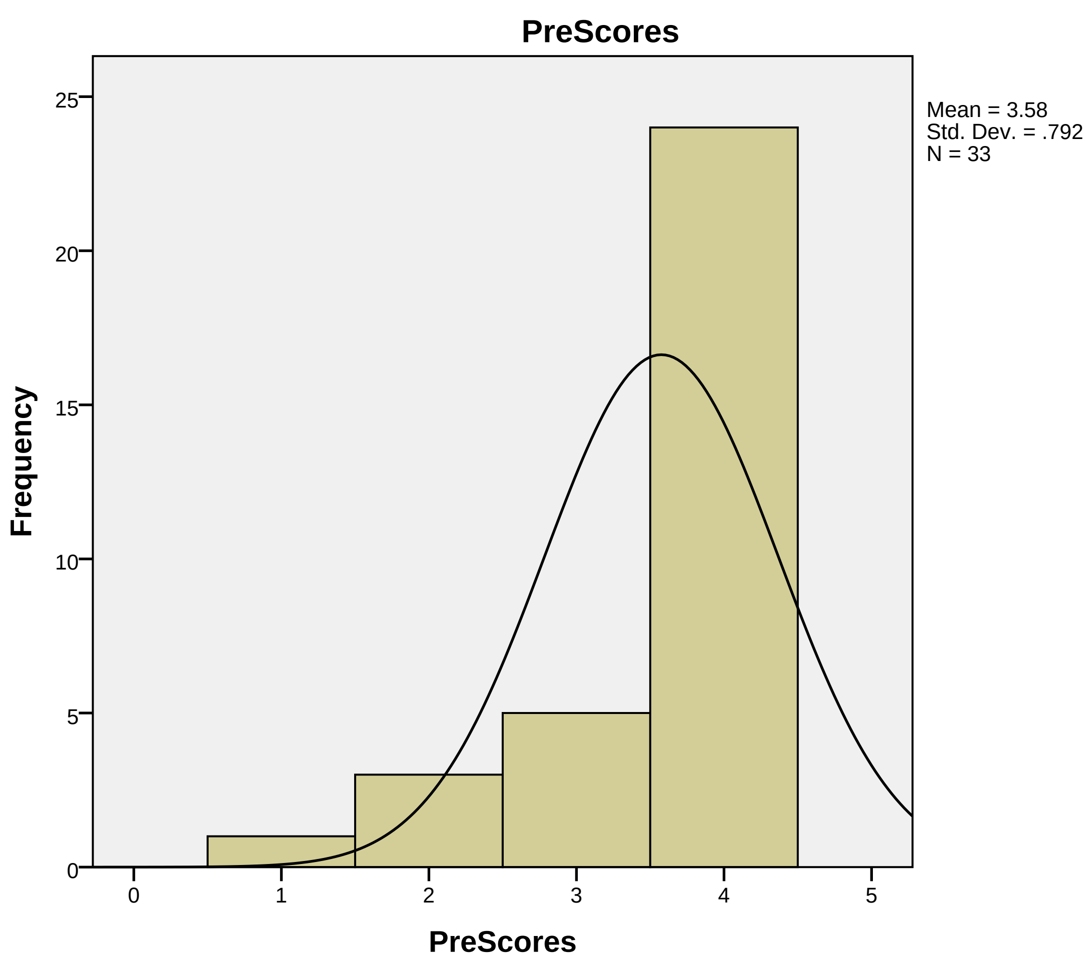
PreScores

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1	1	1.5	3.0	3.0
	2	3	4.5	9.1	12.1
	3	5	7.5	15.2	27.3
	4	24	35.8	72.7	100.0
	Total	33	49.3	100.0	
Missing	System	34	50.7		
Total		67	100.0		

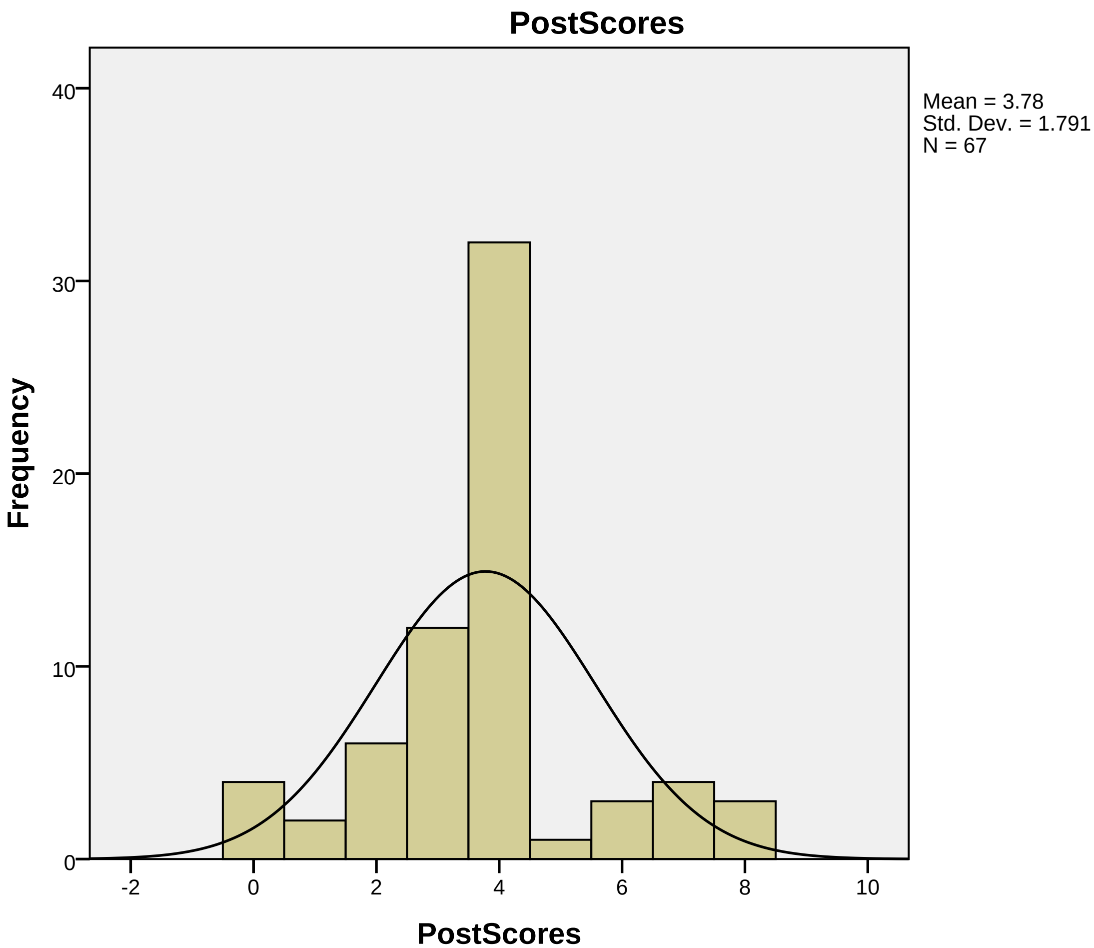
PostScores

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0	4	6.0	6.0	6.0
	1	2	3.0	3.0	9.0
	2	6	9.0	9.0	17.9
	3	12	17.9	17.9	35.8
	4	32	47.8	47.8	83.6
	5	1	1.5	1.5	85.1
	6	3	4.5	4.5	89.6
	7	4	6.0	6.0	95.5
	8	3	4.5	4.5	100.0
	Total	67	100.0	100.0	

Histogram



Page 3


PostScores
Mean = 3.78
Std. Dev. = 1.791
N = 67
Frequency
0
10
20
30
40
-2
0
2
4
6
8
10
PostScores

APPENDIX H

APPENDIX H

POSTTEST TEST OF NORMALITY FROM SPSS 22

```
EXAMINE VARIABLES=PreScores
  /PLOT BOXPLOT STEMLEAF NPPLOT
  /COMPARE GROUPS
  /STATISTICS DESCRIPTIVES
  /CINTERVAL 95
  /MISSING LISTWISE
  /NOTOTAL.
```

Explore

Notes

Output Created		30-MAR-2015 20:18:24
Comments		
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	Split File	<none>
	N of Rows in Working Data File	67
Missing Value Handling	Definition of Missing	User-defined missing values for dependent variables are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any dependent variable or factor used.
Syntax		EXAMINE VARIABLES=PreScores /PLOT BOXPLOT STEMLEAF NPPLOT /COMPARE GROUPS /STATISTICS DESCRIPTIVES /CINTERVAL 95 /MISSING LISTWISE /NOTOTAL.
Resources	Processor Time	00:00:00.42
	Elapsed Time	00:00:00.51

Page 1

Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
PreScores	33	49.3%	34	50.7%	67	100.0%

Descriptives

			Statistic	Std. Error
PreScores	Mean		3.58	.138
	95% Confidence Interval for Mean	Lower Bound	3.30	
		Upper Bound	3.86	
	5% Trimmed Mean		3.67	
	Median		4.00	
	Variance		.627	
	Std. Deviation		.792	
	Minimum		1	
	Maximum		4	
	Range		3	
	Interquartile Range		1	
	Skewness		-1.873	.409
	Kurtosis		2.821	.798

Tests of Normality

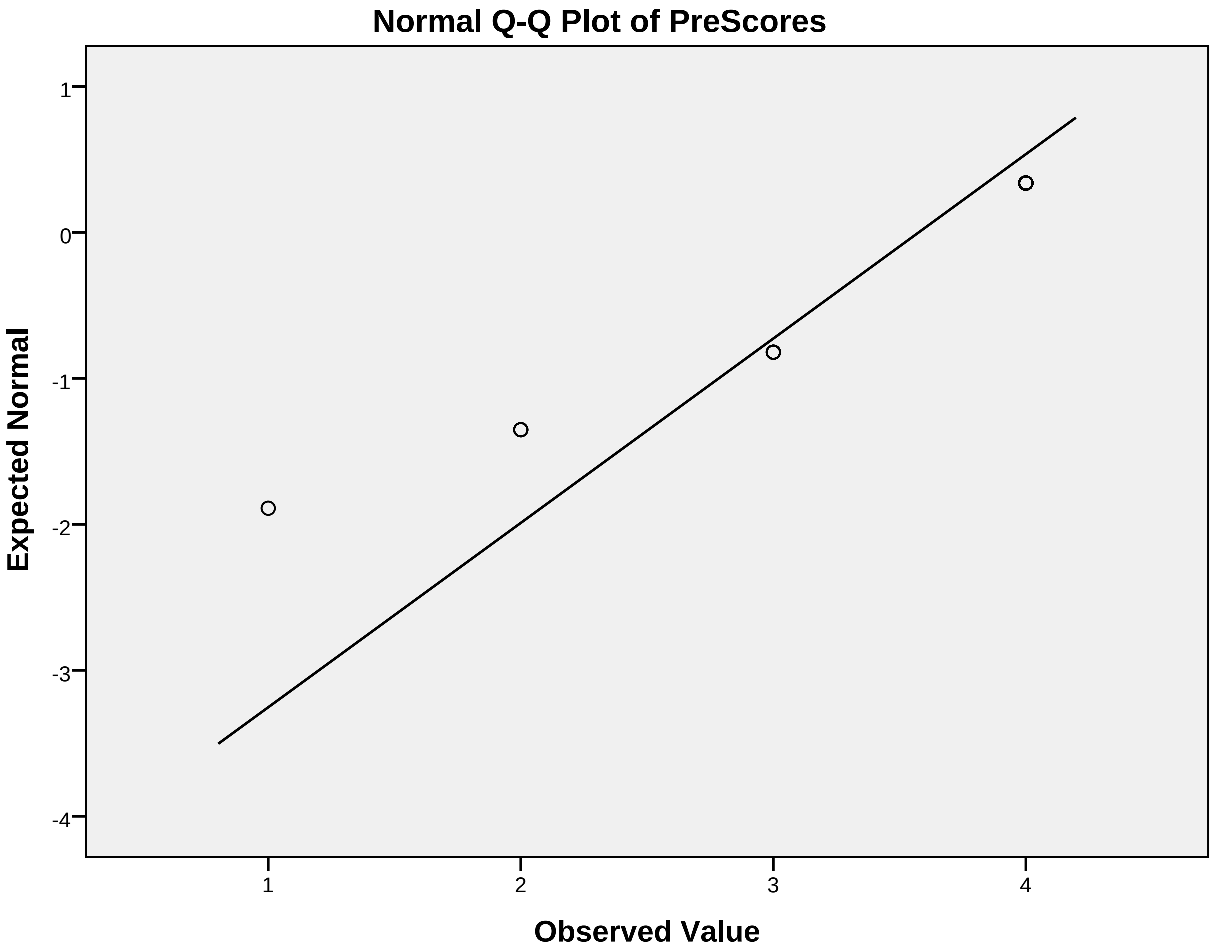
	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
PreScores	.431	33	.000	.603	33	.000

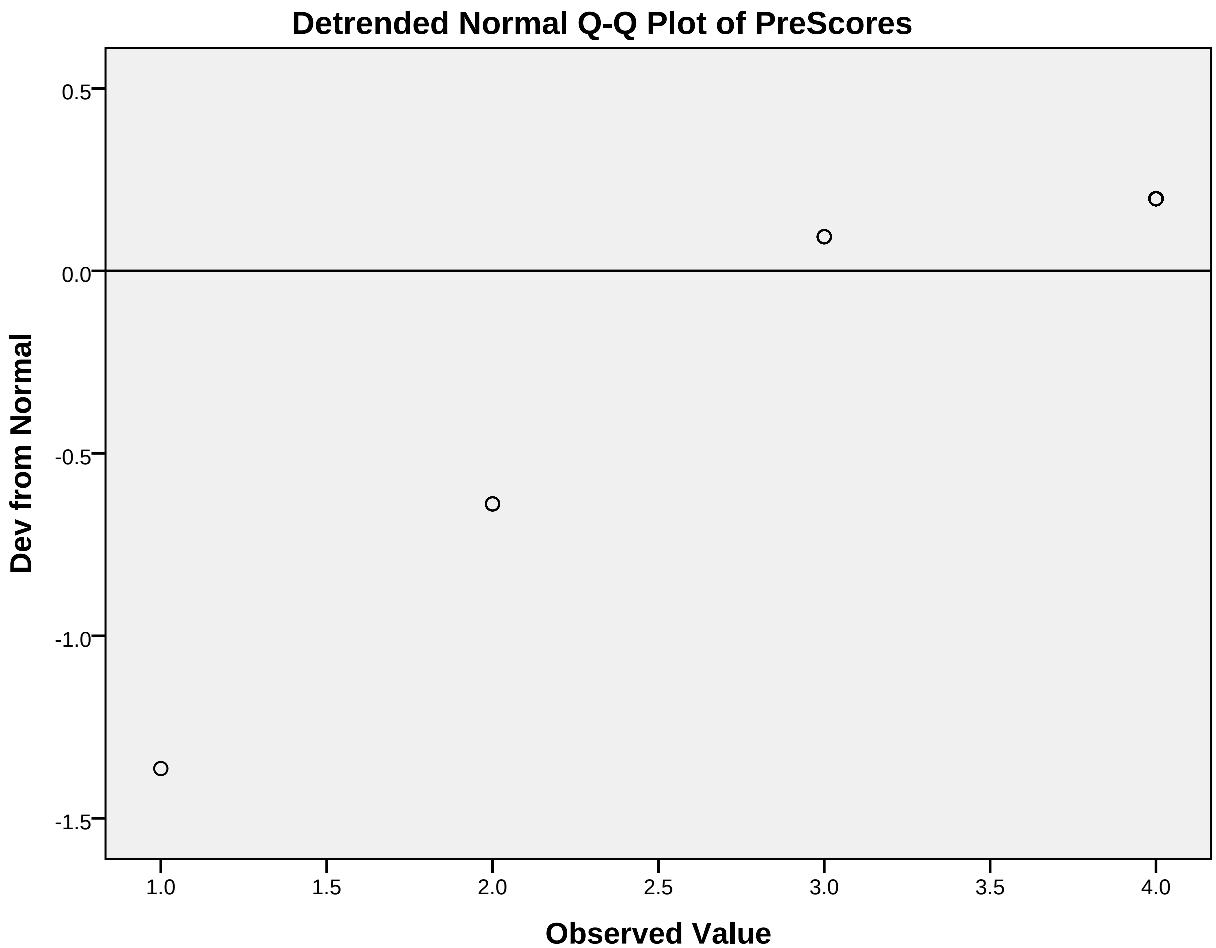
a. Lilliefors Significance Correction

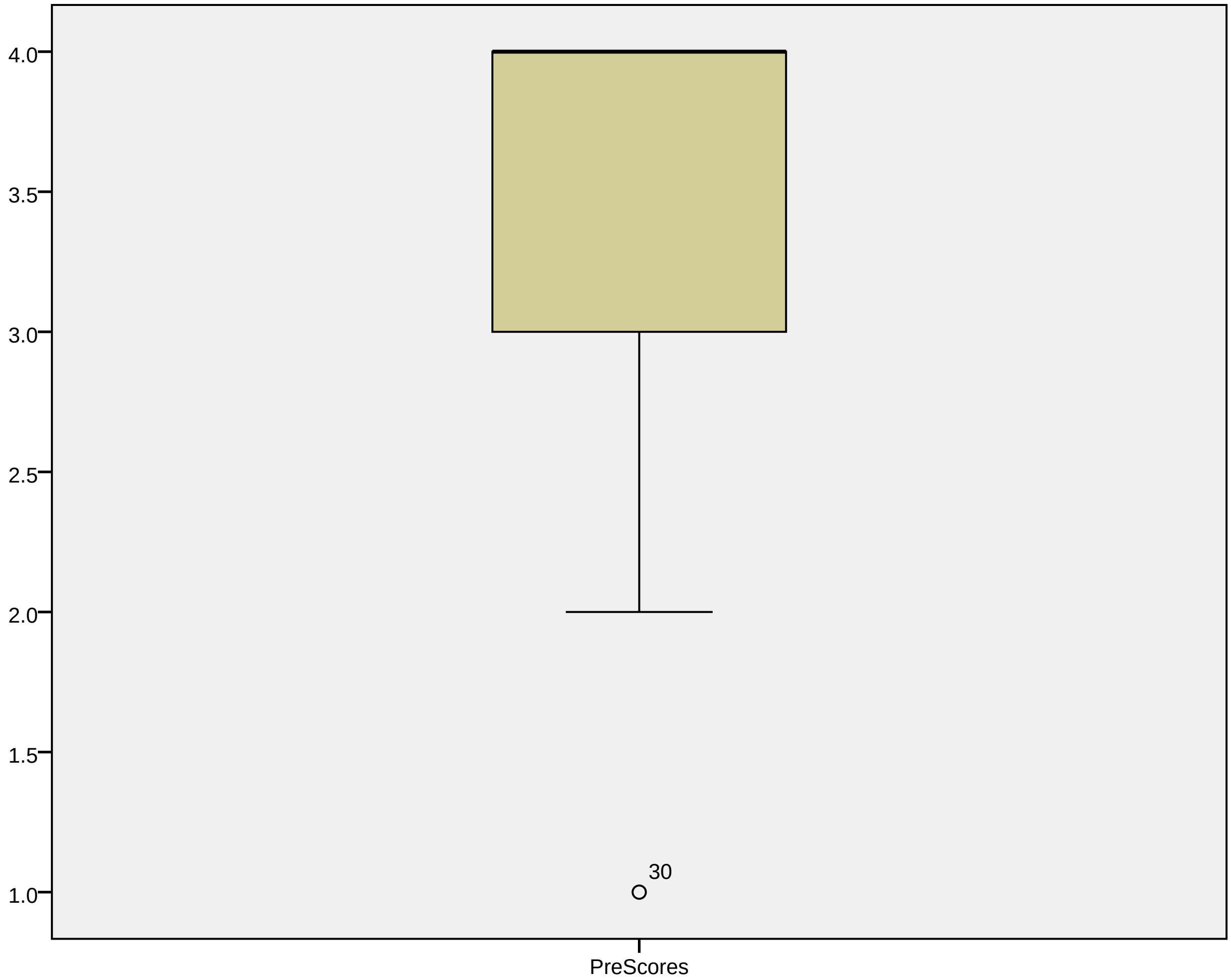
PreScores

```
PreScores Stem-and-Leaf Plot

 Frequency    Stem &  Leaf

     1.00 Extremes    (=<1.0)
     3.00        2 .  000
      .00        2 .
     5.00        3 .  00000
      .00        3 .
    24.00        4 .  000000000000000000000000

 Stem width:        1
```

Each leaf: 1 case(s)




Detrended Normal Q-Q Plot of PreScores
Dev from Normal
Observed Value
0.5
0.0
-0.5
-1.0
-1.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0


4.0
3.5
3.0
2.5
2.0
1.5
1.0
30
PreScores

APPENDIX I

APPENDIX I

POSTTEST TEST OF NORMALITY FROM SPSS 22

```
EXAMINE VARIABLES=PostScores
 /PLOT BOXPLOT STEMLEAF NPPLOT
 /COMPARE GROUPS
 /STATISTICS DESCRIPTIVES
 /CINTERVAL 95
 /MISSING LISTWISE
 /NOTOTAL.
```

Explore

Notes

Output Created		30-MAR-2015 20:21:06
Comments		
Input	Data	C: \Users\admin\Documents\UTPA\Dissertation\Data\SPSS Output and Data Files\Input - Pre and PostTest Results.sav
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	Split File	<none>
	N of Rows in Working Data File	67
Missing Value Handling	Definition of Missing	User-defined missing values for dependent variables are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any dependent variable or factor used.
Syntax		EXAMINE VARIABLES=PostScores /PLOT BOXPLOT STEMLEAF NPPLOT /COMPARE GROUPS /STATISTICS DESCRIPTIVES /CINTERVAL 95 /MISSING LISTWISE /NOTOTAL.
Resources	Processor Time	00:00:00.42
	Elapsed Time	00:00:00.42

Page 1

Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
PostScores	67	100.0%	0	0.0%	67	100.0%

Descriptives

			Statistic	Std. Error
PostScores	Mean		3.78	.219
	95% Confidence Interval for Mean	Lower Bound	3.34	
		Upper Bound	4.21	
	5% Trimmed Mean		3.76	
	Median		4.00	
	Variance		3.207	
	Std. Deviation		1.791	
	Minimum		0	
	Maximum		8	
	Range		8	
	Interquartile Range		1	
	Skewness		.266	.293
	Kurtosis		.827	.578

Tests of Normality

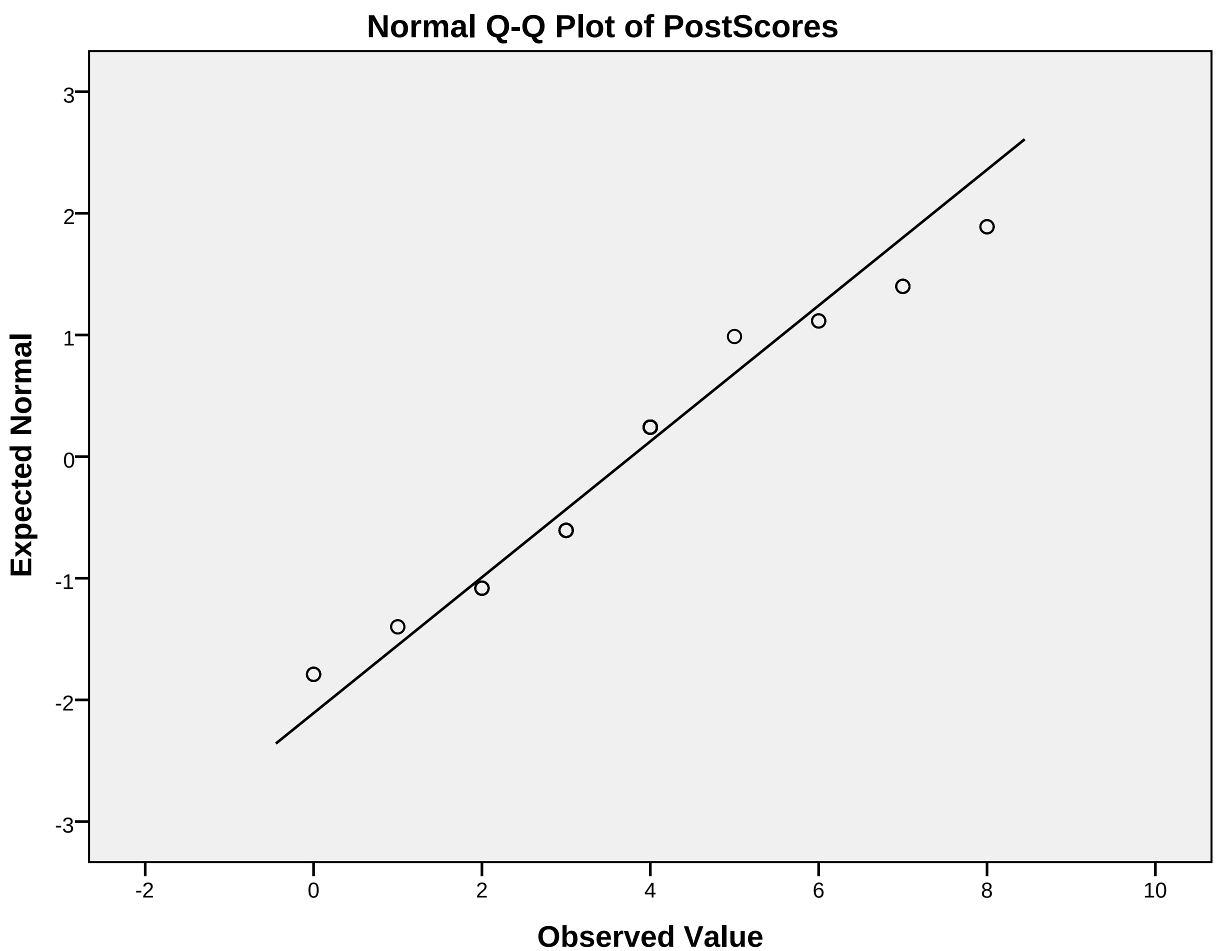
	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
PostScores	.286	67	.000	.886	67	.000

a. Lilliefors Significance Correction

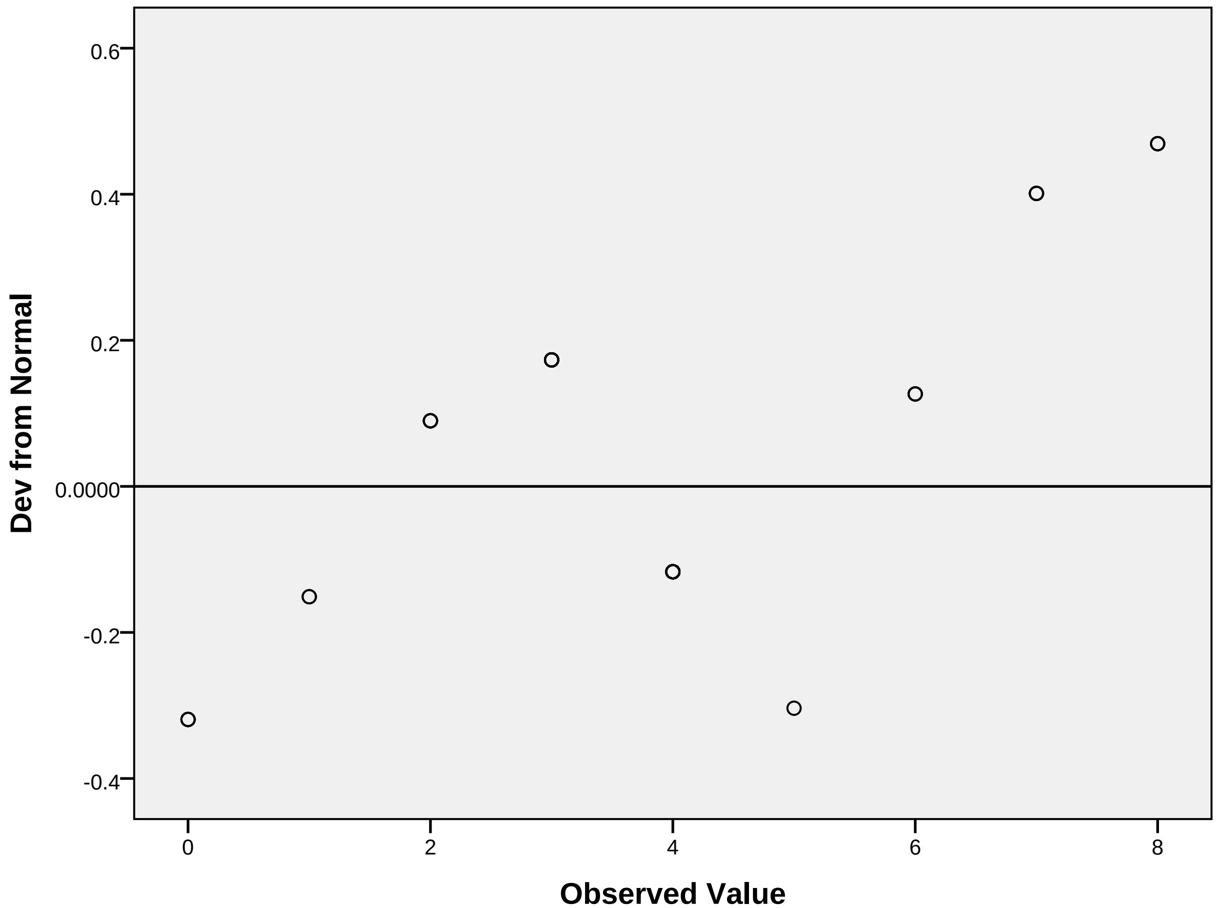
PostScores

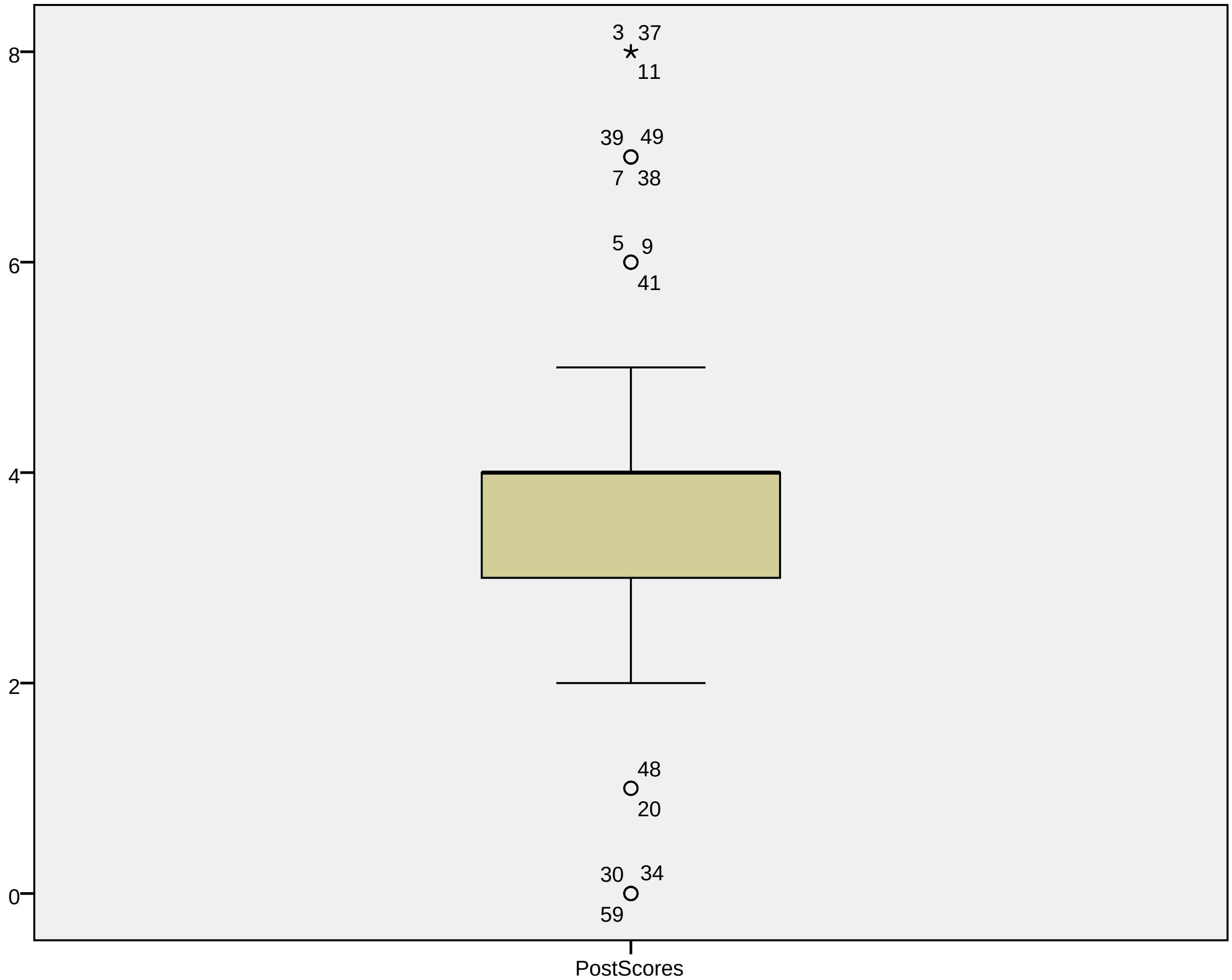
```
PostScores Stem-and-Leaf Plot

 Frequency    Stem &  Leaf

     6.00 Extremes    (=<1.0)
     6.00        2 .  000000
      .00        2 .
    12.00        3 .  000000000000
      .00        3 .
    32.00        4 .  00000000000000000000000000000000
      .00        4 .
     1.00        5 .  0
```

```
   10.00 Extremes    (>=6.0)

Stem width:         1
Each leaf:       1 case(s)
```



Detrended Normal Q-Q Plot of PostScores





Page 5

APPENDIX J

APPENDIX J

PRETEST ONE-SAMPLE T-TEST FROM SPSS 22

```
T-TEST
  /TESTVAL=4
  /MISSING=ANALYSIS
  /VARIABLES=PreScores
  /CRITERIA=CI(.95).
```

T-Test

Notes

Output Created		30-MAR-2015 20:22:57
Comments		
Input	Data	C: \Users\admin\Documents\UTPA\Dissertation\Data\SPSS Output and Data Files\Input - Pre and PostTest Results.sav
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	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	67
Missing Value Handling	Definition of Missing	User defined missing values are treated as missing.
	Cases Used	Statistics for each analysis are based on the cases with no missing or out-of-range data for any variable in the analysis.
Syntax		T-TEST /TESTVAL=4 /MISSING=ANALYSIS /VARIABLES=PreScores /CRITERIA=CI(.95).
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.00

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
PreScores	33	3.58	.792	.138

Page 1

One-Sample Test

	Test Value = 4					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
PreScores	-3.078	32	.004	-.424	-.70	-.14

```
T-TEST
 /TESTVAL=8
 /MISSING=ANALYSIS
 /VARIABLES=PreScores
 /CRITERIA=CI(.95).
```

T-Test

Notes

Output Created		30-MAR-2015 20:23:15
Comments		
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	Active Dataset	DataSet1
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	67
Missing Value Handling	Definition of Missing	User defined missing values are treated as missing.
	Cases Used	Statistics for each analysis are based on the cases with no missing or out-of-range data for any variable in the analysis.
Syntax		T-TEST /TESTVAL=8 /MISSING=ANALYSIS /VARIABLES=PreScores /CRITERIA=CI(.95).
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.02

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
PreScores	33	3.58	.792	.138

One-Sample Test

	Test Value = 8					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
PreScores	-32.100	32	.000	-4.424	-4.70	-4.14

APPENDIX K

APPENDIX K

POSTTEST 2-WAY ANOVA FROM SPSS 22

```
UNIANOVA PostScores BY Treated Pretested
 /METHOD=SSTYPE(3)
 /INTERCEPT=INCLUDE
 /PLOT=PROFILE(Treated Pretested Treated*Pretested)
 /EMMEANS=TABLES(OVERALL)
 /EMMEANS=TABLES(Treated) COMPARE ADJ(LSD)
 /EMMEANS=TABLES(Pretested) COMPARE ADJ(LSD)
 /EMMEANS=TABLES(Treated*Pretested)
 /PRINT=ETASQ HOMOGENEITY DESCRIPTIVE
 /CRITERIA=ALPHA(.05)
 /DESIGN=Treated Pretested Treated*Pretested.
```

Univariate Analysis of Variance

Notes

Output Created		30-MAR-2015 20:26:45
Comments		
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	Active Dataset	DataSet1
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	67
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on all cases with valid data for all variables in the model.

Notes

Syntax		UNIANOVA PostScores BY Treated Pretested /METHOD=SSTYPE(3) /INTERCEPT=INCLUDE /PLOT=PROFILE(Treated Pretested Treated*Pretested) /EMMEANS=TABLES(OVERALL) /EMMEANS=TABLES(Treated) COMPARE ADJ(LSD) /EMMEANS=TABLES(Pretested) COMPARE ADJ(LSD) /EMMEANS=TABLES (Treated*Pretested) /PRINT=ETASQ HOMOGENEITY DESCRIPTIVE /CRITERIA=ALPHA(.05) /DESIGN=Treated Pretested Treated*Pretested.
Resources	Processor Time	00:00:00.42
	Elapsed Time	00:00:00.41

Between-Subjects Factors

		N
Treated	NO	35
	YES	32
Pretested	NO	34
	YES	33

Descriptive Statistics

Dependent Variable: PostScores

Treated	Pretested	Mean	Std. Deviation	N
NO	NO	3.06	1.110	18
	YES	3.47	1.231	17
	Total	3.26	1.172	35
YES	NO	4.06	2.516	16
	YES	4.63	1.784	16
	Total	4.34	2.164	32
Total	NO	3.53	1.942	34
	YES	4.03	1.610	33
	Total	3.78	1.791	67

Levene's Test of Equality of Error Variances[a]

Dependent Variable: PostScores

F	df1	df2	Sig.
4.530	3	63	.006

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept + Treated + Pretested + Treated * Pretested

Tests of Between-Subjects Effects

Dependent Variable: PostScores

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	23.775[a]	3	7.925	2.658	.056	.112
Intercept	966.897	1	966.897	324.242	.000	.837
Treated	19.515	1	19.515	6.544	.013	.094
Pretested	3.992	1	3.992	1.339	.252	.021
Treated * Pretested	.091	1	.091	.030	.862	.000
Error	187.867	63	2.982			
Total	1167.000	67				
Corrected Total	211.642	66				

a. R Squared = .112 (Adjusted R Squared = .070)

Estimated Marginal Means

1. Grand Mean

Dependent Variable: PostScores

Mean	Std. Error	95% Confidence Interval	
		Lower Bound	Upper Bound
3.803	.211	3.381	4.226

2. Treated

Estimates

Dependent Variable: PostScores

Treated	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
NO	3.263	.292	2.680	3.847
YES	4.344	.305	3.734	4.954

Pairwise Comparisons

Dependent Variable: PostScores

(I) Treated	(J) Treated	Mean Difference (I-J)	Std. Error	Sig.[b]	95% Confidence Interval for Difference[b]	
					Lower Bound	Upper Bound
NO	YES	-1.081*	.422	.013	-1.925	-.236
YES	NO	1.081*	.422	.013	.236	1.925

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

Univariate Tests

Dependent Variable: PostScores

	Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Contrast	19.515	1	19.515	6.544	.013	.094
Error	187.867	63	2.982			

The F tests the effect of Treated. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

3. Pretested

Estimates

Dependent Variable: PostScores

Pretested	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
NO	3.559	.297	2.966	4.152
YES	4.048	.301	3.447	4.649

Pairwise Comparisons

Dependent Variable: PostScores

(I) Pretested	(J) Pretested	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a]	
					Lower Bound	Upper Bound
NO	YES	-.489	.422	.252	-1.333	.355
YES	NO	.489	.422	.252	-.355	1.333

Based on estimated marginal means

a. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

Univariate Tests

Dependent Variable: PostScores

	Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Contrast	3.992	1	3.992	1.339	.252	.021
Error	187.867	63	2.982			

The F tests the effect of Pretested. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

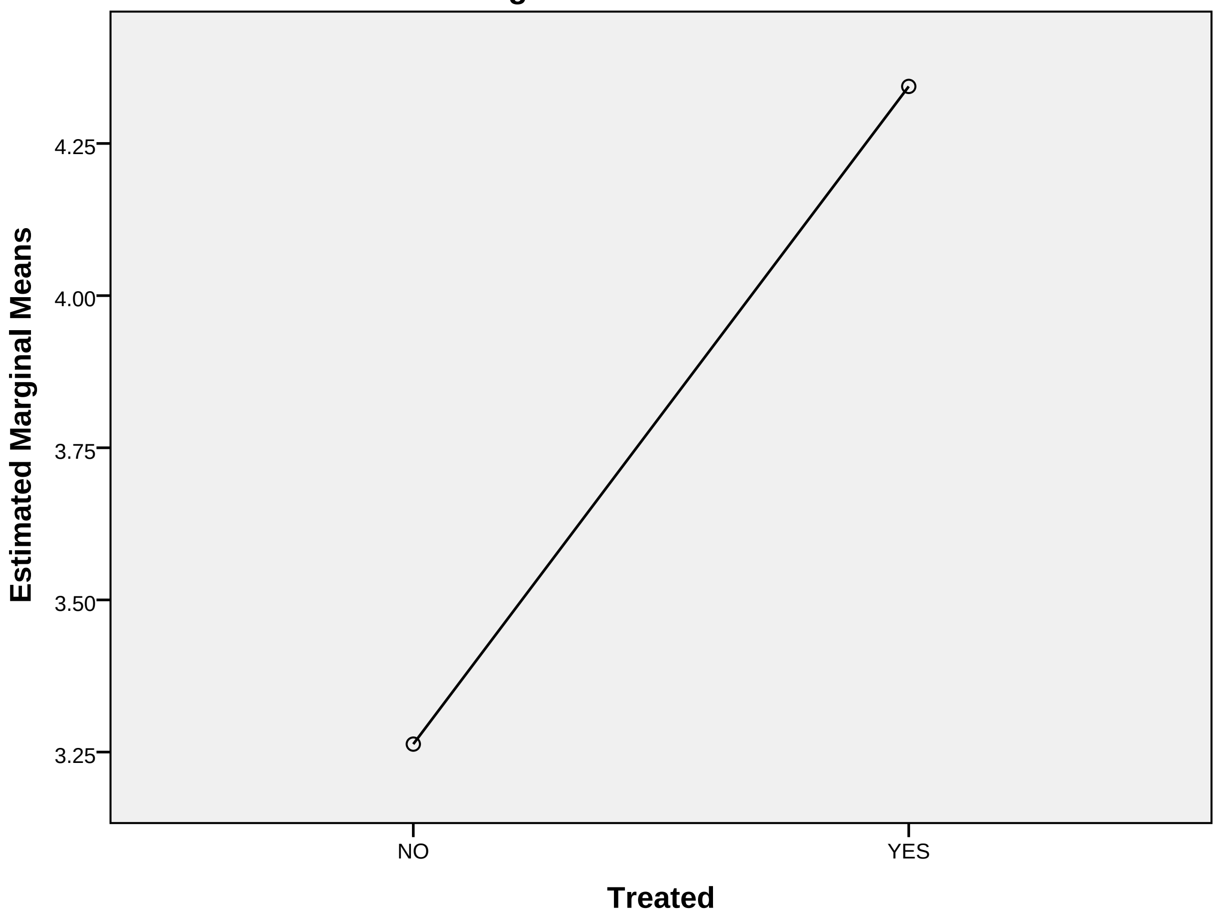
4. Treated * Pretested

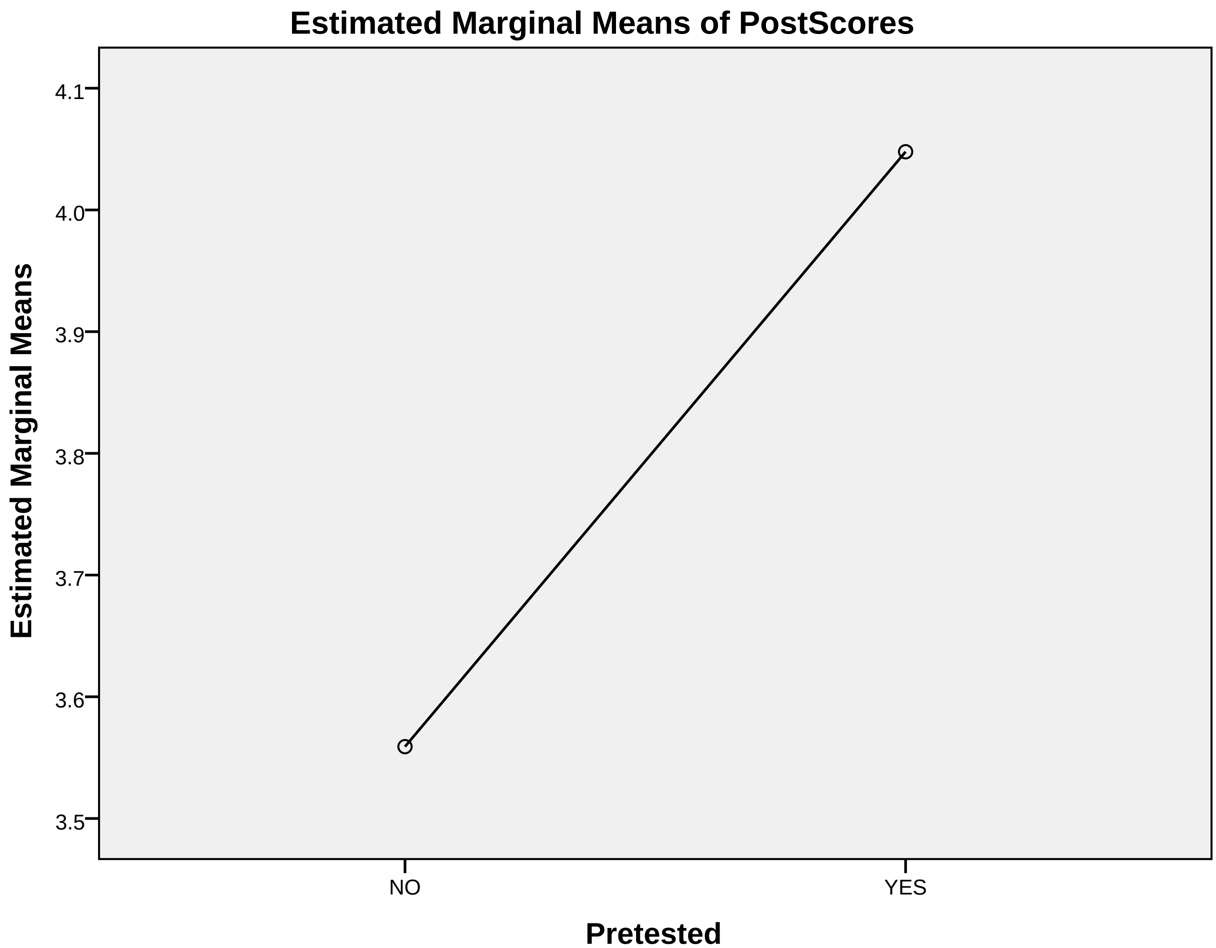
Dependent Variable: PostScores

				95% Confidence Interval	
Treated	Pretested	Mean	Std. Error	Lower Bound	Upper Bound
NO	NO	3.056	.407	2.242	3.869
	YES	3.471	.419	2.634	4.308
YES	NO	4.062	.432	3.200	4.925
	YES	4.625	.432	3.762	5.488

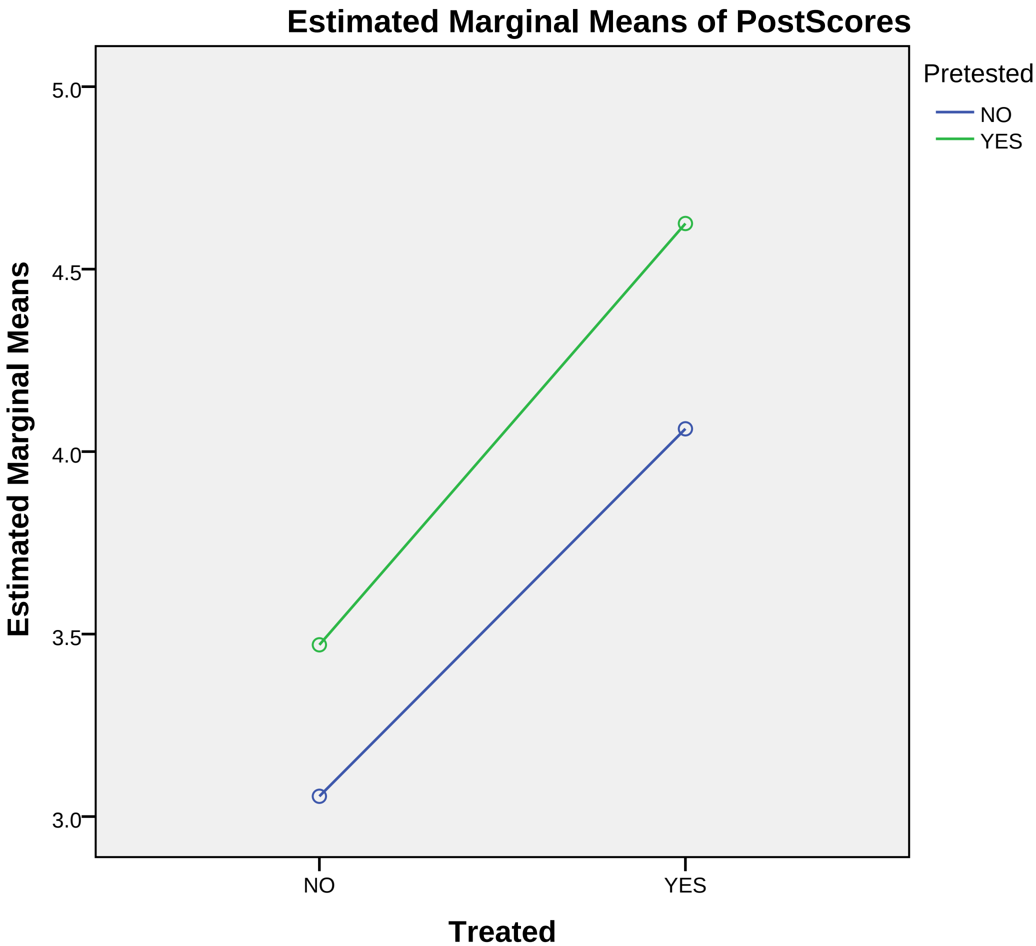
Profile Plots

Estimated Marginal Means of PostScores





Estimated Marginal Means of PostScores
Estimated Marginal Means
4.1
4.0
3.9
3.8
3.7
3.6
3.5
NO
YES
Pretested



Estimated Marginal Means of PostScores
Pretested
NO
YES
5.0
4.5
4.0
3.5
3.0
Estimated Marginal Means
NO
YES
Treated

BIOGRAPHICAL SKETCH

Francisco Rivera has lived in Mission, Texas since 1993. He graduated from Mission High School in 1985. He earned a Bachelor's Degree in Electrical Engineering with Honors specializing in Computer Engineering from the University of Texas at Austin in August 1990. Francisco graduated in May 1997 with a Master's Degree in Education specializing in Educational Administration and earned a Doctor of Education degree in Educational Leadership in July 2015 from the University of Texas-Pan American.

Francisco joined the Army Reserve during the summer between his junior and senior year in high school and served as a Laundry and Bath Specialist in the Army reserve unit in Rio Grande City, Texas. Francisco received a commission as a 2nd lieutenant in the Engineer Branch of the Army Reserve in May 1988 through the Reserve Officer Training Corps (ROTC) program at The University of Texas at Austin. Francisco received an honorable discharge from the Army Reserve in May 1996.

Francisco obtained teaching certificates from the Texas Education Agency in Secondary Mathematics, Secondary Computer Information System, Secondary Science Composite and Mid-Management Administrator in 1992, 1993, 1998 and 1998 respectively. Francisco has worked as a public school educator for 24 years and is currently an administrator. Francisco has been on the Board of the Rio Grande Valley Council of Teachers of Mathematics (RGVCTM) since the year 2000 and is President of the South Texas Association of Testing Specialists (STATS).