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Optimum directional well path design considering collapse and fracture pressures

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Abstract: Well path optimisation is often done based on the wellbore stability where the production related concerns are ignored. In fact, many of the studies carried out in the past have not included hydraulic fractures into their calculations. In fact, an optimum path for wellbore should not only provide the maximum stability during drilling but also offer a relatively low pressure to fracture the formation in the production stage. In this study, attempts are made to provide a methodology to determine an optimum well path for drilling, hydraulic fracturing and production stage using wellbore stability analysis in different stress regimes. An analytical model was proposed and used to determine the collapse pressure and fracture gradient during drilling and hydraulic fracturing at various azimuths and inclinations. The results obtained revealed that the well path does not changes in a normal faulting regime during production. However, the azimuth and inclination of wells may need to be changed in the strike-slip and reverse fault regimes for a better drilling and fracturing. It was also found that deviated wells can be a better option in normal and strike-slip stress regimes, but further studies might be needed to confirm these findings.

uncontrollable factors, while well pressure and well trajectory are taken as controllable parameters in the stability analysis (Khatibi et al., 2018a).

Most of wellbore stability analysis studies focused on the controllable parameters to avoid borehole collapse during drilling and to prevent sanding during the production stage (Alexeyev et al., 2017, Khatibi et al., 2019). In addition to the consideration of collapsing and sanding problems, the study of the optimum well trajectory with minimum fracture gradient during the production phase is essential. Therefore, a path that has the minimum mud pressure to avoid borehole collapse (i.e., collapse pressure) during drilling, and, at the same time provide the minimum fracture gradient during the production will represent the overall optimum well trajectory.

Furthermore, there are many studies that have been conducted on wellbore stability analysis for special drilling operations and conditions (Hareland and Dehkordi, 2007, Khatibi et al., 2018b), for instance, combined an elastoplastic model with a finite-explicit code (FLAC) and estimated an optimum equivalent circulating density (ECD) to avoid instability during underbalanced drilling (UBD). He et al. (2015) covered the impact of fluid seepage on wellbore stability during UBD and proposed a new model for collapse pressure determination in UBD of horizontal wells. Moreover, He et al. (2014) modelled the collapse pressure based on seepage mechanics and adopted a linear elastic theory in the study. From this work, it has been concluded that considering seepage will raise the minimum collapse pressure and narrow the safe mud window. As an example for a study of special conditions, (Lee et al., 2012). Dokhani et al. (2016) modelled wellbore stability in shale formations considering anisotropic strength and shale-fluid interaction.

In addition, for complex stress states, Himmerlberg and Eckert (2013) used a three dimensional mechanical earth model (3D MEM) and developed a new method based on Stereographic projection to determine the well path with the widest mud pressure window. Furthermore, Kadyrov and Tutuncu (2012) coupled a numerical wellbore stability model considering the variations of in situ stresses, temperature alteration, rock and fluid interaction and the effect of induced fluid flow with the field geological model. This study adopted several failure criteria and revealed that Mohr-Coulomb criterion is generally overestimating the safe mud weight.

For drilling horizontal wells, Dutta and Farouk (2008) used a MEM from a nearby offset well and conducted a trajectory sensitivity analysis. In addition, Al-Ajmi and Zimmerman (2009) investigated the impact of in situ stress regimes on wellbore stability to optimise the well path during drilling and production simultaneously. Implying Mogi-Coulomb failure criterion, Zare-Reisabadi et al. (2012) also presented a model that estimates the collapse pressure during drilling and maximum sand free drawdown pressure during production. In this work, it has been found that optimum well trajectory for both drilling and production is the same. Furthermore, Hassani et al. (2016) adopted Mogi-Coulomb failure criterion in a mechanical earth model together with the simulation of pore pressure variation near wellbore to propose a new model for stability analysis in the studied fields.

As a main part of stability analysis, the adopted failure criteria are taking a significant impact in wellbore stability analysis. In this regard, (Al-Ajmi and Zimmerman, 2005, 2006b, 2006a) developed and proposed the Mogi-Coulomb failure law to improve the stability modelling of vertical and directional wells. This recommendation and development had been widely reported in the recent literature. For example, Chabook et al. (2015) found that Mogi-Coulomb law is the best choice to determine both collapse pressure and the optimised well trajectory. Zhang et al. (2010) evaluated different failure criteria based on triaxial test data of five rocks and found that Mogi-Coulomb and 3D Hoek-Brown criteria are the most realistic criteria. Generally, it can be stated that Mogi-Coulomb criterion shows the most realistic results in predicting collapse pressures for all field conditions (Manshad et al., 2014).

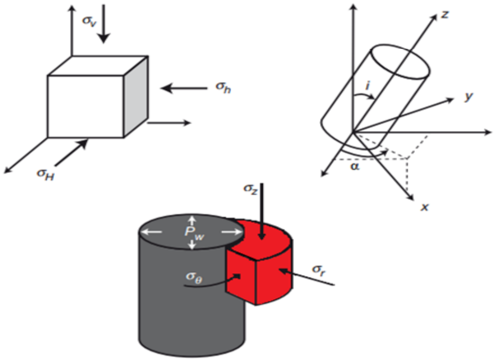
In this paper, Mogi-Coulomb law has been applied in the wellbore stability analysis, and for comparison, Drucker-Prager failure criterion has been utilised. This study is focusing on modelling the optimum well trajectory for different in situ stress regimes, where the potential of borehole collapsing is minimal and, at the same time, hydraulic fracturing can be conducted at a minimum possible pump pressure.

2 Methodology

In this study, both the shear failure represented by collapse of borehole wall and tensile failure indicated by hydraulic fracturing were investigated. First, using Mogi-Coulomb and Drucker-Prager criteria, wellbore collapse pressures (P_{wc}) for different well trajectories (i.e., azimuths and inclinations) were determined. The well path with the minimum collapse pressure was then chosen as the optimum well path. After that, the required hydraulic fracturing pressures (P_{wf}) for different well paths were obtained. The well path with the minimum fracture gradient is considered as the more appropriate trajectory to have a hydraulic fracture with respect to lowering the need in pump power.

To determine the optimised well trajectory, the fracture and collapse pressures of the possible well paths were added together. In this case, the minimum sum of fracture and collapse pressures represents the optimum well path. After determining the optimum well trajectory during drilling and fracturing, a synthetic reservoir with several hydraulic fractured wells was modelled and the changes in the reservoir pressure were studied over 55 years of production, in order to investigate the wellbore stability during the production phase. The results determined by this model were compared to analytical well path optimisation model (Al-Ajmi and Zimmerman, 2009).

Figure 1 The schematic of Cartesian and radial coordinates systems for stresses around the wellbore (see online version for colours)



2.1 Collapse pressure during drilling

Wellbore collapse is one of the most common problems during drilling a well. This phenomenon usually occurs at low wellbore pressures. The magnitude of the field stresses causing borehole collapse in Cartesian coordinates is (Fjar et al., 2008):

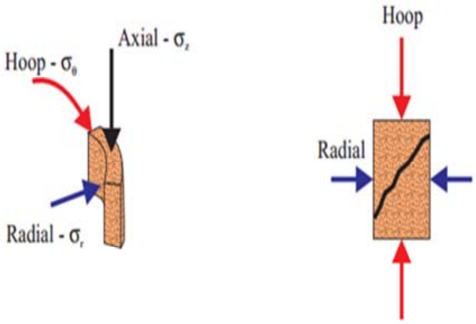
$$\begin{aligned}
\sigma_x &= \left(\sigma_H \cos^2\alpha + \sigma_h \sin^2\alpha\right)\cos^2 i + \sigma_v \sin^2 i,\\
\sigma_y &= \left(\sigma_H \sin^2\alpha + \sigma_h \cos^2\alpha\right),\\
\sigma_z &= \left(\sigma_H \cos^2\alpha + \sigma_h \sin^2\alpha\right)\sin^2 i + \sigma_v \cos^2 i,\\
\tau_{xy} &= 0.5\left(\sigma_h - \sigma_H\right)\sin 2\alpha \cos i,\\
\tau_{xz} &= 0.5\left(\sigma_H \cos^2\alpha + \sigma_h \sin^2\alpha - \sigma_v\right)\sin 2i,\\
\tau_{yz} &= 0.5\left(\sigma_h - \sigma_H\right)\sin 2\alpha \sin i, \qquad (1)
\end{aligned}$$

where σ_v, σ_H and σ_h are vertical, maximum horizontal and minimum horizontal in situ stresses respectively. The parameter i represents well inclination and α is the well azimuth with respect to the maximum horizontal stress. Because of the cylindrical drilling of wells, it is preferred to study the stresses around the wellbore in cylindrical coordinates that is defined as:

$$\begin{aligned}
\sigma_r &= P_w,\\
\sigma_\theta &= \left(\sigma_{xx} + \sigma_{yy}\right) - 2\left(\sigma_{xx} - \sigma_{yy}\right)\cos 2\theta - 4\tau_{xy}\sin 2\theta - P_w,\\
\sigma_z &= \sigma_{zz} - v\left[2\left(\sigma_{xx} - \sigma_{yy}\right)\cos 2\theta + 4\tau_{xy}\sin 2\theta\right],\\
\tau_{\theta z} &= 2\tau_{yz}\cos\theta - 2\tau_{xz}\sin\theta,\\
\tau_{zr} &= \tau_{r\theta} = 0, \qquad (2)
\end{aligned}$$

where θ is the angular position around the wellbore circumference, υ is Poisson's ratio and P_w is the mud pressure at the bottom of the well, which is illustrated in Figure 1. The radial stress (σ_r) and the tangential or hoop stress (σ_θ) are functions of the wellbore pressure. As the well pressure increases, the radial stress will increase while the hoop stress decreases. If the difference between the hoop and radial stresses becomes high, shear failure might occur (see Figure 2).

Figure 2 The schematic of shear failure in the wellbore (see online version for colours)



The application of a failure criterion for stability analysis require the use of principal stresses. In equation (2), σ_z and σ_θ are not principal stresses, and this case the principal stresses during drilling is expressed by (Nauroy, 2011):

$$\sigma_1=\frac{\sigma_\theta+\sigma_z}{2}+\frac{1}{2}\sqrt{(\sigma_\theta-\sigma_z)^2 4\tau_{\theta zz}^2},$$

$$\sigma_2=\frac{\sigma_\theta+\sigma_z}{2}-\frac{1}{2}\sqrt{(\sigma_\theta-\sigma_z)^2+4\tau_{\theta zz}^2},$$

$$\sigma_3=\sigma_r=p_w. \tag{3}$$

The orientation of collapse onset at the wall of the borehole is:

$$\tan 2\theta=\frac{2\tau_{xy}}{\sigma_x-\sigma_y}. \tag{4}$$

To predict shear failure, a comparison between the existing field stresses and rock failure criterion should be made. In this study, Mogi-Coulomb and Drucker-Prager failure criteria are used and recorded in Table 1. The parameters *a*, *b*, *k* and *m* are strength parameters that can be related to the cohesion strength and friction angle of the rocks.

Table 1 Two common shear failure criteria

Mogi-Coulomb criterion	*Drucker-Prager criterion*
$\tau_{oct}=a+b\sigma_{m,2}$	$\tau_{oct}=k+m\sigma_{oct}$
$\tau_{oct}=\left[\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}\right]/3$	$\tau_{oct}=\left[\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}\right]/3$
$\sigma_{m,2}=(\sigma_1+\sigma_3)/2$	$\sigma_{oct}=(\sigma_1+\sigma_2+\sigma_3)/3$

To determine the collapse pressure, a MATLAB program is written to conduct collapse pressure prediction. The input parameters for this program are recorded in Table 2, where C_o is the cohesion strength, ϕ is the friction angle and P_o is the pore pressure. The collapse pressure is determined for different well trajectories where the inclination and azimuth are varied between 0° to 90° and 0° to 180°, respectively.

2.2 Fracture pressure

Fracturing occurs when the wellbore pressure increases significantly, and the minimum principal stress exceeds the tensile strength of the rock formation. Generally, the uniaxial tensile strength of the studied field formations assumed to be equal to zero due to the common presence of natural fractures (Al-Ajmi and Zimmerman, 2006b). The existing three principal stresses for this condition is defined by (Aadnoy and Looyeh, 2011):

$$\begin{aligned} \sigma_1 &= \sigma_r = P_w, \\ \sigma_2 &= \frac{\sigma_\theta + \sigma_z}{2} + \frac{1}{2}\sqrt{\left(\sigma_\theta - \sigma_z\right)^2 + 4\tau_{\theta zz}^2}, \\ \sigma_3 &= \frac{\sigma_\theta + \sigma_z}{2} - \frac{1}{2}\sqrt{\left(\sigma_\theta - \sigma_z\right)^2 + 4\tau_{\theta zz}^2}. \end{aligned} \tag{5}$$

In this case, the radial stress is the maximum principal stress, while for collapse pressure it was the minimum one. Wellbore fracturing occurs when the effective least stress reaches the tensile rock strength which is:

$$\sigma_3' = \sigma_3 - P_o \le \sigma_t, \tag{6}$$

where σ'_3 is effective minimum principal stress and σ_t is the tensile strength. The tensile failure around the boreholes is illustrated in Figure 3.

Figure 3 Tensile failure around boreholes (see online version for colours)

By combining equation (6) and the minimum principal stress, the hoop stress becomes:

$$\sigma_\theta = \frac{\tau_{\theta z}^2}{\sigma_z - P_o} + P_o + \sigma_t. \tag{7}$$

At the same time, the hoop stress is equal to

$$\sigma_\theta = \sigma_x + \sigma_y - 2\left(\sigma_x - \sigma_y\right)\cos 2\theta - 4\tau_{xy}\sin 2\theta - p_w. \tag{8}$$

Accordingly, the wellbore fracture pressure can be estimated by

$$P_{wf} = \sigma_x + \sigma_y - 2\left(\sigma_x - \sigma_y\right)\cos 2\theta - 4\tau_{xy}\sin 2\theta - \frac{\tau_{\theta z}^2}{\sigma_z - P_o} - P_o - \sigma_t, \tag{9}$$

where P_{wf} is the well pressure at the tensile failure of the borehole wall. As per equation (9), fracture may occur in any direction and to get the fracture angle equation (9) should be differentiated which is:

$$\frac{dP_{wf}}{d\theta} = 0 \tag{10}$$

Or

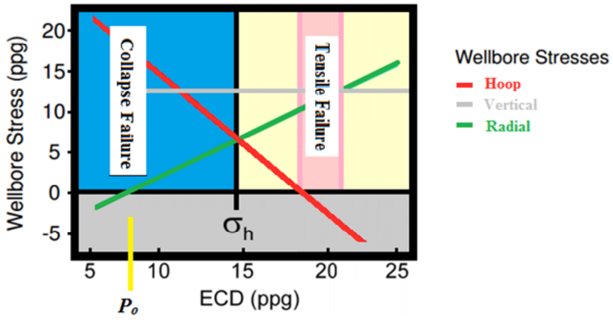
$$\tan 2\theta = \frac{2\tau_{xy}\left(\sigma_z - P_o\right) - \tau_{xz}\tau_{yz}}{\left(\sigma_x - \sigma_y\right)\left(\sigma_z - P_o\right) - \tau_{xz}^2 - \tau_{yz}^2}. \tag{11}$$

Since the shear stresses are negligible compared to the normal stresses, the second order of shear stress can be eliminated and equation (11) will be defined by:

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}. \tag{12}$$

Figure 4 illustrates the potential wellbore collapse and fracture pressures, where the magnitude of radial, axial and hoop stresses are shown versus the equivalent circulation density. It is shown that the wellbore collapse may occur when the difference between the tangential and radial stresses is so high. Furthermore, when the tangential stress becomes zero, tensile failure might take place.

Figure 4 Shear and tensile failure illustration (see online version for colours)



2.3 *Stability analysis during production*

After determining the best well trajectory during drilling, the optimum well path should also be investigated during production. In the production phase, the bottom hole pressure falls below the pore pressure and wellbore stress distribution changes and is expressed by (Ewy et al., 1999):

$$\sigma_r = p_w \sigma_\theta = \sigma_x + \sigma_y - 2\left(\sigma_x - \sigma_y\right)\cos 2\theta - 4\tau_{xy}\sin 2\theta - p_w + \beta_0\left(p_w - p_f\right),$$

$$\sigma_z = \sigma_z - v\left[2\left(\sigma_x - \sigma_y\right)\cos 2\theta - 4\tau_{xy}\sin 2\theta\right] + \beta_0\left(p_w - p_f\right),$$

$$\beta_0 = \frac{1-2v}{1-v}\beta, \tag{13}$$

where P_w is the bottom hole pressure, P_f is reservoir pressure, v is Poisson's ratio and β is Biot's poroelastic constant. When wellbore pressure decreases, the radial stress decreases and the tangential stress increases, and the difference between these stresses may cause rock shear failure. In equation (13), the reservoir pressure is considered constant. However, in real reservoir conditions the reservoir pressure decreases and this reduction will increase the applied horizontal stresses which is given by (Aadnoy and Looyeh, 2011):

$$\sigma_{H^*} = \sigma_{H_{initial}} - \beta\left(\frac{1-2v}{1-v}\right)\left(P_{initial} - P_{current}\right),$$

$$\sigma_{h^*} = \sigma_{h_{initial}} - \beta\left(\frac{1-2v}{1-v}\right)\left(P_{initial} - P_{current}\right), \tag{14}$$

where σ_H^* and σ_h^* are new horizontal stresses, $P_{initial}$ and $P_{current}$ are the initial and current reservoir pressures. So, when the reservoir pressure decreases during the production, the vertical stress usually remains constant and the total horizontal stresses decrease.

To conduct the stability analysis during production, a synthetic reservoir modelled in a commercial software is utilised, and reservoir pressure changes are studied for a hydraulic fractured reservoir. The best well paths for 55 years of production are reported and compared to the initial well path. In addition, the results of the study are compared with Al-Ajmi and Zimmerman analytical model for well path optimisation. They found that in all the stress regimes, the optimum stable path is existing in σ1–σ3 plane and the relative magnitude of in situ stresses mainly affects the optimum path deviation from σ1-direction. To find this deviation angle, they obtained the path with lowest critical stress by differentiating the hoop stress and equating it to zero which gives

$$\sigma_3 \cos^2\gamma + \sigma_1 \sin^2\gamma - \sigma_2 = 0, \tag{15}$$

where γ is the deviation angle of the borehole from the maximum principle in situ stress in the σ1-σ3 plane. This deviation angle is defined as:

$$\gamma = \arcsin\left(\sqrt{n}\right) \tag{16}$$

where n is the stress anisotropy function expressed by:

$$n = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3} \tag{17}$$

3 Results and discussions

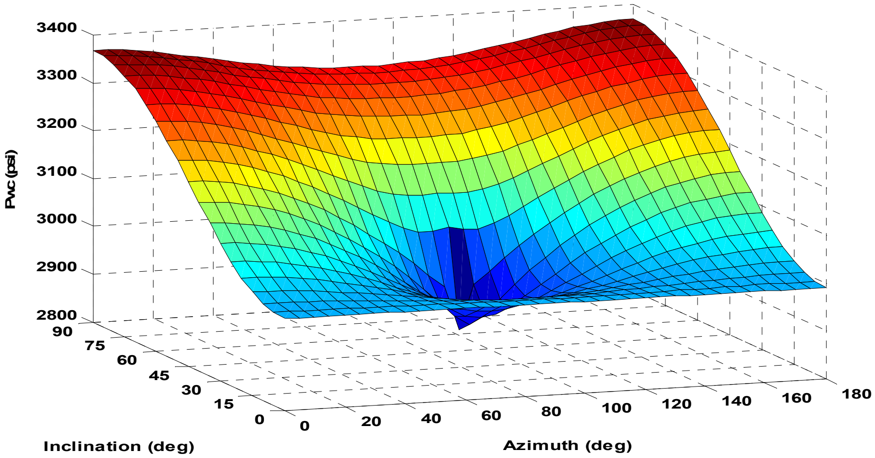
Considering three stress regimes with properties reported in Table 2, the minimum required pressures to avoid borehole collapse are calculated. Mogi-Coulomb law is markedly use for the stability analysis (Chabook et al., 2015). In normal fault stress regime (NF), deviated wells along the minimum horizontal stress where the azimuth is equal to 90° found to be the best trajectory which is totally agree by the results found by

(Zare-Reisabadi et al., 2012) (see Figure 5). Potential of a horizontally drilled well for collapsing in this stress regime is high.

Table 2 Input parameters

Stress regime	*Depth (ft)*	σ_v *(psi/ft)*	σ_H *(psi/ft)*	σ_h *(psi/ft)*	P_o *(psi/ft)*	υ	C_o *(psi)*	ϕ *(deg)*
Normal	7,000	1.0	0.9	0.8	0.45	0.25	1,000	28
Strike-slip	7,000	0.9	1.0	0.8	0.45	0.25	1,000	28
Reverse	7,000	0.8	1.0	0.9	0.45	0.25	1,000	28

Figure 5 Collapse pressures using Mogi-Coulomb law in a normal fault stress regime (see online version for colours)



To increase the well productivity and flow, hydraulic fracturing after drilling the wells is common solution which is highly depend on the values of the *in-situ* stresses and the direction of the well. The required fracture pressure to initiate a frac for different well trajectories in the normal fault stress regime is also calculated. Results in Figure 6. depicted that the horizontal wells along the maximum horizontal stress are the best choice for this operation.

In all the stress regimes, the difference between the maximum and minimum values of the fracture pressures is nearly 4 to 5 times greater than this difference for the collapse pressure. For example, in the normal fault stress regime the difference between the maximum and minimum fracture pressures in different positions is approximately 2,000 psi, however, for collapse pressure this difference is about 400 psi. Therefore, the choice of optimum minimum fracture pressure is essential and should be considered during the drilling programs. For each stress regime, the summations of collapse pressures and fracture pressures for different trajectories are obtained and the best paths are selected. Later, the effect of the failure criterion is investigated in this part of the study.

The optimum inclinations of different azimuths based on the summations of collapse pressures and fracture pressures for normal fault stress regime are shown in Figure 7. The best trajectory is on the direction with an azimuth of 0° and an inclination of 50° or 60°

(marked black) depending on the applied failure criterion. The trends of the optimum well path using the applied rock failure criteria are almost the same. Considering fracture pressures alone, horizontal wells are the best path for breaking the formation. Designating collapse pressures alone, will result in optimum azimuth equal to 90°. But the optimum well path model that used in this study depicts that deviated wells are the best solution for normal fault stress regime (inclination of 50° or 60°).

Figure 6 Fracture pressures in a normal fault stress regime (see online version for colours)

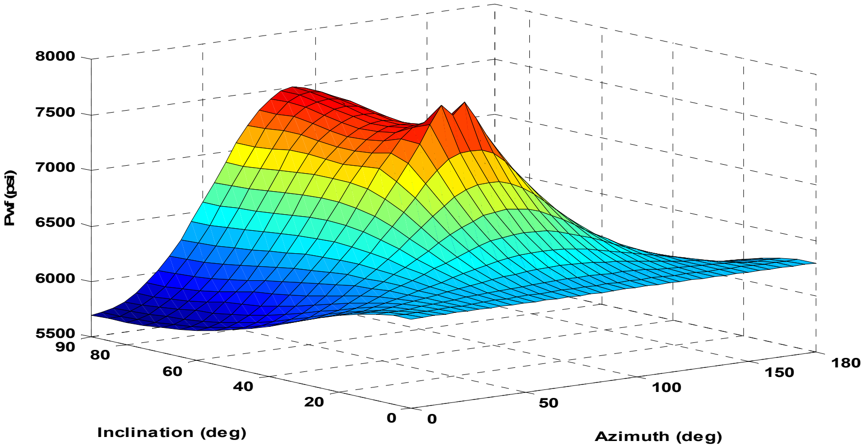
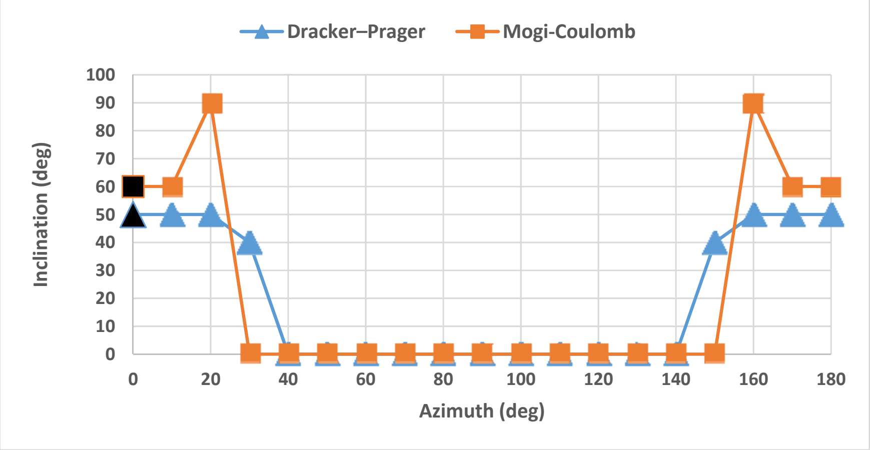


Figure 7 Optimum well trajectories based on the summations of collapse pressures and fracture pressures for a normal fault stress regime using Mogi-Coulomb and Drucker-Prager failure criteria (see online version for colours)



In reverse fault stress regime (RF), minimum mud pressure required to prevent borehole collapse and fracture of the formation are shown in Figure 8 and Figure 9, respectively. Deviated wells with an inclination of about 45° and azimuth of 0° or 180° are the most stable trajectories with respect to borehole collapsing. When it comes to frac the formation, horizontal wells with an azimuth of 90° is favourable. However, considering the summation of collapse and fracture pressures, wells with an inclination of 90° for all azimuths and using both failure criteria were selected as the best directions (see Figure 10).

Figure 8 Collapse pressure for a reverse fault stress regime based on Mogi–Coulomb law (see online version for colours)

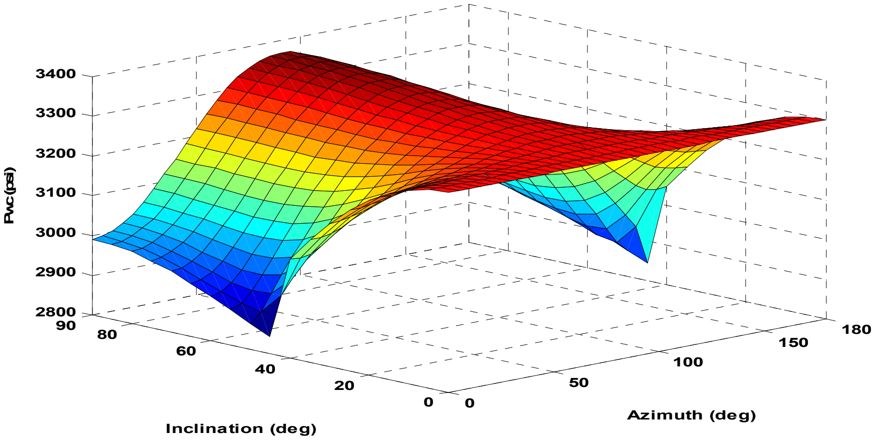


Figure 9 Fracture pressure for a reverse fault stress regime (see online version for colours)

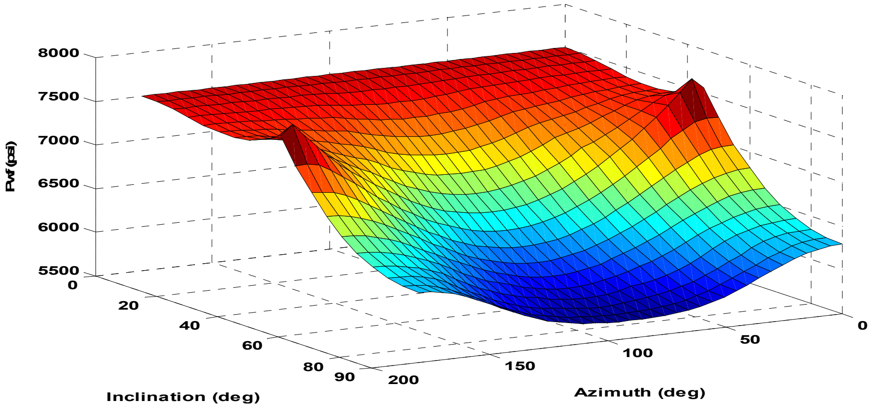
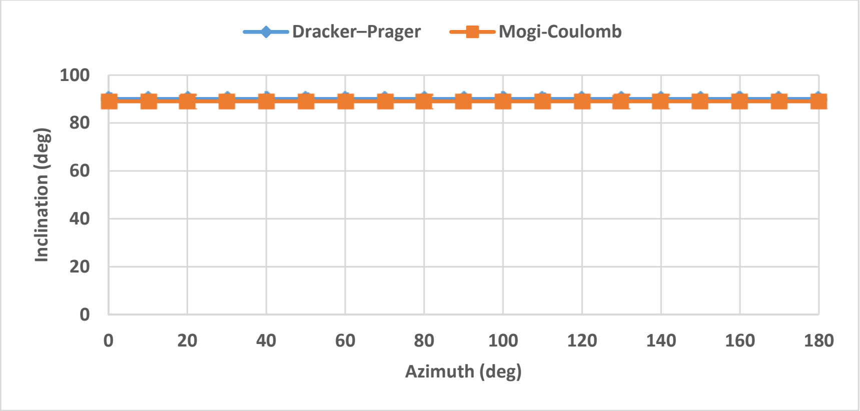


Figure 10 Optimum well trajectories based on the summations of collapse pressures and fracture pressures for a reverse fault stress regime using Mogi-Coulomb and Drucker-Prager failure criteria (see online version for colours)



In strike-slip stress regime (SS), horizontal wells with azimuth of about 40° are the most stable directions during drilling. (see Figure 11). For fracture pressure, vertical wells are the best trajectory for the strike-slip stress regime and horizontal wells should not be drilled in this stress regime in order to fracture the formation later as illustrated in Figure 12. Using the summation of collapse and fracture pressures, the optimum well inclinations for different azimuths and failure criteria are shown in Figure 13. The optimum well path model that used in this study shows results almost the same in which a deviated well along the maximum horizontal stress is the most stable option. In the studied strike-slip stress regime, the optimum inclinations for the two applied rock failure criteria are 30° and 40°.

Figure 11 Collapse pressure based on Mogi–Coulomb law for a strike-slip stress regime (see online version for colours)

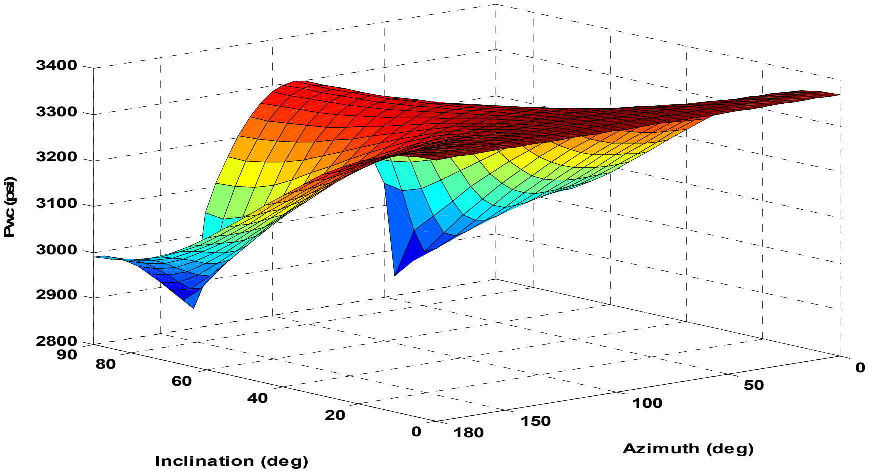


Figure 12 Fracture pressure for a strike-slip stress regime (see online version for colours)

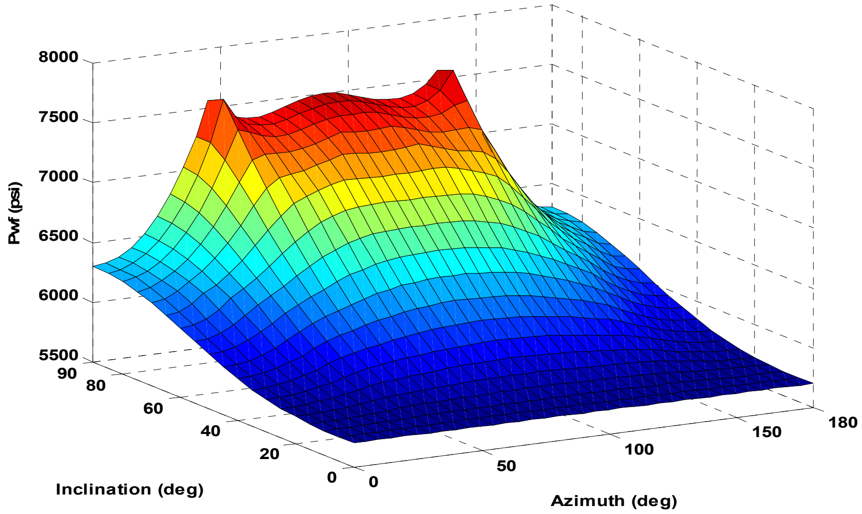
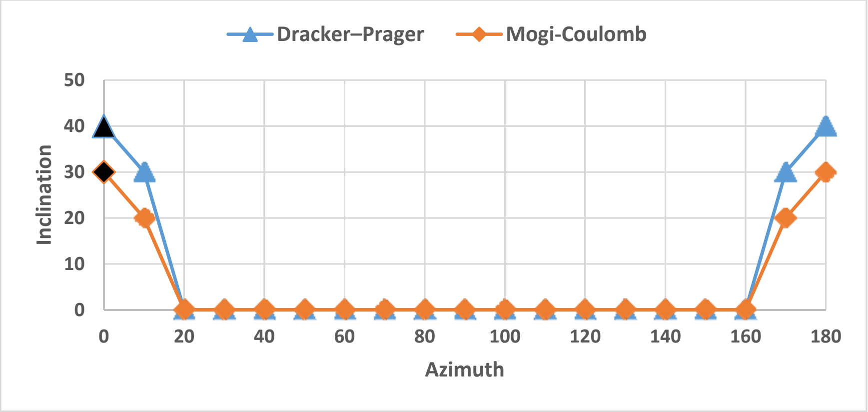


Figure 13 Optimum well trajectories based on the summations of collapse pressures and fracture pressures for a strike-slip stress regime using Mogi-coulomb and Drucker-Prager failure criteria (see online version for colours)



After selecting the best well trajectory for different stress regimes, it is necessary to investigate the wellbore stability during production. Hence, a synthetic oil reservoir with two horizontal and deviated wells that have been fractured is modelled in a commercial software. In the first step, the effect of fracture on the oil flow rate is studied. As shown in Figure 14, over 55 years of production, the cumulative oil production for hydraulic fractured wells are approximately two times the case in which none of the wells were fractured. The schematic of the studied model is shown in Figure 15, where the initial reservoir pressure is set to be equal to 1,350 psi. Three cases of different stress regimes are studied and recorded in Table 3 with other input parameters. The strength parameters, cohesion and friction angle, are set to be equal to 1,000 psi and 30°, respectively.

Figure 14 Cumulative oil production for hydraulic fractured and non-hydraulic fractured reservoir (see online version for colours)

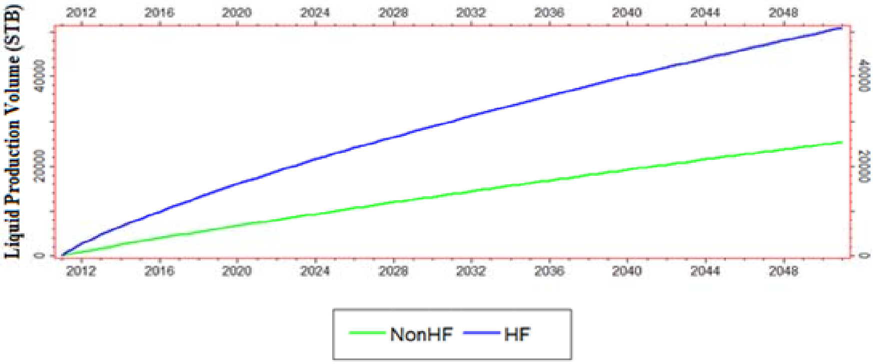


Figure 15 Illustration of the studied reservoir (see online version for colours)

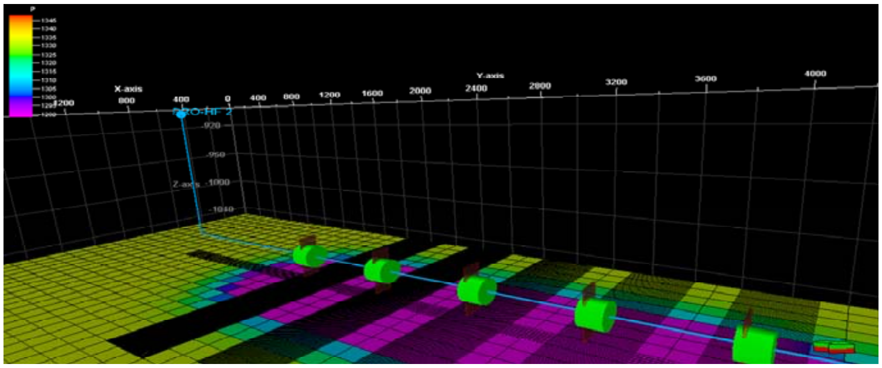


Table 3 Input data for stability analysis during production condition

Stress regime	*Depth(ft)*	*σ_v (psi/ft)*	*σ_H (psi/ft)*	*σ_h (psi/ft)*	*P_0 (psi/ft)*	*υ*
NF	3,000	1	0.9	0.8	0.45	0.25
RF	3,000	0.8	1	0.9	0.45	0.25
SS	3,000	0.9	1	0.8	0.45	0.25

To predict the collapse pressure during production, the reservoir pressure is predicted over the life of the reservoir with and without hydraulic fracturing (see Figure 16). The drop down of the reservoir pressure for fractured wells is much more than when none of the wells are fractured. In addition, the stability of non-fractured wells remains almost constant and for fractured wells the best trajectory changes over the reservoir life. When the reservoir pressure declines, the vertical *in situ* stress remains almost constant and the horizontal *in situ* stresses changes (for example, see Figure 17). To investigate the wellbore stability during production for different stress regimes, wellbore pressure is decreased from the initial reservoir pressure until the wellbore wall breaks out and borehole collapsing take place.

Figure 16 Average reservoir pressure with and without hydraulic fracturing over the reservoir life (see online version for colours)

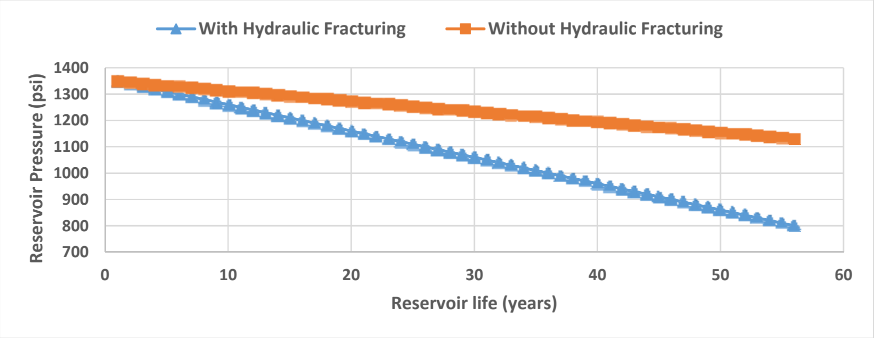
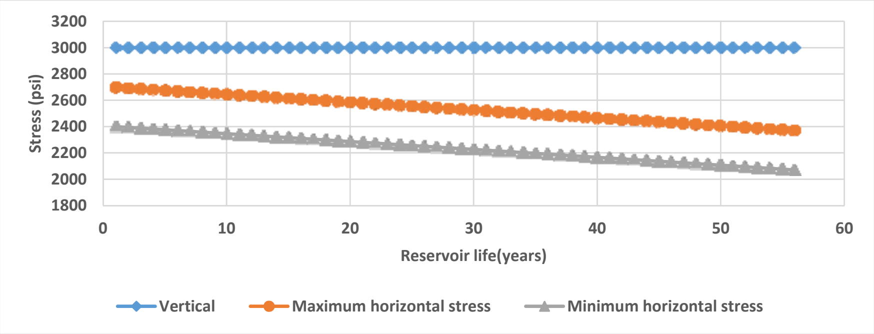


Figure 17 Illustration of stress changes during reservoir life in normal fault stress regime (see online version for colours)



In normal fault stress regime and using Mogi–Coulomb law, at an early stage of the reservoir life deviated wells with an azimuth of 90° are the best trajectories during production, (Figure 18). During production at first years, the trend of the collapse pressure is almost the same as during the drilling phase. After 55 years of production, the reservoir pressure declines and changes the *in situ* horizontal stresses. Wellbore collapse pressure after 55 years of production is shown in Figure 19. At the last stage of the reservoir life, the best well trajectories are still the same for deviated wells along the minimum horizontal stress as model predicted.

Figure 18 Collapse pressure at early reservoir life for NF stress regime using on Mogi–Coulomb law (see online version for colours)

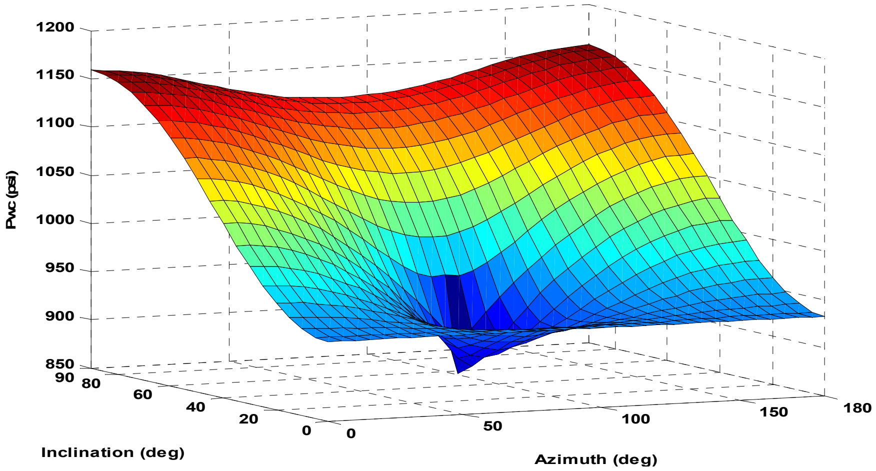
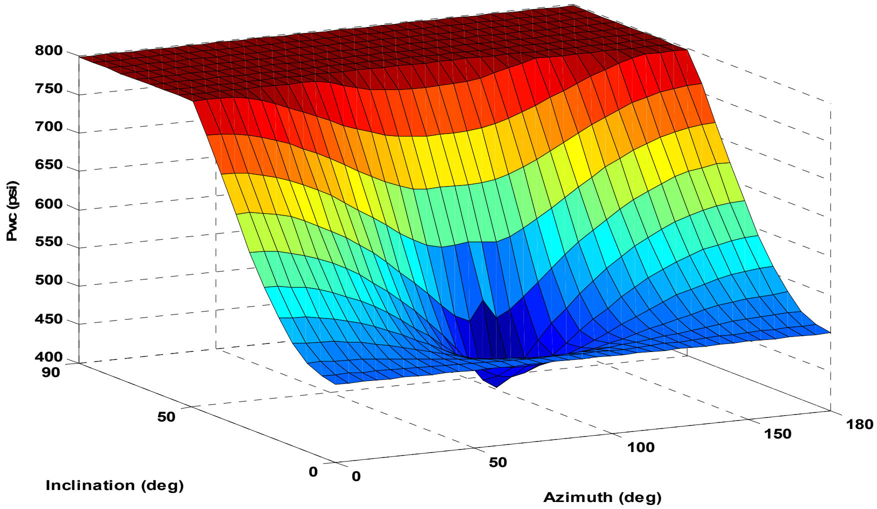
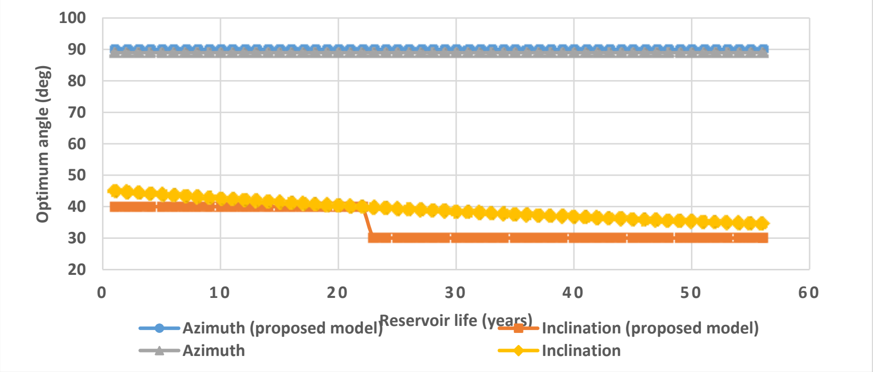


Figure 19 Collapse pressure for NF stress regime at the last stage of the reservoir life using Mogi–Coulomb law (see online version for colours)



The optimum well trajectory during reservoir life by adopting Mogi-Coulomb law for normal fault stress regime is illustrated in Figure 20. Using the proposed model and Al-Ajmi and Zimmerman (2009), the best well trajectory is not that sensitive to the reservoir depletion over 55 years of reservoir life and remains constant at 90°. Furthermore, using both models, the optimum well inclination is almost constant over the reservoir life with a change of approximately 10° which is practically insignificant but more precise.

Figure 20 Optimum well trajectory during reservoir life using Mogi–Coulomb law and Al-Ajmi and Zimmerman's model in NF stress regime (see online version for colours)



Based on the Drucker-Prager criterion in normal fault stress regime, at early and late stages of the reservoir life the vertical wells are the stable option (see Figure 21 and Figure 22). The optimum well trajectory during reservoir depletion has been obtained and illustrated in Figure 23. Based on previous studies different failure criteria give different responses during reservoir life production. The result of the model showed Mogi-Coulomb criterion gives more reasonable results to field cases and Drucker-Prager criterion is generally optimistic which might miss lead the drilling design. The results of the proposed model in this paper is consistent with the results of Al-Ajmi and Zimmerman's model for well path optimisation. Using both failure criteria in normal fault stress regime, horizontal well is the worst well pattern regardless of the adopted azimuth.

Figure 21 Collapse pressure for NF stress regime based on the Drucker-Prager criterion at early stage of the reservoir life (see online version for colours)

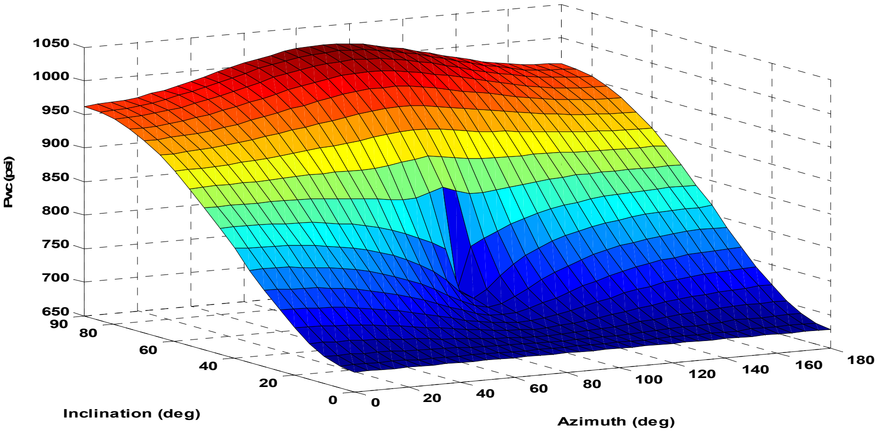


Figure 22 Collapse pressure for NF stress regime based on the Drucker–Prager criterion at last stage of the reservoir life (see online version for colours)

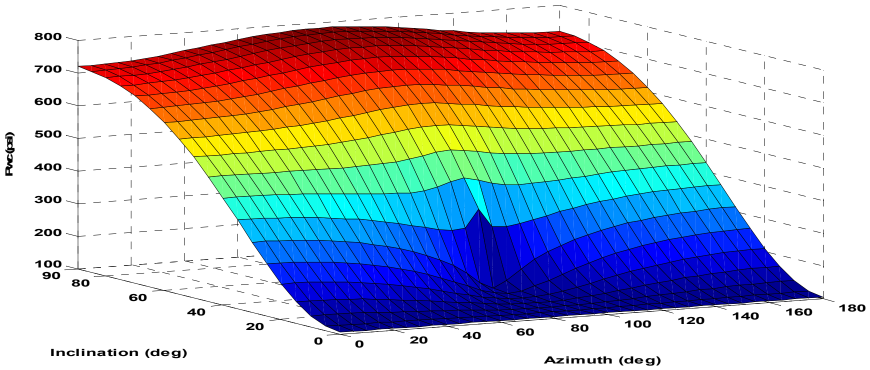


Figure 23 Collapse pressure for NF stress regime based on the Drucker–Prager criterion at last stage of the reservoir life (see online version for colours)

In strike-slip stress regime, using Mogi-Coulomb law, the optimum well azimuth and inclination during reservoir production changed from 40° to 90° and 90° to 70°, respectively (see Figure 24, Figure 25, and Figure 26). Up to about 50 years of production, the optimum well inclination remained constant at 90° with a change in the azimuth from 40° to 90°. Therefore, in the studied strike-slip stress regime, the optimum azimuth found to be much more sensitive to the reservoir depletion compared to the other stress regimes. Model depicts that after 50 years of production, the vertical in situ stress becomes equal to the maximum in situ horizontal stress with a slight increase towards a normal fault stress regime (see Figure 27). This behaviour is mainly due to the reservoir pressure depletion and its effect in the applied in situ horizontal stresses.

Figure 24 Collapse pressure for SS stress regime using Mogi-Coulomb law at the first stage of reservoir life (see online version for colours)`

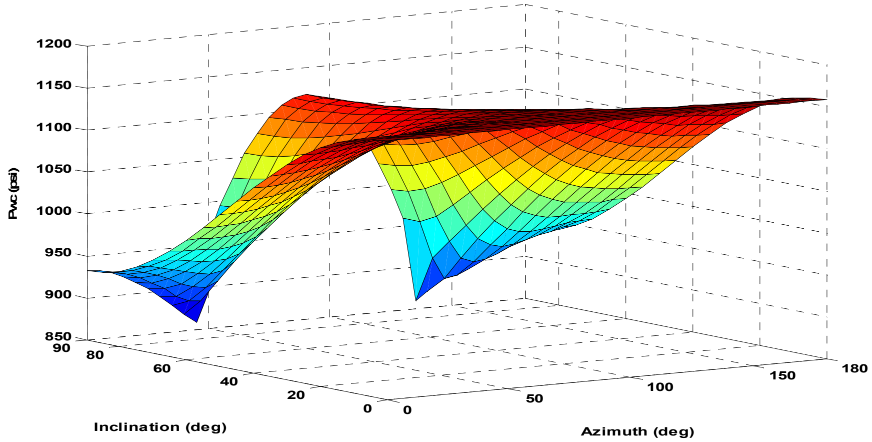


Figure 25 Collapse pressure for SS stress regime using the Mogi-Coulomb law at the last stage of reservoir life (see online version for colours)

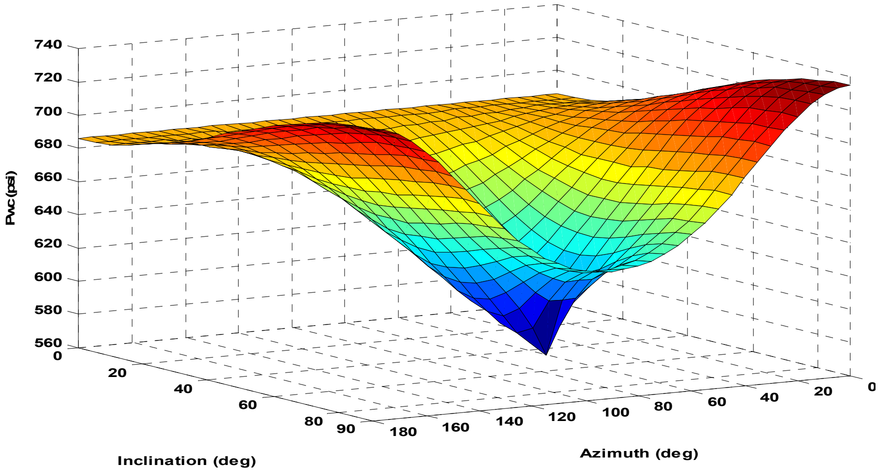


Figure 26 Optimum well path for SS stress regime using the proposed model and Al-Ajmi and Zimmerman's model (see online version for colours)

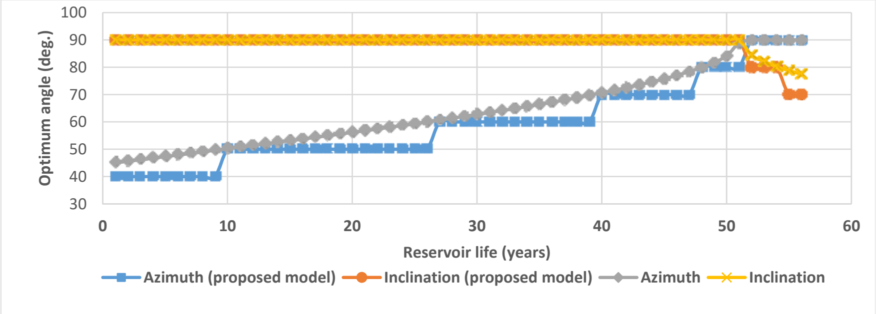


Figure 27 The change of in situ stresses over 55 years of production for an initial stike-slip stress regime (see online version for colours)

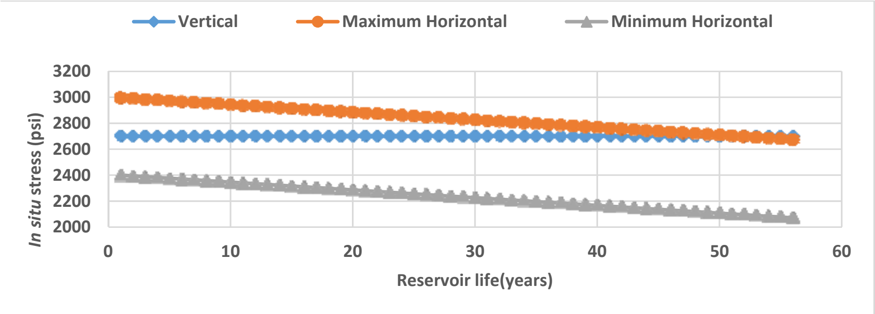


Figure 28 Collapse pressure for SS stress regime using the Dracker-Prager criterion at the first stage of reservoir life (see online version for colours)

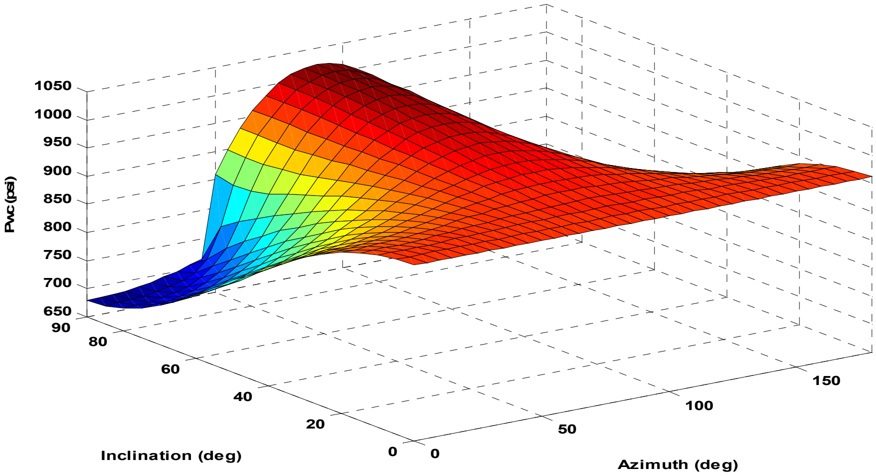


Figure 29 Collapse pressure for SS stress regime using the Dracker-Prager criterion at the last stage of reservoir life (see online version for colours)

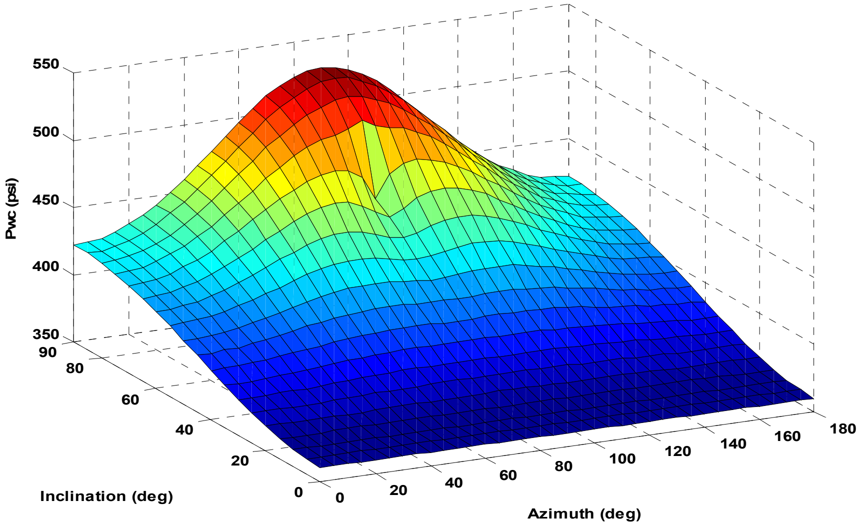
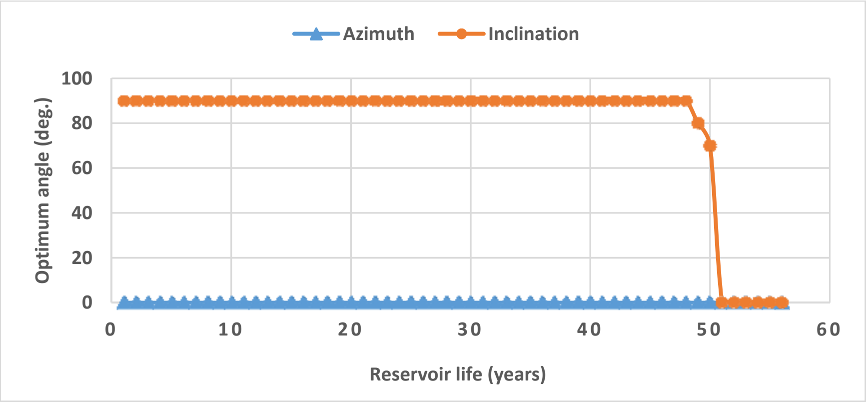


Figure 30 Optimum well trajectory for SS stress regime using the Dracker-Prager criterion (see online version for colours)



If Dracker-Prager failure criterion is considered, the horizontal wells along the maximum horizontal stress are the most stable pattern at the first stage of reservoir life, but a high depletion the vertical wells becomes more stable (see Figure 28, Figure 29, and Figure 30). In this application, the optimum well inclination dropped drastically from 90° to 0° after 48 years of production. Therefore, based on the propose model, the field stability condition is critical at the last stage of the reservoir life. Hence, to ensure stability it is essential to move from utilising a horizontal well to a vertical well, which is economically an expensive option. Using Mogi-Coulomb law, after about 50 years of production, there is a suggestion to change the well inclination slightly to improve the well stability. This recommendation is practically a minor change that can be neglected, and a horizontal well can be kept as a good option all over the life of the reservoir. These results highlight the significant of applying an appropriate rock failure law such as Mogi-Coulomb in wellbore stability analysis. Otherwise, a wrong decision might be taken in the design of the operations.

Figure 31 Collapse pressure for RF stress regime using Mogi-Coulomb law at the first stage of reservoir life (see online version for colours)

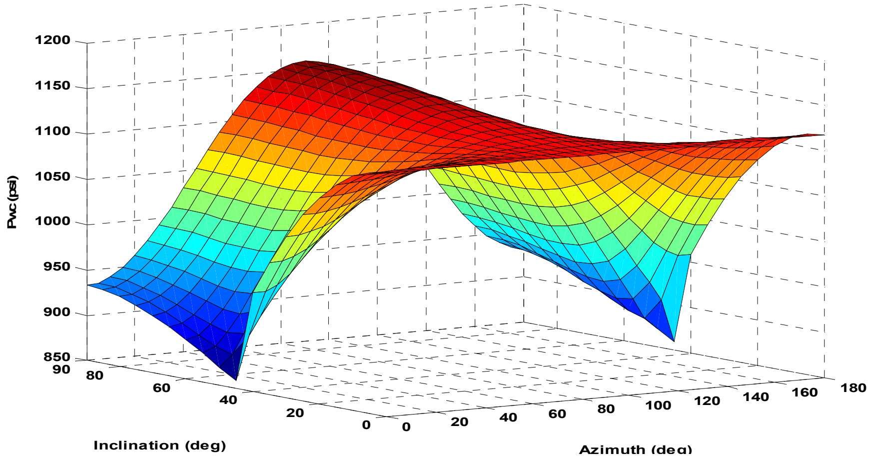
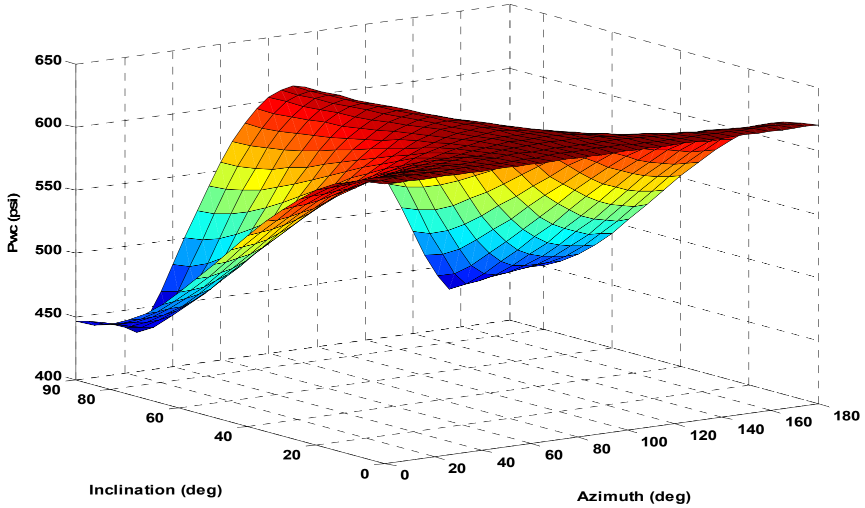


Figure 32 Collapse pressure for RF stress regime using Mogi-Coulomb law at the last stage of reservoir life (see online version for colours)



In the reverse fault stress regime, according to Mogi-Coulomb law, deviated wells with an azimuth of 0° are the best trajectory at the first stage of reservoir depletion (Figure 31). Over the years of production, the optimum well inclination increased from 50° to 90° (see Figure 31, Figure 32, and Figure 33). The results of the proposed model with respect to well path optimisation is consistent with the results obtained by adopting Al-Ajmi and Zimmerman's model as shown in Figure 33. For this stress regime, over the reservoir life the magnitude of in situ stresses change in the same manner as in strike-slip stress regime. By applying the Dracker-Prager failure criterion in the stability analysis, horizontal wells along the maximum horizontal stress are generally the most stable trajectory over the life of the reservoir (see Figure 34 and Figure 35).

Figure 33 Optimum well trajectory for RF stress regime using the proposed model and Al-Ajmi and Zimmerman's model (see online version for colours)

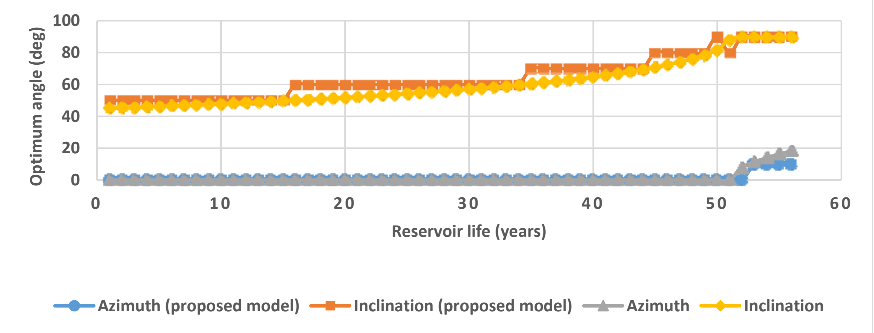


Figure 34 Collapse pressure for RF stress regime using Dracker-Prager criterion at the first stage of reservoir life (see online version for colours)

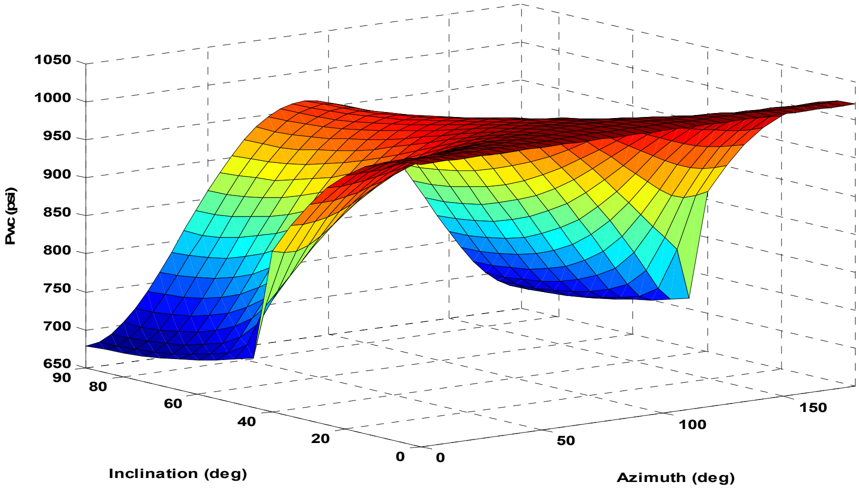
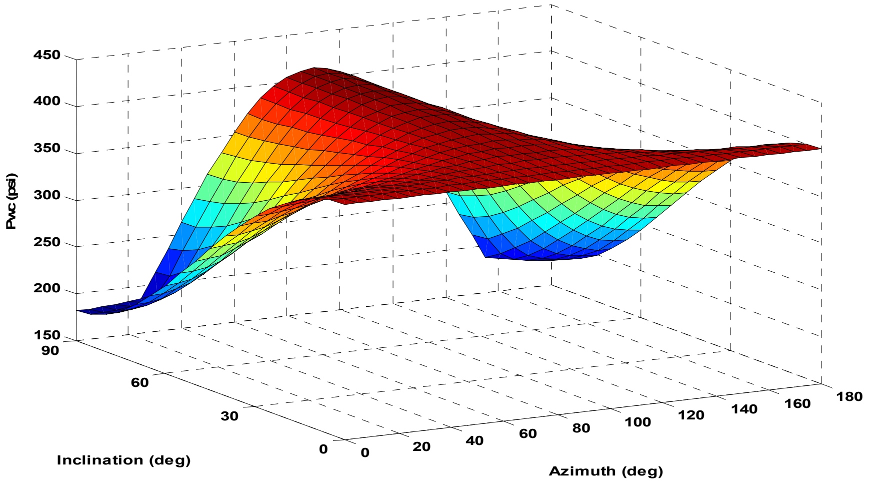


Figure 35 Collapse pressure for RF stress regime using Dracker-Prager criterion at the last stage of reservoir life (see online version for colours)



4 Conclusions

Considering the minimum fracture gradient during the production phase is a very important item in well path design. In all the stress regimes, it has been found that the difference between the maximum and minimum values of the fracture pressures is nearly 4 to 5 times greater than this difference for the collapse pressure. Therefore, the choice of optimum minimum fracture pressure is essential and should be considered during the drilling programs. A path design model for optimum well trajectory which consider the summation of minimum mud pressure to avoid borehole collapse during drilling and minimum fracture gradient during the production is developed in this work.

During the drilling stage, the developed model shows different responses for the optimum well trajectories for different stress regimes. In normal and strike-slip stress regimes, drilling deviated wells parallel to the minimum horizontal stress are the most stable path. For reverse fault stress regime, however, drilling horizontal wells with an azimuth close to the minimum horizontal stress are more stable.

For the production stage, using Mogi-Coulomb law, the optimum well path pattern during the reservoir life for normal fault stress regime was mostly constant. For the other stress regimes, however, the optimum well path changed during the reservoir life. Therefore, in normal fault stress regime there is no need to change the well orientation over the whole life of the reservoir. However, in the other stress regimes well trajectory might be required to be changed to ensure the stability. For strike-slip stress regime, well optimum azimuth showed higher sensitivity for the depletion process than the well optimum inclination. And, for reverse fault stress regime, well optimum inclination was more sensitive to the reservoir depletion than the well optimum azimuth.

List of notations (continued)

β	Biot's poroelastic constant
γ	deviation angle of the borehole from the maximum principle in situ stress in the $\sigma 1$–$\sigma 3$ plane