A Subgame Perfect Approach to a Multi-Period Stackelberg Game with Dynamic, Price-Dependent, Distributional-Robust Demand

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A Subgame Perfect Approach to a Multi-Period Stackelberg Game with Dynamic, Price-Dependent, Distributional-Robust Demand

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Abstract

This paper investigates a multi-periodic channel optimization facing uncertain, price-dependent, and dynamic demand. The picture of the market uncertainty is incomplete, and only the price and time-dependent mean and standard deviation are known and may depend on the price history. The actual demand distribution itself is unknown as is typically the case in real-world problems. An algorithm finding the optimized decentralized channel equilibrium is developed when the downstream member optimizes her expected profit stream by a distributional-robust approach, and the upstream member (leader) considers it as the follower's reaction function. The algorithm allows for strategic decisions whereby the current demand is scaled by the previous price setting.

JEL classification: C61, C62, C63, C72, C73, D81.

Keywords: Multi-Periodic problem, Stochasticity, Stackelberg Game, Subgame Perfect Distributional-Robust Approach, Supply Chain Management, Price-History Dependent Dynamic Demand.

1 Introduction

A supply channel is frequently accompanied by a time-varying and uncertain demand. The upstream member (manufacturer) and the downstream member (retailer) are exposed to this market uncertainty in different manners. The retailer faces the uncertainty directly, while the manufacturer senses it through the order quantity made by the retailer. The uncertainty in demand usually leads the chain to a lost sale or unsold quantity which can be salvaged (Khan & Sarkar, 2021).

The simplest case occurs when the demand is structured from a distribution of price-independent quantities. In reality, demand varies as time goes by. Consequently, demand-boosting strategies, such as period(s) with free or low-price commodities to boost the market in the following periods, expedite the market. On the other hand, customers, aware of the price trend, may change their purchase plan based on the historical product's prices over time. The market demand can be adjusted to increase (decrease) with positive (negative) effects from price history. Strategic pricing policy, hence, occurs where the demand contains current and historical prices. The main challenge in a multi-period discrete-time model with dynamic price-dependent demand is the interdependence of all price values. This nestedness comes into play by the notion of memory or scaling functions that carry the effect of prior prices.

A channel is normally not fully equipped with comprehensive demand distribution information, either the information is unavailable or too costly to be achieved. These cases logically require a distributional-robust (DR) approach. The present work considers a game between upstream and downstream parties of a decentralized channel where the manufacturer is the leader, and the retailer follows him. The demand of this channel is dynamic and price-dependent for a perishable commodity where the demand distribution is unknown. The problem is formulated in a multi-period revenue management setting. In each period (k) the leader starts by deciding the wholesale price (w_k) and the follower immediately follows up by deciding her order quantity (q_k) and retail price (r_k) to the market. Both parties want to optimize their holistic profits over all periods. The commodity cannot be stored for later use. Thus, any unsold item must be salvaged (discarded) at a lower price (cost) s_k.

The retailer encounters the market risk through stochastic demand (D_k) and may endure not meeting the market by missing an opportunity to sell $(D_k - q_k)^+$ more, or salvaging/discarding $(q_k - D_k)^+$ leftovers. In each period the uncertainty is unveiled after the decisions on $w_k, r_k,$ and q_k are made. The main novelty in this paper is to solve such multi-periodic supply chain problems where the means and standard deviations are the only known information about the demand encompassing the effect of antecedent price setting. This approach incorporates the effects on future market demand by previous decisions (price-setting). This implements by setting the means and standard deviations as functions of all previous retail prices, such that previous prices scale the demand, but only the current retail price determines the current coefficient of variation (CV). The potential dependence on previous prices authorizes strategic pricing, e.g., lowering prices to enhance demand by attracting more customers. Over time, prices can be raised if the customer base increases sufficiently.

This work considers the normal flow of retail; The product is supplied (produced) by a wholesaler (manufacturer) and sold by a retailer to customers. Both parties are risk-neutral and want to maximize their expected discounted total profit. The main part of the solution effort is the computation of equilibrium prices leading to order quantities that maximize expected profits. It leads to a type of subgame perfect optimization that can be decomposed into a sequence of connected decisions. A trivial subclass of our approach covers the multi-period supply chain games with stochastic demands that are only dependent on the current price and time. The non-negative property of demand excludes all distributions with compact support not limited from below, e.g. the normal distribution. This is particularly important when the volatility is dependent on decision variables (e.g. the price). Section 4 works out an illustration comparing distributional-robust results with the alternative fully informed cases exemplified by uniform distributions.

2 Literature Review

In 1958, Scarf proposed a method to solve an inventory problem with limited demand information where the only knowledge of demand is the mean and standard deviation and the demand distribution is uncharted (Scarf, 1958). Later, Gallego and Moon revised and extended Scarf's method for a newsvendor problem with three conditions when: a second purchasing opportunity can occur after demand is revealed, a multi-item case, and a random yield case (Gallego & Moon, 1993). In another study, Gallego discussed a minimax distributional-robust approach to acquire order and inventory levels minimizing the cost of holding/shortage in a newsvendor problem (Gallego, 1992). Gallego, in cooperation with Moon, analyzed continuous and periodic inventory models with backorders and lost sales and a price-independent demand by a minimax DR approach to obtain the order volume and retail price (Moon & Gallego, 1994).

Godfrey and Powell optimized the newsvendor problem with a repeated inventory under DR model using the concave adaptive value estimation (CAVE) algorithm (Godfrey & Powell, 2001). Mostard et al. studied the DR newsvendor problem where the returned items, if not damaged, were resold before the end of the season and leftovers would be salvaged. Even though the shortage also can harm their algorithm by a shortage cost on the retailer's side (Mostard, et al., 2005). Pal et al. also inset a DR newsvendor problem and study inventory management with a non-linear holding cost to diminish the inventory level (Pal, et al., 2015). Sarkar et al. investigated the DR Stackelberg newsvendor problem under a make-to-order and consignment policy where both parties carry some share of the holding cost (Sarkar, et al., 2018). Khan and Sarkar presented a DR newsvendor model with back-ordering and stochastic and price-dependent demand (Khan & Sarkar, 2021). Their retailer is required to pay an additional price per product to transfer the unsold items' risk to the manufacturer. Govindarajan et al. solved a DR multi-location newsvendor problem to optimize the inventory level minimizing cost (Govindarajan, 2021).

Our paper contributes to this research area by optimizing the multi-period supply chain Stackelberg game in which demand is time and price-dependent, although the distribution of demand is unknown. The only available information is the mean (μ) and the standard deviation (σ) of the demand as functions of time and prices when the price history impacts the future demand, i.e., price history dependent demand (PHD). Practically speaking, figuring out the stochastic drivers in a time-dependent demand distribution may not be available or economically viable. Hence, the distributional-robust model is a maxmin-optimization to generate a weak lower bound on optimal expected value. This is considered in section 4.

Table 1*:* A literature review on distributional-robust problem

Author(s)	Perishable	Periods	Demand[1]	PHD
Scarf, 1958	✓	1	S	×
Gallego, 1992	×	1	S	×
Gallego & Moon, 1993	✓	1	S	×
Moon & Gallego, 1994	×	1	S	×
Godfrey & Powell, 2001	✓	2	S	×
Mostard, et al., 2005	✓	1	S	×
Pal, et al., 2015	✓	1	S	×
Sarkar, et al., 2018	✓	1	S	×
Khan & Sarkar, 2021	✓	1	S	×
Govindarajan, 2021	✓	1	S	×
This paper	✓	**Any**	**TPD**	✓

[1] S=static and TPD=time and price dependent

3 Model Framework

To the best of our knowledge, the DR supply chain problem has been widely studied in the literature but has not gone beyond 2 periods with newsvendor structure. In this paper, our DR algorithm finds the subgame perfect optimal equilibrium prices and quantities. The problem is stated in a multi-periodic setting with explicit time-dependent model parameters (non-autonomous). In the following, a single-period problem is explained in section 3.1, and a multi-periodic extension in sections 3.2 and 3.3. To follow the rest of this section, the notation list is stated as follows where n is the number of periods.

Notation

$\beta = \{\beta_1, \dots, \beta_n\}$	Discount factor over individual periods[2]
$c^m = \{c_1^m, \dots, c_n^m\}$	Manufacturer cost
$s = \{s_1, \dots, s_n\}$	Salvage price/discarding cost
$w = \{w_1, \dots, w_n\}$	Wholesale price (decisions)
$r = \{r_1, \dots, r_n\}$	Retail price (decisions)
$q = \{q_1, \dots, q_n\}$	Order quantity (decisions)
$k \in \{1, \dots, n\}$	Time or period
$D = \{D_1, \dots, D_n\}$	Demand
$\mu = \{\mu_1, \dots, \mu_n\}$	Mean of demand
$\sigma = \{\sigma_1, \dots, \sigma_n\}$	The standard deviation of demand
$z = \{z_1, \dots z_n\}$	Stochastic and independent drivers with mean 0 and variance 1
$\pi^m = \{\pi_1^m, \dots, \pi_n^m\}$	Manufacturer profit (running value)
$\pi^r = \{\pi_1^r, \dots, \pi_n^r\}$	Retailer profit (running value)
JR^x	The total expected value of player x, in DR model
JD^x	The total expected value of player x, in the model with known distribution

3.1 Single-Period Distributional-Robust Supply Chain Model

In this supply chain under the Stackelberg game, the channel leader, the manufacturer, acts first and offers the price w that maximizes his profit ($E[\pi^m(q, w)]$). Then the follower, the retailer, decides on the optimal volume q and optimal retail price r that maximizes his expected profit

[2] The discount factors related to the start (t=0) are $\alpha_k = \beta_1 \cdot \beta_2 \cdot \dots \cdot \beta_k$. Individual periods may be of different length.

($E[\pi^r(q, D, r, w)]$). It is a single-order opportunity, and the market cannot be replenished; Consequently, the unmet demand is considered backlogged and is not involved in the algorithm. The unsold items, on the other hand, can be salvaged/discarded at a lower price/cost s. We have dropped the time index since it is single-period problem. The general form of demand forms

$$D = \mu(r) + \sigma(r)\, z \geq 0, \tag{1}$$

where μ and σ are deterministic known functions of retail price r, and z is a stochastic variable with a mean and standard deviation of 0 and 1 respectively. Noticing the stochastic demand, the retailer orders q and sells $min\ (D, q)$ at price r to maximize his profit

$$\pi^r \ = \ r \min(D, q) + s\,(q - D)^+ - w\, q \tag{2}$$

The leftovers $(q - D)^+$is salvaged at $s(> 0)$ or discarded at $s(< 0)$. To optimize the problem, the expected value is illustrated as[3]

$$\begin{aligned} E[\pi^r] \ &= \ (r - s)\, E[\min(D, q)] - (w - s)\, q \\ &= \ (r - s)\, E(D - [D - q]^+) - (w - s)\, q \\ &= \ (r - s)\, \mu - (w - s)\, q - (r - s)\, E[D - q]^+ \end{aligned} \tag{3}$$

If the demand is accompanied by a known distribution, the value of $E[D - q]^+$ can be calculated. In general, the following hold

I. $(D - q)^+ \leq |D - q|$,

II. $E[|D - q|] \leq \sqrt{E[(D - q)^2]} = \sqrt{(\mathrm{q} - \mathrm{\mu})^2 + \sigma^2}$ (Cauchy-Schwartz inequality)

III. $(D - q)^+ = \frac{1}{2}\{|D - q| + (D - q)\}$.

A simple consequence of these relations is

$$E[D - q]^+ \leq \frac{\sqrt{\sigma^2 + (q - \mu)^2} - q + \mu}{2}. \tag{4}$$

This inequality gives a tight lower bound on expected retailer profit for any distribution with the same μ and σ. Hence,

$$E[\pi^r] \geq \ (r - s)\, \mu - (w - s)\, q - (r - s)\ \frac{\sqrt{\sigma^2 + (q - \mu)^2} - q + \mu}{2} \equiv \Pi^r. \tag{5}$$

[3] $\min(D, q) = D - (D - q)^+$ and $(q - D)^+ = (q - D) + (D - q)^+$

The DR approach is defined by replacing $E[\pi^r]$ with Π^r. It has been shown that equality holds in Eq. (5) for some special distributions (Gallegol & Moon, 2016) and trivially for a deterministic demand.

The optimal ordering volume follows from optimizing Π^r for q

$$q = \mu + \sigma\Lambda, \qquad \Lambda = \frac{\eta - \frac{1}{2}}{\sqrt{\eta\,(1-\eta)}} \quad \text{and} \quad \eta = \frac{r-w}{r-s} \tag{6}$$

The μ and σ approach zero when prices turn to large values[4]. The manufacturer optimizes his problem to find the optimal price w, manipulating the retailer to order q in the Stackelberg game, such that this pair (w, q) maximizes his profit

$$\pi^m = (w - c^m)\, q = E[\pi^m] \tag{7}$$

To have consistent notation, $E[\pi^m] = \pi^m = \Pi^m$.

3.2 Multi-Period Distributional-Robust Model

In a multi-periodic chain, players endeavor to maximize their total discounted expected profit streams

$$J_k^x = \alpha_k \Pi_k^x + \alpha_{k+1}\,\Pi_{k+1}^x + \alpha_{k+2}\Pi_{k+2}^x + \cdots + \alpha_n\,\Pi_n^x \quad \text{for } x \in \{m, r\}, \\ \text{where } \alpha_k = \beta_1 . \beta_2 \dots \beta_k, \tag{8}$$

and n is the number of periods that may be of different duration and β is the discount rate. ${J_k}^r$ and J_k^m are the present values of the streams for the retailer and manufacturer respectively, from period k and onward. The players optimize their J_k^x at each period (i.e., subgame perfect).

3.3 Multi-Period Distributional-Robust Model with Price-History Dependent Demand

Demand is usually sensitive to price, and this may evolve as time goes by. The current price and time are normally not the only factors impacting the current demand. Previous price settings may scale the market by, e.g., boosting or shrinking the upcoming demands. This impact is likely to be time-dependent. In this work, we assume that the price history only affects the size of the demand while the present price also modifies the coefficient of variation (CV), i.e.,

$$D_k = \Phi_k(r_1, \cdots, r_{k-1})\, d_k(r_k), \quad d_k(r_k) = \hat{\mu}_k(r_k) + \hat{\sigma}_k(r_k) z_k, \quad k \in \{1, \cdots, n\}, \quad \Phi_1 = 1 \tag{9}$$

[4] Real demand is non-negative with compact support on a finite interval.

The Φ_k is a scaling function, representing a cumulative relation of previous market price settings. As an example, if the relevance between periods' memories is multiplicative, the cumulative scaling at each period forms

$$\Phi_k(r_1, \cdots, r_{k-1}) = g_k(r_{k-1})\Phi_{k-1}(r_1, \cdots, r_{k-2}) = \prod_{i=2}^{k} g_i(r_{i-1}), \tag{10}$$

where g_k carries the effect of the previous price r_{k-1}. Strategic pricing to boost future demand may occur optimally in some model specifications. The case $\Phi_k \equiv 1$ for all k implies that the demand in each period only depends on the current price, i.e., $D_k = d_k(r_k)$ and no strategic pricing can occur. Hence, Eq. (8) can be written

$$J_k^x = \sum_{i=k}^{n} \alpha_i \Phi_i \hat{\pi}_i^x, \quad x \in \{m, r\}, \tag{11}$$

where $\hat{\pi}_k^r, \hat{\pi}_k^m$ only depend on decision variables in period k, i.e.,

$$[\hat{\mu}_k(r_k), \hat{\sigma}_k(r_k), \hat{q}_k(r_k, w_k)] = \frac{[\mu_k(r_1, \dots, r_k), \sigma_k(r_1, \dots, r_k), q(r_1, \dots, r_k, w_1, \dots w_k)]}{\Phi_k(r_1, \dots, r_{k-1})} \tag{12}$$

The term $\alpha_k \Phi_k$ is known at the beginning of period k. Viewing the problem from an arbitrary period (k) and onward, Eq. (11) implies maximizing j_k^x for each player,

$$j_k^x = \hat{\pi}_k^x + \beta_{k+1}\, g_{k+1}\, j_{k+1}^x \ \text{ for } \ x \in \{m, r\}$$
$$\text{where } j_k^x = \frac{J_k^x}{\alpha_k \cdot \Phi_k} \tag{13}$$

By starting at the last period (n), the sequence of leader-follower games defined by $\{\{j_n^m, j_n^r\}, \dots, \{j_1^m, j_1^r\}\}$ is optimized. Each of these games has objectives to be maximized in the form $j_k^x = \hat{\pi}_k^x + \beta_{k+1}\, g_{k+1}\, j_{k+1}^x = \pi(r, w, \hat{q}(r, w)) + g(r)A$, with a known constant A which is zero in the last period (at period n, $A = \beta_{n+1} j_{n+1} = 0$), and is a known constant at each period in the backward induction process where it is calculated from a higher period. When the scaled games are solved, and the decisions r^*, w^*, and q^* are known, the Φ^* are determined and then quantities and profits are rescaled to their proper values.

Remark

In a single-period newsvendor (fixed prices) problem, a fully equipped demand creates more expected profit for the retailer compared to the DR model, $\pi R^*(q_R^*) \leq \pi D(q_R^*) \leq \pi D^*(q_D^*)$, where πR and πD represent the expected profit of the distributional-robust model and the model with distribution, respectively. It implies that the DR optimal profit is a lower bound for the problem with distribution. Furthermore, the DR model's policy is not optimal for the model with distribution ($\pi D(q_R^*) \leq \pi D(q_D^*)$).

In a multi-periodic supply chain problem, the relation

$$JR^{r*}(w_R^*, r_R^*, q_R^*) \leq JD^{r}(w_R^*, r_R^*, q_R^*) \leq JD^{r*}(w_D^*, r_D^*, q_D^*) \tag{14}$$

holds. The $JR^{r*}, JD^{r},$ and JD^{r*} are the retailer optimal expected value of the DR model, the model with distribution before optimization, and the model with distribution after optimization respectively, and JR and JD follow Eq. (11). The indexes R and D, used for w, r and q, represent the distributional robust model and the model with distribution respectively. Hence, $JD^{r}(w_R^*, r_R^*, q_R^*)$ solves the model with distribution for the DR policy.

4 Numerical Implementation

The simplest case occurs when the demand depends only on the current price. This family of problems decouples into a series of independent single-period problems. Albeit most markets have some dependency on the price history affecting customers' behavior. In this section, we offer examples with a price history (path) dependent demand to show how to implement the proposed algorithm. One may try to vary the scaling factor or mean and standard deviation functional form to take full advantage. Our numerical illustration is given by applying the algorithm to optimize problems with the scaled mean and standard deviation of demand given by (see Eq. (9))

$$\hat{\mu}_k(r_k) = \frac{1000\left(1+\frac{1}{1+k}\right)}{r_k^2} \text{ and } \hat{\sigma}_k(r_k) = \frac{\hat{\mu}_k(r_k)}{2\sqrt{3}}. \tag{15}$$

To assess the DR results, we assume that uniform distribution (UD) is the true distribution and solve the UD model algorithm with the DR policy and call it UR model results.

From Eq. (10), the scaling factor

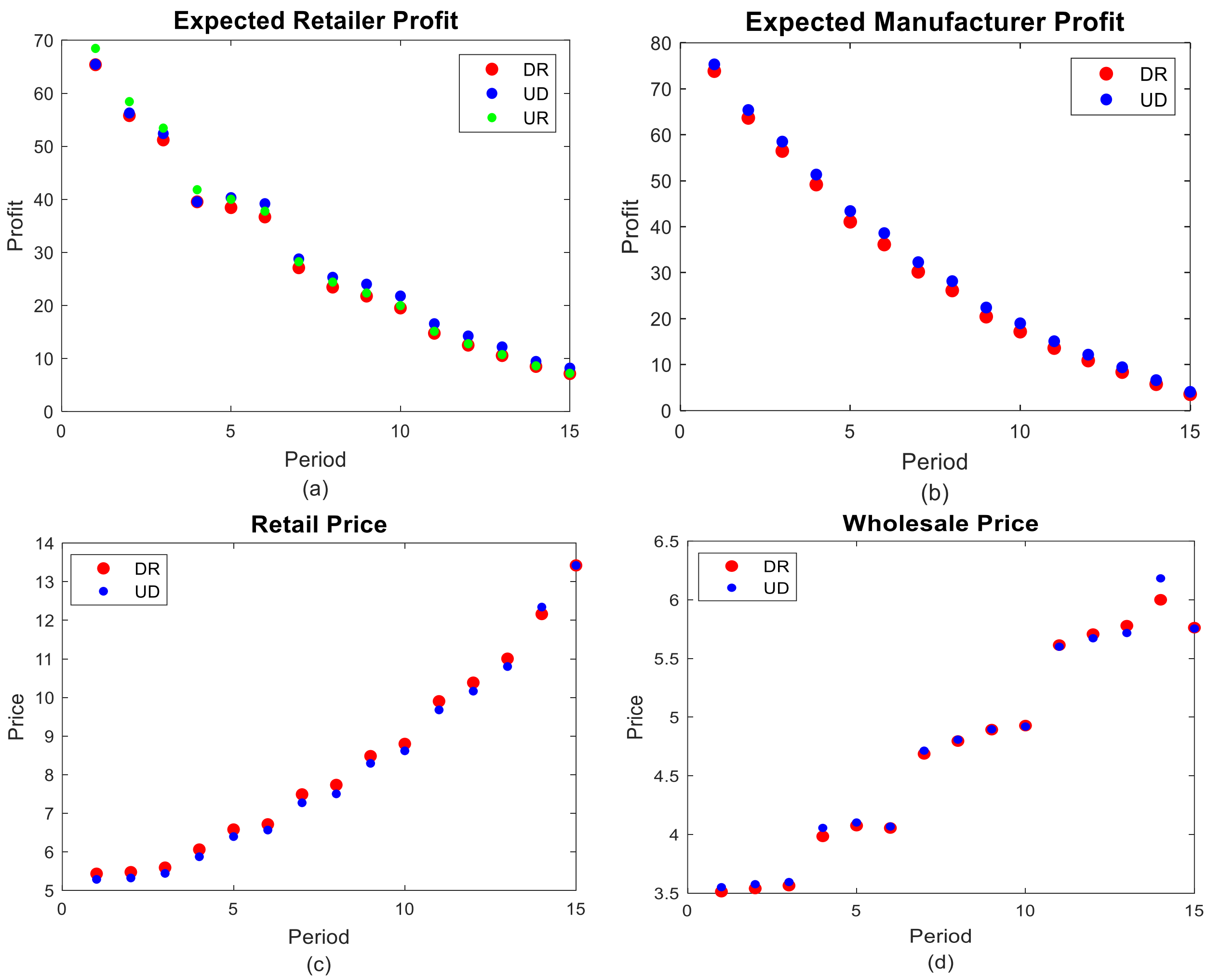
$$g_k(r_k) = e^{\gamma_k(K_k - r_k)} \tag{16}$$

at each period. The time-dependent parameter K_k is a kind of current time preference price and γ_k represents the strength of a current deviation to the future demand. The scale factor g_k acts similarly to a discount factor, though the retailer can manipulate it by setting the price to modify future demand.

To optimize the manufacturer-retailer problem an n-value parameter set has been applied for each time-dependent parameters c^m, s, β, K, γ, where n represents the number of periods (15 in this illustration),

c^m = [2 2 2 2.2 2.2 2.2 2.5 2.5 2.5 2.5 2.8 2.8 2.8 3 3]
s = [1 1 1 1 1 1.2 1.2 1.2 1.2 1.2 1.3 1.3 1.3 1.3 1.3]
β = [1 0.96 0.96 0.96 0.97 0.97 0.97 0.97 0.98 0.98 0.98 0.98 0.98 0.98 0.98]
K = [5.6 5.6 5.4 5.4 5.4 5.3 5.3 5.3 5.3 5.1 5.1 5.1 5.1 5.1]
γ = [0.05 0.05 0.05 0.05 0.04 0.04 0.04 0.04 0.03 0.03 0.03 0.03 0.03 0.03]

The decision variables $(w^*, r^*, q^*(w^*, r^*))$ in the equilibrium state are pictured in Figure 1. The retailer optimal expected profit at each period is illustrated in Figure 1 (a), where the result of the model with the uniform distribution is the blue line (UD), distributional-robust model result (DR) is in red, and implementing the UD model with the DR policy is in green (UR). Figure 1 (b) displays the manufacturer profits in cases DR and UD.



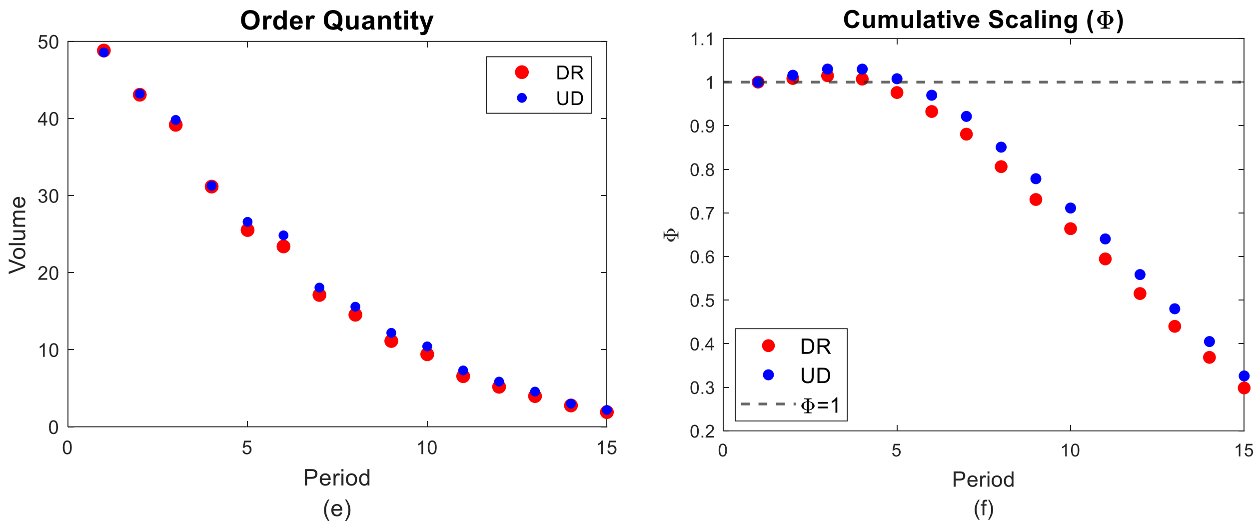


Figure 1: Optimal results

The total retailer expected profit satisfies $JR^{r*}(= 432) \leq JD^{r}(= 449) \leq JD^{r*}(= 454)$, as stated in Eq. (14). The manufacturer does not face the incomplete information consequence directly. Albeit he feels the market volatility due to the retailer's order volume decision. The manufacturer achieves $JR^{m*}(= 456.1) \leq JD^{m*}(= 482.1)$. The difference between $JD^{x*}(w_D^*, r_D^*, q_D^*)$ and $JD^{x}(w_R^*, r_R^*, q_R^*)$ defines the DR profit deviation from UD model. The loss that the incomplete information causes is the Expected Value of Additional Information (EVAI), and in this example

$$EVAI^{r} = 454 - 449 = 5$$

$$EVAI^{m} = 482.1 - 456.1 = 26$$

The retailer deviates 0.94 % and the manufacturer 5.2 % from their actual[5] value if they implement DR policy. The retailer may spend up to 5 units of currency to obtain complete information. However, the DR policy is a very good heuristic.

The retailer price grows by 147 % in the DR and 153.9 % in the UD models (plot (c)) over time. The wholesale price decision improves by 64 % in DR and 62.1 % in UD models (plot (d)). The price decisions lead to a 96.1 % quantity decline in DR model and 95.5 % in UD model (plot (e)) over time. Looking at plot (f), the retail prices exceed the market price preference from period 3 leading to a downward trend in cumulative scaling.

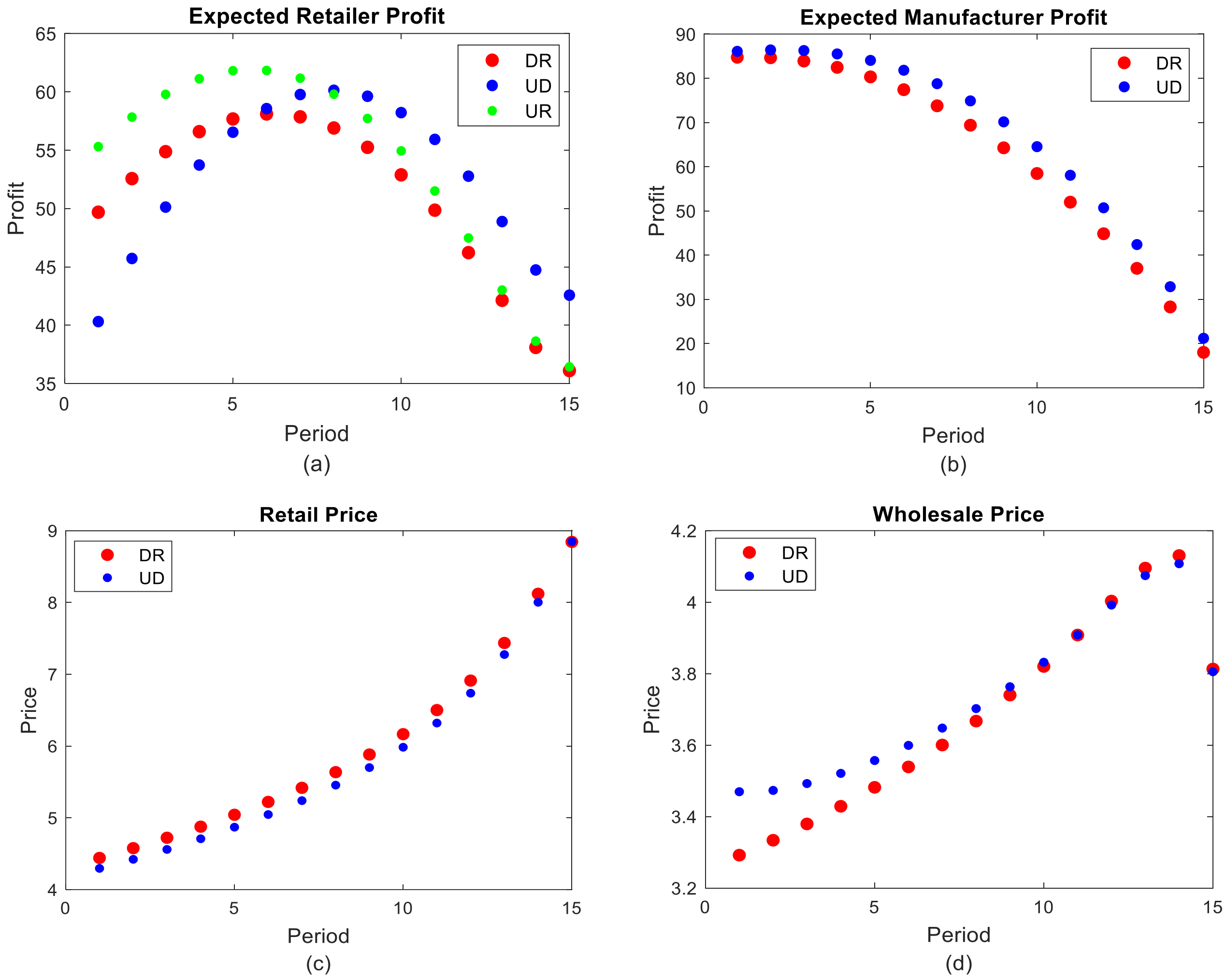
Plot (d) shows that the wholesale price follows the same pattern as the manufacturer's cost vector. However, except for the last stair, an increase in cost usually has been compensated with

[5] If the problem with complete information is considered actual.

a higher increase in the wholesale price. For instance, a 10 % increase in the cost in the first jump (period 3 to 4) raises the wholesale price by 11.8 % and 12.8% in DR and UD models respectively. The reason may stem from the fact that the increase in cost implies both higher cost and salvage loss $(w - s)$ for the retailer, leading to a lower order quantity. Hence, the manufacturer sets a price to also partly compensate for this quantity reduction.

4.1 Time Independent Model Parameters

We have fixed the parameter sets as, $c^m = 2,\ \beta = 0.96, s = 1, K = 5.6, \gamma = 0.05$. This example is time-independent; Hence, it is only the scaling factor that changes the results.



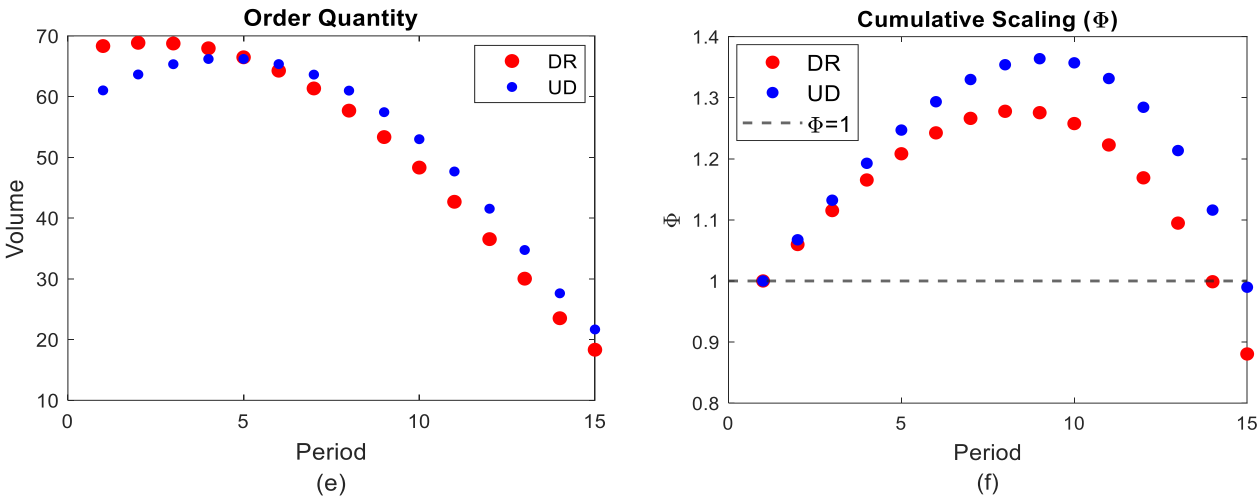


Figure 2: Results with time-independent model parameters

In this experiment, the retailer obtains 769.8 from DR and 787.6 from UD models (plot (a)). Meanwhile, the manufacturer gains 939.5 from implementing DR and 1000.4 from UD models (plot (b)). The wholesale price increases by 25.5 % until period 14, but by 15.8 % overall (plot (d)) and the retail price by 99.1 % (plot (c)) in DR model. These prices have led to a 73.2 % decline in ordering (plot (e)), while the market is in a state of prosperity for a long time (wherever higher than 1 in plot (f)).

4.2 Impact of Key Model Parameters on Total Profit

In this section, we evaluate the effect of changes in salvage value (s), manufacturer cost (c^m), and current time preference price (K) on the players' values in the proposed model (DR approach). Any of these parameters can increase, decrease, or stay unchanged. Table 2 illustrates 27 scenarios, where scenario 27th represents the reference problem (baseline) solved in section 4.

Table 2: The effect of key parameters on outputs

Scenario	c^m(%)	s(%)	K(%)	JM	JR	ΔJM(%)	ΔJR (%)
1	10	10	10	432.9	400	-5.1	-7.4
2	10	10	-10	319.5	315.4	-29.9	-27
3	10	10	0	369.95	353.4	-18.9	-18.2
4	10	-10	10	419.4	387.2	-8.1	-10.4
5	10	-10	-10	309.8	305.7	-32.1	-29.2
6	10	-10	0	358.7	342.5	-21.3	-20.7
7	10	0	10	426	393.4	-6.6	-8.9
8	10	0	-10	314.6	310.5	-31	-28.1
9	10	0	0	364.2	347.8	-20.2	-19.5
10	-10	10	10	703.8	643.8	54.3	49
11	-10	10	-10	505.4	492.4	10.8	14

Scenario	c^m(%)	s(%)	K(%)	JM	JR	ΔJM(%)	ΔJR (%)
12	-10	10	0	593.4	560.2	30.1	29.7
13	-10	-10	10	677.1	617.6	48.5	43
14	-10	-10	-10	468.7	473.1	6.7	9.5
15	-10	-10	0	571.1	537.6	25.21	24.5
16	-10	0	10	690	630.2	51.3	45.9
17	-10	0	-10	495.8	482.5	8.7	11.7
18	-10	0	0	581.9	548.6	27.6	27
19	0	10	10	546.8	501.6	19.9	16
20	0	10	-10	397.9	390	-12.77	-9.8
21	0	10	0	464.1	440	1.75	1.8
22	0	-10	10	528	483.6	15.8	12
23	0	-10	-10	384.9	376.3	-15.6	-13
24	0	-10	0	448.5	424.4	-1.7	-1.7
25	0	0	10	537.2	492.2	17.8	14
26	0	0	-10	391.5	382.9	-14.2	-11.4
27(Baseline)	0	0	0	456.1	432	0	0

The variation is ±10%, otherwise, the parameter stays unchanged, indicated with 0% in the table. The last two columns depict the percentage of value deviation from scenario 27. The results for total expected profits are

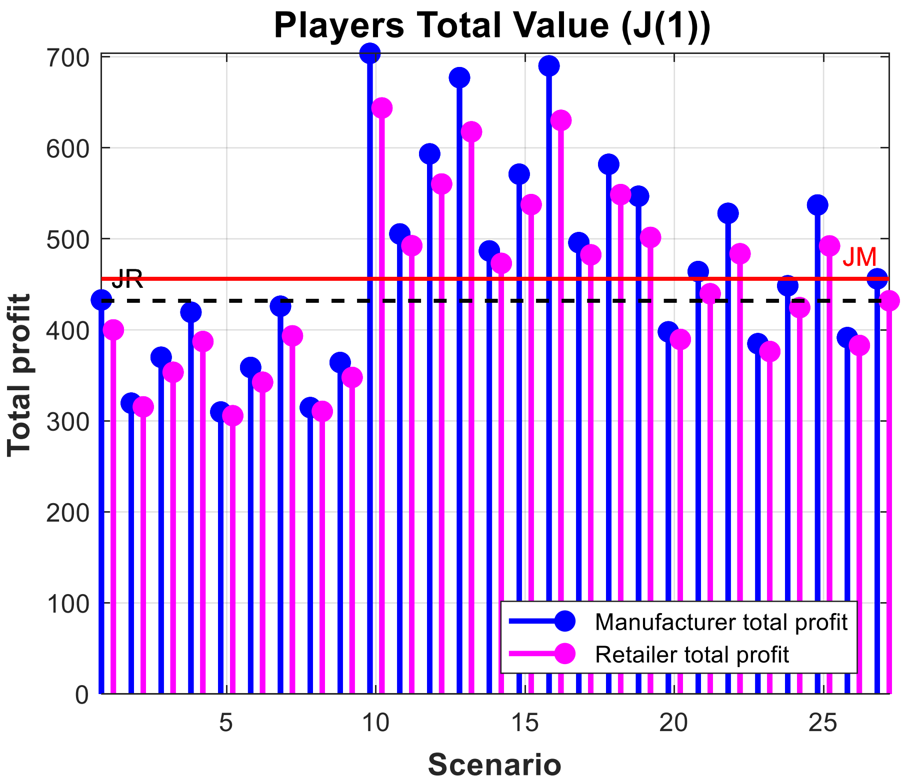


Figure 3: Total expected profits in each scenario

where the horizontal red and black lines show scenario 27 total expected profits for the manufacturer and the retailer respectively. Both players are present in this plot, the blue stems stand for the manufacturer and the magenta for the retailer in each scenario. In this example, scenarios 1-9, 20,23-24, and 26 are detrimental and the others are beneficial for the channel value. Scenario 14 indicates that a cost reduction can compensate for salvage value and preference price reduction. Even though the salvage value and preference price increase cannot compensate for the cost increase, mirrored in scenario 1 and inferring the higher sensitivity to the cost. The next figure (Figure 4) pictures the order quantities generated by different scenarios in which the highest and lowest order quantities, like the total profit, occur in scenarios 10 and 5 respectively. Howbeit the trend of ordering almost follows the same trend.

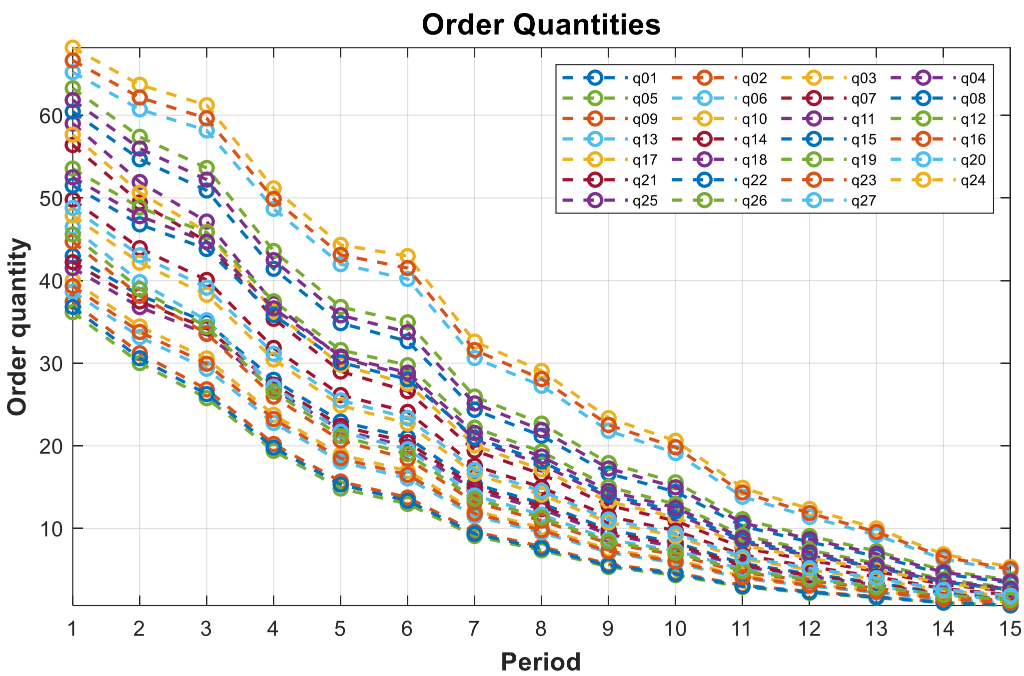


Figure 4: Order quantities in each scenario

5 Concluding Remarks

We have presented a framework to solve multi-periodic manufacturer-retailer games in the presence of a dynamic and stochastic market, which depends on the price history but lacks knowledge about the distribution of the stochastic drivers. All parameters defining the (Stackelberg) game are allowed to be time-dependent. The algorithm solves the distributional-robust (DR) model in a subgame-perfect manner through backward induction. The DR approach creates a weak lower bound on the retailer's expected value.

The players initiate a new contract at each period considering a price history-dependent demand where the structure is sub-game perfect. However, this periodic-contract structure does not allow for any order/production capacity constraint. This problem arises from the way we solve the problem in which a scaled quantity is computed in the main body of the algorithm and in the end the order quantity is rescaled to its actual amount. Furthermore, in the calculation phase, the players decide for each period and move to the next period, without having the chance to modify their policy if needed. This limitation will be of key interest in future research.

We presented an example and evaluated a ± 10 % change in $c^m, s,$ and $K,$ and monitored the model reaction. The outcomes depict that the manufacturer cost alone strongly affects the outcome such that a 10 % increase (decrease) makes a 19.8 % (27.3 %) decrease (increase) in the channel's total profit. The change in preference price results in a 15.9 % (12.8 %) increase (decrease) in the channel value. The salvage value makes the least impact on the value of the channel by a 1.8 % (1.7 %) increase (decrease) followed by the least quantity transition. The sensitivity to cost also emerges in wholesale price where an increase in cost results in a higher increase in the wholesale price. The manufacturer does not face the market stochasticity directly, but he realizes that through the order volume from the retailer. The market is improved by $\Phi > 1$ and whenever $r > K$ the cumulative scaling factor begins to decrease and in $\Phi < 1$ shrinks the market as depicted in the example.

For future research, one may try to incorporate optimal buyback and quantity discount schemes. A very interesting improvement can occur if the players can consider single contracts for the complete time horizon and incorporate realistic constraints not easily incorporated in the multi-periodic contract scheme with the subgame perfect approach.

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