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A LEARNING CURVE MODEL ACCOUNTING FOR THE FLATTENING EFFECT IN PRODUCTION CYCLES

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The authors investigate production cost estimates to identify and model modifications to a prescribed learning curve. Their new model examines the learning rate as a decreasing function over time as opposed to a constant rate that is frequently used. The purpose of this research is to determine whether a new learning curve model could be implemented to reduce the error in cost estimates for production processes. A new model was created that mathematically allows for a "flattening effect," which typically occurs later in the production process. This model was then compared to Wright's learning curve, which is a popular method used by many organizations today. The results showed a statistically significant reduction in error through the measurement of the two error terms, Sum of Squared Errors and Mean Absolute Percentage Error.

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Many manufacturing firms today operate in a fiscally constrained and financially conscious environment. Managers throughout these organizations are expected to maximize the utility from every dollar as budgets and profit margins continue to shrink. Increased financial scrutiny adds greater emphasis on the accuracy of program and project management cost estimates to ensure acquisition programs are sufficiently funded. Cost estimating models and tools used by organizations must be evaluated for their relevance and accuracy to ensure reliable cost estimates. Many of the cost estimating procedures for learning curves were developed in the 1930s (Wright, 1936) and are still in use today as a primary method to model learning. As automation and robotics increasingly replace human touch-labor in the manufacturing process, the current 80-year-old learning curve model may no longer provide the most accurate approach for estimates. New learning curve methods that incorporate automated production and other factors that lead to reduced learning should be examined as an alternative for cost estimators in the acquisition process.

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Since Wright's (1936) original learning curve model was developed, researchers have found other functions to model learning within the manufacturing process (Carr, 1946; Chalmers & DeCarteret, 1949; Crawford, 1944; DeJong, 1957; Towill, 1990; Towill & Cherrington, 1994). The purpose of this research is to address a gap in the literature that fails to account for the nonconstant rate of learning, which results in a flattening effect at the end of production cycles. We will investigate learning curve estimating methodology, develop learning curve theory, and pursue the development of a new estimation model that examines learning at a nonconstant rate.

This research identifies and models modifications to a learning curve model such that the estimated learning rate is modeled as a decreasing learning rate function over time, as opposed to a constant learning rate that is currently in use. Wright's (1936) learning curve model in use today mathematically states that for every doubling of units there will be a constant gain in efficiency. For example, if a manufacturer observes a 10% reduction in labor hours in the time to produce unit 10 from the time to produce unit 5, then it should expect to see the same 10% reduction in labor hours in the time to produce unit 20 from the time to produce unit 10. We propose that

more accurate cost estimates would result if a more flexible exponent were taken into consideration in developing the learning curve model. The proposed general modification would take the form:

$$Cost(x) = Ax^{f(x)} \tag{1}$$

Where:

$Cost(x)$ = cumulative average cost per unit

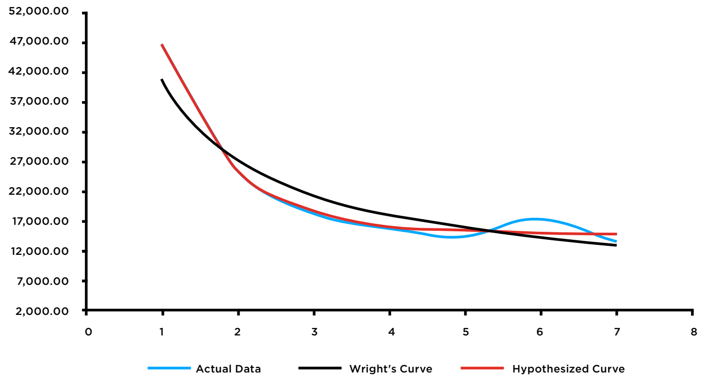
A = theoretical cost to produce the first unit

x = cumulative number of units produced

$f(x)$ = learning curve effect as a function of units produced

The exponent function in Equation 1 will be explored in this article. Figure 1 demonstrates the phenomena this research will examine. The black (flatter) line depicts the traditional curve where learning occurs at a constant rate; the red (steeper) line represents the hypothesized learning structure where the rate of learning changes as a function of the number of units produced; and the blue line represents notional data used to fit the two curves.

FIGURE 1. LEARNING CURVE DEPICTION



To address this research gap, our study aims to model a function that has the added precision of diminishing learning effects over time by introducing a learning curve decay factor that more closely models actual production cycle learning. We will accomplish this by developing a new learning curve model that minimizes the amount of error compared to current estimation models. Learning curves, specifically when estimating the expected cost per unit of complex manufactured items such as aircraft, are frequently modeled with a mathematical power function. The intent of these models is to capture the expected reduction in costs over time due to learning effects, particularly in areas with a high percentage of human touch labor. Typically, as production increases, manufacturers identify labor efficiencies and improve the process. If labor efficiencies are identified, it translates to unit cost savings over time. The general form of the learning curve model frequently used today is based on Wright's theory and is shown in Equation 2. Note that the structure of the exponent b ensures that as the number of units produced doubles, the cost will decrease by a given percentage defined as the learning curve slope (LCS). For example, when LCS is 0.8, then the cost per unit will decrease by 80% between units 2 and 4.

$$Cost(x) = Ax^b \tag{2}$$

Where:

Cost(x) = cumulative average cost per unit

A = theoretical cost to produce the first unit

x = cumulative number of units produced

$b = \frac{\ln Learning\ Curve\ Slope}{\ln 2}$

The cost of a particular production unit is modeled as a power function that decreases at a constant exponential rate. The problem is that the rate of decrease is not likely to be constant over time. We propose that the majority of cost improvements are to be found early in the production process, and fewer revelations are made later as the manufacturer becomes more familiar with the process. As time progresses, the production process should normalize to a steady state and additional cost reductions prove less likely.

> **Our study aims to model a function that has the added precision of diminishing learning effects over time by introducing a learning curve decay factor that more closely models actual production cycle learning.**

For relatively short production runs, the basic form of the learning curve may be sufficient because the hypothesized efficiencies will not have time to materialize. However, when estimating production runs over longer periods of time, the basic learning curve could underestimate the unit costs of those furthermost in the future. The underestimation occurs because the model assumes a constant learning rate, while actual learning would diminish, causing the actuals to be higher than the estimate. Current models may underestimate a significant amount when dealing with high unit cost items such as those in major acquisition programs; a small error in an estimate can be large in terms of dollars. Through the use of curve fitting techniques, a comparison can be made to determine which model best predicts learning within a particular production process. The remainder of this article is organized as follows. A literature review of the most common learning curve processes is presented in the next section, followed by methodology and model formulation. We then provide an in-depth analysis of the learning curve models, followed by future research directions, conclusions, and limitations of this research.

Literature Review

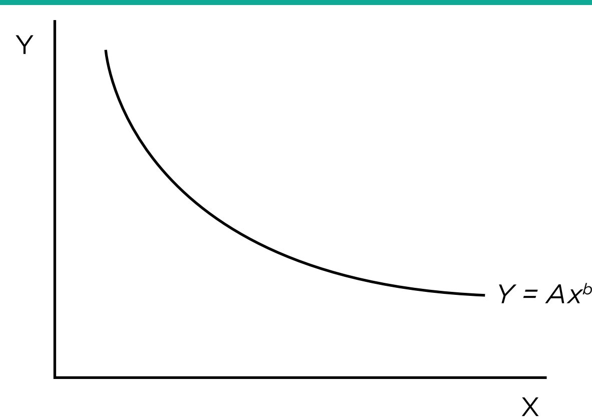
Learning curve research dates back to 1936, when Theodore Paul Wright published the original learning curve equation that predicted the production effects of learning. Wright recognized the mathematical relationship that exists between the time it takes for a worker to complete a single task and the number of times the worker had previously performed that task (Wright, 1936). The mathematical relationship developed from this hypothesis is that as workers complete the same process, they get better at it. Specifically, Wright realized that the rate at which they get better at that task is constant.

The relationship between these two variables is as follows: as the number of units produced doubles, the worker will do it faster by a constant rate. He proposed that this relationship takes the form of:

$$F = N^x \quad \text{or} \quad x = \frac{Log\, F}{Log\, N};$$

"where F = a factor of cost variation proportional to the quantity N. The reciprocal of F then represents a direct percent variation of cost vs. quantity" (Wright 1936). The relationship between these variables can be modified to predict the expected cost of a given unit number in production by multiplying the factor of cost variation by the theoretical cost of the first unit produced—this relationship was stated in Equation 2 and is shown in Figure 2. It is a log linear relationship through an algebraic manipulation. The logarithmic form of this equation (taking the natural log of both sides of the equation) allows practitioners to run linear regression analysis on the data to find what slope best fits the data using a straight line (Martin, n.d.).

FIGURE 2. WRIGHT'S LEARNING CURVE MODEL



Note. (Martin, n.d.)

The goal of using learning curves is to increase the accuracy of cost estimates. Having accurate cost estimates allows an organization to efficiently budget while providing as much operational capability as possible because it can allocate resources to higher priorities. While the use of learning curves focuses on creating accurate cost estimates, learning curves often use the number of labor hours it takes to perform a task. When Wright originated the theory, he proposed the output in terms of time to produce, not production cost. However, many organizations perform learning curve analysis on both production cost and time to produce, depending on the data available. Nevertheless, labor hour cost is relevant because it is based on factors such as labor rates and other associated values. The use of labor hours in learning curve development allows a common comparison over time without the effects of inflation convoluting the results. However, the same goal can be achieved by using inflation-adjusted cost values.

Wright's model has been compared to some of the more contemporary models that have surfaced in recent years since the original learning curve theory was established (Moore et al., 2015). Moore compared the Stanford-B, Dejong, and the S-Curve models to Wright's model to see if any of these functions could provide a more accurate estimate of the learning phenomenon. Both the Dejong and the S-curve models use an incompressibility factor in the calculation. Incompressibility is a factor used to account for the percentage of automation in the production process. Values of the incompressibility factor can range from zero to one where zero is all touch labor and one is complete automation. Moore found that when using an incompressibility factor between zero and 0.1, the Dejong and S-Curve models were more accurate (Moore et al., 2015). In other words, when a production process had very little automation and high amounts of touch labor, the newer learning curve models tended to be more accurate. For all other values of incompressibility, Wright's model was more accurate.

More recently, Johnson (2016) proposed that a flattening effect is evident at the end of the production process where learning does not continue to occur at a constant rate near the end of a production cycle. Using the same models as Moore, Johnson explored the difference in accuracy between Wright's model and contemporary models early in the production process versus later in the production process. He had similar findings to Moore in that Wright's model was most accurate except in cases where the incompressibility factors were extremely low. When the incompressibility factor is low, more touch labor is involved in the process allowing for the possibility of additional learning to occur. He also found that Wright's learning curve was more accurate early in the production process whereas the Dejong and S-Curve models were more accurate later in the production process (Johnson, 2016).

Another key concept in learning curve estimation and modeling is the idea of a forgetting curve (Honious et al., 2016). A forgetting curve explains how configuration changes in the production process can cause a break in learning, which leads to loss of efficiency that had previously been gained. When a configuration change occurs, the production process changes. Changes may include factors such as using different materials, different tooling, adding steps to a process, or might even be attributed to workforce turnover. The new process affects how workers complete their tasks and causes previously learned efficiencies to be lost. If manufacturers fail to take these breaks into account, they may underestimate the total effort needed to produce a product. Honious et al. (2016) found that configuration changes significantly changed the learning curve, and that the new learning curve slope was steeper than the previous steady slope prior to a configuration change. The distinction between pre- and post-configuration change is important to ensure both types of effects are taken into account.

"When a production process had very little automation and high amounts of touch labor, the newer learning curve models tended to be more accurate. For all other values of incompressibility, Wright's model was more accurate.

The International Cost Estimating and Analysis Association (ICEAA) published learning curve training material in 2013. While presenting the basics of learning curve theory, it also presented some rules of thumb for learning. The first rule is that learning curves are steepest when the production process is touch-labor intensive. Conversely, learning curves are the flattest when the production process is highly automated (ICEAA, 2013). Another key piece of information is that adding new work to the process can affect the overall cost. ICEAA states that this essentially adds a new curve for the added work, which increases the original curve by the amount of the new curve (ICEAA, 2013). The equation is as follows:

$$Cost(x) = A_1 x^{b_1} + A_2 (x-L)^{b_2} \tag{3}$$

Where:

$Cost(x)$ = cumulative average cost per unit

A_1 = theoretical cost to produce the first unit prior to addition of new work

x = cumulative number of units produced

L = last unit produced before addition of new work

A_2 = theoretical cost to produce the first unit after addition of new work

$$b_1 = \frac{\ln \textit{Learning Curve Slope prior to additional work}}{\ln 2}$$

$$b_2 = \frac{\ln \textit{Learning Curve Slope prior to additional work}}{\ln 2}$$

(typically same as b_1)

Equation 3 is important to consider when generating an estimate after a major configuration change or engineering change proposal (ECP). For example, while producing the eighth unit of an aircraft, the customer realizes they need to drastically change the radar on the aircraft. Learning has already taken place on the first eight aircraft; the new radar has not yet been installed, and therefore no learning has taken place. To accurately take into account the new learning, the radar would be treated as a second part to the equation, ensuring we account for the learning on the eight aircraft while also accounting for no learning on the new radar.

Lastly, Anderlohr (1969) and Mislick and Nussbaum (2015) write about production breaks and the effects they have on a learning curve. These production breaks can cause a direct loss of learning, which can fully or partially reset the learning curve. For example, a 50% loss of learning would result in a loss of half of the cost reduction that has occurred (ICEAA, 2013). This information is important when analyzing past data to ensure that breaks in production are accounted for.

Thus far, we have laid out the fundamental building blocks for learning curve theory and how they might apply in a production environment. Wright's learning curve formula established the method by which many organizations account for learning during the procurement process. Following Wright's findings, other methods have emerged that account for breaks in production, natural loss of learning over time, incompressibility factors, and half-life analysis (Benkard, 2000). This article adds to the discussion by examining the flattening effect and how various models predict learning at different points in the production process.

When examining learning curve theory and the effects learning has on production, it is critical to understand the production process being estimated. Since Wright established learning curve theory in 1936, factory automation and technology have grown tremendously and continue to grow. Contemporary learning curve methods try to account for this automation. To get the best understanding, we will examine the aircraft industry, specifically how it behaves in relation to the rest of the manufacturing industry.

The aircraft industry has relatively low automation (Kronemer & Henneberer, 1993), especially compared to other industries. Kronemer and Henneberer (1993) state that the aircraft industry is a fairly labor-intensive process with relatively little reliance on automated production techniques, despite it being a high-tech industry. Specifically, they list three main reasons why manufacturing aircraft is so labor-intensive. First, aircraft manufacturers usually build multiple models of the same aircraft, typically for the commercial sector alone. These different aircraft models mean different tooling and configurations are needed to meet the demand of the customer. Second, aircraft manufacturers deal with a very low unit volume when compared to other industries in manufacturing. The final reason

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for low levels of automation is the fact that aircraft are highly complex and have very tight tolerances. To attain these specifications, manufacturers must continue to use highly skilled touch laborers or spend extremely large amounts of money on machinery to replace them (Henneberer & Kronemer, 1993). For these reasons, we should typically see or use low incompressibility factors in the learning curve models when estimating within the aircraft industry.

Although the aircraft industry remains largely unaffected by the shift to machine production from human touch labor, many industries are seeing a rise in the percentage of manufacturing processes that are automated. In a *Wall Street Journal* article posted in 2012, the author showed how companies have been increasing the amount of money spent on machines and software while spending less on manpower. They proposed that part of the reason behind this shift was a temporary tax break "that allowed companies in 2011 to write off 100% of investments in the first year" (Aeppel, 2012). Tax breaks combined with extremely low interest rates provided industry with incentive to invest in future production. Investment in production technology increases the incompressibility factor that should be used when estimating the effects of learning. In a separate article for the *Wall Street Journal*, Kathleen Madigan also pointed out the increase in spending on capital investments in relation to labor. She stated that "businesses had increased their real spending on equipment and software by a strong 26%, while they have added almost nothing to their payrolls" (Madigan, 2011).

Methodology

Model Formulation

Before we can begin the process of developing a new learning curve equation, we need to examine the characteristics of the curve we expected to best fit the data. Our hypothesis is that a learning curve whose slope decreases over time would fit the data better than Wright's curve. To adjust the rate at which the curve flattens, the *b* value from Wright's learning curve, or the exponent in the power function, needs to be adjusted. Specifically, to make the curve move in a flatter direction, the exponent in the power curve must decrease as the number of units produced increases. Initially we modified Wright's existing formula by dividing the exponent by the unit number as shown in Equation 4.

$$Cost(x) = Ax^{b/x} \tag{4}$$

Where:

$Cost(x)$ = cumulative average cost per unit

A = theoretical cost of the first unit

x = cumulative number of units produced

b = Wright's learning curve constant as described in Equation 2

Using Wright's learning curve, b is a negative constant that has a larger magnitude for larger amounts of learning (i.e., as LCS decreases, b becomes more negative). Therefore, in Equation 4, when b is divided by x, the amount of learning is reduced. In fact, the flattening effect is fairly drastic. For example, when applying Equation 4, a standard 80% Wright's learning curve exhibits 90% learning by the second unit and flattens to 97% by the fourth unit. To implement a learning curve that has the flexibility to not flatten as quickly, we instead divide b by $1+x/c$ where c is a positive constant (see Equation 5). The term $1+x/c$ is always greater than 1 and is increasing as x increases; therefore, a flattening effect always occurs (i.e., learning decreases as the number of units produced increases). The choice of the constant c is critical in determining how quickly the learning decreases. For example, when $c = 4$, a standard 80% Wright's learning curve exhibits 86% learning by the second unit and approximately 89% learning by the fourth unit. For the same standard 80% curve when $c = 40$, the learning decreases to 80.9% by the second unit and to 81.6% by the fourth unit. The new equation (which we also refer to as Boone's learning curve hereafter) took the form:

$$Cost(x) = Ax^{b/(1+x/c)} \quad (5)$$

Where:

Cost(x) = cumulative average cost per unit

A = theoretical cost of the first unit

x = cumulative number of units produced

b = Wright's learning curve constant as described in Equation 2

c = decay value (positive constant)

The function that modifies the traditional learning curve exponent in Equation 5—i.e., $1+x/c$ – has a key characteristic—ensures that the rate of learning associated with traditional learning curve theory decreases as each additional unit is produced. Specifically, $1+x/c$ is always greater than 1 since x/c is always positive. Note that c is an estimated parameter and x increases as more units are produced, so the term x/c is decreasing. When c is large, Boone's learning curve would effectively behave like Wright's learning curve. For example, if the fitted value of c is 5,000, then $1+x/c$ equals 1.0002 after the first unit has been produced and 1.004 after the twentieth unit has been produced. This equates to a decrease in the learning rate of the traditional theory (i.e., b) of less than 0.07%. More formally, as c goes to infinity, $b/(1 + x/c)$ goes to b.

Note that the previous discussion assumed that b was the same value for both Wright's and Boone's learning curve to help demonstrate the flattening effect. In practice, nothing precludes each of the learning curves from having different b values. For instance, if we desire a learning curve that possesses more learning early in production and less learning later in production (compared to Wright's curve), then the b parameters could be different—this was shown in Figure 1. In this case, Boone's curve would have a b value less than Wright's curve (i.e., a more negative value representing more learning). Then the flattening effect of dividing by $1+x/c$ as production increases would eventually result in a curve with less learning than Wright's curve. For example, consider an 80% Wright's learning curve and a Boone's learning curve that initially has 70% learning and a decay value of 8; by the eighth production unit, Boone's curve would be at 82% learning.

Population and Sample

To test the new learning curve in Equation 5, we looked at quantitative data from several DoD airframes to gain a comprehensive understanding of how learning affects the cost of lot production. The costs used in this analysis are the direct lot costs and exclude costs for items such as Research, Development, Test, & Evaluation (RDT&E), support items, and spares. These data specifically include Prime Mission Equipment (PME) only as these costs are directly related to labor, and can be influenced directly through learning. To ensure we are comparing properly across time, we used inflation and rate-adjusted PME cost data for each production lot of the selected aircraft systems. The PME cost data were adjusted using escalation rates for materials using Office of the Secretary of Defense (OSD) rate tables, when applicable. We used data from fighter, bomber, and cargo aircraft, as well as missiles and munitions. This diverse dataset allowed comparison among multiple systems in different production environments.

Data Collection

Data used were gathered from the Cost Assessment Data Enterprise (CADE). CADE is a resource available to DoD cost analysts that stores historical data on weapon systems. Some of the older data also came from a DoD research library in the form of cost summary reports. The data used can be broken out by Work Breakdown Structure (WBS) or Contract Line Item Number (CLIN). For this research, the PME cost data were broken out by WBS element, then rolled up into top line, finished product elements and used for the regression analysis. In total, 46 weapon system platforms were analyzed (see Table).

TABLE. RESULTS

PROGRAM	Wright's SSE	Wright's MAPE	Boone SSE	Boone MAPE	SSE Difference	MAPE Difference
Platform A	2.78E+08	5.3%	2.17E+08	4.8%	-22%	-10%
Platform B	4.88E+08	5.4%	4.90E+08	5.6%	0%	5%
Platform C	1.58E+07	10.8%	4.51E+05	2.1%	-97%	-80%
Platform D	6.56E+10	22.1%	6.02E+10	24.5%	-8%	11%
Platform E	1.14E+09	6.2%	1.10E+09	5.6%	-4%	-9%
Platform F	1.94E+06	4.6%	1.95E+06	4.6%	0%	1%
Platform G	7.14E+08	13.6%	6.28E+08	12.9%	-12%	-5%
Platform H	5.49E+06	4.6%	5.00E+06	4.0%	-9%	-13%
Platform I	1.30E+09	18.6%	1.21E+09	23.8%	-7%	28%
Platform J	7.90E+06	3.9%	6.12E+06	3.6%	-23%	-8%
Platform K	2.18E+07	6.0%	7.48E+06	3.2%	-66%	-47%
Platform L	1.06E+08	9.6%	1.05E+08	9.7%	-1%	0%
Platform M	1.49E+07	10.7%	1.48E+07	13.4%	0%	26%
Platform N	9.92E+08	16.3%	7.67E+07	10.0%	-92%	-39%
Platform O	1.81E+08	13.0%	1.78E+08	14.0%	-1%	7%

TABLE. RESULTS (CONTINUED)

PROGRAM	Wright's SSE	Wright's MAPE	Boone SSE	Boone MAPE	SSE Difference	MAPE Difference
Platform P	1.71E+07	6.3%	7.96E+06	4.7%	-53%	-26%
Platform Q	8.00E+06	10.1%	4.11E+06	7.6%	-49%	-25%
Platform R	1.48E+09	18.8%	1.31E+09	18.3%	-12%	-2%
Platform S	5.00E+07	6.2%	4.89E+07	6.1%	-2%	-2%
Platform T	4.01E+07	11.1%	5.45E+06	6.5%	-86%	-41%
Platform U	1.19E+06	8.8%	1.34E+06	7.8%	13%	-11%
Platform V	1.60E+09	10.6%	1.74E+02	0.0%	-100%	-100%
Platform W	1.39E+09	6.4%	1.38E+09	6.4%	-1%	0%
Platform X	7.61E+08	18.1%	3.18E-01	0.0%	-100%	-100%
Platform Y	6.81E+05	3.3%	1.10E+06	4.1%	4.1%	26%
Platform Z	2.12E+06	7.5%	1.57E+06	6.8%	6.8%	-9%
Platform AA	2.66E+07	5.0%	2.73E+07	5.5%	5.5%	10%
Platform AB	1.48E+09	18.8%	1.31E+09	18.3%	18.3%	-2%
Platform AC	3.81E+07	5.9%	2.45E+07	4.5%	4.5%	-24%
Platform AD	3.03E+11	21.9%	1.34E+11	16.7%	16.7%	-24%
Platform AE	1.04E+09	10.0%	1.03E+09	10.3%	10.3%	3%
Platform AF	9.01E+05	5.1%	6.94E+05	4.0%	4.0%	-23%
Platform AG	8.20E+06	5.9%	1.77E+06	3.7%	3.7%	-37%
Platform AH	6.40E+06	10.8%	6.11E+06	9.8%	9.8%	-9%
Platform AI	1.47E+07	8.2%	5.22E+06	5.4%	5.4%	-35%
Platform AJ	4.95E+07	10.0%	4.98E+07	10.7%	10.7%	6%
Platform AK	5.99E+07	19.8%	5.69E+07	20.4%	20.4%	3%
Platform AL	1.50E+10	12.9%	1.43E+10	14.8%	14.8%	15%
Platform AM	1.29E+07	5.5%	1.28E+07	5.4%	5.4%	-3%
Platform AN	4.99E+06	3.7%	3.02E+06	3.4%	3.4%	-9%
Platform AO	9.63E+07	21.9%	9.45E+07	21.5%	21.5%	-2%
Platform AP	1.18E+06	3.1%	1.22E+06	3.4%	3.4%	7%
Platform AQ	2.77E+03	3.4%	1.19E-05	0.0%	0.0%	-100%
Platform AR	1.84E+06	17.3%	1.82E+06	18.0%	18.0%	4%
Platform AS	3.27E+06	1.3%	1.09E+00	0.0%	0.0%	-100%
Platform AT	1.98E+03	2.8%	1.19E+03	1.7%	1.7%	-40%

Note. The actual names of each system and contractor have been removed and replaced with a designator of Platform A…Platform AT.

Analysis

Regression analysis was used to test which learning curve model was most accurate in estimating the data. The goal is to minimize the sum of squared errors (SSE) in the regression to examine how well a model estimates a given set of data. The SSE is calculated by taking the vertical distance between the actual data point (in this case lot midpoint PME cost) and the prediction line (or estimate) (Mislick & Nussbaum, 2015). This error term is then squared and the sum of these squared error terms is the value for comparing which model is a more accurate predictor. However, since an extra parameter is available in fitting the regression for the new model, it should be able to maintain or decrease the SSE in most cases. As previously mentioned, as the decay parameter in Equation 5 approaches infinity, Boone's learning curve approaches Wright's learning curve formula. With this in mind, we also examined the Mean Absolute Percentage Error (MAPE). MAPE takes the same error term as the SSE calculation but then divides it by the actual value; then the mean of the absolute value of these modified error terms is calculated. By examining the error in terms of a percentage, comparisons between different types and sizes of systems are more robust. If Boone's curve reduces both SSE and MAPE when compared to the SSE and MAPE of Wright's curve, it would indicate the new model may be better suited for modeling learning and the associated costs.

As stated previously, Wright's learning curve is suitable for a log-log model. A log-log model is used when a logarithmic transformation of both sides of an equation results in a model that is linear in the parameters. As Wright

proposed, this linear transformation occurs because learning happens at a constant rate throughout the production cycle. If learning happens at a non-constant rate (as in Boone's learning curve), then the curve in log-log space would no longer be linear. This constraint means typical linear regression methods would not be suitable for estimating Boone's learning curve; therefore, we had to use nonlinear methods to fit these curves.

Specifically, we used the Generalized Reduced Gradient (GRG) nonlinear solver package in Excel to minimize the SSE by fitting the *A*, *b*, and *c* parameters from Equation 5. To use this solver, bounds for the three parameters had to be established. These are values that are easy to obtain for any dataset, as they are provided by Microsoft Excel when fitting a power function or by using the "linest()" function in Excel. We used this as a starting point because Wright's curve is currently used throughout the DoD. For the *A* variable, the lower bound was one-half of Wright's *A* and the upper bound was 2 times Wright's *A*. These values were used to give the solver model a wide enough range to avoid limiting the value but small enough to ease the search for the optimal values. Neither of these limits was found to be binding. For the exponent parameter *b*, we chose values between 3 and -3 times Wright's exponent value. In theory, the value of the exponent should never go above 0 due to positive learning leading to a decrease in cost, but in practice some datasets go up over time and we wanted to be able to account for those scenarios, if necessary. Again, these values between 3 and -3 times Wright's exponent value were never found to be binding limits for the model. Finally, for the decay parameter *c*, fitted values were bounded between 0 and 5,000; the 5,000 upper bound was found to be a binding constraint in the solver on several occasions. In practice, analysts could bound the value as high as possible to reduce error, but in the case of this research, we used 5,000 as no significant change was evidenced from 5,000 to infinity—relaxing this bound would have only further reduced the SSE for Boone's learning curve.

Statistical Significance Testing

Once the SSE and MAPE values were calculated for each learning curve equation, we tested for significance to determine whether the difference between the error values for the two equations were statistically different.

Specifically, we conducted t-tests on the differences in error terms between Wright's and Boone's learning curve equations. This *t*-test was conducted for both SSE and MAPE values separately. A nonsignificant *t*-test indicates no statistically significant difference between the two learning curves.

Analysis and Results

The Table shows the SSE and MAPE values for both Wright's and Boone's learning curve for each system in the dataset. The last two columns are the percentage difference in SSE and MAPE between the two learning curve methods. This percentage was calculated by taking the difference of Boone's error term minus Wright's error term divided by Wright's error term. Negative values represent programs where Boone's learning curve had less error than Wright's learning curve, and positive values represent programs where Wright's curve had less error than Boone's curve.

Based on this analysis, we observed that Boone's learning curve reduced the SSE in approximately 84% of programs and reduced MAPE in 67% of programs. The mean reduction of SSE and MAPE was 27% and 17%, respectively. As previously mentioned, these values were based on using both learning curve equations to minimize the SSE for each system in the dataset. This is standard practice in the DoD as prescribed by the U.S. Government Accountability Office (GAO, 2009) *Cost Estimating and Assessment Guide* when predicting the cost of subsequent units or subsequent lots.

> **"The incompressibility factor represents the amount of automation in the production process, which limits how much learning can occur (Badiru et al., 2013).**

We conducted additional tests to determine if a statistical difference existed between the means of both curve estimation techniques. On average, programs estimated using Boone's learning curve had a lower error rate (M = 4.73, SD = 2.15) than those estimated using Wright's learning curve (M = 8.64, SD = 4.55). Additionally, the difference between these two error rates expressed as a percentage and compared to a hypothesized value of 0 (no difference) was significant, $t(46) = -4.87$, $p < .0001$, and represented an effect of $d = 1.10$. We then applied the same test to the difference in the MAPE values from Boone's learning curve and Wright's learning curve. On average, programs estimated using Boone's learning curve had a lower MAPE value (M = .08, SD = .07) than those estimated using Wright's MAPE value (M =

.10, SD = .06). The difference between these two estimates has a mean of -.17, which translates to Boone's curve reducing MAPE by 17% more on average. Additionally, the difference between these two error rates expressed as a percentage and compared to a hypothesized value of 0 (no difference) was significant, $t(46) = -3.48$, $p < .0005$, and represented an effect of $d = .22$. The results indicate that in both SSE and MAPE, Boone's learning curve reduced the error, and that each of those values was statistically significant when using an alpha value of 0.05.

Discussion

As stated previously, an average of a 27% reduction in the SSE resulted from among the 46 programs analyzed. These results were statistically significant. Also, a 17% reduction in the MAPE resulted from among the programs analyzed, which was also found to be statistically significant. Based on these results, we can conclude that Boone's learning curve equation was able to reduce the overall error in cost estimates for our sample. This information is critical to allow the DoD to calculate more accurate cost estimates and better allocate its resources. These conclusions help answer our three guiding research questions. Specifically, we were looking for the point where Wright's model became less accurate than other models. We found that adding a decay factor caused the learning curve to flatten out over time, which resulted in less error than Wright's model. Additionally, we found that Boone's learning curve was more accurate throughout the entire production process, not just during the tail end when production was winding down. Boone's learning curve was steeper during the early stages of production when it's hypothesized that the most learning occurs. Toward the end of the

> **"Future research should identify decay values for different types of weapon systems—similar to the way learning curve rates are established for different categories of programs.**

production process, Boone's curve flattens out more than Wright's curve, supporting our contention that learning toward the end of the production cycle yields diminishing returns. While Wright's curve assumes constant learning throughout the entire process, Boone's curve treats learning in a nonlinear fashion that slows down over time. By reducing the error in the estimates and properly allocating resources, the DoD could potentially minimize risk for all parties involved. The benefit of Boone's learning curve is accuracy in the estimation process. If labor estimates aren't accurate in the production process, risks escalate, such as schedule slip, cost overruns, and increased costs for all involved. Accuracy in the cost estimate should be the goal of both the contractor and government, thereby facilitating the acquisition process with better data.

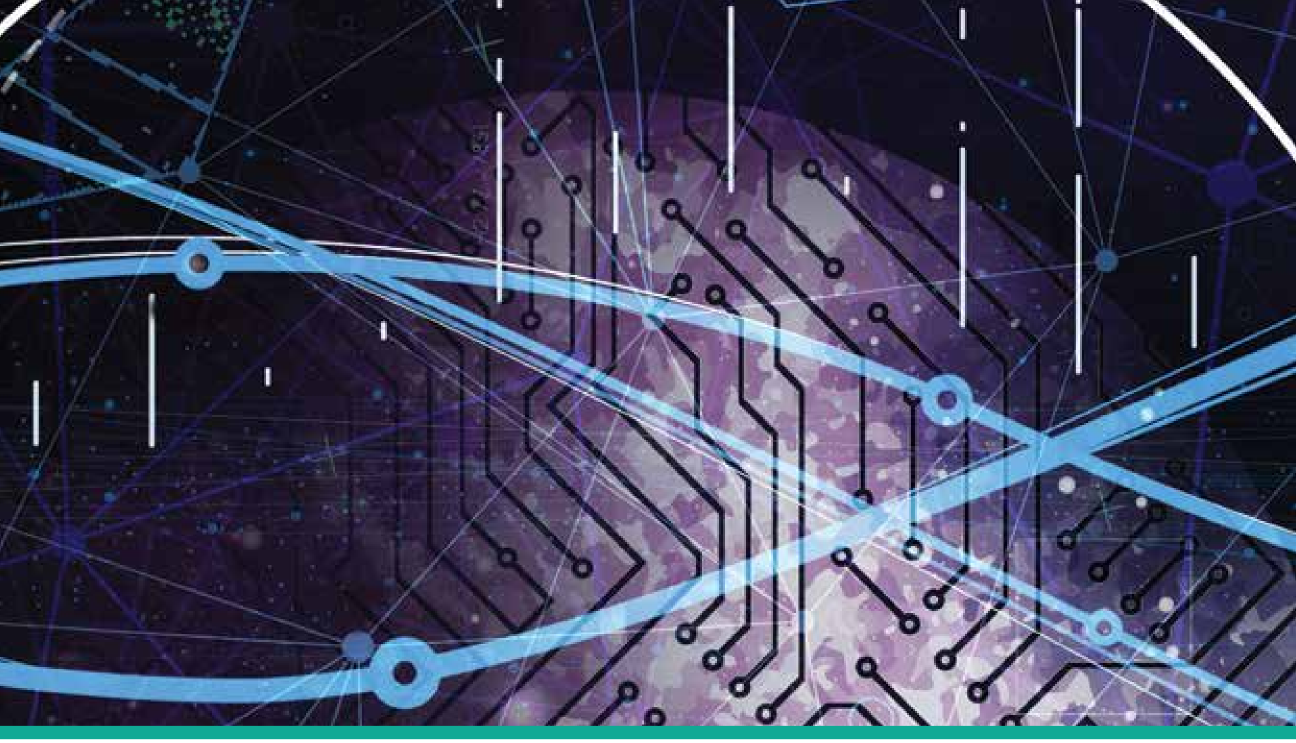
Limitations

One limitation of this study is that all 46 of the weapon systems analyzed were U.S. Air Force systems. While the list included many platforms spanning decades, we hesitate to draw conclusions outside of the U.S. Air Force without further research and analysis. That said, we see no reason our model wouldn't apply equally well in any aircraft production environment, both within and outside the DoD. Another limitation in this research is the use of PME cost as opposed to labor hours. Labor-hour data are not readily available across many platforms, which led to the use of PME cost. Contractor data provided to the government normally come in the form of lots, which is the lowest level tracked by cost estimators. To compare learning curves across multiple platforms, the same level of analysis is required to ensure a fair comparison. Future research should attempt to examine data at the individual level of analysis between systems and exclude those where only lot data are available. Because there are inherently less lots than units, this may affect how the equation behaves when applied at the unit level. For this research, we used the lot midpoint formula/method (Mislick & Nussbaum, 2015), but further research should be conducted to evaluate the performance of Boone's learning curve with unitary data. Finally, we only performed a comparison to Wright's learning curve since that is a primary method of estimation in the DoD. A comparison with other learning curve models may yield different results, although previous research found those curves were not statistically better than Wright's.

Recommendations for Future Research

Data outside of the U.S. Air Force should be examined to test whether this equation applies broadly to programs, and not just to Air Force programs. Also, conducting the analysis with unitary data could confirm that this works for predicting subsequent units as well as subsequent lots, while reducing error over Wright's method. We also made an attempt to select weapon systems that had minimal automation in the production process. However, DeJong's Learning Formula is another derivation from Wright's original in which an incompressibility factor is introduced. The incompressibility factor represents the amount of automation in the production process, which limits how much learning can occur (Badiru et al. 2013). Other models such as the S-Curve model (Carr, 1946) and a more recent version (Towill, 1990; Towill & Cherrington, 1994) also account for some form of incompressibility. Additional research could also include modifications to Boone's formula to try and further reduce the error types listed in this research. Furthermore, fitting Boone's curve in this analysis was based on past data whereas cost estimates are used to project future costs. Therefore, future research should identify decay values for different types of weapon systems—similar to the way learning curve rates are established for different categories of programs. Lastly, further research could examine whether the incorporation of multiple learning curve equations at different points in the production process would be beneficial to reducing additional error in the estimates.

We developed a new learning curve equation utilizing the concept of learning decay. This equation was tested against Wright's learning equation to see which equation provided the least amount of error when looking at both the SSE and MAPE. We found that Boone's learning curve reduced error in both cases and that this reduction in error was statistically significant. Follow-on research in this field could lead to further discoveries and allow for broader use of this equation in the cost community.



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