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Connected end anti-fuzzy equitable dominating set in anti-fuzzy graphs

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Abstract

In this paper, the notion of connected end anti-fuzzy equitable dominating set of an anti-fuzzy graph is discussed. The connected end anti-fuzzy equitable domination number for some standard graphs are obtained. The relation between anti-fuzzy equitable domination number, end anti-fuzzy equitable domination number and connected end anti-fuzzy equitable domination number are established. Theorems related to these parameters are stated and proved.

Keywords: dominating set; end anti-fuzzy equitable dominating set; connected end anti-fuzzy equitable dominating set

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1. Introduction

A graph is an advantageous method for representing the data including connection between objects. The objects are represented by nodes and relations by arcs. Whenever there is uncertainty or vagueness in the description of items or in its connections or in both, it is common that we have to plan an anti-fuzzy graph model. A fuzzy set, as a superset of a crisp set, owes its origin to the work of Zadeh [14] in 1965 that has been introduced to deal with uncertainty. M. Akram [1] defined the concept of anti-fuzzy graph structures in 2012. A. Somasundaram and S.Somasundaram [12] presented several types of domination parameters such as independent domination, total domination, connected domination and domination in Cartesian product and composition of fuzzy graphs. R.Muthuraj and A. Sasireka [7] introduced domination in anti-fuzzy graphs. The concept of equitable domination in graphs was introduced by Swaminathan and Dharmalingam [13]. The end equitable domination number in graph has introduced by J.H.Hattingh and M.H.Henning [8]. Further results were extended by Murthy and Puttaswamy [10]. Some works in extension of fuzzy graphs can be found in [5, 6, 9]. S.Firthous Fatima and K.Janofer [2, 3, 4] introduced the concept of anti-fuzzy equitable dominating set, connected anti-fuzzy equitable dominating set and end anti-fuzzy equitable dominating set of an anti-fuzzy graphs. In this paper, the connected end anti-fuzzy equitable domination set of an anti-fuzzy graph is introduced. The connected end anti-fuzzy equitable domination number of an anti-fuzzy graphs is also obtained.

2. Preliminaries

Definition 2.1[1] A fuzzy graph is said to be an anti-fuzzy graph with a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$, where for all $u, v \in V$, we have $\mu(u,v) \geq \sigma(u) \vee \sigma(v)$ and it is denoted by $G_{AF}(\sigma,\mu)$ or $G(\sigma,\mu)$.

Definition 2.2 [1] The order p and size q of an anti-fuzzy graph $G_{AF} = (\sigma,\mu)$ are defined to be $p = \sum_{u\in V} \sigma(u)$ and $q = \sum_{uv\in E} \mu(u,v)$. It is denoted by $O(G)$ and $S(G)$.

Note 2.1 In all the examples, σ is chosen suitably and the function μ considered as reflexive and symmetric and G is an undirected anti-fuzzy graph.

Definition 2.3 An anti-fuzzy graph G_{AF} is said to be bipartite if the vertex set V can be partitioned into two sets σ_1 on V_1 and σ_2 on V_2 such that $\mu(v_1,v_2) = 0$ if $(v_1,v_2) \in V_1 \times V_1$ or $(v_1,v_2) \in V_2 \times V_2$.

Definition 2.4 A bipartite anti-fuzzy graph G_{AF} is said to be complete bipartite anti-fuzzy graph if $\mu(v_1,v_2) = \sigma(v_1) \vee \sigma(v_2)$ for all $v_1 \in V_1$ and $v_2 \in V_2$ and is denoted by K_{σ_1,σ_2}.

Definition 2.5 A path in an anti-fuzzy graph G_{AF} is a sequence of distinct vertices $u_0, u_1, u_2, \dots, u_n$ such that $\mu(u_{i-1}, u_i) = \sigma(u_{i-1}) \vee \sigma(u_i)$, $1 \leq i \leq n$, $n > 0$ is called the length of the path. The path in an anti-fuzzy graph is called an anti-fuzzy cycle if $u_0 = u_n$, $n \geq 3$.

Definition 2.6 An anti-fuzzy graph G_{AF} is said to be cyclic if it contains at least one anti-fuzzy cycle, otherwise it is called acyclic.

Definition 2.7 An anti-fuzzy graph G_{AF} is said to be connected if there exists at least one path between every pair of vertices. A connected acyclic anti-fuzzy graph is said to be an anti-fuzzy tree.

Definition 2.8 [4] Let G_{AF} be an anti-fuzzy graph and let $u, v \in V$. If $\mu(u,v) = \sigma(u) \vee \sigma(v)$ then u dominates v (or v dominates u) in G_{AF}. A set $D \subseteq V$ is said to be a dominating set of an anti-fuzzy graph G_{AF} if for every vertex $v \in V - D$ there exists $u \in D$ such that u dominates v.

Definition 2.9 [4] A dominating set D of an anti-fuzzy graph G_{AF} is called a minimal dominating set if there is no dominating set D' such that $D' \subset D$.

Definition 2.10 [5] The maximum scalar cardinality taken over all minimal dominating set is called anti-fuzzy domination number of an anti-fuzzy graph G_{AF} and is denoted by $\gamma_{AFG}^{d}(G_{AF})$.

Definition 2.11 [2] Let G_{AF} be an anti-fuzzy graph. Let v_1 and v_2 be two vertices of G_{AF}. A subset D of V is called a anti-fuzzy equitable dominating set if every $v_2 \in V - D$ there exist a vertex $v_1 \in D$ such that $v_1 v_2 \in E$ and $|d(v_1) - d(v_2)| \le 1$ where $d(v_1)$ denotes the degree of vertex v_1 and $d(v_2)$ denotes the degree of vertex v_2 with $\mu(v_1, v_2) = \sigma(v_1) \vee \sigma(v_2)$.

Definition 2.12 [2] An anti-fuzzy equitable dominating set D of an anti-fuzzy graph G_{AF} is called a minimal anti-fuzzy equitable dominating set if there is no anti-fuzzy equitable dominating set D' such that $D' \subset D$. The maximum scalar cardinality taken over all minimal anti-fuzzy equitable dominating set is called anti-fuzzy equitable domination number and is denoted by γ_{AFG}^{ed}.

Definition 2.13 [4] An anti-fuzzy equitable dominating set S of a connected anti-fuzzy graph G_{AF} is called the end anti-fuzzy equitable dominating set if S contains all the terminal vertices.

Definition 2.14 [4] The maximum scalar cardinality taken over all minimal end anti-fuzzy equitable dominating set is called end anti-fuzzy equitable domination number of G_{AF} and it is denoted by γ_{AFG}^{eed}.

Definition 2.15 If for each $x \in V - S$ there exist a vertex $y \in S$ such that $xy \in E(G_{AF})$ and either one of the vertex x or y is with degree k and other vertex is with degree $k + 1$, then G_{AF} is called a bi-regular anti-fuzzy graph.

3. Connected end anti-fuzzy equitable dominating set

Definition 3.1 An end anti-fuzzy equitable dominating set S of an anti-fuzzy graph G_{AF} is called the connected end anti-fuzzy equitable dominating set (CEAFED-set) if induced anti-fuzzy subgraph $< S >$ is connected.

Definition 3.2 The maximum scalar cardinality taken over all minimal connected end anti-fuzzy equitable dominating set is called connected end anti-fuzzy equitable domination number of G_{AF} and it is denoted by γ_{AFG}^{ceed}.

Example 3.3

Consider the following anti-fuzzy graph G_{AF},

In the anti-fuzzy graph G_{AF}, given in figure 3.1, the minimal connected end anti-fuzzy equitable dominating sets are $S_1 = \{v_3, v_4, v_5, v_6, v_7\}$ and $S_2 = \{v_2, v_3, v_4, v_5, v_6\}$.
The scalar cardinality of $S_1 = |\{v_3, v_4, v_5, v_6, v_7\}| = 0.4 + 0.2 + 0.3 + 0.6 + 0.6 = 2.1$
The scalar cardinality of $S_2 = |\{v_2, v_3, v_4, v_5, v_6\}| = 0.5 + 0.4 + 0.2 + 0.3 + 0.6 = 2$
$\gamma_{AFG}^{ceed} = \max\{|S_1|, |S_2|\} = \max\{2.1, 2\} = 2.1$

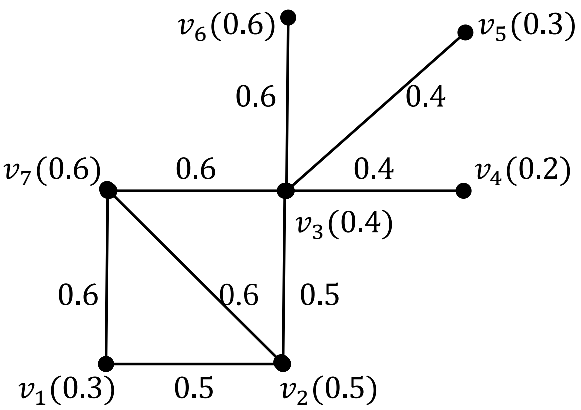

Figure 3.1: Anti-fuzzy graph G_{AF}

Therefore, connected end anti-fuzzy equitable domination number is $\gamma_{AFG}^{ceed} = 2.1$ corresponding to the connected end anti-fuzzy equitable dominating set S_1.

Theorem 3.4
Let G_{AF} be any connected anti-fuzzy graph. Then
$$\gamma_{AFG}^{d}(G_{AF}) \leq \gamma_{AFG}^{ed}(G_{AF}) \leq \gamma_{AFG}^{eed}(G_{AF}) \leq \gamma_{AFG}^{ceed}(G_{AF}).$$
Proof:
Let G_{AF} be any connected anti-fuzzy graph.
Let $S \subseteq V(G_{AF})$ be any γ_{AFG}^{ceed} −set in G_{AF}.
Then obviously, S is also an end anti-fuzzy equitable dominating set in G.
Therefore, $\gamma_{AFG}^{eed}(G_{AF}) = |S| \leq \gamma_{AFG}^{ceed}(G_{AF})$ (1)
Suppose let S' be any γ_{AFG}^{eed} −set of G_{AF}.
By definition of end anti-fuzzy equitable dominating set, S' is also an anti-fuzzy equitable dominating set of G_{AF}.
Therefore, $\gamma_{AFG}^{ed}(G_{AF}) = |S'| \leq \gamma_{AFG}^{eed}(G_{AF})$ (2)
We know that, every anti-fuzzy equitable dominating set is an anti-fuzzy dominating set. Therefore, $\gamma_{AFG}^{d}(G_{AF}) \leq \gamma_{AFG}^{ed}(G_{AF})$ (3)
Hence from (1), (2) and (3), we have

$$\gamma_{AFG}^{d}(G_{AF}) \leq \gamma_{AFG}^{ed}(G_{AF}) \leq \gamma_{AFG}^{eed}(G_{AF}) \leq \gamma_{AFG}^{ceed}(G_{AF}).$$

Remarks 3.5

The equality of theorem 3.4 can be hold when the anti-fuzzy graph G has no isolated vertices. For example, anti-fuzzy cycle and complete anti-fuzzy graphs can hold the equality condition.

Theorem 3.6

For any connected anti-fuzzy graph G_{AF}, $\gamma_{AFG}^{ced}(G_{AF}) \leq \gamma_{AFG}^{ceed}(G_{AF})$.

Proof:

Let $S \subseteq V$ be the minimum connected anti-fuzzy equitable dominating set of a connected anti-fuzzy graph G_{AF}. Then S is an anti-fuzzy dominating set of G and the induced anti-fuzzy subgraph $< S >$ is connected.

Therefore, S is also a connected anti-fuzzy dominating set.

Clearly, any connected end anti-fuzzy equitable dominating set is also connected equitable dominating set.

Hence, $\gamma_{AFG}^{ced}(G_{AF}) \leq \gamma_{AFG}^{ceed}(G_{AF})$.

Theorem 3.7

For any k-regular anti-fuzzy graph for $k > 1$ then $\gamma_{AFG}^{ced}(G_{AF}) = \gamma_{AFG}^{ceed}(G_{AF})$

Proof:

Let us assume that G_{AF} be a k-regular anti-fuzzy graph.

Then each vertex of G_{AF} has a same degree k.

Let S be the minimal connected anti-fuzzy equitable dominating set of G_{AF}

Then cardinality of $S = \gamma_{AFG}^{ced}(G_{AF})$.

If $u \in V - S$ then S is connected anti-fuzzy equitable dominating set, then there exists $v \in S$ and uv be the effective edge, also $d(u) = d(v) = k$.

Therefore $|d(u) - d(v)| = |k - k| = 0 \leq 1$.

Hence S is a connected end anti-fuzzy equitable dominating set of G such that

$$\gamma_{AFG}^{ced}(G_{AF}) \geq \gamma_{AFG}^{ceed}(G_{AF}) \qquad (1)$$

By theorem 3.6, we have $\gamma_{AFG}^{ced}(G_{AF}) \leq \gamma_{AFG}^{ceed}(G_{AF})$. (2)

Hence, from (1) and (2),

$$\gamma_{AFG}^{ced}(G_{AF}) = \gamma_{AFG}^{ceed}(G_{AF})$$

Corollary 3.8

Let G_{AF} be $(k, k+1)$ bi-regular anti-fuzzy graph. Then, $\gamma_{AFG}^{ced}(G) = \gamma_{AFG}^{ceed}(G)$.

Proof:

By theorem 3.6, $\gamma_{AFG}^{ced}(G) \leq \gamma_{AFG}^{ceed}(G)$. Now, let S be minimum connected end anti-fuzzy equitable set of $(k, k+1)$ bi-regular anti-fuzzy graph. By the definition, the connected end anti-fuzzy equitable dominating set S is also an anti-fuzzy equitable dominating set and $\langle S \rangle$ is connected, since G is $(k, k+1)$ bi-regular anti-fuzzy graph. Therefore, S is also a connected end anti-fuzzy equitable dominating set.

Theorem 3.9

Let G_{AF} be an anti-fuzzy graph with n vertices then $\gamma_{AFG}^{ceed}(G) = \gamma_{AFG}^{eed}(G)$ if and only if G_{AF} has no end vertex and there is atleast one vertex $v \in V$ adjacent to $(n-1)$ vertices in G_{AF}.

Proof:

Let G_{AF} be an connected anti-fuzzy graph with n vertices and without end vertex.

Then, every vertex adjacent to atleast two vertices and there exists a one vertex say $u \in V(G_{AF})$ adjacent to $(n-1)$ vertices, then the set $S = \{u\}$ is connected end anti-fuzzy equitable dominating set G.

By theorem 3.4, we get $\gamma_{AFG}^{eed}(G_{AF}) = \gamma_{AFG}^{ceed}(G_{AF})$.

Conversely, suppose G_{AF} is connected anti-fuzzy graph and $\gamma_{AFG}^{ceed}(G_{AF}) = \gamma_{AFG}^{eed}(G_{AF})$ then G has no end vertex and there is $S = \{v\}$ which is connected end anti-fuzzy equitable dominating set. Therefore atleast any one vertex adjacent to $(n-1)$ vertices in G.

Corollary 3.10

Let G_{AF} be an anti-fuzzy cycle with order p then $\gamma_{AFG}^{ceed}(G_{AF}) = p - \max\{min_{uv\in E}\{\sigma(u), \sigma(v)\}\}$.

Proof:

Since $\gamma_{AFG}^{ced}(G_{AF}) = p - \max\{min_{uv\in E}\{\sigma(u), \sigma(v)\}\}$ and $S = V - \{u, v\}$ is any subset of the vertices on the anti-fuzzy cycle G_{AF} such that u and v are adjacent vertices. Clearly S is connected end anti-fuzzy equitable set of G it means $\gamma_{AFG}^{ceed}(G_{AF}) \leq p - \max\{min_{uv\in E}\{\sigma(u), \sigma(v)\}\}$ and by the theorem 3.6, we have, $p - \max\{min_{uv\in E}\{\sigma(u), \sigma(v)\}\} = \gamma_{AFG}^{ceed}(G_{AF})$.

Hence $\gamma_{AFG}^{ceed}(G) = p - \max\{min_{uv\in E}\{\sigma(u), \sigma(v)\}\}$.

Theorem 3.11

For any complete bipartite anti-fuzzy graph then

$$\gamma_{AFG}^{ced}(G_{AF}) = \gamma_{AFG}^{ceed}(G_{AF}) = \begin{cases} \max\limits_{u\in V_1}\{\sigma(u)\} + \max\limits_{v\in V_2}\{\sigma(v)\}, & |d(u)+d(v)| \leq 1 \\ p+q & , |d(u)+d(v)| > 1 \end{cases}$$

Proof:

Case (1) : If $G \cong K_{m,n}$ and $|d(u)+d(v)| > 1$ for all $u \in V_1$ and $v \in V_2$ then the anti-fuzzy graph G_{AF} is totally anti-fuzzy equitable disconnected.

Therefore, $\gamma_{AFG}^{ced}(G_{AF}) = \gamma_{AFG}^{ceed}(G_{AF}) = p+q$.

Case (2): If $G \cong K_{m,n}$ and $|d(u)+d(v)| \leq 1$ then if V_1 and V_2 be the partite sets of G_{AF}, be selecting one vertex $u \in V_1$ and $v \in V_2$ then $S = \{u, v\}$ is connected end anti-fuzzy equitable dominating set.

$\gamma_{AFG}^{ced}(G_{AF}) = \gamma_{AFG}^{ceed}(G_{AF}) \leq max_{u\in V_1}\sigma(u) + max_{v\in V_2}\sigma(v)$,

but $\gamma_{AFG}^{ceed}(G_{AF}) \neq max_{u\in V}\sigma(u)$.

$\gamma_{AFG}^{ced}(G_{AF}) = \gamma_{AFG}^{ceed}(G_{AF}) = \max_{u\in V_1}\{\sigma(u)\} + \max_{v\in V_2}\{\sigma(v)\}$.

Theorem 3.12
Let G_{AF} be connected anti-fuzzy graph with order m and N be the set of all end anti-fuzzy vertices of G_{AF} then $\gamma_{AFG}^{ceed}(G_{AF}) \geq |N| + \max\{\sigma(x)\}$, where $x \in V$ which is not an end anti-fuzzy vertex.
Proof:
Clearly if G_{AF} is connected anti-fuzzy graph with order m.
Let N be the set of all end anti-fuzzy vertices of G_{AF}. Let S be any connected end anti-fuzzy equitable dominating set of G_{AF} then all the end vertices and also supporting vertices must along to end anti-fuzzy equitable dominating set.
Hence $\gamma_{AFG}^{ceed}(G_{AF}) \geq |N| + \max\{\sigma(x)\}$.

4 Conclusions

A mathematical model helps to accomplish the problem in a complex situation. The possible solution is to convert the problem into a graph model. Anti-fuzzy graph theory has been used to model many decision making problems in uncertain situations. It have numerous applications in modern science in technology, computer science, especially in the field of information theory, neural network, cluster analysis, diagnosis and control theory etc., In this paper, the connected end anti-fuzzy equitable dominating set of an anti-fuzzy graph is defined. The relation between anti-fuzzy equitable domination number, end anti-fuzzy equitable domination number and connected end anti-fuzzy equitable domination number are established. Theorems related to these parameters are established. In future, we are going to establish these types of parameters in edge dominating sets of anti-fuzzy graphs.

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