New Characterization Of (1, 2)S_P-Kernel in Bitopological Spaces

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Abstract

Let $J(G) = (V, E)$ be a jump graph. Let D be a nominal prevailing (dominating) set in a jump graph $J(G)$. If $V - D$ contains a prevailing set $D'$of $J(G)$, then D' is called an inverse prevailing set with respect to D. The nominal cardinality of an inverse prevailing set of a jump graph $J(G)$ is called inverse domination number of $J(G)$. In this paper, we computed some interconnections betwixt inverse domination number of jump graph for some graphs.

Keywords:(1,2)semi-open, (1,2)pre-open, (1,2)pre-closed, (1,2)S_p-open sets, (1,2)S_p-closed sets, (1,2)S_p-kernel sets, (1,2)S_p-derived sets, (1,2)S_p-shell sets.

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1. Introduction

In 1963, Kelly [3] initiated the study of a triplet (X,τ_1,τ_2), where X is a non-empty set and τ_1,$\tau_2$are the two topologies on X. The notion of (1,2)α-open set [4] in a bitopological space was introduced by Lellis Thivagar in 1991. Raja Rajeswari [6] defined and studied the concepts of ultra-kernel in bitopological spaces. In 2017, (1,2)S_p-open set [2] in bitopological spaces was introduced by Hardi Ali Shareef et.al. In this paper, a new class of sets in bitopological spaces called (1,2)S_p-kernel is introduced and some of its properties are derived.

2. Preliminaries

Definition 2.1. [5] A subset A of a bitopological space X is called a
(i) (1, 2) semi-open if A⊆ $\tau_1\tau_2$Cl (τ_1Int (A)).
(ii) (1, 2) pre-open if A⊆ τ_1Int ($\tau_1\tau_2$Cl (A)).
(iii) (1, 2) regular-open if A = τ_1Int ($\tau_1\tau_2$Cl (A)).
The collection of all (1, 2) semi-open, (1, 2) pre-open and (1, 2) regular-open sets are denoted by (1, 2) SO(X), (1, 2) PO(X) and (1, 2) RO(X) respectively.

Definition 2.2 [5] A subset A of a bitopological space X is called a
(i) (1, 2) α-closed if τ_1Cl($\tau_1\tau_2$Int(τ_1Cl(A))) ⊆ A.
(ii) (1, 2) semi-closed if $\tau_1\tau_2$Int(τ_1Cl(A)) ⊆ A.
(iii) (1, 2) pre-closed if τ_1Cl($\tau_1\tau_2$Int(A)) ⊆ A.
(iv) (1, 2) regular-closed if A = τ_1Cl($\tau_1\tau_2$Int(A)).
The set of all (1, 2) α-closed, (1, 2) semi-closed, (1, 2) pre-closed and (1, 2) regular-closed sets are denoted as (1, 2) αCL (X), (1, 2) SCL (X), (1, 2) PCL (X) and (1, 2) RCL (X) respectively. Also, for any subset A of X, the (1, 2) α-closure, (1, 2) semi-closure, (1, 2) pre-closure and (1, 2) regular-closure of A is denoted as (1, 2) αCl (A), (1, 2) SCl (A), (1, 2) PCl (A) and (1, 2) RCl (A) respectively.
Definition 2.3. [2] A (1, 2) semi-open set A of a bitopological space X is called (1, 2) S_p-open set if for each $x \in$ A, there exists a (1, 2) pre-closed set F such that $x \in$ F ⊆ A.

Remark 2.4. If A and B are (1, 2) α-open sets of X, then A ∪B is also a (1, 2) α-open set.

Definition 2.5. A subset A of a bitopological space X is said to be (1, 2) S_p-locally closed (briefly (1, 2) S_pLC) if A = C ∩ D, where C is a (1, 2) S_p-open set and D is a (1, 2) S_p-closed set in X. The family of (1, 2) S_p-locally-closed sets is denoted by (1, 2) S_pLC(X).

3. (1, 2)S_P-Kernel in Bitopological Spaces

Definition 3.1. Let A be a non-empty subset of a bitopological spaces X. Then (1, 2) S_p-kernel of A is denoted by (1, 2) S_p-Ker ({A}) and it is defined as (1, 2) S_p-Ker ({A}) = ∩ {G ∈(1, 2) S_pO(X)/ A ⊆ G}.

Definition 3.2. Let x ∈ X. Then the (1, 2)S_p-kernel of x is defined by (1, 2)S_p-Ker ({x}) = ∩ {G ∈(1, 2) S_pO(X) / x ∈ G}.

Definition 3.3. A subset N of a bitopological space X is said to be (1, 2) S_p-neigborhood ((1, 2) S_p-nbhd) of a point x ∈ X, if there exists a (1, 2) S_p-open set U such that x ∈U ⊆N.

Lemma 3.4. Let X be a bitopological space. Then for any non-empty subset A of X, (1, 2)S_p-Ker ({A}) = {x ∈ X / (1, 2) S_p-Cl ({x}) ∩ A ≠ ϕ}.
Proof: Let x ∈ (1, 2)S_p-Ker ({A}) and (1, 2)S_p-Cl ({x)} ∩ A≠ϕ. Then A ⊆ [X – (1, 2) S_p-Cl ({x})] and [X – (1, 2) S_p-Cl ({x})] is a (1, 2) S_p-open set containing Abut not x, which is a contradiction. Hence (1, 2) S_p-Cl ({x}) ∩ A ≠ ϕ.
Also, let x ∉ (1, 2) S_p-Ker ({A}) and (1, 2) S_p-Cl ({x}) ∩ A ≠ ϕ. Then there exists a (1, 2) S_p-open set D containing A but not x and there exists an element y ∈(1, 2)S_p-Cl ({x}) ∩ A. Hence we get a (1, 2)S_p-nbhd of y, say D with x ∉D, which is a contradiction. Hence x ∈ (1, 2) S_p-Ker (A).

Definition 3.5. In a bitopological space X, a subset A of X is said to be weakly (1, 2) S_p-separated from a subset B of X if there exists a (1, 2) S_p-open set G of X such that A ⊆ G and G ∩ B = ϕ or A ∩ (1, 2)S_p-Cl (B) =ϕ. It is shown in the following example.

Example 3.6. Let X = {a, b, c, d}. τ_1 = {ϕ, X, {a, c}, {a, c, d}}. τ_2= {ϕ, X}. $\tau_1\tau_2$Cl = {X, ϕ, {b, d}, {b}}. (1, 2) SO (X) = {ϕ, X, {a, c}, {a, b, c}, {a, c, d}}. (1, 2) PCL (X) = {X, ϕ, {b, c, d}, {a, b, d}, {c, d}, {b, d}, {b, c}, {a, d}, {a, b}, {d}, {c}, {b}, {a}}. (1, 2) S_pO (X) = {ϕ, X, {a, c}, {a, b, c}, {a, c, d}}. (1, 2) S_pCL (X) = {X, ϕ, {b, d}, {d},{b}}. Let A = {b}, B = {d} and G = {a, b, c} ∈ (1,2)S_pO(X). Here A ⊆G and G ∩ B = ϕ or A ∩ (1,2)S_p-Cl(B) = ϕ. Hence a subset A of X is weakly (1, 2) S_p-separated from a subset B of X.

Theorem 3.7. Suppose X is a (1, 2) S_p-space and A, B ∈ (1, 2) S_pLC (X). If A and B are weakly (1, 2) S_p-separated, then A ∪B ∈ (1, 2) S_pLC (X).
Proof: Assume A, B ∈ (1, 2) S_pLC (X) and let A = G ∩ (1, 2) S_p-Cl(A) and B = E ∩ (1, 2) S_p-Cl (B), where G and E are (1, 2) S_p-open sets of X. Put U = G ∩ [X – (1, 2) S_p-Cl (B)] and V = E ∩ [X – (1, 2) S_p-Cl (A)]. Then U ∩ (1, 2)S_p-Cl(A) = G ∩ [X – (1, 2) S_p-Cl (B)] ∩ (1, 2) S_p-Cl(A) = [G ∩ X – (1, 2) S_p-Cl (A)] ∩ [X – (1, 2)S_p-Cl (B)] = A ∩ [X – (1, 2) S_p-Cl (B)] = A.
Similarly, V ∩ (1, 2) S_p-Cl (B) = [E ∩ (X – (1, 2) S_p-Cl({A}))] ∩ (1, 2) S_p-Cl{A})

= [E ∩ [X – (1, 2) S_p-Cl ({B})] ∩ [(X – (1, 2)S_p-Cl ({A})] = B ∩ [(X – (1, 2) S_p-Cl ({A})] = B and hence U ∩ (1, 2) S_pCl (B) = V ∩ (1, 2)S_p-Cl (A) = ϕ. Moreover U and V are (1, 2) S_p-open in X. By Remark 2.4, (U ∪ V) is also (1, 2) S_p-open. Then (U ∪ V) ∩ (1, 2)S_p-Cl(A ∪ B) = [U ∩ (1,2)S_p-Cl(A)] ∪ [U ∩ (1,2)S_p-Cl(B)] ∪ [V ∩ (1,2)S_p-Cl(A)] ∪ [V ∩ (1,2)S_p-Cl(B)] = A ∪ B. Hence A ∪ B is (1,2)S_p-locally closed set.

Lemma 3.8. In view of Lemma 3.4 and Definition 3.5, we have for x, y in X of a bitopological space,
(i) (1, 2) S_p-Cl ({x}) = {y: y is not weakly (1, 2) S_p-separated from x}.
(ii) (1, 2) S_p-Ker ({x}) = {y: x is not weakly (1, 2) S_p-separated from y}.

Definition 3.9. For any point x of a bitopological space X,
(i) The derived set of x is denoted by (1, 2) S_p-d ({x}) and is defined to be the set (1, 2) S_p-d ({x}) = (1, 2) S_p-Cl ({x}) – {x} = {y: y ≠ x and y is not weakly (1, 2) S_p-separated from x}.
(ii) the shell of a point x of X is denoted by (1,2)S_p-shl({x}) and is defined to be the set (1, 2) S_p-shl ({x}) = (1, 2) S_p-Ker({x}) – {x} = {y : y ≠ x and x is not weakly (1, 2) S_p-separated from x}.

Definition 3.10: Let X be a bitopological space. Then we define
(i) (1, 2) S_p-N-D = {x: x ∈X and (1, 2)S_p-d ({x}) =ϕ}.
(ii) (1, 2) S_p-N-Shl = {x: x ∈ X and (1, 2)S_p-shl ({x}) =ϕ}.
(iii)(1, 2) S_p-⟨x⟩ = (1, 2) S_p-Cl({x}) ∩ (1, 2)S_p-Ker ({x}).

Theorem 3.11. Let x, y∈X. Then the following conditions hold good:
(i) y ∈(1,2)S_p-Ker({x}) if and only if x ∈(1,2)S_p-Cl({y}).
(ii) y ∈(1,2)S_p-shl({x}) if and only if x ∈(1,2)S_p-d({y}).
(iii)y∈(1,2)S_p-Cl({x}) implies (1,2)S_p-Cl({y}) ⊆(1,2)S_p-Cl({x}).
(iv)y∈(1,2)S_p-Ker({x}) implies (1,2)S_p-Ker({y}) ⊆(1,2)S_p-Ker({x}).
Proof: The proof of (i) and (ii) are obvious from the Lemma 3.8.
(iii) Let z ∈(1,2)S_p-Cl({y}). Then z is not weakly (1, 2)S_p-separated from y which implies there exists a (1, 2)S_p-open set G containing x such that G ∩ {y}≠ϕ. Hence y∈G and by assumption G ∩ {x} ≠ϕ. Hence z is not weakly (1, 2)S_p-separated from x which implies z ∈ (1, 2)S_p-Cl ({x}). Therefore (1, 2)S_p-Cl ({y}) ⊆(1, 2)S_p-Cl ({x}).
(iv) Let z ∈ (1, 2)S_p-Ker ({y}). Then y is not weakly (1, 2)S_p-separated from z which implies y ∈ (1, 2)S_p-Cl ({z}). Hence (1, 2)S_p-Cl ({y}) ⊆(1, 2)S_p-Cl ({z}). By assumption y ∈ (1, 2) S_p-Ker ({x}) that implies x ∈ (1, 2) S_p-Cl ({y}). Then (1, 2)S_p-Cl ({x}) ⊆ (1, 2) S_p-Cl ({y}). Ultimately, (1, 2) S_p-Cl ({x}) ⊆ (1, 2)S_p-Cl ({z}) which implies x ∈ (1, 2) S_p-Cl ({z}), that is z∈(1, 2) S_p-Ker ({x}). Therefore (1, 2)S_p-Ker ({y}) ⊆ (1, 2)S_p-Ker ({x}).

Theorem 3.12. Let x, y be in X. Then

(i) for every x ∈X, (1,2)S_p-shl({x}) is a degenerate set if and only if for all x , y ∈X, $x \neq y$, (1,2)S_p-d({x}) ∩ (1,2)S_p-d({y}) = ϕ.
(ii) for every x ∈X, (1,2)S_p-d({x}) is a degenerate set if and only if for every x , y ∈X, $x \neq y$, (1,2)S_p-shl({x}) ∩ (1,2)S_p-shl({y}) = ϕ.

Proof. (i) Let (1,2)S_p-d({x}) ∩ (1,2)S_p-d({y}) ≠ϕ. Then there exists a z∈X such that z∈(1,2)S_p-d({x}) and that z∈(1,2)S_p-d({y}). Then z ≠y ≠x and z ∈(1,2)S_p-Cl({x}) and z ∈ (1,2)S_p-Cl({y}). That is x, y ∈(1,2)S_p-Ker({z}). Hence (1,2)S_p-Ker({z}) implies (1,2)S_p-shl({z}) is not a degenerate set, which is a contradiction. Hence (1,2)S_p-d({x}) ∩ (1,2)S_p-d({y}) = ϕ.

Also, let x, y∈(1,2)S_p-shl({z}) is not a degenerate set. Then x ≠y≠z and x, y∈(1,2)S_p-Ker({z}). Then z is an element of both (1,2)S_p-Cl({x}) and (1,2)S_p-Cl({y}) which implies (1,2)S_p-Cl({x}) ∩ (1,2)S_p-Cl({y}) ≠ϕ which is a contradiction. Hence (1,2)S_p-shl({z}) is a degenerate set.

The proof of (ii) is similar that of (i).

Theorem 3.13. If y∈(1,2)S_p-⟨x⟩, then (1,2)S_p-⟨x⟩ = (1,2)S_p-⟨y⟩,

Proof. If y∈(1,2)S_p-⟨x⟩, then y∈(1,2)S_p-Cl({x}) and y∈(1,2)S_p-Ker({x}) and by Theorem 3.10, (1,2)S_p-Cl({y}) ⊆(1,2)S_p-Cl({x}) and (1,2)S_p-Ker({y}) ⊆(1,2)S_p-Ker({x}). which implies (1,2)S_p-Cl({y}) ∩ (1,2)S_p-Ker({y}) ⊆(1,2)S_p-Cl({x}) ∩ (1,2)S_p-Ker({x}). Thus (1,2)S_p-⟨y⟩⊆ (1,2)S_p-⟨x⟩. Now, y∈(1,2)S_pCl({x}) implies x ∈(1,2)S_p-Ker({y}) and y∈(1,2)S_p-Ker({x}) which implies x ∈(1,2)S_p-Cl({y}). Which implies (1,2)S_pCl({x}) ∩ (1,2)S_p-Ker({x}) ⊆(1,2)S_p-Ker({y}) ∩ (1,2)S_p-Cl({y}). Thus (1,2)S_p-⟨x⟩⊆(1,2)S_p-⟨y⟩. Hence (1,2)S_p-⟨x⟩ = (1,2)S_p-⟨y⟩.

Theorem 3.14. For all x ,y∈X, either (1,2)S_p-⟨x⟩∩ (1,2)S_p-⟨y⟩ = ϕ or (1,2)S_p-⟨x⟩ = (1,2)S_p-⟨y⟩.

Proof. Let (1,2)S_p-⟨x⟩∩ (1,2)S_p-⟨y⟩≠ϕ, then there exists z ∈X such that z∈(1,2)S_p-⟨x⟩ and z∈(1,2)S_p-⟨y⟩. By theorem 3.13, (1,2)S_p-⟨z⟩ = (1,2)S_p-⟨x⟩ = (1,2)S_p-⟨y⟩. Hence (1,2)S_p-⟨x⟩ = (1,2)S_p-⟨y⟩.

Theorem 3.15. For any two points x, y in X, the following statements are equivalent.
(i) (1,2)S_p-Ker({x}) ≠(1,2)S_p-Ker({y}).
(ii) (1,2)S_p-Cl({x}) ≠(1,2)S_p-Cl({y})

Proof. (i) ⇒ (ii) Let us assume that (1,2)S_p-Ker({x}) ≠(1,2)S_p-Ker({y}). Then there exists a point z ∈(1,2)S_p-Ker({x}) but z ∉(1,2)S_p-Ker({y}). As z ∈(1,2)S_p-Ker({x}), x ∈(1,2)S_p-Cl({z}) and (1,2)S_p-Cl({x}) ⊆(1,2)S_p-Cl({z}). Also since z ∉(1,2)S_p-Ker({y}). By Lemma 3.4,(1,2)S_p-Cl({z}) ∩ {y} = ϕ which implies (1,2)S_p-Cl({x}) ∩ {y} = ϕ and y is weakly (1,2)S_p-separated from x, that is y ∉(1,2)S_p-Cl({x}). Hence (1,2)S_p-Cl({y}) ≠ (1, 2) S_p-Cl({x}).

(ii) ⇒ (i) Suppose (1, 2) S_p-Cl({x}) ≠(1,2)S_p-Cl({y}). Then there exists a point z ∈ (1, 2)S_p-Cl ({x}) but z ∉(1, 2)S_p-Cl({y}). Also a (1, 2)S_p-open set containing z and x but not y implies y ∉(1,2)S_p-Ker({x}). Hence (1,2)S_p-Ker({y}) ≠(1,2)S_p-Ker({x}).

4. Conclusions

In this paper, the new characterization of (1,2)S_p-kernel was introduced and some of its properties are discussed. Later on Research be reached out with certain applications.

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