Restoring Quantum Communication Efficiency over High Loss Optical Fibers

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In the absence of quantum repeaters, quantum communication proved to be nearly impossible across optical fibers longer than $\gtrsim 20$ km due to the drop of transmissivity below the critical threshold of $1/2$. However, if the signals fed into the fiber are separated by a sufficiently short time interval, memory effects must be taken into account. In this Letter, we show that by properly accounting for these effects it is possible to devise schemes that enable unassisted quantum communication across arbitrarily long optical fibers at a fixed positive qubit transmission rate. We also demonstrate how to achieve entanglement-assisted communication over arbitrarily long distances at a rate of the same order of the maximum achievable in the unassisted noiseless case.

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Introduction.—Reliably transmitting qubits across long optical fibers is crucial for establishing a global quantum internet [1,2], an architecture that would allow unconditionally secure communication, entanglement distribution over long distances, quantum sensing improvements (e.g., in telescope observations [3] and clock synchronization [4]), distributed quantum computing [5], and private remote access to quantum computers [6]. The main technological hurdle in establishing a global quantum internet is the fact that the transmissivity of an optical fiber decreases rapidly —typically, exponentially—with its length. This is generally a serious problem, because the (unassisted) quantum capacity of an optical fiber—i.e., its ability to reliably transmit quantum messages—drops to zero if the overall transmissivity of the communication line falls below the critical value of $1/2$. A similar effect applies also to the two-way-assisted quantum capacity of these models—i.e., their ability to reliably transmit quantum information with the help of free classical communication—which is known to vanish for sufficiently low transmissivities [7]. Typical optical fibers employed nowadays attenuate the signal by 0.2 dB/km, the absolute record being 0.14 dB/km [8,9]; this means that in absence of quantum repeaters [10–12] the quantum capacity vanishes for fibers longer than 15 km or at most 21.5 km.

Here, we present a conceptually simple scheme that overcomes this problem and, in principle, enables quantum communication at a constant rate over arbitrarily long distances, i.e., for arbitrarily low nonzero values of the transmissivity. We further prove that this same scheme can be combined with entanglement assistance to neutralize the effect of noise in classical communication altogether. We achieve these results by studying information transmission preceded by a trigger pulse: if the time interval between trigger pulse and signal is sufficiently short, then the memoryless assumption commonly invoked in the quantum capacity analysis of these models breaks down, and one can effectively alter the environment before the actual transmission begins [13,14]. Since memory effects have been experimentally observed in optical fibers [15,16], the above scheme may offer a concrete route to enable quantum communication over long distances.

At the quantum level, an optical fiber is typically described by a memoryless thermal attenuator Φ_{λ,τ_ν}, i.e., a bosonic quantum channel [17] that mixes the input signal with a thermal environment τ_ν through a beam splitter (BS) of transmissivity $\lambda \in [0,1]$. Our findings build on those of [18], where the phenomenon of "die-hard quantum communication" (D-HQCOM) was uncovered. Such an effect consists of the observation that, if we replace the thermal state τ_ν of the environment with a suitable state $\sigma = \sigma(\lambda)$ (possibly dependent upon the transmissivity of the model), the quantum capacity of the modified attenuator channel $\Phi_{\lambda,\sigma(\lambda)}$ stays above a fixed positive constant $c > 0$. The reason why this result cannot be applied directly to improve the quantum communication capabilities of long optical fibers is that neither the sender (Alice) nor the receiver (Bob) can realistically have access to the initial state of the environment. Prior to the present work, D-HQCOM thus seemed mostly a mathematical oddity. Overturning this view, our main conceptual contribution is to recognize that it can instead be turned into a potentially viable technology by exploiting memory effects, paving new avenues to quantum communication beyond current limitations.

Notation.—The set of quantum states on a Hilbert space $\mathcal{H}$ is denoted by $\mathfrak{S}(\mathcal{H})$. Given $\rho_1, \rho_2 \in \mathfrak{S}(\mathcal{H})$, the trace norm of their difference is denoted as $\|\rho_1 - \rho_2\|_1$. The information-carrying signal we consider is a single mode of electromagnetic radiation with definite frequency and polarization. This system, associated with the Hilbert space $\mathcal{H}_S := L^2(\mathbb{R})$, is described as a quantum harmonic oscillator. A BS of transmissivity $\lambda \in [0,1]$ acting on two single-mode systems S_1 and S_2 is defined as the unitary transformation $U_\lambda^{S_1 S_2} := \exp[\arccos\sqrt{\lambda}\,(a_1^\dagger a_2 - a_1 a_2^\dagger)]$, where a_1 and a_2 are the annihilation operators on S_1 and S_2, respectively. Let $\mathcal{H}_E := L^2(\mathbb{R})$ denote a single-mode system, dubbed "environment," and let b denote its annihilation operator. Fixed $\lambda \in [0,1]$ and $\sigma \in \mathfrak{S}(\mathcal{H}_E)$, a general attenuator $\Phi_{\lambda,\sigma}: \mathfrak{S}(\mathcal{H}_S) \mapsto \mathfrak{S}(\mathcal{H}_S)$ is a quantum channel defined by $\Phi_{\lambda,\sigma}(\rho) := \mathrm{Tr}_E[U_\lambda^{SE}\rho \otimes \sigma (U_\lambda^{SE})^\dagger]$. $\Phi_{\lambda,\sigma}$ is completely noisy for $\lambda = 0$, in the sense that $\Phi_{0,\sigma}(\rho) = \sigma$ for all ρ; on the contrary, it is noiseless, i.e., it coincides with the identity channel Id, for $\lambda = 1$. As mentioned in the Introduction, a thermal attenuator Φ_{λ,τ_ν} is a special case of general attenuator obtained by identifying σ with a thermal state $\tau_\nu := \frac{1}{\nu+1}\sum_{n=0}^{\infty}(\frac{\nu}{\nu+1})^n |n\rangle\langle n|$, with $|n\rangle$ being the nth Fock state of the model.

The energy-constrained (EC) classical capacity $C(\Phi, N)$ [respectively, quantum capacity $Q(\Phi, N)$] of a quantum channel Φ is the maximum rate of bits (respectively, qubits) that can be reliably transmitted through Φ, assuming that Alice has access to a limited amount (N) of input energy to build her coding signals. In addition, if an unlimited amount of preshared entanglement can be exploited by Alice and Bob, the corresponding maximum achievable bit (respectively, qubit) transmission rate is called the EC entanglement-assisted (EA) classical (respectively, quantum) capacity $C_{\mathrm{EA}}(\Phi, N)$ [respectively, $Q_{\mathrm{EA}}(\Phi, N)$] [19–24]—the two quantities being related by the identity $C_{\mathrm{EA}}(\Phi, N) = 2Q_{\mathrm{EA}}(\Phi, N)$ thanks to quantum teleportation 25]] and superdense coding [26]. The EC EA classical capacity can be expressed as [20,24,27]

$$C_{\mathrm{EA}}(\Phi, N) = \max_{\rho \in \mathfrak{S}(\mathcal{H}_S):\ \mathrm{Tr}[\rho a^\dagger a] \le N} [S(\rho) + I_{\mathrm{coh}}(\Phi, \rho)], \qquad (1)$$

where $S(\rho) := -\mathrm{Tr}[\rho \log_2 \rho]$, $I_{\mathrm{coh}}(\Phi,\rho) := S[\Phi(\rho)] - S[\Phi \otimes \mathrm{Id}_P(|\psi\rangle\langle\psi|)]$, with $|\psi\rangle \in \mathcal{H}_S \otimes \mathcal{H}_P$ being a purification of ρ [17,28,29], $\mathcal{H}_P$ being the purifier Hilbert space, and Id_P being the identity superoperator on $\mathcal{H}_P$. In addition, the EC quantum capacity can be written as [17,21,30–32]:

$$Q(\Phi, N) = \lim_{n\to\infty} \frac{1}{n} Q_1(\Phi^{\otimes n}, nN) \ge Q_1(\Phi, N),$$

$$Q_1(\Phi^{\otimes n}, N) := \max_{\rho \in \mathfrak{S}(\mathcal{H}_S^{\otimes n}):\ \mathrm{Tr}[\rho \sum_{i=1}^n a_i^\dagger a_i] \le N} I_{\mathrm{coh}}(\Phi^{\otimes n}, \rho).$$

The quantities $C_{\mathrm{EA}}(\Phi_{\lambda,\tau_\nu}, N)$ [33–35], $C(\Phi_{\lambda,\tau_\nu}, N)$ [36], and $Q(\Phi_{\lambda,|0\rangle\langle 0|}, N)$ [7,21,33,37–39] have been determined exactly. For $\nu > 0$, sharp bounds on $Q(\Phi_{\lambda,\tau_\nu}, N)$ are known if $\lambda > 1/2 + \frac{\nu}{2(\nu+1)}$ [33,39–44], while it is known that $Q(\Phi_{\lambda,\tau_\nu}, N) = 0$ otherwise [41].

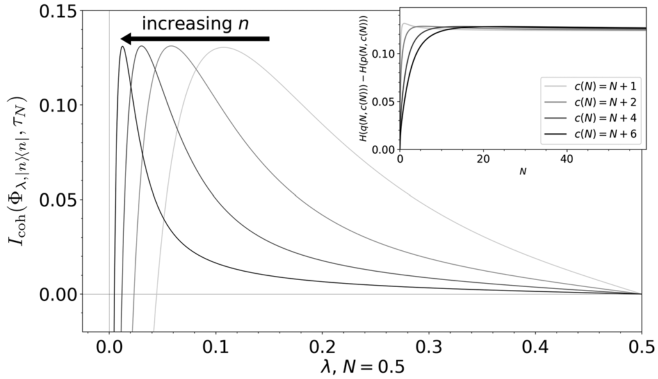

FIG. 1. The quantity $I_{\mathrm{coh}}(\Phi_{\lambda,|n\rangle\langle n|}, \tau_N)$ plotted with respect to λ for $N = 0.5$ and for several values of n from 10 to 100. In the inset we plot the function $H(q(N, c(N))) - H(p(N, c(N)))$ (see Lemma 3) with respect to N for several choices of $c(N)$ of the form $c(N) = N + \alpha$.

Environment control and EA imply noise neutralization.—In [18] it was shown that, even for arbitrarily low values of the transmissivity $\lambda > 0$, suitable choices of the environmental state make the quantum capacity of the corresponding general attenuator bounded away from zero. Specifically:

Theorem 1 [18].—For all $\lambda \in (0,1]$ there exists $\sigma(\lambda)$ such that

$$Q(\Phi_{\lambda,\sigma(\lambda)}) \ge Q(\Phi_{\lambda,\sigma(\lambda)}, 1/2) > \eta, \qquad (2)$$

where $\eta > 0$ is a universal constant. More specifically, for $\varepsilon \ge 0$ sufficiently small and for all $\lambda \in (0, 1/2 - \varepsilon)$ it holds that $Q(\Phi_{\lambda,|n_\lambda\rangle\langle n_\lambda|}) \ge Q(\Phi_{\lambda,|n_\lambda\rangle\langle n_\lambda|}, 1/2) > c(\varepsilon)$, where $c(\varepsilon) \ge 0$ is a constant with respect to λ and $n_\lambda \in \mathbb{N}$ satisfies $1/\lambda - 1 \le n_\lambda \le 1/\lambda$. Moreover, it holds that $c(0) = 0$, and $c(\bar{\varepsilon}) \ge 5.133 \times 10^{-6}$ for an appropriate $\bar{\varepsilon}$ such that $0 < \bar{\varepsilon} \ll 1/6$ (see the Supplemental Material of [18]).

Here, we provide an extension of this result for the EC EA capacities. We start by observing that, for all $N > 0$, $\lambda \in [0,1]$, and $n \in \mathbb{N}$, by choosing τ_N as an ansatz for ρ in (1), one obtains

$$C_{\mathrm{EA}}(\Phi_{\lambda,|n\rangle\langle n|}, N) \ge C(\mathrm{Id}, N) + I_{\mathrm{coh}}(\Phi_{\lambda,|n\rangle\langle n|}, \tau_N), \qquad (3)$$

where we used $S(\tau_N) = C(\mathrm{Id}, N)$ [45]. The quantity $I_{\mathrm{coh}}(\Phi_{\lambda,|n\rangle\langle n|}, \tau_N)$ is calculated in Lemma S9 in the Supplemental Material [46] and expressed in a simple form by exploiting the "master equation trick" introduced in [47]. By plotting it (see, for example, Fig. 1), one can notice that the lower end point of the λ range for which $I_{\mathrm{coh}}(\Phi_{\lambda,|n\rangle\langle n|}, \tau_N) > 0$ seems to converge to zero as n grows. This leads to:

Conjecture 2.—For all $N > 0$, $\lambda \in (0, 1/2)$, if $n \in \mathbb{N}$ is sufficiently large, then $I_{\mathrm{coh}}(\Phi_{\lambda,|n\rangle\langle n|}, \tau_N) > 0$.

Our investigation (Sec. III [47], Lemma 3, and Fig. 1) presents overwhelming numerical evidence that Conjecture

2 is true. Notice that if such statement is valid then (3) will imply that, irrespective of how small λ is, by choosing n sufficiently large one can make $C_{\mathrm{EA}}(\Phi_{\lambda,|n\rangle\langle n|}, N)$ larger than or equal to the classical capacity of the noiseless channel $C(\mathrm{Id}, N)$ [and, equivalently, $Q_{\mathrm{EA}}(\Phi_{\lambda,|n\rangle\langle n|}, N) > Q(\mathrm{Id}, N)/2$]. While we are not able to prove Conjecture 2 in full generality, in what follows we shall see that it holds at least in the most significant regime where $\lambda \to 0^+$. For this purpose, we introduce the following (see the Supplemental Material [46]):

Lemma 3.—For all $N, c > 0$ it holds that

$$\liminf_{n\to\infty} I_{\mathrm{coh}}(\Phi_{\frac{c}{n},|n\rangle\langle n|}, \tau_N) \geq H(q(N,c)) - H(p(N,c)),$$

where $\{q_k(N,c)\}_{k\in\mathbb{Z}}$ and $\{p_k(N,c)\}_{k\in\mathbb{Z}}$ are two probability distributions defined as

$$q_k(N,c) := e^{-c(2N+1)}\left(\frac{N}{N+1}\right)^{k/2} I_{|k|}(2c\sqrt{N(N+1)}),$$

$$p_k(N,c) := \begin{cases} \frac{e^{-c/(N+1)}N^k}{(N+1)^{k+1}} L_k\left(-\frac{c}{N(N+1)}\right) & \text{if } k \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

with $I_k(\cdot)$ and $L_k(\cdot)$ being the kth Bessel function of the first kind and the kth Laguerre polynomial, respectively. $H(\cdot)$ denotes the Shannon entropy.

If we could find a value of c that makes the quantity $H(q(N,c)) - H(p(N,c))$ positive, then Conjecture 2 would be proved for $\lambda \to 0^+$. Let us choose, for example, $c(N) := N + \alpha$, with $\alpha > 0$ fixed. The plot of the function $H(q(N, N+\alpha)) - H(p(N, N+\alpha))$, shown in the inset of Fig. 1, demonstrates that such function is numerically verified to be positive for all N. Consequently, thanks to (3), we can conclude that

$$\liminf_{n\to\infty} C_{\mathrm{EA}}(\Phi_{\frac{c(N)}{n},|n\rangle\langle n|}, N) > C(\mathrm{Id}, N). \tag{4}$$

This effectively proves Conjecture 2 in the regime where $\lambda \to 0^+$, establishing that environment control and entanglement assistance enable communication performance comparable to that achievable in the case in which $\lambda = 1$. This phenomenon of noise neutralization can be regarded as an EA version of the phenomenon of D-HQCOM uncovered in [18]. Let us remark that the possibility of extending the above analysis beyond the infinitesimal values of λ to the finite case is prevented by the (rather surprising) fact that typically $C_{\mathrm{EA}}(\Phi_{\lambda,\sigma}, N)$ is not monotonically increasing in λ [47].

Control of the environment.—The D-HQCOM effect (uncovered in [18]) and its EA version (uncovered in this Letter) guarantee that very good communication performances are possible even in the limit of vanishing transmissivities, under the assumption that the environment state can be suitably chosen. Here we remove this assumption, by providing a fully consistent protocol that exploits memory effects in order to control the environment.

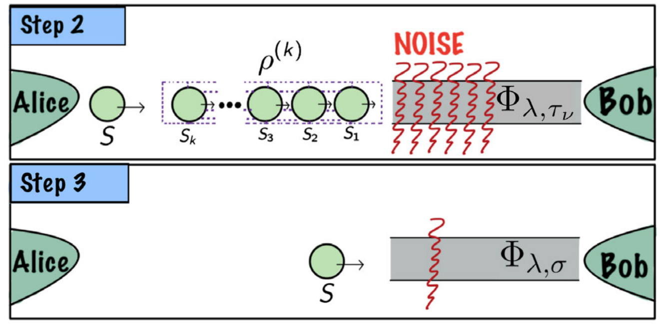

FIG. 2. Steps 2 and 3 of the noise attenuation protocol. At the beginning of step 2, the environment is initialized in τ_ν. By sending the signals $S_1, S_2, \ldots, S_k$, Alice aims to turn the environment into a state σ, where the latter is such that $\Phi_{\lambda,\sigma}$ is less noisy than Φ_{λ,τ_ν}. Right after the environment has transformed into σ, step 3 starts with Alice sending the information-carrying signal S.

Memory effects in quantum communication can be described by the collisional model formulated in [14]. Translating it into the case we are analyzing here, it consists of splitting the channel environment of the fiber in two components: a local term E initialized into a thermal state τ_ν that couples directly with Alice's signals via the BS interaction U_λ^{SE}, and a remote term R, which instead only interacts with E trying to reset its state to τ_ν via a thermalization process characterized by a timescale t_E. This process is described by a one-parameter family of quantum channels $\{\xi_{\delta t}\}_{\delta t\geq 0}$ such that for any state σ of E it holds that (a) $\xi_{\delta t}(\sigma) = \tau_\nu$ for $\delta t \geq t_E$ and (b) $\xi_{\delta t}(\sigma) \simeq \sigma$ for $\delta t \ll t_E$. We assume (a) since in this way if the time interval δt between signals sent by Alice is such that $\delta t \geq t_E$ the above model reduces to the best studied model of bosonic quantum communication across optical fibers, where the attenuation noise that affects each signal is represented by the same quantum channel, i.e., by the memoryless thermal attenuator Φ_{λ,τ_ν}. If the time interval between signals satisfies $\delta t \ll t_E$, the thermalization induced by R can be neglected and the dynamical evolution of E will be dominated by its interactions with the transmitted signals: in this regime, Alice has hence the possibility of exerting a certain level of control on the transmission line. Building upon this observation, we can hence introduce a protocol that enables the communicating parties to effectively move from the memoryless channel description Φ_{λ,τ_ν} into a new effective memoryless channel $\Phi_{\lambda,\sigma}$ (see Fig. 2):

Noise attenuation protocol: Step 1: Alice waits for a time t_E (so that the thermalization resets E into τ_ν). Step 2: Alice sends k suitable signals, dubbed "trigger signals," that alter E into the chosen state σ. Step 3: Alice sends an information-carrying signal. Then, she goes back to step 1, unless the communication is complete.

Let $\mathcal{H}_{S_i}$ denote the Hilbert space of the ith trigger signal. Suppose that Alice sends k trigger signals $S_1, S_2, \ldots, S_k$ separated by a time interval δt and initialized into the state

$\rho^{(k)}$. For simplicity, suppose that δt is also the time interval between the kth trigger signal and the information-carrying signal S of step 3. Then the state σ of E, which interacts with S, can be expressed as

$$\sigma = \mathrm{Tr}_{S_1,\ldots,S_k}[\xi_{\delta t}\circ\mathscr{U}_\lambda^{S_kE}\ldots\circ\xi_{\delta t}\circ\mathscr{U}_\lambda^{S_1E}(\rho^{(k)}\otimes\tau_\nu)], \quad (5)$$

where $\mathscr{U}_\lambda^{S_iE}$ is a quantum channel defined by $\mathscr{U}_\lambda^{S_iE}(\cdot) = U_\lambda^{S_iE}(\cdot)U_\lambda^{S_iE\dagger}$.

From now on, suppose that $\delta t \ll t_E$ so that we can use the approximation $\xi_{\delta t} \simeq \mathrm{Id}$. Hence, (5) reduces to

$$\sigma = \mathrm{Tr}_{S_1,\ldots,S_k}[U_\lambda^{S_kE}\ldots U_\lambda^{S_1E}\rho^{(k)}\otimes\tau_\nu(U_\lambda^{S_kE}\ldots U_\lambda^{S_1E})^\dagger]. \quad (6)$$

The phenomena of D-HQCOM can be activated if E is altered in a suitable Fock state $|n\rangle\langle n|$. Unfortunately, for all $n \in \mathbb{N}^+$ there does not exist $\rho^{(k)}$ that alters E into $|n\rangle\langle n|$ ([47], Theorem 13). However, there exists $\rho^{(k)}$ that alters E into a state that is as close (in trace distance) to $|n\rangle\langle n|$ as desired if k is large, as established by the following theorem (see the Supplemental Material [46]).

Theorem 4.—Let $\nu \geq 0$, $n \in \mathbb{N}$, and $\lambda \in (0,1)$. There exists a suitable state of k trigger signals $\rho_{\lambda,n}^{(k)}$ such that it can alter E into a state $\sigma_{\lambda,\nu,n,k}$, given by (6), which satisfies $\lim_{k\to\infty}\|\sigma_{\lambda,\nu,n,k} - |n\rangle\langle n|\|_1 = 0$.

Furthermore, let $\lambda \in (0,1/2)$ and set $n = n_\lambda \in \mathbb{N}$ with $1/\lambda - 1 \leq n_\lambda \leq 1/\lambda$, then for k sufficiently large it holds that $Q(\Phi_{\lambda,\sigma_{\lambda,\nu,n_\lambda,k}}) > 0$.

In addition, if Conjecture 2 holds, then for all $N > 0$, $\lambda \in (0,1/2)$, and for $\bar{n} \in \mathbb{N}$ sufficiently large, it holds that

$$\begin{aligned} &\lim_{k\to\infty} C_{\mathrm{EA}}(\Phi_{\sigma_{\lambda,\nu,\bar{n},k}}, N) > C(\mathrm{Id}, N), \\ &\lim_{k\to\infty} Q_{\mathrm{EA}}(\Phi_{\lambda,\sigma_{\lambda,\nu,\bar{n},k}}, N) > Q(\mathrm{Id}, N)/2, \\ &\lim_{k\to\infty} Q(\Phi_{\lambda,\sigma_{\lambda,\nu,\bar{n},k}}, N) > 0. \end{aligned} \quad (7)$$

Theorem 4 implies that the D-HQCOM effect can be activated by applying the noise attenuation protocol with a sufficiently large number k of trigger signals initialized in the state $\rho_{\lambda,n}^{(k)}$: an explicit construction to produce such pulses using Fock states and linear optics can be found in [47]. Applying this protocol achieves a dramatic improvement of the communication performance of an optical fiber. Indeed, if an optical fiber with transmissivity $0 < \lambda \ll 1/2$ is used as usual—i.e., by sending signals separated by a time interval $\delta t \gtrsim t_E$—then it is described by a thermal attenuator that has zero quantum capacity and vanishing (two-way or entanglement)-assisted capacities. A drawback of this construction, however, is that, counting the transmission of the trigger signals as channel uses, the rate it achieves is equal to $Q(\Phi_{\lambda,\sigma})/(k+1)$, which can be small if the construction in Theorem 4 yields a large k. Fortunately, for $\lambda > 0$ sufficiently small, just $k = 2$ is enough to guarantee nonzero quantum capacity [46].

Theorem 5.—Let $\nu \geq 0$. Suppose that Alice sends two trigger signals in $U_{1/(1+\lambda)}^{S_1S_2}|0\rangle_{S_1}|n_\lambda\rangle_{S_2}$, with $n_\lambda \in \mathbb{N}$ such that $1/\lambda - 1 \leq n_\lambda \leq 1/\lambda$. Then, E is altered into a state $\sigma_{\lambda,\nu}$ such that for $\lambda > 0$ sufficiently small it holds that $Q(\Phi_{\lambda,\sigma_{\lambda,\nu}}) \geq Q(\Phi_{\lambda,\sigma_{\lambda,\nu}}, 1/2) \geq c$, where $c > 0$ is a fixed positive constant.

Theorem 5 shows that it is possible to reliably transmit qubits at a fixed positive rate if $\lambda > 0$ is sufficiently low, i.e., if the optical fiber is sufficiently long. Theorems 4 and 5 are valid not only if the equilibrium state of the thermalization process is a thermal state τ_ν, but also if it is any state σ_0 such that $\langle(b^\dagger b)^2\rangle_{\sigma_0} < \infty$ [47].

The analysis presented in this section is based on the expression of the state of E in (6), which is an approximation valid for $\delta t \ll t_E$. A similar analysis can also be carried out using the exact expression in (5). The latter reduces to (6) if $\xi_{\delta t} = \mathrm{Id}$. It turns out that, under suitable continuity properties of $\xi_{\delta t}$, if $\rho^{(k)}$ is such that the general attenuator with environment state in (6) has strictly positive quantum capacity, then $\rho^{(k)}$ is such that the general attenuator with environment state in (5) has strictly positive quantum capacity for δt sufficiently short ([47], Theorem 19). In other words, any scheme for quantum communication that works with the approximation $\xi_{\delta t} = \mathrm{Id}$ presented in this section will also work without such an approximation for δt sufficiently short.

Conclusions.—This Letter shows how memory effects can be engineered in order to improve the communication performance of an optical fiber. This is done by exploiting the noise attenuation protocol with trigger signals initialized in a suitable state. This protocol allows arbitrarily long optical fibers to reliably transmit (a) qubits at a fixed positive rate and (b) bits and qubits at a rate of the same order of the maximum achievable in the ideal case of absence of noise, provided that preshared entanglement is consumed. An interesting development of our analysis is to take into account the decoherence process suffered by the signals as they wait to be fed into the optical fiber, due to imperfections in Alice's quantum memory, e.g., by exploiting the recently proposed queuing framework [48–50].

We encourage experimental research on memory effects in optical fibers. For instance, one could test the model we have presented and estimate the thermalization time t_E. Since the latter may be not much larger than the shortest time interval between subsequent signals allowed by present-day devices, an interesting development of this work is to analyze the noise attenuation protocol without our simplifying hypothesis $\delta t \ll t_E$.

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[45] It holds $C(\mathrm{Id}, N) = Q(\mathrm{Id}, N) = C_{\mathrm{EA}}(\mathrm{Id}, N)/2 = Q_{\mathrm{EA}}(\mathrm{Id}, N) = S(\tau_N)$.

[46] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.129.180501 for complete proofs of some of the results discussed in the main text.

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