Attitude Control of a 2-DOF Helicopter System with Input Quantization and Delay

1st Siri Marte Schlanbusch
Department of Engineering Sciences
University of Agder
Grimstad N-4879, Norway
siri.schlanbusch@uia.no

2nd Ole Morten Aamo
Department of Engineering Cybernetics
Norwegian University of Science and Technology
Trondheim N-7491, Norway
ole.morten.aamo@ntnu.no

3rd Jing Zhou
Department of Engineering Sciences
University of Agder
Grimstad N-4879, Norway
jing.zhou@uia.no

***Abstract*—In this paper the attitude tracking control problem of a 2 degrees-of-freedom helicopter system with network induced constraints is studied. A predictor feedback control law is developed to compensate a known delay in the communication, where the inputs are quantized before transmitted over the network. Stability of the closed-loop system is established, where tracking is achieved with bounded tracking errors due to the network issues. The developed predictor-based controller is experimentally tested on the helicopter system, where we demonstrate that tracking is achieved in presence of both input delay and quantization.**

***Index Terms*—Attitude control, quantization, delay**

I. Introduction

Air vehicles such as unmanned aerial vehicles (UAVs) and helicopters provide great accessibility and have a wide range of applications such as transport, search and rescue, inspection, monitoring and photography. Unmanned aircraft are controlled by a human operator from ground or fully autonomously by electronic systems, where remote controlled systems are sensitive to time delays and also the sampling and quantization of signals before transmitted in the communication network affect the performance of such systems.

For an attitude tracking control problem where signals are sent through a network, both quantization and delay have impact on the tracking performance. Quantization naturally exists in networked control systems (NCSs), where a quantizer can be considered as a devise that converts a continuous signal into a piecewise constant signal, which leads to quantization errors that are nonlinear. These errors can not be ignored when the resolution in the network is low, since it will affect the performance and stability of the system. Quantization can also be considered as useful, from the advantage of reducing occupation rate of transmission bandwidth in the communication channel [1]. Tracking control of systems with input quantization has been investigated in e.g. [2]–[5] for uncertain nonlinear systems, in [6] for a group of unmanned aerial vehicles with unknown parameters, in [7] for under-actuated autonomous underwater vehicles (AUVs) and in [8] for a 2 degrees-of-freedom (DOF) helicoper system.

One of the first tools for handling delays was the Smith predictor used for compensating a pure time-delay for open-loop stable plants. A modified Smith predictor compensates for both the predicted effect of the control input and of the future evolution of the system state, and also works for unstable plants [9]. Several predictor based approaches have been proposed to compensate input delays for linear systems in [10]–[12] and nonlinear systems in [13]–[19] where a backstepping transformation was introduced in the control design in [13], which makes it possible to show stability of the closed-loop system using a Lyapunov functional. In [20] the attitude stabilization of a quadrotor with a known input delay was considered where a predictor feedback controller was developed to compensate the delay. Compared to stabilization to a desired attitude, the problem of tracking a changing reference signal with time is more difficult. Unless knowing the reference signal in advance, and by sending the reference signal the delayed-time units ahead to the controller, it is not possible to track the desired signal perfectly in presence of a delay. In [21], the tracking control problem of nonlinear networked and quantized control systems was studied. In [22] a predictor feedback controller was developed for trajectory tracking where both input delay and parameters were unknown.

In this paper we are focusing on the problem of tracking a given reference attitude for a nonlinear multiple-input multiple-output (MIMO) helicopter system with 2 DOF, when there is a known constant time-delay of D-time units for the inputs and at the same time, the inputs are quantized before transmitted over the network. The main contributions in this paper are dealing with the simultaneous issues caused by quantization and delay, where the effect of the delay is compensated for by the design of a predictor feedback controller, and where the effect of quantization is analytically shown to be related to the tracking error. A higher quantization level increases the tracking error. Simulations and experiments are carried out to illustrate the proposed control scheme.

The paper is organized as follows. In Section II, the dynamical model of the helicopter system, the control problem and the considered quantizer are presented. Section III provides the predictor-feedback control design, in Section IV a proof of stability on the basis of a Lyapunov functional is given and in Section V experimental results of the proposed method implemented on the helicopter system are presented and Section VI sums up the paper in a conclusion.

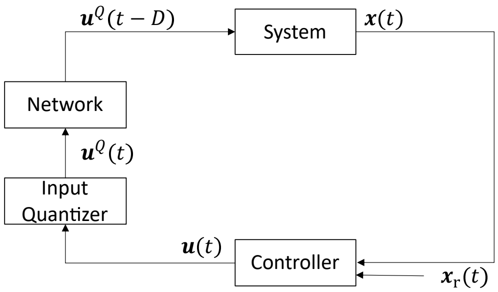

Fig. 1: Control system with input quantization and delay over a network.

II. Dynamical model and problem statement

A. Notations

Vectors are denoted by small bold letters and matrices with capitalized bold letters. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote the maximum and minimum eigenvalue of the matrix $(\cdot)$, and $\|\cdot\|$ denotes the $\mathcal{L}_2$-norm and induced $\mathcal{L}_2$-norm for vectors and matrices, respectively. For vector functions, the norm $\|\boldsymbol{u}(t)\|_2 = \sqrt{\int_0^D \boldsymbol{u}(x,t)^\top \boldsymbol{u}(x,t)dx}$ denotes the spatial $\mathcal{L}_2$ norm.

B. Problem Statement

We are considering a control problem as shown in Fig. 1, where the input vector $\boldsymbol{u}$ is quantized before transmitted in the communication network and there is a time-delay D in the network. The system is assumed noiseless, so that the quantized signals are recovered after transmission, and so the system receives the quantized delayed input $\boldsymbol{u}^Q(t-D)$.

The control objective is to develop a predictor based control law to compensate for a constant known input delay for a multi input nonlinear helicopter system to track a given reference attitude signal. From the derived error dynamics, we will design a controller so that stability of the origin of the error system is maintained in the presence of both quantization and delay of the input.

C. Quantizer

In this paper we consider a uniform quantizer for the inputs, where the quantizer for each input signal is modeled as

$$Q(u) = u^Q = \begin{cases} u_i \operatorname{sgn}(u), & u_i - \frac{l}{2} < |u| \le u_i + \frac{l}{2} \\ 0, & |u| \le u_0 \end{cases}, \quad (1)$$

where $Q(\cdot)$ is a quantizer, $u_0 > 0, u_1 = u_0 + \frac{l}{2}$, $u_{i+1} = u_i + l$, $l > 0$ is the length of the quantization interval, $\operatorname{sgn}(u)$ is the sign function. The uniform quantization $u^Q \in U = \{0, \pm u_i\}$, and a map of the quantization for $u_i > 0$ is shown in Fig. 2.

The following property holds for the uniform quantizer

$$|u^Q - u| \le \delta, \quad (2)$$

where $\delta > 0$ denotes the quantization bound. Clearly, the property in (2) is satisfied with $\delta = \max\{u_0, \frac{l}{2}\}$. When a vector is quantized, we have

$$\boldsymbol{u}^Q = \begin{bmatrix} u_1^Q & u_2^Q & \cdots & u_n^Q \end{bmatrix}^\top, \quad (3)$$

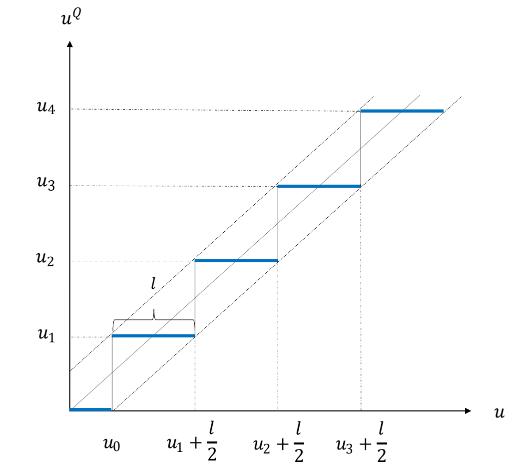

Fig. 2: Map of the uniform quantizer for $u > 0$.

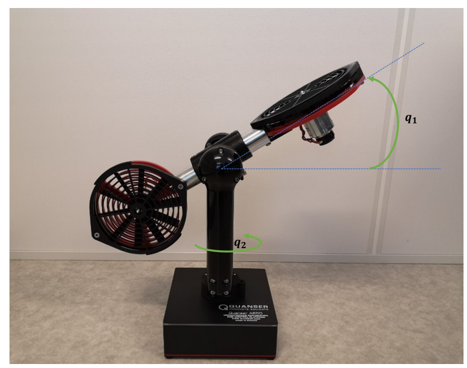

Fig. 3: Quanser Aero helicopter system.

and so each vector element is bounded by (2), and we have

$$\|\boldsymbol{u}^Q - \boldsymbol{u}\| = \|\boldsymbol{d}\| \le \|\boldsymbol{\delta}\| \triangleq \delta_u, \quad (4)$$

where $\boldsymbol{d}$ is the quantization error.

D. Mathematical Model

The helicopter system shown in Fig. 3 is a two-rotor laboratory equipment for flight control-based experiments. With a horizontal position of the main thruster and a vertical position of the tail thruster, this resembles a helicopter with two propellers driven by two DC motors. The helicopter is a MIMO system with 2 DOF, and can rotate around two axes. This is considered as a rigid body and a mathematical model is derived using Euler-Lagrange equations and expressed as:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u}^Q(t-D), \quad (5)$$

where

$$\boldsymbol{M}(\boldsymbol{q}) = \begin{bmatrix} I_p + mr^2 & 0 \\ 0 & I_y + mr^2 \sin^2 q_1 \end{bmatrix}, \quad (6)$$

$$\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \begin{bmatrix} 0 & -mr^2 \sin q_1 \cos q_1 \dot{q}_2 \\ mr^2 \sin q_1 \cos q_1 \dot{q}_2 & mr^2 \sin q_1 \cos q_1 \dot{q}_1 \end{bmatrix}, \quad (7)$$

$$\boldsymbol{g}(\boldsymbol{q}) = \begin{bmatrix} mgr \sin q_1 & 0 \end{bmatrix}^\top, \quad \boldsymbol{q} = \begin{bmatrix} q_1 & q_2 \end{bmatrix}^\top, \quad (8)$$

and where $\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \in \mathbb{R}^2$ are angles, angular velocities and accelerations, $\boldsymbol{M}(\boldsymbol{q})$, $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}}),\boldsymbol{D} \in \mathbb{R}^{2\times 2}$ are the inertia, Coriolis and damping matrices, respectively, where $\boldsymbol{D}$ is a constant matrix, $\boldsymbol{g}(\boldsymbol{q}) \in \mathbb{R}^2$ is a vector of gravitational loading, r is the distance between the center of mass and the origin of the body-fixed frame, I_p and I_y are the moments of inertia of q_1 and q_2 respectively, g is the gravitational acceleration, and m is the total mass of the Aero body.

Defining $\boldsymbol{x} = [\boldsymbol{q}^\top,\dot{\boldsymbol{q}}^\top]^\top = [\boldsymbol{x}_1^\top,\boldsymbol{x}_2^\top] \in \mathbb{R}^4$, and $\boldsymbol{u} \in \mathbb{R}^2$, the system can be written in state space form as

$$\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t),\boldsymbol{u}^Q(t-D)). \tag{9}$$

For tracking of a reference signal $\boldsymbol{x}_r(t)$, the error states are defined as

$$\boldsymbol{z}_1 = \boldsymbol{x}_r - \boldsymbol{x}_1, \tag{10}$$
$$\boldsymbol{z}_2 = \dot{\boldsymbol{z}}_1 + \boldsymbol{A}\boldsymbol{z}_1, \tag{11}$$

where $\boldsymbol{A}$ is a constant positive definite matrix, and the error dynamics is given as

$$\dot{\boldsymbol{z}}(t) = f(\boldsymbol{z}(t),\boldsymbol{u}^Q(t-D)) = \begin{bmatrix} \boldsymbol{z}_2 - \boldsymbol{A}\boldsymbol{z}_1 \\ \boldsymbol{A}\boldsymbol{z}_2 + \boldsymbol{h} - \boldsymbol{M}^{-1}\boldsymbol{u}^Q(t-D) \end{bmatrix}, \tag{12}$$

$$\boldsymbol{h} = \ddot{\boldsymbol{x}}_r - \boldsymbol{A}^\top\boldsymbol{A}\boldsymbol{z}_1 + \boldsymbol{M}^{-1}[(\boldsymbol{C}+\boldsymbol{D})(\dot{\boldsymbol{x}}_r + \boldsymbol{A}\boldsymbol{z}_1 - \boldsymbol{z}_2) + \boldsymbol{g}]. \tag{13}$$

The change of coordinates (10)–(11) are chosen by following the backstepping design procedure [23], where a similar design is given in e.g. [24]. To achieve the control objective, the following assumption regarding the reference signal is imposed:

Assumption 1. *The desired angles, angular velocities and accelerations, $\boldsymbol{x}_r(t),\dot{\boldsymbol{x}}_r(t),\ddot{\boldsymbol{x}}_r(t) \in \mathbb{R}^2$, are known, continuous and bounded for all $t \geq t_0 \geq 0$.*

III. Predictor-feedback control design

To compensate for the input delay, we derive a predictor-feedback controller for the system. A nominal controller for the error system (12) without quantization and delay $D = 0$, can be formulated as

$$\boldsymbol{u}(t) = \kappa\left(\boldsymbol{z}(t),\boldsymbol{x}_r(t)\right) = \boldsymbol{M}(\boldsymbol{h} + (\boldsymbol{A}+\boldsymbol{B})\boldsymbol{z}_2 + \boldsymbol{z}_1), \tag{14}$$

where $\boldsymbol{B}$ is a positive definite matrix, and makes the origin exponentially stable in the absence of delay and quantization. System (12) can be equivalently modeled by a cascade of ODE-PDE [17]

$$\dot{\boldsymbol{z}}(t) = f(\boldsymbol{z}(t),\boldsymbol{u}(0,t)), \tag{15}$$
$$\boldsymbol{u}_t(x,t) = \boldsymbol{u}_x(x,t), \tag{16}$$
$$\boldsymbol{u}(D,t) = \boldsymbol{u}^Q(t), \tag{17}$$

where the actuator state is modeled by a transport PDE and where the solution to (16)–(17) is given by $\boldsymbol{u}(x,t) = \boldsymbol{u}^Q(t + x - D)$ for all $x \in [0,D]$.

The predictor feedback controller is defined as [14]

$$\boldsymbol{u}^Q(t) = Q\left(\kappa[\boldsymbol{p}(D,t),\boldsymbol{x}_r(t+D)]\right), \tag{18}$$

where the predictor state is given as

$$\boldsymbol{p}(x,t) = \boldsymbol{z}(t) + \int_0^x f(\boldsymbol{p}(y,t),\boldsymbol{u}(y,t))dy, \forall x \in [0,D], \tag{19}$$

where, assuming perfect model f, $\boldsymbol{p}(x,t) = \boldsymbol{z}(t+x)$ $\forall x \in [0,D]$, and so $\boldsymbol{p}(D,t) = \boldsymbol{z}(t+D)$ is the D-time units ahead predictor of $\boldsymbol{z}(t)$. Then the delayed input

$$\boldsymbol{u}(0,t) = \boldsymbol{u}^Q(t-D) = Q(\kappa[t,\boldsymbol{p}(0,t)]) = \kappa(t,\boldsymbol{z}(t)) + \boldsymbol{d}(t), \tag{20}$$

where $\boldsymbol{d}(t)$ is the quantization error which satisfies (4).

IV. Stability analysis

To analyze the closed-loop stability, we first establish some preliminary results as stated in the following Lemma.

Lemma 1. *The open loop system $\dot{\boldsymbol{z}} = f(\boldsymbol{z},\boldsymbol{\omega})$ is forward complete.*

Proof. Consider the nonnegative-valued, radially unbounded, smooth Lyapunov function and its derivative [18]

$$V_1(\boldsymbol{z}) = \frac{1}{2}\boldsymbol{z}^\top\boldsymbol{z}, \tag{21}$$

$$\begin{aligned} \dot{V}_1 &= \boldsymbol{z}_1^\top(\boldsymbol{z}_2 - \boldsymbol{A}\boldsymbol{z}_1) + \boldsymbol{z}_2^\top(\boldsymbol{A}\boldsymbol{z}_2 + \boldsymbol{h} - \boldsymbol{M}^{-1}\boldsymbol{\omega}) \\ &\leq c_1 V_1 + \frac{1}{2}\boldsymbol{\omega}^\top\boldsymbol{\omega} + c_2(\boldsymbol{x}_r^\top\boldsymbol{x}_r + \dot{\boldsymbol{x}}_r^\top\dot{\boldsymbol{x}}_r + \ddot{\boldsymbol{x}}_r^\top\ddot{\boldsymbol{x}}_r) \\ &\leq c_1 V_1 + c_3, \quad \forall \boldsymbol{z} \in \mathbb{R}^4, \boldsymbol{\omega} \in \mathbb{R}^2, \end{aligned} \tag{22}$$

where $c_{(\cdot)}$ are positive constants, Assumption 1 is used, and where $\boldsymbol{\omega}$ is a bounded input. Then, the system $\dot{\boldsymbol{z}}$ is forward complete and solutions exist globally. □

A definition of forward completeness is given in e.g. [14]. Since the system is forward complete, the problem of a finite escape phenomenon is avoided, and ensures that for every initial condition and every bounded input signal, the corresponding solution is defined for all $t \geq 0$.

Following [17], we define the direct and inverse backstepping transformation

$$\boldsymbol{w}(x,t) = \boldsymbol{u}(x,t) - Q\left(\kappa[x+t,\boldsymbol{p}(x,t)]\right), \tag{23}$$
$$\boldsymbol{u}(x,t) = \boldsymbol{w}(x,t) + Q\left(\kappa[x+t,\boldsymbol{\pi}(x,t)]\right), \tag{24}$$

where for all $x \in [0,D]$,

$$\begin{aligned} \boldsymbol{\pi}(x,t) =& \boldsymbol{z}(t) + \int_0^x f(\boldsymbol{\pi}(y,t), Q\left(\kappa[t+y,\boldsymbol{\pi}(y,t)]\right) \\ &+ \boldsymbol{w}(y,t))dy, \end{aligned} \tag{25}$$

where $\boldsymbol{\pi}(x,t)$ are used to generate the target predictor state $\boldsymbol{\pi}(D,t)$.

By [17, Lemma 1], the transformation (23) maps the closed loop system consisting of the error system (15)–(17) and the control law (18)–(19) into the target system

$$\begin{aligned} \dot{\boldsymbol{z}}(t) &= f(\boldsymbol{z}(t),\boldsymbol{w}(0,t) + \kappa(t,\boldsymbol{z}(t)) + \boldsymbol{d}(t)) \\ &= \begin{bmatrix} \boldsymbol{z}_2(t) - \boldsymbol{A}\boldsymbol{z}_1(t) \\ -\boldsymbol{z}_1(t) - \boldsymbol{B}\boldsymbol{z}_2(t) - \boldsymbol{M}^{-1}\boldsymbol{w}(0,t) - \boldsymbol{M}^{-1}\boldsymbol{d}(t) \end{bmatrix}, \end{aligned} \tag{26}$$

$$\boldsymbol{w}_x(x,t) = \boldsymbol{w}_t(x,t),\ \forall x \in [0,D], \tag{27}$$
$$\boldsymbol{w}(D,t) = 0. \tag{28}$$

By [17, Lemma 2], (24) is the inverse of (23).

We now state our main result in the following theorem.

Theorem 1. *Consider the closed-loop system consisting of the error dynamics of the helicopter system* (15)–(17)*, the control law* (18)–(19) *with input quantization satisfying the bounded property* (4)*, and the reference signal* $\boldsymbol{x}_r(t)$ *satisfying Assumption 1. If the gain matrices* $\boldsymbol{A}$ *and* $\boldsymbol{B}$ *are chosen to satisfy the inequality*

$$\min\{2\lambda_{\min}(\boldsymbol{A}), 2\lambda_{\min}(\boldsymbol{B}) - 2, 1\} > c_4 > 0, \tag{29}$$

where c_4 *is a positive constant, then for all initial conditions* $\boldsymbol{z}(t_0) \in \mathbb{R}^4$*,* $\boldsymbol{u}(x, t_0) \in \mathbb{R}^2\ \forall x \in [0, D]$ *and for all* $t \geq t_0 \geq 0$*, the following holds:*

$$\|\boldsymbol{z}(t)\| + \|\boldsymbol{w}(t)\|_2 \leq c_6 (\|\boldsymbol{z}(t_0)\| + \|\boldsymbol{w}(t_0)\|_2)\, e^{-\frac{c_4}{2}(t-t_0)} + c_5\delta_u, \tag{30}$$

where

$$c_5 = \sqrt{\frac{2k}{c_4}} > 0, \qquad c_6 = \sqrt{2ke^D} > 0, \tag{31}$$

where $k = \max\{1, \lambda_{\max}(\boldsymbol{M}^{-1})^2\}$.

Proof. Due to forward completeness of (15) (Lemma 1), the predictor state (19) is well defined and therefore $\boldsymbol{w}(x, t)$ in (23) is well defined. It follows that the target system (26)–(28) is well defined and that we can select the Lyapunov function candidate

$$V_2(t) = \frac{1}{2}\boldsymbol{z}(t)^\top \boldsymbol{z}(t) + \frac{k}{2}\int_0^D e^x \boldsymbol{w}(x, t)^\top \boldsymbol{w}(x, t)dx, \tag{32}$$

that satisfies

$$\frac{1}{2}E(t) \leq V_2(t) \leq \frac{1}{2}ke^D E(t), \tag{33}$$

where

$$E(t) = \boldsymbol{z}(t)^\top \boldsymbol{z}(t) + \int_0^D \boldsymbol{w}(x, t)^\top \boldsymbol{w}(x, t)dx. \tag{34}$$

The derivative of (32) is

$$\begin{aligned}
\dot{V}_2 = &-\boldsymbol{z}_1^\top \boldsymbol{A}\boldsymbol{z}_1 - \boldsymbol{z}_2^\top \boldsymbol{B}\boldsymbol{z}_2 - \boldsymbol{z}_2^\top \boldsymbol{M}^{-1}\boldsymbol{w}(0, t) \\
&- \boldsymbol{z}_2^\top \boldsymbol{M}^{-1}\boldsymbol{d}(t) + k\int_0^D e^x \boldsymbol{w}(x, t)^\top \boldsymbol{w}_t(x, t)dx \\
\leq &-\boldsymbol{z}_1^\top \boldsymbol{A}\boldsymbol{z}_1 - \boldsymbol{z}_2^\top \boldsymbol{B}\boldsymbol{z}_2 + \boldsymbol{z}_2^\top \boldsymbol{z}_2 \\
&- \frac{k}{2}\boldsymbol{w}(0, t)^\top \boldsymbol{w}(0, t) - \frac{k}{2}\int_0^D e^x \boldsymbol{w}(x, t)^\top \boldsymbol{w}(x, t)dx \\
&+ \frac{k}{2}\left(\boldsymbol{w}(0, t)^\top \boldsymbol{w}(0, t) + \delta_u^2\right),
\end{aligned} \tag{35}$$

where Young's inequality and integration by parts are used. By choosing matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ such that (29) holds, we have

$$\dot{V}_2 \leq -c_4 V_2 + \frac{k}{2}\delta_u^2. \tag{36}$$

From (36) and by using the comparison lemma [25, Lemma 3.4], then for all $t \geq t_0 \geq 0$,

$$\begin{aligned}
V_2(t) &\leq V_2(t_0)e^{-c_4(t-t_0)} + \frac{k}{2c_4}\delta_u^2(1 - e^{-c_4(t-t_0)}) \\
&\leq V_2(t_0)e^{-c_4(t-t_0)} + \frac{k}{2c_4}\delta_u^2.
\end{aligned} \tag{37}$$

From (37) and (33) we have

$$E(t) \leq \frac{k}{c_4}\delta_u^2 + ke^D E(t_0)e^{-c_4(t-t_0)}, \tag{38}$$

and by using the inequality $(\|\boldsymbol{z}(t)\| + \|\boldsymbol{w}(t)\|_2)^2 \leq 2E(t)$ we get estimate (30). This shows that the target system is uniformly ultimately bounded with an ultimate bound that is directly related to the value of the quantization parameter. □

Remark 1. *From* (30)*, tracking is achieved with a bounded error proportional to the quantization.*

TABLE I: Helicopter Parameters.

Symbol	Value	Units
I_p, I_y	0.0217	kgm^2
m	1.075	kg
g	9.81	m/s^2
r	0.0038	m
$\boldsymbol{D}$	[0.007 0;0 0.0095]	kgm^2/s

V. Simulation and Experimental Results

In this section, the attitude tracking control problem is considered for the Quanser Aero helicopter system, where both simulation using MATLAB/Simulink and experiments on the helicopter system have been carried out. The initial values were set to $\boldsymbol{x}(t_0) = \boldsymbol{0}$, where t_0 defines the start of experiment. The parameters used for simulation and experiment are shown in Table I, and the design parameters were chosen as $\boldsymbol{A} = 3\boldsymbol{I}$ and $\boldsymbol{B} = 1.6\boldsymbol{I}$ and satisfies the inequality (29). The objective in the experiment was to track a given sinusoidal signal for the attitude $\boldsymbol{x}_r(t)$ in presence of both quantization and delay of the inputs.

To illustrate the performance of the proposed predictor-based controller, we first tested without the predictor and without quantization when there was a delay for the input, and so the system received the delayed inputs $\boldsymbol{u}(t - D)$, where the input vector is defined in (14). By increasing the delay, the system had more oscillation, and when $D = 0.1s$, the oscillations increased during the experiment and was stopped after about $4s$. Figs. 4–5 show the tracking of angle $q_1(t)$ and the inputs $\boldsymbol{u}(t - D)$, respectively, from this experiment. This shows that the closed-loop system becomes unstable without the predictor for delays greater or equal to $0.1s$.

The proposed control law was then tested for different delays and quantization parameters. The initial condition of the actuator state was set to $\boldsymbol{u}(x, t_0) = \boldsymbol{0}\ \forall\ x \in [0, D]$, and so the system received zero input until $t = t_0 + D$. The results from simulation and experiment, where the quantization parameters were set to $l = 0.01$, $u_0 = l/2$ and with a time delay

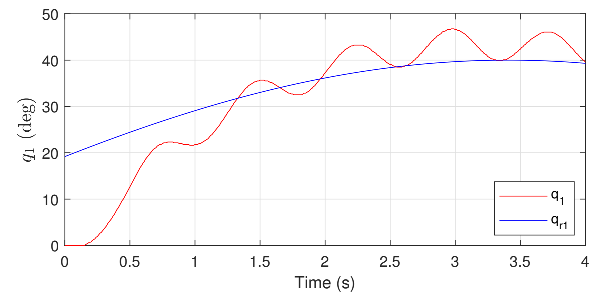

Fig. 4: Tracking of angle $q_1(t)$, with delay, without predictor.

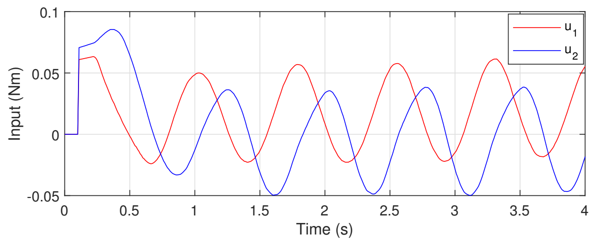

Fig. 5: Input $\boldsymbol{u}(t-D)$ without predictor.

$D = 0.2s$, are shown in Figs. 6–11, showing tracking of the angles $\boldsymbol{q}(t)$, the tracking errors $\boldsymbol{z}(t)$ and the inputs $\boldsymbol{u}^Q(t-D)$, respectively, where the red plots are from simulation and the blue plots are from experiment.

From Figs. 6–9 we can see that the desired trajectory can be followed both in simulation and when tested on the helicopter, illustrating our main results in Theorem 1. From the simulation, there are only small tracking errors that are due to the quantization. From the experiment, the tracking errors are higher relative to the simulations due to several other disturbances to the system such as unmodeled dynamics and sensor noise that affects the performance, and the helicopter have a practical stabilization with this controller.

To compare results for different delays and quantization

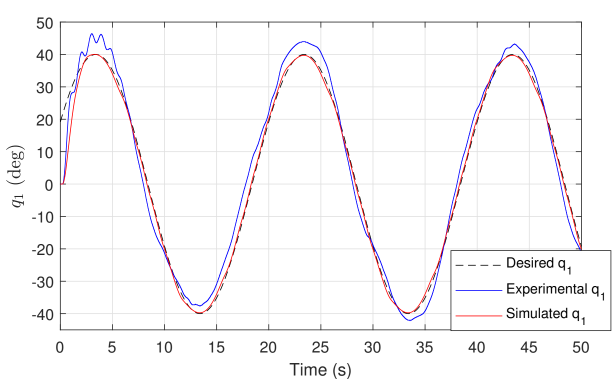

Fig. 6: Tracking of angle $q_1(t)$ from simulation and experiment with delay $D = 0.2$s and quantization.

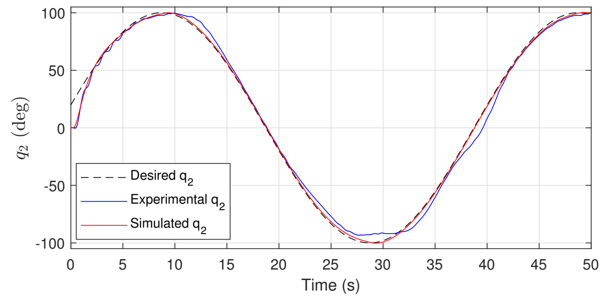

Fig. 7: Tracking of angle $q_2(t)$ from simulation and experiment with delay $D = 0.2$s and quantization.

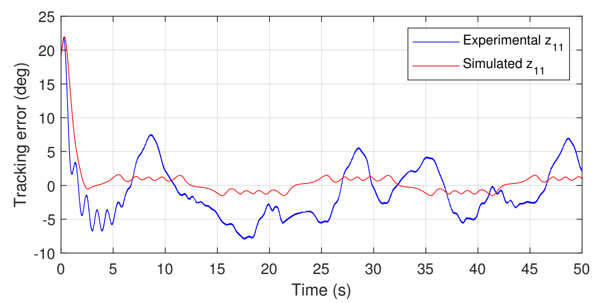

Fig. 8: Tracking error $z_{11}(t)$ from simulation and experiment with delay $D = 0.2$s and quantization.

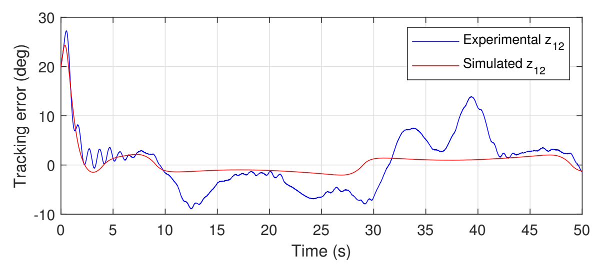

Fig. 9: Tracking error $z_{12}(t)$ from simulation and experiment with delay $D = 0.2$s and quantization.

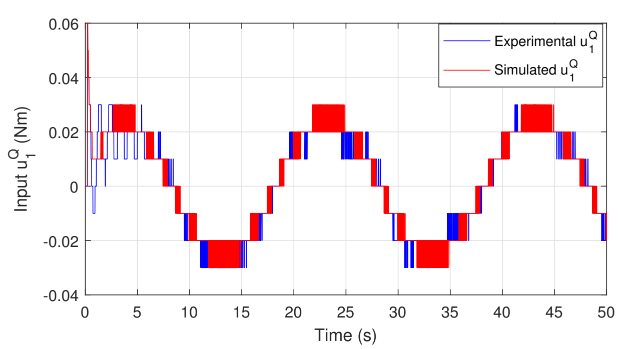

Fig. 10: Input $u_1^Q(t-D)$ from simulation and experiment with delay $D = 0.2$s and quantization.

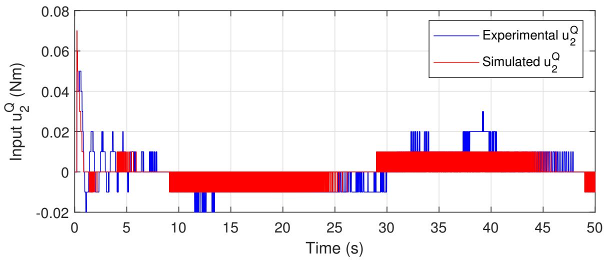

Fig. 11: Input $u_2^Q(t-D)$ from simulation and experiment with delay $D = 0.2$s and quantization.

TABLE II: Total tracking error from experiment with and without delay and quantization. System receives input $\boldsymbol{u}^Q(t-D)$.

Experiment		**Quantization**			
z_{track} $\times 10^{-4}$		No q.	$l = 0.010$	$l = 0.012$	$l = 0.014$
Delay	$D = 0$	3222	3271	3305	3490
	$D = 0.1$	4866	5278	5741	5689
	$D = 0.2$	7274	8692	8518	9114
	$D = 0.3$	13748	11788	13868	14038

parameters, the total tracking error was defined as

$$z_{\text{track}} = \int_{t_0}^{t_f} \boldsymbol{z}_1^\top \boldsymbol{z}_1 d\tau, \tag{39}$$

where t_0 and t_f define start and end of experiment, respectively, and the experiments were run for 50 s where results are provided in Table II. From the results we see that by increasing the delay, the total tracking error also increases for the helicopter system, for mainly two reasons. First, since the system receives no input until D seconds after the start of experiment, the total tracking error increases during an initial time period, since the reference signal is changing while the helicopter remains stationary. Then, the system receives control input by the predictor based controller and starts tracking the desired signal. So increasing D, increases the time before control kicks in, and z_{track} increases initially. Secondly, because the model is not perfect and from other effects such as measurement errors, the tracking error increases by an increase in the delay. From a perfect model without quantization, the total tracking error will not increase after an initial time period since then $\boldsymbol{z}_1$ becomes zero.

The effect of quantization is also shown, where by increasing the quantization, the measurement of the total tracking error increases. This is also affected by other disturbances.

VI. Conclusion

In this paper, the attitude tracking control problem of a nonlinear system with networked induced delay and quantization for the inputs has been considered. A predictor-feedback controller is proposed to compensate for the input delay. Based on a Lyapunov approach, stability of the closed loop system is ensured and tracking of a desired reference signal is achieved with a bounded tracking error that is directly related to the quantization parameter. Simulations and experiments illustrate the proof.

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