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# Decisions under Ignorance and the Individuation of States of Nature

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**Abstract:** How do you make decisions under ignorance? That is, how do you decide when you lack subjective probabilities for some of your options' possible outcomes? One answer is that you follow the Laplace Rule: you assign an equal probability to each state of nature for which you lack a subjective probability (that is, you use the Principle of Indifference) and then you maximize expected utility. The most influential objection to the Laplace Rule is that it is sensitive to the individuation of states of nature. This sensitivity is problematic because the individuation of states seems arbitrary. In this paper, however, I argue that this objection proves too much. I argue that all plausible rules for decisions under ignorance are sensitive to the individuation of states of nature.

**Key words:** ignorance, the Principle of Indifference, states of nature, individuation, Pierre-Simon Laplace, John Milnor

How do you make decisions under ignorance? That is, how do you decide when you lack subjective probabilities for some of your options' possible outcomes (so you can't assess their expected utility)?[1] A classic answer is that you choose an option that is at least as preferred as any available option after applying

> *The Laplace Rule*: In the absence of any subjective probabilities for some states of nature, assign an equal probability to each of these states (*the Principle of Indifference*). Then, using these assigned probabilities, option $x$ is at least as preferred as option $y$ if and only if $x$ has at least as great expected utility as $y$.[2]

The Laplace rule, however, suffers from a well-known problem: It is sensitive to how states of nature are individuated. The problem is that the individuation of states of nature seems arbitrary, so, if the Laplace Rule depends on that individuation, its ranking of options will be arbitrary too.[3] More formally, the Laplace Rule violates

> *State-Individuation Invariance*: Whether option $x$ is at least as preferred as option $y$ does not depend on whether a state of nature is split into two duplicate states.[4]

Consider, for instance, the following decision under ignorance, where the numbers represent the utilities of the outcomes of the available options $o_1$ and $o_2$ in the epistemically possible states of nature $S_1$ and $S_2$ for which the agent lacks subjective probabilities:

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*Situation One*

| | $S_1$ | $S_2$ |
|---|---|---|
| $o_1$ | 8 | 2 |
| $o_2$ | 1 | 7 |

Applying the Principle of Indifference, we assign an equal probability to each epistemically possible state of nature: $P(S_1) = P(S_2) = 1/2$. The Laplace Rule then entails that $o_1$ is preferred to $o_2$, because the expected utility of $o_1$ is $8 \cdot 1/2 + 2 \cdot 1/2 = 5$ and the expected utility of $o_2$ is $1 \cdot 1/2 + 7 \cdot 1/2 = 4$. But suppose we individuate states of nature more finely—so that we split $S_2$ into two separate states, $S_2'$ and $S_2''$:

*Situation Two*

| | $S_1$ | $S_2'$ | $S_2''$ |
|---|---|---|---|
| $o_1$ | 8 | 2 | 2 |
| $o_2$ | 1 | 7 | 7 |

This time, when we apply the Principle of Indifference and assign an equal probability to each epistemically possible state of nature, we get $P(S_1) = P(S_2') = P(S_2'') = 1/3$. And then the Laplace Rule entails that $o_2$ is preferred to $o_1$, because the expected utility of $o_1$ is $8 \cdot 1/3 + 2 \cdot 1/3 + 2 \cdot 1/3 = 4$ and the expected utility of $o_2$ is $1 \cdot 1/3 + 7 \cdot 1/3 + 7 \cdot 1/3 = 5$. Hence splitting $S_2$ into two duplicate states of nature reverses the Laplace Rule's ranking of the options.

Ken Binmore takes this sensitivity to individuation to be a fatal problem for the Laplace Rule. If we're completely ignorant about the states of nature, he asks, why should we accept a principle that tells us that duplicated states correspond to different but equi-probable states of nature?[5] State-Individuation Invariance is, according to Binmore, one of the indispensable axioms for decisions under ignorance, along with the following principles:[6]

> *Expansion Consistency*: Whether option $x$ is at least as preferred as option $y$ does not change if another option is added to the situation.[7]

> *Pairwise State Symmetry*: If $x$ and $y$ are the only available options and the outcome of $x$ is just like the outcome of $y$ except for a permutation of two states of nature such that neither state is subjectively more probable than other, then $x$ is equally preferred as $y$.[8]

> *Transitivity*: If option $x$ is at least as preferred as option $y$ and $y$ is at least as preferred as option $z$, then $x$ is at least as preferred as $z$.[9]

> *The Weak Principle of Statewise Dominance*: If the outcome of option $x$ is preferred to the outcome of option $y$ in every epistemically possible state of nature, then $x$ is preferred to $y$.[10]

In this paper, I will show that, given a compelling strengthening of the Weak Principle of Statewise Dominance, these indispensable axioms rule out State-Individuation Invariance. So it's not only the Laplace Rule that is sensitive to the individuation of states of nature: *Any* plausible principle for how to make decisions under ignorance is so too. Hence the main objection to the Laplace Rule proves too much, since it also rules out all plausible rivals.

Instead of the Weak Principle of Statewise Dominance, we will rely on

> *The Strong Principle of Statewise Dominance*: If the outcome of option $x$ is at least as preferred as the outcome of option $y$ in every epistemically possible state of nature and the outcome of $x$ is preferred to the outcome of $y$ in some epistemically possible state of nature, then $x$ is preferred to $y$.[11]

This principle is stronger than the Weak Principle of Statewise Dominance, but both principles are supported by the same kind of thought: An option that is at least as preferred as another option in all epistemically possible states of nature and strictly preferred in at least one of these states to the other option has no disadvantage compared to the other option but at least one advantage; hence it should be preferred.

We will prove the following impossibility theorem:

> The following conditions cannot all be true:
>
> - Expansion Consistency,
> - Pairwise State Symmetry,
> - State-Individuation Invariance,
> - the Strong Principle of Statewise Dominance, and
> - Transitivity.

So no rule could satisfy all of these conditions. Therefore, even if we reject the Laplace Rule, it's still implausible to accept State-Individuation Invariance, because Expansion Consistency, Pairwise State Symmetry, the Strong Principle of Statewise Dominance, and Transitivity are all plausible.[12]

For the proof, we will consider a series of choice situations.[13] Each one is a decision under ignorance—the agent lacks subjective probabilities for the epistemically possible states of nature.

First, consider the following situation:

*Situation Three*

| | $S_1$ | $S_2$ |
|---|---|---|
| $o_1$ | 2 | 1 |
| $o_2$ | 1 | 2 |

We have, by Pairwise State Symmetry,

(1) In Situation Three, $o_1$ is equally preferred as $o_2$.[14]

Next, consider the following situation, which is just like Situation Three except that we have split $S_2$ into two separate states of nature, $S_2'$ and $S_2''$:

*Situation Four*

| | $S_1$ | $S_2'$ | $S_2''$ |
|---|---|---|---|
| $o_1$ | 2 | 1 | 1 |
| $o_2$ | 1 | 2 | 2 |

From (1), we have, by State-Individuation Invariance,

(2) In Situation Four, $o_1$ is equally preferred as $o_2$.

Now, consider the following situation, which is just like Situation Four except that we have added another option:

*Situation Five*

| | $S_1$ | $S_2'$ | $S_2''$ |
|---|---|---|---|
| $o_1$ | 2 | 1 | 1 |
| $o_2$ | 1 | 2 | 2 |
| $o_3$ | 2 | 2 | 1 |

From (2), we have, by Expansion Consistency,

(3) In Situation Five, $o_1$ is equally preferred as $o_2$.

Finally, consider the following situation, which is just like Situation Five except that we have removed $o_1$:

*Situation Six*

| | $S_1$ | $S_2'$ | $S_2''$ |
|---|---|---|---|
| $o_2$ | 1 | 2 | 2 |
| $o_3$ | 2 | 2 | 1 |

We have, by Pairwise State Symmetry,

(4) In Situation Six, $o_2$ is equally preferred as $o_3$.

From (4), we have, by Expansion Consistency,

(5) In Situation Five, $o_2$ is equally preferred as $o_3$.

From (3) and (5), we have, by Transitivity,

(6) In Situation Five, $o_1$ is equally preferred as $o_3$.

But (6) violates the Strong Principle of Statewise Dominance. So we find that Expansion Consistency, Pairwise State Symmetry, State-Individuation Invariance, the Strong Principle of Statewise Dominance, and Transitivity cannot all be true.

This result, of course, does not dissolve the problem of sensitivity to the individuation of states of nature. It just shows that it is a problem for all plausible rules for decisions under ignorance. The problem is that, if the rule's ranking of options depends on the individuation of states of nature and this individuation is arbitrary, then the rule's ranking of options is arbitrary too. This problem would be solved if we found a non-arbitrary principle of individuation for states of nature. Could we find one? A compelling idea is to adopt the following principle for the individuation of outcomes:

> *The Principle of Individuation by Rational Indifference*: Outcome $x$ should be treated as the same as outcome $y$ if and only if it is rationally required to be indifferent between $x$ and $y$.[15]

Then we could adopt the following principle for the individuation of states of nature:

> *The Principle of Individuation by Outcomes*: State of nature $S$ should be treated as distinct from state of nature $S'$ if and only if some possible option has different outcomes in $S$ and $S'$.

We need *possible* here rather than *available*, because if we claimed that *states of nature should be distinguished as different if and only if some available option has different outcomes in the states* then, given that our preferences over options are sensitive to the individuation of states of nature, we would violate Expansion Consistency. We would do so, because the individuation of states of nature would depend on what options are available in the situation and our preferences over options would depend on the individuation of states of nature.

A problem is that this proposal would probably explode the number of states of nature, because, for any arbitrary split of a state of nature, there would probably (depending on what it's rational to prefer) be some possible act that has different outcomes in the new states. The point of this paper, however, isn't to solve the problem of how to individuate states of nature. The point is merely that that problem is a problem for all plausible proposals for how to make decisions under ignorance—and not just the Laplace Rule.[16]

## NOTES

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1. Decisions under ignorance are sometimes called decisions under *uncertainty* (see, for example, Knight 1921: 20 and Luce and Raiffa 1957: 13). This is an unfortunate terminology, because 'decisions under uncertainty' merely suggest decisions that aren't decisions under certainty (where the agent is certain what outcome each option would result in). This would seem to include both decisions under ignorance and decisions under risk (where the agent has subjective probabilities for the possible outcomes of the options).
2. Milnor 1954: 49–50. The rule is named after Laplace (1774: 653; 1986: 378) who proposed the Principle of Indifference. See also Keynes 1921: 41–42.
3. See the examples in Bertrand 1889: 4–5, Keynes 1921: 42–44, and van Fraassen 1989: 302–317.
4. Milnor 1954: 52, Arrow and Hurwicz 1972: 5, and Binmore 2009: 158. Milnor and Binmore call this principle 'Column Duplication.' Barberà and Jackson (1988: 37) call it 'Independence of Duplicate States.'
5. Binmore 2009: 159. See also Rawls 1974: 649–650.
6. Binmore 2009: 158.
7. Milnor (1954: 51) and Binmore (2009: 157) call this principle 'Row Adjunction.' See also Nash 1950: 159, Radner and Marschak 1954: 63, and Sen 1969: 384.
8. Milnor's (1954: 51) and Binmore's (2009: 157) 'Symmetry' principle is slightly stronger than Pairwise State Symmetry, because it lacks the weakening to pairwise permutations. (The stronger condition follows from the weaker one given Expansion Consistency and Transitivity.) See also Chernoff 1954: 429 and Arrow and Hurwicz 1972: 5. The idea behind Pairwise State Symmetry is that the agent doesn't care whether a pay-off occurs in one of two states of nature rather than the other if none of these states are subjectively more probable than the other. Strictly, Milnor's (1954: 51) principle only requires that the *ordering* (that is, the transitive and complete ranking) of options is unaffected by the permutation of states, so his condition wouldn't say anything about the case where *x* and *y* are unordered—that is, related by a preferential gap. So, without assuming Completeness (see note 9), could *x* and *y* be related not by indifference but by a preferential gap? It seems that they couldn't be so rationally. When outcomes only differ with respect to a permutation of two states of nature (neither of which is subjectively more probable than the other), any advantage or detriment of one of the outcomes is perfectly matched by an equivalent advantage or detriment for the other. So there seems to be no rational basis for a preferential gap in this case. Nevertheless, we could avoid this issue by weakening Pairwise State Symmetry so that, rather than that *x* is equally preferred as *y*, it only requires that neither *x* nor *y* is strictly preferred to the other. And then we could assume Completeness separately. This wouldn't change the overall dialectic, because Binmore (2009: 157) regards 'Ordering' (which entails Completeness) as indispensable. Likewise, the main rivals to the Laplace Rule—among others, the Maximin Rule, the Minimax-Regret Rule, and the Hurwicz Rule—also entail Completeness.
9. Arrow 1951: 13 and Jensen 1967: 171. Milnor (1954: 51) and Binmore (2009: 157) rely on the stronger 'Ordering' principle, which also entails *Completeness*—that is, the principle that option *x* is at least as preferred as option *y* or *y* is at least as preferred as *x*. See Chang 1997 for an overview of the main worries about Completeness.
10. Milnor 1954: 51 and Binmore 2009: 157. Milnor and Binmore call this principle 'Strong Domination.' This allows for an ambiguity whether 'Strong'/'Weak' modifies the strength of the dominance or the logical strength of the principle. I prefer to call the principle 'the *Weak Principle* of Statewise Dominance,' because it is logically

weaker than the Strong Principle of Statewise Dominance. In my usage, it should be clear that 'Strong'/'Weak' modifies the logical strength of the principle.

11. This is a widely accepted principle. See, for example, Savage 1951: 58, Milnor 1954: 55, Luce and Raiffa 1957: 287, Nozick 1969: 118, and Barberà and Jackson 1988: 37.
12. The Maximin Rule (Wald 1950: 18) violates the Strong Principle of Statewise Dominance. The Leximin Rule (Sen 1970: 138n12 and Pattanaik and Peleg 1984: 114–117) violates State-Individuation Invariance. The Protective Criterion (Barberà and Jackson 1988: 37) violates Transitivity. The Hurwicz Rule (Milnor 1954: 50) violates the Strong Principle of Statewise Dominance. And the Minimax-Regret Rule (Savage 1951: 59) violates Expansion Consistency and the Strong Principle of Statewise Dominance.
13. So, technically, we also need to assume the existence of these choice situations.
14. It may perhaps seem that Pairwise State Symmetry implicitly assumes the Laplace Rule, because, *if* we had subjective probabilities for the states of nature in Situation Three, we should only be indifferent between the options in case we regarded the states as equally likely. So it may seem that Pairwise State Symmetry requires that we form our preferences as if we assigned equal probabilities to the states. But note that the main rivals to the Laplace Rule—among others, the Maximin Rule, the Minimax-Regret Rule, and the Hurwicz Rule—also entail Pairwise State Symmetry (see note 12 and Milnor 1954: 52–55). And the fact that these theories tell us to act as if we assigned equal probabilities to the states of nature in Situation Three (and in Situation Six) doesn't mean that they tell us to act as if we assigned equal probabilities to the states of nature in other situations where we decide under ignorance. So we see that Pairwise State Symmetry does not implicitly assume the Laplace Rule in general.
15. Gustafsson 2022: 14. This is a variation of Broome's (1990: 140; 1991: 103) following proposal:

    *The Principle of Individuation by Justifiers*: Outcome $x$ should be treated as distinct from outcome $y$ if and only if it is rationally permitted to have a preference between $x$ and $y$.

    The main advantage of the Principle of Individuation by Rational Indifference over the Principle of Individuation by Justifiers is that it allows that outcomes are distinct in case it is rationally required to have a preferential gap between them.
16. It may be objected that we could sidestep this problem for the Laplace Rule by instead relying on an outcome-focused variation of the Principle of Indifference that says that, in the absence of any subjective probabilities for some possible outcomes of an option, assign an equal probability to each of these outcomes. But consider Situation Five. Suppose that the outcomes with utility 1 are the same and, likewise, that the outcomes with utility 2 are the same. Then, by this variant of the Laplace Rule, we find that $o_1$ is equally preferred as $o_3$. So it implausibly violates the Strong Principle of Statewise Dominance.

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