NUST Journal of Engineering Sciences, Vol. 1, No. 1, 2008, pp.12-17

Analysis and Simulation of a Pan Tilt Platform Based on Linear Model

Imran S. Sarwar, Afzaal M. Malik
College of Electrical & Mechanical Engineering
National University of Sciences and Technology, Islamabad, Pakistan
imrande20@hotmail.com, mafzmlk@hotmail.com

Abstract

This research paper deals with the modeling, analysis and simulation of a two degree of freedom pan tilt platform (PTP) for rapidly positioning a camera. The PTP, with 2-revolute joints, is a device that makes possible for the camera to point in a desired direction. The objective of this paper is to derive a mathematical model of the PTP to point in a desired direction. To achieve the objective, a feedback control system with PD controller was analyzed and simulated in section III. The PTP was used on unmanned aerial vehicle to perform visual tracking experiments for moving objects. There is a growing demand for control systems with motion tracking ability. The practicality of this project can be extended to a broad range of applications. Of these, the most apparent use is in defense, where there is a strong emphasis on reliably neutralizing threats without risking human life. Unmanned aerial systems can provide significant reductions in manpower and risk to humans for critical security and defense roles. The applications of PTP in defense and security applications have been increased significantly in recent years. Such applications include target acquisition, intelligence, surveillance and reconnaissance, border patrol, maritime security, search and rescue, and environmental monitoring. Also, several applications for this device exist both in law enforcement and in entertainment.

Keywords: Linear Control System, PD Controller, Pan Tilt Platform, Pan Mechanism, Tilt Mechanism, Simulation.

Introduction

The schematic of the PTP has two degree of freedom as shown in fig. 1. The movement about the vertical axis, called pan and the movement about horizontal axis, called tilt are explained in the figure. The complete geometric structure of the PTP is presented in fig. 2. It can continuously revolve about the pan axis and 90 degrees of motion range in the tilt axis.

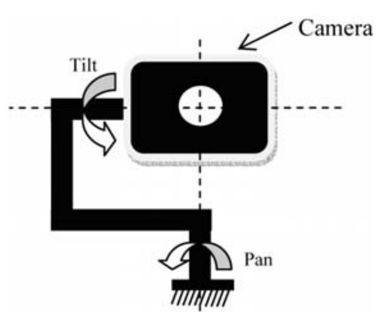


Fig. 1. PTP Schematics

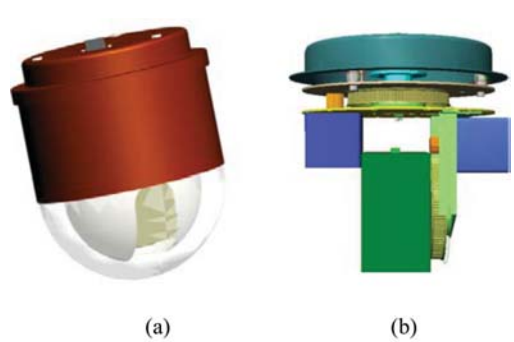


Fig. 2. The PTP with (a) cover (b) without cover

We have built linear models for pan and tilt mechanisms. In the linear model Coulomb friction, centrifugal and coriolis forces have been neglected. The parameters (friction, inertia) were identified from experiments or obtained from computer aided design for modeling and simulation purposes. The feedback control systems along with proportional-derivative (PD) controllers were designed according to requirements, analyzed and simulated in Section III. The controllers were designed according to the following allowable specifications

1. The settling time (Ts) is expected to be achieved within .1 to .5 seconds.
2. The steady state error (ess) can be tolerated within ± 2%.
3. The %overshoot (%OS) is expected to be kept below 22%.

The results are presented in section IV.

System's linear model

The basis for the model of the system is the Lagrange-Euler equation [2]:

$$u = M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + F(\dot{\theta}) + g(\theta) \quad (1)$$

Here $u = \tau$ (torque) is control input and θ is a feed-back input vector representing the joint orientation, $M(\theta)$ is a symmetric inertia matrix and $C(\theta,\dot{\theta})$ accounts for centrifugal and coriolis forces. The term

Corresponding Author: Afzaal M. Malik
NUST Publishing, © (2008), ISSN: 2070-9900

$F\dot{\theta}$ accounts for the viscous friction. The term $g(\theta)$ accounts for gravity forces.

The term $C(\theta,\dot{\theta})$ is effective when the mass centre begins to move away from the centre of rotation. The effect of centrifugal force becomes evident on the distributed mass of the body. Therefore, centre of PTP is located on the rotational axis of the body and term $C(\theta,\dot{\theta})$ is neglected in linear model. The non-linear equation is given as

$$\tau = J_{eff}\ddot{\theta} + f_v\dot{\theta} + f_c\,\mathrm{sgn}(\dot{\theta}) \tag{2}$$

Here,, Coulomb friction. τ = Torque, J_{eff} = Effective Inertia, f_v = Viscous Friction, f_c = Coulomb Friction

This is a general equation for a joint and represents the non-linear model of pan / tilt mechanism. The viscous friction is a frictional force that resists objects in motion. The viscous friction is actually a property of the medium in which the motion of the object is occurring. Any fluid medium, such as the grease in the bearings or the air, has an internal resistance to flow, which is represented by the viscous friction f_v . The effective inertial loads of the system are computed from the following relation [2].

$$J_{eff} = \frac{J_a + J_m}{n} + nJ_L \tag{3}$$

J_a = Actuator Inertia. J_m = Gear Inertia, J_L = Load Inertia, n = Gear Ratio

Equations (2) and (3) lead to

$$\tau = \left(\frac{J_a + J_m}{n} + nJ_L\right)\ddot{\theta} + f_v\dot{\theta} + f_c\,\mathrm{sgn}(\dot{\theta}) \tag{4}$$

This nonlinear equation needs to be linearized in order to develop a controller for the system. The nonlinear term in equation (3) is Coulomb friction. It is a frictional force that exists between two objects that are in contact. The close proximity of their surfaces acts to prevent the motion of the objects. Therefore, it is not present in the linear model of the system. Linearization simplifies the pan/tilt model by focusing on each angle (θ_{pan}, θ_{tilt}) independently, the linearized model is

$$\tau = J_{eff}\ddot{\theta} + f_v\dot{\theta} \tag{5}$$

This second order differential equation can be expressed in state-space form by introducing the following state variables , $x_1 = \theta$, $x_2 = \dot{\theta}$ with the derivatives as $\dot{x}_1 = \dot{\theta}$, $\dot{x}_2 = \ddot{\theta}$. Thus the state equations are

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{f_v}{J_{eff}}x_2 + \frac{u}{J_{eff}} \tag{6}$$

The output equation is

$$y = x_1 \tag{7}$$

Hence the state space model in vector matrix form is as follows

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{f_v}{J_{eff}} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{J_{eff}} \end{bmatrix} u \tag{8}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \tag{9}$$

Here

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}, \quad u = \tau, \quad A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{f_v}{J_{eff}} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ \frac{1}{J_{eff}} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The unknowns in the state space model are f_v and . J_{eff} The viscous friction has been determined experimentally. A script was run to determine the viscous friction. The values of viscous friction are presented in table 1. The forward and reverse frictions values are averaged to one value.

Table 1. Viscous friction

	Positive	Negative
f_v (tilt) (Nms/rad)	0.0019	0.0017
f_v (pan) (Nms/rad)	0.005	0.005

To find the effective inertial loads in the system, we have used a combination of calculation and measurement. The load of the basic pan and tilt system was provided in the CAD model. We were able to determine the load on each axis in CAD software by adding our additional components with the proper weights/densities. The gear inertia was measured separately in CAD software and the actuator inertia was determined from the motor datasheet. Effective load on each axis is calculated by using equation (2), as in the table 2

Table 2. Inertial loads for pan and tilt mechanism

	J_L (Kg m^2)	J_m (Kg m^2)	J_a (Kg m^2)	n	J_{eff} (Kg m^2)
Pan mechanism	0.0750	4×10^{-7}	1.6×10^{-6}	$\frac{120}{21}$	**0.4286**
Tilt mechanism	0.0055	4×10^{-7}	1.6×10^{-6}	$\frac{120}{21}$	**0.0314**

The transfer function for tilt mechanism is found using (8), (9), table 1, 2, and $G(s) = C(sl - A)^{-1}B$, [9] is given below

$$G(s) = \frac{31.6188}{s^2 + 0.1581s} \tag{10}$$

Similarly, the transfer function for pan mechanism is presented below

$$G(s) = \frac{2.3333}{s^2 + 0.0042s} \tag{11}$$

Linear control system

The linear tilt system and linear pan system were controlled by PD controllers as shown in fig. 3.

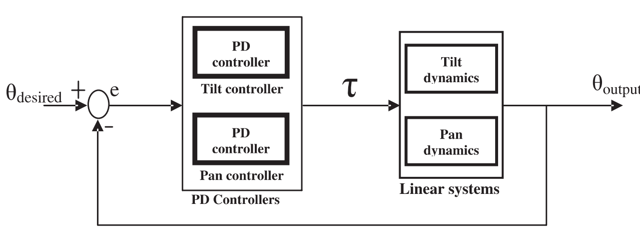


Fig. 3. PTP feedback control system

The tilt system analysis and simulation is shown in section III (a). Similarly, the pan system analysis and simulation is presented in section III (c). The compensated tilt system (tilt mechanism and PD controller) analysis and simulation via the root locus method is presented in section III (b). The compensated pan system (pan mechanism and PD controller) analysis and simulation via the root locus method is presented in section III (d).

Tilt system response

According to the root locus technique [9], tilt system (10) has two branches of root locus, symmetrical with respect to the real axis, real-axis segment is [0,-.1581], starting points are the open-loop poles at 0 and -.1581, ending points are the open-loop zeros at ∞ (infinity),-∞, real-axis Intercept is at -0.079, angle of asymptotes are 90°, 270° and breakaway point is at -0.079. The result of root-locus technique, based on simulation performed in Matlab is shown in fig. 4.

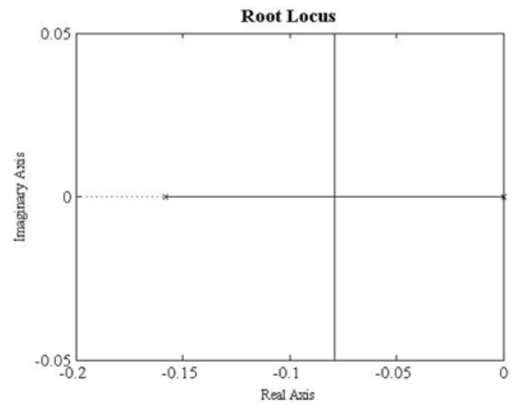


Fig. 4. Root locus of pan system

According to the allowable specification; the %OS is less than 22%. We had selected %OS equal to 5% for which damping ratio (ξ) is equal to 0.6901. The gain (K) has to be designed as shown schematically in fig. 5. The open-loop transfer function using equation (10) is given as

$$KG(s) = \frac{31.6188K}{s^2 + 0.1581s} \tag{12}$$

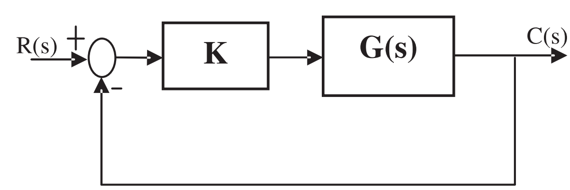


Fig. 5. Closed loop system with gain K

The root locus for the uncompensated system with a damping ratio of 0.69 is represented by a radial line shown in fig. 6(a). We have found dominant pair of poles at -0.0790 ± 0.0830i along the damping ratio line for a gain K = 0.00041494. The uncompensated system step response is shown in fig. 6(b). The closed loop transfer function (T(s)) based on (12) is as follows

$$T(s) = \frac{0.01314}{s^2 + 0.1581s + 0.01314} \tag{13}$$

The tilt system response was calculated, and the simulated results are Closed-loop poles (CLP), K, T_s, Peak time (T_p), natural frequency (w_n), K_v and e_{ss} equal to -0.0790 ± 0.0830i, 4.1548e-004, 50.6009 seconds, 37.8513 seconds, 0.1146 rad/sec, 0.0831 and 12.0342 respectively. As noticed in fig. 6(b), the setting time (50.6 sec) and the steady-state error (12.0342) far exceed the desired performance values. This deficiency has to be compensated by designing the control system.

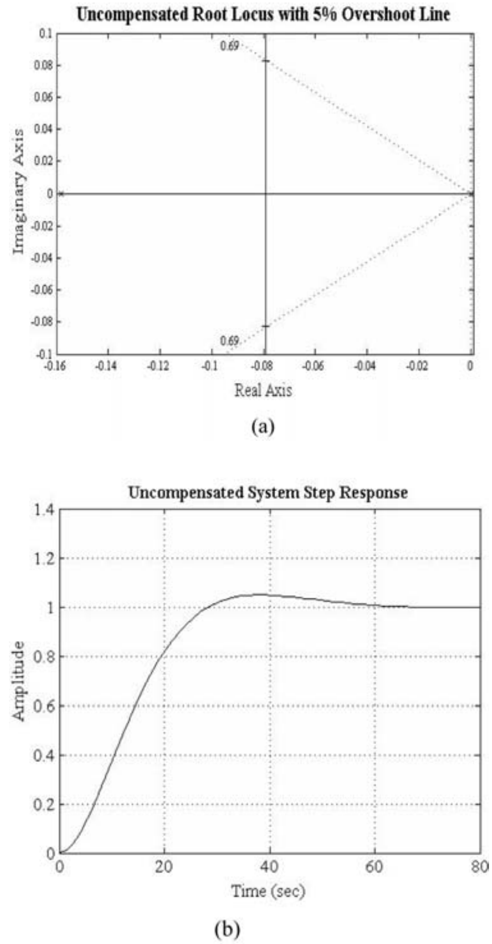


Fig. 6. Uncompensated system (a) Root locus with radial line (b) step response

Response of tilt system with PD controller

The objective of a PD controller is to drive the T_s to less than 0.5 sec for the unity feedback system. The compensated system is schematically shown in figure 7.

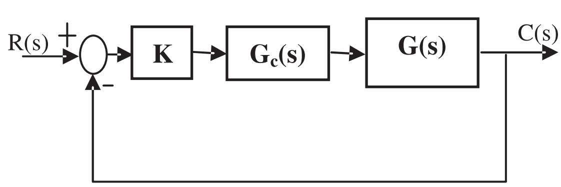


Fig. 7. Closed loop system with compensator

The PD controller is found by using [9]. For the PD controller, the calculated w_n, the desired dominant pole location (DPL), the PD angle contribution (AC) and zero location are presented in table 3.

Table 3. Calculated values

	w_n (rad/sec)	DPL	AC	**Zero location** (z_c)
Tilt mech-anism	28.9855	$-20.0028 \pm 20.9799i$	87.0 501°	**21.0808**

The PD controller ($G_c(s)$) in Laplace domain is equal to

$$G_c(s) = s + z_c$$
$$G_c(s) = s + 21.0808$$

The open-loop transfer function is given as

$$KG(s)G_c(s) = K\frac{31.62s + 666.6}{s^2 + 0.1581s} \tag{14}$$

The root locus for the compensated system is shown in fig. 8 (a). A damping ratio of 0.69 is represented by a radial line drawn on the root-locus. We have found dominant pair of poles at -19.3414 ±20.9295i along the damping ratio line for a gain K = 12.184. The compensated system step response is shown in fig. 8 (b). The closed loop transfer function (T(s)) based on (14) is as follows

$$T(s) = \frac{38.21s + 805.6}{s^2 + 38.37s + 805.6} \tag{15}$$

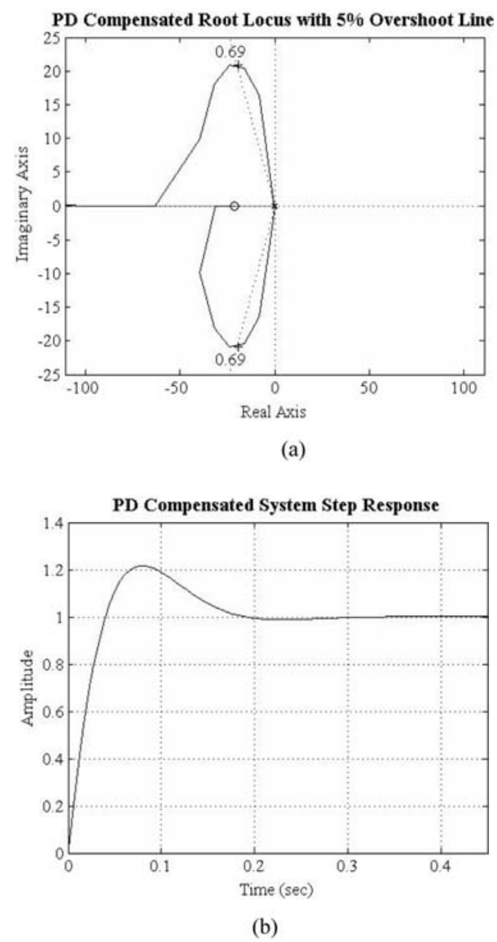


Fig. 8. Compensated system (a) Root locus with radial line (b) step response

The tilt system with PD controller response was simulated; results are CLP, K, T_s, T_p, w_n, K_v and e_{ss} equal to -19.3414 ±20.9295i, 12.184, 0.2068 seconds, 0.1501 seconds, 28.4979 rad/sec, 5.1368×10^{3} and 1.9467×10^{-4} respectively. As noticed in fig. 8(b), the setting time (0.2068 sec) and the steady-state error (1.9467×10^{-4}) meet the desired performance values.

Pan system response

According to the root locus technique [9], pan system (11) has two branches of root locus, symmetrical with respect to the real axis, real-axis segment [0,-.0042] , starting points are open-loop poles at 0, -.0042, ending points are open-loop zeros at ∞, -∞, real-axis intercept was at -0.0021, angle of asymptotes are 90°,270° and breakaway point is at -0.0021. The result of root-locus method, based on simulation performed in Matlab is shown in fig. 9.

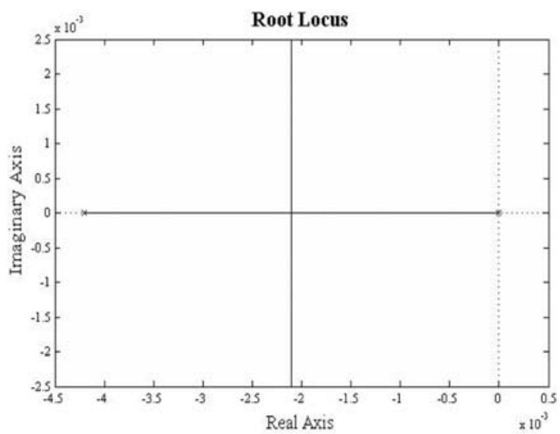


Fig. 9. Root locus of pan system

According to the allowable specification, the %OS is less than 22%. We had selected %OS equal to 5% for which damping ratio (ξ) is equal to 0.6901. The gain (K) has to be designed as shown schematically in fig. 5. The open-loop transfer function using equation (11) is given as:

$$KG(s) = \frac{2.3333K}{s^2 + 0.0042s} \tag{16}$$

We have found dominant pair of poles -0.0021± 0.0022i along the damping ratio line for a K equal to 3.9526×10^{-6}. The uncompensated system step response is shown in fig. 10 (b). The uncompensated system step response is shown in fig. 10(b). The closed loop transfer function (T(s)) based on (12) is as follows

$$T(s) = \frac{9.223\times10^{-6}}{s^2 + 0.0042s + 9.223\times10^{-6}} \tag{17}$$

Pan system response was simulated; results are CLP, K, T_s, T_p, w_n, K_v and e_{ss} equal to -0.0021± 0.0022i, 3.9526×10^{-6}, 1.9048×10^{3} seconds, 1.4320e+003 seconds, 0.0030 rad/sec, 0.0022 and 455.3986 respectively. As noticed in fig. 10 (b), the setting time (1.9048×10^{3} sec) and the steady-state error (455.3986) far exceed the desired performance values. This deficiency has to be compensated by designing the control system.

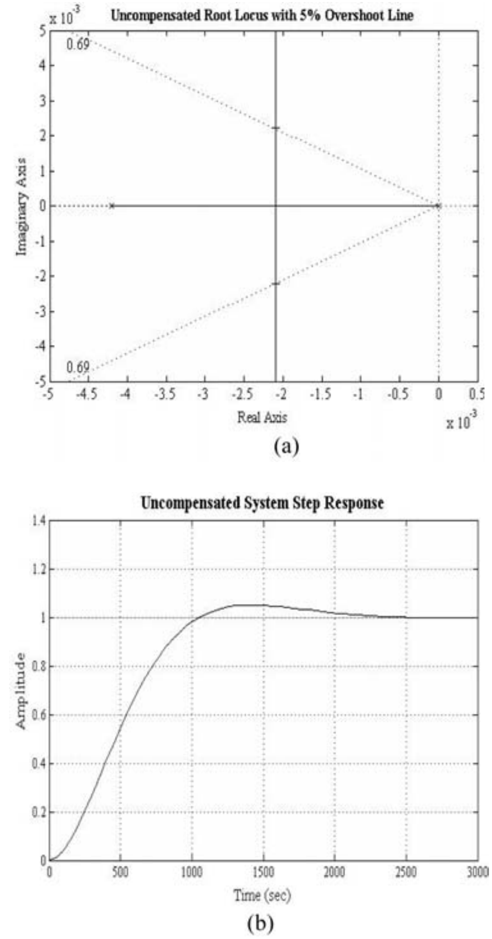


Fig. 10. Uncompensated system (a) root-locus with radial line (b) step response

Response of Pan system with PD Controller

The objective of a PD controller is to derive the T_s less than 0.5 sec for the unity feedback system. The compensated system is shown in figure 7. The PD controller is found by using [9]. For the PD controller w_n DPL, AC and zero location are calculated and presented in table 4.

Table 4. Calculated values

	w_n (rad/sec)	*DPL*	*AC*	**Zero location (z_c)**
Pan mech-anism	28.9855	$-20.0028 \pm 20.9799i$	87.2711°	21

The PD controller ($G_c(s)$) in Laplace domain is equal to

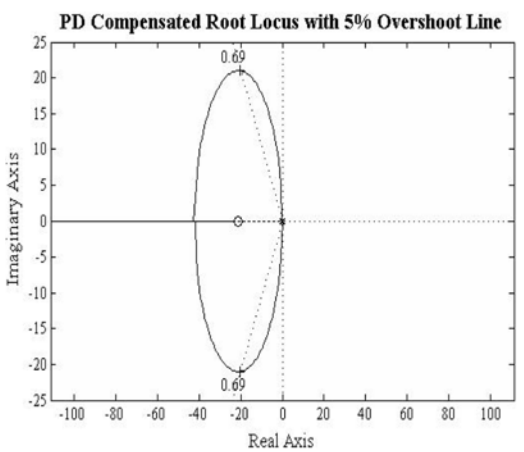
$$G_c(s) = s + z_c$$
$$G_c(s) = s + 21$$

The open-loop transfer function is given as:

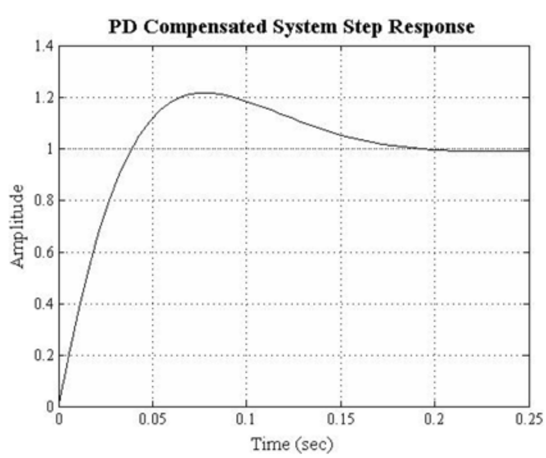
$$KG(s)G_c(s) = K\frac{2.333s + 49}{s^2 + 0.0.0042s} \quad (18)$$

The root locus for the compensated system is shown in fig. 11(a). A damping ratio of 0.69 is represented by a radial line drawn on the s-plane. We have found dominant pair of poles -20.0502 ±20.9761i along the damping ratio line for a gain K = 17.1843. The compensated system step response is shown in fig. 11 (b). The closed loop transfer function (T(s)) is given as

$$T(s) = \frac{40.1s + 842}{s^2 + 40.1s + 842} \quad (19)$$



(a)



(b)

Fig. 11. Compensated system (a) root-locus with radial line (b) step response

Pan system response with PD controller response was simulated; results are CLP, K, T_s, T_p, w_n, K_v and e_{ss} equal to -20.0502±20.9761i, 17.1843, 0.1995 sec, 0.1498 sec, 29.0174 rad/sec, 2.0048×10^5 and 4.9881×10^{-6} respectively. As noticed in fig. 11 (b), the setting time (0.1995 sec) and the steady-state error (1.9467×10^{-4}) meet the desired performance values.

Results

Linear modeling of the PTP, and its analysis and simulation based on feedback control system using the PD controllers for both mechanisms leads to the following conclusions.

- The linear system model with PD controller was calculated and simulated; results are presented in table 5.

Table 5. System model with PD controller

	T_S (seconds)	e_{SS}
Tilt Mechanism	0.2068	1.9467×10^{-4}
Pan Mechanism	0.1995	4.9881×10^{-6}

The calculated results are verified by the simulated results.

- The linear model of system with PD controller gives us desired orientation of camera.

REFERENCES

1. R. J. Schilling, *Fundamental of robotics analysis and control*, Prentice Hall, 1990.
2. Hassan K. khalil, *Nonlinear Systems*, 3rd edition, Prentice Hall, 2002, P. 24.
3. Akın Delibas, Türker Türker and Galip Cansever, "Real–time DC motor position control by fuzzy logic and PID controllers using Labview", *IEEE*, 2004.
4. Daniel D. Morris and James M. Rehg, "Singularity analysis for articulated object tracking", *CVPR '98*, Santa Barbara, CA, 1998.
5. W. R. Evans, "Control system synthesis by root locus method", *AIEE Transactions*, Vol. 69, 1950, pp. 547 – 551.
6. Stuart Bennett, "The past of PID controllers", *Elsevier Science Ltd*, Annual reviews in control Vol. 25, 2001, pp. 43-53.
7. Bob Rice and Doug Cooper, "Design and tuning of PID controllers for Integrating (non-self regulating) processes", *Proc. ISA 2002 Annual Meeting,* P057, Chicago, IL, pp. 424.
8. Fariborz Behi, "Kinematic analysis for a six-degree-of-freedom 3-PRPS parallel mechanism", *IEEE Journal of Robotics and Automation"*, Vol. 4, Oct 1988.
9. Norman S. Nise, *Control Systems Engineering*, Wiley, student Fourth ed., 2004, pp. 515-524.
10. Imran S. Sarwar, "Design, modeling and control of Pan Tilt Platform for unmanned aerial vehicle", M. S. thesis, Dept. of Mechatronics Engineering, National University of Sciences and Technology, Islamabad, Pakistan, 2006.