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Assessing the information content of option-based volatility forecasts using fuzzy regression methods

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Abstract

Volatility is a key variable for portfolio selection models, option pricing models and risk management techniques. Volatility can be estimated and forecasted by using either historical information or option prices. The present paper focuses on option-based volatility forecasts for three main reasons. First, for the forward looking nature of option-based forecasts (as opposed to the backward looking nature of historical information); second, for the average superiority, documented in the literature, of option-based estimates in forecasting future realized volatility; third, for the widespread use of option prices in the computation of the most important market volatility indexes (see e.g. the VIX index for the Chicago Board Options Exchange).

The aim of this paper is to assess the information content of future realised volatility of different option-based volatility forecasts, through the use of fuzzy regression methods. The latter methods offer a suitable tool to handle both imprecision in measurements and fuzziness of the relationship among variables. Therefore, they are particularly useful for volatility forecasting, since the variable of interest (realised volatility) is unobservable and a proxy for it is used. Moreover, measurement errors in both realised volatility and volatility forecasts may affect the regression results. Fuzzy regression methods have not yet been used in volatility forecasting. Our case study is based on intra-daily data on the DAX-index options market.

Keywords: Fuzzy regression methods, linear programming, least squares, volatility forecasting.
JEL classification: C61; C53; G17.

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1. Introduction.

Volatility measuring and forecasting with option prices is a very promising area of research, a relatively new field compared to historical measures of volatility, with important implications both at the micro level for portfolio selection models and risk management and at the macro level for policy makers. Volatility can be estimated either by using historical data (historical volatility) or by looking at market prices of options (implied volatility). In the first case the estimation is backward looking, while in the second the estimate is forward looking since the volatility implied by option prices represents the expectation of volatility between the evaluation date and the maturity of the option. The present paper focuses on option-based volatility forecasts for three main reasons. First, for the forward looking nature of option-based forecasts (as opposed to the backward looking nature of historical information); second, for the average superiority, documented in the literature, of option-based estimates in forecasting future realized volatility; third, for the widespread use of option prices in the computation of the most important market volatility indexes (see e.g. the VIX index for the Chicago Board Options Exchange).

Among option-based volatility forecasts we have the Black-Scholes (B-S) implied volatility, which is a "model-dependent" forecast since it relies on the Black and Scholes model, and the so-called "model-free" implied volatility, proposed by Britten-Jones and Neuberger [2], which does not rely on a particular option pricing model. B-S implied volatility is usually extracted from a single option, by inverting the Black and Scholes formula by means of a numerical method such as the bisection method. A drawback of using B-S implied volatility is clearly its dependence on the strike price of the option (the so-called smile effect), time to maturity of the option (term structure of volatility) and option type (call versus put). Model-free implied volatility is derived by using a cross-section of option prices differing in strike price and option type. Therefore, theoretically, model-free implied volatility should be more informative than the implied volatility backed out from a single option. However, for the computation of model-free implied volatility we need a continuum of strike prices ranging from zero to infinity. As this assumption is not verified in the reality of financial markets, the computation of model-free implied volatility is faced with both truncation and discretization errors.

In the literature many contributions have tried to assess the predictive power of implied volatility (either using the Black-Scholes formula or the model-free implied volatility definition) with respect to realised volatility. Most of the studies concentrate on Black-Scholes implied volatility (for a review see Poon and Granger [22]), and assess on average the superiority of Black-Scholes implied volatility (extracted from at-the-money option prices, i.e. options which have a zero payoff if exercised) on other historical volatility forecasts. Limited and mixed is instead the evidence about the forecasting power of model-free implied volatility. In particular some studies question the superiority of model-free implied volatility in predicting future realised volatility with respect to Black-Scholes implied volatility (see e.g. Andersen and Bondarenko [1] and Muzzioli [17]).

The aim of this paper is to assess the information content on future realised volatility of different option-based volatility forecasts through the use of fuzzy regression methods. The latter methods offer a suitable tool to handle both imprecision in measurements and fuzziness of the relationship among variables. Therefore, they are particularly useful for volatility forecasting, since the variable of interest (realised volatility) is unobservable and a proxy for it is used. Moreover, measurement errors in both realised volatility and volatility forecasts may affect the regression results. In fact, for realised volatility, the length of the time series, the frequency and the estimation methodology may lead to different estimates. For implied volatility the choice of the options to be used in the computation of Black-Scholes implied volatility as well as the truncation and discretization errors in model-free implied volatility may affect the estimation results. Therefore, the crisp estimate we use in ordinary least squares (OLS) regression for the volatility forecasts comes indeed from a vast range of data and can be considered as a sort of "average" around which the true value for the forecast may lie. Therefore, fuzzy regression methods can be particularly useful in order to represent the fuzzy relationship among the variables, even if the data set used are crisp. Up to now fuzzy regression methods have not been used in volatility forecasting. In order to allow for a direct comparison with ordinary least squares regression, we use the same data set of Muzzioli [17], consisting of intra-daily data on the DAX-index options market.

2. Ordinary linear regression and fuzzy linear regression.

Ordinary linear regression is one of the most powerful tools in order to model linear dependence between a given variable and one or more other variables. It is used in order to explain changes in the dependent variable (y) based on changes in the independent variable(s) (x_i):

$$y_t = a_0 + a_1 x_{1t} + \ldots + a_n x_{nt} + \varepsilon_t \tag{1}$$

The dependent variable is assumed to be random, i.e. to have a probability distribution, while the variables x_i are assumed to be non-stochastic and to take fixed values in repeated samples. In particular, the ordinary least squares method fits a line to the data by minimising the sum of squared residuals (the squared difference between actual values and fitted values from the regression line). The disturbance term captures the errors in the way y is measured and is also used to explain the absence of some determinants of y which may have been omitted or are non-measurable.

The ordinary linear regression model is based on strong hypotheses on the error term (the errors have zero mean, constant and finite variance, the errors are linearly independent of one another and there is no relationship between the errors and the corresponding variable *x*). Moreover, in order to make valid inference on the population parameters from the sample parameters it is also required that the error term is normally distributed.

On the other hand, in fuzzy regression, the same errors are viewed as the fuzziness of the model structure, therefore no hypothesis concerning the errors is made and the errors are thus not part of the fuzzy regression model:

$$Y = A_0 + A_1 x_1 + \ldots + A_n x_n \quad (2)$$

where Y is a fuzzy output, $x = (x_1, x_2, \ldots , x_n)$ is a non-fuzzy input vector and A_i, $i=0,\ldots, n$, are the fuzzy coefficients. The goal of fuzzy regression is to determine a linear fuzzy model which includes all the given (x,y) pairs.

Ordinary linear regression can be used to fit only crisp data. Fuzzy regression can be used to fit both crisp and fuzzy data, such as linguistic data for qualitative terms ("good", "bad", "excellent"), which can be better and easier represented by membership functions than crisp values. Moreover, fuzzy regression can be used when ordinary regression is not suitable: when only a limited number of observations is available and it is not possible to pursue enough experiments to derive a valid statistical relationship, when the data is imprecise, when there are difficulties in verifying the distributional assumptions, when the linear relationship between the variables is inappropriate and the relationship is subject to inaccuracy or vagueness, when human estimation, which cannot be modelled by crisp quantities, is important (see e.g. Kim, Moskowitz and Koksalan [15], Kahraman, Beşkese and Bozbura [13]).

Among fuzzy regression models we distinguish between models where the relationship between the variables is fuzzy and models where the variables themselves are fuzzy. In the first case, we have crisp inputs, crisp outputs (CICO) and a fuzzy system structure, while in the second case, the system structure is fuzzy, the output is fuzzy and the input can be fuzzy or crisp (crisp input and fuzzy output (CIFO) and fuzzy input and fuzzy output (FIFO)). A second classification employs the two basic approaches used in fuzzy regression: the so-called fuzzy possibilistic regression which aims at minimizing the fuzziness in the model (Tanaka, Uejima and Asai [24], a linear programming approach), and the fuzzy least squares regression, which uses least squares of errors as a fitting criterion (Diamond [8]).

In the following we concentrate mainly on the CICO case, which is of interest for the financial application and briefly review some methods of the two basic approaches. The fuzzy possibilistic approach has been first proposed by Tanaka, Uejima and Asai [24], and aims at minimizing the fuzziness in the model. In equation (2) the fuzzy coefficients are determined in such a way that the estimated Y has minimum fuzzy width at a target degree of belief h. In other words, each y of the data set (y can be crisp or fuzzy) must lie within the estimated Y at the h-level set. The parameter h can be chosen by the decision maker (see also Moskowitz and Kim [16] for the assessment of h) and represents the degree of confidence (or degree of belief) desired: if the degree of confidence is set to zero, the fuzzy output will exactly embed all the observations at the 0-level set; if a higher degree of confidence ($h>0$) is set, upper and lower fuzzy bands are widened in order to embed all the observations at the h-level set. As a consequence, the 0-level set of the fuzzy output in the latter case will be wider than that in the first case.

The Tanaka, Uejima and Asai method is one of the most used given its simplicity, although it has a number of drawbacks. In particular, the method is too sensitive to outliers, which influence the upper or the lower boundaries of the fuzzy regression model, adversely affecting the results.

Several extensions of the Tanaka, Uejima and Asai method have been proposed in order to solve the problem of outliers. Among others, Peters [21] assumes that the bounds of the fuzzy regression model are themselves fuzzy: the observations may belong to the upper and lower bound with a given degree of membership. Therefore some compensation between good and bad data becomes possible. Omrani, Aabdollahzadeh and Alinaghian [20] computes two separate linear programming models for the estimation of the upper and lower fuzzy bands. The upper fuzzy band is estimated based on the upper points and the central points and the lower fuzzy band is estimated based on the lower points and the central points. Given that central points are less sensitive to outliers, the resulting bands should be less sensitive to outliers. Other critics to the fuzzy possibilistic approach are that it does not allow all the observations to contribute to the estimation of the fuzzy model and as more data is collected the estimated fuzzy interval becomes wider. Several papers (e.g. Nasrabadi, Nasrabadi and Nasrabadi [18] have proposed multi-objective fuzzy regression to cope with these problems.

In the fuzzy possibilistic approach the minimization of fuzziness is the fitting criterion and the problem is solved by linear programming. The second approach, fuzzy least squares regression, integrates the least squares criterion into fuzzy regression. The aim is to minimize the sum of squared errors and the various contributions differ basically on the metric used in order to specify the errors (see e.g. Diamond [8]). Instead of minimising the errors, Celmiņš [4] maximizes the compatibility between the data and the fitted model. Savic and Pedrycz [23] integrate ordinary least squares regression (for the central values) and the minimum fuzziness criterion by using a two-stage procedure. D'Urso and Gastaldi [9] propose a doubly linear adaptive fuzzy regression method in which both a core regression for the central values and a spread regression for the spreads of the dependent variable are used. All the above-mentioned contributions have the drawback that as fuzziness decreases, the model does not collapse to the ordinary least squares regression (see Chang and Ayyub [6]). The first contribution in this sense is Chang [5], who derives a hybrid fuzzy least squares regression by using weighted fuzzy arithmetic. Ishibuchi and Nii [2011], by using asymmetric fuzzy numbers, proposed a hybrid method which computes the central values of the linear fuzzy model by means of ordinary least squares regression and the upper and lower bounds of the fuzzy model by minimizing the total width of the fuzzy output. Kao and Chyu [14] defuzzify the data in order to compute the center of the model with ordinary least squares, and use the estimated parameters in order to derive the fuzzy error term.

Given that they minimize the difference between observed and estimated values, the least squares fuzzy regression methods usually result in a better fit to the data, but have a higher computational complexity compared to possibilistic regression methods. For a comprehensive literature review of fuzzy regression and applications see Kahraman, Beşkese and Bozbura [13] and Nather [19].

3. The possibilistic and the least squares fuzzy regression methods.

In this section we briefly recall the basics of the possibilistic regression method of Tanaka, Uejima and Asai and the least squares fuzzy regression method of Savic and Pedrycz which are used in the financial application. The Tanaka, Uejima and Asai method for equation (2) assumes the fuzzy coefficients to have a symmetric triangular membership function: $A_i = (a_i^c, a_i^w)$, where a_i^c and a_i^w are the center and the spread respectively of the symmetric triangular fuzzy number A_i. The computation of the right-hand side is pursued by using fuzzy arithmetic. As the A_i are symmetric triangular fuzzy numbers, it follows that also $F(x) = A_0 + A_1 x_1 + ... + A_n x_n$ is a symmetric triangular fuzzy number. Starting from m non-fuzzy input-output pairs (x_p, y_p), where $x_p=(x_{p1}, x_{p2}, \ldots, x_{pn})$, $p=1,\ldots,m$, the aim of fuzzy possibilistic regression is to include all the given data pairs in the linear fuzzy model at level h, where $[F(x)]_h$ is the alpha-cut of the fuzzy output $F(x)$, therefore the condition which should be satisfied is $y_p \in [F(x_p)]_h$. The fuzzy coefficients are determined in such a way that the estimated $F(x)$ has minimum fuzzy width at a target degree of belief h.

In order to determine the symmetric triangular fuzzy coefficients A_i, the following linear programming problem has to be solved. Minimize the total spread of the fuzzy output:

$$\min z = \sum_{p=1}^{m} \left\{ a_0^w + a_1^w \left| x_{p1} \right| + ... + a_n^w \left| x_{pn} \right| \right\} \tag{3}$$

Subject to:

$$a_0^c + \sum_{i=1}^{n} a_i^c x_{pi} - (1-h)\left[a_0^w + \sum_{i=1}^{n} a_i^w \left| x_{pi} \right| \right] \le y_p \, , \; p = 1, \ldots, \; m$$

$$a_0^c + \sum_{i=1}^{n} a_i^c x_{pi} + (1-h)\left[a_0^w + \sum_{i=1}^{n} a_i^w \left| x_{pi} \right| \right] \ge y_p \, , \; p = 1, \ldots, \; m$$

$$a_i^w \ge 0$$

Note that in problem (3) the function to be minimized is the total spread of the fuzzy output, as proposed by Tanaka [25], instead of the total spread of the fuzzy coefficients as in the original Tanaka, Uejima and Asai method.

The fuzzy least squares regression method of Savic and Pedrycz combines ordinary least squares regression and the minimum fuzziness principle, by pursuing a two-stage methodology. In the first stage only the center of the fuzzy model is fixed by using ordinary least squares regression. In the second stage the minimum fuzziness criterion is used in order to find the width of the fuzzy regression coefficients, by solving model (3), where the center of each fuzzy coefficient is imposed to be equal to the ordinary least squares coefficient computed in the first stage.

4. The data set and the computation of the volatility measures.

Muzzioli [17] investigated the information content of different option-based forecasts in the DAX-index options market, by using ordinary linear regression. In this paper, for comparative purposes, we use the data set of Muzzioli [17] which consists of intra-daily data on DAX-index options, and DAX-index, recorded from 1 January 2001 to 31 December 2005. As for the risk-free rate, the one-month Euribor rate is recorded in the same time period at a daily frequency.

The option data set has been filtered in order to eliminate stale prices, illiquid trades and avoid no-arbitrage opportunities as detailed in Muzzioli [17]. Moreover, monthly non-overlapping samples are used in order to avoid the telescoping problem described in Christensen, Hansen and Prabhala [7].

Four volatility measures are computed: realised volatility (σ_r), historical volatility (σ_h), B-S implied volatility (σ_{BS}) and model-free implied volatility (σ_{mf}). Realised volatility is computed, in annual terms, as the squared root of the sum of five-minute frequency squared index returns over the life time of the option:

$$\sigma_r = \sqrt{\sum_{t=1}^{n} \left[\ln\left(\frac{S_{t+1}}{S_t} \right) \right]^2 * 12}$$

where $n+1$ is the number of index prices S_t, $t=1,\ldots,n+1$, spaced by five minutes in the one-month period. As a proxy for historical volatility (σ_h), in line with previous studies (see e.g. Canina and Figlewski [3]) we use lagged (one month before) realized volatility. B-S implied volatility (σ_{BS}) is computed as a weighted average of the two implied volatilities that correspond to the two strikes that are closest to being at-the-money, with weights inversely proportional to the distance to the moneyness. Model-free implied volatility (σ_{mf}) is computed by the Britten-Jones and Neuberger [2] formula adapted to the particularities of the German financial market. For more details on the computation of model-free and Black-Scholes implied volatility we refer the interested reader to Muzzioli [17]. Here we briefly recall only the features of the data set which call for the use of fuzzy regression methods. Both implied volatility estimates may suffer from errors in measurement and are obtained as averages of different estimates. In particular, the data set is made of tick-by-tick data on option trades and to reduce the computational burden only the last hour of trades has been used. In order to compute implied volatilities it is very important to have synchronous prices between the option and the underlying index, and this has been obtained by using a one-minute window to do the matching. However, in one hour several realisations for implied volatility of options with the same strike price are possible, therefore, for each strike price the different implied volatilities are averaged in order to have a single estimate. This may affect the estimation of both implied volatility estimates. Moreover, for Black-Scholes implied volatility the choice of the options to be used may lead to different estimates; for model-free implied volatility, possible errors in measurement are given by the choice of the truncation and discretization parameters. In fact, the theoretical definition of model-free implied volatility requires a continuum of option prices with strike ranging from zero to infinity. As in the reality of financial markets only a limited and discrete number of strikes is used, interpolation and extrapolation mechanisms are needed in order to artificially create the required option prices and alleviate the truncation and discretization errors. For these reasons we deem fuzzy regression methods particularly useful in order to assess the information content of option-based volatility forecasts.

5. The methodology

As there are two approaches in pursuing fuzzy regression, we use both the possibilistic regression method of Tanaka, Uejima and Asai and the least squares regression method of Savic and Pedrycz which are two of the most used and most simple methods in the two categories. In this way we are able to compare the results across different methodologies. In this application we have m=58 observations of the three input-output pairs (x_i,y_i), $i=1,\ldots m$, where x is the volatility forecast (which corresponds to either historical volatility, or Black-Scholes volatility, or model-free volatility) and y is realised volatility.

The information content of implied volatility is examined both in univariate and in encompassing regressions. The series are examined in logarithmic terms, since as shown in Muzzioli [17] they conform more to normality (as a robustness test, they are examined also in the levels in Section 8). In order to examine the forecasting ability of each volatility forecast, in univariate regressions realised volatility is regressed against each of the three volatility forecasts: Black-Scholes implied volatility (σ_{BS}), model-free implied volatility (σ_{mf}), or historical volatility (σ_h). The univariate regressions are the following:

$$\ln(\sigma_r) = \alpha + \beta \ln(\sigma_i) \quad (5)$$

where σ_r = realised volatility and σ_i= volatility forecast, i= BS, mf, h.

In order to distinguish which forecast has the highest explanatory power and to address whether a single volatility forecast subsumes all the information contained in the others we rely on encompassing regressions where realised volatility is regressed against two or more volatility forecasts. First, we compare pairwise one option-based forecast with historical volatility in order to see if the information content of historical volatility is subsumed in the option-based forecast. Second we compare the two implied volatility forecasts, in order to see which one has the highest information content on future realised volatility. Lastly, we regress realised volatility against the three volatility forecasts in order to distinguish the relative importance of each forecast.

The encompassing regressions used are the following:

$$\ln(\sigma_r) = \alpha + \beta \ln(\sigma_i) + \gamma \ln(\sigma_h) \quad (6)$$

where σ_r = realised volatility, σ_i= implied volatility, i = BS, mf and σ_h = historical volatility,

$$\ln(\sigma_r) = \alpha + \beta \ln(\sigma_{BS}) + \gamma \ln(\sigma_{mf}) \quad (7)$$

where σ_r = realised volatility, σ_{BS}= B-S implied volatility and σ_{mf} = model-free implied volatility,

$$\ln(\sigma_r) = \alpha + \beta \ln(\sigma_{BS}) + \gamma \ln(\sigma_{mf}) + \delta \ln(\sigma_h) \quad (8)$$

where σ_r = realised volatility, σ_{BS}= B-S implied volatility and σ_{mf} = model-free implied volatility and σ_h = historical volatility.

6. The results for univariate regressions.

The results for the Tanaka, Uejima and Asai and the Savic and Pedrycz fuzzy regression methods for univariate regressions (5) are reported in Table 1 for the confidence level h=0, and in Table 2 for the confidence level h=0.5. Figure 1 illustrates the fuzzy regression results. For both methodologies, a higher degree of confidence h implies a wider band, since by imposing a higher h we are requiring that all the crisp pairs should be included in the h-cut of the fuzzy output. Therefore h=0 yields the narrowest band. Moreover, we can observe that the center of the fuzzy output, for both methods remains unchanged if we change the degree of belief h, which follows from the assumption of symmetric spreads for the fuzzy output.

The center of the Tanaka, Uejima and Asai method is different from the center of the Savic and Pedrycz method, which corresponds to the ordinary least squares regression equation. In particular for the Tanaka, Uejima and Asai method the slope coefficients are lower than the corresponding OLS coefficients: for historical volatility the difference is marginal, while for Black-Scholes and model-free implied volatilities the coefficients are only half of the corresponding OLS coefficients. The observed difference is due to the different fitting criteria in the two methods. For the Tanaka, Uejima and Asai method the aim is to embed all the observations in the h-cut of the fuzzy output, by minimising the width of the fuzzy output, and the center is by construction imposed in the middle of the upper and lower boundaries. In the Savic and Pedrycz method, the center is fixed and the objective is to minimize the width of the fuzzy output, by using symmetrical spreads. Therefore the upper and lower bands are wider for the Savic and Pedrycz method, and this is apparent if we look at the last column of Tables 1 and 2, which reports the fit of the two methods. The fit measure corresponds to the minimum value attained by the objective function: therefore the lower the value, the better the fit. For the Tanaka, Uejima and Asai method the fit is better than for the Savic and Pedrycz method. As expected, the fit deteriorates if we increase the degree of belief, since we impose wider upper and lower bands.

It is worth noting that for historical volatility, for both methods, the slope coefficient has no spread, while for Black-Scholes and model-free implied volatilities the spread of the slope coefficients are pretty high if compared to the central value. On the other hand, for historical volatility all the fuzziness is captured by the intercept coefficient, which has a very high spread, while for Black-Scholes and model-free implied volatilities the intercept coefficient has no spread and all the fuzziness is captured by the slope coefficient. For both implied volatilities the results are very similar, in the Tanaka, Uejima and Asai method the slope coefficient of model-free implied volatility is a little lower than the slope coefficient of Black-Scholes, the opposite holds for the Savic and Pedrycz method. Therefore we may conclude that for historical volatility we may be pretty sure about the value of the slope coefficient, while for Black-Scholes and model-free implied volatilities, the intercept coefficient presents no significant fuzziness. The slope coefficient of implied volatilities in the Savic and Pedrycz method are very close to one, even if they present a substantial spread, therefore we expect them to be unbiased predictors of realised volatility. On the other hand, in the Tanaka, Uejima and Asai method, they are substantially lower than one.

The central values of the intercept coefficients in the regressions with implied volatilities as explanatory variables are all sizeable and negative and imply the presence of a substantial volatility risk premium (see e.g. Jiang and Tian [12]) i.e. option-based forecasts over predict realised volatility. This means that investors are averse to variations of volatility and are therefore willing to pay a high price for realised volatility (represented by the higher implied volatility) in order to be hedged against peaks in volatility which typically represent bad market conditions. As for the fit of the three forecasts, for the Tanaka, Uejima and Asai method the best one is Black-Scholes, followed by model-free implied volatility; the worst performance is obtained by historical volatility. For the Savic and Pedrycz method, historical volatility obtains the best performance, followed by Black-Scholes implied volatility, and model-free implied volatility obtains the worst performance.

7. The results for multivariate regressions.

The results for the Tanaka, Uejima and Asai and the Savic and Pedrycz fuzzy regression methods for bivariate regressions (6-7) and multivariate regression (8) are reported in Table 3 for the confidence level h=0 and in Table 4 for the confidence level h=0.5. In all regressions, the center of the Tanaka, Uejima and Asai method is different from the center of the Savic and Pedrycz method, which corresponds to the ordinary least squares regression equation. As it is the case for univariate regressions, also for each multivariate regression the fit for the Savic and Pedrycz method is on average worse than that of the Tanaka, Uejima and Asai method, since upper and lower bands are wider for the Savic and Pedrycz method. Moreover, by looking at the results for h=0 and h=0.5, the minimum value attained by the objective function increases, i.e. the fit deteriorates, if we increase the degree of belief, since we impose wider upper and lower bands, the central values remain unchanged, only the spreads of the coefficients increase. As for the fit of the bivariate regressions for the Tanaka, Uejima and Asai method the best one is the one with as explanatory variables the historical and model-free implied volatility, followed by historical and Black-Scholes implied volatility, while the worst performance is obtained by the one with as explanatory variables the two implied volatility estimates. On the other hand, for the Savic and Pedrycz method, the ranking is the opposite: the latter bivariate regression is the one which obtains the best fit, followed by the bivariate regression with historical volatility and Black-Scholes implied volatility as explanatory variables. The regression with as explanatory variables the historical and model-free implied volatility obtains the worst performance. Moreover, if we compare the fit of the bivariate regressions with the one of univariate regressions, the fit improves for the bivariate regressions in the Tanaka, Uejima and Asai method. However, the fit does not always improve for the Savic and Pedrycz method, since the exogenous choice of the central values does not allow the fit to improve if another explanatory variable is introduced, as it happens for ordinary least squares. For example, in multivariate regression (8) the fit is better than in bivariate regressions in the Tanaka, Uejima and Asai method, but it is worse than in one of the bivariate regressions in the Savic and Pedrycz method.

The following comments apply to both the Tanaka, Uejima and Asai and the Savic and Pedrycz methods. First, the central values of the intercept coefficients in all regressions are negative and the spread is close to zero (except for the bivariate regression with Black-Scholes and model-free implied volatilities as explanatory variables, which in fact mix the explanatory power of two implied volatility forecasts and cause the intercept coefficient to be fuzzier). Second, for all regressions, the central value of the slope coefficient of historical volatility is lower than in the univariate regression, while the spread is still equal to zero. Third, the slope coefficient of Black-Scholes implied volatility in bivariate regression with model-free implied volatility and in multivariate regression (8) has spread equal to zero. The two latter facts mean that the slope coefficients of historical volatility and of Black-Scholes implied volatility do not contribute to the fuzziness of the output. Fourth, when we compare Black-Scholes (or model-free) implied volatility with historical volatility all the fuzziness is captured by the slope coefficient of Black-Scholes (or model-free) implied volatility. When we compare the two implied volatility estimates, all the fuzziness is captured by the slope coefficient of model-free implied volatility.

Some important differences between the two methods (Tanaka, Uejima and Asai and Savic and Pedrycz) are apparent. In the Savic and Pedrycz method the slope coefficient of historical volatility is very close to zero (we expect the efficiency of the two implied volatility forecasts, since they subsume all the information conveyed by historical volatility). In the Tanaka, Uejima and Asai method the slope coefficient of historical volatility is positive and higher than the slope coefficients of implied volatilities, therefore we roughly expect to reject the hypothesis of efficiency of the two implied volatility forecasts. Moreover in the Tanaka, Uejima and Asai method the central values of the slope coefficients of implied volatility are substantially lower than the corresponding estimates in the Savic and Pedrycz method, even if they have a lower spread. Another important difference is the sign of the slope coefficient of model-free implied volatility in the regression equations which embed also Black-Scholes implied volatility. In the Tanaka, Uejima and Asai method the slope coefficient is negative, while in the Savic and Pedrycz method it is positive. It seems that in the Tanaka, Uejima and Asai method, when we aggregate very similar information (given by the two implied volatility estimates), the method compensates for that by giving opposite signs to the slope coefficients, while in the Savic and Pedrycz method there is no compensation since the central values obey the least squares principle.

8. The results of univariate and multivariate regressions in the volatility levels.

The results for the Tanaka, Uejima and Asai and the Savic and Pedrycz fuzzy regression methods for univariate regressions (5) are reported in Table 5 for the confidence level h=0 (and in Table 7 for the confidence level h=0.5). Figure 2 illustrates the fuzzy regression results. For univariate regressions, overall, the results are very similar to the findings in Section 6, where the natural logarithm of the variables is used. One noticeable difference is in the spread of the coefficients: while for the logarithmic case the slope coefficient of historical volatility was the only one with spread equal to zero, in this case both in the Tanaka, Uejima and Asai and in the Savic and Pedrycz methods, the slope coefficients of the three forecasts display zero spread: all the fuzziness is captured by the intercept coefficients. Moreover, in the Savic and Pedrycz method, the central value of all the intercepts is close to zero and the central values of the slope coefficients are smaller than in the logarithmic case. The Savic and Pedrycz method provides the same ranking obtained for variables in logs. The Tanaka, Uejima and Asai method switches historical with model-free implied volatility at the second place, while Black-Scholes implied volatility remains the preferred one.

In the Tanaka, Uejima and Asai method the slope coefficient of model-free implied volatility is smaller than the slope coefficient of Black-Scholes, which in turn is smaller than the slope coefficient of historical volatility. In the Savic and Pedrycz method the highest slope coefficient is attained by Black-Scholes implied volatility, while the lowest one by model-free implied volatility. The slope coefficient of implied volatilities in the Savic and Pedrycz method are very close to one, therefore we expect them to be unbiased predictors of realised volatility. On the other hand, in the Tanaka, Uejima and Asai model, they are substantially lower than one.

The results for the Tanaka, Uejima and Asai and the Savic and Pedrycz fuzzy regression methods for bivariate and multivariate regressions (6-7) are reported in Table 6 for the confidence level h=0 (and in Table 8 for the confidence level h=0.5). There are some noticeable differences with the analysis conducted in Section 7, with the variables in logarithmic terms. In bivariate and multivariate regressions the fit with respect to univariate regressions improves for both fuzzy regression methods. In all the regressions and in both methods all the slope coefficients display zero spread and all the fuzziness is captured by the intercept value. Therefore the regressions in the volatility levels seem to better capture a clear linear relationship between variables than the regressions in logarithmic terms. For the rest, the results substantially corroborate the results in Section 7. In the Savic and Pedrycz method it is evident that the explanatory power of historical volatility is very low (the slope coefficient is close to zero), while the variation in realised volatility is explained by both Black-Scholes and model-free implied volatility (the latter to a lesser extent).

In the Tanaka, Uejima and Asai method, all three volatility forecasts display substantial forecasting power, but when the two implied volatility estimates are used jointly, model-free implied volatility displays a negative relationship with subsequent realised volatility, as the slope coefficient is negative.

9. Conclusions.

In this paper we have assessed the information content of option-based forecasts of volatility by using fuzzy regression methods. We have implemented both the possibilistic regression method of Tanaka, Uejima and Asai and the least squares fuzzy regression method of Savic and Pedrycz.

In univariate regressions, all three volatility forecasts present forecasting power on subsequent realised volatility. For historical volatility all the fuzziness is captured by the intercept coefficient, while for Black-Scholes and model-free implied volatilities all the fuzziness is captured by the slope coefficient. The slope coefficient of implied volatilities in the Savic and Pedrycz method are very close to one, even if they present a substantial spread, therefore we expect them to be unbiased predictors of realised volatility. On the other hand, in the Tanaka, Uejima and Asai method, they are substantially lower than one. As for the fit of the three forecasts, the two models provide a different ranking. For the Tanaka, Uejima and Asai method the best one is Black-Scholes, followed by model-free implied volatility, the worst performance is obtained by historical volatility. For the Savic and Pedrycz method, historical volatility obtains the best performance, followed by Black-Scholes implied volatility, and model-free implied volatility obtains the worst performance. The two methods agree in assessing the superiority of Black-Scholes implied volatility to model-free implied volatility.

In multivariate regressions, the information content of the different volatility forecasts is disentangled. When we compare Black-Scholes (or model-free) implied volatility with historical volatility all the fuzziness is captured by the slope coefficient of Black-Scholes (or model-free) implied volatility. When we compare the two implied volatility estimates, all the fuzziness is captured by the slope coefficient of model-free implied volatility. In the Savic and Pedrycz method the slope coefficient of historical volatility is very close to zero (we expect the efficiency of the two implied volatility forecasts, since they subsume all the information conveyed by historical volatility). In the Tanaka, Uejima and Asai method the slope coefficient of historical volatility is positive and higher than the slope coefficients of implied volatilities, therefore we roughly expect to reject the hypothesis of efficiency of the two implied volatility forecasts.

The results of univariate and multivariate regressions in the volatility levels complete the analysis and further confirm the results. One noticeable difference is in the spread of the coefficients: in this case the slope coefficients of the three forecasts display zero spread and all the fuzziness is captured by the intercept coefficients. Moreover, the fit improves with respect to the analysis in logarithmic terms, for both fuzzy regression methods. Therefore the regressions in the volatility levels seem to capture even better than the regressions in logarithmic terms a clear linear relationship between the variables.

If compared to the OLS results obtained in Muzzioli [17], the application of fuzzy regression methods yields a "confidence" band in which all the possible regression lines are embedded. In the Savic and Pedrycz method the center of the model coincides with the OLS regression, while in the Tanaka, Uejima and Asai method the center is chosen as the midline between the upper and lower limits of the fuzzy output. The fuzziness of the model can be captured by the intercept, by the slope coefficient, or both. When the fuzziness of the model is captured mainly by the intercept, it means that the relationship between the dependent and the independent variable is clear and the vagueness regards the scale adjustment which has to be made in order to shift up or down the regression line. On the other hand, when the fuzziness of the model is captured mainly by the slope coefficient, it means that there is a clear constant adjustment that has to be made to the independent variable, while the relationship between the two variables is not sharp. When the fuzziness of the model is captured by both the intercept and the slope coefficients, we face the fuzziest relationship between independent and dependent variables.

Differently from ordinary regression, the coefficients of the fuzzy regression model are represented by fuzzy numbers, and each real number in the support of the fuzzy number has a given degree of membership. Therefore the fuzzy coefficients provide a natural interpretation of the most possible value within the interval (the peak value) and of the interval of possible values around the most possible (we may think at the support of the fuzzy number as a sort of "confidence interval" around the most possible value). Another degree of freedom in fuzzy regression, which constitutes a difference with OLS regression, is given by the choice of the degree of belief that the decision maker desires, which can be set on the base of her experience and the reliance on the data (see). In both fuzzy regression methods, a higher degree of belief (h) implies a higher spread, similarly to the case in statistics, where a higher confidence level yields a wider confidence interval. We remark that in both methods the central value is not affected by the choice of the h-level.

Therefore we can conclude that the fuzzy regression methods used provide a more informative interpretation of the relationship among the variables, which can be tailored to the experience of the decision maker. Moreover fuzzy regression methods are able to disentangle the contribution of each regression coefficient to the overall fuzziness of the model.

Since the fuzzy regression methods used in this paper are non-statistical in nature, inference methods and tests on the parameters are hard to implement (see e.g. Kim, Moskowitz and Koksalan, [15]). Therefore, the present study should be complemented by using different fuzzy regression methodologies which provide inferential procedures for testing the parameters and assessing the unbiasedness and efficiency of the different volatility forecasts. Along this line, an interesting extension is the use of fuzzy random variables in order to model both randomness and the imprecision in volatility estimation (see e.g. Ferraro, Coppi, Gonzales Rodriguez and Colubi, [2010], who use LR fuzzy random variables in order to model random experiments and derive a linear regression model with an imprecise response).

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References.

[1] T. G. Andersen, O. Bondarenko, Construction and Interpretation of Model-Free Implied Volatility, In: I. Nelken (Ed.), Volatility as an Asset Class, Vol. Risk Books, London, 2007, pp. 141-18.

[2] M. Britten-Jones, A. Neuberger, Option prices, implied price processes, and stochastic volatility, Journal of Finance 55 (2000) 839-866.

[3] L. Canina, S. Figlewski, The informational content of implied volatility, Review of Financial studies 6 (1993) 659-681.

[4] A. Celmiņš, Least Squares Model Fitting to Fuzzy Vector Data, Fuzzy Sets and Systems 22 (1987) 245-269.

[5] Y-H. O. Chang, Hybrid fuzzy least-squares regression analysis and its reliability measures, Fuzzy Sets and Systems 119 (2001) 225-246.

[6] Y-H. O. Chang, B.M. Ayyub, Fuzzy regression methods - a comparative assessment, Fuzzy Sets and Systems 119 (2001) 187-203.

[7] B.J. Christensen, C.S. Hansen, N.R. Prabhala, The telescoping overlap problem in options data, Working paper, University of Aarhus and University of Maryland (2001).

[8] P. Diamond, Fuzzy least squares, Information Sciences 46 (1988) 141-157.

[9] P. D'Urso, T. Gastaldi, A least-squares approach to fuzzy linear regression analysis, Computational Statistics & Data Analysis 34 (2000) 427-440.

[10] M.B. Ferraro, R. Coppi, G. Gonzales Rodriguez, A. Colubi, A linear regression model for imprecise response, International Journal of Approximate Reasoning 51 (2010) 759-770.

[11] H. Ishibuchi, M. Nii, Fuzzy regression using asymmetric fuzzy coefficients and fuzzified neural networks, Fuzzy Sets and Systems 119 (2001) 273-290.

[12] G. J. Jiang, Y. S. Tian, The model-free implied volatility and its information content, Review of Financial Studies 18 (2005) 1305-1342.

[13] C. Kahraman, A. Beşkese, F.T. Bozbura, Fuzzy Regression Approaches and Applications, Studies in Fuzziness and Soft Computing 201 (2006) 589-615.

[14] C. Kao, Chin-Lu Chyu, A fuzzy regression model with better explanatory power, Fuzzy Sets and Systems 126 (2002) 401-409.

[15] K.J. Kim, H. Moskowitz, M. Koksalan, Fuzzy versus statistical linear regression, European Journal of Operational Research 92 (1996) 417-434.

[16] H. Moskowitz, K.J. Kim, On assessing the H value in fuzzy linear regression, Fuzzy Sets and Systems 58 (1993) 303-327.

[17] S. Muzzioli, Option-based forecasts of volatility: an empirical study in the DAX-index options market, European Journal of Finance 16 (2010) 561-586.

[18] M.M. Nasrabadi, E. Nasrabadi, A.R. Nasrabadi, Fuzzy linear regression analysis: a multi objective programming approach, Applied Mathematics and Computation 163 (2005) 245-251.

[19] W. Nather, Regression with fuzzy random data, Computational Statistics and Data Analysis 51 (2006) 235-252.

[20] H. Omrani, S. Aabdollahzadeh, M. Alinaghian, A Simple and Efficient Goal Programming Model for Computing of Fuzzy Linear Regression Parameters with Considering Outliers, Journal of Uncertain Systems 5 (2011) 62-71.

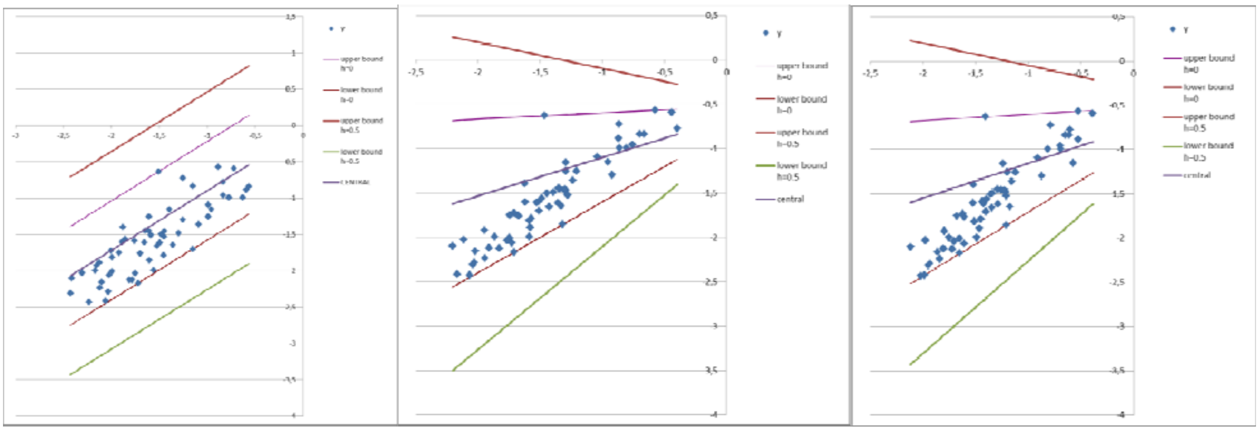
[21] G. Peters, Fuzzy linear regression with fuzzy intervals, Fuzzy Sets and Systems 63 (1994) 45-55.

[22] S. Poon, C.W. Granger, Forecasting volatility in financial markets: a review, Journal of Economic Literature 41 (2003) 478-539.

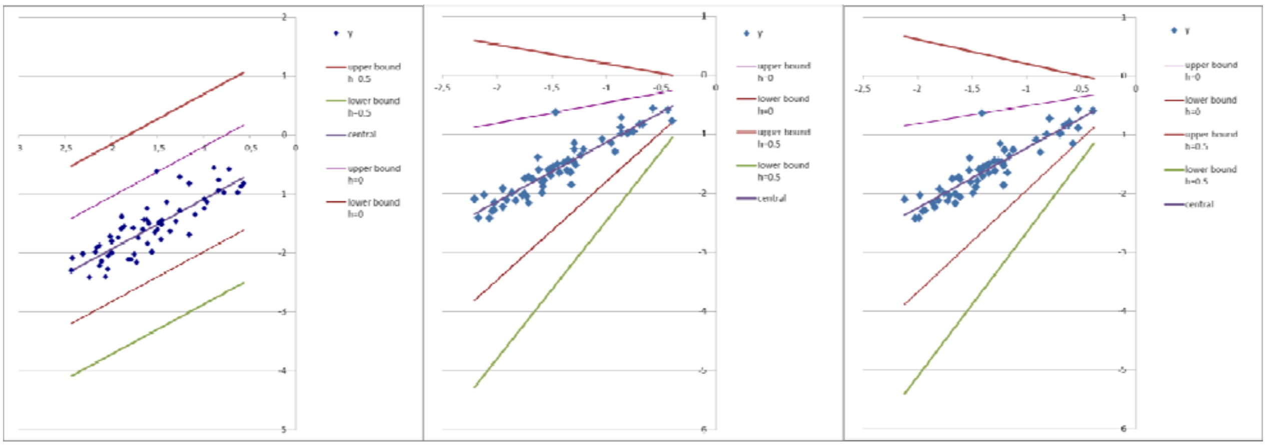
[23] D.A. Savic, W. Pedrycz, Evaluation of fuzzy linear regression models, Fuzzy Sets and Systems 39 (1991) 51-63.

[24] H. Tanaka, S. Uejima, K. Asai, Linear regression analysis with fuzzy model, IEEE Transactions on Systems, Man and Cybernetics 12 (1982) 903-907.

[25] H. Tanaka, Fuzzy data analysis by possibilistic linear models, Fuzzy Sets and Systems 24 (1987) 363-375.

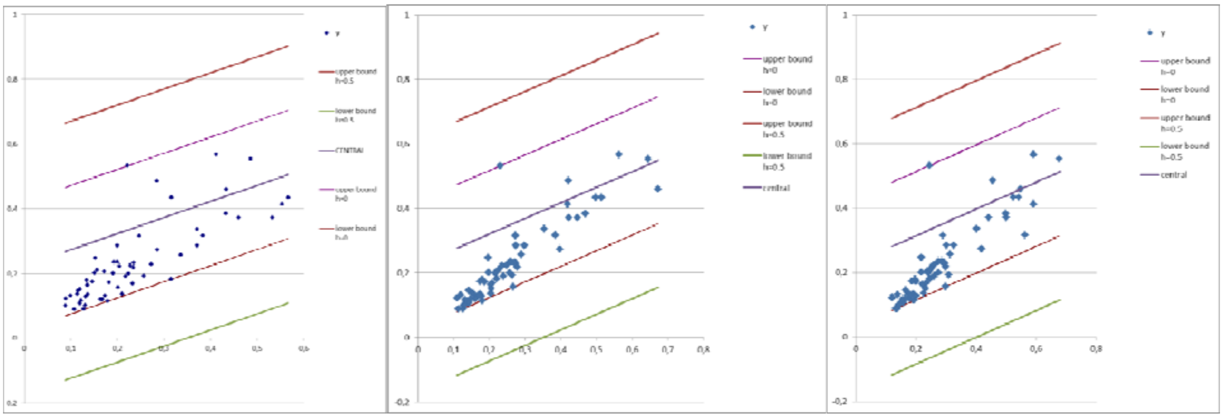


(a) Historical / TU&A (b) Black-Scholes / TU&A (c) Model-free / TU&A

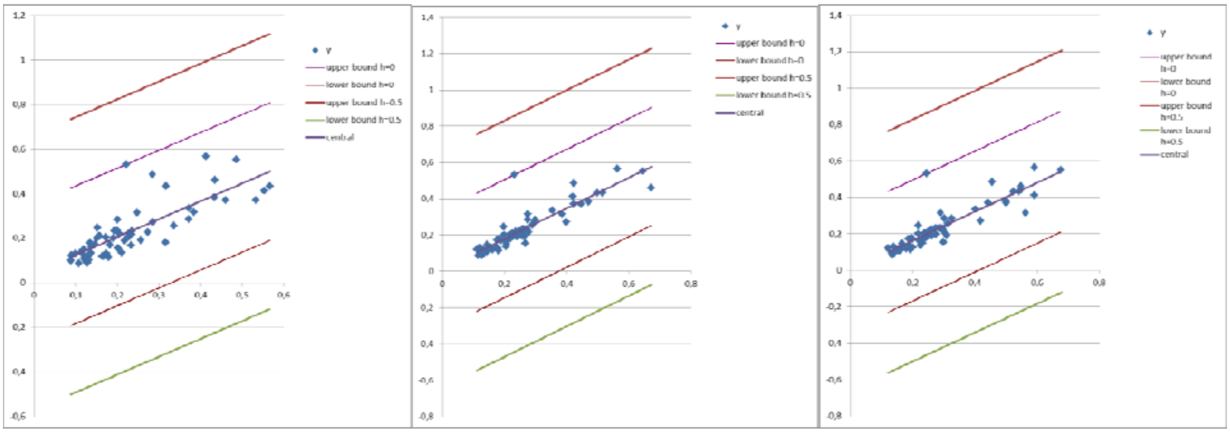


(d) Historical / S&P (e) Black-Scholes / S&P (f) Model-free / S&P

Figure 1. Univariate regression representation for the three volatility forecasts, variables in logs. The figures show the possibilistic fuzzy regression model of Tanaka, Uejima and Asai (TU&A) ((a) for historical volatility, (b) for Black-Scholes and (c) for model-free) and the least squares fuzzy regression model of Savic and Pedrycz (S&P) ((d) for historical volatility, (e) for Black-Scholes and (f) for model-free). Each figure reports the crisp data pairs (y), the central value of the fuzzy output and the upper and lower bounds of the fuzzy output for two different degrees of belief: h=0 and h=0.5.



(a) Historical TU&A (b) Black-Scholes TU&A (c) model-free TU&A



(d) Historical S&P (e) Black-Scholes S&P (f) model-free S&P

Figure 2. Univariate regression representation for the three volatility forecasts, variables in levels. The figure show the possibilistic fuzzy regression model of Tanaka, Uejima and Asai (TU&A) ((a) for historical volatility, (b) for Black-Scholes and (c) for model-free) and the least squares fuzzy regression model of Savic and Pedrycz (S&P) ((d) for historical volatility, (e) for Black-Scholes and (f) for model-free). Each figure reports the crisp data pairs (y), the central value of the fuzzy output and the upper and lower bounds of the fuzzy output for two different degrees of belief: h=0 and h=0.5.

h=0	Tanaka, Uejima and Asai				
	A_0		A_1		
	central	spread	central	spread	fit
historical	-0.07	0.68	0.82	0.00	39.47
Black-Scholes	-0.67	0.14	0.43	0.36	38.22
model-free	-0.76	0.23	0.40	0.33	38.39
	Savic and Pedrycz				
	A_0		A_1		
	central	spread	central	spread	fit
historical	-0.24	0.89	0.85	0.00	51.70
Black-Scholes	-0.13	0.00	1.01	0.66	55.11
model-free	-0.20	0.00	1.02	0.72	55.65

Table 1. The results for univariate regressions (variables in logs, h=0).

h=0.5	Tanaka, Uejima and Asai				
	A_0		A_1		
	central	spread	central	spread	fit
historical	-0.07	1.36	0.82	0.00	78.94
Black-Scholes	-0.67	0.28	0.43	0.73	76.44
model-free	-0.76	0.45	0.40	0.65	76.77
	Savic and Pedrycz				
	A_0		A_1		
	central	spread	central	spread	fit
historical	-0.24	1.78	0.85	0.00	103.40
Black-Scholes	-0.13	0.00	1.01	1.33	110.23
model-free	-0.20	0.00	1.02	1.43	111.30

Table 2. The results for univariate regressions (variables in logs, h=0.5).

Tanaka, Uejima and Asai								
h=0								
	A_0		historical		Black-Scholes		model-free		fit
	central	spread	central	spread	central	spread	central	spread	
	-0.13	0.00	0.56	0.00	0.20	0.43			35.81
	-0.29	0.03	0.54	0.00			0.13	0.44	35.56
	-0.38	0.21			1.70	0.00	-1.12	0.32	36.90
	-0.10	0.00	0.55	0.00	0.84	0.00	-0.63	0.45	34.76
Savic and Pedrycz									
h=0									
	A_0		historical		Black-Scholes		model-free		fit
	central	spread	central	spread	central	spread	central	spread	
	-0.13	0.00	-0.01	0.00	1.01	0.66			54.63
	-0.21	0.00	-0.03	0.00			1.05	0.72	55.81
	-0.10	0.00			0.83	0.00	0.20	0.69	53.36
	-0.14	0.00	-0.03	0.00	0.83	0.00	0.21	0.69	53.85

Table 3. The results for multivariate regressions (variables in logs, h=0).

Tanaka, Uejima and Asai									
h=0.5									
	A_0		historical		Black-Scholes		model-free		fit
	central	spread	central	spread	central	spread	central	spread	
	-0.13	0.00	0.56	0.00	0.20	0.86			71.62
	-0.29	0.06	0.54	0.00			0.13	0.87	71.12
	-0.38	0.42			1.70	0.00	-1.12	0.64	73.80
	-0.10	0.00	0.55	0.00	0.84	0.00	-0.63	0.90	69.52
Savic and Pedrycz									
h=0.5									
	A_0		historical		Black-Scholes		model-free		fit
	central	spread	central	spread	central	spread	central	spread	
	-0.13	0.00	-0.01	0.00	1.01	1.32			109.26
	-0.21	0.00	-0.03	0.00			1.05	1.44	111.63
	-0.10	0.00			0.83	0.00	0.20	1.37	106.71
	-0.14	0.00	-0.03	0.00	0.83	0.00	0.21	1.39	107.69

Table 4. The results for multivariate regressions (variables in logs, h=0.5).

h=0	Tanaka, Uejima and Asai				
	A_0		A_1		
	central	spread	central	spread	fit
historical	0.22	0.20	0.50	0.00	11.51
Black-Scholes	0.22	0.20	0.48	0.00	11.42
model-free	0.23	0.20	0.42	0.00	11.55
h=0	Savic and Pedrycz				
	A_0		A_1		
	central	spread	central	spread	fit
historical	0.05	0.31	0.80	0.00	17.90
Black-Scholes	0.01	0.33	0.84	0.00	18.87
model-free	0.01	0.33	0.79	0.00	19.25

Table 5. The results for univariate regressions (variables in levels, h=0).

Tanaka, Uejima and Asai									
h=0									
	A_0		historical		Black-Scholes		model-free		fit
	central	spread	central	spread	central	spread	central	spread	
	0.17	0.19	0.38	0.00	0.38	0.00			11.08
	0.18	0.19	0.37	0.00			0.32	0.00	11.18
	0.21	0.19			1.02	0.00	-0.44	0.00	11.31
	0.16	0.19	0.39	0.00	0.67	0.00	-0.24	0.00	11.02
Savic and Pedrycz									
h=0									
	A_0		historical		Black-Scholes		model-free		fit
	central	spread	central	spread	central	spread	central	spread	
	0.01	0.32	0.07	0.00	0.78	0.00			18.81
	0.01	0.33	-0.04	0.00			0.83	0.00	19.30
	0.01	0.33			0.55	0.00	0.28	0.00	19.03
	0.01	0.33	-0.03	0.00	0.55	0.00	0.30	0.00	19.06

Table 6. The results for multivariate regressions (variables in levels, h=0).

h=0.5	Tanaka, Uejima and Asai				
	A_0		A_1		
	central	spread	central	spread	fit
historical	0.22	0.40	0.50	0.00	23.01
Black-Scholes	0.22	0.39	0.48	0.00	22.83
model-free	0.23	0.40	0.42	0.00	23.10
	Savic and Pedrycz				
	A_0		A_1		
	central	spread	central	spread	fit
historical	0.05	0.62	0.80	0.00	35.80
Black-Scholes	0.01	0.33	0.84	0.00	18.87
model-free	0.01	0.33	0.79	0.00	19.25

Table 7. The results for univariate regressions (variables in levels, h=0.5).

Tanaka, Uejima and Asai								
h=0.5								
	A_0		historical		Black-Scholes		model-free		fit
	central	spread	central	spread	central	spread	central	spread	
sto bs	0.17	0.19	0.38	0.00	0.38	0.00			11.08
sto mf	0.18	0.19	0.37	0.00			0.32	0.00	11.18
bs mf	0.21	0.19			1.02	0.00	-0.44	0.00	11.31
sto bs mf	0.16	0.19	0.39	0.00	0.67	0.00	-0.24	0.00	11.02
Savic and Pedrycz									
h=0.5									
	A_0		historical		Black-Scholes		model-free		fit
	central	spread	central	spread	central	spread	central	spread	
	0.01	0.32	0.07	0.00	0.78	0.00			18.81
	0.01	0.33	-0.04	0.00			0.83	0.00	19.30
	0.01	0.33			0.55	0.00	0.28	0.00	19.03
	0.01	0.33	-0.03	0.00	0.55	0.00	0.30	0.00	19.06

Table 8. The results for multivariate regressions (variables in levels, h=0.5).

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