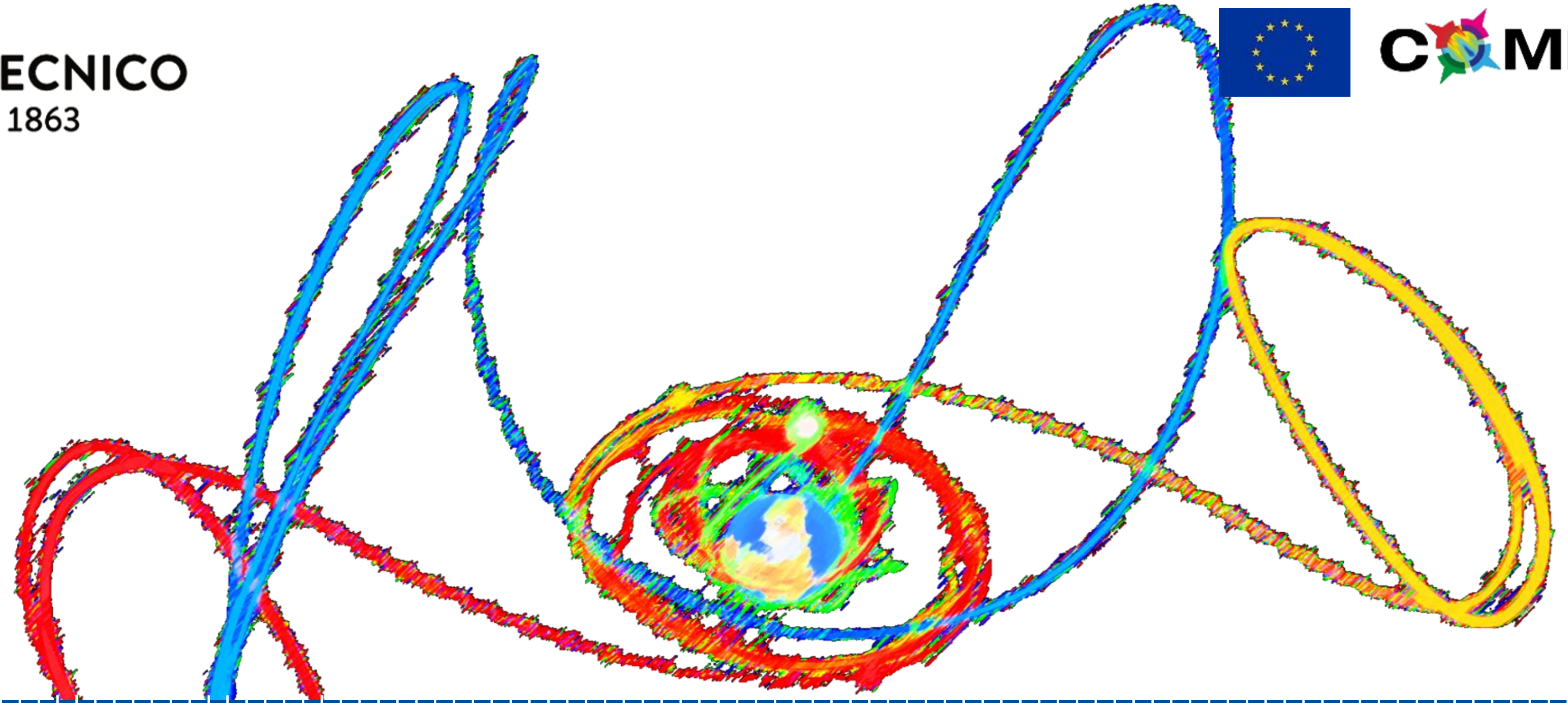

Reachable domain analysis for analytical design of end-of-life disposal

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Politecnico di Milano

New Frontiers of Celestial Mechanics: theory and applications

- Background and motivations
- Modelling of the long-term evolution of s/c
- Phase space structure of orbital elements
- Post-mission disposal
- Conclusion and future work

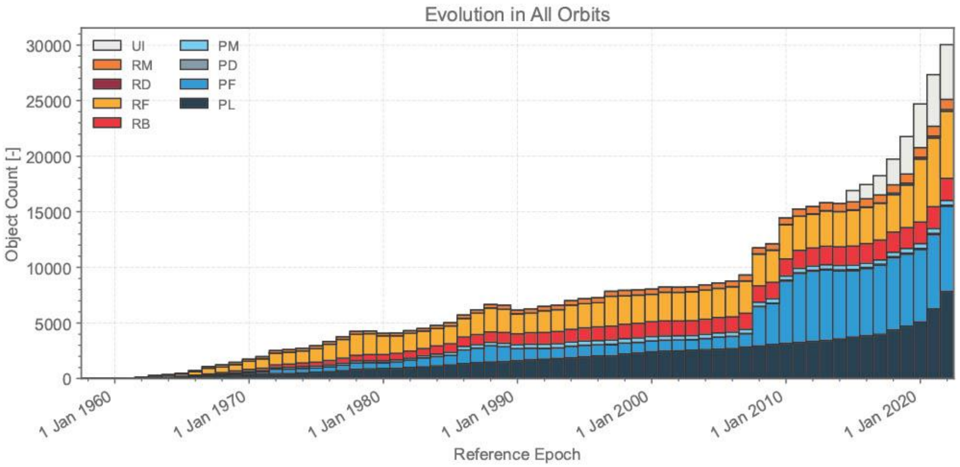
Background and motivation

- Number of debris increases
- Earth’s orbit region becomes crowded

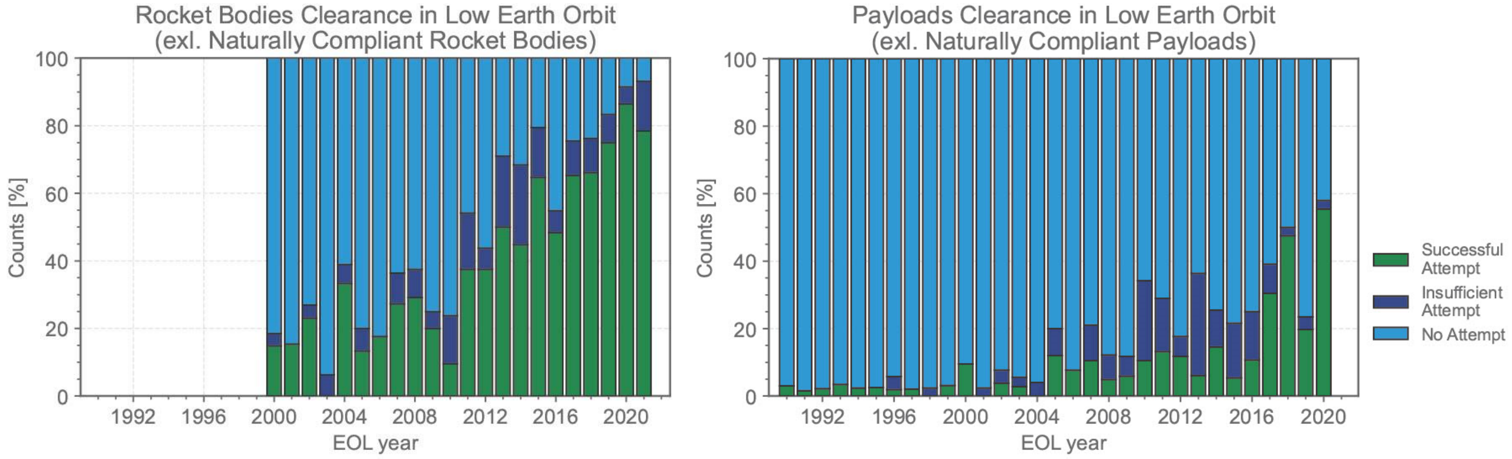
- Necessities of mitigation measures
  - Post-mission disposal, largest contribution
  - …

- Trade-off of mitigation measures
  - Space environment improvements
  - Feasibility and cost

- Post-mission disposal
  - Technically feasible
  - Economical in the sense of fuel consumption



Objects in space increasing rapidly



Making progress in clearance, but nowhere near enough

- ESA’S Annual Space Environment Report, April 2022

Background and motivation

- Aim: end-of-life disposal design techniques leveraging perturbations to decrease the fuel consumption during the disposal
  - Follow the natural evolution of the s/c orbit
  - Enhance the natural perturbations by manoeuvres
    - Impulsive
    - Continuous

  - semi-analytical models of s/c dynamics
  - Hamiltonian formalism
    - Phase space structure of orbital elements

Modelling of the long-term evolution of s/c

Semi-analytical dynamics model based on averaging technique

- Lagrange planetary equations

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2}{na}\frac{\partial R}{\partial M}$$
$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{b}{na^3e}\frac{\partial R}{\partial \omega} + \frac{b^2}{na^4e}\frac{\partial R}{\partial M}$$
$$\frac{\mathrm{d}i}{\mathrm{d}t} = -\frac{1}{nab\sin i}\frac{\partial R}{\partial \Omega} + \frac{\cos i}{nab\sin i}\frac{\partial R}{\partial \omega}$$
$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{1}{nab\sin i}\frac{\partial R}{\partial i}$$
$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = -\frac{\cos i}{nab\sin i}\frac{\partial R}{\partial i} + \frac{b}{na^3e}\frac{\partial R}{\partial e}$$
$$\frac{\mathrm{d}M}{\mathrm{d}t} = n - \frac{2}{na}\frac{\partial R}{\partial a} - \frac{b^2}{na^4e}\frac{\partial R}{\partial e}$$

- Filtering out the short period effects

$$\overline{F}(\alpha) = \frac{1}{T}\int_{t_0}^{t_0+T} F(\alpha)\mathrm{d}t = \frac{1}{2\pi}\int_0^{2\pi} F(\alpha)\mathrm{d}M.$$

- Possible reference frame choices:
  - Equatorial
  - Lunar plane
  - Ecliptic

- Single average, over period of s/c
  - J_2 perturbation

$$\overline{R}_{J_2} = \frac{\mu J_2 R_\oplus^2}{4a^3\eta^3}\left(2 - 3\sin^2 i\right).$$

  - Third-body attraction

$$\overline{R}_{3b} = \sum_{l\geq 2}\frac{\mu_3}{r_3}\left(\frac{a}{r_3}\right)^l F_l(A,B,e).$$

- Double average, over period of the third body

$$\overline{\overline{R}}_{3b} = \sum_{l\geq 2}\frac{\mu_3 a^l}{a_3^{l+1}\eta_3^{2l}}\sum_k\left(F_{l,k}\cos k\omega_3 + G_{l,k}\sin k\omega_3\right)$$

- Total disturbing potential
  - Single-averaged

$$\overline{R} = \overline{R}_{J_2} + \overline{R}_{Moon} + \overline{R}_{Sun}$$

  - Double-averaged

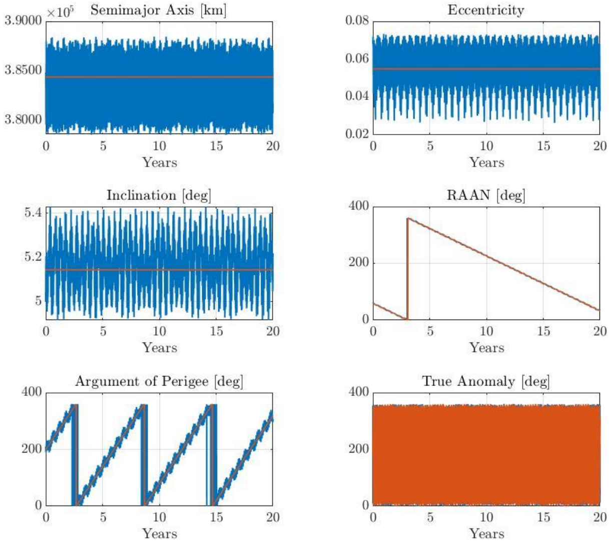
$$\overline{\overline{R}} = \overline{R}_{J_2} + \overline{\overline{R}}_{Moon} + \overline{\overline{R}}_{Sun}$$

- Kaufman and Dasenbrock, Higher order theory for long-term behavior of earth and lunar orbiters, 1972
- Colombo, C. “Long-Term Evolution of Highly-Elliptical Orbits: Luni-Solar Perturbation Effects for Stability and Re-Entry”, 2019

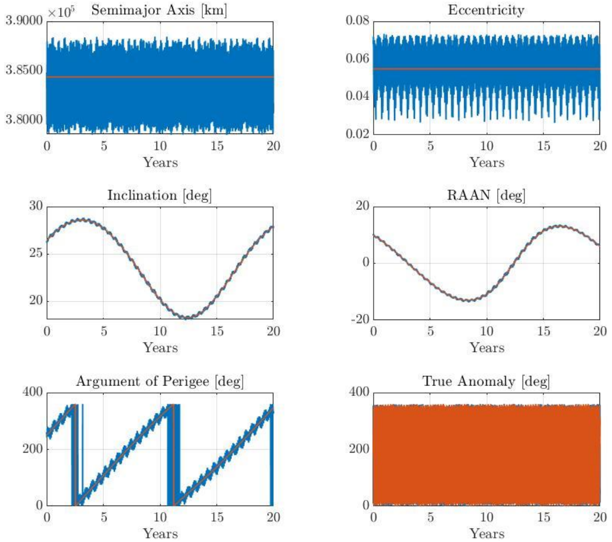
Modelling of the long-term evolution of s/c

Model for lunar and solar ephemerides

- Sun: circular Earth’s orbit
- Moon:
  - Vallado, 2013
  - Simplified model used in double-averaged dynamics
    - Constant semimajor, eccentricity, inclination referred to the ecliptic
    - Linear regression of the lunar node
    - Linear advance of the lunar perigee



Ephemerides of the Moon: refer to the ecliptic.
Orange: Simplified model
Blue: Model from Vallado 2013



Ephemerides of the Moon: refer to the equator
Orange: Simplified model
Blue: Model from Vallado 2013

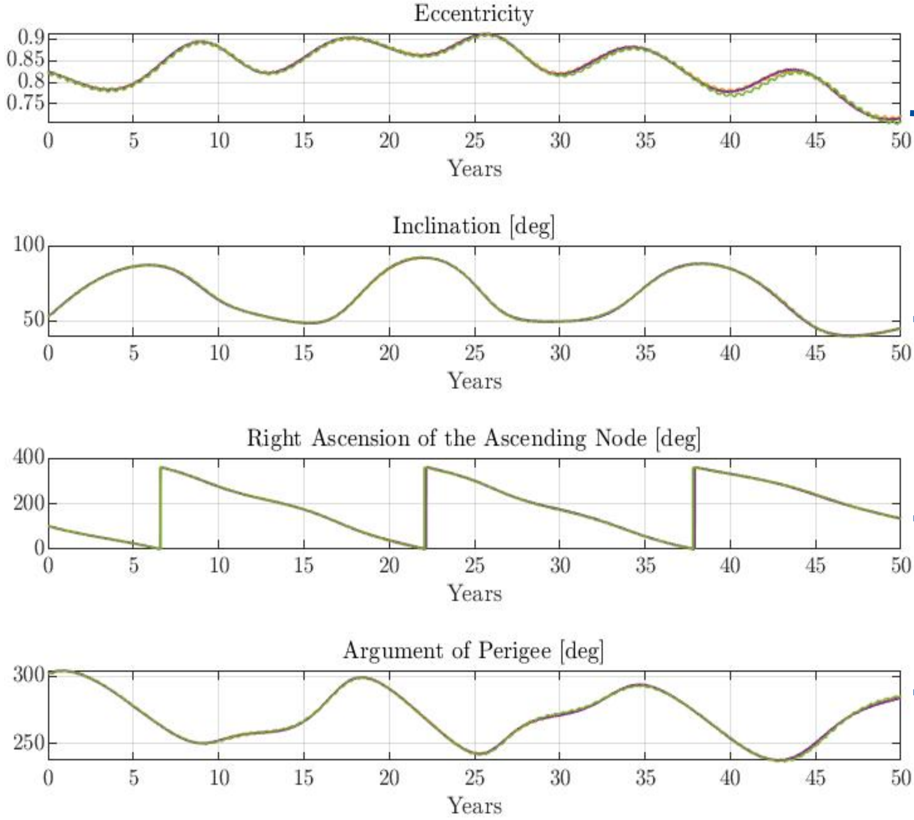
Modelling of the long-term evolution of s/c

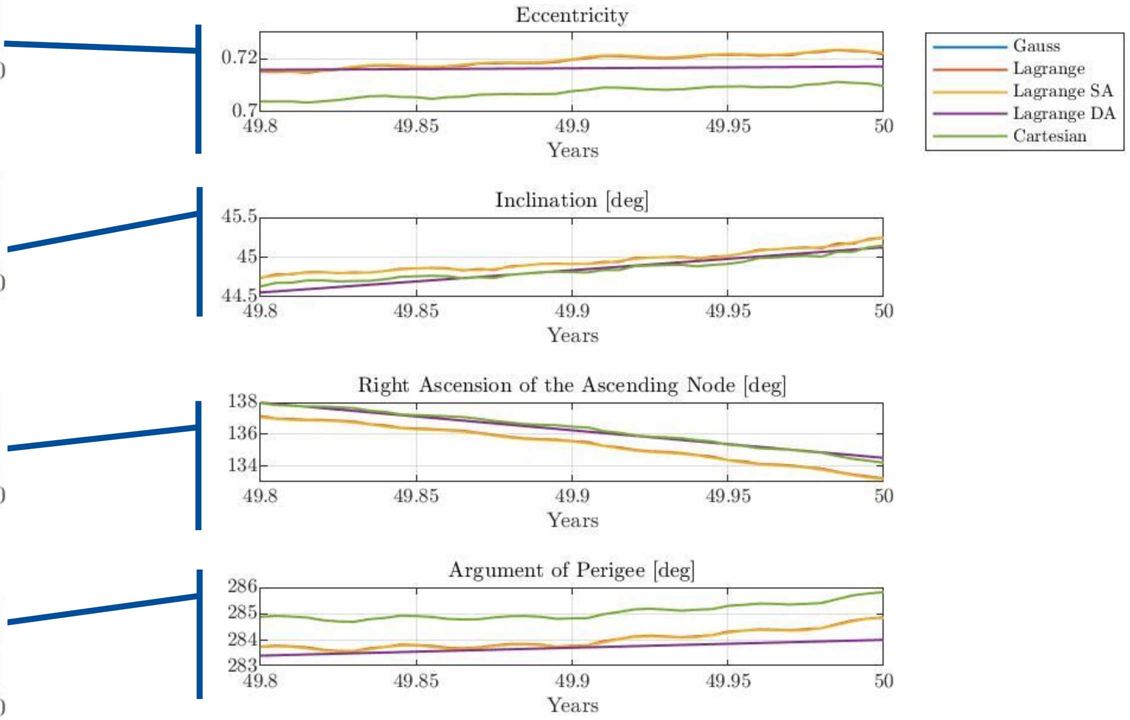
Validation of the model

- INTEGRAL
  - $[87736\ \text{km}, 0.824, 53\ \text{deg}, 102\ \text{deg}, 302\ \text{deg}, 83\ \text{deg}]$
  - From 13-Nov-2002, propagate for 50 years
  - Including J2, and lunisolar perturbations

- Gauss and Cartesian
  - Exact force model for third body attraction

- Lagrange, single-averaged and double-averaged
  - Truncated up to 4th order of the Legendre form

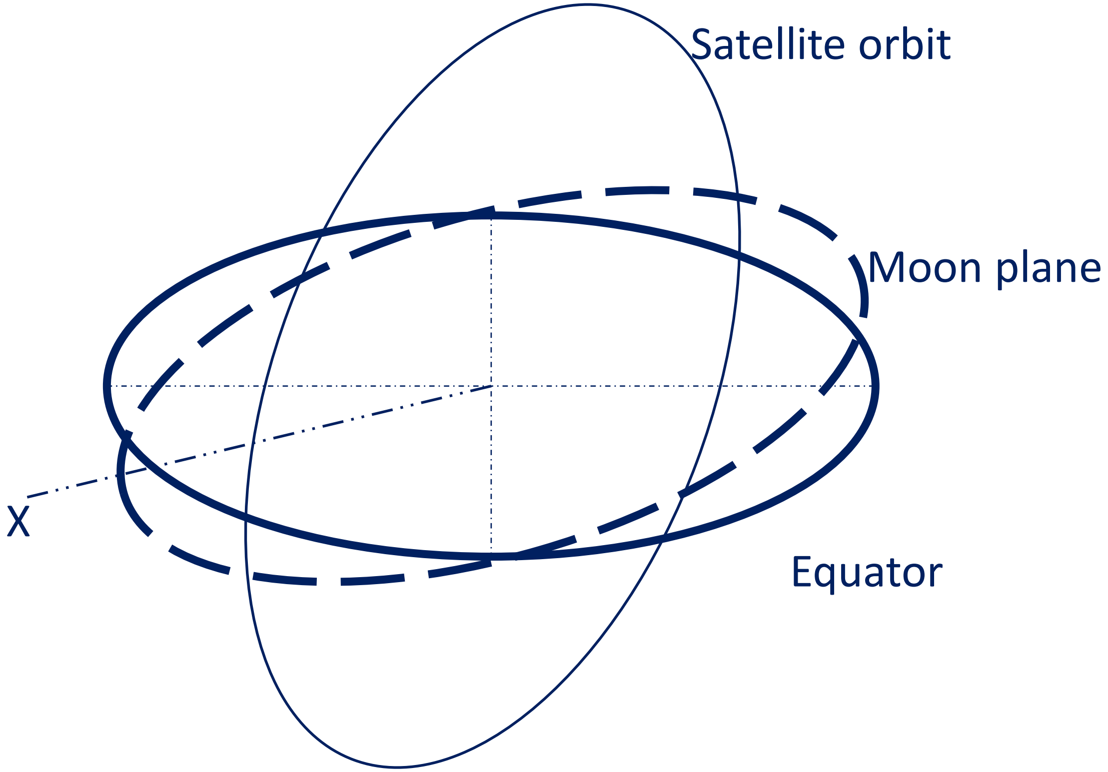




Phase space structure of orbital elements

Double-averaged elements referred to Moon plane

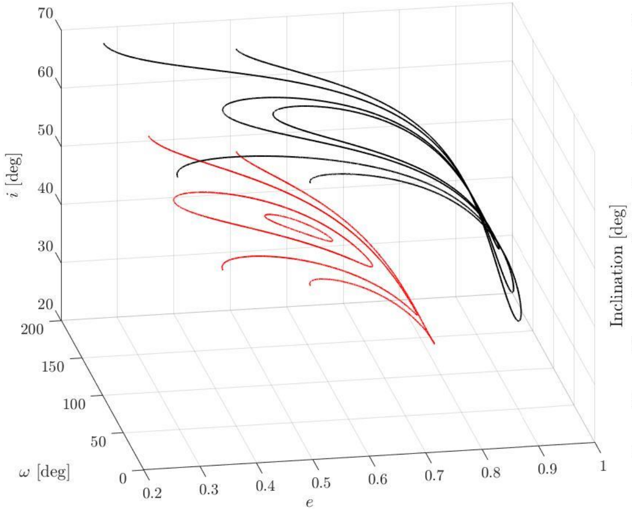
- Constructing the phase space maps
  - Propagate the orbital elements using double averaged equations
  - Transfer to the orbital elements referred to the equatorial plane to those referred to the Moon plane
  - Construct the $e - \omega - i$ maps

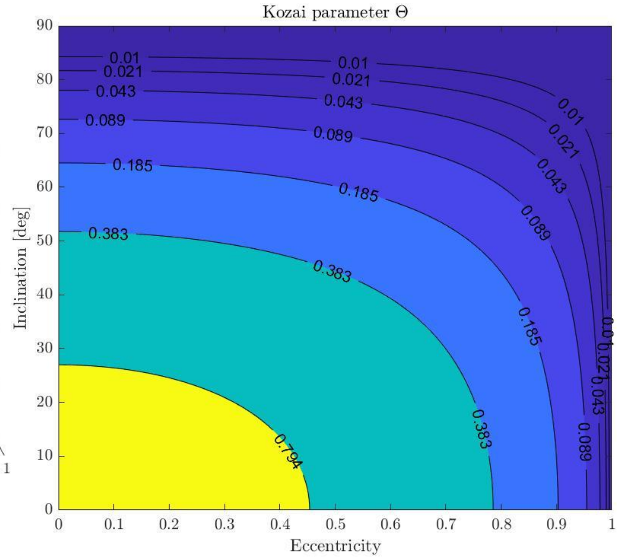


Phase space structure of orbital elements

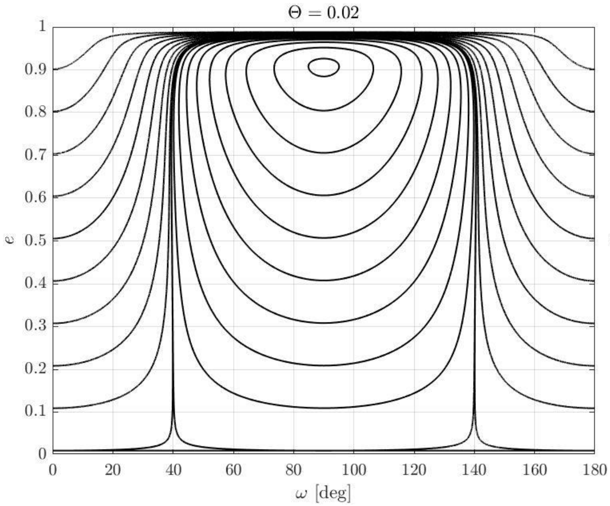
- $e - \omega - i$ map, double-averaged elements referred to Moon plane

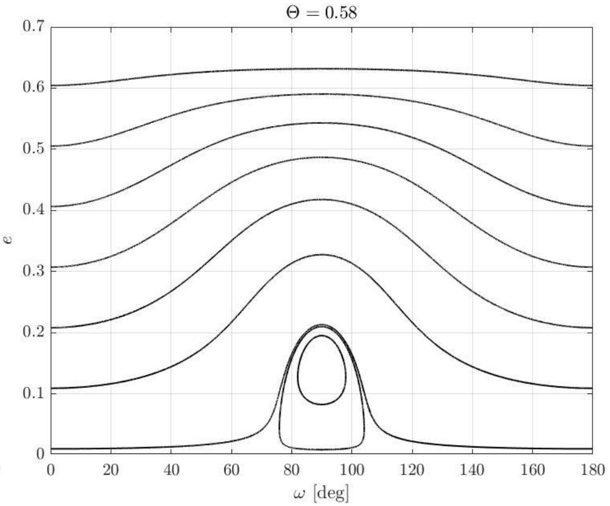
- Lunar perturbation only
  - Layer structure for different (e, i) pairs
  - Corresponding to Kozai parameter, $\Theta = (1 - e^2)\cos^2 i$
    - integral corresponding to the z-component of the angular momentum
  - Smaller Θ, libration regime
    - Large variations for e, ω, i
    - Libration centre at $\omega = 90$ deg
  - Θ increases
    - Libration centre move towards lower eccentricity and lower inclination
  - Higher Θ, rotation regime
    - Almost no libration centre
    - Rotating apsidal line
  - Variation of a, ω only cause movement within the same layer of the phase portrait





Black: $\Theta = 0.1$ Red: $\Theta = 0.3$



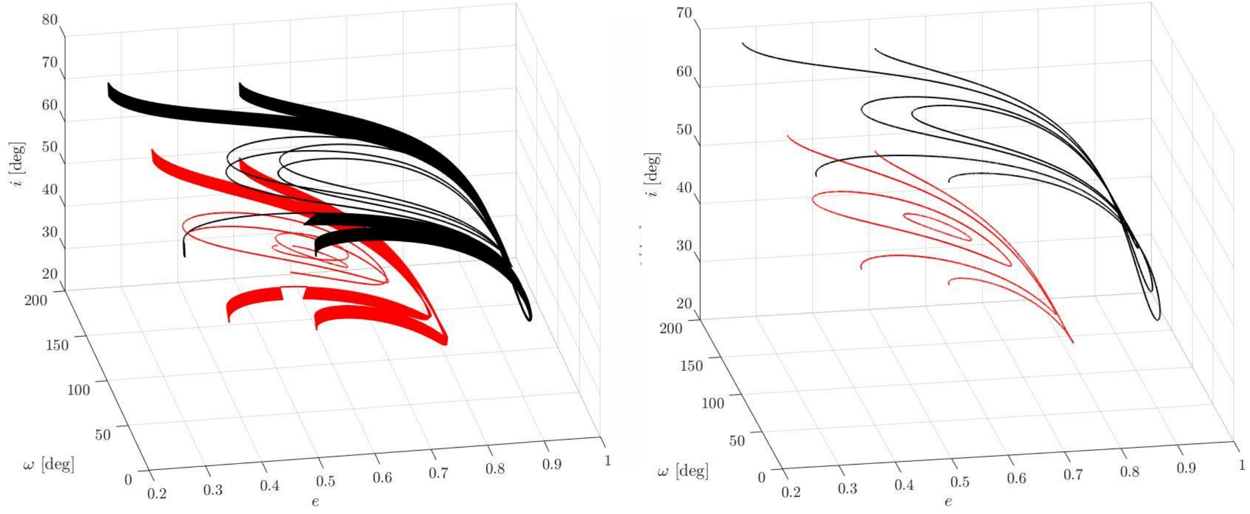


- Kozai Y. “Secular perturbations of asteroids with high inclination and eccentricity”, 1962

Phase space structure of orbital elements

- $e - \omega - i$ map, double-averaged elements referred to the Moon plane

- Lunisolar perturbations
  - Slightly deviate from the case with only lunar perturbation
  - Libration regime
    - Similar to lunar case
    - periodic
  - Rotation regime
    - quasi-periodic
  - Due to small inclination of the lunar orbit with respect to the ecliptic, $i_{Moon} = 5.145 \text{ deg}$
- J_2 and lunisolar perturbations
  - Deviate from the lunisolar case



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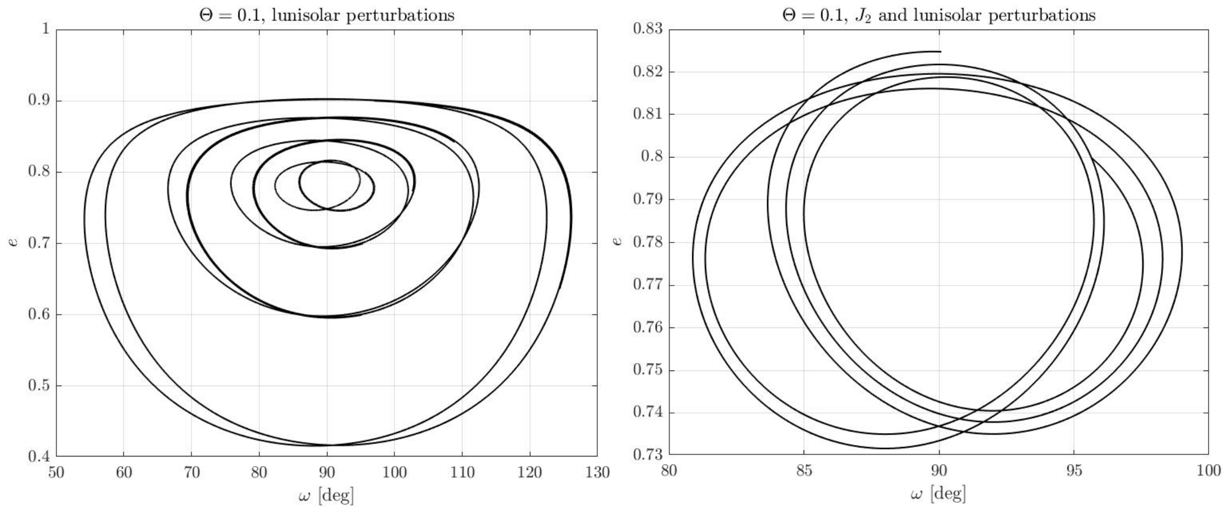
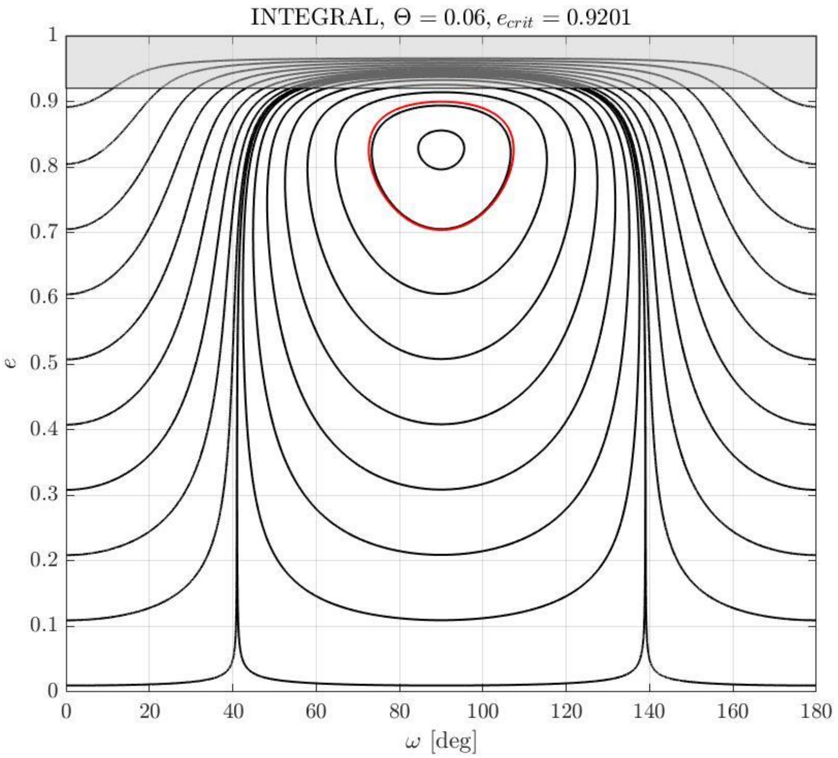


Post-mission disposal

- Possible re-entry solution aided by natural perturbations

- Post-mission disposal for s/c in HEO
  - INTEGRAL
  - Targeting a re-entry



- Phase space structure provide possible solutions for re-entry.
  - Give an initial impulsive manoeuvre
  - $kep = kep_0 + \Delta kep$
  - $\Delta kep = \text{Gauss' equations}(kep_0, \Delta v)$
  - $\Delta v = ||\Delta v|| \begin{bmatrix} \cos\alpha\cos\beta \\ \sin\alpha\cos\beta \\ \sin\beta \end{bmatrix}$, in TNH frame
  - Propagate for 50 years using double averaged dynamics
  - Maximal eccentricity

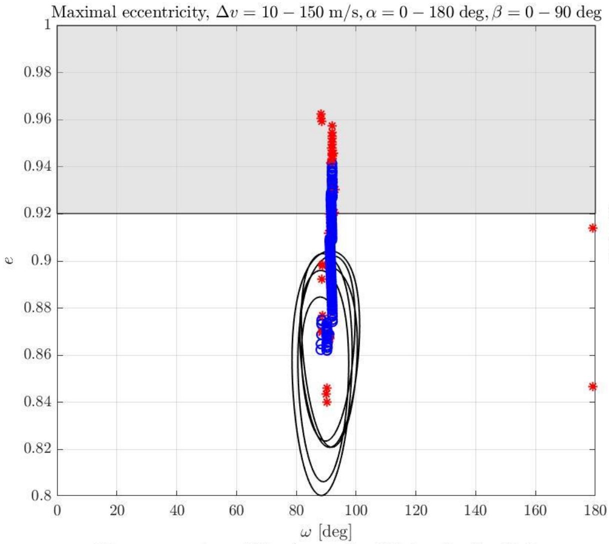
- Criterion for re-entry
  - Perigee height $h_p \leq 100$ km
  - Critical eccentricity $e_{crit} = 1 - \frac{R_\oplus + h_p}{a}$
  - $e_{max} \geq e_{crit}$, possible to re-enter

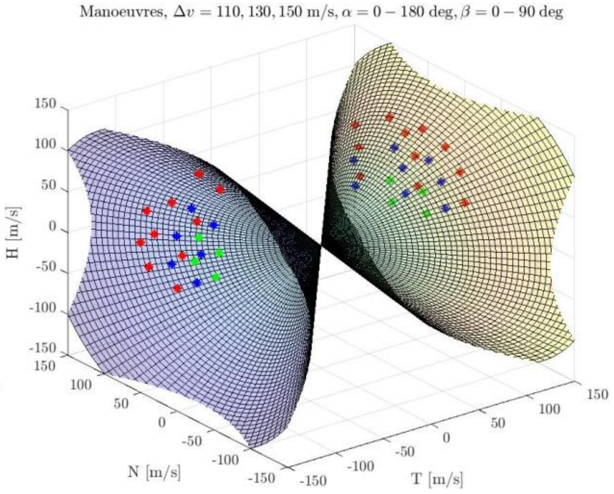
- Colombo, C., End-of-life re-entry for highly elliptical orbits: the INTEGRAL mission, 2014
- Scala F., Analytical design of end-of-life disposal manoeuvre for Highly Elliptical Orbits under the influence of the third body's attraction and planet's oblateness", 2018

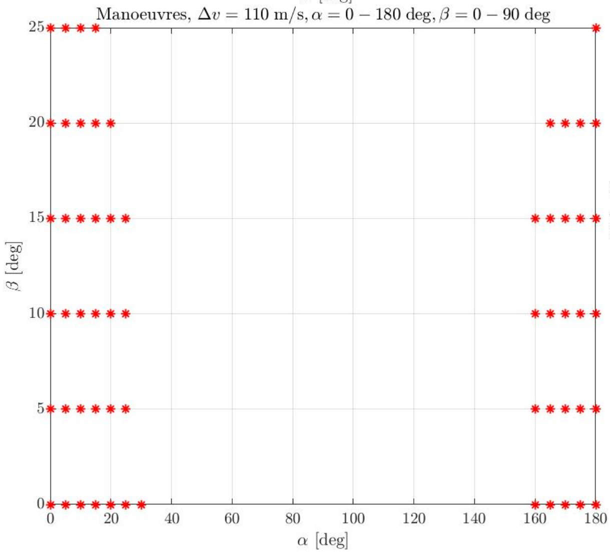
Post-mission disposal

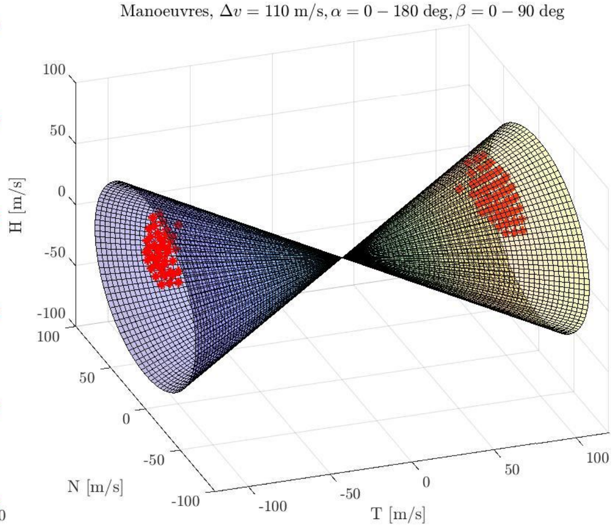
Possible re-entry solution aided by natural perturbations

- Grid search for possible manoeuvre for INTEGRAL’s re-entry
- Initial Keplerian elements of INTEGRAL
  - [81120 km, 0.8977, 55.8956 deg, 109.4648 deg, 293.0959 deg, 0 deg]
  - Date = [2020 12 8]
- 1st search
  - $||\Delta v|| = 100\sim150\ \mathrm{m/s}$
  - $\alpha = 0\sim180\ \mathrm{deg}, \beta = 0\sim90\ \mathrm{deg}$
  - Possible solutions lay in the cone with axis T
  - Either increase or decrease the velocity along the tangential axis
- 2nd search
  - $||\Delta v|| = 110\ \mathrm{m/s}$
  - $\alpha = 0\sim30\ 150\sim180\ \mathrm{deg}, \beta = 0\sim30\ \mathrm{deg}$







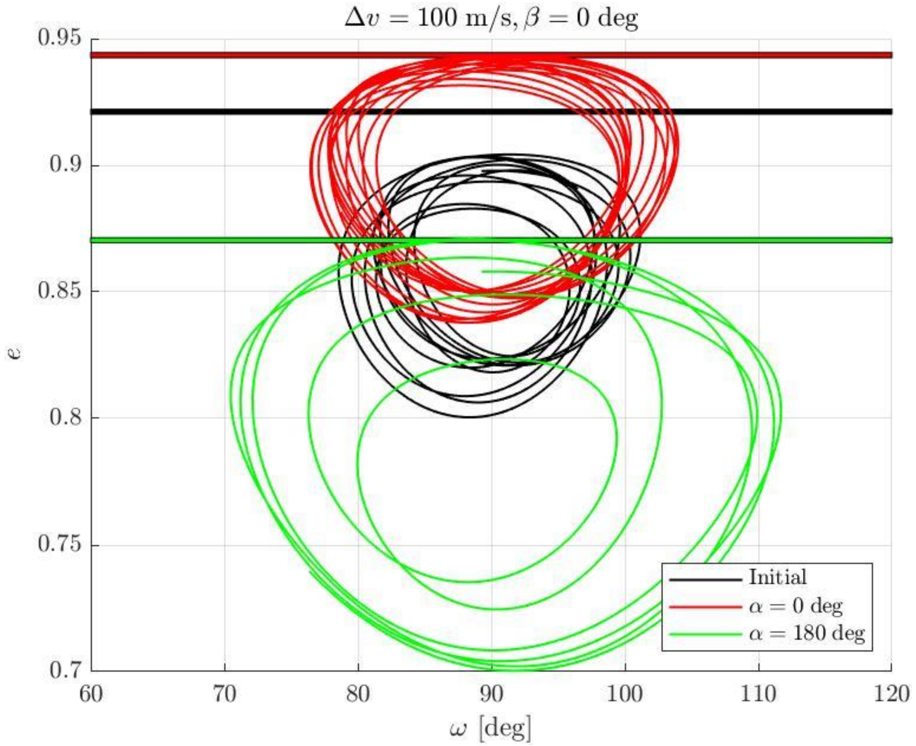


Post-mission disposal

- Possible re-entry solution aided by natural perturbations

- Summary of the results
  - successful re-entry manoeuvres lie in the cone of the tangential direction
  - most effective: T or -T direction

- In the phase space
  - Get the lowest perigee height
  - Increase the maximal eccentricity within 50 years
  - Move the evolution of elements to another layer of the phase portrait, corresponding to different values of the Kozai parameter
    - Change inclination, hard and consuming more fuel
    - Change eccentricity, feasible, along the tangential direction



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- Scala F., Analytical design of end-of-life disposal manoeuvre for Highly Elliptical Orbits under the influence of the third body’s attraction and planet’s oblateness”, 2018

Conclusion and Future work

- Conclusion
  - Implemented the averaged dynamics for s/c under J_2 and lunisolar perturbations
  - Constructed and analysed the phase space structure of Keplerian elements
    - Especially $e - \omega - i$ maps
  - Post-mission disposal analysis targeting an atmospheric re-entry
  - Serve as preliminary work for future optimisations

- Future work
  - Optimisation of the enhancing manoeuvre for the post-mission disposal
  - Analytical design of the post-mission disposal using the phase space structure

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