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On measurable uncertainty
and the fight for taking uncertainty seriously in economics

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Abstract

This paper discusses the engagement of economists with the issue of the measurability of uncertainty. Since Knight's seminal distinction between risk, intended as measurable uncertainty, and unmeasurable uncertainty, the question has been to what extent the extension of the theory of choice from certainty to risk through von Neumann and Morgenstern's expected utility hypothesis would allow dealing with uncertain events. The paper develops from a study of the rationale underlying the theories of those authors who objected to the mainstream view that the axiomatic approach developed in the early 1950s, mainly through Leonard Savage's generalization of expected utility, makes it, indeed, possible to reduce uncertainty to risk. After a summary of the meaning attributed by authors such as Knight, Keynes, Shackle and Ellsberg to the contention that uncertainty is irreducible to risk and unmeasurable, the paper aims to investigate why this view did not emerge as a significant alternative to the mainstream up until recently. A main reason, at times alluded to but never openly discussed in the literature, is shown to be the close link between Savage and the group of decision theorists at the Cowles Commission for Research in Economics under the directorship of Jacob Marschak and Tjalling Koopmans. Archival evidence suggests that arguing that a theory of decision under uncertainty could be developed on the basis of "axioms that seem unobjectionable," as Koopmans put it, was indeed an integral part of the attempt undertook at Cowles to move forward in economic theory by prioritizing scientific rigour in the form of mathematical models engaging with new mathematical tools.

Keywords: probability, uncertainty, decision-making

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1. Introduction

This paper deals with the engagement of economists with uncertainty and in particular with the issue of its measurability. Although the history of the subject is long and the reference to uncertainty has pervaded the development of economic thought since its early days, no proper analysis of uncertainty as distinguished from risk existed before Frank Knight and John Maynard Keynes. The paper takes the classic definition by Knight (1921) as the point of entry of its analysis. Knight famously considered uncertainty as an all-encompassing notion and then distinguished risk, intended as measurable uncertainty, from "true," unmeasurable uncertainty. Knight argued that it was because of the missing consideration of true uncertainty that no convincing explanation for entrepreneurial profit could be found in economics. Similarly, Keynes (1937, p. 214) accused "classical economics" of dealing with the present "by abstracting from the fact that we know very little about the future." As Keynes ([1921] 1973) had provided an entire treatise on the limits of classic probabilistic thinking, the logic of degrees of beliefs he proposed is the other starting point of what follows.

Since uncertainty re-emerged in this fashion, the question of whether it was a measurable entity became of concern for scholars involved in the process of mathematization taking place in economics in the 1940s, particularly after von Neumann and Morgenstern's (1947) *Theory of Games*. For decision theorists, the question was to what extent the extension of the theory of choice from certainty to risk through von Neumann and Morgenstern's expected utility hypothesis would also satisfactorily deal with Knightian (and Keynesian) uncertainty.

After pointing out the similarities between Knight's analysis of uncertainty and Keynes's analysis of probability, the paper provides a brief summary of the rationale underlying the theories of those authors who, admittedly sporadically, tried to keep this view of uncertainty alive by objecting to the mainstream view that the axiomatic approach developed in the early 1950s. The meaning attributed by authors such as George Shackle (1949) and Daniel Ellsberg (1961) to the contention that uncertainty is irreducible to risk, and is therefore unmeasurable, is then contrasted with that proposed by Leonard Savage's (1954) generalization of expected utility to *subjective* expected utility, which makes it indeed possible to think of uncertainty as if reduced to risk through the use of a probability measure representing degrees of belief.

The paper mainly focuses on why the unorthodox view did not emerge as a significant alternative to the mainstream until recently (Gilboa *et al.* 2008, Binmore 2009). A main reason, at times alluded to but never openly discussed in the literature, is shown to be the close link between Savage and the group of decision theorists at the Cowles Commission for Research in Economics under the directorship of Jacob Marschak and Tjalling Koopmans. Archival evidence suggests that arguing that a theory of decision under uncertainty could be developed on the basis of "postulates that seem unobjectionable," as Koopmans (1957) put it, was indeed an integral part of the attempt undertaken at Cowles to move forward in economic theory by prioritizing scientific rigour in the form of mathematical models engaging with new mathematical tools.

2. Unmeasurable uncertainty and non-numerical probabilities

Notwithstanding two different philosophical approaches to probabilities, Knight and Keynes had remarkably similar views about the significance of uncertainty, which they both placed at the very

heart of their respective theories. As noted in Runde (2001) and Feduzi, Runde and Zappia (2014), both authors distinguished between situations in which numerically definite probabilities can be determined from those in which they cannot. Knight is usually remembered for putting forward a widely known distinction between risk and uncertainty, but Keynes's *Treatise on Probability* is also of major interest. The *Treatise* presents the rationale for Keynes's insistence on uncertainty in his subsequent works in economics, providing a sound probabilistic perspective to the issue, something not attempted by Knight.

Knight (1921, pp. 224-225) distinguished between risk and uncertainty on the basis of a taxonomy of "probability situations." When probabilities can be calculated by means of either a priori considerations and general principles − as in fair games of chance − or the identification of classes of homogeneous trials for empirically determined frequencies, the situation is one of risk. Situations of "*true uncertainty*," on the contrary, are those in which only "estimates" can be formulated. Estimates are described as a "third type of probability judgement," but they are not made the subject matter of probabilistic inquiry, since "the processes of intuition or judgement, being unconscious, are inaccessible to study" (Knight 1921, p. 230).[1]

Knight's claim, therefore, is that there are decision-making contexts in which individual agents cannot but formulate estimates, which do not have the status of a probability measure, as there is nothing "objective" in their determination. Against this contention, the subjectivists later argued that estimates may correspond to numerically definite subjective probabilities, since probability is an epistemic notion, a property of the way individuals think about the events conditioning the outcomes of their choices. This view amounted to conflating uncertainty into risk, thus making it measurable, as implicitly assumed in mainstream economics even today (Blume and Easley 2008). However, that Knight's estimates do not necessarily coincide with subjective probabilities, even from an epistemic viewpoint, is a point that can indeed be made, as suggested in the approach to epistemic probability independently put forward by Keynes.

In his *Treatise on Probability*, Keynes interpreted probability as something different from chance or frequency, treating it as a measure of the strength of the logical relation between two propositions, namely a conclusion and the evidence for it, with the term "probable" describing the degree of belief about a proposition which a certain amount of knowledge justifies. The subject matter of the theory of probability, as a result, is defined by Keynes as the relation between two sets of propositions such that "if we know the first, we can attach to the latter some degree of rational belief" (Keynes [1921] 1973, pp. 6-7). In this way, Keynes explicitly aimed at formulating an epistemic approach to probability.[2]

The distinctiveness of Keynes's epistemic approach is that the probability of a conclusion given certain evidence corresponds to the degree of belief that is "rational" to hold, that is, it

[1] As it is well known, Knight illustrated his point in his analysis of insurance. Insurance markets fail to cover uncertain contingencies, events for which no (objective) probability is available. Insurance activity strictly depends "upon the measurement of probability on the basis of a fairly accurate grouping into classes," and it is thus impossible to provide insurance when the events to be insured against "are far too unique, generally speaking, for any sort of statistical tabulation to have any value for guidance" (Knight 1921, p. 231).

[2] In his critique of the frequentist approach, Keynes ([1921] 1973, p. 103) argued: "the identification of probability with statistical frequency is a very grave departure from the established use of words; for it clearly excludes a great number of judgements which are generally believed to deal with probability."

corresponds to what can be logically deduced from that evidence.[3] This aspect of the *Treatise* was harshly criticized by Frank Ramsey ([1931] 1964) and, after the emergence of the subjectivist approach, became a minority viewpoint in discussions of the philosophical foundations of probability (Kyburg and Smokler 1964). Nevertheless, Keynes's theory remains relevant for its discussion of the potential of the epistemic as opposed to the frequency approach. On these grounds, of particular relevance is Keynes's rejection of the idea that probabilities can always be given a numerical representation.

Keynes ([1921] 1973, p. 21) restricted the range of applicability of frequency probability theory, and argued against the generally accepted opinion that "a numerical comparison between the degrees of any pair of probabilities is not only conceivable but it is actually within our power." Very much like Knight, Keynes understood the determination of numerical probabilities as something largely restricted to the epistemic analogue of chance-based set-ups. However, what is peculiar about Keynes's view is that it contemplates probabilistic reasoning even in the absence of numerical probabilities. Keynes discussed an ordinal notion of probability in which the ordering is not necessarily complete, as became common in subsequent decision theory. Of some probabilities, he argued, it cannot be said that one is either equal to, greater than, or less than another (Keynes [1921] 1973, p. 33). In these instances, typical of many ordinary situations, degrees of belief can be represented by what he called "non-numerical" or "numerically indeterminate" probabilities: these special probabilities may be comparable in qualitative terms. What is more, he also attempted to provide a mathematical structure for the non-numerical probabilities set up.[4]

Well before the developments suggested by Savage and the emergence of the new mainstream in the 1950s, therefore, Keynes's view appears critical of an approach in which initial subjective probabilities are precise numbers, something first noted − and objected to − by Ramsey in his seminal presentation of the subjectivist approach. However, Keynes's critical stance presented a framework for imprecise probabilistic reasoning with characteristics that made it in principle suitable for a generalization of the Bayesian approach. As a matter of fact, those subjectivist scholars who were critical of what they saw as a too restricted version of subjectivism in the works of Savage explicitly referred to Keynes's approach in his *Treatise* (Smith 1961, Ellsberg [1962] 2001, Good 1962).

Differences in philosophical foundations aside, it is evident that the peculiar Keynesian construct for probabilistic reasoning is well suited to justify Knight's distinction between risk and uncertainty. The analogy between Keynesian and Knightian uncertainty comes out even more strongly in Keynes's analysis of financial markets in Chapter 12 of his *General Theory* (Basili and Zappia 2021) and in the 1937 *Quarterly Journal of Economics* article, famously indicating that the revolutionary character of the *General Theory* was rooted in its dealing with uncertainty (Backhouse 2006). Both Knight and Keynes insisted on the importance of allowing vagueness to pervade the

[3] Unlike subjectivists, Keynes argued that different individuals sharing the same evidence are constrained by these principles to agree on definite probability judgements. This view originated the so-called logical approach to probability (Gillies 2000).

[4] In his formal logic of probability, Keynes pointed to the possibility of inexact numerical comparisons, highlighting the partial ordering that emerges in situations in which it is not possible to translate degrees of beliefs into sharp numerical probabilities (Brady 1993; Basili and Zappia 2009a). His attempt to develop what he called a "systematic method of approximation" was later taken up by Bernard Koopman (1940), who provided an axiomatization of Keynes's ideas that allows for ranges of subjective probability priors, sometimes called interval-valued probabilities, and defended among subjective probabilists by Irving Good (1952).

decision environment individual agents usually confront, something that makes uncertainty in principle unmeasurable.[5]

However, Keynes's more detailed treatment provides a firmer foundation for developing a formal theory of probability. Keynes's attempt to provide a formal logic for qualitative probability constituted a step forward noted by probabilists and decision theorists (Zappia 2021b). As we shall see, both Kenneth Arrow and Savage took Keynes's theory of probability into account while championing the development of the subjectivist approach. But, in their choice to limit rational decision-making to a specifically defined design, they deliberately restricted the representation of beliefs to what they saw as an axiomatically manageable notion for consistency, while Keynes's analysis of probability indicates that he preferred a wider notion of measurability.[6]

3. The 1950s mainstream view

The establishment of a new mainstream view in decision-making under risk and uncertainty was largely accomplished in the mid-1950s. As argued by Luce and Raiffa (1957, p. 37), Savage's (1954) extension of von Neumann and Morgenstern's (1947) expected utility approach to dealing with uncertainty constituted the backbone of a "modern" theory of utility, deemed as the basis of much decision theory.

Building on what had already become usual in the theory of choice under certainty (Hicks and Allen 1934; Samuelson 1938), the accepted approach was to develop a set of assumptions devoid of reference to the psychology of the economic agent, but considered "plausible" by the decision theorist, who would also consider their testability in controlled environments. Rationality as consistency had started to become the trademark of post-war neoclassical rationality (Giocoli 2003). In particular, von Neumann and Morgenstern had shown how to proceed in providing the axiomatic bases for the use of utility even in choices under risk, and these were reformulated by Jacob Marschak in order to clarify what requirements choosing among risky prospects would entail. After Marschak's (1950) reinterpretation of von Neumann and Morgenstern, the typical axioms for choice under certainty − mainly transitivity and completeness, plus technical conditions such as continuity − were

[5] As recently restated in various contributions to the 100th anniversary of their books, the kind of arguments that Knight and Keynes were making about the nature of uncertainty continue to be raised against the subjective expected utility model, both in economics and decision theory (Faulkner *et al*. 2021). An extended analysis of communalities and diversities in Knight and Keynes on uncertainty is provided by Gerrard (2022).

[6] The same attention to Keynes's probabilistic views cannot always be attributed to economists. In his 1937 article summarizing the importance of uncertainty in his General Theory, Keynes (1937, pp. 113-114) wrote that by "uncertain" knowledge he did not refer to situations like "the game of roulette … or the prospect of a Victory bond to be drawn," but to situations like "the prospect of a European war … or the obsolescence of a new invention." And then he famously concluded: "about these matters there is no scientific basis to form any calculable probability whatever." While the emphasis on an uncertainty fundamentally different from risk is clear, whether this amount to a statement that uncertainty is unmeasurable *tout court* is, on the contrary, controversial. The possible analogy between "calculable" and "numerical" probabilities in the just quoted excerpt suggests that there can be a non-numerical treatment of uncertainty that admits formal representation, although of a kind different from that of the mainstream (Zappia 2016).

supplemented by the so-called independence axiom, thus accounting for risky prospects under the same theoretical construct.[7]

Marschak's (1950) aim was to extend the logic of choice from certainty to "uncertainty" − indeed merely risky prospects, not truly uncertain ones *à la* Knight.[8] His contribution consisted in presenting an axiomatic system that, by virtue of the independence axiom, made it possible to restrict economic behaviour to consistent action, even when the consequences of individual choice were not uniquely determined. The validity of expected utility theory − mentioned by Jevons and other marginalist thinkers, although criticized by Marshall in light of people's willingness to engage in unfair lotteries (Friedman and Savage 1948) − was now based on the restrictions imposed by axioms on basic choices, choices that do satisfy both theoretically imposed reasonable properties and empirical verification. The observation of individuals' choice behaviour over lotteries could be used to test their (implicit) use of expected utility, in accordance with revealed preference theory.

But it was Savage's version of the independence axiom, which he termed the "sure-thing principle," which made it possible to use the maximization of expected utility even when probabilities are not "objectively" given. Only after Savage's extension from the dealing with decisions among "prospects" to decisions among "acts" did it become possible to allow the assignment of both utilities and probabilities to choices.[9] Rational choice theory translated into an axiomatic treatment of utility in which the subjective probabilistic viewpoint made it possible to analyse uncertainty in the same vein as risk. Because of Savage's and the subjectivists' insistence on the viability of using a sharp probability prior in any instance − to be updated in view of new information following the Bayes Theorem − this approach has been since called Bayesian decision theory.

Soon after Savage's *Foundations* presented subjective expected utility in a consistent axiomatic setting, the acclaim among decision theorists was almost unanimous. Luce and Raiffa (1957, p. 304) opened their chapter on individual decision-making under uncertainty with an endorsement of Savage's representation of the decision problem in terms of "states of nature," "acts," and "consequences," praising that in his approach "no concept of objective probability is assumed; rather, a subjective probability measure arises as a consequence of his axioms." Raiffa and Schlaifer (1961, p. vii) claimed that applied statistical decision theory should consist of identifying a course of action "that is logically consistent with the decision maker's own preferences for consequences, as expressed by numerical utilities, and with the weights he attaches to the possible states of the world, as expressed by numerical probabilities." In interpreting business decision-making under uncertainty

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[7] Von Neumann and Morgenstern's (1947) second edition of *Theory of Games* had offered a proof that individuals obeying a series of axioms about their preferences over indifference classes of lotteries would act as if following expected utility theory. Their axiomatization of the expected utility hypothesis included the completeness, transitivity and continuity of preferences, but did not feature an assumption corresponding to what today, after Marschak (1950) presented it as postulate IV, is called the independence axiom (Fishburn and Wakker 1995, Moscati 2018).

[8] Marschak used to distinguish the case of "complete information" from that of "incomplete information." The former applies to individual choices when the individual agent "thinks he knows certain relevant probability distributions," the latter when the individual "does not think so." Marschak (1950, p. 113) maintains the "uncertainty in Knight's sense is presumably identical with incomplete information," and clarifies from the outset that his analysis of rational behaviour does not apply to incomplete information.

[9] As it became conventional in decision theory after Anscombe and Aumann (1963), the distinction between choice among prospects (lotteries constituted by different outcomes with known probabilities) and choice among acts (gambles with outcomes attached to events of which probability is "unknown") can be seen as one between "roulette lotteries" and "horse lotteries."

this way, they depended on "the logical and philosophical justification for this statement of the problem … fully developed by Savage in his *Foundations of Statistics*."[10]

The fully receptive attitude shown in decision theory smoothly translated into economics (Arrow 1958, Baumol 1958, Hirshleifer 1965, Samuelson 1966).[11] The broad acts / states / consequences framework suggested by Savage provided a natural way to structure decision-making problems in economic theorizing. In particular, it has since been assumed that, in order to guide their actions, decision-makers employ such a highly simplified cognitive representation to deal with any kind of outside environment. Through its application to game theory with incomplete information (Harsanyi 1967), this subjectivist viewpoint informed the economics of information (Hirshleifer and Riley 1979). Eventually it became standard in microeconomics textbooks.

As Savage (1961, p. 576) himself summarized, in the subjectivist viewpoint, probability simply is "an index, in an operational sense …, of a person's opinion about an event." More relevantly, following Ramsey ([1931] 1964) and de Finetti ([1937] 1964), Savage showed that such an index satisfies the role of probabilities, making it possible to turn personal degrees of beliefs into a probability measure, a result he obtained through an emphasis on what having "consistent" beliefs implies. It was therefore on the basis of such a subjective perspective that it was possible to think of decision-making under uncertainty as if it were under risk, defying Knight's view that there is a behavioural distinction between risk and uncertainty based on what can be probabilistically measured and what cannot. From the acceptance of Savage's axioms system on, most of the economics profession has taken for granted that all uncertainties can indeed be measured through a probability measure, and therefore can be dealt with as one would deal with risk. Only a minority of heterodox economists insisted on the relevance of Knight's distinction, with the Post Keynesians being the most vocal in view of Keynes's similar position (Davidson 1991).[12]

Surely, such an immediate conversion to considering uncertainty as a measurable entity has its origins in the peculiar process of mathematization taking place in economics in the 1940s, of which von Neumann and Morgenstern's expected utility theory represents a major outcome (Giocoli 2009). But it was the endorsement of Savage's subjectivist viewpoint which originated a definitive shift in the theory of choice. Not only were the neoclassical individual utility maximizers transformed into "rational" agents satisfying consistency requirements in their choices, but also those requirements could be applied to their beliefs, transformed into a theoretical object suitable for modelling.[13]

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[10] Giocoli (2013) argues that the endorsement of applied statistical decision-making in American business schools in the 1960s is a relevant element explaining the widespread adoption of Savage's perspective.

[11] Moscati (2016) has shown how convincing Samuelson that the sure-thing principle was normatively reasonable was a fundamental step in the formation of the new mainstream.

[12] A revealing episode of the pervasiveness of subjective expected utility is the exchange between Joseph Stiglitz and Paul Davidson while commenting Robert Skidelsky's book on the need to a return to Keynes after the 2008-9 financial crises. While appreciative of Skidelsky's main argument about returning to use Keynes's insights, in his review the self-proclaimed *new Keynesian* Stiglitz criticizes an excess of emphasis placed by Skidelsky on uncertainty, claiming that it provides "little insight" into the causes of the Great Recession. To this point the self-proclaimed *post Keynesian* Davidson reacted arguing that Stiglitz's comment shows "his own imperfect understanding" of Keynes (Stiglitz 2010). In the background, it looms large the acceptance versus the denial of subjective expected utility in the Keynesian camp.

[13] Marschak had already imagined a "homo oeconomics" conflated into "homo statisticus" to such an extent as to be able to use expected utility under both risk and uncertainty. Cherrier (2010, p. 449) reports an illuminating 1947 exchange with Karl Polanyi in which Marschak claims: "I also agree that to identify 'economic' with 'rational' narrows down the field to an extent that realistic economics becomes almost impossible. . . . To construct a rational man acting under conditions of uncertainty, would mean to make the

Savage's (1954, p. 6) goal was to obtain a characterization of rational behaviour devoid of any psychological contamination by extending logic "to bear more fully on uncertainty," as already suggested by Marschak.[14] Soon after Savage proved that this further extension of logic was indeed possible, the Bayesian creed, the claim that individuals are able to provide a probabilistic assessment on which to act in every instance − from chance games to never-before-experienced uncertain situations such as research and development investment activities − became an agreed standard.

It is therefore of interest to delve into the history of how having a numerical representation of uncertainty, one that hinges on the notion that degrees of belief are indeed ("personal") probability measures, became an indisputable building block of economics research. In fact, as a survey of alternative contemporary analyses reminds us, this was by no means an obvious or inevitable development. The next section offers a brief recap of why this is so.

4. Shackle's representation of uncertainty and Ellsberg's ambiguity

We have seen in the previous sections that the traditional view was that uncertainty was unmeasurable. However, the effort to show that economics was a science of rational choice made it necessary to deal with uncertainty in a formalized manner. Savage's success was related to his ability to find axioms justifying dealing with uncertainty as if it were risk. But the meshing of a subjective probability view *à la* Ramsey-de Finetti with the axiomatic approach *à la* von Neumann-Morgenstern was not obvious at all in the late 1940s. Indeed, the rejection of the possibility of representing degrees of beliefs through a probability measure was the norm rather than the exception. Even Paul Samuelson, who had shown how to reconstruct preferences from choices, was not ready to accept that a similar procedure could be applied to beliefs (Samuelson 1966).

The discussion about criteria for decision-making under uncertainty centred on Abraham Wald's theory of statistical decision functions. In his approach to statistics as a field of decisions to be made rather than inferences to be drawn, Wald (1945) had suggested that in absence of a meaningful probabilistic set-up it was sensible to concentrate on the minimum consequence associated to each possible event and then to choose a course of action guaranteeing the maximum consequence among the minima. Leonid Hurwicz (1951) suggested that a criterion for choice under extreme uncertainty, which he identified with ignorance, could not concentrate only on worst consequences − Wald's maxmin being too pessimistic − but should combine both the worst and the best consequence associated with each course of action. These criteria would have been a step forward in dealing with situations of uncertainty, but appeared ad hoc in the way they selected a course of action, because they were not based on a set of postulates of rational behaviour associated with the suggested decision function (Arrow 1951, Chernoff 1954).

man not only a 'homo economicus' but also a 'homo statisticus;' he would have to be supposed to make rational decisions based on incomplete information, as it is being done by sampling statisticians." Unlike Savage, though, Marschak's statistician was a classic statistician, not a Bayesian one endorsed with a subjective probability prior (see also Marschak 1951).

[14] Marschak's classic justification reads as follows: "If advisable decisions must obey rules of logic and arithmetic, they are not themselves defined by those rules alone. The fulfilment of rules of conventional logic and arithmetic is a necessary but not a sufficient condition for a decision to be advisable. We need additional definitions and postulated rules, to 'prolong' logic and arithmetic into the realm of decision" (Marschak 1950, p. 112).

The fact that there are situations in which an individual agent (or Wald's statistician) does not assign probability to events, mostly considered as technical questions in debate over criteria for decision-making, was a philosophical and methodological starting point for George Shackle. Shackle (1949) objected to the representation of individual beliefs through a probability measure on the grounds that probabilities do not apply to singular decisions that are crucial to the decision-maker. He distinguished between distributional uncertainty variables, which can be used if the list of states of the world is exhaustive, and non-distributional uncertainty variables, which must be used when the list is not complete, that is, under uncertainty. Although initially his reference to Knight and Keynes was sparse, he stressed that crucial decisions are in the realm of true uncertainty and probabilities cannot be used to represent degrees of belief in such situations.

What is peculiar to Shackle, and makes him a significant figure in the history of decision theory, is that he proposed a formal theory of his non-probabilistic approach. Shackle detailed the axioms that, in his view, would justify the use of a decision criteria based on what he called focus values, a selection of the possible consequences associated with a particular course of action.[15] Arrow's (1951) influential survey of the state of contemporary decision theory, to which we shall return, singled out Shackle's as the sole formalized theory deserving of examination among those aiming to discard the probability framework. Although he objected to the viability of his theory, Arrow identified Shackle as the main representative of the tradition of thought initiated by Knight and Keynes and presented his theory as part of state-of-the-art theories of decision under uncertainty.

In the 1950s, decision theorists, mathematical statisticians and psychologists analyzed Shackle's theory as an alternative to the then consolidating mainstream. Some of them showed a keen interest in the viability of a formally structured alternative to those theories of behaviour aiming to describe uncertainty through probability statements (Graaff and Baumol 1949).[16] Shackle took part to the 1952 Paris Conference on Risk, invited by Maurice Allais to confront the "American School" of expected utility theory supporters. As late as 1955, Shackle was invited to give the introductory theoretical paper at a conference in Pittsburgh on expectations and business decisions, a central episode which we shall come back to later. Daniel Ellsberg, a decision theorist who was to become famous for his attack on Bayesian decision-making, quoted Shackle among the authors inspiring his claim that Savage's framework was unable to deal with the problem of choice when information is perceived to be limited and ambiguous (Ellsberg [1962] 2001, pp. 19-20).

However, Shackle's proposal to take uncertainty seriously was soon marginalized. To be sure, Shackle can be considered part of a lively school of thought, but only if seen as a post Keynesian and only from a 1970s perspective. He is still quoted as the archetypal dissenting Keynesian regarding decision-making and the limits of probability calculus (Chick and Dow 2012). But when he confronted the subjectivist systematization of decision theory, Shackle was an isolated figure and the absence of a well articulated group of people supporting him partly accounts for his failure in the 1950s to stand against his critics, notwithstanding the fact that part of his intuition proved significant

[15] On Shackle's theory, see Zappia (2014), on which this section partly draws.

[16] As reported in Zappia (2014) a letter from William Baumol to Shackle testifies to Baumol's fascination with Shackle's discussion of decisions under uncertainty. In a short remark added to his 1950 Christmas wishes to Shackle, Baumol wrote: "I am now taking issue with Marschak who in order to 'measure' utility argues that rationality requires us to maximise the actuarial value of the utility of a prospect even on a non-repeated decision! Will the irrelevance of probability features for such decisions never be understood?" The story of Baumol's conversion to the mainstream is presented in Moscati (2018).

in later developments, even some of its formal aspects. Shackle never tried to compare the formal structure of his axioms system with the contemporary effort made by decision theorists such as Marschak and Savage, but aspects of decision-making on which he concentrated were of special relevance. In particular, after rejecting probability calculus he was forced to introduce new rules of conditioning for his notion of degrees of improbability, which he famously called potential surprise. Although rejected at the time, Shackle's rules of conditioning were re-evaluated by Levi (1972) as being appropriate to a measure that embraces a generalized Bayesian perspective with multiple priors and have also since reviewed in recent assessments of non-Bayesian decision theory.[17]

Above all, Ellsberg's attack on Bayesian decision-making constituted the most well-know and significant step towards a rehabilitation of uncertainty as different from risk, and shows that there was a persistent, if minority, view of the Savage viewpoint. Ellsberg (1961) questioned Savage's subjective probability approach on the grounds that it was not rich enough for dealing with many instances of actual decision environments, thus allowing the unwarranted reduction of decision-making under uncertainty into a framework more properly devised for risk. On the basis of observed "violations" by a number of colleagues he tested in the late 1950s at the RAND Corporation and Harvard Economics Department, Ellsberg (1961) claimed that vagueness about probabilities could lead individuals to breach the axioms of consistent behaviour upon which Bayesian decision theory was based. Savage's sure-thing principle, in particular, was violated by individuals acting in situations of what Ellsberg (1961, p. 646) called "ambiguity." Ellsberg identified a class of very simple choice-situations in which "many otherwise reasonable people neither wish nor tend to conform to the Savage postulate." Since in many cases the violations were deliberate − that is, knowledgeable individuals such as his colleagues did not change their choices even after thorough reconsideration of the significance of their deviation from the theory − he concluded that the use of sharp probability priors could be inappropriate even from a normative point of view. In economics literature, these results are well known as the Ellsberg Paradox.

The Ellsberg Paradox has become the subject matter of many experimental studies since the 1980s (Camerer and Weber 1992) and it remains the fundamental starting point for every study aiming to show that decision theory under uncertainty implies a wider spectrum of decision criteria than the maximization of subjective expected utility (Gilboa 2009). Although with different emphases, its relevance in a re-appraisal of Keynes's analysis of uncertainty has been noted among Keynesian authors (Dequech 2000, Fontana and Gerrard 2004). But its influence was very limited at the time of publication. Ellsberg's results were granted empirical interest, but were deemed irrelevant from a normative viewpoint. Very much as with Shackle, the inability to provide a sound axiomatic structure for his results made his interpretation of uncertainty as an unmeasurable entity descriptively appealing, but this was ultimately rejected on the basis that it was impossible to support it normatively. Notwithstanding Ellsberg's proximity to Savage, the uncertainty he alluded to was classified among the kind of paradoxical results every theoretical corpus may confront without requiring its demise (Zappia 2021a).

[17] As recalled in Basili and Zappia (2009b), Shafer (1976) noted that Shackle's properties pertain to a non-additive probability function like the one he was investigating in his theory of evidence. Eventually, these properties have been extensively investigated in a statistical literature relying on the notion of fuzzy measures, and indeed they are part of the foundations of modern possibility theory (Dubois and Prade 1988) and represent a link to the non-additive probability approach (Machina and Siniscalchi 2014).

But it is now time to come back to the reasons why queries such as Shackle's and Ellsberg's, trying to give formal content to an idea of uncertainty inspired by Knight's and Keynes's suggestions, have remained marginalized for so long.

5. Creating a mainstream view

Trained as a mathematician at the University of Michigan, where he received his Ph. D. in 1941, after a year at the Institute for Advanced Studies in Princeton as an assistant to John von Neumann, Leonard Jimmie Savage started working on statistics during World War II. He was part of the Statistical Research Group, a think tank at Columbia University providing statistical analysis for the US Army that included people such as Milton Friedman, Harold Hotelling, Frederick Mosteller, Abraham Wald and Allen Wallis. He then went to the University of Chicago as Research Associate to the Department of Mathematics in 1947, where he contributed with Wallis to the foundation of the Statistics Department, initially named the Committee on Statistics, in 1949. Since the University of Chicago had given in those years "increased support and recognition" to the Cowles Commission (Cowles 1949, p. 4) it comes as no surprise that he was involved with people and research at Cowles.

As noted by his disciple and co-author David Lindley (1980, p. 2), possibly as a result of his period in Princeton, Savage was very excited by the axiomatization of utility provided by von Neumann and Morgenstern and developed an interest in the possible extension to an axiomatization of decision-making embracing both probability and utility. In addition, he was already known among economists for his joint paper with Friedman, an assessment of von Neumann and Morgenstern's expected utility theory introducing the notion of risk aversion (Friedman and Savage 1948). The turn at Cowles toward a normative view of decision-making suited perfectly Savage's attitude to move forward from the empirical justification of expected utility offered in the 1948 paper toward a proper axiomatic inquiry (Friedman and Savage 1952).

Italian mathematical statistician Bruno de Finetti played a crucial role in the development of Savage's ideas. In search of support and inspiration, Savage invited him to Chicago in 1950 and they started collaborating. Amongst the increasingly conspicuous correspondence over the years is a March 1961 letter from Savage to de Finetti highlighting the process through which Savage definitively turned towards a subjective approach to probability and decision-making.

While commenting in the letter on the English translation of a paper by de Finetti he was editing, Savage describes the "personal history" of how his ideas developed in late 1940s.[18] During an expository lecture given at the Cowles Commission as part of a seminar series on game theory early in 1949, Savage wrote, "it occurred to me that a concept of personal probabilities analogous to the utility concept as developed by Neumann and Morgenstern might be developed, and I said so during the lecture." Tjalling Koopmans was in the audience, indeed was possibly chairing the seminar in his capacity as Director at Cowles, while Savage offered his thoughts during the lesson. Savage reports:

> T. C. Koopmans … was enthusiastic about this idea of mine and convened a seminar to discuss it the very next morning. … Frankly, I was resentful of Koopmans trying to organize a group of people on an idea that I had announced, without giving me the opportunity to work on it

[18] L. J. Savage to B. de Finetti, March 3, 1961, Leonard Jimmie Savage Papers, Manuscripts and Archives Department of Yale University Library, Box 7, Folder 192.

> myself. His position was that I was a notorious neglecter of my own good ideas and that something should be done about this. In compromise, he had extorted me a promise to think about the matter and report on it publicly at Boulder.

It was in preparation for the Boulder meeting that Savage studied de Finetti's foundational paper in *Annales de l'Institut Henri Poincaré*, and indeed, he adds to de Finetti, "my talk there was sort of a review of your Poincaré paper."[19]

As just mentioned, the talk referred to by Savage in his letter to de Finetti was given at the Cowles Commission on March 31 1949, as part of a seminar series organized at Cowles on the application of the theory of games. The Boulder meeting, a meeting of the Econometric Society held on Aug 29 - Sept 2, 1949, featured a talk by Savage titled "The role of personal probabilities in statistics." In his summary, Savage (1950, pp. 183-184) claims that "finding satisfactory rules for acting in the face of uncertainty" would mean using a novel concept of probability, similar to that suggested by de Finetti ([1937] 1964). Building on de Finetti,

> plausible assumptions about the behavior of a 'reasonable' individual faced with uncertainty about future events imply that he associates numbers with the events, which from the purely mathematical point of view are probabilities. These personal probabilities are in principle measurable by experiments which may be performed on the individual either by himself or by others and their interpretation is such as to make clear (in view of the von Neumann-Morgenstern theory of utility) how the individual should act in the face of uncertainty

The active role of Koopmans highlighted by Savage in his letter testifies to the keen interest at the Cowles Commission in a step forward in the theory of choice. As a general remark, it must be remembered that the shift from political economy to economics as a science of scarcity usually associated with the 1930s included a focus on analyzing individual behaviour that accords well with the development of marginalist microeconomic analysis towards a rational choice theory. But, as Backhouse and Medema (2009, p. 227) noted, even in the early 1950s the only unambiguous endorsements of the Lionel Robbins's (1935) definition of economic science in the journal literature were by members of Cowles, while this definition became widely accepted only by the end of the 1960s, when mathematical methods − including the axiomatic approach − became more pervasive in economics. More specifically, it was around Cowles, and in particular in the works of Marschak, that utility maximization was given an axiomatic justification, namely, that a simple and "reasonable" set of axioms imply expected utility theory as a theorem, a shift in emphasis that, as already noted, was otherwise contested, at the time, also by some mathematically oriented economists such as Samuelson and Baumol (Moscati 2016).

The history of the Cowles Commission and its enduring influence on the subsequent evolution of economics has been comprehensively recounted in the last twenty years or so (among others, see Mirowski 2002, Duppe and Weintraub 2014, Dimand 2019). But the way the issue of uncertainty was dealt with at Cowles has received relatively little attention compared to other topics such as activity analysis, econometric modelling and general equilibrium theory. Herfeld (2018) notes that, despite a

[19] In the letter, Savage adds: "As soon as I saw your paper I knew very clearly how to blend utility and probability into one theory, and that was before I had seen Ramsey's papers … but when I saw them, it was clear to me that he anteceded me in the whole matter." Savage was making reference to de Finetti ([1937] 1964) and Ramsey ([1931] 1964), the papers unanimously considered as the classics of the subjective approach to probability.

strong focus on theories of economic behaviour initiated under Marschak's directorship, an improved theory of choice under uncertainty was perceived at Cowles as remaining far away even after von Neumann and Morgenstern's expected utility theory. In Cowles's yearly activity report of June 1949, Koopmans, who had succeeded Marschak as director, acknowledged that uncertainty had not been satisfactorily integrated into decision theory. While von Neumann and Morgenstern's justification of expected utility theory offered a crucial starting point, finding a theory of economic behaviour under uncertainty became an increasing concern at Cowles. The seminar series centred on game theory hosted at Cowles early in 1949 was meant to "clarify the fundamental issues of human choices under uncertainty" using "rigorous logical tools" (Cowles 1949, p. 12), but in his 1950 crucial paper on rational behaviour Marschak had limited his analysis to complete information and "objective" probabilities. The interest at Cowles in the issue is further evidenced by the kind of funding sought by the Commission: in July 1951, the Office of Naval Research began funding Cowles for research on choice under uncertainty (Cowles 1951, p. 39).

As we have seen, Savage's investigation was intended as the step forward towards a coherent model of incomplete information and uncertainty, using "subjective" probabilities for the representation of degrees of belief. Savage's 1954 *Foundations of Statistics* introduced a new set-up for choice under uncertainty and provided the crucial representation theorem showing that maximizing subjective expected utility was implied by being consistent in choices among gambles. However, it is important to note that these subjectivist developments were appreciated and sponsored by researchers at Cowles well before Savage had elaborated them in full. Indeed, although Savage was not a Cowles research associate, unlike Arrow, Herman Chernoff, Marschak and Herman Rubin, to quote those contributing to the topic, he was actively involved as a founding member of the Committee on Statistics, whose birth is very much related to the connection between Cowles and the Department of Mathematics at the University of Chicago.[20]

In a Cowles Commission discussion paper of November 30, 1950 (reproduced in *Econometrica* as a review article) illustrating the state of the art in the theory of choice in "risk-taking situations," Arrow (1951, p. 406) presented the "dramatic break in continuity" constituted by von Neumann and Morgerstern's approach. But in order to show the advance it represented, he discussed the various contemporary attempts to represent degrees of beliefs, both in probabilistic settings and not, and the related criteria for decision-making under uncertainty, still not all conflated into the maximization of expected utility. Arrow reviewed and criticized approaches as different as Wald's maxmin and, as already noted, Shackle's potential surprise. More relevantly, he showed distrust about Knight's claim that not all types of risk can be described by probability statements, and that there are unmeasurable uncertainties that can be dealt with only by means of "estimates," and not by proper probabilities. On the basis of a "degree-of-belief theory of probability" in the making, Arrow concluded that "Knight's uncertainties seem to have surprisingly many of the properties of ordinary probabilities, and it is not clear how much is gained by the distinction."

The reference Arrow provided for this subjective approach to probability was "a new stage, … originating with Ramsey, in which *a priori* probabilities are derived from behavioural postulates," substantiated by the ongoing work on "setting down axioms for rational behavior under uncertainty" investigated by Chernoff (1949), Rubin (1949) and Savage (1950). Of Savage, in particular, Arrow

[20] Weintraub and Mirowski (1994) suggest that, disillusioned by the reaction of the Economics Department to his call for the application of new mathematical tools, Koopmans had begun making overtures to the Mathematics Department, with the aim of establishing a mathematical statistics unit.

added that he had made "important oral contributions to this development." As a matter of fact, the reference to subjective probability in Chernoff's and Rubin's papers is terse, to say the least, while the paper abstracted in Savage (1950) was not circulated (and indeed it could not be retrieved by the present author in the Savage Papers at Yale University Library). Arrow's reference to Savage's "oral contributions" is then to be interpreted literally, although notes presenting the axiomatic core of the 1954 *Foundations* were already circulated by him in mid-1950 (Moscati 2016) and probably presented at another Cowles seminar, which Savage gave early in November 1950 under the title "De Finetti's theory of subjective probability with reference to the statistical decision problem."[21]

It is also worth noting that, while quite dismissive of Knight's distinction between risk and uncertainty, Arrow's treatment of Keynes was more nuanced. Arrow attributed to Keynes's *Treatise on Probability* the merit of having revived the camp of those scholars who interpret probability as a measure of degrees of belief, objecting to the predominant frequentist view among statisticians that "probability statements cannot describe all kind of ignorance." He also stressed that Keynes's *Treatise* features a significant role in the whole discussion of how to represent uncertainty through probabilities, and that, unlike Knight, "Keynes is anxious to describe all uncertainties as probabilities." However, Arrow concluded, Keynes's "full theory of the description of uncertainty is … to be more properly classed with those which hold that probability statements alone are insufficient," among which Arrow singled out as most relevant, and targeted for criticism, Shackle's theory of potential surprise (Arrow 1951, pp. 413, 416). This kind of attention for Keynes's *Treatise* was not unusual among decision theorists at the time, as testified by both philosophically oriented analyses such as de Finetti's (1938) and mathematically oriented ones such as Savage's (1954).[22]

Among the reasons that may explain Arrow's earlier endorsement of Savage, the fact that it was perceived as an outgrowth of the research programme put forward at Cowles must have been crucial. Savage's axioms system, and the ensuing maximization of *subjective* expected utility, reinvigorated a view that at Cowles was sponsored as mainstream in addressing decision-making. As already noted, Marschak's idea had been to provide a normative interpretation of von Neumann and Morgenstern's analysis, an interpretation that could support the programme to extend "rules of conventional logic … into the realm of decision." The meaning of the independence axiom was to provide the grounds on which a certain behavioural norm could enjoy the same measure of acceptance of fundamental rules of logic and arithmetic when applied to probabilistic environments.

In specifying a divide that became conventional in the following years, Marschak (1951, p. 493) argued that "to discuss a set of norms of reasonable behaviour … is a problem in logic, not in psychology … a normative, not a descriptive, problem." On normative grounds, Marschak argued, the issue was to define "reasonable" postulates to extend the logic of choice to risky environments.

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[21] The first formal version of Savage's axioms system was published in French in Savage (1953), a four page contribution to the famous 1952 Paris Conference on Risk. A summary of what Savage will call the sure-thing principle in his *Foundations* had already been presented in Friedman and Savage (1952).

[22] Savage devoted significant attention to Keynes. About approaches admitting vagueness and not allowing for the complete ordering among acts he was assuming, Savage (1954, 39) observed: "It would however be disingenuous not to mention that some who have worked on a closely related concept of probability, notably Keynes and Koopman, would object to axiom P6' [the axiom granting compactness of the choice space] precisely because it implies that the agreement between numerical probability and qualitative probability is strict." Keynes ([1921] 1973, p. 33) explicitly rejected compactness in his *Treatise*. Keynes's proposal to represent the uncertain environment through a partial, not a complete, ordering was well understood by Savage.

Savage endorsed this view in the introductory chapters of his *Foundations* presenting the meaning of his enterprise, and further developed this idea of extending logic, providing the justification for expected utility maximisation even under uncertainty.[23]

That these developments were largely considered an outgrowth of Cowles research is repeatedly confirmed in the reports of the Chicago years, with increasing emphasis after Koopmans stepped in as director. The 1951 report described decision-making under uncertainty as an "area in which the Commission is continuing and expanding its research program." Reporting on the extensive effort to provide studies of "rational, or optimal behaviour," it is noted that in most of them "the decision-maker was supposed to act on the basis of probabilities that are known." But at least as important, the report clarifies, should be studies in which probabilities are "believed," since "no rational decision is possible without some knowledge of, or belief about, the probabilities of the alternative 'states of natures' that influence the action's outcome" (Cowles 1951, p. 12).

At the time, Koopmans was mostly concentrating on his own research on activity analysis. The pivotal role of the 1949 Conference on "Activity analysis of Production and Allocation" in creating a community of scholars in which mathematicians were supposed to bring "new tools of analysis essential to the progress of economics" − as Koopmans put it in the introduction to the proceedings − has been stressed by Duppe and Weintraub (2014, p. 99). But Koopmans's wider interests had a strong methodological component that was grounded in the need for elaborated theories of individual behaviour. In his classic critique of the measurement of economic phenomena made without reference to theory, an attack on the National Bureau of Economic Research's approach to empirical analysis, Koopmans (1947, p. 163) had argued that the study of business fluctuations could not be pursued without reference to the "toolkit of the theoretical economist." He noted that Burns and Mitchell (1946) made use neither of supply and demand schedule nor of any equation expressing individual behaviour. Koopmans (1947, p. 166) placed emphasis on the need for economists to possess well-established theories of economic behaviour, that he assessed as "an indispensable element in understanding in a quantitative way the formation of economic variables." In stressing the importance of theories of behaviour, though, he also admitted that "much of these theories is incomplete and in need of reformulation and elaboration (particularly in regard to behavior over time under conditions of uncertainty)." And, in fact, as director overseeing the entire programme at Cowles, he placed great emphasis on the need for advance in the theory of decision under uncertainty. His acknowledgement of Savage's contribution in his published writings only confirms this.

For Koopmans, moving forward in economic theory would mean to prioritize scientific rigour in the form of mathematical models engaging with new mathematical tools. In particular, the mathematical theory of linear spaces provided the "new tools" for analysis of the "old problems" of the allocation of resources and a price system compatible with efficiency and Pareto optimality, as in the general equilibrium approach *à la* Arrow-Debreu. On these issues, Koopmans offered an elaborate view in his famous systematization of the axiomatic approach. It has been highlighted that Koopmans's volume *Three Essays on the State of Economic Science* constitutes the main piece that

[23] It remains usually unnoticed that Marschak's first discussion of uncertainty proper is but a presentation of how his use of preference relations among "prospects" for risky situations had been turned by Savage into preference relations among "acts" for uncertain situations (Marschak 1954). Marschak (1954, p. 22) admitted that, in the discussion on von Neumann and Morgenstern's expected utility, "the necessarily subjective nature of probabilities was somewhat neglected" before Savage.

defended the new theoretical approach in methodological terms at the time (Mirowski 2002, Duppe and Weintraub 2014). What has been less noted is that, in Koopmans's view, Savage's developments constitute the main example of how fruitful the use of axioms is in the subsequent development of more realistic and precise models.

Koopmans's second essay in the 1957 volume, addressing "The construction of economic knowledge," is an attempt to show, *contra* Friedman, that "nothing is lost, and much may be gained, in … broadening the postulational basis of economic theory." Koopmans (1957, pp. 142-143) argued in terms of a "sequence of models" in which the study of a simple model can be protected from "the reproach of unreality" considering that this model may be a prototype of a "more realistic, but also more complicated" subsequent model. For Koopmans (1957, pp. 147-149), "perceptions of additional aspects of reality must necessarily precede their recognition in model formulation," but only the axiomatic method makes it possible to know "which conclusions or recommendations depend on which postulates, and which postulates depend for their validity on which verifications of their implications by accumulated experience." Therefore, Koopmans indicated that the advances in economic theorizing could be pursued only insofar as "aspects of reality" constituting "problems" are recognized by "suitable postulates to make discussion of each problem possible or meaningful."[24]

The idea that the "logical core of economics" could be enlarged by resolving the "apparent conflict between rigor and realism," was substantiated by a main example. For Koopmans, the "core problem" of the economic organization of a society on which economic theory has not yet shed much light with adequate rigor was "how to face and deal with uncertainty." Koopmans (1957, p. 155) saw in the recognition of uncertainty "the turning point beyond which empirical verification of postulates should become the main preoccupation of economists," and presented a "verbal translation" of Savage's axiomatic system. Following on von Neumann and Morgenstern, Savage's study of the logic of one person decision-making under uncertainty has derived "surprisingly strong and detailed conclusions from postulates that seem unobjectionable as a statement of mere consistency and sharpness of preferences." In his generalization to uncertainty, Savage has provided a result of "considerable normative value," that consistency of choices is best achieved by making beliefs as well as valuations "numerically explicit" (Koopmans 1957, p. 159).

Koopmans stressed that a model of decision-making under uncertainty, in which a series of axioms imply the existence of both a subjectively numerical probability associated to states of nature and a utility associated to consequences, was of fundamental value. Moreover, Savage's model was likely to represent a valuable starting point in the discussion of "realistic" issues such as decisions over time and the problem of balancing cost of information gathering against its value for correct decisions. The parallel path of the normative and the descriptive in economic theory, already indicated by Marschak and followed by Savage, is clearly endorsed by Koopmans.

At least another significant episode illustrates Koopmans's determination in establishing Savage's as the mainstream view supported by Cowles. In October 1955, a conference on

[24] Koopmans preached a methodology that became conventional practice among economists objecting to Friedman (Blaug 1992, chap. 15). As documented in Zappia (2021a), in private correspondence with his critics, Savage followed exactly this line of argument in his defence against the paradoxical examples provided by Ellsberg. Pressed by William Fellner, who sided with Ellsberg, he admitted to finding Ellsberg's and Fellner's (1961) results of interest, but he claimed not to be ready to abandon his own theory simply on the basis of counter-arguments and appeals to realism, until the critics were able provide an axiomatically grounded alternative.

"Expectations, Uncertainty and Business Behaviour" took place at the Carnegie Institute of Technology in Pittsburgh. The conference was organized by the Committee on Business Enterprise Research of the Social Science Research Council under the supervision of Mary Jean Bowman. The list of authors who contributed a paper to the volume collecting its proceedings includes Albert Hart, George Katona, Herbert Simon (all members of the Committee), Kenneth Boulding, Ward Edwards, Robert Eisner, Nicholas Georgescu-Roegen and Franco Modigliani, among others.

The approach to individual behaviour at Carnegie was more diverse than at Cowles, concentrated on actual rather than on rational behaviour. At Carnegie, Herbert Simon valued the plurality of mathematical models of rationality to such an extent that he distanced himself from Cowles (where he had been a research associate since 1947) after Cowles's turn to axiomatics (Petracca 2021, Assous *et al*. 2022). His aim to construct theories in which the requirement of rationality was progressively dispensed with in order to account for the limitations of actual behaviour was well in line with the group of scholars interested in "business behaviour" who gravitated around the Graduate School of Industrial Organization. And, indeed, the 1955 conference was mainly an interdisciplinary effort delving into issues of administrative and firm behaviour.

In her introduction to the conference volume Bowman (1958, p. 1) clarified that although contributions were mainly empirically oriented, with special emphasis on the psychological aspects of business expectations, the organizers thought that an opening section presenting current theoretical developments was essential. The study of business decision-making drew upon several distinct fields of learning, but economics and the mathematics of probability had central relevance. The theoretical section consisted of two papers: Georgescu-Roegen's survey of the theories of probability and expectations as well as Shackle's presentation of the potential surprise approach with application to liquidity preference.

As documented in Zappia (2014), the fact that Shackle's non-probabilistic approach to decision-making was given such a crucial role at the conference was severely criticized by Koopmans, who argued in favour of the potentialities of Savage's approach. Bowman (1958, pp. 2-3) quotes a sentence from Koopmans's comment during the discussion: "I would like to have seen more attention given to the axiom system for decision-making under uncertainty recently developed by Leonard Savage, continuing and extending the work by Ramsey and de Finetti … concerned with decision-making in general and as such as fundamental to economics as to statistics." Shackle was unable to attend the conference, but his paper, circulated in advance, was targeted by Koopmans and others in the discussion. In a letter to Shackle summarizing the debate, Bowman wrote:[25]

> I wish you could have been present for the discussion of your paper at the conference … Unfortunately Marschak had to back out at the last minute. We were fortunate in getting Tjalling Koopmans to take his place … . Koopmans suggested that we need empirical studies to discriminate between your approach with the two focus points and the Savage approach which would emphasize a central value … . You may be assured that it [your paper] stirred quite a bit of excitement, even though the fire was aimed at your general theory and not this application [to liquidity preference] per se.

Among others, even Albert Hart complained that "the notion that human beings simply cannot set up a meaningful 'subjective probabilities' and apply them to the likelihood that events may diverge

[25] Letter from M. J. Bowman to G. L. S. Shackle, November 8, 1955 (George Shackle Papers, Cambridge University Library, Add. MS 7669, 9/1/148).

from a central estimate strikes me as unacceptable … it would leave us with such very intractable models that analysis would be paralysed" (Bowman 1958, p. 6).

The rejection of Shackle's theory was across the board, in view of Savage's alternative. Shackle was defended only by Ward Edwards, a psychologist at the University of Michigan today widely considered among the founders of behavioural decision-making. In a note appended to Shackle's paper, Edwards (1958, p. 44), commented that "Shackle's refusal to add subjective improbabilities … seems to me to be the most important and desirable feature of his system." Edwards referred to experimental evidence he had already collected that argued against the additive property "so strongly that I do not see how it is possible any longer to defend that property." Anticipating future developments, he suggested that it might be possible "to develop a utility-subjective probability model that is mathematically satisfactory and that does not require subjective probabilities to add to one or anything else" (Edwards 1958, p. 44).

It is interesting to note that Edwards's comments shared Shackle's distrust for conventional probabilities as a measure of uncertainty. Given that Savage's theory emphasized the internal consistency of individual choice, showing that individuals do not care about such kinds of consistency in certain situations was a significant counter-example to the subjectivist approach. Indeed, this was to be Ellsberg's main theme in his 1961 article, when arguing that individuals asked to make choices within the framework of his urn problems were unrepentant violators of Savage's axioms.[26] But Edwards's appreciation hinted at the descriptive, rather than the normative value of Shackle's theory. Notwithstanding his criticism of Savage's assumption, he did not question the idea that Savage's developments had a normative status in principle untouchable from the reproach of actual behaviour diverging from the maximization of subjective utility. This view, favoured by Koopmans and the Cowles Commission in the 1950s, flourished in the years to follow even among behavioural economists (Edwards 1992).

6. Concluding remarks

Due to the endorsement of Savage's subjective perspective by decision theorists, the traditional Knightian distinction between risk and uncertainty lost relevance. After Savage, probabilistic reasoning, typically limited to dealing only with risk, started to be used to address uncertainty, with the latter now considered as a measurable entity. The new mainstream in decision theory then came to be characterized as subjective expected utility.

While statisticians never really accepted the Bayesian revolution proposed by Savage, economists wholeheartedly approved Savage's proposal to use subjective probabilities to characterize individual agents' behaviour under uncertainty. Notwithstanding early criticism by Maurice Allais (1953), the mainstream approval of subjective expected utility was so immediate that Savage did not have to defend it. And even after Ellsberg's (1961) attack against the normative value of his sure-thing principle, the protective belt was provided by those who followed his approach (Raiffa 1961).

This paper has documented that the fact that Savage's work was perceived as an outcome of the Cowles Foundations must have been crucial. At Cowles, particularly under the directorship of

[26] That Edwards's comments were of the utmost interest is confirmed by the fact that Kahneman and Tversky (1979) referred to Edwards's experimental results in order to introduce the non-additive probabilistic weights on which prospect theory is based.

Koopmans, Savage's axiomatic system was understood as making it possible to provide a theoretical environment in which the theory of choice could be made more "realistic," without losing in "rigour." On such grounds, the problem of extending rational choice theory to "proper" uncertainty was thought to be solved, and new problems could be addressed while still adhering to the axiomatic approach.

What appeared to be a period of investigation of multiple alternatives in the analysis of uncertainty turned out to be the definitive consolidation of a clear-cut approach about how to measure uncertainty, an approach that became dominant for decades to come.

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