



## ACTIVE LEARNING IN MATHEMATIC FOR STEM: REAL-LIFE ENGINEERING APPLICATIONS

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### ABSTRACT

An opinion piece in Scientific American [1] discusses how a fraction of students ultimately complete a STEM degree and cites research [2] that disengagement with traditional calculus courses as one of the causes. It goes on to highlight examples of several promising calculus reforms and recommends that STEM faculty take the lead in introducing changes by collaborating and co-creating across disciplines to make mathematics more relevant and interesting to students.

Feedback from module surveys indicate that students learn much better when the link between theoretical and practical knowledge is captured and echoes pedagogical literature.

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The author introduces past experiences of active learning approaches to enhance the teaching of mathematics to first-year engineering students. Class discussions incorporate real-life engineering applications highlighting example problems from a wide variety of core engineering modules such as Fluid Mechanics, Vibration, and Mechanics of Materials.

The impact of this approach has not been directly measured and documented for the module being discussed here and is motivated by encouraging student feedback where they shared that they find the teaching interesting, fun, engaging, and interactive. The present concept paper therefore outlines how past pedagogical practice have influenced the enhancements in the delivery of engineering mathematics with a particular focus on interdisciplinary approach. It then goes on to demonstrate some examples of implementation and offers initial reflections based on student feedback. Finally, the author proposes future steps of detailing the effect on student learning experience via class surveys, interviews and making comparisons to comparably taught modules.



## 1 INTRODUCTION

### 1.1 Motivation

Galileo wrote: *“Philosophy is written in this grand book, the universe... [But the book] is written in the language of mathematics.”* The study of engineering requires a substantial grounding in engineering principles, science, and mathematics. The gradual development of students' critical thinking and analytical ability to solve real engineering problems is the key to their future success [3]. A strong foundation in mathematics is therefore necessary to produce competent engineers who can confidently analyse and solve problems, employ analytical tools and techniques, design and innovate, and communicate their results.

Research about why students abandon degrees [2] suggests that traditional calculus courses are one of the main reasons. This tallies with anecdotal observations of declining competencies of mathematical ability in senior undergraduate students, with several engineering students struggling to do simple calculus in higher levels of studies, based on discussions and comments from academic colleagues. And alarmingly, a recent report, *‘Charting a New Course: Investigating Barriers on the Calculus Pathway to STEM’* [4] adds that traditional approaches to calculus are partly responsible for the large proportions of women and students from minoritised backgrounds getting discouraged from pursuing STEM careers.

It's generally suspected that one underlying factor to these issues can be attributed to the lack of engaging and relevant material in describing, solving, and understanding the significance of mathematics in an engineering context. A publication recommends that STEM faculty prioritise collaborations and co-creation across disciplines to transform math classes [1]. The same article concludes by proposing that “math learning is fundamental to all STEM fields, but the opposite also appears to be true: the STEM fields may be central to making math learning effective for more students.

Student-centered pedagogies like problem-based learning, collaborative learning, process-oriented guided inquiry learning, and peer-led learning have been extensively developed and tested in response to the tried-and-test approaches to match the way we teach to the way students learn [5]. Studies have shown that active learning environments are more effective than traditional lectures [6]. And Mark Deakin [7] concludes that students value the link between teaching and research, placing particular weight on research led teaching and the bearing which it has on the quality of their learning experiences.

With these in mind, the present concept paper first briefly describes prior successes of employing CDIO framework in the design and delivery of engineering modules such as Computational Fluid Dynamics and Solid Mechanics. The positive impact of the past implementation motivated the use of the CDIO approach in the teaching of the current mathematics module. The paper outlines how this method has been adopted in the existing engineering mathematics module to make it more relevant, attractive, and interesting to learners and reports early results from observations and



module surveys. Finally further work is proposed to structurally measure the effects of the introduced enhancements on students learning and progression.

## 1.2 CDIO

The paper will start with a summary of the CDIO approach and its benefits. CDIO is an educational framework that stresses engineering fundamentals set in the context of Conceiving, Designing, Implementing and Operating (CDIO for short) real-world systems and products. CDIO advocates active learning techniques such as problem-based learning to equip engineering students with technical knowledge as well as communication and professional skills. These techniques collectively promotes active and integrated learning experiences. The core philosophy of CDIO is to prepare engineering students who can engineer. Readers are encouraged to visit [www.cdio.org](http://www.cdio.org) to find out more. This has a bank of useful resources including standards, syllabus, and case studies.

## 1.3 Past Implementation and Student Feedback

The author has experimented with and implement different educational frameworks and has continuously adapted and refined his teaching on reviewing class performance and feedback. He has received very encouraging student feedback over the past decade such as *“The only enjoyable class throughout my studies is Solid Mechanics Class. Additionally, we were given the opportunity to work on an assignment in which, for the first time, I was able to apply my engineering knowledge to design a given task. This has never been done in any other Engineering classes and should strongly be taken into consideration”* referencing the CDIO approach employed. Another student from a subsequent cohort agreed: *“He’s very good in explaining the theory behind each mechanics applied in engineering as well as in real life situation. He makes us think out of the box which is very good and indeed challenging. Keep up the good work!”*

The author’s colleague attests: *“I believe his plan for assessment strategies proved very effective in motivating the students to develop and apply their techniques...and to act as real engineers in a way that would be sought by future employers”*.

The knowledge, skills, and experiences gained over the years backed by positive student feedback and observable improvements in class performance helped refine the author’s teaching practice and build his confidence. The next section describes how the past implementation and experience of CDIO principles influenced the teaching of mathematics. Then goes on to reflect on the impact on student learning initial feedback received from the cohorts.

## 2 ACTIVE LEARNING IN MATHEMATICS

### 2.1 Adopting CDIO in engineering mathematics

Pedagogical literature, student feedback and discussions with colleagues have highlighted the benefits of integrating theoretical knowledge with practical application which is the underpinning principle of CDIO. Drawing from cross-discipline research and engineering subject expertise the mathematic lessons are updated with real-life



applications to introduce mathematical concepts, solve class examples, and demonstrate applications of the new contents being presented and discussed. The intention is to help learners relate to the subject matter, appreciate the relevance of mathematics in the wider engineering context, and in the process excite them to discover more and transfer knowledge.

## 2.2 Outline of the engineering mathematics module

The mathematics module (titled 'Engineering Analysis 2') is taught to 1<sup>st</sup> year undergraduate engineering students in their second semester and covers:

- Vectors
- Complex Numbers
- Ordinary Differential Equations
- Multivariate Functions
- Series and Sequences

The module's main aim is to provide the essential grounding in mathematical analysis techniques for engineering students with a focus on calculus which is the mathematical study of change.

## 2.3 Delivery plan

In the very first lesson students are introduced to the Navier-Stokes Equations as one of the unsolved Millennium Prize Problems by the Clay Mathematics Institute<sup>2</sup>. This is followed by discussion about various ways of solving engineering problems: analytical, experimental, and computational and the class finally go through the use of the Navier-Stokes Equations in the context of Computational Fluid Dynamics (CFD) with some real-life examples from senior student projects over the years. The intention of this is to encourage the first-year students to see the bigger picture and appreciate the end goal of learning all the mathematical techniques and how it will gradually equip them with the tools and skills to analyse and solve engineering problems in later level of studies.

The beauty of the Navier-Stokes Equations - which are fundamentally the conservation equations of mass (continuity), momentum (Newton's second law) and energy - are that they are multidisciplinary, and learners will come across them repeatedly in various forms and iterations in the other core modules of Thermodynamics, Fluid Dynamics, Solid Mechanics and so on, throughout their engineering studies.

As we progress through the 10 weeks of classes, we revisit the different aspects of the Navier-Stokes equation and relate them to the mathematic concepts we are learning thus threading all the topics together.

Two examples are presented below:

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<sup>2</sup> <https://www.claymath.org/millennium-problems>

**Example 1: Vectors (Chapter 1)**

Fig.1. is a screenshot of the material hosted on Microsoft OneNote presented to the class when introducing the ‘Vectors’ topic. It includes example of student projects using CFD and linked to the Navier-Stokes Equations. The left image is of a student modelling the ventilation in a lecture theatre to evaluate the indoor air quality in response to the covid outbreak. The other two are simulations of a previous intake manifold used by the Formula Student race team and their proposed design change to enhance the volumetric efficiency. This gets the students interested, motivated and encourages them to appreciate the reason they are learning vectors.

Introduction to Vectors

- Use of vectors
- What will we learn in this chapter?
  - o Basic definitions: Vector vs Scalar
  - o Coordinate Systems: Cartesian, Cylindrical and Spherical (3D). Polar (2D)
  - o Products: Dot and Cross Products
  - o Equation of a Line
  - o Equation of a Plane

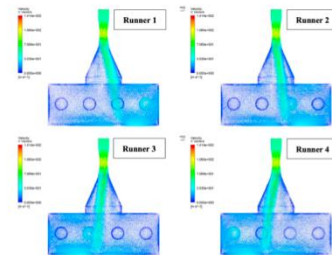
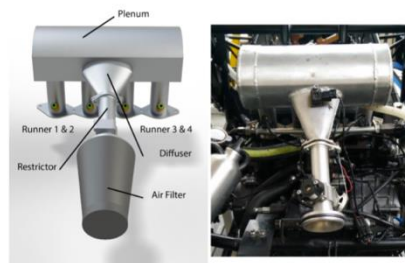
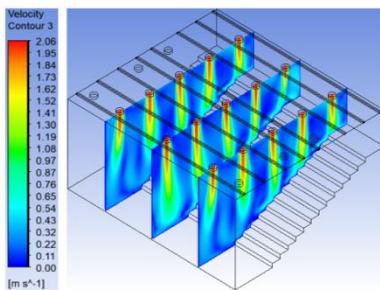


Fig. 1. Introduction to Vectors

**Example 2: Multivariate Functions (Chapter 4)**

Half-way through the ‘Multivariate Functions’ topic, we revisit the Navier-Stokes equation to demonstrate that acceleration (in  $F = ma$ ) is a multivariate function of  $x$ ,  $y$ ,  $z$  and  $t$  and we proceed to employ the chain rule to derive the material derivative. This also links well with ‘vectors’ topic where we come across the dot product again. The other example to the left illustrates the conversion between cartesian and polar coordinate systems also covered in vectors and discusses composite functions and the chain rule.





Example: Find  $\frac{\partial T}{\partial r}$  and  $\frac{\partial T}{\partial \theta}$  when  
 $T(x, y) = x^2 + 2xy + y^3$   
 and  $x = r \cos \theta$  and  $y = r \sin \theta$

Handwritten solution for Example 1:

$$T(x, y) = x^2 + 2xy + y^3$$

$$x = r \cos \theta \quad y = r \sin \theta$$

$$\frac{\partial T}{\partial r} = \frac{\partial T}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial T}{\partial y} \frac{\partial y}{\partial r}$$

$$= [2x + 2y + 2xy^2] \cos \theta + [0 + 2x + 3y^2] \sin \theta$$

$$\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial x} \frac{\partial x}{\partial \theta} + \frac{\partial T}{\partial y} \frac{\partial y}{\partial \theta}$$

$$= -[2x + 2y + 2xy^2] r \sin \theta + [2xy^2] r \cos \theta$$

Example: Find  $\frac{dR}{ds}$  when  
 $R(x, y) = \cosh(x^2 + 3y)$   
 and  $x(s) = s^2 + 3s$  and  $y(s) = \sin s$ .

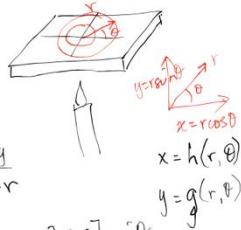
Handwritten solution for Example 2:

Example: Find  $\frac{dR}{ds}$  when  
 $R(x, y) = \cosh(x^2 + 3y)$   
 and  $x(s) = s^2 + 3s$  and  $y(s) = \sin s$ .  
 Solution: For this example,  $x$  and  $y$  are functions of  $s$  only so

$$\frac{dR}{ds} = \frac{\partial R}{\partial x} \frac{dx}{ds} + \frac{\partial R}{\partial y} \frac{dy}{ds}$$

which gives

$$\begin{aligned} \frac{dR}{ds} &= 2s(2s + 3) \sinh(x^2 + 3y) + 3 \cos s \sinh(x^2 + 3y) \\ &= 2(s^2 + 3s)(2s + 3) \sinh((s^2 + 3s)^2 + 3 \sin s) \\ &\quad + 3 \cos s \sinh((s^2 + 3s)^2 + 3 \sin s) \\ &= 2(2s^2 + 9s^2 + 9s) \sinh((s^2 + 3s)^2 + 3 \sin s) \\ &\quad + 3 \cos s \sinh((s^2 + 3s)^2 + 3 \sin s) \end{aligned}$$



**Navier-Stokes Equations**  
 3-dimensional - unsteady

Coordinates (x, y, z)    Time: t    Pressure: p    Heat Flux: q  
 Velocity Components (u, v, w)    Density:  $\rho$     Stress:  $\tau$     Reynolds Number: Re  
 Total Energy: Et    Prandtl Number: Pr

Continuity:  $\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$

X-Momentum:  $\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2)}{\partial x} + \frac{\partial (\rho uv)}{\partial y} + \frac{\partial (\rho uw)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}$

Y-Momentum:  $\frac{\partial (\rho v)}{\partial t} + \frac{\partial (\rho uv)}{\partial x} + \frac{\partial (\rho v^2)}{\partial y} + \frac{\partial (\rho vw)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}$

Z-Momentum:  $\frac{\partial (\rho w)}{\partial t} + \frac{\partial (\rho uw)}{\partial x} + \frac{\partial (\rho vw)}{\partial y} + \frac{\partial (\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$

Energy:  $\frac{\partial (\rho E)}{\partial t} + \frac{\partial (\rho u E)}{\partial x} + \frac{\partial (\rho v E)}{\partial y} + \frac{\partial (\rho w E)}{\partial z} = -\frac{\partial (p u)}{\partial x} - \frac{\partial (p v)}{\partial y} - \frac{\partial (p w)}{\partial z} + \frac{\partial (q_x)}{\partial x} + \frac{\partial (q_y)}{\partial y} + \frac{\partial (q_z)}{\partial z}$

The Navier-Stokes Equations

$$m \mathbf{a} = \sum \mathbf{F}$$

$$\rho \left( \frac{d\mathbf{v}}{dt} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = -\nabla p + \mu \nabla^2 \mathbf{v}$$

Newton's 2<sup>nd</sup> Law of Motion

Handwritten notes on vector calculus:

$\mathbf{F} = m \mathbf{a}$      $\mathbf{v} = (x, y, z, t)$     but also  $x = h(t)$   
 $y = g(t)$

Total Differentiation

$$\mathbf{a} = \frac{D\mathbf{v}}{Dt} = \frac{\partial v_x}{\partial x} \frac{dx}{dt} + \frac{\partial v_x}{\partial y} \frac{dy}{dt} + \frac{\partial v_x}{\partial z} \frac{dz}{dt} + \frac{\partial v_x}{\partial t} \frac{dt}{dt}$$

$$= v_x \frac{\partial v}{\partial x} + v_y \frac{\partial v}{\partial y} + v_z \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$= \frac{\partial v}{\partial t} + \mathbf{v} \cdot (\nabla \mathbf{v})$$

Fig. 2. Chain Rule for Differentiation of Multivariate Functions

### 3 REFLECTIONS

I've included student comments collected as part of the formal module feedback exercise from the past and current academic year for the module. This was obtained via a standardised survey deployed across the faculty using evasys+ tool (<https://evasysplus.co.uk/>). The comments below are in response to the open-ended question:

**Q: Please name the one thing in the module which had the most impact on your learning.**

*I enjoy these lectures I feel like the lecturer has a passion for mathematics and it shows through his teaching.*

*I like the variety of ways he teaches his lectures (example classes, quiz classes, polls etc) as this helps me stay engaged and focused.*

*I like the use of the class notebook [Microsoft OneNote], it makes it easier to go back and revise some of the example problems covered.*

*I think sir is quite interactive. I understand very clearly and when I am stuck, he supports us.*

*Lecturer is very likable and approachable, and this shows in his lectures. Everyone pays more attention because he also makes the lectures more interesting and fun to attend.*

*Really enjoying the module so far. Good level of teaching and good structured live zoom lectures!*



*The lecturer is very engaging. He makes the lectures interesting and his passion for the subject is very clear, making lectures more enjoyable. The lectures are very interactive.*

*Love coming to this module. The lecturer is really enthusiastic and passionate about what we are being taught. The content is being taught at a good pace. I really appreciate the chances to ask and answer questions throughout the lectures and the fact it is made sure everyone understands before we move on.*

The keywords: passion, interactive, engaging, fun, interesting and of course the best compliment: love reassures me that students are benefiting from and enjoying the teaching approach. The author attributes these positive comments to the use of real-life engineering examples to explain and bring to life the mathematical techniques. He draws in content and applications from other engineering subjects and research activities making the module truly inter- and multi-disciplinary.

The positive student feedback support the opinion piece in Scientific American [1] which suggest that “math education researchers consider more relevant and engaging curriculum to be an important strategy for increasing persistence rates particularly among students traditionally excluded from STEM fields, such as Black, Latinx and Indigenous students, as well as women.” The same article gives an example from Wright State University’s where academics focused on preparing students for calculus by emphasising ‘engineering motivation for math’ rather than changing the module content.

#### **4 FUTURE WORK**

The Scientific American publication continues by arguing that “the shift toward more practical applications of calculus is missing one key academic endorsement: publication in widely-read journals, if the success of the courses is examined academically at all.”

One can deduce from the student feedback that the adopted teaching approach of incorporating material from other core engineering subjects and emphasising real-life applications very likely contributed to the positive satisfaction with the currently discussed module. It will therefore be interesting to conduct more pedagogical research via surveys and interviews and comparing against similar modules (the teaching of the engineering mathematics is split in to 3 separate cohorts due to large numbers of engineering students – over 600 in total).

The author also proposes to investigate to what extent the revised teaching delivery plan has had an impact on the students learning by interviewing completed cohorts as they progress to their next level of studies (Year 2 and 3, respectively) and will include questions that draws comparisons to related modules, particularly the preceding engineering maths module delivered in the previous semester and to pre-university maths studies such as A-levels and/or Foundation programmes.

There will be specific questions related to the inter- and multidisciplinary approach and real-life examples, and whether this has better prepared them for modules such





as Fluid Mechanics and Mechanics of Materials that they have encountered in the later years of studies.

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