

Nodal and Mesh Analysis Simplification by Introducing a Theorem-Based Preliminary Step

Ferran Reverter^{1b}

Abstract—This brief proposes a new and simplified method for the analysis of linear circuits that combines the classical mesh-current or node-voltage method with a recently-stated theorem. The beforehand application of such a theorem, which involves the insertion of an open or short circuit, in a circuit with N meshes or nodes results in a system of $N - 1$ linear equations for $N - 1$ unknowns, instead of N equations for N unknowns obtained using the conventional approach. Therefore, if the circuit is analyzed in matrix form, the resulting coefficient matrix is square of order $N - 1$, instead of N , thus facilitating the hand calculations. Examples of circuit analysis are provided to demonstrate the applicability and advantages of the proposed analysis method in comparison with the conventional approach.

Index Terms—Circuit theory, circuit analysis, linear circuits, mesh-current method, node-voltage method.

I. INTRODUCTION

THE MESH-CURRENT and node-voltage methods are commonly-employed systematic techniques, based on the Kirchhoff's voltage and current laws respectively, for the analysis of linear circuits [1], [2]. Thanks to these methods, many different circuits can be methodically evaluated without the need of creativity in approaching the analysis. The application of these two methods provides information about the current and/or voltage in any element of the circuit under analysis. These are also employed to determine the transfer function and, hence, the poles of a circuit, although other approximated and more intuitive approaches have also been suggested in the literature [3].

In order to solve some limitations of the two before-mentioned analysis methods, alternatives techniques have been proposed. For example, the modified nodal analysis (MNA) has been suggested to better handle voltage sources and current-controlled elements [4], in comparison with the basic nodal analysis. Using MNA, the current through the previous elements is introduced as an additional variable in the matrix equation, thus resulting in a coefficient matrix of higher order. This makes hand calculations more complex [5], but circuit simulators more efficient [6], [7]. It is worthy to remark that analysis methods intended for computer implementation usually do not consider the suitability for hand calculations [8].

Manuscript received 8 June 2022; accepted 19 July 2022. Date of publication 21 July 2022; date of current version 28 October 2022. This brief was recommended by Associate Editor S. Bose.

The author is with the e-CAT Research Group, Department of Electronic Engineering, Universitat Politècnica de Catalunya—BarcelonaTech, 08860 Barcelona, Spain (e-mail: ferran.reverter@upc.edu).

Digital Object Identifier 10.1109/TCSII.2022.3193034

With the aim of simplifying the analysis of circuits, several theorems have also been stated. In general, these theorems make the analysis more intuitive, but less methodical than the nodal and mesh methods. Some of these theorems are of general purpose, for example: superposition [1], [2], Thévenin, and Norton [9], [10] theorems. Other theorems are more specific, such as Millman's theorem [11], Miller's theorem [12], [13], extra-element theorem [14], cut-insertion theorem [15], [16], Foster's theorem [17], and reciprocal power theorem [18].

With the background of the Thévenin and Norton theorems, a novel general-purpose theorem was recently stated for the analysis of linear circuits [19], which is also applicable to the measurement field for loading-effect reduction [20]. According to this theorem, any current can be determined as an equivalent voltage divided by an equivalent impedance, and any voltage as an equivalent current multiplied by an equivalent impedance. The equivalent voltage must be determined by blocking the current path of interest through an open circuit, whereas the equivalent current by applying a short circuit between the terminals of the voltage under study. The equivalent impedance is the impedance between the terminals of the intended open circuit in the former case, and between the terminals of the voltage under study (without the short circuit) in the latter, but in both cases with the independent sources set to zero. A similar approach to determine the voltage of a circuit node was proposed in [21] using the concept of driving-point impedance instead of equivalent impedance, but with some variations since it involved the application of auxiliary voltage sources.

Taking into account the previous context, this brief proposes to combine the classical nodal and mesh analysis methods with the theorem stated in [19]. As demonstrated in the following sections, the beforehand application of such a theorem, which involves the insertion of an open or short circuit, reduces by one the number of equations and unknowns of the circuit. As a consequence of that, solving the system of equations by hand, with either algebraic or matrix methods, becomes more direct and straightforward.

II. PROPOSED ANALYSIS METHOD

Since the analysis method explained next relies on a theorem [19] that in turn relies on the Thévenin and Norton theorems, a circuit can be analyzed using the proposed approach provided that Thévenin and Norton theorems are applicable. This means that the circuit must be linear and can

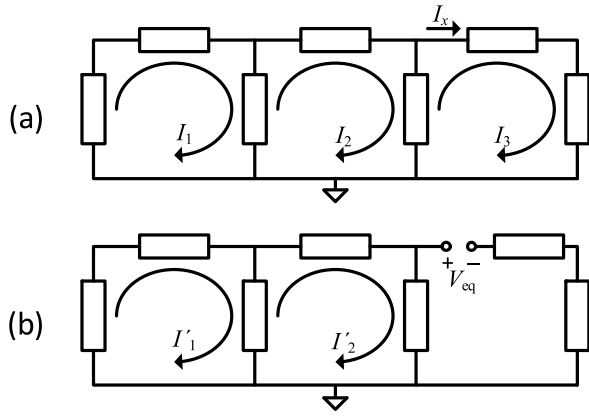


Fig. 1. Three-mesh generic linear circuit analyzed using (a) the conventional mesh-current method, and (b) complemented by the new theorem that involves an open circuit.

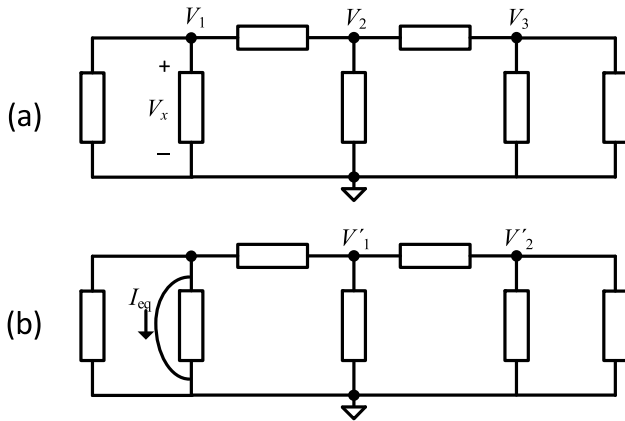


Fig. 2. Three-node generic linear circuit analyzed using (a) the conventional node-voltage method, and (b) complemented by the new theorem that implies a short circuit.

include passive (resistances, capacitances, and/or inductances) and/or active (independent and/or linear dependent sources) elements. Any of these elements is represented by a rectangular symbol in the circuits shown in Figs. 1 and 2, which are identified as *generic linear circuits*.

A. Mesh-Current Method

Let us consider the generic linear circuit with three meshes represented in Fig. 1(a); for the sake of simplicity, here we assume that active elements are independent voltage sources. If the circuit in Fig. 1(a) is systematically analyzed using the mesh-current method, the result is a system of three linear equations for three unknowns (I_1 , I_2 , and I_3) that can be generically described in matrix form as

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \quad (1)$$

where Z_{ii} is the equivalent impedance of mesh i , Z_{ij} (with $i \neq j$) is the impedance common to meshes i and j , and V_i is the equivalent source voltage in mesh i , assuming that $i, j = 1, 2, 3$.

TABLE I
CORRESPONDENCE BETWEEN THE IMPEDANCES OF THE COEFFICIENT MATRICES IN (1) AND (2), DEPENDING ON THE MESH WHERE THE OPEN CIRCUIT IS APPLIED

Eq. impedance in (2)	Corresponding eq. impedance in (1) when...		
	Open circuit applied to mesh 1	Open circuit applied to mesh 2	Open circuit applied to mesh 3
Z'_{11}	Z_{22}	Z_{11}	Z_{11}
Z'_{12}	Z_{23}	Z_{13}	Z_{12}
Z'_{21}	Z_{32}	Z_{31}	Z_{21}
Z'_{22}	Z_{33}	Z_{33}	Z_{22}

From the statements formulated in [19], any current (e.g., I_x in Fig. 1(a)) can be determined as an equivalent voltage (V_{eq}) divided by an equivalent impedance (Z_{eq}). The former must be calculated by blocking the current path through an open circuit, as represented, for example, in Fig. 1(b) for the calculation of I_x . Thanks to this open circuit, the current of mesh #3 in Fig. 1(a) is known and equal to zero. Consequently, the number of meshes is reduced by one, and a system of two equations for two unknowns (I'_1 and I'_2 in Fig. 1(b)) is obtained. This can be generically expressed in matrix form as:

$$\begin{bmatrix} Z'_{11} & Z'_{12} \\ Z'_{21} & Z'_{22} \end{bmatrix} \begin{bmatrix} I'_1 \\ I'_2 \end{bmatrix} = \begin{bmatrix} V'_1 \\ V'_2 \end{bmatrix} \quad (2)$$

where Z'_{ii} is the equivalent impedance of mesh i , Z'_{ij} (with $i \neq j$) is the impedance common to meshes i and j , and V'_i is the equivalent source voltage in mesh i , with $i, j = 1, 2$. Note that if the current to be determined is exclusively equal to any of the three original mesh currents, there is a direct correspondence between the equivalent impedances of the coefficient matrices in (1) and (2), as indicated in Table I. For example, if $I_x = I_3$, as occurs in Fig. 1 with an open circuit applied to mesh 3, then the equivalent impedances in (2) are equal to those in (1) according to the fourth column in Table I.

Solving (2), which is clearly more direct and straightforward than (1), provides the expression/value of both I'_1 and I'_2 in Fig. 1(b). Using them and by inspection of the circuit, V_{eq} can be easily determined. On the other hand, Z_{eq} can be simply calculated as the impedance between the terminals of the intended open circuit when the independent sources are turned off. With V_{eq} and Z_{eq} , the current of interest is then determined as V_{eq}/Z_{eq} .

B. Node-Voltage Method

Let us assume the generic linear circuit with three main nodes, excluding the reference node, shown in Fig. 2(a); for simplification purposes, here we consider that active elements are independent current sources. Applying the node-voltage method to systematically analyze the circuit in Fig. 2(a) provides a system of three linear equations for three unknowns

TABLE II
CORRESPONDENCE BETWEEN THE ADMITTANCES OF THE COEFFICIENT MATRICES IN (3) AND (4), DEPENDING ON THE NODE WHERE THE SHORT CIRCUIT IS APPLIED

Eq. admittance in (4)	Corresponding eq. admittance in (3) when...		
	Short circuit applied to node 1	Short circuit applied to node 2	Short circuit applied to node 3
Y'_{11}	Y_{22}	Y_{11}	Y_{11}
Y'_{12}	Y_{23}	Y_{13}	Y_{12}
Y'_{21}	Y_{32}	Y_{31}	Y_{21}
Y'_{22}	Y_{33}	Y_{33}	Y_{22}

(V_1 , V_2 , and V_3) that can be expressed in matrix form as

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \quad (3)$$

where Y_{ii} is the sum of admittances connected to node i , Y_{ij} (with $i \neq j$) is the equivalent admittance that directly interconnects nodes i and j , and I_i is the equivalent source current that directly goes towards node i , assuming that $i, j = 1, 2, 3$.

According to [19], any voltage (e.g., V_x in Fig. 2(a)) can be determined as an equivalent current (I_{eq}) multiplied by an equivalent impedance (Z_{eq}). The former must be calculated by applying a short circuit between the terminals of the voltage under study, as represented, for example, in Fig. 2(b) for the calculation of V_x . Thanks to this short circuit, the voltage of node #1 in Fig. 2(a) is known and equal to zero. Therefore, the number of nodes is reduced by one, and the result is a system of two equations for two unknowns (V'_1 and V'_2 in Fig. 2(b)) that can be described in matrix form as

$$\begin{bmatrix} Y'_{11} & Y'_{12} \\ Y'_{21} & Y'_{22} \end{bmatrix} \begin{bmatrix} V'_1 \\ V'_2 \end{bmatrix} = \begin{bmatrix} I'_1 \\ I'_2 \end{bmatrix} \quad (4)$$

where Y'_{ii} is the sum of admittances connected to node i , Y'_{ij} (with $i \neq j$) is the equivalent admittance that directly interconnects nodes i and j , and I'_i is the equivalent source current that directly goes towards node i , with $i, j = 1, 2$. If the voltage to be determined equals any of the three original node voltages, a direct correspondence exists between the equivalent admittances of the coefficient matrices in (3) and (4), as summarized in Table II. For instance, if $V_x = V_1$, as happens in Fig. 2 with a short circuit applied to node 1, then the equivalent admittances in (4) are equal to those in (3) following the second column in Table II.

The expression/value of both V'_1 and V'_2 can be calculated by solving (4), which is simpler than (3). Then, by circuit inspection, I_{eq} can be determined in Fig. 2(b) without difficulty. In addition, by turning off the independent sources, Z_{eq} can be calculated as the impedance between the nodes of the voltage under study without the intended short circuit. With I_{eq} and Z_{eq} , the voltage of interest can be estimated as $I_{eq} \cdot Z_{eq}$.

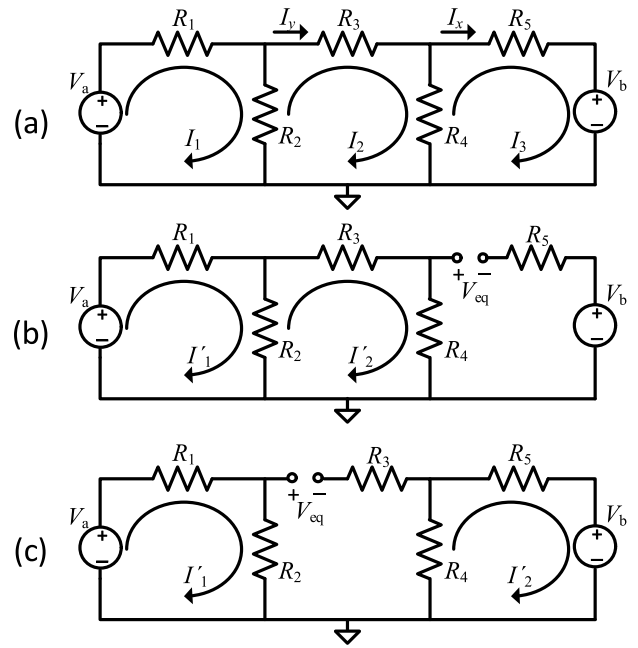


Fig. 3. Example of a three-mesh linear circuit analyzed using (a) the mesh-current method, and simplified by the new theorem to find (b) I_x , and (c) I_y .

C. Discussion

According to the preceding explanation, the beforehand application of the theorem proposed in [19] in a linear circuit with N meshes (or N main nodes) results in a system of $N - 1$ linear equations for $N - 1$ unknowns, instead of N equations for N unknowns obtained using the traditional approach. In case the system is expressed in matrix form, the resulting coefficient matrix is square of order $N - 1$, instead of N , thus facilitating the solving process by hand. It is true that the proposed method then requires the determination of the equivalent voltage (or current) and impedance, but these can be obtained by easy inspection of the circuit. Therefore, the proposed method involves two steps but these are simpler to be solved. In the next two sections, two different examples with $N = 3$ are given to demonstrate the applicability of the new analysis approach, but of course it could be applied to circuits with $N \neq 3$.

III. EXAMPLE 1

As an example of how the mesh-current method can be complemented and simplified using the new theorem, let us consider the three-mesh linear circuit with resistive elements shown in Fig. 3(a), where the current of interest to be determined is I_x . If this circuit is systematically analyzed exclusively applying the mesh-current method, the result is the following matrix equation:

$$\begin{bmatrix} R_1 + R_2 & -R_2 & 0 \\ -R_2 & R_2 + R_3 + R_4 & -R_4 \\ 0 & -R_4 & R_4 + R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ -V_b \end{bmatrix} \quad (5)$$

Since I_x equals the current of mesh #3 in Fig. 3(a), this can be determined by solving (5) for I_3 , thus resulting in:

$$I_x = I_3 = \frac{R_2 R_4 V_a - R_A^2 V_b}{R_A^2 R_5 + R_4(R_1 R_2 + R_1 R_3 + R_2 R_3)} \quad (6)$$

where $R_A = (R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_3 + R_2 R_4)^{0.5}$.

Let us now analyze the same circuit but first of all applying the theorem. Accordingly, the current path of I_x must be blocked through an open circuit, as represented in Fig. 3(b), thus obtaining a two-mesh circuit. The analysis of this circuit applying the mesh-current method results in the ensuing matrix equation:

$$\begin{bmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 + R_4 \end{bmatrix} \begin{bmatrix} I'_1 \\ I'_2 \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \end{bmatrix} \quad (7)$$

whose coefficients agree with those in (5) following the correspondence indicated in the fourth column in Table I. On the other hand, an easy inspection of the circuit in Fig. 3(b) provides the equivalent voltage and resistance, thus resulting in the following expression of I_x :

$$I_x = \frac{V_{eq}}{R_{eq}} = \frac{I'_2 R_4 - V_b}{[R_4 || (R_3 + (R_1 || R_2))] + R_5} \quad (8)$$

where I'_2 can be easily obtained by solving (7) as:

$$I'_2 = \frac{R_2 V_a}{R_A^2} \quad (9)$$

Of course, replacing (9) in (8) provides the same expression for I_x obtained in (6), but the suggested methodology involves operating with matrices of order 2 rather than 3, which is more straightforward, thanks to the open circuit (Fig. 3(b)) applied beforehand.

The approach proposed herein becomes even simpler if the current of interest involves an open circuit that makes the two resulting meshes independent from each other. This is the case, for example, when the current to be determined is I_y in Fig. 3(a), which involves the open circuit depicted in Fig. 3(c). Here, the two meshes are independent and, therefore, the resulting matrix equation has a diagonal-type coefficient matrix, as follows:

$$\begin{bmatrix} R_1 + R_2 & 0 \\ 0 & R_4 + R_5 \end{bmatrix} \begin{bmatrix} I'_1 \\ I'_2 \end{bmatrix} = \begin{bmatrix} V_a \\ -V_b \end{bmatrix} \quad (10)$$

By easy inspection of the circuit in Fig. 3(c), I_y can be expressed as

$$I_y = \frac{V_{eq}}{R_{eq}} = \frac{I'_1 R_2 + I'_2 R_4}{(R_1 || R_2) + (R_4 || R_5) + R_3} \quad (11)$$

where I'_1 and I'_2 can be simply found by solving (10) as:

$$I'_1 = \frac{V_a}{R_1 + R_2} \text{ and } I'_2 = \frac{-V_b}{R_4 + R_5} \quad (12)$$

Consequently, the combination of the mesh-current method and the new theorem transforms a system of three equations for three unknowns (5) to two independent systems of one equation for one unknown (10), whose solving is clearly much simpler.

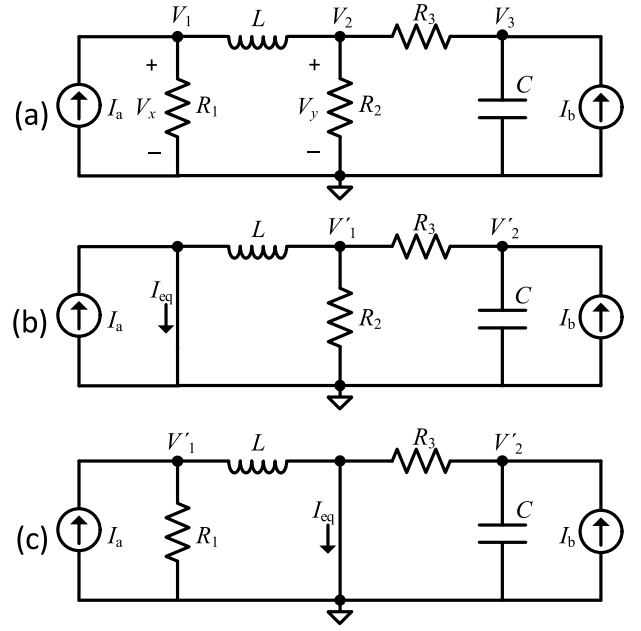


Fig. 4. Example of a three-node linear circuit analyzed using (a) the node-voltage method, and simplified by the new theorem to find (b) V_x , and (c) V_y .

IV. EXAMPLE 2

To illustrate how the node-voltage method can be completed and simplified using the new theorem, it is suggested to use the three-node linear circuit with resistive, capacitive, and inductive elements shown in Fig. 4(a) as an example, where the voltage of interest to be determined is V_x . Applying exclusively the node-voltage method to systematically analyze this circuit provides the ensuing matrix equation:

$$\begin{bmatrix} G_1 + Y_L & -Y_L & 0 \\ -Y_L & Y_L + G_2 + G_3 & -G_3 \\ 0 & -G_3 & G_3 + Y_C \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_a \\ 0 \\ I_b \end{bmatrix} \quad (13)$$

where G_i is the conductance associated to R_i (i.e., $G_i = 1/R_i$) with $i = 1, 2, 3$, and Y_L and Y_C are the admittances related to L and C , respectively, with $Y_L = 1/(L \cdot s)$ and $Y_C = C \cdot s$, s being the Laplace variable. Given that V_x in Fig. 4(a) is equal to the voltage of node #1, this can be found by solving (13) for V_1 , thus obtaining:

$$V_x = V_1 = \frac{I_a Y_A^2 + I_b Y_L G_3}{Y_A^2 G_1 + Y_L (G_2 G_3 + G_2 Y_C + G_3 Y_C)} \quad (14)$$

where $Y_A = (Y_L G_3 + Y_L Y_C + G_2 G_3 + G_2 Y_C + G_3 Y_C)^{0.5}$.

Let us now proceed with the analysis of the same circuit but firstly considering the theorem. In such a case, a short circuit must be applied between the terminals of the voltage under study (i.e., across R_1 , which becomes superfluous), as shown in Fig. 4(b). If the resulting two-node circuit is analyzed through the node-voltage method, the following matrix equation is obtained:

$$\begin{bmatrix} Y_L + G_2 + G_3 & -G_3 \\ -G_3 & G_3 + Y_C \end{bmatrix} \begin{bmatrix} V'_1 \\ V'_2 \end{bmatrix} = \begin{bmatrix} 0 \\ I_b \end{bmatrix} \quad (15)$$

whose coefficients agree with those in (13) following the correspondence indicated in the second column in Table II. In addition, V_x can be expressed as a function of Z_{eq} and I_{eq} , which can be determined by easy inspection of the circuits in Figs. 4(a) and 4(b), respectively, as:

$$V_x = I_{eq}Z_{eq} = (I_a + V'_1 Y_L) \left(R_{1||} \left[Y_L^{-1} + \left(R_{2||} \left(R_3 + Y_C^{-1} \right) \right) \right] \right) \quad (16)$$

where V'_1 can be simply obtained by solving (15) as:

$$V'_1 = \frac{I_b G_3}{Y_A^2} \quad (17)$$

Substituting (17) in (16) results in an expression for V_x equal to (14). Note, however, that again the proposed analysis method is simpler since it involves operating with matrices of order 2 rather than 3, thanks to the short circuit (Fig. 4(b)) applied beforehand.

As also explained in Section III, the proposed analysis method becomes even more straightforward when the voltage of interest implies a short circuit that makes the two remaining nodes independent from each other. This is what occurs, for instance, when the voltage to be determined is V_y in Fig. 4(a), which implies the short circuit represented in Fig. 4(c). In such conditions, the two nodes are independent and the circuit is characterized by the ensuing matrix equation with a diagonal-type coefficient matrix:

$$\begin{bmatrix} G_1 + Y_L & 0 \\ 0 & G_3 + Y_C \end{bmatrix} \begin{bmatrix} V'_1 \\ V'_2 \end{bmatrix} = \begin{bmatrix} I_a \\ I_b \end{bmatrix} \quad (18)$$

Expressing now V_y as a function of Z_{eq} and I_{eq} , which can be easily obtained by circuit inspection, the result is:

$$V_y = I_{eq}Z_{eq} = \left(V'_1 Y_L + \frac{V'_2}{R_3} \right) \left(R_{2||} \left(Y_L^{-1} + R_1 \right) \right) \left(R_3 + Y_C^{-1} \right) \quad (19)$$

where V'_1 and V'_2 can be easily determined by solving (18) as:

$$V'_1 = \frac{I_a}{G_1 + Y_L} \quad \text{and} \quad V'_2 = \frac{I_b}{G_3 + Y_C} \quad (20)$$

As also highlighted before in Section III, the combination of the node-voltage method and the new theorem has converted a system of three equations for three unknowns (13) to two independent systems of one equation for one unknown (18), with the corresponding benefits in terms of mathematical operations.

V. CONCLUSION

A novel and simplified method for the analysis of linear circuits combining the traditional mesh or nodal method with a recently-stated theorem has been suggested in this brief. Before proceeding with the mesh or nodal analysis, it has been proposed to first apply the theorem to the current/voltage of interest so that an open/short circuit is introduced and, therefore, the number of meshes/nodes of the resulting circuit is

reduced by one. In case the circuit is analyzed in matrix form, a circuit with N meshes (or N main nodes) results in square coefficient matrix of order $N - 1$, instead of N , thus facilitating the solving process by hand. For example, a circuit with $N = 3$ can be solved by operating matrices of order 2, with $N = 4$ by matrices of order 3, and so on. In the author's opinion, the proposed combination of methods and theorem is easy to remember and to apply and, therefore, it is expected to become a basic tool in circuit analysis.

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