TREBALL FINAL DE GRAU

Avoiding instabilities in short gap car-followings with connected autonomous vehicles

Work done by:
Mohamed Khaled Abdelrahim Mohamed

Directed by:
Marcel Sala Sanmarti
Margarita Martínez Díaz

Bachelors in:
Civil Engineering

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DECA - Department of Civil and Environmental Engineering

Abstract

The image of driverless vehicles cruising on highways has been coming closer to reality over the last years thanks to the constant investigations in the technologies used in Connected and Automated Vehicles (CAV). Platooning of CAVs has been the focal point of investigations due to the potential benefits that can be reaped from the proper implementation of platoons on highways. One of the challenges facing the successful operation of platoons is the bullwhip phenomenon that causes propagating perturbations in platoons hindering its stability. The present study focuses on eliminating or reducing the bullwhip effect suffered by vehicles in a platoon. The aforementioned platoon algorithm will be governed by the formula of the Desired Space Gap (DSG) as followers will have to maintain a gap equal to the DSG to follow the leader. This algorithm managed to successfully carry out any increase or decrease in velocity of the platoon however, in the case of braking it has proved to be extremely unstable and suffers from effect of the bullwhip phenomenon. The average cumulative gap was used as a solution to trigger an instant response from all vehicles down the platoon to the actions of the leader as it was observed that vehicles in the end of the platoon tend to approach the leaders at high velocities because they only start decreasing their velocities once the vehicle ahead of them decelerates rather than the leader. This solution has been effective in significantly reducing the bullwhip effect on some vehicles in the platoon only, mainly the ones at the end of the platoon. Furthermore, this solution has given positive results only in cases of great changes in velocity.

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1. Introduction

Over the years, technology has revolutionized our living environment and has been incorporated in our daily life's activities. The world is now moving rapidly forward towards innovative technologies specifically in the fields of digitalization and automation. The dependence on machines, AI and computers is increasing recently as they have proven to be more economic and efficient. This has sparked a great deal of interest from several sectors in the modern economy to research and develop new technologies to adapt to current practices and exploit the benefits of automation.

One of the most important sectors that is actively exploring the field of automation is the automobile industry. The industry faces several problems that automation and the use of technology can aid in solving. The current climate change crisis that the world is facing has spurred a movement towards the reduction of fuel consumption. The continuous increase in fuel prices globally alongside with the negative environmental effects of burning them has led the automobile industry to seek solutions using technology. (Alexander, et al., 2018) Automation can help in the development of fuel saving solutions such as platooning of connected and automated vehicles.

A platoon is a group of vehicles that can safely travel at high speeds with very small spacings by utilizing vehicle-to-vehicle (V2V) communication technologies. Platooning has shown positive results in commercial trucks with a reduction of fuel consumption up to 12%. (Texas A&M Transportation Institute, 2016). The issue of safety has also been a great concern for the manufacturers. Throughout the past 50 years a lot of safety features have been added to commercial cars many of which are integrated technology systems such as ABS (Anti-Lock Brakes system) and ADAS (Advanced drivers assistance system) (Holland, 2022).

The concept of platooning has been of great interest to both manufacturers and governments as studies have proven that with successful implementation of platooning for cars the capacities of highways can be increased. There have been several initiatives of platooning recently mainly focused on the platooning of trucks such as EDDI: Electronic Drawbar – Digital Innovation project run by MAN Truck & Bus, DB Schenker, and the Hochschule Fresenius. Another project worth mentioning is "Sweden 4 Platooning (S4P) (Schenker, et al., 2018; Axelsson, et al., 2020).

2. State of the Art

2.1. Automated vehicles (AV)

Automated vehicles (AV) are vehicles that have computer systems and mechanical equipment incorporated in them allowing them to pose varying levels of control over multiple functions of the vehicles to safely maneuver itself without human interference through traffic. This is achieved through a range of equipment such as sensors, GPS, cameras, ultrasounds, and other communications devices that provide information about the surrounding environment to the installed computer systems (Synopsys, 2022). AVs can formulate and continuously update a map of the car's surroundings using the numerous types of sensors and cameras onboard. All the gathered data in input into onboard computers that analyze it to create a safe passage for the vehicle and send commands to the car to accelerate or decelerate and instructions on how to steer too (Synopsys, 2022).

There have been several developments of automation technologies in cars in the past several years that are now implemented in commercial vehicles in the market. Some examples of these technologies are listed below (Choudhury, 2020; Synopsys, 2022).

- **Advanced Driver Assistance Systems (ADAS)**, the key role of this system is to prevent human injuries or death by reducing accidents. ADAS combines several systems under it such as pedestrian/blind spot detection, Automatic Emergency Braking (AEB) and lane keeping assist.

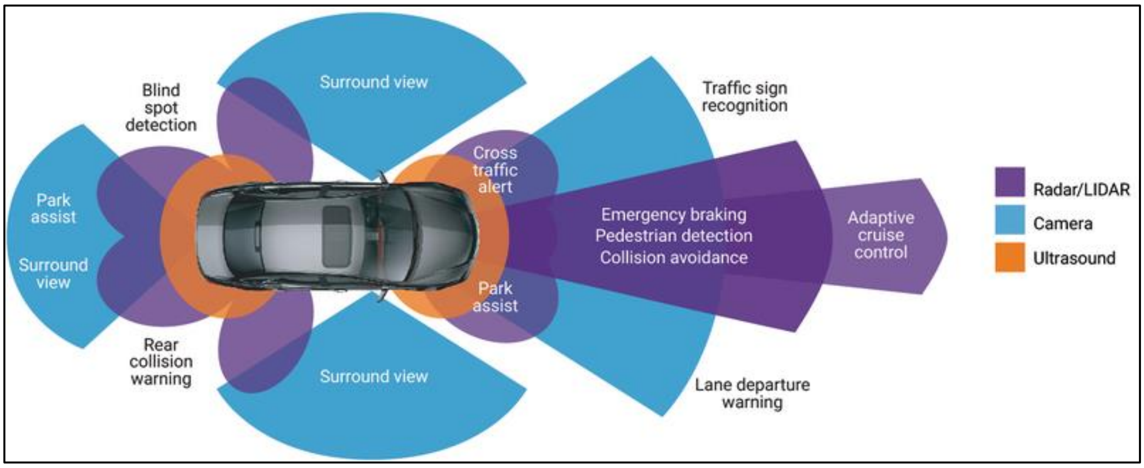

Figure 1: scheme of different systems and their coverage areas in the ADAS (Synopsys, 2022)

- **Adaptive Cruise Control (ACC)**, it is a smart type of cruise control where it adapts the speed of the vehicle automatically depending on the vehicle Infront of it thus reducing chances of collisions.
- **Electronic Stability Control (ESC)**, it is a feature that allows computers to brake individual wheels in the vehicle to maintain control over it in dangerous situations.

There are also other technologies that increase the autonomy of the car without playing a key role in increasing safety such as automatic parking. As technology advanced and a lot of new features started getting added to cars it was important to create some sort of scale that states the level of autonomy that a vehicle actually has. The Society of Automotive Engineers (SAE) defines six levels of automation as shown in Figure 2 from 0 (no automation) to 5 (full automation)

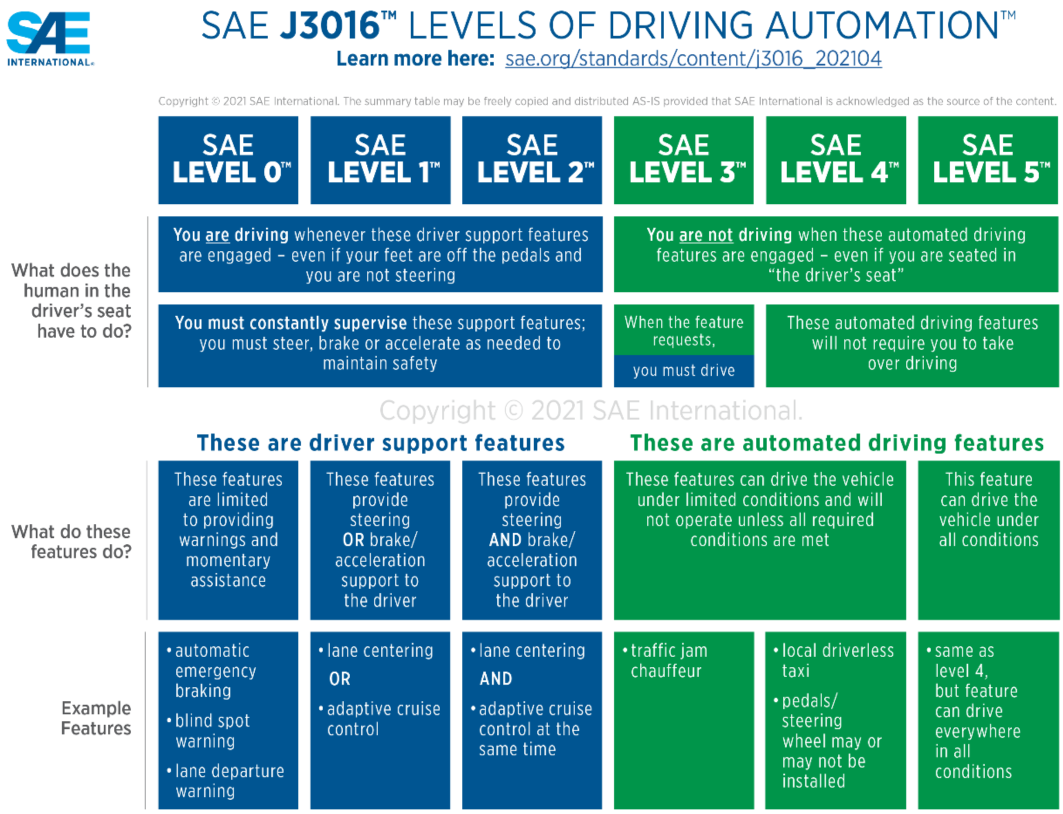

Figure 2: SAE Automation levels definition (SAE, 2021)

According to statistics in 2019 around 1.4 million vehicles with level 3 automation have been sold worldwide (Placek, 2022). Major manufacturers around the world are racing towards releasing new models with high levels of autonomy. There are several commercially available level 3 cars on the market now such as the Audi A8 2019 model equipped with the AI traffic jam pilot allowing the car to travel on specific highways at speeds up to 60 km/h (Martínez-

Díaz, et al., 2019). There is also the Mercedes Benz “Drive Pilot” that is available in the S-class and EQS models which is allowed to drive in conditionally automated mode at speeds of up to 60 km/h in heavy traffic or congested situations on 13,000 km of highways in Germany (Mercedes-Benz, 2021).

There are several reasons behind the interest of car manufacturers in automation. The first one is safety. According to the World Health Organization report on Global Status of road safety 1.35 million people die annually on roadways (WHO, 2018). This number reflects the size of this global issue which results in companies continuously trying to improve safety features in their vehicles. Automation has been one of the major solutions to reducing the numbers of deadly accidents. The United States Department of Transportation (USDOT) believes that autonomous vehicles would reduce road deaths and injuries as 94% of deadly accidents occur due to human error. Furthermore, the house Energy and commerce committee states “ Self-driving cars are projected to reduce traffic deaths by 90%, saving 30,000 lives a year.” (Goldin, 2018). In 2015 a model was created to see the impact of AVs on car crash injuries and fatalities, it was estimated that if AVs represent 90% of cars in the United States around 25,000 lives would be saved annually and also save 200 billion $ in economic losses resulting from the crashes (Luttrell, et al., 2015).

As the world suffers from the consequences of climate change there has been a great focus on reducing the consumption of fossil fuels and the emission of carbon dioxide. Fuel based transportation modes are one of the major polluters in 2020 producing around 7.3 billion metric tons of carbon dioxide emissions 41% of which directly comes from cars (Tiseo, 2021). Automation has been proven to have a green fingerprint on the environment both directly and indirectly. The indirect effects are by-product of the reduction of accidents and traffic congestion. Accidents cause huge congestions that increase the emissions of cars. Ohio University published “Future of Driving report” stating that the transition to autonomous cars can cause a 60% decrease in emissions (Ohio univeristy, 2021). Furthermore, due to reduction in traffic congestions the improvement of traffic flow can increase the fuel economy of vehicles leading to 23% - 29% increase in fuel efficiency (Fagnent & Kockelman, 2015). As for the direct effect according to Rand Corporation guide “Autonomous vehicle Technology” autonomous cars are expected to reduce fuel consumption by 4-10 percent as a result of smoother acceleration and deceleration (Anderson, et al., 2016).

The use of automation can enhance the performance of transportation systems . It can help reduce traffic congestions by the elimination of stop and go waves. According to research done by the University of Illinois at Urbana-Champaign experiments show that a 5% of autonomous vehicles would eliminates the stop and go waves and thus reduce congestions caused by human driving behaviour (Stern, et al., 2018). Another study conducted by the university of Texas estimates that at a 10% AV penetration rate a 15% reduction in congestion is expected as a result of smoother traffic flow and elimination of bottlenecks. Furthermore, by increasing the penetration rate up to 90% congestions reductions of around 60% is expected accompanied by an almost doubling in the highway capacity and extreme crash reductions (Fagnent & Kockelman, 2015).

AVs can also increase lane capacity of highways and intercity roads. Studies have shown the capacity can be increased by 500% by combining the use of autonomous vehicles and platooning (Anderson, et al., 2016). Other studies showed that by the use of purely autonomous vehicles in city traffic would result in a 40% increase in capacity while in highways the increase in capacity can reach eighty Percent (Friedrich, 2016). In a study conducted to Analise the impact of autonomous vehicles on urban roads the centra areas of seongnam city was studied. The studied road network was 4.5 km long and had thirteen intersections. The results of the study were that AVs reduced pressure of increasing traffic volumes experience throughout the day and at 100% penetration rate the capacity of the roads increase by 50% (Park, et al., 2021).

Lastly, in a study carried out in the UK it was deduced that with automation travel times can decrease which besides reducing congestion and emission will save billion pounds in increase productivity of people time (KPMG, 2015). It is also important to point out that the safety, environment, and transportation benefits are all economic benefits too as they save money by reducing costs and consumption.

2.2. Connected vehicles (CV)

Connected vehicles are vehicles equipped with data transmission technologies such as an Internet of Things instruments (IoT) that allows it to communicate with other objects in its surrounding environment to exchange information. There are different families or types of communication. The main three are (Keenan, 2019):

- **Vehicle to vehicle (V2V):** vehicles are able to share data with other vehicles such as their positions, speed, size, and mechanical properties such as their braking limits. This type of connectivity is vital for safety as vehicles can share live conditions of the road ahead for example in a case of a crash information is passed back for incoming vehicles to slow down. This type of communication also allows the formation of platoons discussed later.
- **Vehicle to Infrastructure (V2I):** Vehicles can communicate with traffic control signals and other traffic infrastructures to receive information about the road conditions, speed limits and state of traffic. Vehicles benefit from knowing the colour of traffic light signals to determine accordingly their speed whether they will stop or pass the light avoiding sudden human reactions at the traffic light. Also Transport authorities can use real time data of incoming flow of vehicle and optimize their signals accordingly to improve fluidity on the roads and impose suitable speed limits to the traffic conditions.
- **Vehicle to Everything (V2X):** Vehicles can share data with any type of device that has the same technology making it able to send and receive information such as smart phones.

However, recently two more types were created which are more of a subsection to the V2X communication.

- **Vehicle to Pedestrian (V2P):** with modern technologies installed in vehicles such as pedestrian detection vehicles can detect pedestrians better using their mobile phones and alert drivers to avoid collisions.
- **Vehicle to network (V2N) / Vehicle to cloud (V2C):** this technology facilitates the exchange of information between all sorts of vehicles and communication systems that solves compatibility issues of different devices used on different models of cars thus enhancing vehicle connectivity.

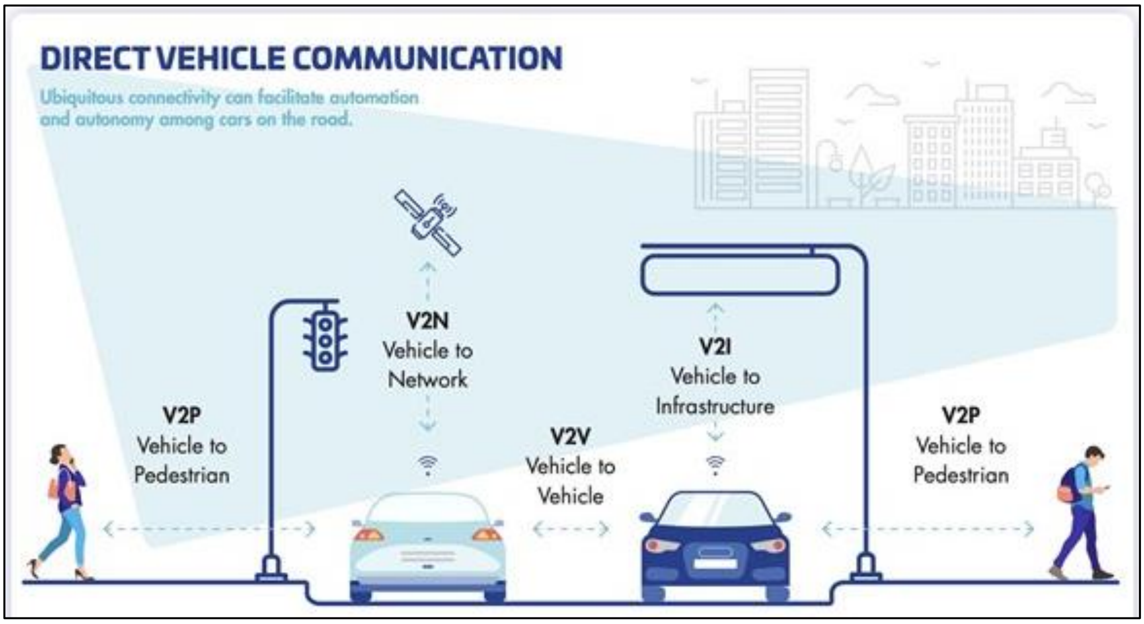

Figure 3:Types of communications (Haider, 2022)

It is important to note that there is a difference between Connected and Autonomous Vehicles. The main difference is that an AV can take decisions on its own without human interference while a CV only provides the information received from infrastructure or other sources to help the human driver in taking actions. Nevertheless, both types of vehicles contribute towards the safety of humans, improving of transportation systems and environmental benefits. CVs are being studied for use in Variable Speed Limit systems and studies show that the use of CVs with the VSL system results in decrease of average total time of travel by 1.5 hours in 84% of simulations run in the study (F.Grumert & Tapani, 2017).

A study conducted by Colombia university estimates that the use of sensor equipped CVs that use V2V communication can increase the highway capacity by around 273% (8,200 cars per hour per lane) compared to a 43% increase if vehicles used sensors only. The study also estimates that with 100% CVs using V2V communications capacity could reach 12,000 cars per hour per lane that around a 545% increase from the US Highway capacity manual capacity of 2,200 cars per hour per lane. Those CVs would be able to travel safely with speeds of 120 km/h with a following gap of 6 m (Tientrakool, et al., 2011). Research has shown that Cooperative Adaptive Cruise Control (CACC) which is an extension of the regular ACC that uses V2X communication shows more significant results in increasing highway capacity than the traditional ACC. Results showed that the ACC is unlikely to produce significant change in highway capacity as drivers are only comfortable with ACC gaps similar to their own gaps while driving manually while the V2X using CACC showed maximum lane capacity of around

4000 cars per hour per lane almost doubling the highway capacity at high penetration rates (Shladover, et al., 2012).

2.3. Connected and Automated Vehicles (CAV)

Connected and automated vehicles are ones the combine both systems and equipment of AVs and CVs therefore increasing the performance levels of the vehicles and rounding off each other's limitations. CAVs have the power to receive information from the infrastructure or the vehicles ahead of it and immediately conduct appropriate manoeuvres in faster times than it would take a human. The AI would also be more efficient in choosing the most important data received and base its manoeuvre on it rather than secondary data that is not as vital which is an error that a human brain can make. CAVs also lay the foundation for the particularly important field of platooning. CAVs travelling in platoons presents a promising management strategy exploiting the most out of Automation technology.

According to a study conducted in Virginia, United States of America that was designed to assess the changes to highway capacities due to the introduction of AVs and CAVs at 100% penetration rates of AVs the highways capacity increased by 28%. However, when running the same simulation with CAVs, the capacity increased by ninety-two% (Heaslip, et al., 2020). This shows how powerful the combinations of CVs and AVs technology together.

2.4. Platoons

Platoons are defined as a group of CAVs that can exchange information using V2V communication and others in order to drive in a coordinated way allowing the possibility of having small spacings at high velocities while ensuring safe conditions. Typically, a platoon consists of one leader and an unlimited number of followers which adapt their speed according to the pace and actions of the leader. Platoons can be classified according to five primary features (Martínez-Díaz, et al., 2021).

- **Type of vehicle:** a platoon can be said to be "homogenous" if all the vehicles have identical or similar characteristics such as but not limited to their mechanical capabilities, size, and automation levels. Otherwise, when several types of vehicles exist in the same platoon such as combining trucks and vehicles it is a "heterogeneous" platoon (Feng, et al., 2019). The diversity in the mechanical features of vehicles in a heterogenous platoon can cause problems due to the difference in acceleration and braking rates. Furthermore, it can imply comfort issues for some drivers such as having a car stuck between two vehicles (Martínez-Díaz, et al., 2021).
- **Platoon length:** the number of vehicles in a platoon. A platoon can be called "finite" when it has a limited number of vehicles in it. Most studies like (Ge & Orosz, 2014) usually have finite platoons to serve the purpose of the study however, platoon can also be called "infinite" where it has an infinite number of vehicles. The infinite case is argued to be a useful paradigm to understand large platoons (Jovanovic & Bamieh, 2005). It is always aimed to create long platoons as benefits of platooning increase with larger platoons. It is still unclear how feasible large platoons can be and this field still requires further investigations.
- **Formation policies:** refers to the method that a platoon is formed. Depending on the type of formation policy there are conditions that have to be satisfied for a car to join a platoon. For example, in cooperative platoons any CAVs that is within a specific range can join the platoon. On the other hand, in opportunistic platoons only CAVs that are in the same lane can join the platoon. Inside every formation policy there is merging policy which states how the new vehicle will join the existing platoon. For example, the vehicle can speed up to catch up with the platoon

or the platoon can slightly decelerate to make it easier for the vehicle to join and there are other hybrid methods.

- **Following policies:** they are car following models that basically define how the following vehicles in the platoon will follow the leader and thus maintain the formation of the platoon. There are many kinds of car following models such as constant space gap, constant time gap and others.
- **Information / communication topology:** this is one of the main methods platoons are classified and it defines the path at which information travels between vehicles in the platoon. Topology (d) also known as Bidirectional-leader topology (BDL) will be the type adapted for the simulations carried out in section 4.

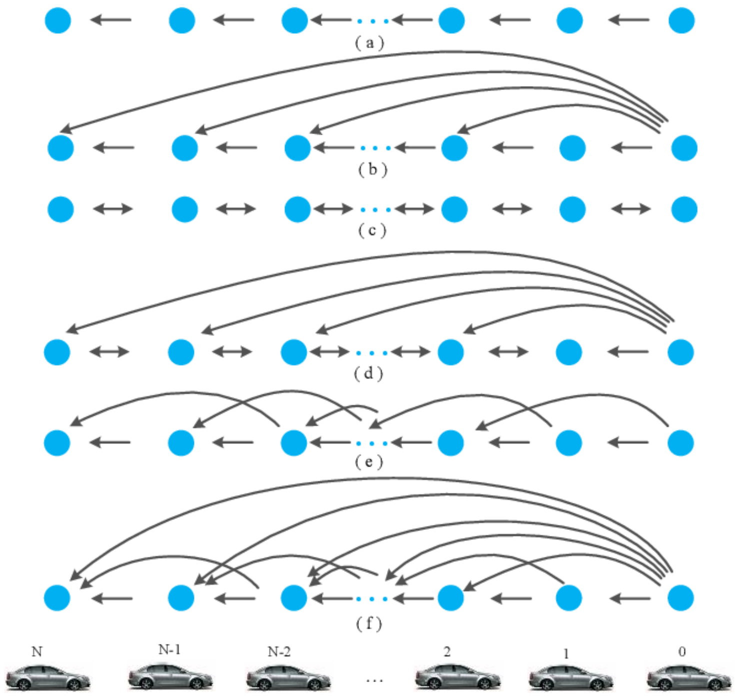

Figure 4: Example of information flow topologies in a platoon (Zheng, et al., 2014)

Platooning of CAVs has proven to be beneficial in multiple aspects. There are several studies that conclude the increase of capacity of highways by either introducing CAV platoons or assigning a platooning lane, in some cases CAVs were proven to enhance capacity of expressways by 500pcu/h (Liu, et al., 2022). A previous study done by colleague at UPC shows a direct relation between increasing numbers of platoons and platoon lengths on the capacity of highways. For cases of 50 % penetration rate of CAVs a huge increase in platooning lane is observed (Herrera, 2019). According to research conducted by (Sala & Soriguera, 2021) platooning can be a promising strategy to increase current infrastructure capacities. At 100% CAV penetration rate with optimistic platooning parameters and a platoon length limited to a realistic value of 20 vehicles capacity of a highway can exceed 10,000 vehicles /h/lane which is five times the normal capacity without platooning.

Furthermore, platoons help in reducing fuel consumptions of cars in them due to the reduction of air drag by values carrying from 20% to 60% depending on the type of vehicles in the platoon and the platoon length (Wadud, et al., 2016). Moreover, there is a qualitative relationship between penetration rate and average fuel consumption reduction rate, reduction rate increases as the penetration increase reaching 18.9% reduction rate for most prominent cases (Zhou, et al., 2021). On a macroscopic level the capacity of a highway has been found to increase with the increase of the platoon length. At 100% CAV penetration platoons of length two vehicles would increase highway capacity by 25 % and by increasing the platoon length to ten vehicles the capacity can increase by around 80% (Chen, et al., 2017)

There have been several real life application of platooning but mainly focused on the freight sector on Trucks the two most promising initiatives are “Sweden 4 Platooning S4P” (Axelsson, et al., 2020) by Scania CV AB, Volvo Technology Corporation, The Royal Institute of Technology (KTH), and “Electronic Drawbar - Digital Innovation EDDI” (Schenker, et al., 2018) by MAN Truck & Bus, DB Schenker, and the Hochschule Fresenius. Both initiatives obtained positive results in terms of fuel consumption and emissions reduction .

2.5. The bullwhip phenomenon

Bullwhip phenomenon is a type of instability that CAVs can experience in a platoon during acceleration or deceleration phases. It mainly affects vehicles located farther back in the platoon as they become highly reactive to manoeuvres carried out by the leader. The way that these vehicles tend to react to the leader's manoeuvres results in greatly amplified actions compared to other vehicles in the platoon. With modern communication technology as the leader carries out a manoeuvre the information is transmitted down the platoon almost instantly. However, the n^{th} vehicle in a platoon adjusts its behaviour according to the vehicle ahead of it and therefore the farther the vehicle is in the platoon with respect to the leader the more amplified response it might have.

To visualize the effects of the bullwhip effect we consider the following scenario. A platoon of twenty vehicles is preforming a deceleration manoeuvre represented by Figure 5 and Figure 6. It is important to note that while these values of acceleration are achievable by current vehicles in the market, but it is extremely uncomfortable for the passengers inside the vehicle.

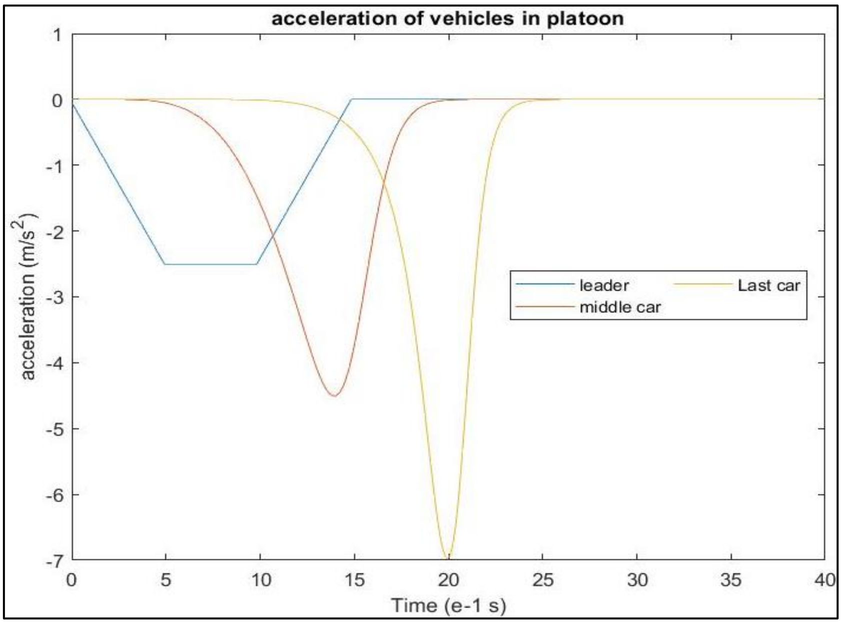

Figure 5: Acceleration-Time graphs of vehicles in a platoon

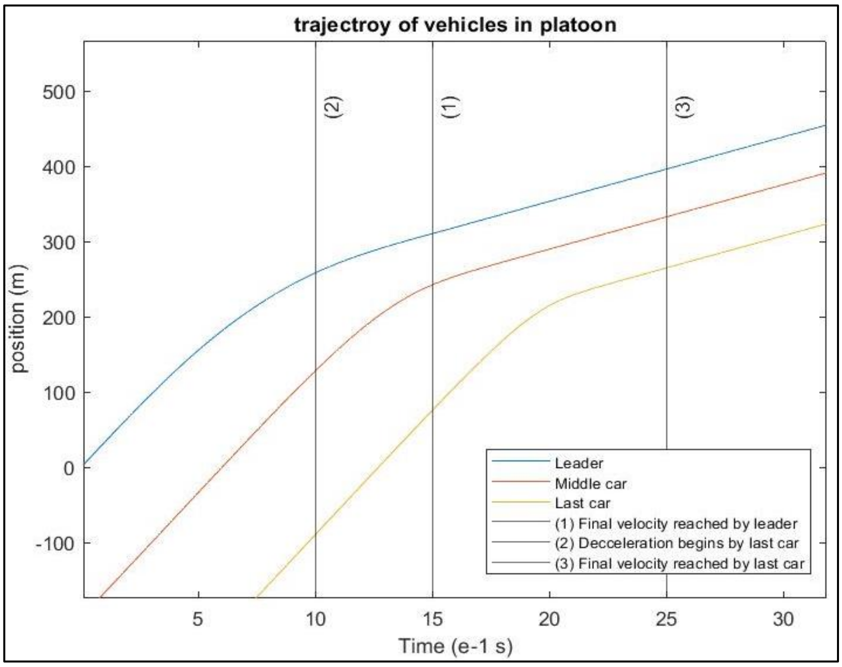

Figure 6: Trajectories of Vehicles in a platoon

As the leader starts decelerating the followers one by one start decreasing their velocities too however, for the last car in the platoon it only stars decreasing its velocity once the vehicle in front of it does. So, for some time the final car in the platoon is still travelling at the initial speed of the platoon thus at a velocity higher than of the leader and we can see that in Figure 6 with the vertical line (2) the represents the begging of deceleration of the last car, for 10s the last car has been traveling at the initial velocity of the platoon while the leader and vehicles up in the platoon have been decreasing their velocities.

As the last car gets closer to the leader it is required to brake more aggressively to reach the required final velocity, we can visualize that using the curves of the trajectories in Figure 6 as we can see the trajectory of the leader has a smooth curve between the initial and final velocity with a bigger radius while the curve of the last car is much sharp and has a small radius showing an abrupt change in velocities. As we can see in Figure 5 the last car in the platoon decelerates much more aggressive that the car in the middle of the platoon which shows us the amplification of response of every vehicle down the platoon. The bullwhip effect often causes followers to have values of jerk and acceleration greater than the maximum values established for the platoon that ensure comfort of passengers.

2.6. Car following models.

An especially important aspect of platooning is car following models, they dictate how one vehicle will follow another vehicle without interrupting the flow of traffic and ensuring safety. First attempt of a model was (Pipes', 1953) model which was fairly simple, the following vehicle has to maintain a minimum distance equal to the length of a car for every 10 mph of speed. This would formulate a linearly increasing minimum distance as speed increase. Later on, (Forbes, 1963) introduced the concept of reaction time which is time needed for driver to notice and event and start decelerating. So, vehicles have to maintain a time gap equal to the reaction time plus time needed to cover distance between the vehicle and the rear end of the leading vehicle (Mishra, 2014).

The General Motor's car following model is one of the most popular models. It was continuously developed throughout the years creating five generations which all took the form of response is a function of sensitivity and a stimulus. This means that the change in acceleration is a function of the difference between two vehicles which is called "the stimulus" and the reaction of the driver to the stimulus which is called "the sensitivity". The five generations had different interpretations to the sensitivity factors every time adding a new factor and finally the expression of the fifth-generation model was the following (Mishra, 2014).

$$\ddot{x}_{n+1}(t+\Delta t) = \frac{\alpha_{l,m}[\dot{x}_{n+1}(t+\Delta t)]^m}{[x_n(t) - x_{n+1}(t)]^l}[\dot{x}_n(t) - \dot{x}_{n+1}(t)] \rightarrow \mathbf{(1)}$$

2.7. Previous works

This work is a continuation of several Works conducted by Camins colleagues who were also investigating CAV platoons. The work of (Boukhellouf, 2019) discovered a problem in the previous works on the topic which was that the models would calculate speeds first and then obtain the acceleration from these speeds. However, in reality acceleration is what constitutes the motion of the vehicles. He then created a model similar in concept to the GM car following model which depends on acceleration calculations based on a desired distance that will be named the DSG (Desired Space Gap) which depends on the leader's speed as they define the movement of the platoon.

$$DSG_n(t) = g_0 + v_0(t)\delta + \frac{v_0(t)^2}{2b} \times \frac{\alpha}{1-\alpha} + L_{n-1} \rightarrow (\mathbf{2})$$

The formula states that the desired gap to ensure safety for a vehicle n at the time instant t is the sum of four terms.

$\boldsymbol{g_0}$	The minimum safety gap that vehicles should have at rest
$\boldsymbol{v_0}$	velocity of leader
$\boldsymbol{\delta}$	Time increment (latency in communication)
$\boldsymbol{b}$	is the average maximum deceleration
$\boldsymbol{\alpha}$	is the maximum variation in value of b representing the maximum variation of the braking capabilities of the vehicles.
$\boldsymbol{L_{n-1}}$	length of the leading car
Terms in the expression	
$\boldsymbol{v_0(t)\delta}$	This term accounts for the change of gap due to latency in communication (δ). It basically is the distance travelled by the leader ($\boldsymbol{v_0}$) during the latency time. It is one of the conditions needed to ensure safety during emergency braking.
$\boldsymbol{\frac{v_0(t)^2}{2b} \times \frac{\alpha}{1-\alpha}}$	This Term is the second condition for safety during emergency braking and it account for the different braking capabilities of cars

Table 1: Parameters of equation (2)

Figure 7 shows how the evolution of the DSG with the increase of velocity.

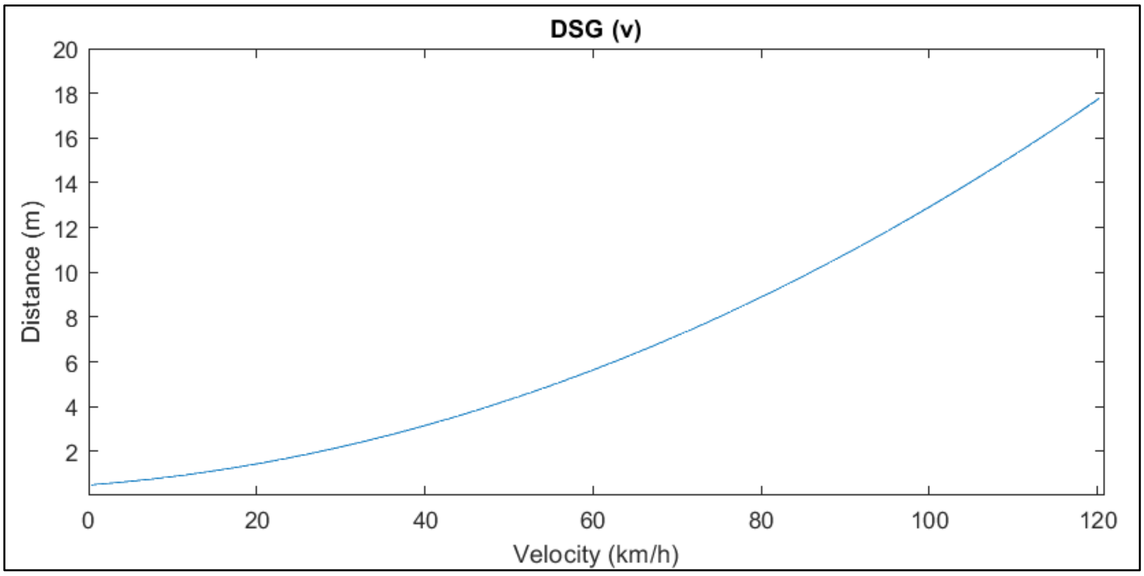

Figure 7: The evolution of DSG with Velocity

Boukhellouf proposed the following acceleration formula.

$$a_n(t+\delta) = a_0(t) + k_1\big(x_{n-1}(t) - x_n(t) - DSG_n(t)\big) + k_2(k_2(v_{n-1}(t) - v_n(t)) \rightarrow \mathbf{(3)}$$

$\boldsymbol{a_n}$	Acceleration of vehicle n in platoon
$\boldsymbol{a_0}$	Acceleration of leader of platoon
$\boldsymbol{v_n}$	Velocity of vehicle n in platoon
$\boldsymbol{x_n}$	Position of vehicle n in platoon
$\boldsymbol{DSG_n}$	Desired Space Gap for vehicle n in platoon
$\boldsymbol{t}$	Time
$\boldsymbol{\delta}$	Time increment (latency in communication)
$\boldsymbol{k_2}$ **and** $\boldsymbol{k_1}$	Calibration parameters.

Table 2: Parameters of equation (3)

Using this formula alongside the algorithm for the platoon creation Boukhellouf managed to create a functional behaviour for the platoon however, for some manoeuvres instabilities phenomena such as the bullwhip occurs which is very undesirable and was the topic of the next study.

The work of (Delate, 2021) tried to tackle the problem of bullwhip in a platoon using a non-dynamic approach of sequential acceleration. The idea is to pre-save manoeuvres in the CAVS so that every follower knows how to react more precisely every follower knows the acceleration pattern they will adapt immediately and not wait for the vehicle in front of it to react. The platoon would then divide itself temporarily to smaller groups and every small group acts like a small platoon and conducts the manoeuvre thus avoiding the bullwhip effect due to the smaller size of the platoon. This is repeated until all vehicles reach the required stable regime. The model was successful in preventing bullwhip effects during acceleration phases however, the results of this method yielded extremely big gaps greater than the DSG and the time of manoeuvres was long ranging from 2-3 minutes making it inefficient. Lastly this model can only be applied to cases in acceleration and cannot be used for braking scenarios.

3. Methodology

The objective of this study is to mitigate or decrease the instability caused by the bullwhip phenomenon in platoons created using models in which the acceleration of the following vehicles is determined to meet a specific DSG. This would be done by first of all by generating a smooth leader trajectory. After the generation of the leader trajectory the trajectories of the followers will be computed using the previously mentioned DSG by implementing that the actual gaps between the vehicles in the platoon are equal to a modified DSG discussed later. If bullwhip effects are detected some solutions will be explored to see their level of effectiveness in eliminating the undesirable phenomenon.

3.1. Generating leader trajectory

To generate the leader trajectory a trapezoidal acceleration-time graph will be studied. There are 4 main inputs to create the acceleration-time graph and they are shown in Table 3.

Δv	The change in velocity caused by the manoeuvre that can be an increase or a decrease in speed. In theory this value represents the area under the curve of the acceleration-time graph.
a_{max}	The maximum value acceleration that is allowed for the vehicles due to mechanical limitations. In theory this value represents the maximum value of the curve in the acceleration-time graph.
J_{max}	the maximum value of Jerk that would abide to safety and comfort regulations. In theory this value represents the slope of the curve at various stages in the acceleration-time graph
t_{acc}	which is the time that the vehicle will be undergoing acceleration until it reaches the desired final velocity.

Table 3: Acceleration-Time graph input parameters

In order to carry out different manoeuvres with different change in velocities the values of the maximum acceleration and jerk had to be adaptable depending on the value of the change in velocity. However, they should still abide the ultimate max values defined by the mechanical properties of vehicles and the comfort of passengers inside the vehicle which are discussed later in section 4.1.2. Therefore, using the simple geometry of the graphs formulas where derived using the physical properties of the graphs such as the area and slope. Given the change of velocity several accelerations are assumed with values ranging from 1 $m \cdot s^{-2}$ to 2.5 $m \cdot s^{-2}$ with increments of 0.5 $m \cdot s^{-2}$. For every acceleration, a value of jerk is computed. The code would then check every pair of acceleration and jerk and compare the value of jerk with max jerk allowed. Once it finds the closest jerk to the max jerk it will adopt this pair of jerk and acceleration as the max jerk and acceleration used in the simulation.

3.1.1. Trapezoidal Acceleration-Time Graph

In this graph the leader will experience the max jerk until the max acceleration is reached then will enter a phase of constant max acceleration with zero jerk and lastly a negative value of max jerk until zero acceleration is reached. It is imposed that these three phases have the exact same time equal to $\frac{t_{acc}}{3}$ to be able to create a symmetric graph that is simple to solve. The change of velocity is equal to the area under the curve that can be divided into two identical triangles and one square.

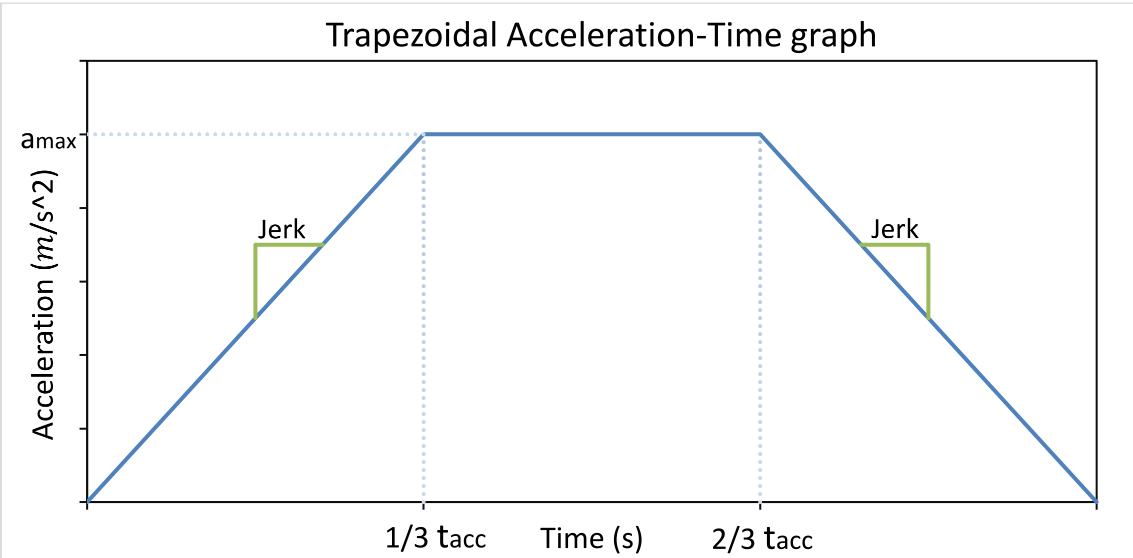

Figure 8: Illustration of Trapezoidal Acceleration-Time graph

$$\Delta v = 2 \times \left(\frac{1}{2} \times \frac{t_{acc}}{3} \times a_{max}\right) + a_{max} \times \frac{t_{acc}}{3} \rightarrow (\mathbf{4})$$

From the expression of the area, we can find the formula for the time of acceleration t_{acc}.

$$t_{acc} = \frac{\Delta v}{a_{max}} \times \frac{3}{2} \rightarrow (\mathbf{4.1})$$

Using the equation of the slope we find the expression of the jerk.

$$J = \frac{3 \times a_{max}}{t_{acc}} \rightarrow (\mathbf{4.2})$$

Now we have the t_{acc} & J in terms of a_{max} as mentioned before we can now compute different values of jerk by substituting values of acceleration from 1 $m.s^{-2}$ to 2.5 $m.s^{-2}$ with increments of 0.5 $m.s^{-2}$ in a_{max}.

3.2. Computing follower's trajectory

After the generation of a smooth leader trajectory the Trajectory of the follower vehicles was also created using the Desired Space Gap formula (DSG) found in (Boukhellouf, 2019).To compute the trajectory of the following vehicles the speed of the follower will be calculated by solving the following equation:

$$DSG(v_n) = gap_n(t + \Delta t) \rightarrow \mathbf{(5)}$$

To define $gap\ (t)$ we will ned to define two other values which are $X(t)\ and\ Y(t)$.

- $X_n(t)$ is defined as the distance between the from bumper of vehicle n and the rear bumper of the leader of the platoon.

$$X_n(t) = Position\ _{leader}(t) - Position\ _n(t) - (n-1) \times L_{car} \rightarrow \mathbf{(6)}$$

$Position\ _n(t)$	Is a variable that represents the distance from the front bumper of the vehicle n at time t from the starting position of the vehicle in the platoon and is measured in meters (m). It will also be used to plot the trajectories of the vehicles in the platoon
L_{car}	Is the length of the vehicle in meters (m)

Table 4: Parameters defining the formula of distance to platoon leader.

- $Y_n(t)$ is defined as the cumulative sum of gaps of all vehicles between vehicle n and the leader.

$$Y_n(t) = X_n(t) - (n-1) \times L_{car} \rightarrow \mathbf{(7)}$$

Finally, now we can define the gap as the following:

$$gap_n\ (t) = Y_n(t) - Y_{n-1}(t)\ \rightarrow \mathbf{(8)}$$

Looking back at equation 5 we define $gap(t + \Delta t)$ as the following:

$$gap_n(t + \Delta t) =\ gap_n(t) + \delta \times (v_{n-1}(t + \Delta t) - v_n(t + \Delta t)) \rightarrow \mathbf{(9)}$$

Where v_{n-1} is the velocity of the leader and v_n is the velocity of the follower.

The idea is that as the leader accelerates the real gap between the leader and follower will change and then the follower will adjust its speed in order to make the real gap equal to the DSG. A minor adjustment to the DSG will be experimented, the DSG formula now will be in terms of the follower's speed and not the leader. Now since both equations of the DSG and the gap are functions of the follower's speed there will exists a unique solution (speed) that will fulfil the condition of $DSG = gap$. In this study we consider the gap to be the distance from the back bumper of the leader to the front bumper of the follower thus L_{n-1} term found in equation 2 can be eliminated from the formula.

$$DSG(t + \Delta t) = g_0 + v_n(t + \Delta t)\delta + \frac{v_n(t + \Delta t)^2}{2b} \times \frac{\alpha}{1 - \alpha}$$

$$gap(t + \Delta t) = gap(t) + \delta(v_0(t + \Delta t) - v_n(t + \Delta t))$$

Since the trajectory of the leader is already known therefore the only unknown in the two previous functions is the velocity of the follower $v_n(t + \Delta t)$. So, the velocity of the follower can be computed such that the condition $DSG = gap$ is satisfied.

$$g_0 + v_n(t + \Delta t)\delta + \frac{v_n(t + \Delta t)^2}{2b} \times \frac{\alpha}{1 - \alpha} = gap(t) + \delta(v_0(t + \Delta t) - v_n(t + \Delta t)) \rightarrow (\mathbf{9})$$

After isolating the velocity of the follower, we are left with the following equation where the only unknown is the follower speed v_n:

$$\left(\frac{1}{2b} \times \frac{\alpha}{1 - \alpha}\right) v_n(t + \Delta t)^2 + (2\delta)v_n(t + \Delta t) - gap(t) - \delta\, v_0(t + \Delta t) + g_0 = 0 \rightarrow (\mathbf{9.1})$$

Let us assume:

$$A = \frac{1}{2b} \times \frac{\alpha}{1 - \alpha} \qquad B = 2\delta \qquad C = gap(t) + \delta\, v_0(t + \Delta t) - g_0$$

So, we end up with the following polynomial:

$$A\, v_n(t + \Delta t)^2 + B\, v_n(t + \Delta t) - C = 0 \rightarrow (\mathbf{9.2})$$

Solving this polynomial gives two possible solutions for the follower's velocity however, only one solution would be close to the values of the speed of both the follower and leader. This is the speed that will be assigned for the follower for the time step calculated.

4. Numerical testing results

To carry out the testing of this proposed model a code was developed using MATLAB. The main code along with all the used functions is available in the appendix of the TFG. The code creates a platoon of a given number of vehicles N with an initial speed v_0 and initial gap DSG_0 computed using the DSG formula for $v = v_0$. The code can conduct both increases and decreases of velocity however, there are some issues that arise in the case of decreasing of velocity “braking” especially for full stop simulations where $v_f = 0$.

A few different configurations with different velocity changes are run and the graphs of the trajectory, speed, acceleration, and jerk and generated. Also, other types of graphs are generated to highlight specific aspects of the model such as the graph that illustrates the satisfaction of the condition. $DSG - gap = 0$ at every time step.

4.1. Defining parameters of the simulations

<table>
<tr><td>N</td><td>The platoon length</td><td>20</td></tr>
<tr><td>L</td><td>The vehicles length</td><td>4.5 m</td></tr>
<tr><td>t</td><td>Time of simulation</td><td>60 s</td></tr>
<tr><td>δ</td><td>Time step equal to the latency of Communications</td><td>0.1 s</td></tr>
<tr><td>v_0</td><td>Initial velocity of platoon</td><td rowspan="6">Changes for every case run</td></tr>
<tr><td>v_f</td><td>Final velocity of platoon</td></tr>
<tr><td>a_{max}</td><td>Max Acceleration</td></tr>
<tr><td>J</td><td>Jerk</td></tr>
<tr><td>DSG_0</td><td>DSG corresponding to initial velocity</td></tr>
<tr><td>DSG_f</td><td>DSG corresponding to final velocity</td></tr>
</table>

Table 5: parameters of simulation

4.1.1. Parameters of the DSG formula

Firstly, some parameters have to be assigned to the constants of the DSG formula. Recalling the DSG formula shown in Equation (5) the values used for the DSG parameters are the same ones recommended by (Boukhellouf, 2019).

$$DSG_n(t) = g_0 + v_0(t)\delta + \frac{v_0(t)^2}{2b} \times \frac{\alpha}{1-\alpha}$$

$\boldsymbol{g_0}$	$0.5\ m$
$\boldsymbol{\delta}$	$0.1\ s$
$\boldsymbol{b}$	$10\ ms^{-2}\ maximum\ deceleration\ (g\ force)$
$\boldsymbol{\alpha}$	20% Maximum possible variation of braking capabilities

Table 6: Values of parameters of DSG formula

4.1.2. Comfort parameters & mechanical limits of vehicles

There should be some restrictions to the values of jerk and acceleration because in reality there are mechanical limits to the vehicles and not all accelerations can be achieved. However, usually the factor of the passenger's comfort inside the vehicle are more restricting than the mechanical capabilities of the vehicles. Passengers are subject to the forces of inertia caused by the accelerations of the vehicles and therefore in this study the maximum jerk accepted will be 0.9 ms^{-3} and the maximum acceleration 2.5 ms^{-2}. Abiding by these values would ensure the comfort of the passengers. These values are the maximum allowed so they are reference values for the simulations but not the exact values used in the simulations. The exact values will be obtained from the method explained in section 3. A table of the values of maximum acceleration and jerks is available in the appendix.

4.2. Case 1: 80 km/h to 120 km/h

In this Case a platoon will perform an acceleration from an initial velocity of 80 km/h to the desired velocity of 120 km/h which is the speed limit of Spanish highways. These values can represent the change of velocity occurring from leaving a conventional road to join a highway. When applying the formula explained before in section (3.1.1) using the change of velocity $\Delta v = 40\ km/h$ the maximum acceleration and jerk for the platoon is calculated and is displayed in Table 7 alongside other parameters. With these parameters the code is run, and the following graphs are obtained.

Parameters	Value	Units
$\boldsymbol{a_{max}}$	2.00	ms^{-2}
$\boldsymbol{J}$	0.72	ms^{-3}
$\boldsymbol{v_0}$	80	km/h
$\boldsymbol{v_f}$	120	km/h
$\boldsymbol{DSG_0}$	8.90	m
$\boldsymbol{DSG_f}$	17.72	m
Time of simulation	60	s

Table 7: Parameters of simulation from 80km/h to 120km/h

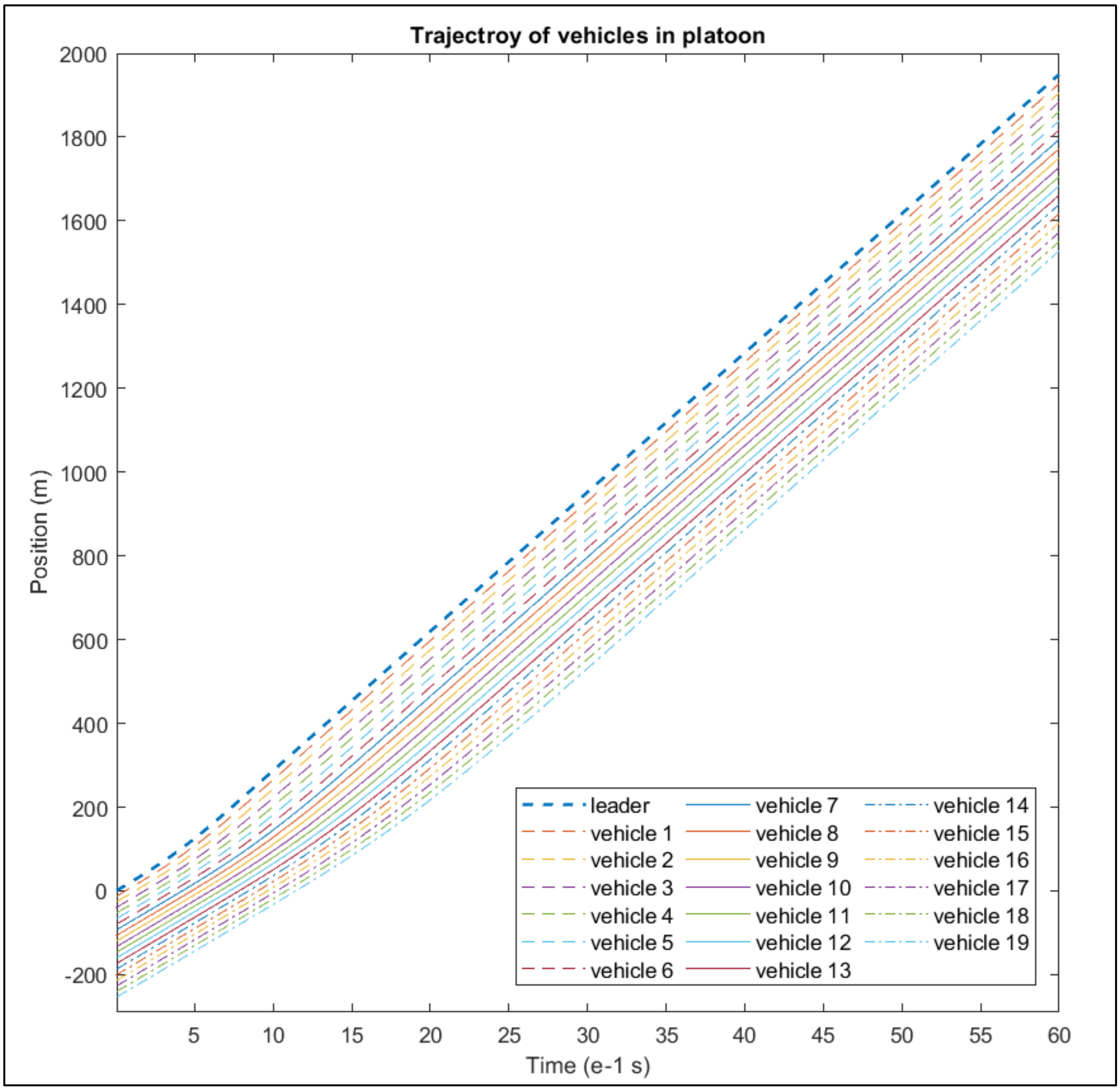

Figure 9: Trajectory of platoon for case of 80km/h to 120km/h

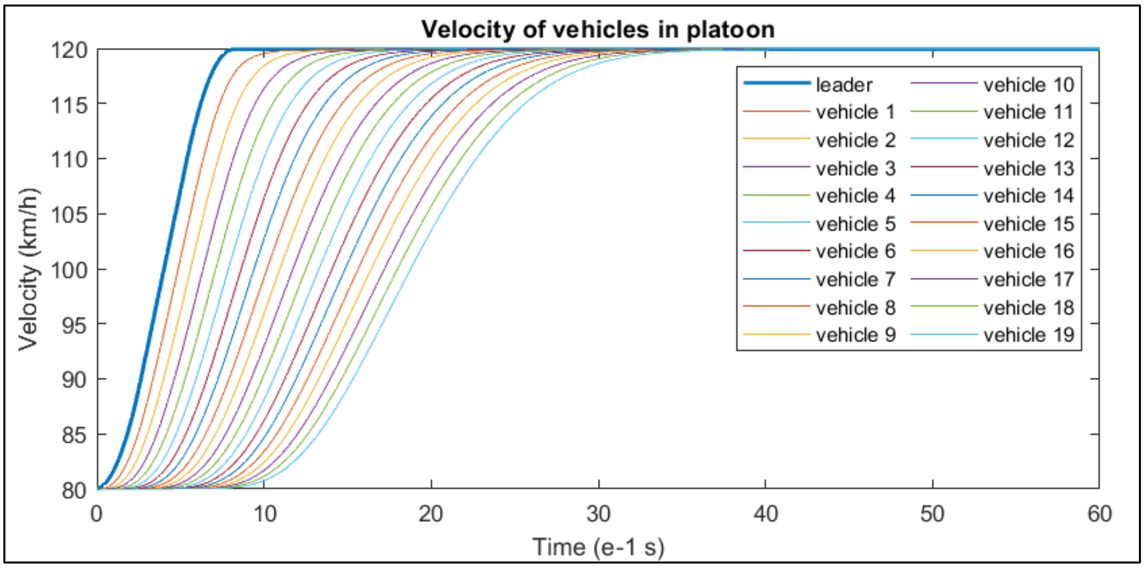

Figure 10: Velocity-Time Graph of platoon for case of 80km/h to 120km/h

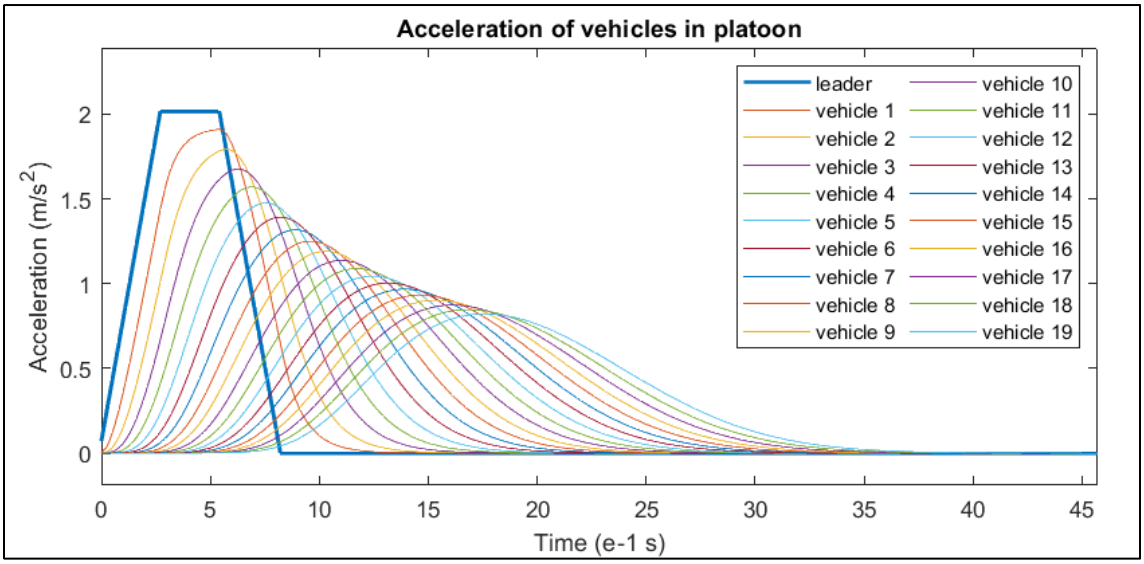

Figure 11: Acceleration-Time Graph of platoon for case of 80km/h to 120km/h

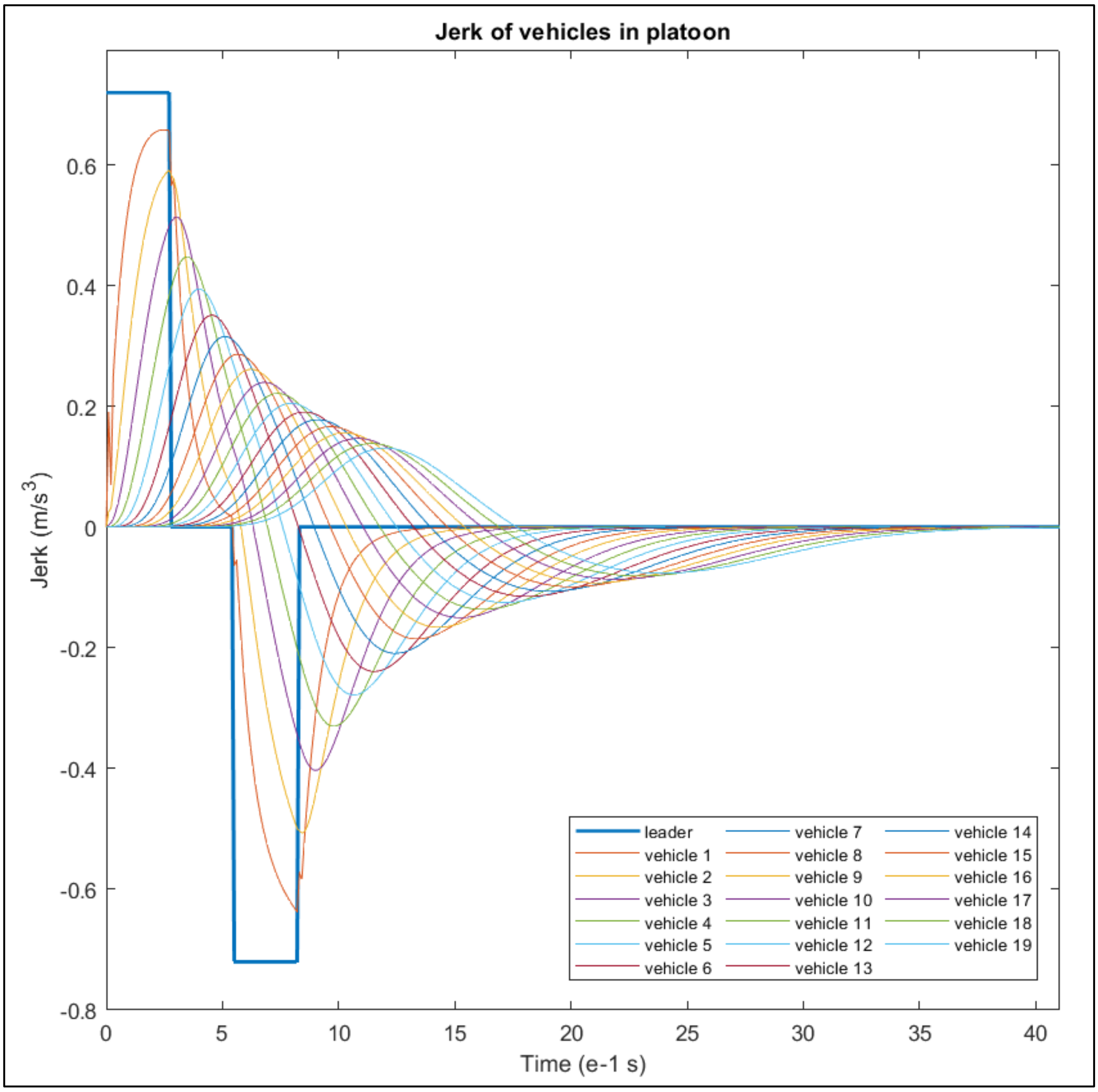

Figure 12: Jerk-Time Graph of platoon for case of 80km/h to 120km/h

From the first look at Figure 10 we can see that all cars successfully increase their velocities from 80 km/h to 120 km/h, we can also observe the same results in Figure 9 from the change of slope of the trajectories of all vehicles in the platoon. Furthermore, by looking at Figure 9 we observe the change of gaps between the trajectory of each vehicle. Figure 13 shows the exact change in values of the gaps of each vehicle in the platoon. We notice that initially the vehicles had smaller gaps between them however they gradually increase to larger values as their velocities increase. This increase is gap is a result of the increase in the DSG that we can see in Figure 14.

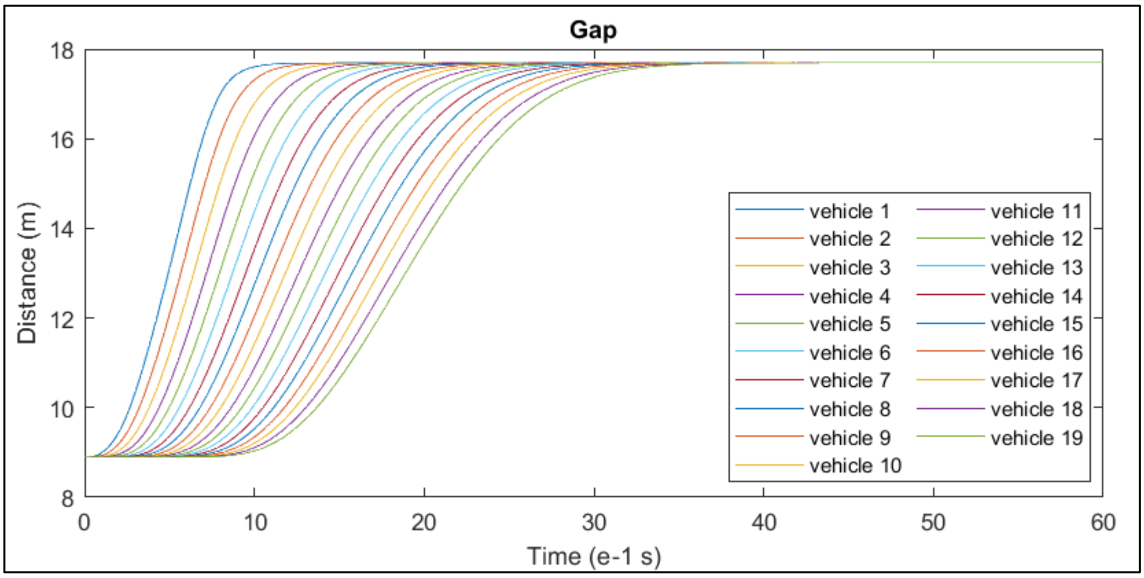

Figure 13: Gap-Time graph of platoon for case of 80km/h to 120km/h

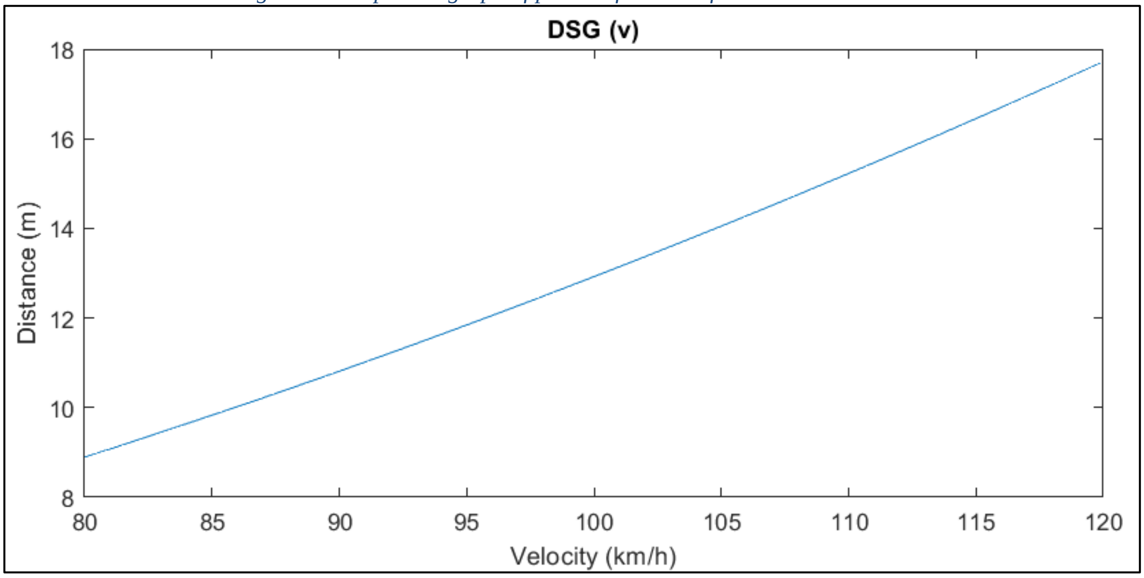

Figure 14: The Desired Space Gap as a function of velocity

When comparing Figure 13 and Figure 14 we notice that all the vehicles have a gap at the final velocity equal to the DSG (120 km/h) equal to 17.69 m which shows the successful implementation of the condition $gap = DSG$.

There seems to be no signs of instabilities such as the bullwhip effect and that is concluded from Figure 11 and Figure 12 as all the following vehicles have values of accelerations and jerks that are lower than their leading vehicles suggesting a smooth increase in velocity down the platoon with no propagation of amplified response to the change of velocity of the leader.

4.3. Case 2: 0 km/h to 120km/h

In this Case a platoon will perform an acceleration from rest to 120 km/h.

Parameters	Value	Units
a_{max}	2.5	ms^{-2}
J	0.375	ms^{-3}
v_0	0	km/h
v_f	120	km/h
DSG_0	0.5	m
DSG_f	17.72	m
Time of simulation	60	s

Table 8: Parameters of simulation from 0km/h to 120km/h

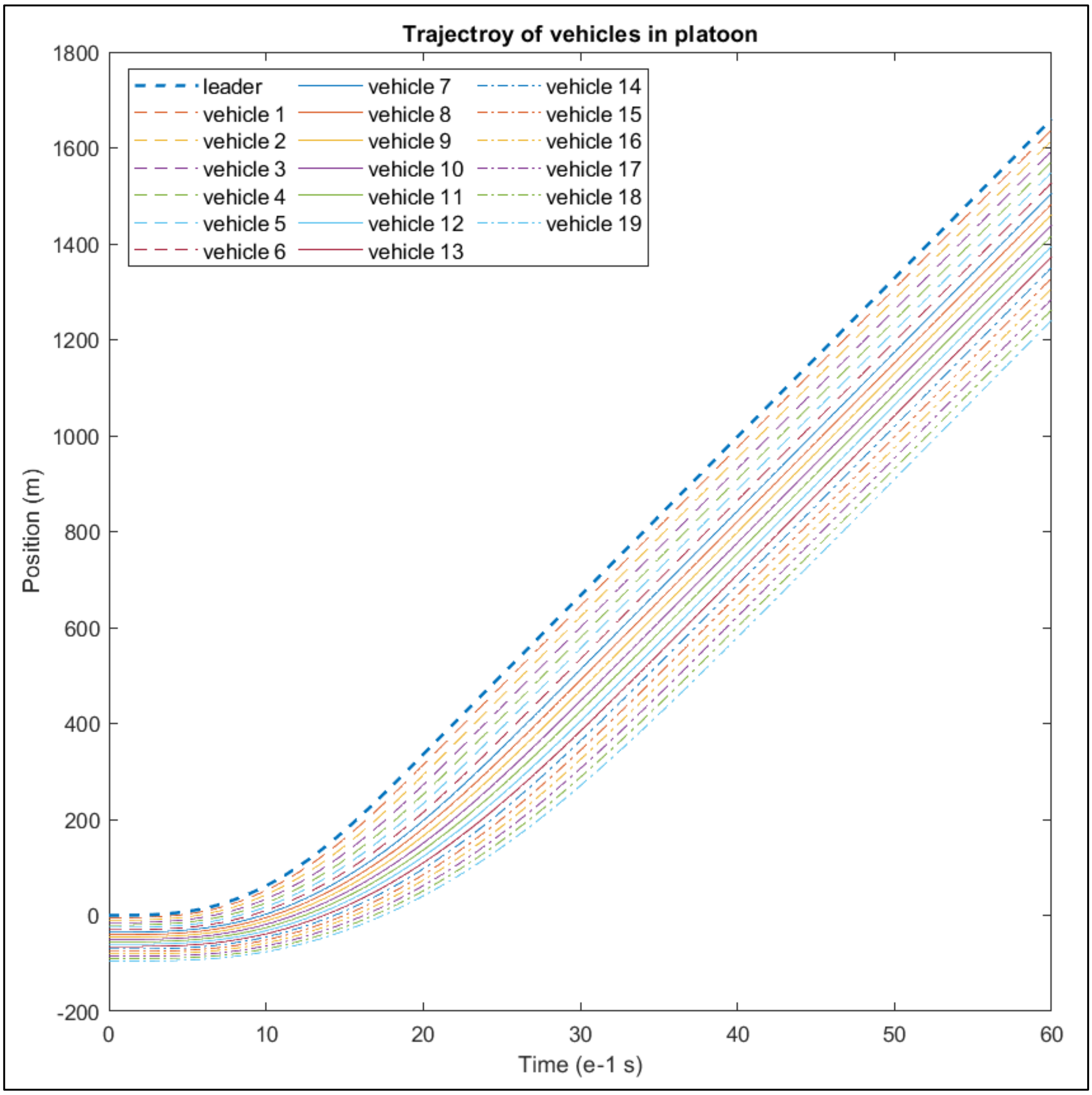

Figure 15: Trajectory of platoon for case of 0km/h to 120km/h

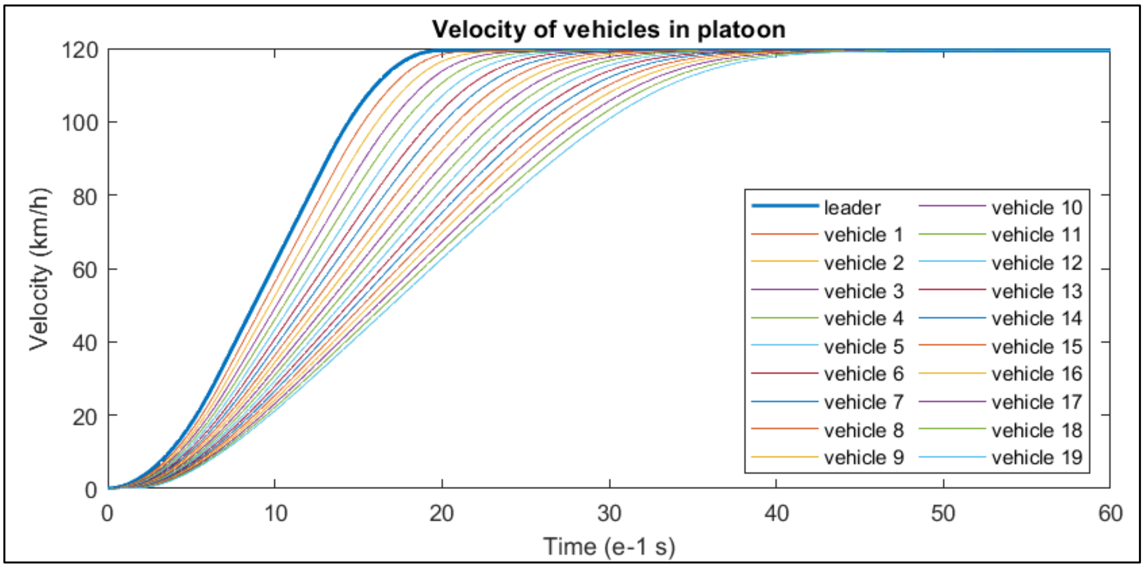

Figure 16: Velocity-Time graph of platoon for case of 0km/h to 120km/h

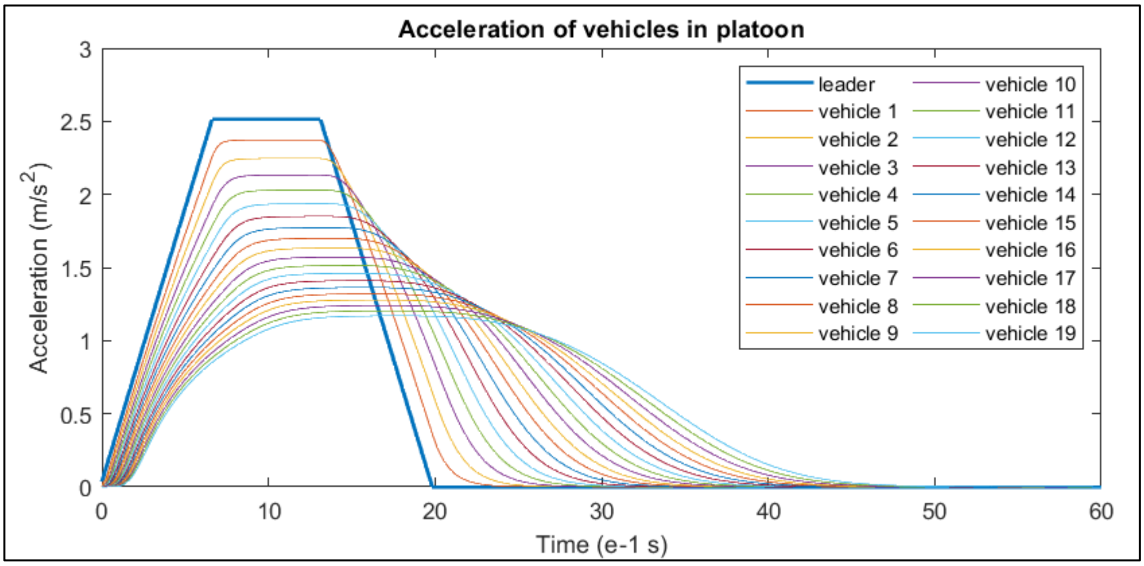

Figure 17: Acceleration-Time graph of platoon for case of 0km/h to 120km/h

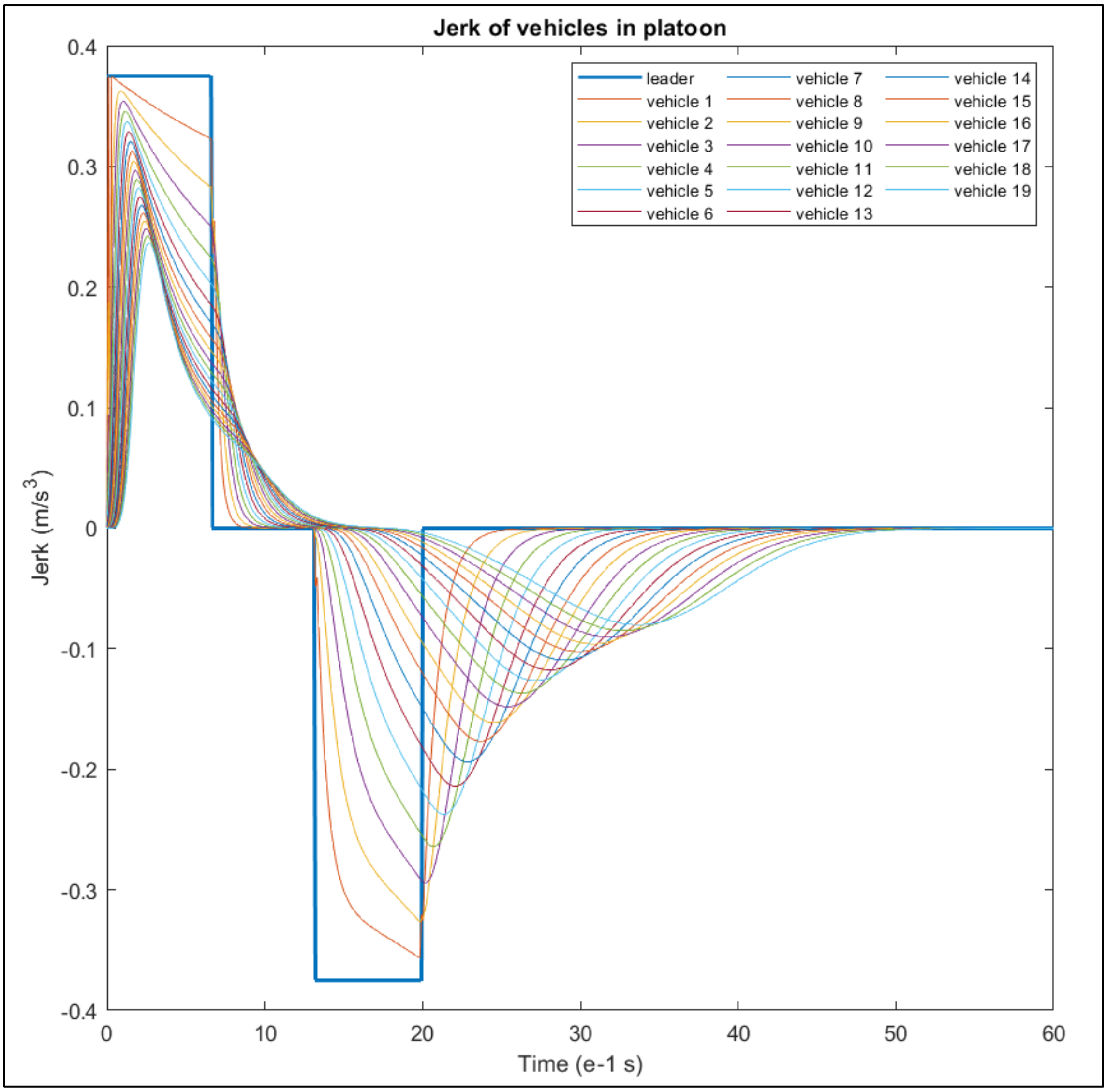

Figure 18: Jerk-Time graph of platoon for case of 0km/h to 120km/h

As we can see that even in the case of great increase in velocity from rest to 120km/h the platoon successfully carries out the manoeuvre reaching the desired final velocity. No signs of instabilities are observed in Figure 17 andFigure 18 as all the followers maintain values of acceleration and jerk lower than of the leader. One slight difference observed is the larger time needed to successfully reach the final velocity which is normal due to the big change of the velocity that needs to be achieved.

4.4. Case 3: 120km/h to 80km/h

This case is considered the reverse of case 1 where a deceleration occurs representing a braking condition or slowing down similar to which a vehicle would encounter to exit a highway. By applying the same steps in the last case, we obtain the following results.

Parameters	Value	Units
a_{max}	2.00	ms^{-2}
J	0.72	ms^{-3}
v_0	120	km/h
v_f	80	km/h
DSG_0	17.72	m
DSG_f	8.90	m
Time of simulation	60	s

Table 9: Parameters of simulation from 120km/h to 80km/h

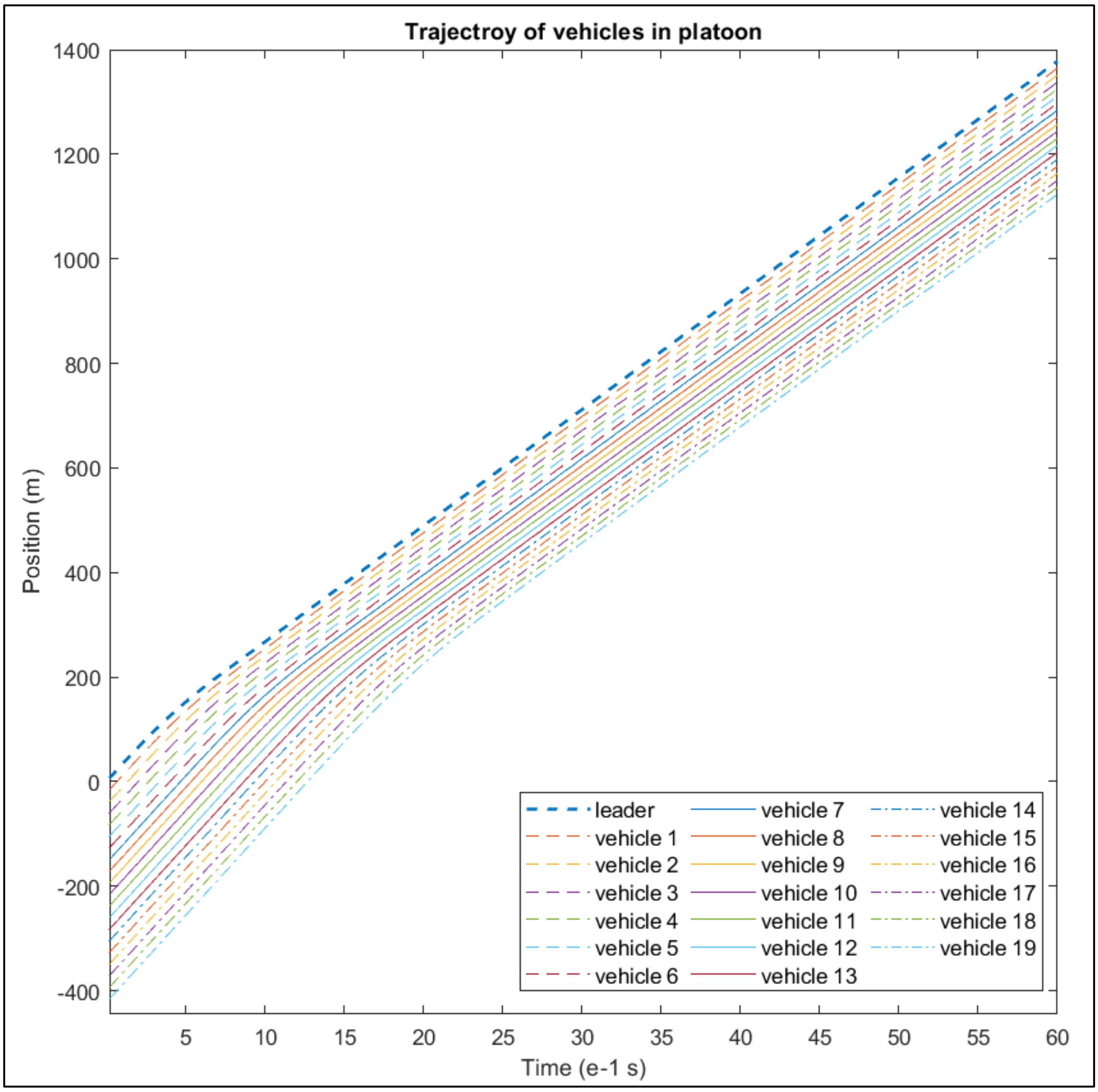

Figure 19: Trajectory of platoon for case of 120km/h to 80km/h

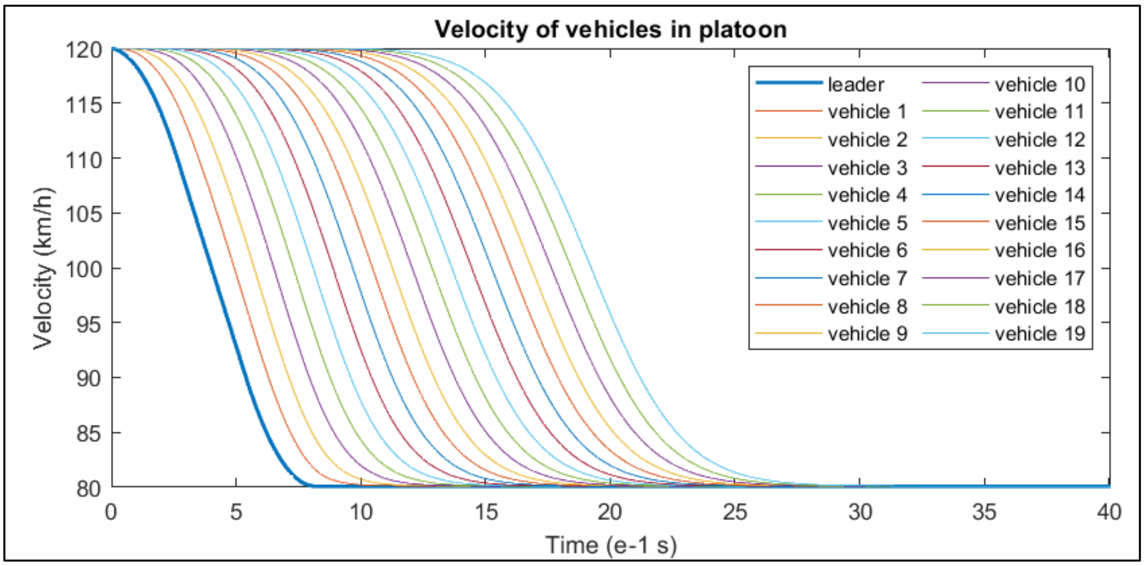

Figure 20: Velocity-Time Graph of platoon for case of 120km/h to 80km/h

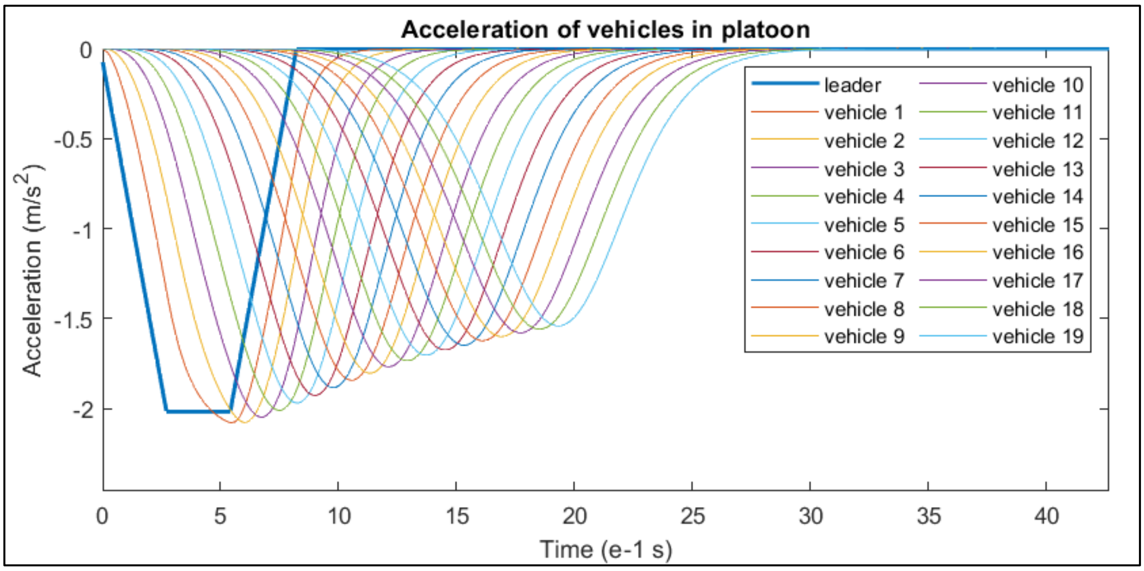

Figure 21: Acceleration-Time Graph of platoon for case of 120km/h to 80km/h

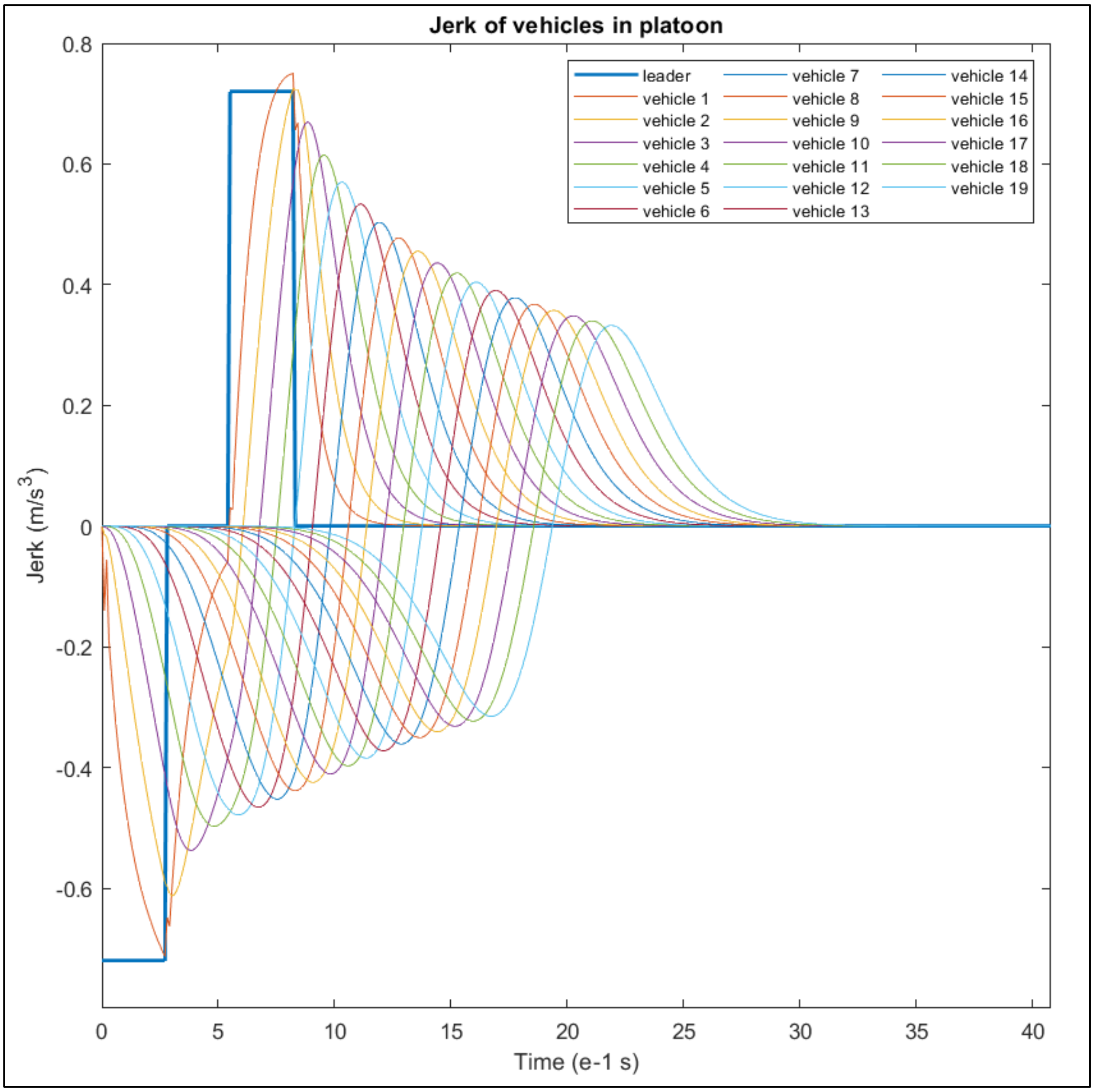

Figure 22:Jerk-Time Graph of platoon for case of 120km/h to 80km/h

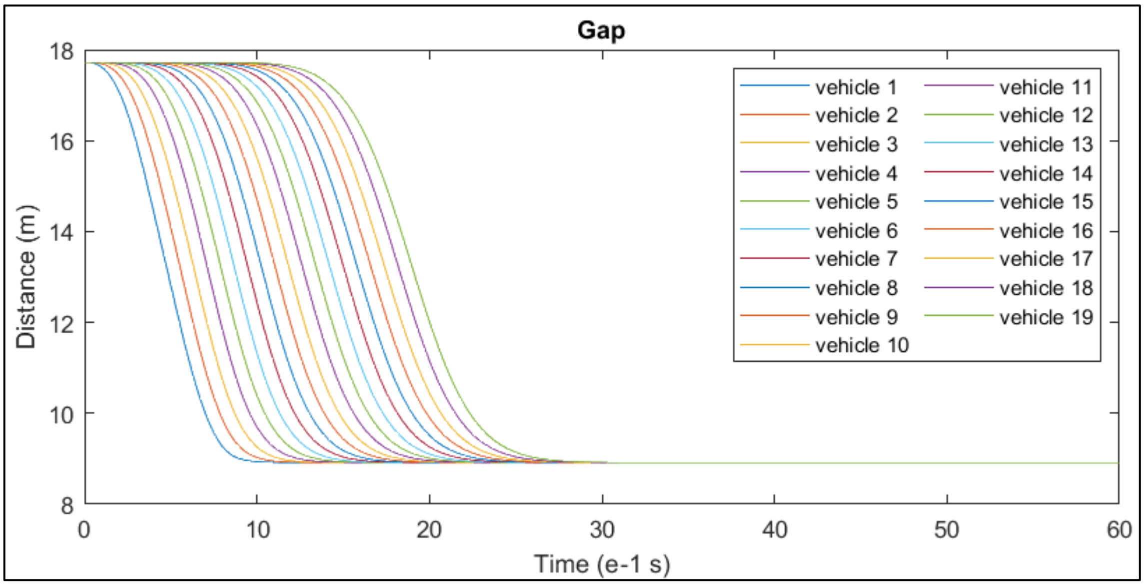

Figure 23: Gap-Time graph of platoon for case of 120km/h to 80km/h

Once again, the platoon is successful in the implementation of the braking manoeuvre as we observe the decrease in velocity in Figure 20 and the change of the slope of the trajectories in Figure 19. Another change we observe in Figure 19 is the decrease of the gaps that is furtherly illustrated in Figure 23. In this scenario as the velocity of the vehicles decrease so does the DSG as shown in Figure 24. The gaps of the platoon reach a value of 8.91 m at the end of the manoeuvre which is equal to the DSG (80 km/h)

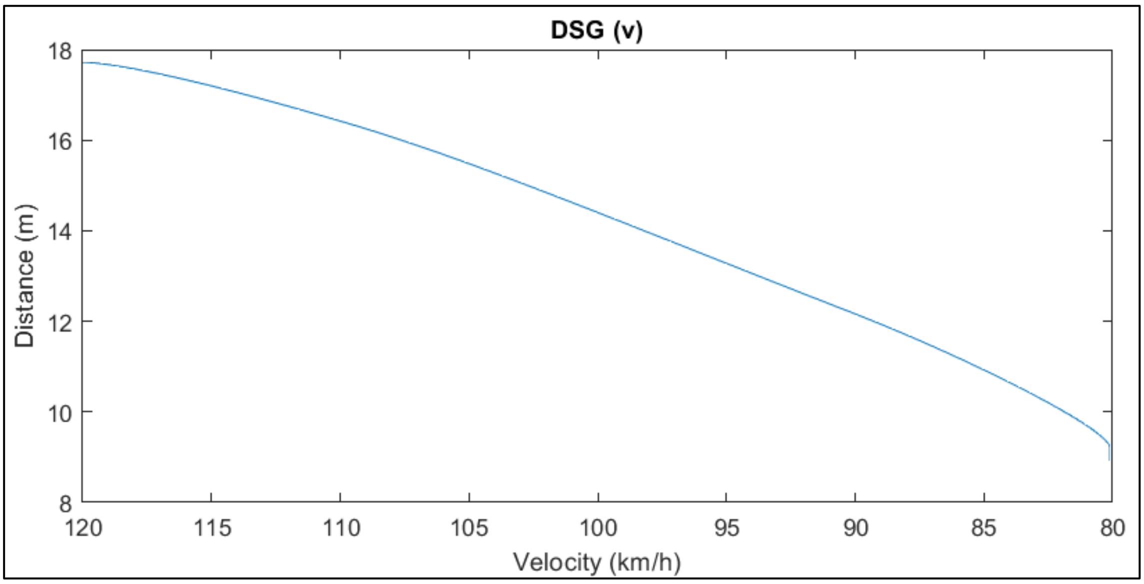

Figure 24: The Desired Space Gap as a function of velocity

However, when looking at Figure 21 andFigure 22 we quickly notice that a few followers have values for jerk and acceleration larger than the leader by a very small value. Despite the difference being very small this shows the existence of a great problem which is the bullwhip phenomenon. Some of the followers are having an aggressive response towards the change of velocity of the leader resulting in them having acceleration and jerk values greater than of the leader. Despite implementing the same absolute change of velocity as in case 1 ($\Delta\ 40\frac{km}{h}$) the braking case exhibits signs of instability.

We also notice that the initial amplified response that is experienced by the few first vehicles tend to dissipate down the platoon unlike the typical behaviour of bullwhip effect where vehicles further back in the platoon have stronger response than the ones in the front. This suggests that the bullwhip effect here seems to be small and will probably increase with further reduction of velocity.

4.5. Case 4: 120km/h to 0km/h

The idea behind this case to see the behaviour of the model in extreme conditions such as an accident on the road that would force the platoon to come to a full stop.

Parameters	Value	Units
a_{max}	2.5	ms^{-2}
J	0.37	ms^{-3}
v_0	120	km/h
v_f	0	km/h
DSG_0	17.72	m
DSG_f	0.5	m
Time of simulation	60	s

Table 10: Parameters of simulation from 120km/h to 0km/h

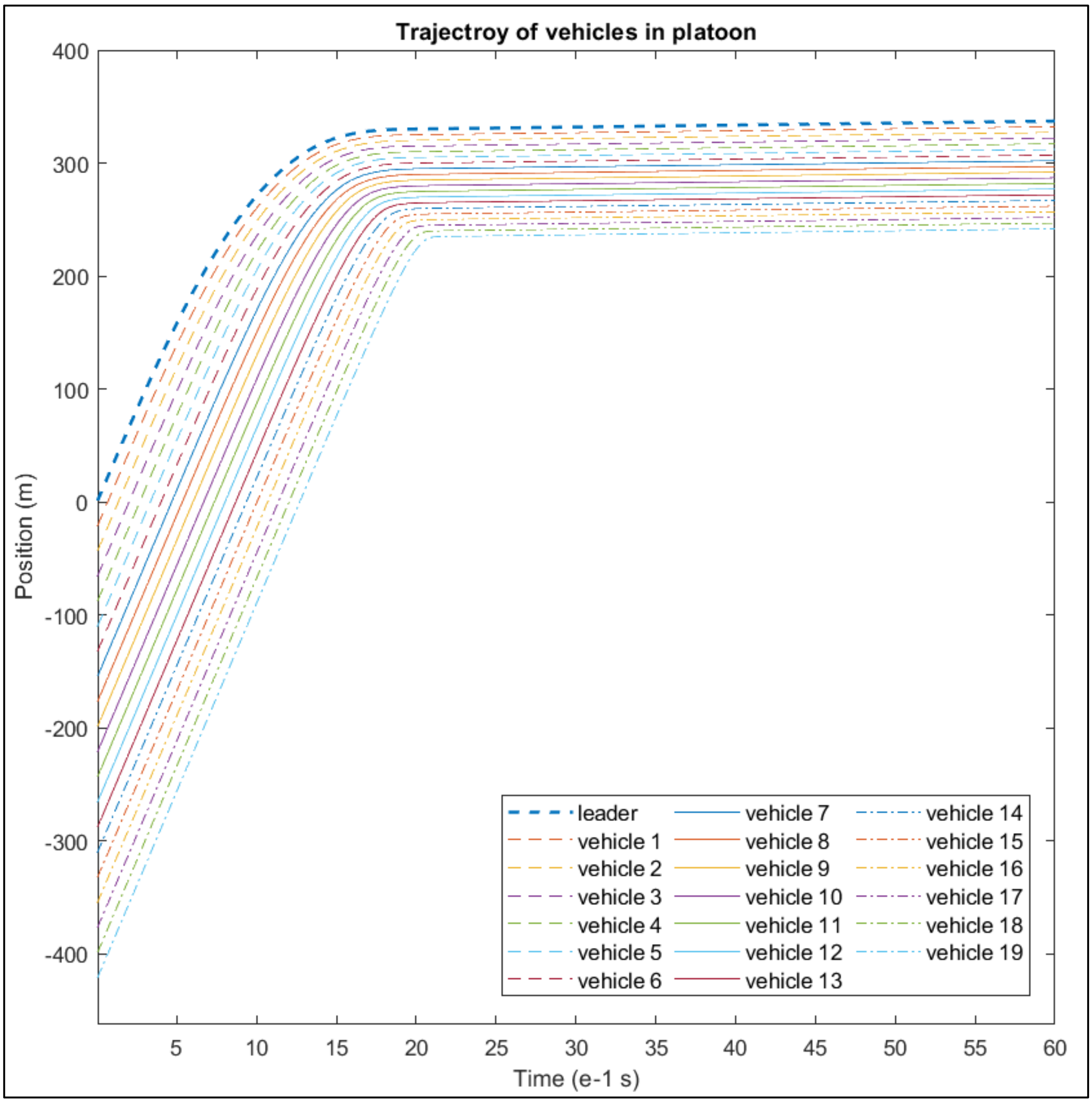

Figure 25: Trajectory of platoon for case of 120km/h to 0km/h

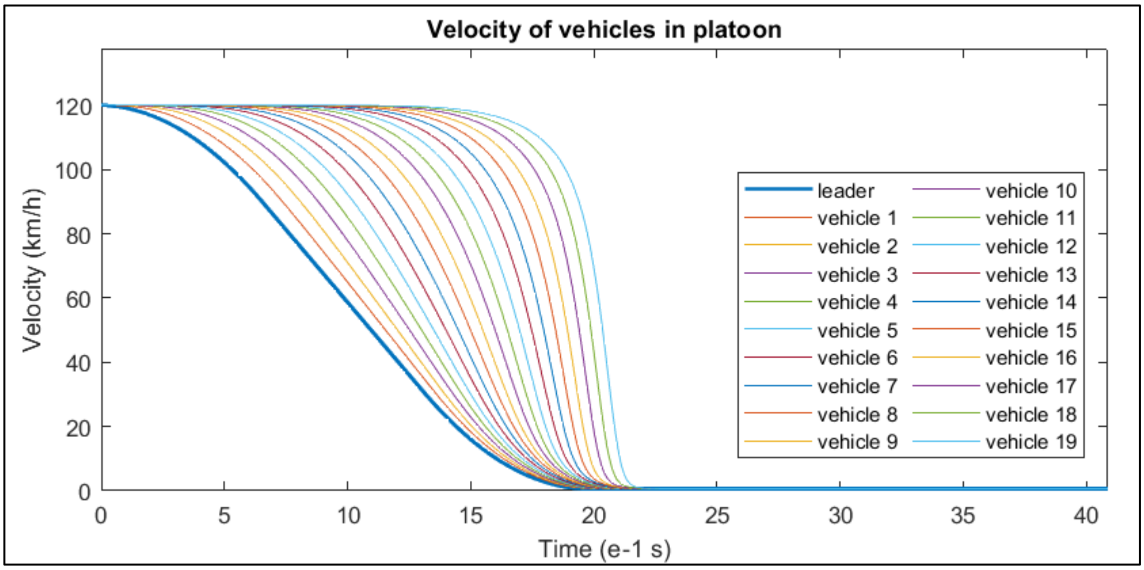

Figure 26: Velocity-Time Graph of platoon for case of 120km/h to 0km/h

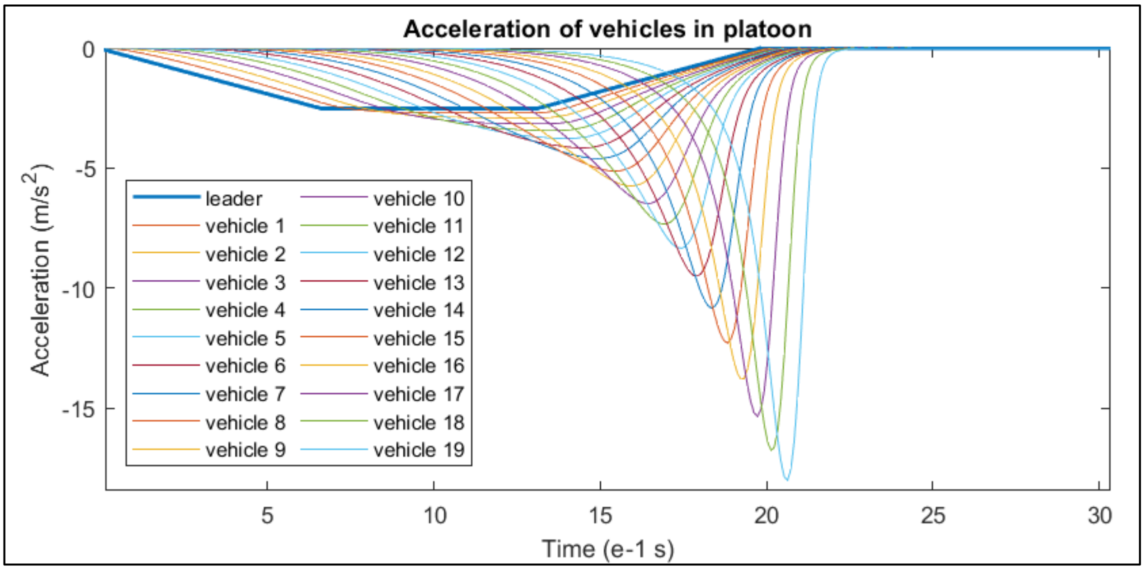

Figure 27: Acceleration-Time Graph of platoon for case of 120km/h to 0km/h

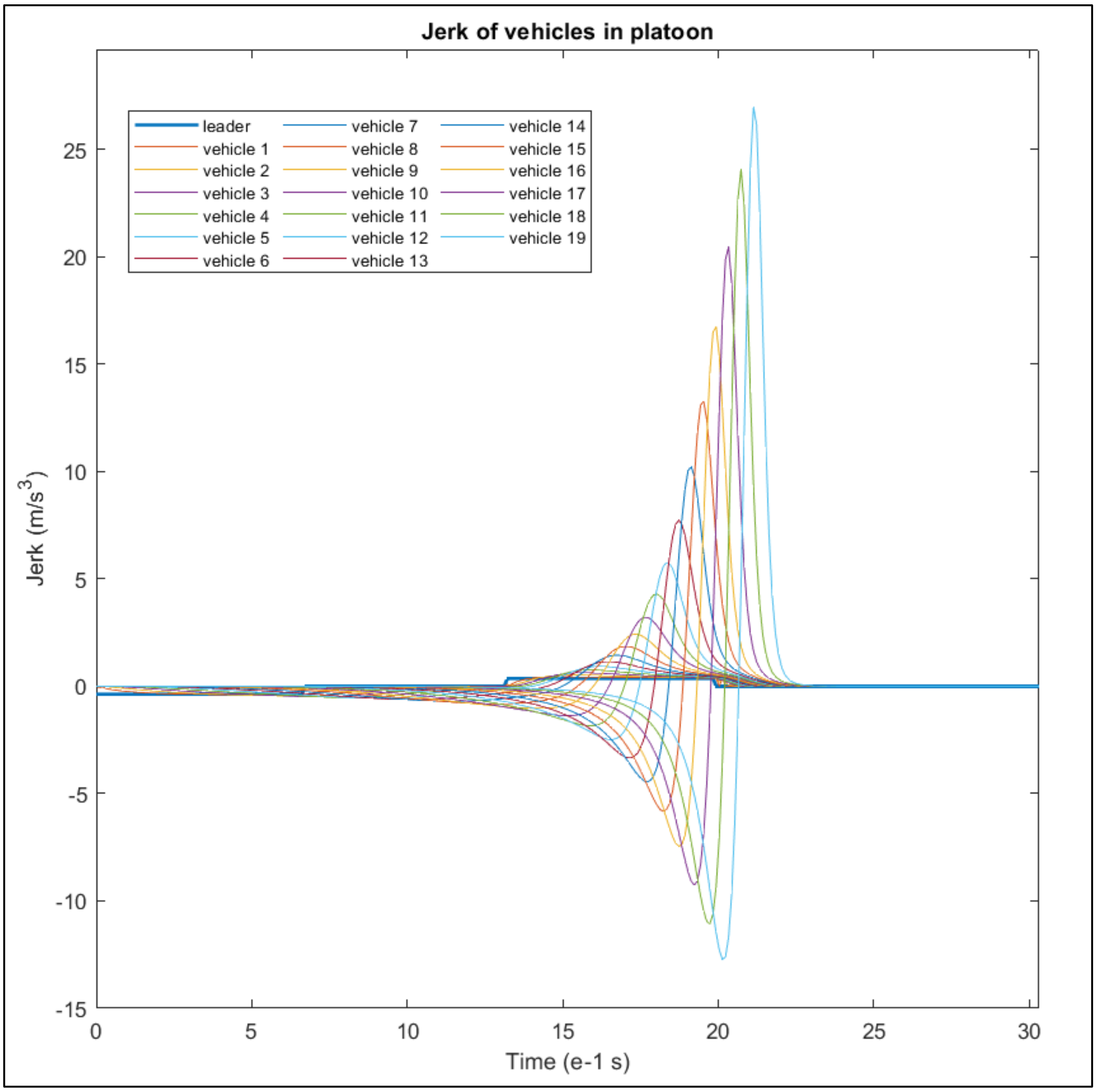

Figure 28: Jerk-Time Graph of platoon for case of 120km/h to 0km/h

It is immediately noticed that the bullwhip behaviour observed in the previous braking conditions has been extremely amplified in the case of the full-stop to values that are not even mechanically achievable by any vehicles on the market. All of the following vehicles in the platoon end up having values of jerk and maximum acceleration higher than that of the leader. Unlike the previous case this time all of the vehicles in the platoon responded aggressively to the change of the velocity and that response is clearly amplified down the platoon as we observe vehicles further down the platoon have huge values of acceleration and jerk compared to the ones in the front. This kind of behaviour shows that the model is extremely unstable in braking situations. Therefore, it is necessary to introduce some changes to eliminate this behaviour.

4.6. Solving the braking instability

In this model under ideal conditions the intervehicle gap is always equal to the DSG. Under these conditions as the change in velocity increases it reaches a value great enough that each vehicle in the platoon will need to brake harder than the one ahead of it. This constitutes the bullwhip effect. The problem that needs to be tackled is that as the DSG is reduced when braking is applied the followers approach the leader at high speed. This causes the followers to brake very strongly as they approach the leader.

The solutions suggested involve gradually reducing the speeds at which vehicles approach the leader by triggering an instant response for all the vehicles in the platoon as soon as the leader starts changing its speed rather than followers only react to the change in the velocity of only the vehicle ahead of them. This could be achieved using the term of cumulative gap defined previously in equation **(7)**.

$$Y_n(t) = X_n(t) - (n-1) \times L_{car}$$

Figure 29 shows the cumulative gaps of every vehicle in the platoon studied in case 3. The value of the cumulative gap for vehicle n represents the sum of all gaps ahead of vehicle n in the platoon. In other words, it represents the free space that is not occupied by vehicles between vehicle n and the leader and that is the space that the vehicle has available to perform the braking manoeuvre.

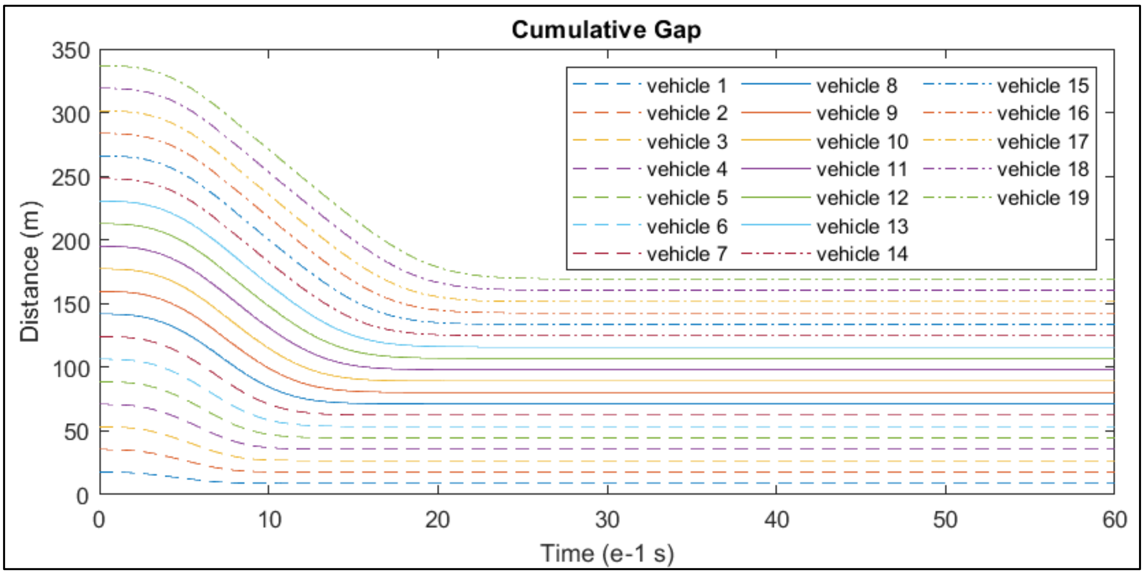

Figure 29: The sum of cumulative gaps for vehicles in case 3

At braking followers at the end of the platoon are travelling with high speeds maintaining a suitable DSG with the vehicle ahead of them in the platoon, but as they approach the leader these speeds are too fast and consequently, their response becomes really aggressive in order to reach safety distances. Therefore, we need to take into consideration that vehicles in the front of the platoon are travelling at slower speeds with already smaller gaps.

So, for a vehicle n in braking conditions the following happens. When a vehicle is travelling at a constant speed in a platoon it can be assumed that the sum of all the free space ahead of the vehicle is $n \times DSG_n$. However, in reality the free space ahead of vehicle n is smaller than that because while vehicle n is still traveling at the initial speed of the platoon vehicles ahead of it have already started decelerating and thus decreasing their DSG and the actual free space ahead of vehicle n is $\sum_{k=1}^{n} DSG_k$. Figure 30 shows the actual values of the free space ahead of vehicle 20 in the platoon studied in case 3.

$$n \times DSG_n > \sum_{k=1}^{n} DSG_k$$

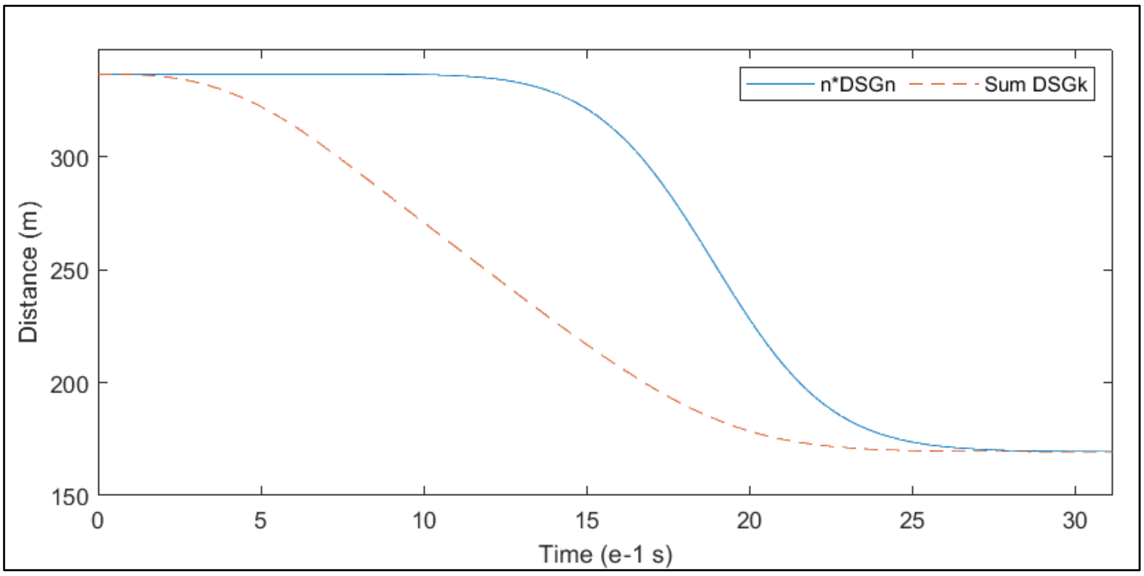

*Figure 30: Comparison between n*DSGn and Sum DSGk for vehicle n=20 in the platoon of case 3*

To force vehicles further back in the platoon to react to the stimulus instantly and avoid having a delayed response caused by only adapting their velocities once the vehicle ahead of them does, we will adapt the formula used (Equation 5) and change the gap used in that equation by assuming that the available gap ahead of vehicle n is the minimum between its current gap & the average cumulative gap in front of the follower until the leader.

$$v_n\ st.\left(gap_n; \frac{Y_n}{n}\right)$$

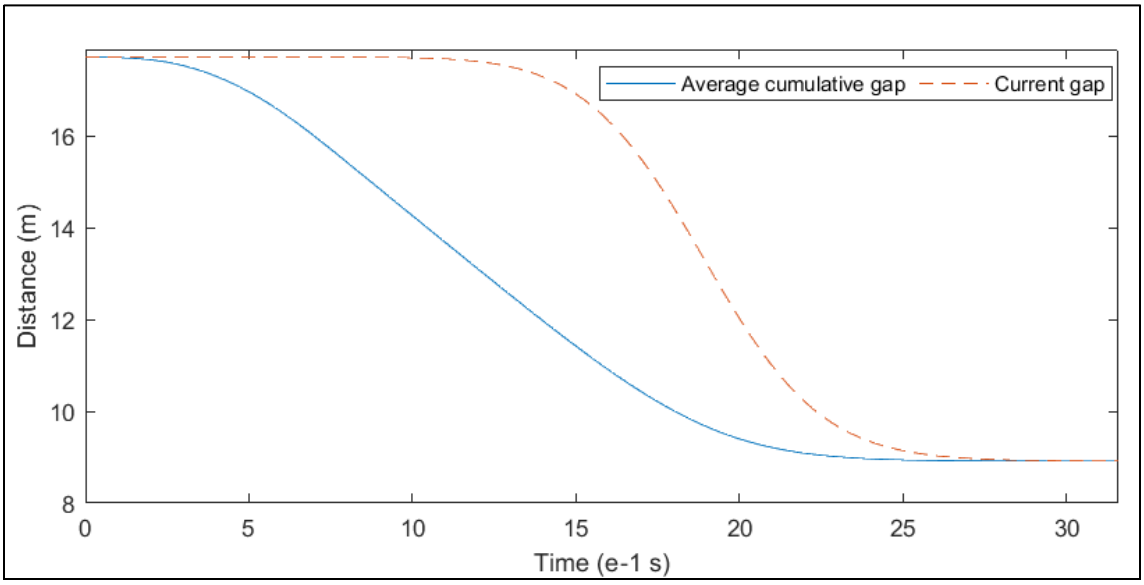

Figure 31: Average Cumulative Gap vs Current gap of vehicle 20 in platoon of case 3

As we can see in Figure 31 the average cumulative gap is always smaller than the current gap of the last vehicle in the platoon in case 3. Using the average cumulative gap instead of the current gap in Equation 5 means that the moment the leader starts decelerating the last vehicle will also decelerate because the average cumulative gap decreases instantly the moment the deceleration starts. In Equation 9.2 the smaller the gap that we introduce to the equation the less aggressive response the follower will have as it approaches the leader.

4.6.1. Case 5: 120km/h to 0 km/h with braking instability solution

Parameters	Value	Units
a_{max}	2.5	ms^{-2}
J	0.375	ms^{-3}
v_0	120	km/h
v_f	0	km/h
DSG_0	17.72	m
DSG_f	0.5	m
Time of simulation	60	s

Table 11: Parameters of simulation from 120km/h to 0km/h with braking solution

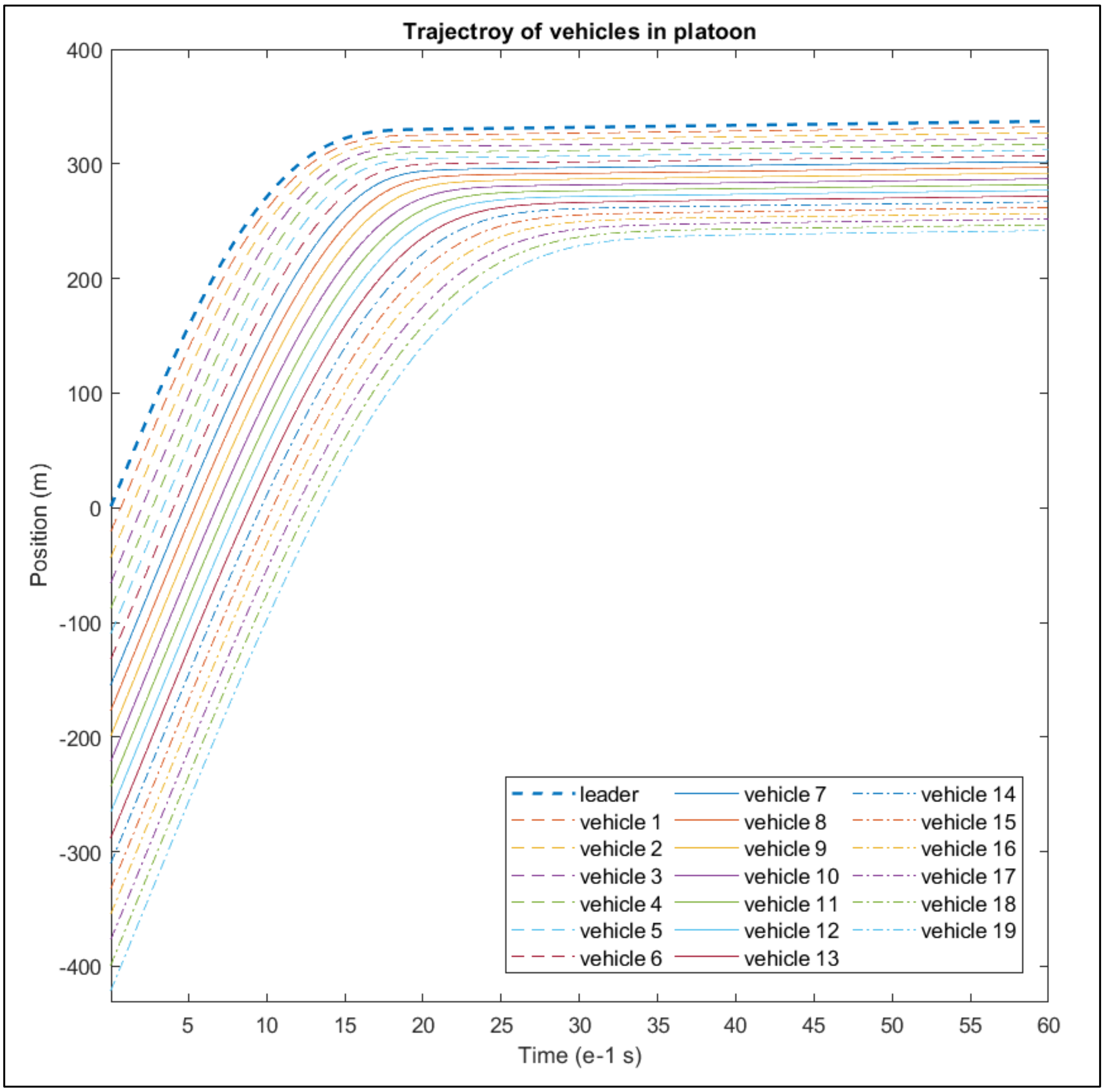

Figure 32:Trajectory of platoon for case of 120km/h to 0km/h with braking solution

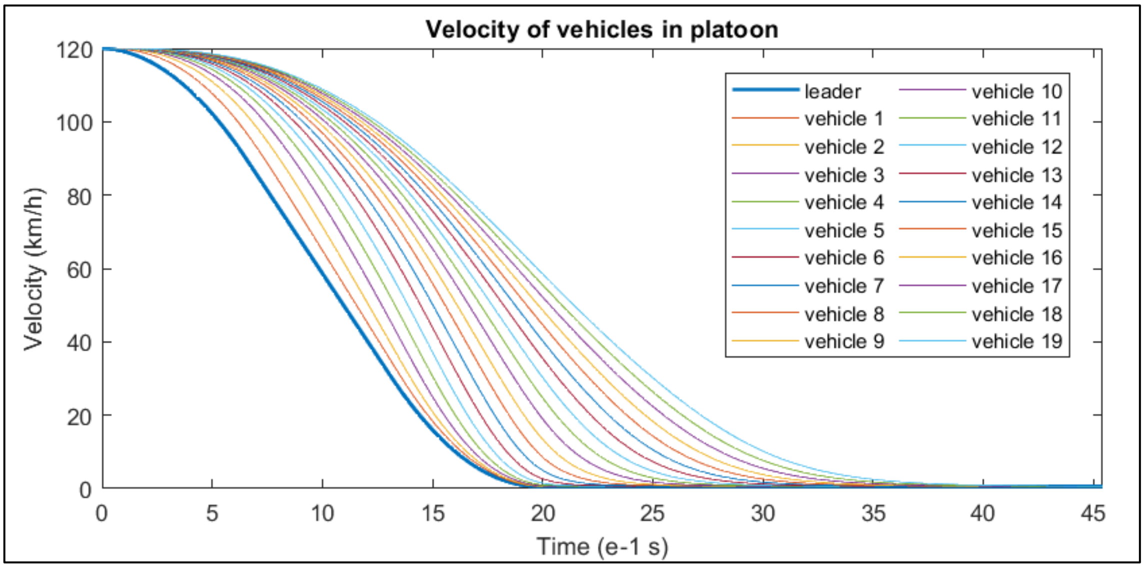

Figure 33: Velocity-Time Graph of platoon for case of 120km/h to 0km/h with braking solution

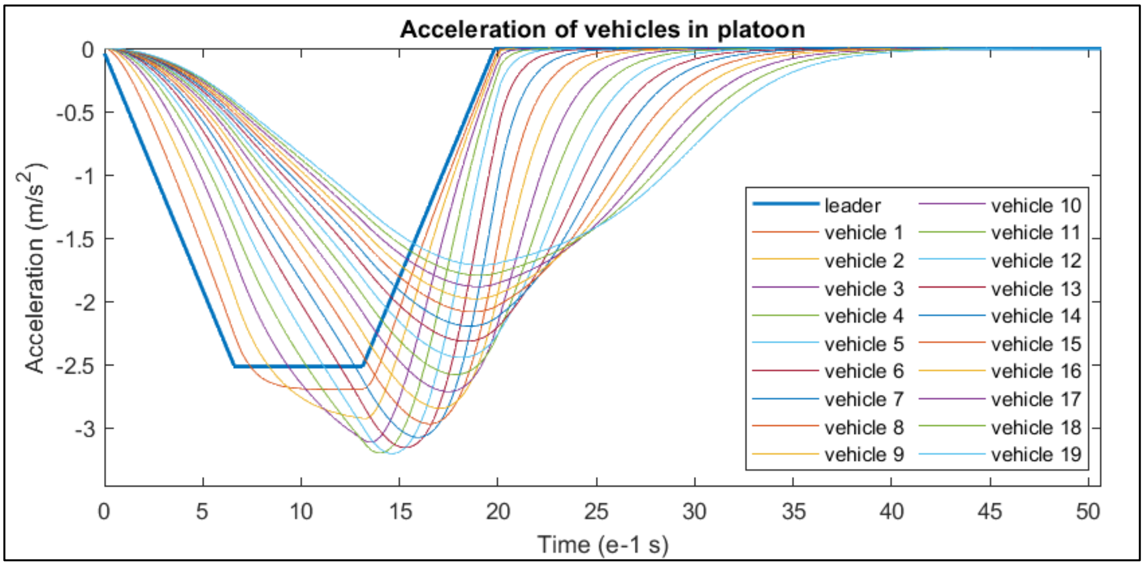

Figure 34: Acceleration-Time Graph of platoon for case of 120km/h to 0km/h with braking solution

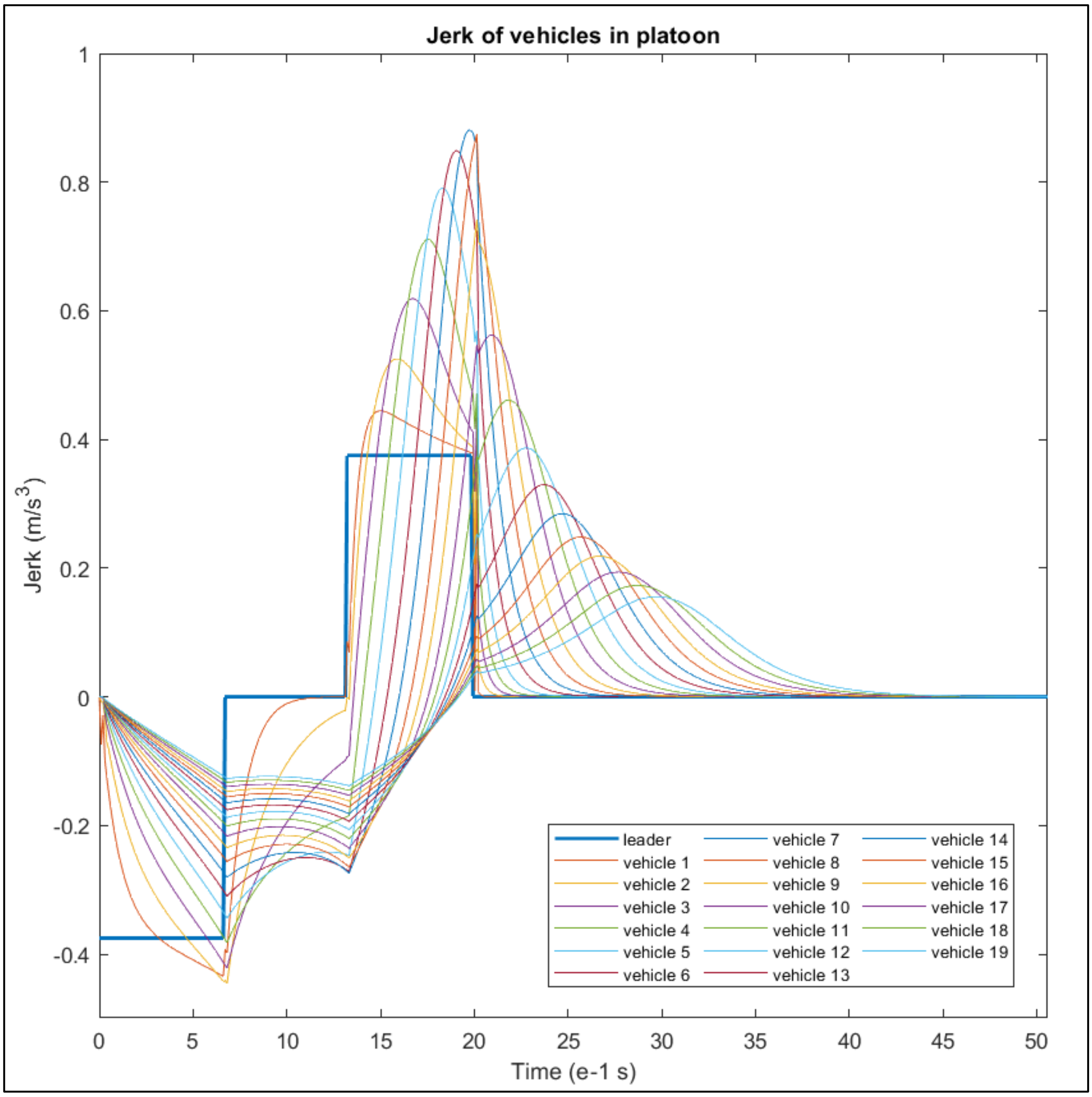

Figure 35: Jerk-Time Graph of platoon for case of 120km/h to 0km/h with braking solution

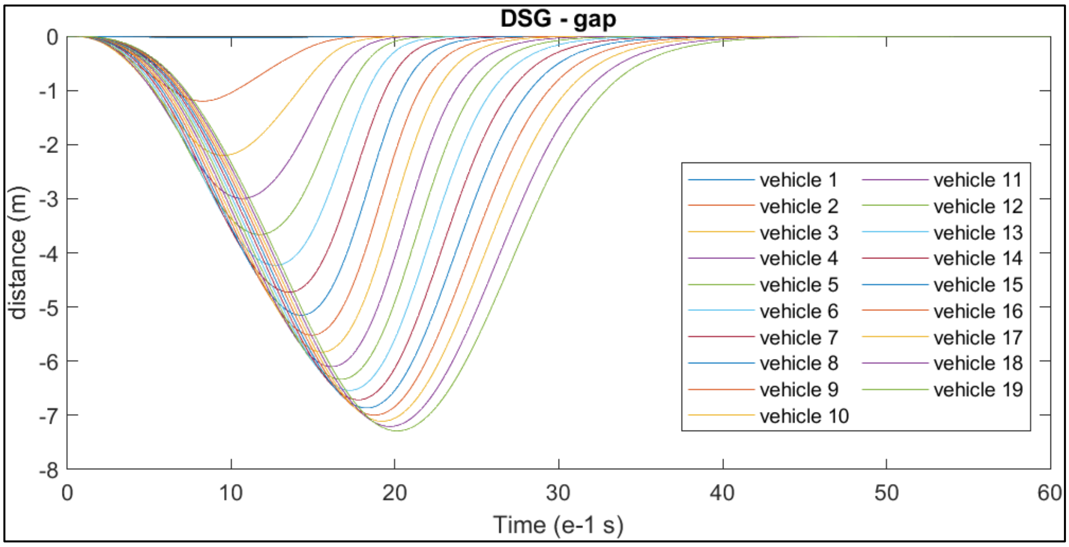

Figure 36: DSG-Real Gap Graph of platoon for case of 120km/h to 0 km/h with braking solution

Several changes are observed in all the graphs of this simulation after implementing the braking instability solution. The most important one to focus on would be Figure 34 and Figure 35 which are the acceleration and jerk graphs, respectively. An extremely noticeable change occurs to the graphs as we observe a huge reduction in the bullwhip effect when compared to Figure 27 and Figure 28. The new braking model significantly reduced the bullwhip effect however, it was not able to eliminate the problem and the platoon still experiences a bullwhip effect. We also observe that the vehicles farther back in the platoons are the ones who had the most reduced bullwhip effects and have now values of acceleration and jerk lower than of the leader however, vehicles in the middle of the platoon still have an aggressive response to the leader's change in velocity and their values of acceleration and jerk exceed that of the leader.

Since now we are computing the followers velocity using Equation 9.2 with the average cumulative gap and not the current gap of the vehicle we can justify that the vehicles in the back of the platoon have more reduced bullwhip effects than the ones in front of them due to the fact that the difference between the average accumulative gap and a vehicles real gap decreases as you get closer to the leader. For example, if we look at Figure 37 that shows vehicle number 10 in the platoon studied in case 3 and compare it to Figure 31 which is the same graph but for vehicle 20 (the last vehicle in the platoon) we notice that the difference between the current gap of the vehicle and its average cumulative gap is lower in vehicle number 10 than vehicle number 20.

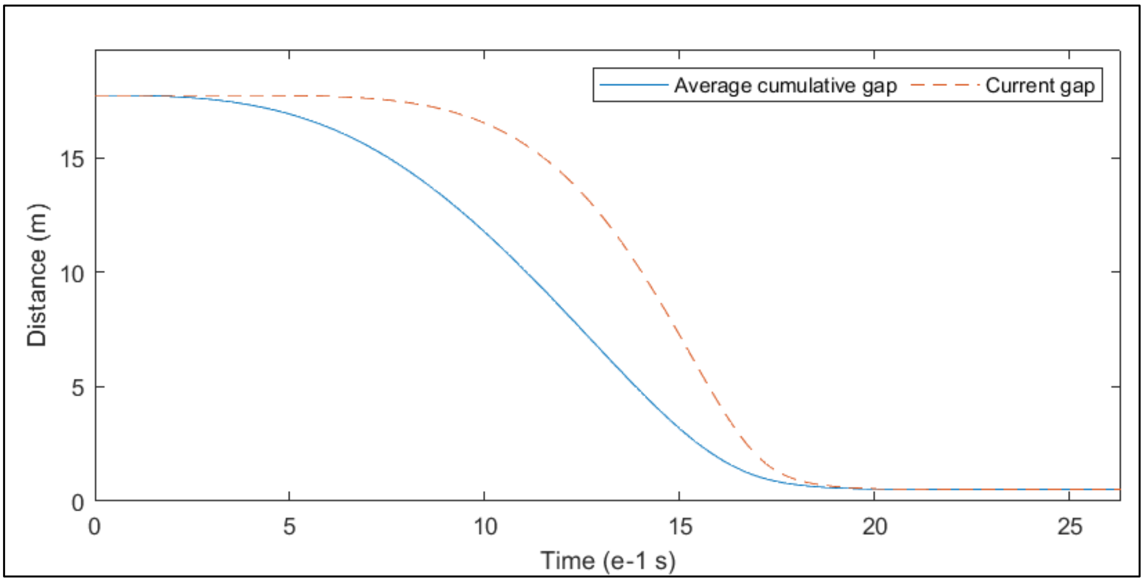

Figure 37: Average Cumulative Gap vs Current gap of vehicle 10 in platoon of case 3

The new braking model reaches values of maximum acceleration less than 3.5 $m.s^{-2}$ in the case of full stop from an initial velocity of 120 km/h. These new values are much more sensible and achievable in real life compared to the unrealistic values achieved earlier up to 18 $m.s^{-2}$. The new braking model shows a reduction in the maximum value of around 80%. A similar effect to the new braking method is observed with the Jerk-Time graph in Figure 35 as we notice a reduction in the maximum jerk by around 97% from a value of $29\ m.s^{-3}\ to\ 0.8\ m.s^{-3}$.

As a result, to this new braking method, we also observe that the time needed to execute the full stop manoeuvre increase from around $23s\ to\ 40s$. This increase in time can be interpreted from differences between the Velocity-Time graphs shown in Figure 26 and Figure 33. When the new braking model was adopted, we notice that the whole platoon starts decreasing their velocities instantly with the leader as shown in Figure 33. While in the old model shown in Figure 26 the vehicles further down the platoon take longer time to respond to the change of the velocity of the platoon leader.

Lastly one last important change is observed in Figure 36. Due to the changing of the gap used to compute the follower's velocity to a value smaller than the actual one the condition $DSG = gap$ is no longer satisfied. The graph shows negative values up to almost $-8m$. This means that during the transitional phase of the platoon the condition $DSG = gap$ is not satisfied however, at the end of the manoeuvre the condition is satisfied, and the vehicles return to having the safety distances stated by the DSG formula. This does not cause a safety issue as it actually acts as an extra safety factor however, there is a trade-off between safety and efficiency as larger gaps in the platoon means less efficient use of space on the highway.

5. Analysis of the results and Conclusions

In view of the results shown in the numerical testing section it is possible to evaluate the performance of the model and identify its problems and limitations. First of all, when examining the functionality of the model it has proven to be successful in carrying out the requested manoeuvres. For any given initial and final velocity, the platoon can successfully reach the required final velocity and its designated DSG respecting the condition of $DSG = gap$. The model also ensures that no vehicles inside the platoon crash into each other in both cases of acceleration and deceleration. The $DSG = gap$ model has proven to be successful in the case of accelerating showing no signs of instabilities such as the bullwhip effect for any given change in velocity as shown in cases 1 and 2. Further simulation results of other changes in velocity can be found in annex (2). It successfully generated smooth trajectories for platoons of 20 vehicles with an increase of velocity of $40 km/h$ in around 35 s and for the increase of velocity of $120 km/h$ in around 50 s.

When it comes to braking manoeuvres, the model had negative results as it proved to be extremely unstable. Annex 1 contains the acceleration-time graphs of vehicles in a platoon for several other braking cases from a Δv of 50 km/h to 110 km/h . by observing the behaviour of followers as Δv increase we observe that the model seems to suffer from bullwhip effects that increase with the magnitude of Δv. The main problem was that the model forces the vehicles to adapt their velocities depending only on the vehicle ahead of them in the platoon. This resulted in scenarios where the leader would be close to reaching the final velocity while the last vehicle in the platoon would still be travelling at the initial velocity thus approaching the leader at extremely high speed that causes a huge mechanically impossible reaction. However, in cases of change of velocity smaller than or equal to 30 km/h the model successfully carries out the manoeuvre without the need of the braking instability solution proposed as it shows no signs of bullwhip effects. Figure 38,Figure 39 andFigure 40 in annex 1 exhibit no signs of the bullwhip phenomenon as all the followers have values of acceleration lower than the leader.

The proposed solution to eliminating the bullwhip effect was successful in reducing the effect greatly, however, was not able to eliminate the problem completely. Furthermore, this solution would only reduce the bullwhip effect with great changes of velocity only and would have very small effects on smaller braking scenarios. This is because for larger changes of velocity there is a greater decrease in the gaps in the front of the platoon which automatically results in a decrease in the average cumulative gap in the platoon. But for rather smaller changes of velocity the average gap does not change a lot than the real gap.

Moreover, this solution only reduces the bullwhip effects on the vehicles located in the end of the platoon which are the ones suffering the most amplified response. This is because the difference between the average cumulative gap and the real gap in vehicles at the end of the platoon is much greater than of the vehicles located closer to the leader. For vehicles in the beginning of the platoon the value of average cumulative gap is influenced by how many cars are ahead of vehicle n. For example, for vehicle $n = 20$ In the platoon there are 19 vehicles ahead of it that slow down and decrease their gaps and thus the average cumulative gap decreases. However, for $n = 5$ there are only 4 vehicles ahead of it slowing down so only 4 gaps decreasing and therefore the average cumulative gap value also decreases yet not as much as in the case of vehicle $n = 20$.

The utilization of the cumulative gap has shown very positive results in reducing the bullwhip effects suffered by vehicles in the platoon. However these results were only limited to vehicles in the end of the platoon which do in fact suffer the most from the bullwhip phenomenon as they suffer from a very amplified response towards the actions of the leader. In my opinion the use of the cumulative gap should be paired with another solution aimed towards vehicles upfront and in the middle of the platoon.

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7. Annexes

Annex 1: Acceleration-time graphs for braking manoeuvres with increasing Δv

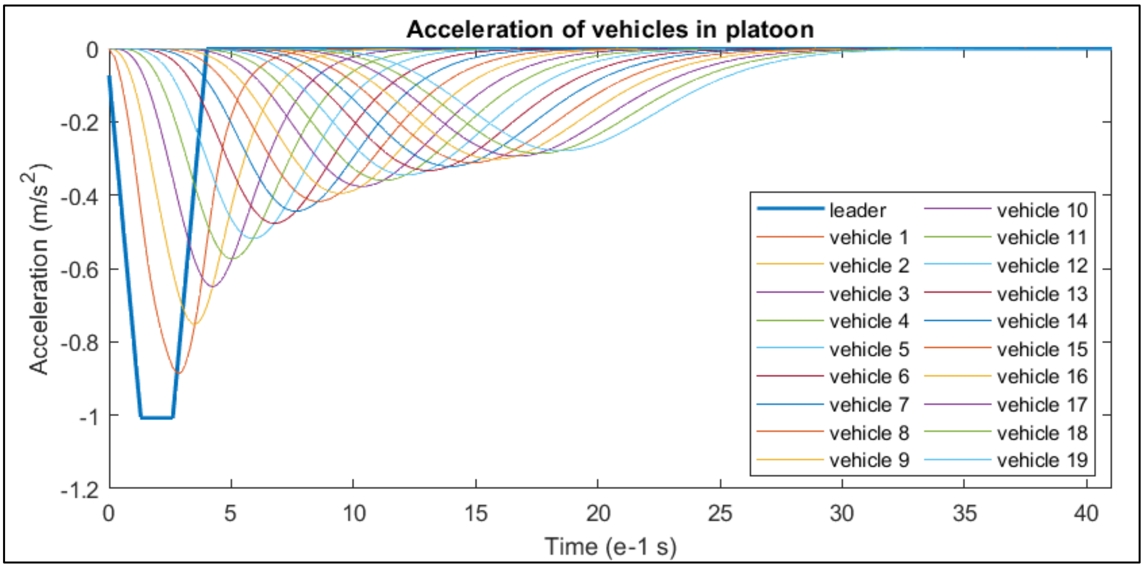

Figure 38: Acceleration time graph for case of 120 km/h to 110 km/h

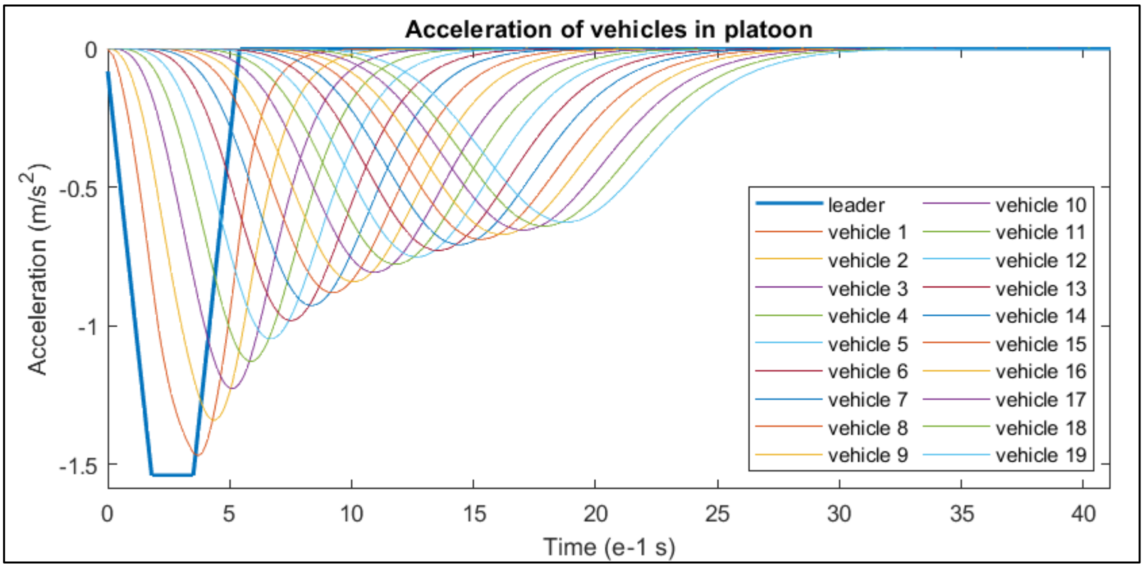

Figure 39: Acceleration time graph for case of 120 km/h to 100 km/h

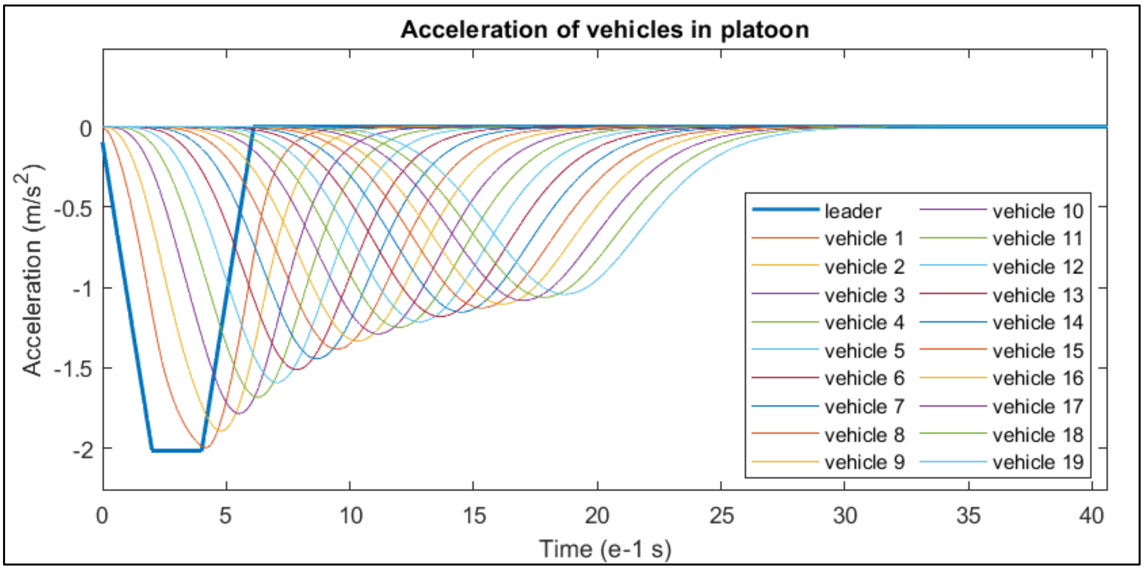

Figure 40: Acceleration time graph for case of 120 km/h to 90 km/h

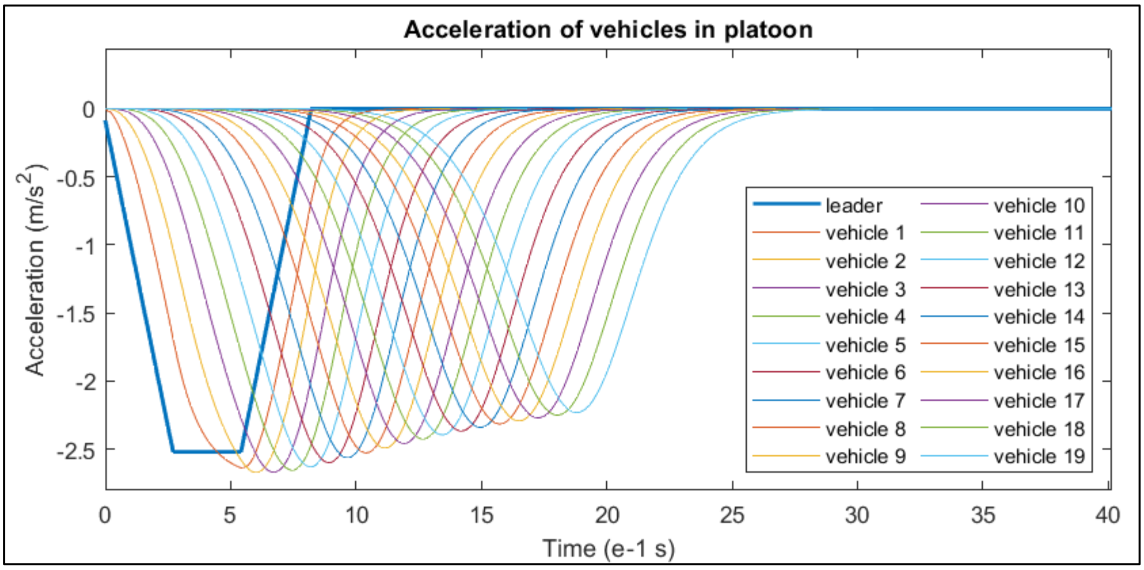

Figure 41: Acceleration time graph for case of 120 km/h to 70 km/h

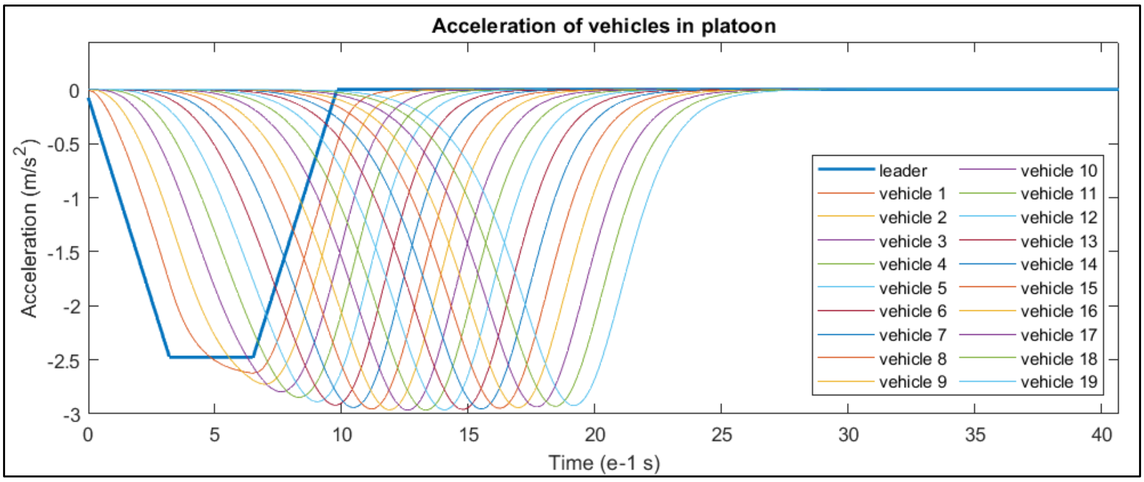

Figure 42: Acceleration time graph for case of 120 km/h to 60 km/h

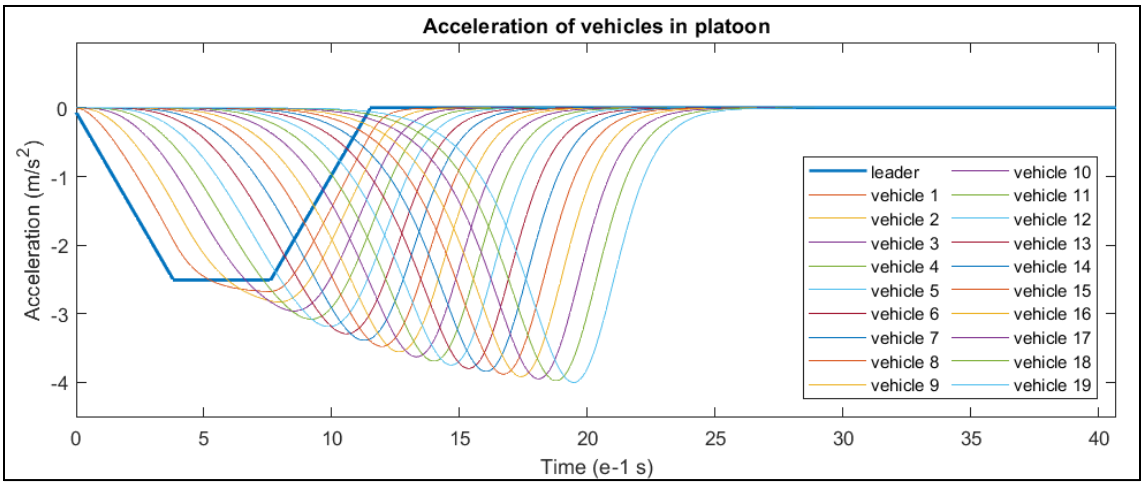

Figure 43: Acceleration time graph for case of 120 km/h to 50 km/h

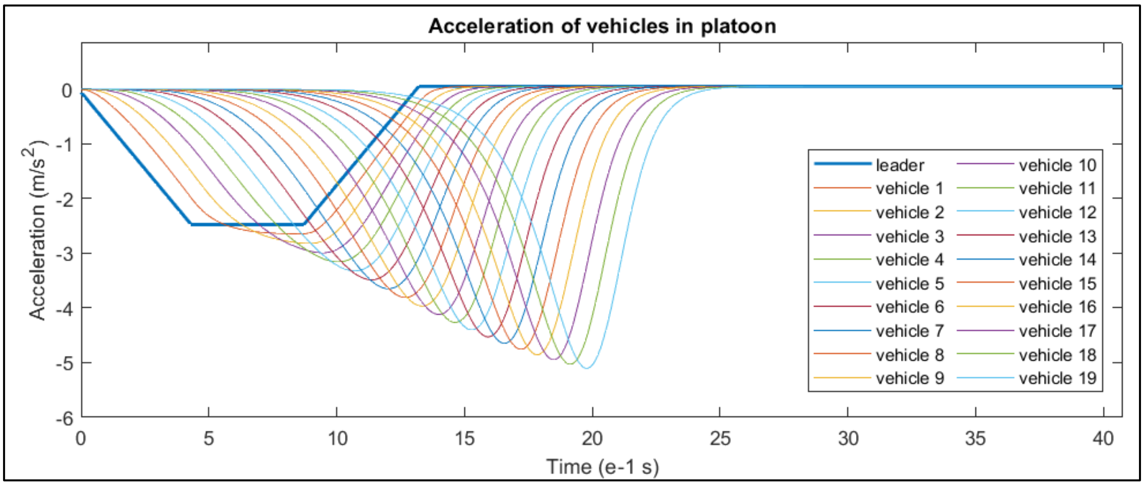

Figure 44: Acceleration time graph for case of 120 km/h to 40 km/h

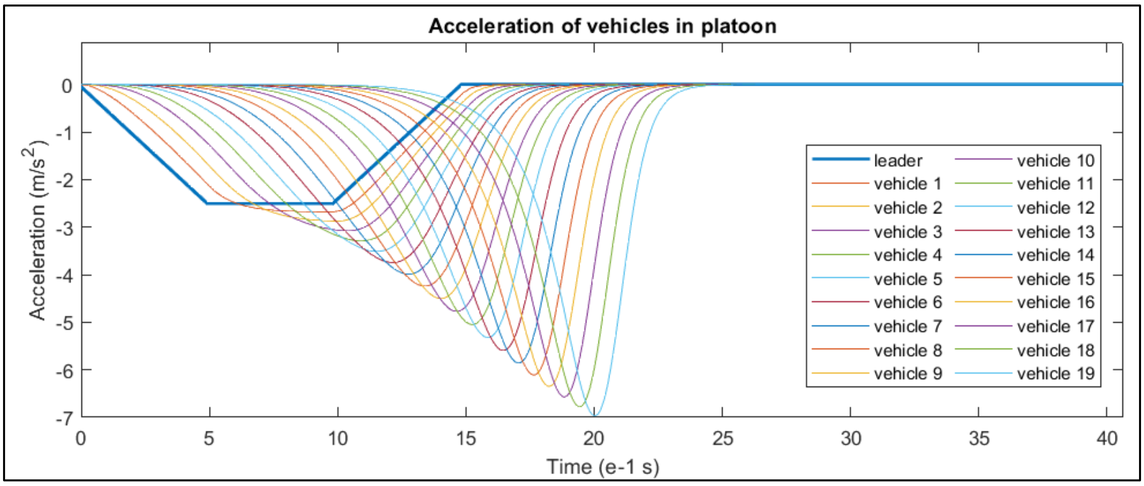

Figure 45: Acceleration time graph for case of 120 km/h to 30 km/h

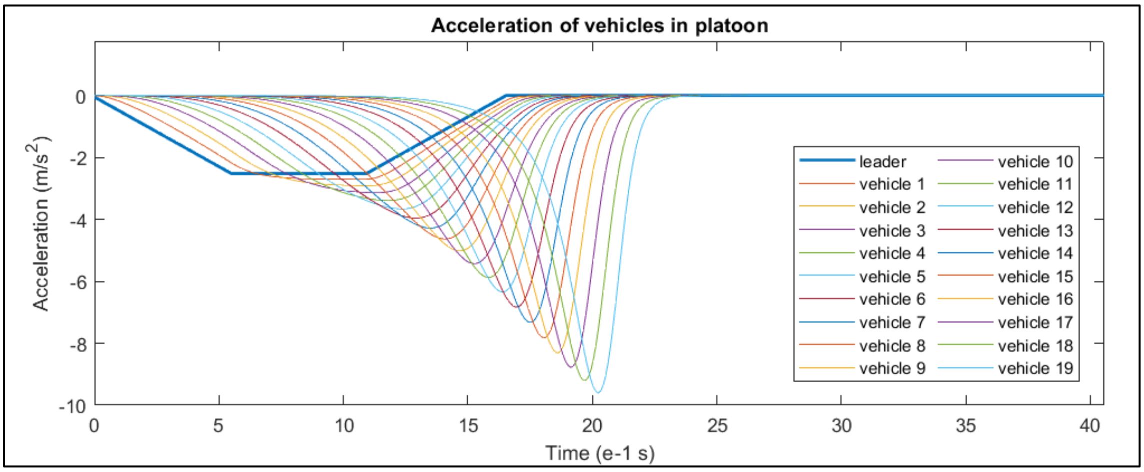

Figure 46: Acceleration time graph for case of 120 km/h to 20 km/h

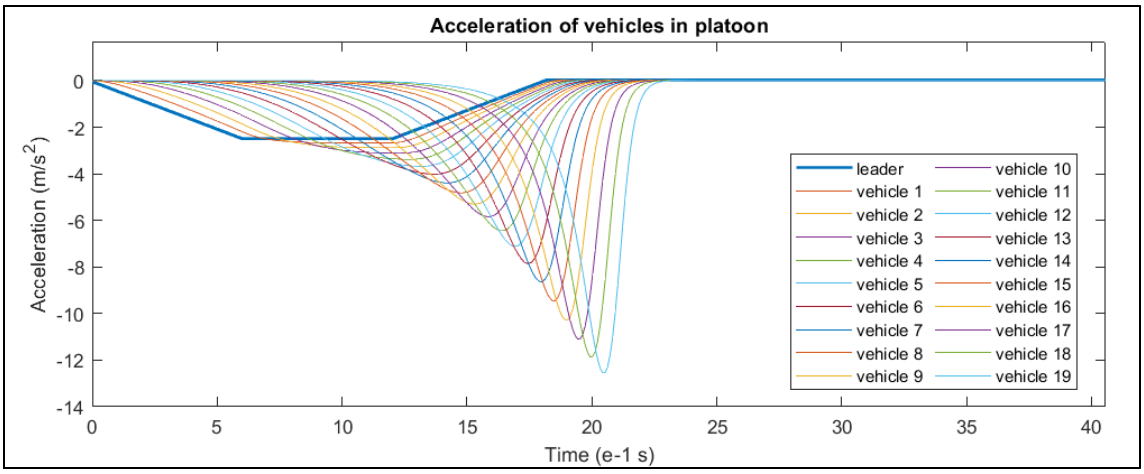

Figure 47: Acceleration time graph for case of 120 km/h to 10 km/h

Annex 2: Results of simulation for acceleration manoeuvres for different changes in velocity.

Change in velocity from 100 km/h to 120 km/h

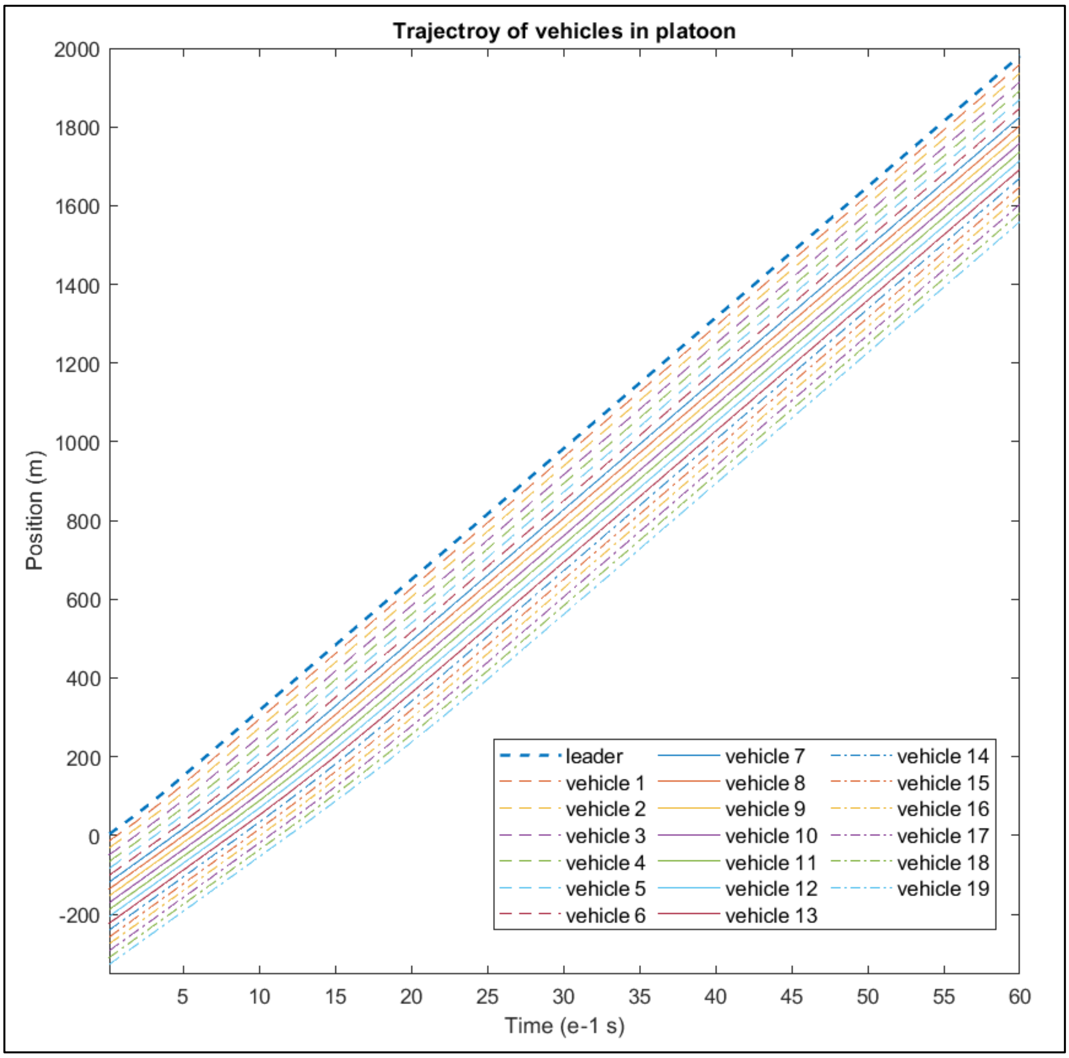

Figure 48: Trajectories of vehicles in platoon for case of 100km/h to 120km/h

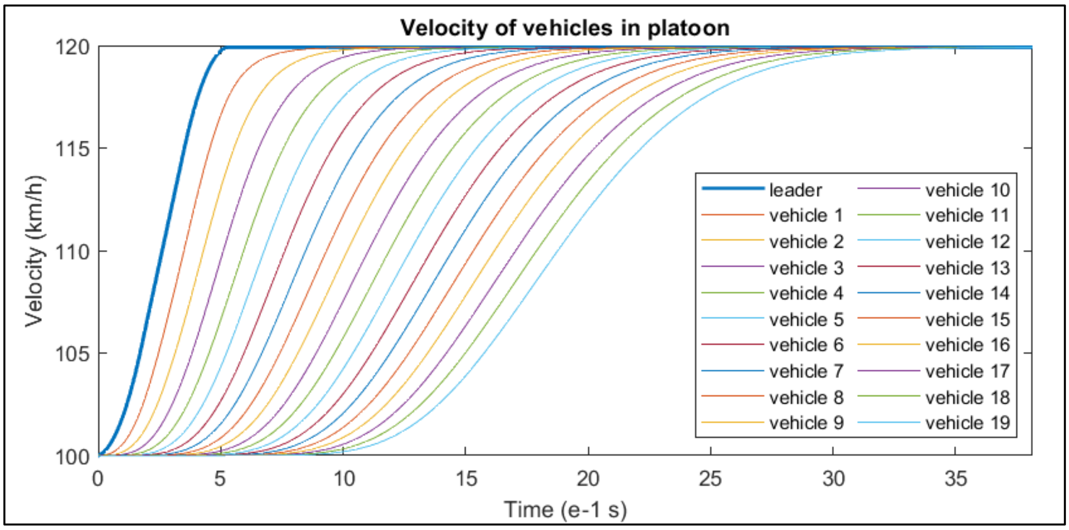

Figure 49: Velocity-Time graph for vehicles in platoon for case of 100km/h to 120 km/h

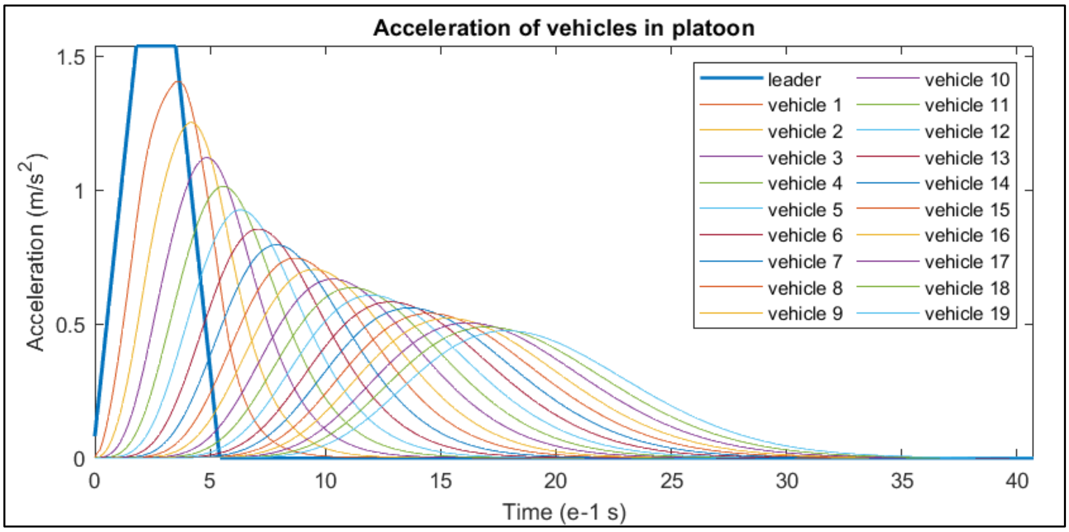

Figure 50: Acceleration-Time graph for vehicles in platoon for case of 100km/h to 120 km/h

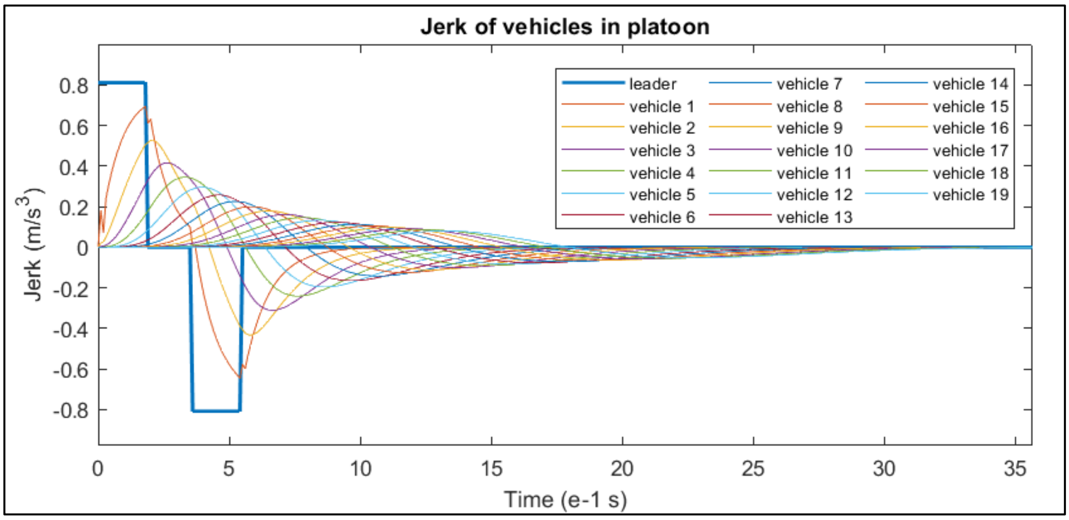

Figure 51: Jerk-Time graph for vehicles in platoon for case of 100km/h to 120 km/h

Change in velocity from 60 km/h to 120 km/h

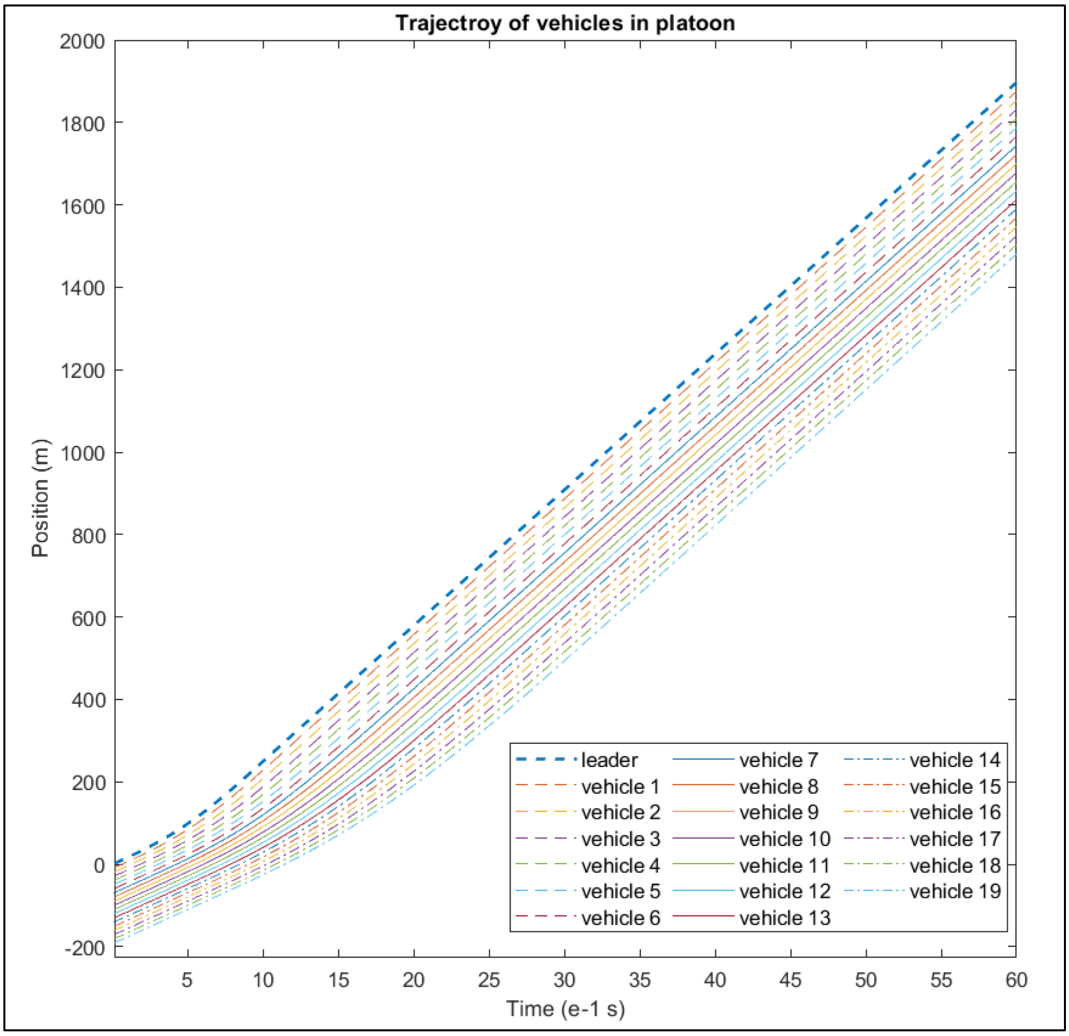

Figure 52: Trajectories of vehicles in platoon for case of 60km/h to 120km/h

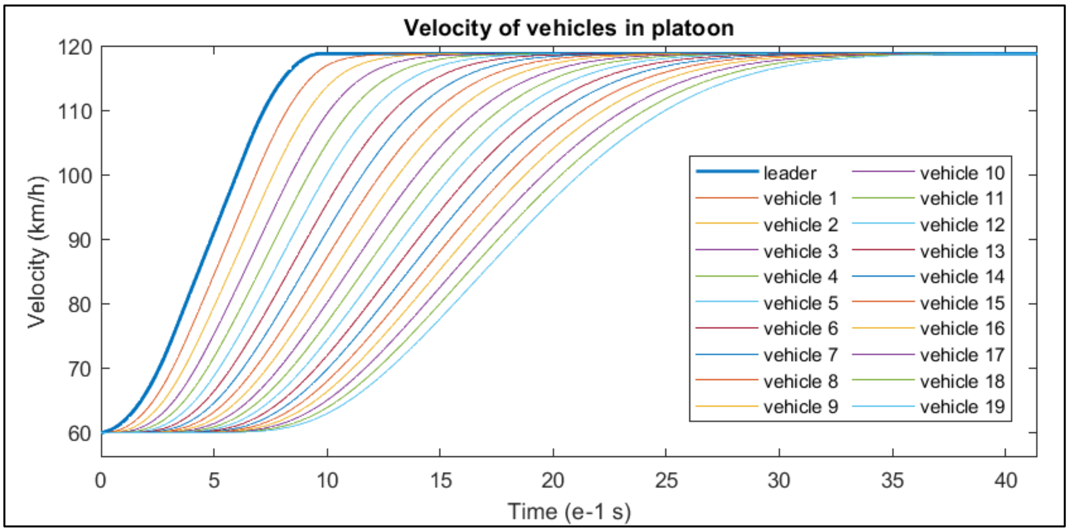

Figure 53: Velocity-Time graph for vehicles in platoon for case of 60km/h to 120 km/h

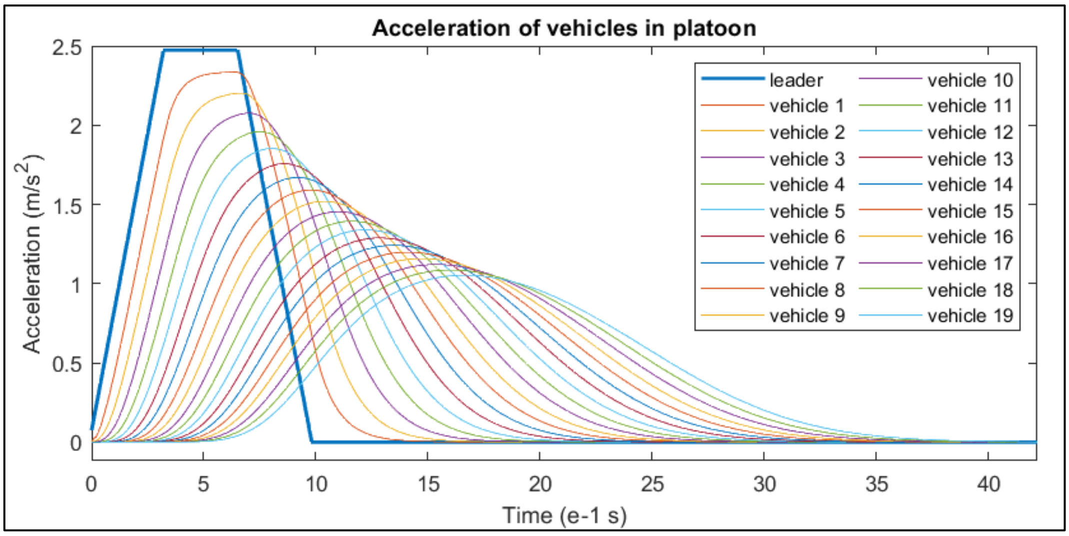

Figure 54: Acceleration-Time graph for vehicles in platoon for case of 60km/h to 120 km/h

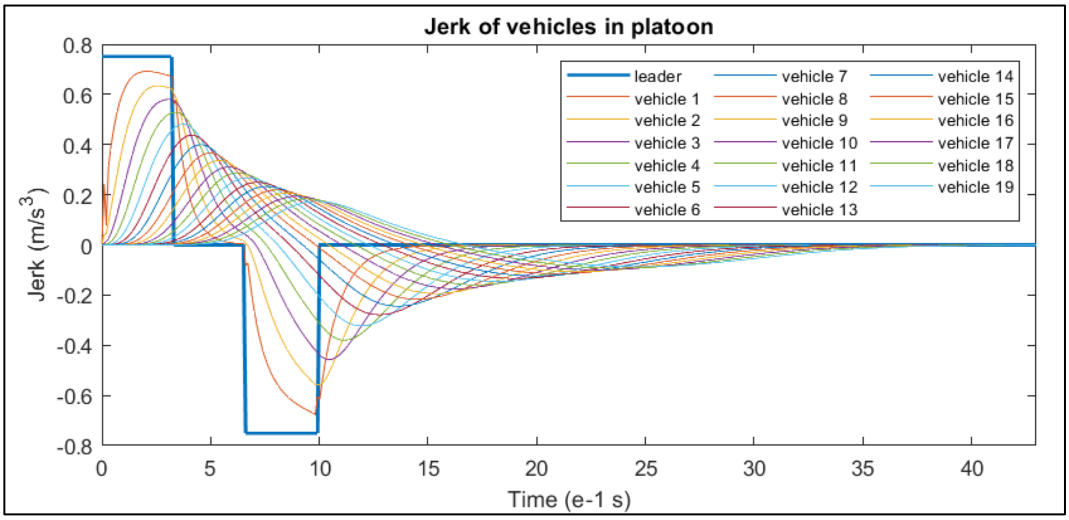

Figure 55: Jerk-Time graph for vehicles in platoon for case of 60km/h to 120 km/h

Change in velocity from 40 km/h to 120 km/h

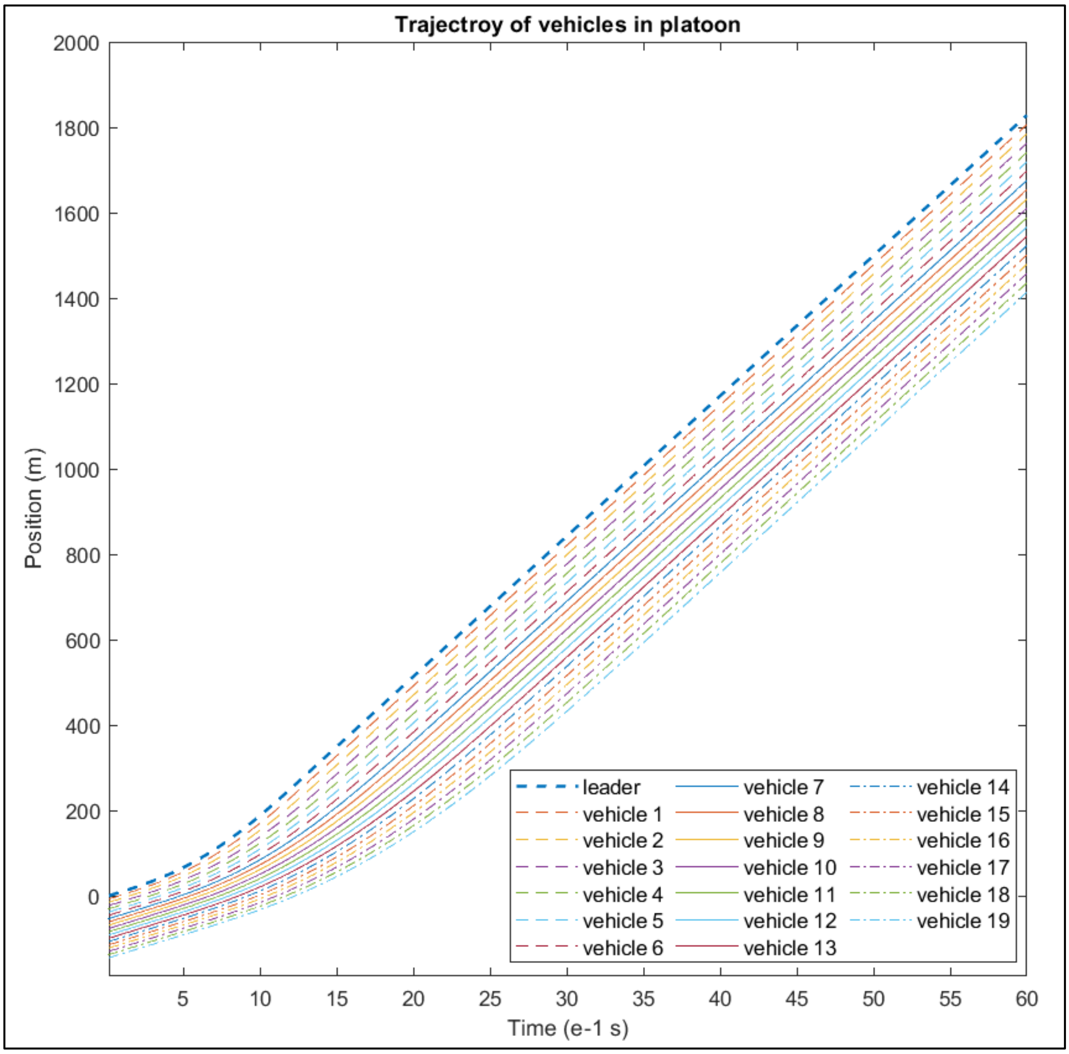

Figure 56: Trajectories of vehicles in platoon for case of 40km/h to 120km/h

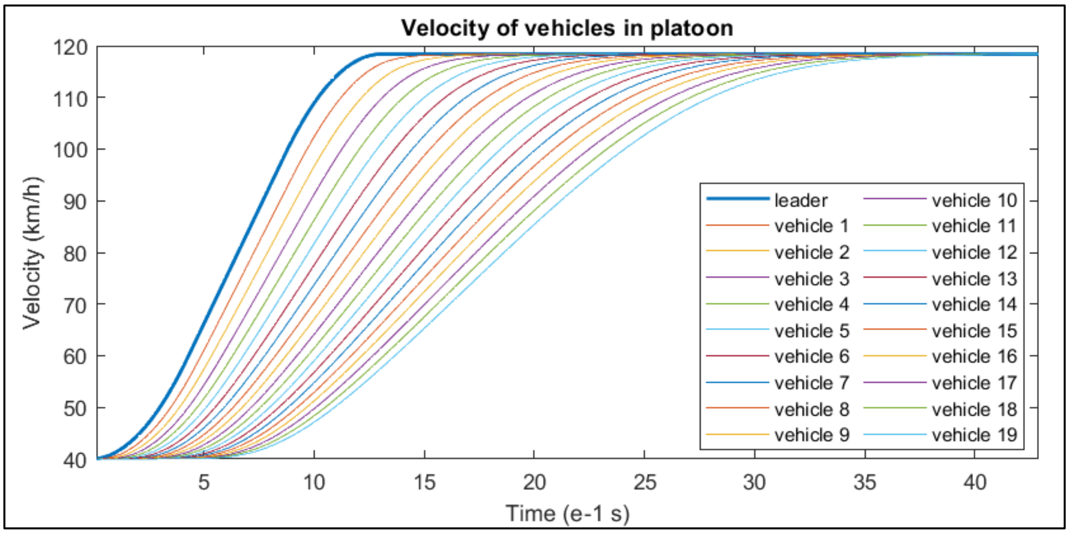

Figure 57: Velocity-Time graph for vehicles in platoon for case of 40km/h to 120 km/h

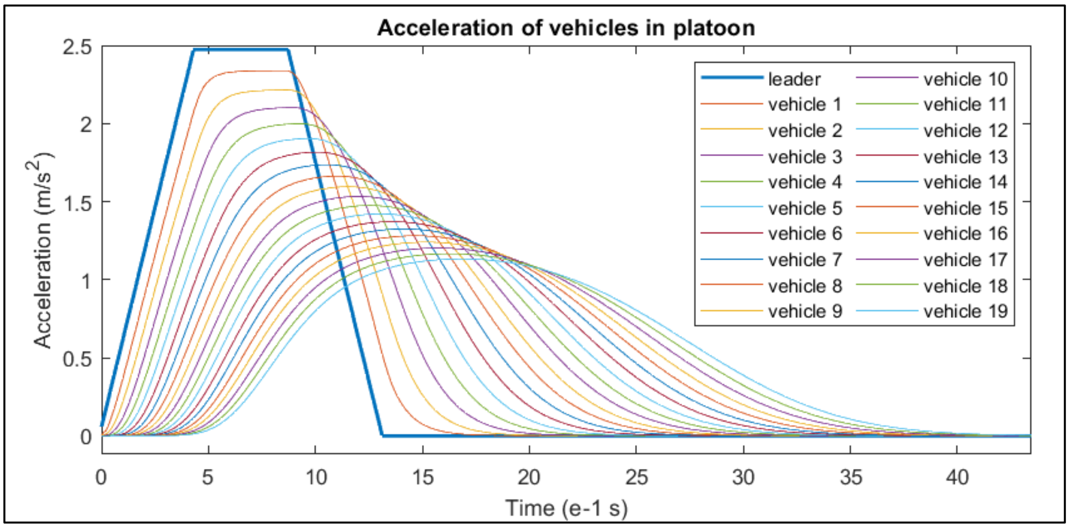

Figure 58: Acceleration-Time graph for vehicles in platoon for case of 40km/h to 120 km/h

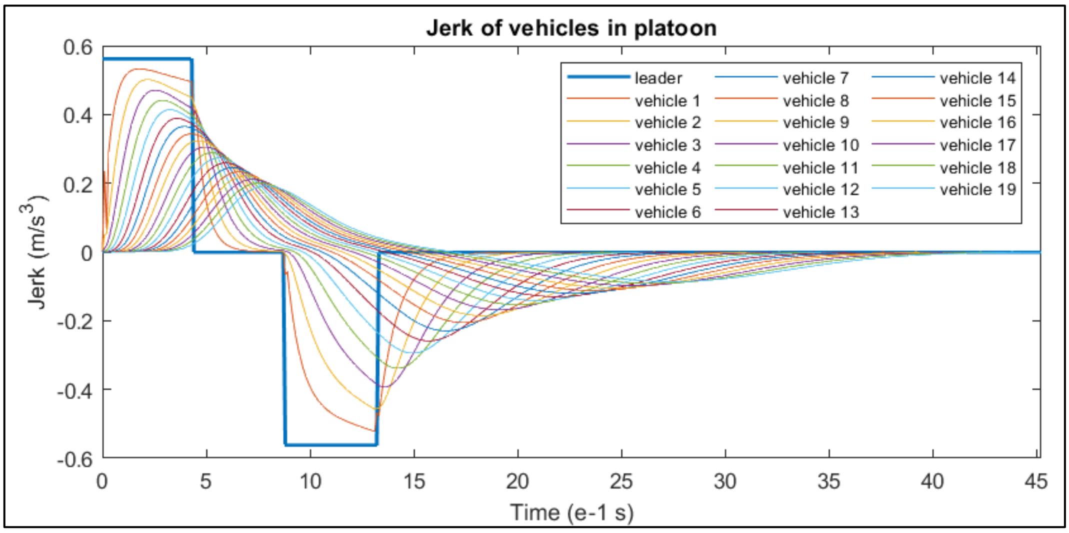

Figure 59: Jerk-Time graph for vehicles in platoon for case of 40km/h to 120 km/h

Annex 3: spreadsheet of values of maximum acceleration and their corresponding jerks for a given change in velocity.

Trapezoidal Acceleration-Time graph

Lim J	0.900	m/s^3

Delta V	10.000	km/h	
	2.778	m/s	
Max Acc (m/s^2)	t acc (s)	J (m/s^3)	ABS(lim J-j)
1.000	4.167	0.720	0.180
1.500	2.778	1.620	0.720
2.000	2.083	2.880	1.980
2.500	1.667	4.500	3.600

Delta V	20.000	km/h	
	5.556	m/s	
Max Acc (m/s^2)	t acc (s)	J (m/s^3)	ABS(lim J-j)
1.000	8.333	0.360	0.540
1.500	5.556	0.810	0.090
2.000	4.167	1.440	0.540
2.500	3.333	2.250	1.350

Delta V	30.000	km/h	
	8.333	m/s	
Max Acc (m/s^2)	t acc (s)	J (m/s^3)	ABS(lim J-j)
1.000	12.500	0.240	0.660
1.500	8.333	0.540	0.360
2.000	6.250	0.960	0.060
2.500	5.000	1.500	0.600

Delta V	40.000	km/h	
	11.111	m/s	
Max Acc (m/s^2)	t acc (s)	J (m/s^3)	ABS(lim J-j)
1.000	16.667	0.180	0.720
1.500	11.111	0.405	0.495
2.000	8.333	0.720	0.180
2.500	6.667	1.125	0.225

Delta V	50.000	km/h	
	13.889	m/s	
Max Acc (m/s^2)	t acc (s)	J (m/s^3)	ABS(lim J-j)
1.000	20.833	0.144	0.756
1.500	13.889	0.324	0.576
2.000	10.417	0.576	0.324
2.500	8.333	0.900	0.000

Delta V	60.000	km/h	
	16.667	m/s	
Max Acc (m/s^2)	t acc (s)	J (m/s^3)	ABS(lim J-j)
1.000	25.000	0.120	0.780
1.500	16.667	0.270	0.630
2.000	12.500	0.480	0.420
2.500	10.000	0.750	0.150

Delta V	70.000	km/h	
	19.444	m/s	
Max Acc (m/s^2)	t acc (s)	J (m/s^3)	ABS(lim J-j)
1.000	29.167	0.103	0.797
1.500	19.444	0.231	0.669
2.000	14.583	0.411	0.489
2.500	11.667	0.643	0.257

Delta V	80.000	km/h	
	22.222	m/s	
Max Acc (m/s^2)	t acc (s)	J (m/s^3)	ABS(lim J-j)
1.000	33.333	0.090	0.810
1.500	22.222	0.203	0.698
2.000	16.667	0.360	0.540
2.500	13.333	0.563	0.338

Delta V	90.000	km/h	
	25.000	m/s	
Max Acc (m/s^2)	t acc (s)	J (m/s^3)	ABS(lim J-j)
1.000	37.500	0.080	0.820
1.500	25.000	0.180	0.720
2.000	18.750	0.320	0.580
2.500	15.000	0.500	0.400

Delta V	100.000	km/h	
	27.778	m/s	
Max Acc (m/s^2)	t acc (s)	J (m/s^3)	ABS(lim J-j)
1.000	41.667	0.072	0.828
1.500	27.778	0.162	0.738
2.000	20.833	0.288	0.612
2.500	16.667	0.450	0.450

Delta V	110.000	km/h	
	30.556	m/s	
Max Acc (m/s^2)	t acc (s)	J (m/s^3)	ABS(lim J-j)
1.000	45.833	0.065	0.835
1.500	30.556	0.147	0.753
2.000	22.917	0.262	0.638
2.500	18.333	0.409	0.491

Delta V	120.000	km/h	
	33.333	m/s	
Max Acc (m/s^2)	t acc (s)	J (m/s^3)	ABS(lim J-j)
1.000	50.000	0.060	0.840
1.500	33.333	0.135	0.765
2.000	25.000	0.240	0.660
2.500	20.000	0.375	0.525

Annex 4: codes & function created on MATLAB.

Gap formula function

```
function dsg= gapformula(d_min,v0,dec_max,delta,alpha)

dsg=d_min+v0*delta+ v0^2/(2*dec_max)*(alpha/(1-alpha));
end
```

Optimum max acceleration & jerk function

```
function [acc_max,t_acc,J] =OptAcc_Max_Trap (Lim_J,delta_v)

% finding the optimum maximum acceleration that has the closest Jerk
to the threshold limit

%forming matrix of data
data=zeros(4,3);
for j=1:4
    data(j,1)= 1+((j-1)*0.5);% acceleration with increments of 0.5
    data(j,2)=((delta_v)/data(j,1))*(3/2);% time of acceleration
    data(j,3)=(3*data(j,1))/data(j,2);% jerk corresponding to
acceleration
end

z=abs(data(:,3)-Lim_J);
[x,i]=min(z);
acc_max=data(i,1);
t_acc=data(i,2);
J=data(i,3);
end
```

Accelerating function

```
function [position, speed, acceleration,
jerk]=Accelerating_Trap(delta,step,v0,t_acc,J,vf)

%inititaiton of vecotrs for data of leader
position = zeros(1,step);
speed = zeros(1,step);
acceleration = zeros(1,step);
jerk= zeros(1,step);

for i=1:step % assuming a trapezoidal acceleration time graph
    if i <= round((t_acc/3)/delta)
        jerk(i)=J;
    elseif i> round((t_acc/3)/delta) && i<round((t_acc/delta)*(2/3))
        jerk (i)=0;
    elseif i>=round((t_acc/delta)*(2/3)) && i<=t_acc/delta
        jerk(i)=-J;
    end
end

%initial conditions
acceleration(1)=J*delta;
speed(1)=v0;

% calculation of the trajectory
```

```
for i=2:step
    acceleration(i)= acceleration(i-1)+(jerk(i)*delta);
    speed(i)=speed(i-1)+(acceleration(i)*delta);
    position(i)=position(i-1)+(speed(i-1)*delta);
end
end
```

Braking function

```
function [position, speed, acceleration,
jerk]=Braking_Trap(delta,step,v0,t_acc,J,vf)

%inititaiton of vecotrs for data of leader
position = zeros(1,step);
speed = zeros(1,step);
acceleration = zeros(1,step);
jerk= zeros(1,step);

for i=1:step % assuming a trapezoidal acceleratio time graph
    if i <= round((t_acc/3)/delta)
        jerk(i)=-J;
    elseif i> round((t_acc/3)/delta) && i<round((t_acc/delta)*(2/3))
        jerk (i)=0;
    elseif i>=round((t_acc/delta)*(2/3)) && i<=t_acc/delta
        jerk(i)=J;
    end
end

%initial conditions
acceleration(1)=-J*delta;
speed(1)=v0;

%calculation of the trajectory
for i=2:step
    acceleration(i)= acceleration(i-1)+(jerk(i)*delta);
    speed(i)=speed(i-1)+(acceleration(i)*delta);
    position(i)=position(i-1)+(speed(i-1)*delta);
    if i==(t_acc/delta)
        jerk(i)=0;
        acceleration(i)=0;
    end
end
```

Braking solution gap function

```
function Min_G=Min_Gap(Y,gap,i,j)
%Finding the min between gap and average gap
Mean_Gap= Y(i-1,j-1)/(i-1);
if gap(i-1,j-1)<=Mean_Gap
    Min_G=gap(i-1,j-1);
else
    Min_G=Mean_Gap;
end
end
```

Main code

```
clear all; close all;clc
%% Simulation time paramters

delta=0.1;%latency
time=60;% time of simulation [s]
step=time/delta;% number of increments

%% platoon parameters

v0=40/3.6;% initial velocity [m/s]
vf=120/3.6;% final velocity [m/s]
N=20;% cars in platoon including leader
Lcar=4.5; % length of car [m]

%% DSG formula parameters
d_min=0.5; % minimum safety gap of cars in platoon [m]
dec_max=10;% max deccelartion = 10 Gravity [m/s^2]
alpha=0.2;% max variation in acceleration
dsg0=gapformula(d_min,v0,dec_max,delta,alpha);% desired gap for
intial velocity [m]

%% Formation of Leader Trajectory

delta_v=abs(vf-v0);% Represents area under the acceleration time
graph of Leader
Lim_J=0.9;% The max value for jerk to ensure comfort [m/s^3]
[acc_max,t_acc,J]=OptAcc_Max_Trap(Lim_J,delta_v);
% acc_max: maximum acceleration leader will reach [m/s^2]
% t_acc: time needed to execute the change in velocity [s]
% J: jerk [m/s^3]

if vf-v0>0
    [position, speed, acceleration,
jerk]=Accelerating_Trap(delta,step,v0,t_acc,J,vf);
else
    [position, speed, acceleration,
jerk]=Braking_Trap(delta,step,v0,t_acc,J,vf);
end

%% Computing followers Trajectories

%expanding existing matrices of data and creating new necesarry ones
position =[position; zeros(N-1,step)];
speed = [speed; zeros(N-1,step)];
acceleration =[acceleration; zeros(N-1,step)];
jerk=[jerk; zeros(N-1,step)];
gap=zeros(N-1,step);
DSG=zeros(N-1,step);
Y=zeros(N-1,step);% accumulated gap (free distnace infront of
vehicles)
X=zeros(N-1,step);% distance to leader for each car

%inputting the initial conditions of the platoon
for i= 2:N
```

```
    position(i,1)= position(i-1,1)-(Lcar+dsg0);
    speed(i,1)=v0;
end
for i=1:N-1
    gap(i,1)=dsg0;
    DSG(i,1)=dsg0;
    X(i,1)=position(1,1)-position(i+1,1);
    Y(i,1)=i*dsg0;
end
% A and B and C are constants of a polynmial in the formula of
calculating the velocity of the follower
A=((1/(2*dec_max))*(alpha/(1-alpha)));
B=2*delta;
for i=2:N
    for j=2:step
         if vf-v0>0
            C=-(gap(i-1,j-1)+(delta*speed(i-1,j))-d_min);
         else
             Min_G=Min_Gap(Y,gap,i,j);
             C=-(Min_G +(delta*speed(i-1,j))-d_min);% with the
breaking stability solution
         end
       p=[A B C];
       v=roots(p);
       if speed(i,j-1)<=v(1)&&v(1)<=speed(i-1,j)
           speed(i,j)=v(1);
       else
           speed(i,j)=v(2);
       end
       acceleration(i,j)=(speed(i,j)-speed(i,j-1))/delta;
       jerk(i,j)=(acceleration(i,j)-acceleration(i,j-1))/delta;
       position(i,j)=position(i,j-1)+(speed(i,j)*delta);
       X(i-1,j)=position(1,j)-position(i,j);
       Y(i-1,j)=X(i-1,j)-((i-1)*Lcar);
       DSG(i-1,j)=gapformula(d_min,speed(i,j),dec_max,delta,alpha);
       if i==2
           gap(i-1,j)= Y(i-1,j);
       else
           gap(i-1,j)=Y(i-1,j)-Y(i-2,j);
       end
    end
end
Z=DSG-gap; % gap and DSG difference should be 0


%% plotting final results
T=linspace(0,time,step);

figure(1)

subplot(2,2,1)
plot(T,position(1,:))
title('Trajectory of leader')
xlabel('Time (e-1 s)')
ylabel('osition (m)')
```

```
subplot(2,2,2)
plot(T,speed(1,:)*3.6)
title('Speed vs Time for leader')
xlabel('Time (e-1 s)')
ylabel('velocity (km/h)')


subplot(2,2,3)
plot(T,acceleration(1,:))
title('Acceleration vs Time for leader')
xlabel('Time (e-1 s)')
ylabel('Acceleration (m/s^2)')


subplot(2,2,4)
plot(T,jerk(1,:))
title('Jerk vs Time for leader')
xlabel('Time (e-1 s)')
ylabel('Jerk (m/s^3)')

figure (2)

for i=1:N
    if i==1
         txt=['leader'];
         plot(T,position(i,:),'--
','lineWidth',1.5,'displayName',txt);
         hold on;
    elseif i<=7
         txt=['vehicle ',num2str(i-1)];
    plot(T,position(i,:),'--','displayName',txt);
    hold on;
    elseif i>7 && i<= 14
    txt=['vehicle ',num2str(i-1)];
    plot(T,position(i,:),'-','displayName',txt);
    hold on;
    elseif i>14
            txt=['vehicle ',num2str(i-1)];
    plot(T,position(i,:),'-.','displayName',txt);
    hold on;
    end
end
legend(gca,'show','NumColumns',3,'FontSize',10)
title('Trajectroy of vehicles in platoon')
xlabel('Time (e-1 s)')
ylabel('Position (m)')

figure(3)
for i=1:N
     if i==1
         txt=['leader'];
         plot(T,speed(i,:)*3.6,'lineWidth',1.5,'displayName',txt);
         hold on;
     else
    txt=['vehicle ',num2str(i-1)];
```

```
    plot(T,speed(i,:)*3.6,'displayName',txt);
    hold on;
     end
end
legend(gca,'show','NumColumns',2)
title('Velocity of vehicles in platoon')
xlabel('Time (e-1 s)')
ylabel('Velocity (km/h)')

figure(4)

for i=1:N
     if i==1
         txt=['leader'];

plot(T,acceleration(i,:),'lineWidth',1.5,'displayName',txt);
         hold on;
     else
    txt=['vehicle ',num2str(i-1)];
    plot(T,acceleration(i,:),'displayName',txt);
    hold on;
     end
end
legend(gca,'show','NumColumns',2)
title('Acceleration of vehicles in platoon')
xlabel('Time (e-1 s)')
ylabel('Acceleration (m/s^2)')

figure (5)
for i=1:N
     if i==1
         txt=['leader'];
         plot(T,jerk(i,:),'lineWidth',1.5,'displayName',txt);
         hold on;
     else
    txt=['vehicle ',num2str(i-1)];
    plot(T,jerk(i,:),'displayName',txt);
    hold on;
     end
end
legend(gca,'show','NumColumns',3,'FontSize',8)
title('Jerk of vehicles in platoon')
xlabel('Time (e-1 s)')
ylabel('Jerk (m/s^3)')

figure(6)

for i=1:N-1
    if i <=7
        txt=['vehicle ',num2str(i)];
        plot(T,Y(i,:),'--','lineWidth',1.5,'displayName',txt);
        hold on;
    elseif i>7 && i<14
        txt=['vehicle ',num2str(i)];
        plot(T,Y(i,:),'-','displayName',txt);
        hold on;
```

```
    else
     txt=['vehicle ',num2str(i)];
     plot(T,Y(i,:),'-.','displayName',txt);
     hold on;
    end
end
legend(gca,'show','NumColumns',3)
title('Cumulative Gap')
xlabel('Time (e-1 s)')
ylabel('Distance (m)')

figure (7)
for i=1:N-1
    txt=['vehicle ',num2str(i)];
    plot(T,X(i,:),'displayName',txt);
    hold on;
end
legend(gca,'show','NumColumns',2)
title('Distance to platoon leader')
xlabel('Time (e-1 s)')
ylabel('Distance (m)')

figure(8)
for i=1:N-1
    txt=['vehicle ',num2str(i)];
    plot(T,Z(i,:),'displayName',txt);
    hold on;
end
legend(gca,'show','NumColumns',2)
title('DSG - gap')
xlabel('Time (e-1 s)')
ylabel('distance (m)')

figure(9)
for i=1:N-1
    txt=['vehicle ',num2str(i)];
    plot(T,gap(i,:),'displayName',txt);
    hold on;
end
legend(gca,'show','NumColumns',2)
title('Gap')
xlabel('Time (e-1 s)')
ylabel('Distance (m)')

figure(10)

plot(speed (1,:)*3.6,DSG(1,:),'displayName',txt);
title('DSG (v)')
set(gca, 'XDir','reverse')
xlabel('Velocity (km/h)')
ylabel('Distance (m)')


figure(11)
txt1=['Average cumulative gap'];
plot(T,Y(N-1,:)/(N-1),'displayName',txt1);
```

```
hold on;
txt2=['Current gap'];
plot(T,gap(N-1,:),'--','displayName',txt2);
legend(gca,'show','NumColumns',2)
xlabel('Time (e-1 s)')
ylabel('Distance (m)')


W=sum(DSG,1);% sum of DSGk
figure (12)
txt1=['n*DSGn'];
plot(T,(N-1)*DSG(N-1,:),'displayName',txt1);
hold on;
txt2=['Sum DSGk '];
plot(T,W(:,:),'--','displayName',txt2);
legend(gca,'show','NumColumns',2)
xlabel('Time (e-1 s)')
ylabel('Distance (m)')

figure (13)
for i=1:N-1
    if i<=7
        txt1=['Vehicle ',num2str(i+1)];
        plot(T,Y(i,:)/i,'--','displayName',txt1);
        hold on;
    elseif i>7 && i<14
        txt1=['Vehicle ',num2str(i+1)];
        plot(T,Y(i,:)/i,'-','displayName',txt1);
        hold on;
    else
        txt1=['Vehicle ',num2str(i+1)];
        plot(T,Y(i,:)/i,'-.','displayName',txt1);
        hold on;

    end
    txt2=['Current gap'];
    plot(T,gap(N-1,:),'--','displayName',txt2);
    hold on;
end
legend(gca,'show','NumColumns',3)
title('Average Cumulative Gap')
xlabel('Time (e-1 s)')
ylabel('Distance (m)')
```