Inverse-design of non-Hermitian potentials for light management

Muriel Botey*[a], Waqas W. Ahmed[b], Ramon Herrero[a], Ying Wu[b], Kestutis Staliunas[a,c],
[a]Departament de Física, Universitat Politècnica de Catalunya (UPC), Barcelona, Catalonia;
[b]Division of Computer, Electrical and Mathematical Sciences and Engineering, King Abdullah University of Science and Technology (KAUST), Thuwal, Saudi Arabia;
[c]Institució Catalana de Recerca i Estudis Avançats (ICREA), Barcelona, Catalonia

ABSTRACT

In this paper, we propose a general inverse-design strategy based on genetic algorithm optimization to achieve 'on demand' manipulation of light in one-dimensional (1D) and two-dimensional (2D) non-Hermitian systems. The optimization process faithfully creates non-Hermitian potentials from any given arbitrary real (or imaginary) permittivity distribution for the desired frequency selective and broadband asymmetric response in 1D multilayer structures. As a demonstration in 2D, we design periodic and aperiodic complex permittivity spatial distributions to create "sink-type" concentrators of light around a desired area. The proposed inverse-design approach to generate non-Hermitian potentials represents an alternative to the Hilbert Transform (HT) generalizing the Kramers Kronig relations in space, additionally being selective in spectrum.

Keywords: Non-Hermitian Photonics, Metamaterials, PT-symmetry, Hilbert transform

1. INTRODUCTION

In recent years, different approaches have been proposed to manipulate the scattering properties of linear and nonlinear materials for light management. Among them, the most common are relaying in structured metamaterials, transformation optics [1] and more recently in non-Hermitian photonics [2]. Non-Hermitian photonics has attracted noticeable interest due to several unprecedented features such as asymmetric light propagation, unidirectional invisibility, among other counterintuitive effects [3-6]. Initially, it was demonstrated that particular non-Hermitian potentials holding Party-Time (PT-) symmetry may suppress the reflections [7]. Indeed, a unidirectional or at least asymmetric response, apart from breaking of conventional wave propagation laws of closed conservative systems, would be desirable for unidirectional communication in technological photonic devices. More recently, in the pursuit of a potential that determines an arbitrary directionality of the field, a Hilbert Transform (HT) was proposed [8] as a generalization of the Kramers Kronig (KK) relations in space [10,11]. Such generalized HT relates the real/imaginary an imaginary spatial distributions, ε_I and ε_{II}, of the complex permittivity to design a background potential to tailor the flow of light following any arbitrary unitary vector field, $\vec{p}(\vec{r})$:

$$\varepsilon_I(\vec{r}) = \frac{1}{\pi} P \iint \frac{\delta(\vec{r}-\vec{r}_1)\vec{q}(\vec{r}-\vec{r}_1)\varepsilon_{II}(\vec{r}_1)}{\vec{p}(\vec{r})(\vec{r}-\vec{r}_1)} d\vec{r}_1 \qquad \varepsilon_{II}(\vec{r}) = -\frac{1}{\pi} P \iint \frac{\delta(\vec{r}-\vec{r}_1)\vec{q}(\vec{r}_1)\varepsilon_I(\vec{r}_1)}{\vec{p}(\vec{r}_1)(\vec{r}-\vec{r}_1)} d\vec{r}_1 \qquad \vec{p}(\vec{r})\cdot\vec{q}(\vec{r}) = 1, \left|\vec{q}(\vec{r})\right| = 1 \quad (1)$$

In the simplest 1D case, a scalar function $p(x)=\pm 1$ recovers the KK relations which lead to the cancellation of half-spectrum of permittivity and unidirectional reflectivity. In higher dimensions, the HT leads to an asymmetric spectral distribution at every spatial point, see Fig.1(iv) for a 2D potential following a vector field in the form of a sink. In either case, the material properties in the complex permittivity profile are critical for asymmetric non-Hermitian potentials that obey either PT-symmetry or the generalized HT [10,11]. While it is indeed possible to restrict the generalized HT to the use of all-dielectric materials or within a given range [12,13], it requires iterative methods. However, as the degrees of freedom increase or if we intend to achieve a selectively multifrequency light transportation, such methods may become computationally expensive. Thus, an alternative approach is to solve the inverse-design problem for the desired light manipulation, which may be fundamental for the development of particular functionalities in optical devices. However, no systematic study has been conducted to achieve asymmetric frequency response in inverse design of non-Hermitian

*muriel.botey@upc.edu; donll.upc.edu

potentials. Yet different optimization methods have been used to face such kind of inverse problems, such as the particle swarm optimization [14] and genetic algorithm [15].

Therefore, in this paper, we present a feasible method based on genetic optimization to uncover non-Hermitian potentials for desired asymmetric reflectivity. However, the present work is intended in gaining fundamental insights into the structures while being optimized for desired optical properties. In this sense, we create inverse-designed non-Hermitian potentials either a broadband or a frequency selective asymmetric reflectivity with perfect transmission both from periodic and aperiodic background potentials [16]. Once the potentials are generated, we inspect the resulting permittivity distributions to search for underlying fundamental insights about the designed structures, and connection to the KK relation in 1D or the HT in higher dimensions. The designed potentials provide a connection between asymmetric reflectivity and the closed loop area occupied by complex permittivity distributions in complex planes. In the simplest 1D case, we consider a planar multilayer structure with different real permittivity in each layer as shown in Fig. 1. The genetic algorithm (GA) takes a real permittivity profile as a seed and provides the optimal imaginary part of the complex permittivity profile for desired unidirectional frequency selective behavior. We aim to design periodic and non-periodic non-Hermitian structures, based on arbitrary permittivity profiles, that hold generalized different asymmetric light propagation as illustrated in Figs. 1(i)-(iii); for either common asymmetric response for discrete frequencies, opposite asymmetric response for different discrete frequencies or broadband asymmetric unidirectionality. Note that for the frequency selective directionality. We explore either common asymmetric response for discrete frequencies (i), opposite asymmetric response for different discrete frequencies or broadband asymmetric unidirectionality. Note that for the frequency selective directionality, Fig 1(ii) would correspond to a multicolor HT and we do not expect the half-spectrum cancellation. For the 2D case, we consider a concentric-ring or hexagonal profiles as seed for the real part of permittivity and generate the associated imaginary distribution with GA for light concentration around the center following the sink behavior [see Fig. 1(b)].

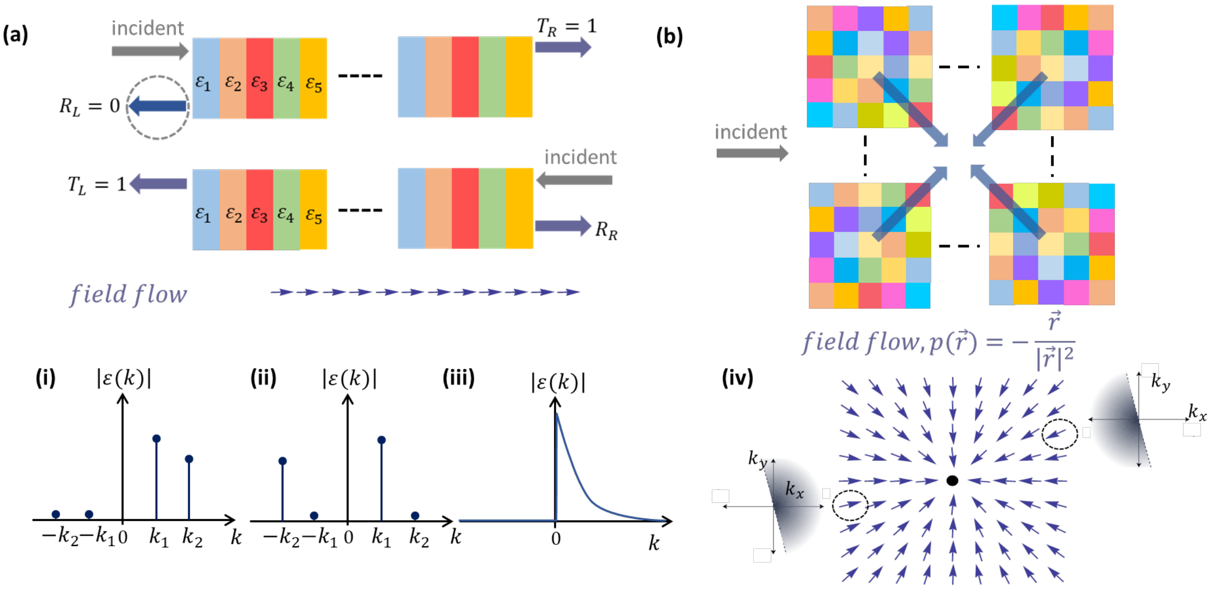

Figure 1. (a) Schematic of multilayer 1D periodic configuration with complex permittivity in each spatial domain for frequency selective asymmetric reflection with perfect transmission. The modulus of asymmetric spectra corresponding to the periodic structures depicts the unidirectional reflection at two given discrete frequencies in the same (i), opposite direction (ii) and broadband frequency range (iii). (b) Scheme to design the spatial distribution of permittivity in 2D configuration to provide a sink power flow. The different colors show the different permittivity value of each spatial region in 2D. (iv) spectrum cancellation at each spatial point depending on the directionality arrow, as it derives from the generalized HT.

*muriel.botey@upc.edu; donll.upc.edu

2. ONE-DIMENSIONAL SYSTEM

We shall start with a 1D system consisting of multilayer periodic structure as shown in Fig. 1(a). The aim is to design the imaginary (real) part of the complex permittivity of each layer for a given arbitary real (imaginary) permittivity values for asymmetric light propagation. We employ the Genetic Algorithm (GA) optimization and design the frequency-depend target function based on either left or right incidence, to provide the desired asymmetric reflectivity from 0% to 100 % along with perfect symmetric transmission. We assume a standard transfer matrix approach to calculate the scattering properties of the system. Decomposing the field into the forward and backward propagating waves of wavevector, k, as: $E(z) = E_f e^{ikz} + E_b e^{-ikz}$, the interaction of waves across different interfaces are combined to form the transfer matrix, *M*, relating the amplitudes of field outside the scattering region. By applying the boundary conditions, the transmittance $T_{R,L} = |t_{R,L}|^2$ and reflectance $R_{R,L} = |r_{R,L}|^2$ are determined from the transmission the coefficients of transfer matrix, *M*:

$$t_R = \frac{1}{M_{22}},\ t_L = \frac{M_{11}M_{22} - M_{12}M_{21}}{M_{22}},\ r_R = \frac{M_{12}}{M_{22}}, \text{and}\ \ r_L = -\frac{M_{21}}{M_{22}}.$$

Here, we employ the GA as a tool to tailor the spectral properties for arbitray given distribution, by minimizing a pre-specified 'target function' in a process that mimics natural evolution. This function, *F*, can be defined in a variety of ways. In our case, the objective is to maximize the asymmetric reflectivity between left and right reflection while keeping the transmittance unity from both sides. We can achieve perfect transmission with desired asymmetric reflection with the following frequency-dependent target function expressed as:

$$F(\omega) = \frac{1}{N}\int_{\omega_1}^{\omega_2}\left[\left|1 - T_R(\omega)\right| + \left|1 - T_R(\omega)\right| + \left|A(\omega)\right|\right] d\omega \qquad (2)$$

where $A(\omega)$ is the measure of "on-demand" asymmetry in reflection.

2.1 1D Periodic structures

We consider three, five and seven-layer periodic structure with given arbitrary real part of permittivity values, and look for the corresponding imaginary parts for unidirectional light propagation with reflection asymmetry: $A(\omega) = 2R_L(\omega)/[R_L(\omega) + R_R(\omega)]$. The choice of the target bandwidth in our optimization depends on the complexity of structure. In the unit cell, with increasing elements, higher wave vectors are introduced, and their coupling must be canceled out to achieve the unidirectional behavior. We select the target bandwidth in different layered system such that the complex permittivity follows the HT. In other words, the zeroing of half spatial spectrum of the designed permittivity profile decides the target bandwidth. The genetic algorithm efficiently finds the imaginary part of complex permittivity distributions for asymmetric light effects and the results are summarized are shown in Fig. 2.

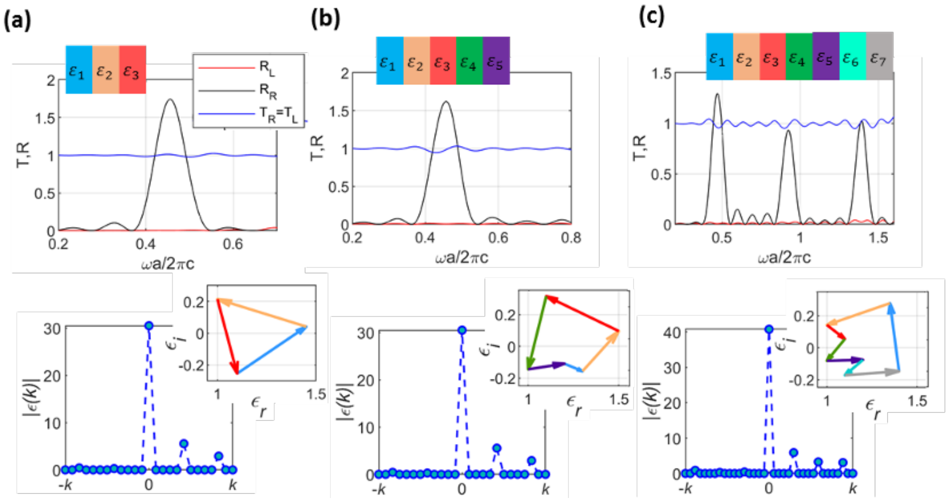

Figure 2. (Top row) Transmission and reflection properties of designed (a) three-layer, (b) five-layer, and (c) seven-layer non-Hermitian structures. (Bottom row) the modulus of spatial spectra for the corresponding designed complex primitivities. The insets depict the vectorial representation of permittivity values in complex plane which form the closed loop due to destructive interference. In all cases, an arbitrary real vector of permittivity is seeded to GA to produce the corresponding imaginary part of complex permittivity that show zero left-reflection with perfect transmission at desired frequencies.

*muriel.botey@upc.edu; donll.upc.edu

The optimized imaginary parts allow perfect transmission from both sides in all cases, but the reflection from the left incident wave is zero, as expected by the target function. In addition, we computed the spatial spectrum of optimized structure to show an asymmetric coupling between wave vectors, which is responsible for asymmetric light propagation. The filtering out of discrete spatial frequency components in left-half spectrum confirms the unidirectional propagation behavior. Interestingly, the optimized permittivity values in the complex plane form a closed loop in the counterclockwise direction, allowing the optimized structure to achieve zero-reflection via destructive interference.

Next, we show that asymmetric reflectivity in periodic structures can be changed from left to right direction (or vice versa) by switching the operating frequency from ω to 2ω. In order to demonstrate that, we consider a five-layer periodic structure and combined the left asymmetry $A_L(\omega) = 2R_L(\omega)/[R_L(\omega) + R_R(\omega)]$ and right asymmetry with step function for target function as: $A(\omega) = H(\omega - \omega_0)A_L(\omega) + H(\omega_0 - \omega)A_R(\omega)$ where $H(\omega)$ is the Heaviside step function. The imaginary part of permittivity is determined with GA for arbitrary given real permittivity, and the results show frequency selective unidirectional in Fig. 3. Inspecting the he spatial spectrum we do not observe half-spectrum suppression but an asymmetric frequency cancellation leading to frequency selective unidirectional left/right propagation. In turn, the insets in Fig. 3 show again the closed loop in the complex permittivity plane.

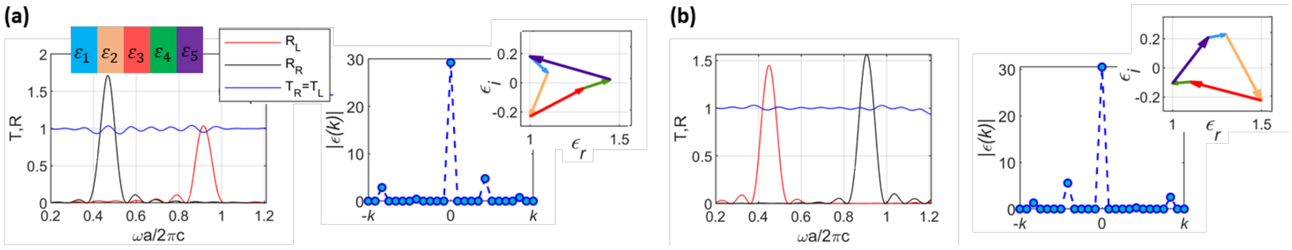

Figure 3. (a) and (b) show two different examples of discrete non-Hermitian five-layer structures holding different directionality at two discrete frequencies, namely $\omega a/2\pi c \approx 0.45$ and $\omega a/2\pi c \approx 0.45$ as shown by the transmission and reflection spectral response. In both examples the frequancy selective assymmetric response is to switch the unidirectional behavior from left to right (or vice versa) is taken: $\omega_o a/2\pi c = 0.7$. The left/right unidirectional behavior can be switched by changing the operating frequency. For both cases, the plots on the right show the spatial spectra of the corresponding complex permittivity where the insets depict the vector representation of the designed primitivities in complex plane.

2.2 1D non-periodic structures

Furthermore, we may develop non-Hermitian planar structures that hold generalized HT with asymmetric reflectivity over a broad frequency range.

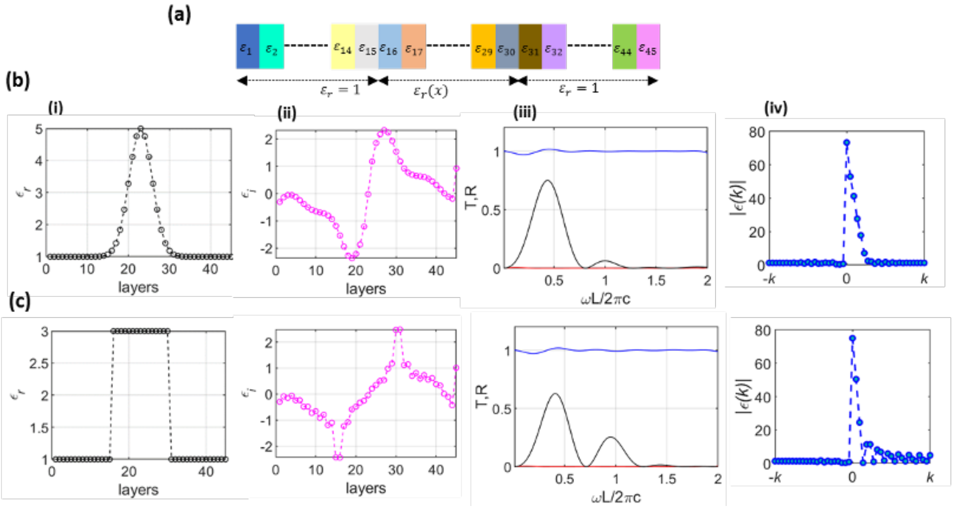

Figure 4. (a) Schematic of forty-five-layers1D non-periodic configuration with different complex permittivity in each spatial domain. GA takes the real permittivity of (b) Gaussian and (c) squared-shape profiles as an input and generate the corresponding imaginary part for broadband unidirectional behavior. (i) real part of given permittivity profile (ii) imaginary

*muriel.botey@upc.edu; donll.upc.edu

part of permittivity (iii) transmission and reflection of designed complex permittivity distributions (iv) respective spatial spectra of the complex primitivity profiles.

To suppress backscattering in the left direction, the environment around the structure is altered by selecting appropriate gain-loss values and results are shown in Fig. 4(b,c). The obtained imaginary permittivity values through genetic optimization confirm that the left reflection is suppressed with perfect transmission as depicted Figs. 4a(iii) and b(iii). The corresponding spectra also reveal that spatial frequencies that contribute to backward propagation are removed [see Fig. 4a(iv) and b(iv)]. The zeroing out of the entire half spectra of the complex permittivity is in agreement with the generalized HT.

3. TWO-DIMENSIONAL SYSTEM

The proposed inverse design approach can be also extended to two dimensional (2D) and higher dimensional non-Hermitian systems. Since in 2D and 3D, there is larger freedom in space, the optimization also allows to construct a "local Hilbert transform" working to different directions in different space positions. Here, we did extend the idea to 2D and demonstrate an example of designing desired directional power to support the validity of our method. By engineering the spatial distribution of the complex refractive index in 2D, there are many directions to break the space symmetry that allow designing the desired direction flow. For instance, if we want to design a 2D non-Hermitian potential $V(\vec{r}) = n_R(\vec{r}) + n_I(\vec{r})$ for "sink-type" directionality i.e., $p(\vec{r}) = -\vec{r}/|\vec{r}|^2$, we define the target function for sink flow, i.e., $F(\vec{r}) = i\left[\left(V(\vec{r})\nabla V^*(\vec{r}) - V^*(\vec{r})\nabla V(\vec{r})\right)/|V(\vec{r})|^2\right] - p(\vec{r})$, which needs to be minimized during optimization. For demonstration, we consider a concentric-ring or hexagonal profiles as seeds for the real part of permittivity and generate the corresponding imaginary distribution with GA. Note that real and imaginary parts form the Hilbert pair in spatial domain for sink directionality. The paraxial model is used to numerically validate the designed sink directionality with non-Hermitian potentials, expressed as;

$$\partial_t A(\vec{r},t) = i\nabla^2 A(\vec{r},t) + iV(\vec{r})A(\vec{r},t) \tag{3}$$

where $A(\vec{r},t)$ is the slowly varying envelop of electric field in 2D space, $\vec{r}(x,y)$, propagating in time, *t*. The complex non-Hermitian potential $V(\vec{r}) = n_R(\vec{r}) + n_I(\vec{r})$ with the given real refractive index $n_R(\vec{r})$ and designed the gain-loss profile $n_I(\vec{r})$ is introduced into Eq. 3 to study the propagation of field. We assume a Gaussian source, initially placed at an arbitrary position within the structure. We observe that for a sufficient integration time the field is efficiently localized around the center resulting in a centered steady state, irrespectively of the initial source location, see Fig. 5.

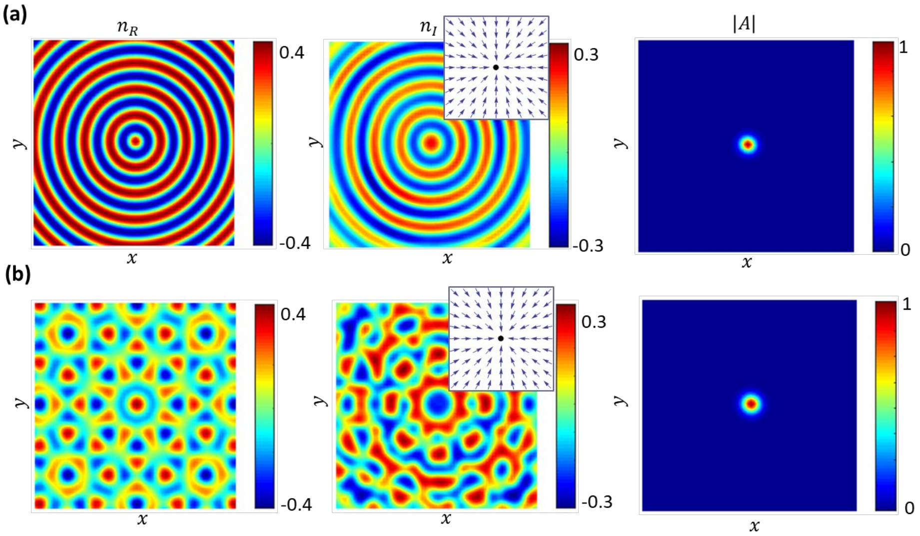

Figure 5. Designed of non-Hermitian potentials for sink-type directionality with two different given seeds (a) concentric-ring pattern (b) hexagonal background pattern. The left column depicts the real part of refractive index, the middle one the imaginary part generated with GA following the directionality field in the inset, and the right column the normalized steady state field profile calculated from paraxial model, for numerical verification. The size of the computational region is 25 λ_0 × 25 λ_0, with operating wavelength λ_0.

4. CONCLUSION

We present a genetic algorithm-assisted inverse design approach for "on demand" light management in periodic and non-periodic non-Hermitian systems. We specifically design various non-Hermitian structures with optimal complex permittivity distribution for unidirectional light propagation in a frequency selective and broad frequency range. We perform spatial spectral analysis to confirm that reflection is suppressed in the desired directions and frequency ranges. The designed structures hold the generalized HT, ensuring a unique solution for any given arbitrary permittivity distribution. In particular, a controllable and frequency selective 1D transport of light along preferred direction is highly desired in photonic integrated circuits. In turn, 2D arrangements particular application on VCSELS and edge-emitting lasers [17]. Beyond optics, the proposed design scheme may be used to inversely design non-Hermitian structures with desired spectral properties in acoustics, plasmonics and elastic wave systems.

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