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RESEARCH ARTICLE

On classifying the non-Tits P-critical posets

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Abstract. In 2005, the authors described all introduced by them P-critical posets (minimal finite posets with the quadratic Tits form not being positive); up to isomorphism, their number is 132 (75 if duality is considered). Later (in 2014) A. Polak and D. Simson offered an alternative way of proving by using computer algebra tools. In doing this, they defined and described the Tits P-critical posets as a special case of the P-critical posets. In this paper we classify all the non-Tits P-critical posets without complex calculations and without using the list of all P-critical ones.

1. Introduction

Quadratic Tits forms play an important role in modern representation theory and its applications. They were first introduced by P. Gabriel for quivers [1].

Let Q be a finite quiver with the set of vertices Q_0 and the set of arrows Q_1. By definition, the *quadratic Tits form of the quiver* Q is the quadratic form $q_Q : \mathbb{Z}^n \to \mathbb{Z}$, $n = |Q_0|$, given by the following equality:

$$q_Q(z) = \sum_{i \in Q_0} z_i^2 - \sum_{i \to j} z_i z_j,$$

where $i \to j$ runs through the set Q_1 (i.e., multiplied by -1, the difference between the number of parameters of all representations of any fixed

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vector-dimension $z = \{z_i \mid i \in Q_0\}$ and the number of parameters of the matrix group acting on these representations). For an undirected graph G the quadratic Tits form q_G is, by definition, the quadratic Tits form of a quiver $Q = Q(G, \varepsilon)$ with some orientation ε on the edges of G (q_G does not depend on choice of ε).

In [1] P. Gabriel proved that, for a connected quiver Q, the following conditions are equivalent:

($1q$) Q is of finite representation type over a field k;

($2q$) the quadratic Tits form of Q is positive;

($3q$) the underlying graph of Q is a (simply faced) Dynkin diagram.

Now let S be a finite poset (without an element 0). By analogy with the Tits quadratic form of a quiver (the counting the number of parameters for matrix representations of posets), *the Tits quadratic form $q_S(z)$ of S* has the following form:

$$q_S(z) := z_0^2 + \sum_{i \in S} z_i^2 + \sum_{i<j, i,j \in S} z_i z_j - z_0 \sum_{i \in S} z_i$$

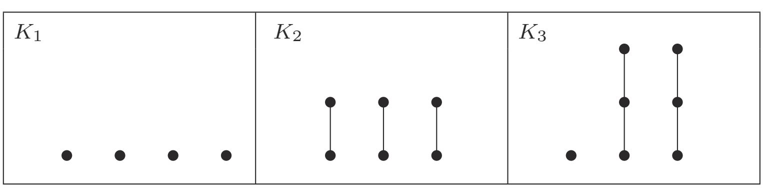
(matrix representations were introduced by L. A. Nazarova and A. V. Roiter [2]; see for more details [3]).

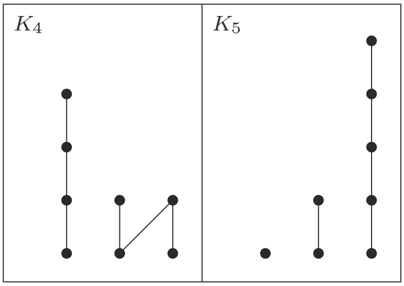
If one talks on finite representational type of posets, the main role is played by weakly positive forms i.e. positive on the set of vectors with non-negative coordinates. Namely, Yu. A. Drozd [4] proved that, for a poset S, the following conditions are equivalent:

($1p$) S is of finite representation type over a field k;

($2p$) the quadratic Tits form of S is weakly positive.

On the other hand, M. M. Kleiner [5] proved that a poset S is of finite representation type if and only if it does not contain (full) subposets K_1–K_5 with the Hasse diagrams of the form





So, for a poset S, the following conditions are equivalent:

$(2p)$ the quadratic Tits form of S is weakly positive;

$(3p)$ S does not contain as subposets the Kleiner's posets K_1–K_5.

A poset S is said to be *WP-critical* if its quadratic Tits form is not weakly positive, but the Tits form of any proper subposet of S is weakly positive. The equivalence of conditions $(2p)$ and $(3p)$ implies that the Kleiner's posets K_1–K_5 form a complete system of WP-critical posets.

Since for posets, in contrast to quivers, the sets of those with weakly positive and with positive Tits forms do not coincide, research related to the positivity of the quadratic Tits form of posets are natural. In particular, posets with positive quadratic Tits form (that are analogs of the Dynkin diagrams).were studied by the authors in [6] – [9] (all such posets were classified in [7]).

In [7] the author also classified (introduced by them) the P-critical posets as the minimal posets with non-positive quadratic Tits form (their number is 132 up to isomorphism and 75 up to isomorphism and duality). More precisely, a poset S is called *P-critical* if the following conditions hold:

(a) the quadratic Tits form $q_S(z)$ of S is not positive;

(b) the quadratic Tits form of any proper subposet of S is positive.

Condition (b) means that if $q_S(z)$ is considered with $z_i = 0$ for any fixed $0 \neq i \in S$, then it is positive.

Later A. Polak and D. Simson [10] offered an alternative way of describing the P-critical posets by using computer algebra tools (mainly symbolic computation in Maple and numeric computation in $C\#$). In doing this, they defined and described the Tits P-critical posets as a special case of the P-critical ones. Namely, a P-critical poset S is said to be a *Tits P-critical* if the quadratic form $q_S(z)$ with $z_0 = 0$ is positive[1].

[1]For an obvious reason, in [10] P-critical posets are called also as almost Tits P-critical posets.

Using a relationship between P-critical and WP-critical posets [7], in this article we describe the non-Tits P-critical posets without using the (previously obtained in [7]) list of all P-critical ones.

2. Main result

Throughout the paper, all posets are assumed to be finite.

A poset T is called *dual* to a poset S and is denoted by S^{op} if $T = S$ as usual sets and $x < y$ in T if and only if $x > y$ in S. When T and T^{op} are isomorphic, the poset T is called *self-dual* (otherwise, *non-self-dual*).

The P-critical posets were described by the authors in [7]. Their number is 132 up to isomorphism and 75 up to isomorphism and duality. Later A. Polak and D. Simson [10] offered an alternative way of describing the P-critical posets by using computer algebra tools; in doing this, they also described the Tits P-critical posets.

In this paper we classify all the non-Tits P-critical posets without using the list of all P-critical ones. Their number is 17 up to isomorphism and 11 up to isomorphism and duality (this is briefly written in the first cell of Table 1 below; self-dual posets are marked by *sd*).

Theorem 1. *Up to duality, the non-Tits P-critical posets are given by Table* 1*.*

From this theorem (and what was said above on all P-critical posets) it follows that the number of the Tits P-critical posets is 115 up to isomorphism and 64 up to isomorphism and duality.

The table of the Tits P-critical posets (as indicated in [7] Table 1 of all the P-critical posets without the posets of the above Table 1) is given in the last section.

3. P-critical posets and minimax equivalence

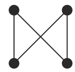
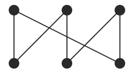
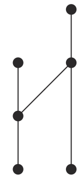
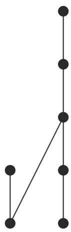
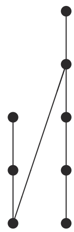
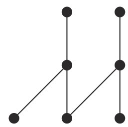
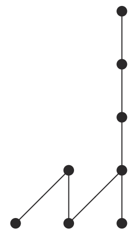
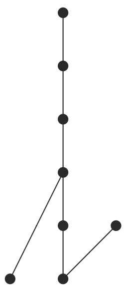
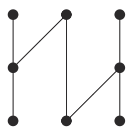
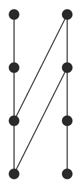
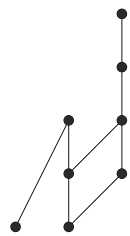
3.1. Minimax equivalence

In this section we recall notation and results from [7].

By a subposet we always mean a full one, and singletons are identified with the elements themselves. Although, by definition, posets are of order $n > 0$, sometimes (in definitions and statements) we admit empty posets which are or may be later subposets of some posets.

Let S be a poset. For a minimal (resp. maximal) element a of S, denote by $T = S_a^{\uparrow}$ (resp. $T = S_a^{\downarrow}$) the following poset: $T = S$ as usual

TABLE 1.

Up to duality=11: self-dual=5 non-self-dual=6 All=17	$nTPC1$ sd 	$nTPC2$ sd 	$nTPC3$ sd 
$nTPC4$ 	$nTPC5$ 	$nTPC6$ 	$nTPC7$ 
$nTPC8$ 	$nTPC9$ sd 	$nTPC10$ sd 	$nTPC11$ 

sets, $T \setminus a = S \setminus a$ as posets, the element a is maximal (resp. minimal) in T, and a is comparable with x in T if and only if they are incomparable in S. A poset T is called *minimax equivalent* or (min, max)-*equivalent* to a poset S, if there are posets $S_1, \ldots, S_p$ ($p \geqslant 0$) such that, if one puts $S = S_0$ and $T = S_{p+1}$, then, for every $i = 0, 1, \ldots, p$, either $S_{i+1} = (S_i)^{\uparrow}_{x_i}$ or $S_{i+1} = (S_i)^{\downarrow}_{y_i}$(this notion was introduced in [11]).

The notion of minimax equivalence can be naturally continued to the notion of *minimax isomorphism*: posets S and S' are minimax isomorphic if there exists a poset T, which is minimax equivalent to S and isomorphic to S'.

The definition of posets of the form $T = S^{\uparrow}_a$ (resp. $T = S^{\downarrow}_a$) can be extended to subposets. Namely, let S be a poset and A its lower (resp.

upper) subposet, i.e. $x \in A$ whenever $x < y$ (resp. $x > y$) and $y \in A$. By $T = S_A^{\uparrow}$ (resp. $T = S_A^{\downarrow}$) we denote the following poset: $T = S$ as usual sets, partial orders on A and $S \setminus A$ are the same as before, but comparability and incomparability between elements of $x \in A$ and $y \in S \setminus A$ are interchanged and the new comparability can only be of the form $x > y$ (resp. $x < y$). Note that S and $S_A^{\uparrow}$ (resp. S and $S_A^{\downarrow}$) are minimax equivalent.

From the definitions we have the following lemma.

Lemma 1. (a) $S_A^{\downarrow} = S_{S\setminus A}^{\uparrow}$; (b) $(S_A^{\downarrow})^{\mathrm{op}} = (S^{\mathrm{op}})_{A^{\mathrm{op}}}^{\uparrow}$.

Corollary 1. *If a poset S is self-dual, then, for any lower subposet B, $S_B^{\uparrow} = (S_{S\setminus B^{\mathrm{op}}}^{\uparrow})^{\mathrm{op}}$.*

Indeed $A = B^{\mathrm{op}}$ is an upper subposet of $S^{\mathrm{op}} = S$ and from equality (b) one has $(S_{B^{\mathrm{op}}}^{\downarrow})^{\mathrm{op}} = S_B^{\uparrow}$, whence, by equality (a) $S_B^{\uparrow} = (S_{S\setminus B^{\mathrm{op}}}^{\uparrow})^{\mathrm{op}}$.

We write $S_{XY}^{\uparrow\downarrow}$ (resp. $S_{YX}^{\downarrow\uparrow}$) instead of $(S_X^{\uparrow})_Y^{\downarrow}$ (resp. $(S_Y^{\downarrow})_X^{\uparrow}$). It is easy to see that $S_{XY}^{\uparrow\downarrow} = S_{YX}^{\downarrow\uparrow}$ if $X < Y$ i.e. $x < y$ for any $x \in X$, $y \in Y$.

The main motivation for introducing the notion of minimax equivalence is the fact that the quadratic Tits forms of minimax equivalent posets are $\mathbb{Z}$-equivalent. This follows from the next proposition.

Proposition 1. *Let S be a poset and let $T = S_A^{\uparrow}$ or $T = S_A^{\downarrow}$. Then $q_S(z) = q_T(z')$, where $z'_0 = z_0 - \sum_{a \in A} z_a$, $z'_x = -z_x$ for $x \in A$ and $z'_x = z_x$ for $x \notin A$.*

In [7] we indicated a three-step algorithm for finding all (up to isomorphism) posets minimax isomorphic to a given poset S:

Step I. Describe the set $L = L(S)$ of lower subposets $X \neq S$ of S ($\varnothing \in L$), and for all such X to build the posets $S_X^{\uparrow}$.

Step II. Describe the set $LU_< = LU_<(S)$ of pairs (X, Y) consisting of proper lower and upper subposets X, Y of S such that $X < Y$, and for them to build the posets $S_{XY}^{\uparrow\downarrow} (= S_{YX}^{\downarrow\uparrow})$.

Step III. Among the posets constructed in I and II choose a complete system of pairwise non-isomorphic ones.

This algorithm is denoted by MM-ALG.

We call subposets X and Y (of a poset S) having the form, indicated in I, *Aut-equivalent* if $\varphi(X) = Y$ for some automorphism φ of S. Similarly, the indicated in II pairs (X, Y) and (X', Y') are called *Aut-equivalent* if, for some automorphism φ, $\varphi(X) = X'$ and $\varphi(Y) = Y'$.

If elements of the sets L and $LU_<$ at the Steps I and II of Algorithm MM-ALG are taken up to *Aut*-equivalent (we will denote them $L^\circ = L^\circ(S)$

and $LU^\circ_< = LU^\circ_<(S)$), the corresponding algorithm is denoted by MM°-ALG; the modified steps of the new algorithm are denoted I°, II° and III°.

3.2. Relationship between *P*-critical and *WP*-critical posets

P-critical posets were introduced and studied in detail for the first time in [7][2]. Recall also that a poset S is called WP-critical if its quadratic Tits form is not weakly positive, but the Tits form of any proper subposet of S is weakly positive. The Kleiner's posets K_1–K_5 form a complete system of WP-critical posets. In practice, it is natural to single out one representative from each isomorphism class. Up to isomorphism, the Kleiner's posets are of the form

$K_1 = \{1, 2, 3, 4\}$ (without relations),
$K_2 = \{1, 2, 3, 4, 5, 6 \,|\, 1 \prec 2, 3 \prec 4, 5 \prec 6\}$,
$K_3 = \{1, 2, 3, 4, 5, 6, 7 \,|\, 2 \prec 3 \prec 4, 5 \prec 6 \prec 7\}$,
$K_4 = \{1, 2, 3, 4, 5, 6, 7, 8 \,|\, 1 \prec 2 \prec 3 \prec 4, 5 \prec 6, 7 \prec 8, 5 \prec 8\}$,
$K_5 = \{1, 2, 3, 4, 5, 6, 7, 8 \,|\, 2 \prec 3, 4 \prec 5 \prec 6 \prec 7 \prec 8\}$[3].

For a poset S of order n, define the *kernel of* $q_S(z)$ as follows: $\operatorname{Ker} q_S(z) := \{u \in \mathbb{Z}^{n+1} \,|\, q_S(u) = 0\}$. The following statement shows that, for each Kleiner's poset K, the kernel of $q_K(z)$ is an infinite cyclic group.

Proposition 2. *Put* $q_i(z) := q_{K_i}(z)\,(1 \leqslant i \leqslant 5)$. *The quadratic forms* $q_i(z)$ *are non-negative and*

$\operatorname{Ker} q_1(z) = (2, 1, 1, 1, 1)\mathbb{Z}$,
$\operatorname{Ker} q_2(z) = (3, 1, 1, 1, 1, 1, 1)\mathbb{Z}$,
$\operatorname{Ker} q_3(z) = (4, 2, 1, 1, 1, 1, 1, 1)\mathbb{Z}$,
$\operatorname{Ker} q_4(z) = (5, 1, 1, 1, 1, 1, 2, 2, 1)\mathbb{Z}$,
$\operatorname{Ker} q_5(z) = (6, 3, 2, 2, 1, 1, 1, 1, 1)\mathbb{Z}$.

Indeed, the relations $\supseteq$ are verified by direct calculations, and then the relations $=$ follows from the results of [13]; non-negativity also follows from [13][4] (see also [15]).

Corollary 2. *Let* $v = (v_0, v_1, \ldots, v_m) \in \operatorname{Ker} q_K(z)$ *with* K *to be a Kleiner's poset. Then* $2v_0 = v_1 + v_2 + \ldots + v_m$.

[2] In fact, a little earlier under the name of critical posets they were first considered in [12].

[3] K_4 and K_5, as notation, often change places in other papers.

[4] See in both cases Theorem 2 of Sect. 1.0 [14], in which the main results of this paper are summarized.

Theorem 2 (Theorem 2 [7]). *A poset S is P-critical if and only if it minimax isomorphic to a Kleiner's poset.*

From Propositions 1, 2 and Theorem 2, we have the following corollary.

Corollary 3. *Let S be a P-critical poset of order n. Then the quadratic Tits form of S is non-negative and* $\operatorname{Ker} q_S(z) = v\mathbb{Z}$ *for some vector $v = (v_i)_{i\in 0\cup S} \in \mathbb{Z}^{n+1}$ with $v_i \neq 0$ for any $i \neq 0$, which is determined by S uniquely up to the sign,*

4. Proof of Theorem 1

4.1. 0-balanced subposets

The vector $v = (v_i)_{i\in 0\cup S} \in \mathbb{Z}^{n+1}$ specified in Corollary 3 is denoted by $v^S = (v_i^S)_{i\in 0\cup S}$[5]. A subposet X of a P-critical poset S is said to be *small* if $|X|$ does not exceed the integer part of $(|S|+1)/2$, and 0-*balanced* if $v_0^S = \sum_{i\in X} v_i^S$. From these defininitions and Proposition 2 it follows the next two lemmas.

Lemma 2. *For any Kleiner posets K_i $(1 \leqslant i \leqslant 5)$, there are no 0-balanced subposets of the form $X = A \cup B$, $A < B$.*

Lemma 3. *For Kleiner posets K_i, small 0-balanced lower subposets A_{ij} are exhausted, up to automorphism (of K_i), by the following ones:*

(1) $A_{11} = \{1,2\}$;
(2) $A_{21} = \{1,2,3\}$, $A_{22} = \{1,3,5\}$;
(3) $A_{31} = \{1,2,3\}$, $A_{32} = \{1,2,5\}$;
(4) $A_{41} = \{1,2,3,7\}$, $A_{42} = \{1,2,5,6\}$, $A_{43} = \{1,2,5,7\}$,
$A_{44} = \{1,5,7,8\}$, $A_{45} = \{5,6,7\}$;
(5) $A_{51} = \{1,2,4\}$, $A_{52} = \{1,4,5,6\}$, $A_{53} = \{2,3,4,5\}$.

The following lemma can be proved by complex enumeration of all cases (using the previous lemma), but it follows directly from Corollaries 1 and 2 (by Corollaries 2, for any Kleiner's poset K, subposets X and $K \setminus X$ at the same time are or are not 0-balanced).

Lemma 4. *Let X be a non-small 0-balanced lower subposet of a Kleiner's poset $K = K_i$. Then there is a small 0-balanced lower subposet Y of K such that $K_X^{\uparrow} = (K_Y^{\uparrow})^{\mathrm{op}}$.*

[5]A little ambiguity of the vector v^S is not essential (one can, for example, fix $v_s \neq 0$ and choose v^S with $v_s^S > 0$).

4.2. The application of Algorithm MM°-ALG

From Corollary 3 (taking into account the definitions of P-critical and Tits P-critical posets) it follows the next statement.

Proposition 3. *A P-critical poset S is not Tits P-critical if and only if $v_0^S = 0$.*

By Theorem 2 and Proposition 1, 3, to prove Theorem 1 it is enough to show that the set of all posets obtained as a result of applying Algorithm MM°-ALG to all 0-balanced lower subposets X (see Step I°) and 0-balanced pairs of subposets (X, Y) (see Step II°) of all the Kleiner's posets $K = K_1, \dots, K_5$ coincides, up to isomorphism and duality, with the set of all posets from Table 1.

It is follows from Lemma 2 that Step II° of the algorithm is in this case empty, and by Lemma 4 on the Step I° one can take only small subposets. So, it remains for us to calculate the posets K_A with $K = K_1, \dots, K_5$ and A running the subposets indicated in Lemma 3, and then compare them with the posets of Table 1. We have:

for $K = K_1$, $A = A_{11}$, the poset K_A is isomorphic to $nTPC1$;
for $K = K_2$, $A = A_{21}$, the poset K_A is isomorphic to $nTPC3$;
for $K = K_2$, $A = A_{22}$, the poset K_A is isomorphic to $nTPC2$;
for $K = K_3$, $A = A_{31}$, the poset K_A is isomorphic to $nTPC4^{\mathrm{op}}$;
for $K = K_3$, $A = A_{32}$, the poset K_A is isomorphic to $nTPC6^{\mathrm{op}}$;
for $K = K_4$, $A = A_{41}$, the poset K_A is isomorphic to $nTPC11$;
for $K = K_4$, $A = A_{42}$, the poset K_A is isomorphic to $nTPC10$;
for $K = K_4$, $A = A_{43}$, the poset K_A is isomorphic to $nTPC9$;
for $K = K_4$, $A = A_{44}$, the poset K_A is isomorphic to $nTPC11^{\mathrm{op}}$;
for $K = K_4$, $A = A_{45}$, the poset K_A is isomorphic to $nTPC8^{\mathrm{op}}$;
for $K = K_5$, $A = A_{51}$, the poset K_A is isomorphic to $nTPC7^{\mathrm{op}}$.
for $K = K_5$, $A = A_{52}$, the poset K_A is isomorphic to $nTPC5^{\mathrm{op}}$;
for $K = K_5$, $A = A_{53}$, the poset K_A is isomorphic to $nTPC5$.

Thus, the set of all posets of the form K_A coincides, up to isomorphism and duality, with the set of posets from Table 1.

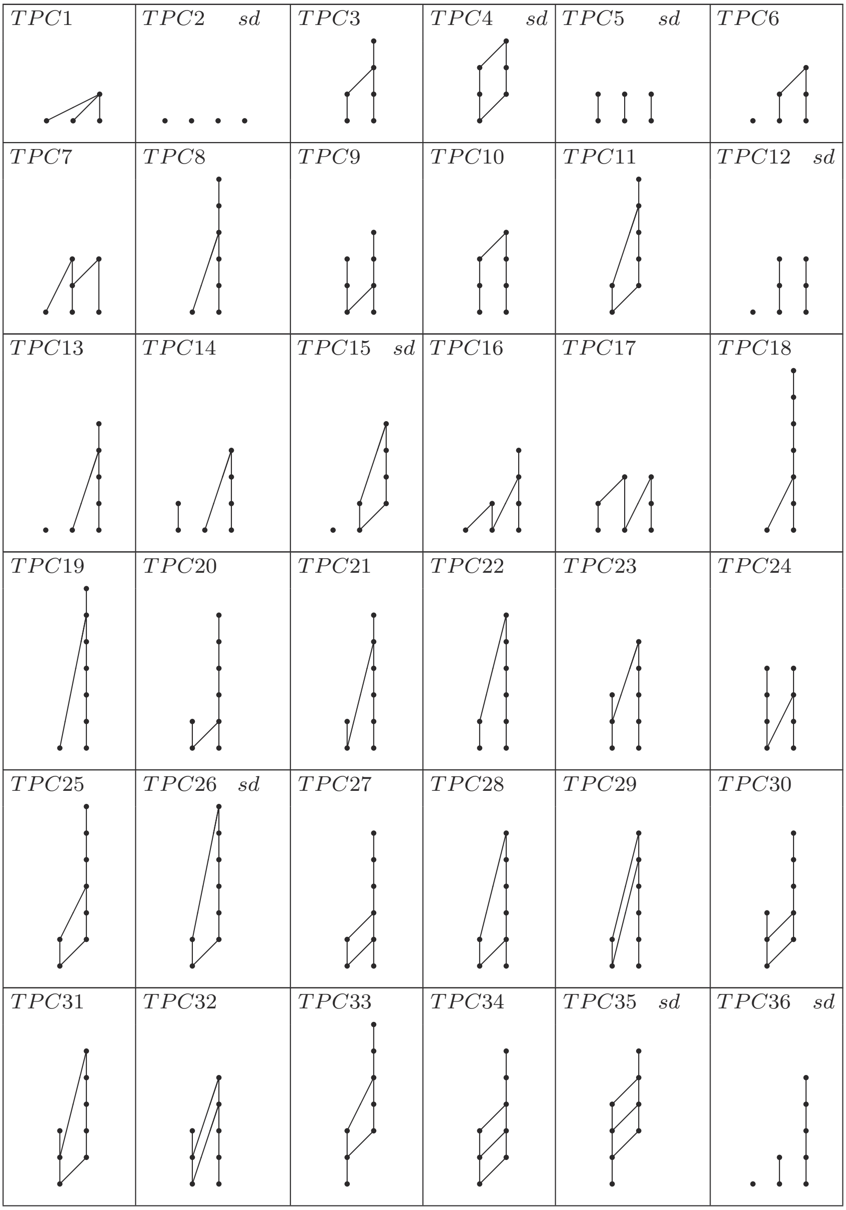
Theorem 1 is proved.

5. The table of the Tits P-critical posets

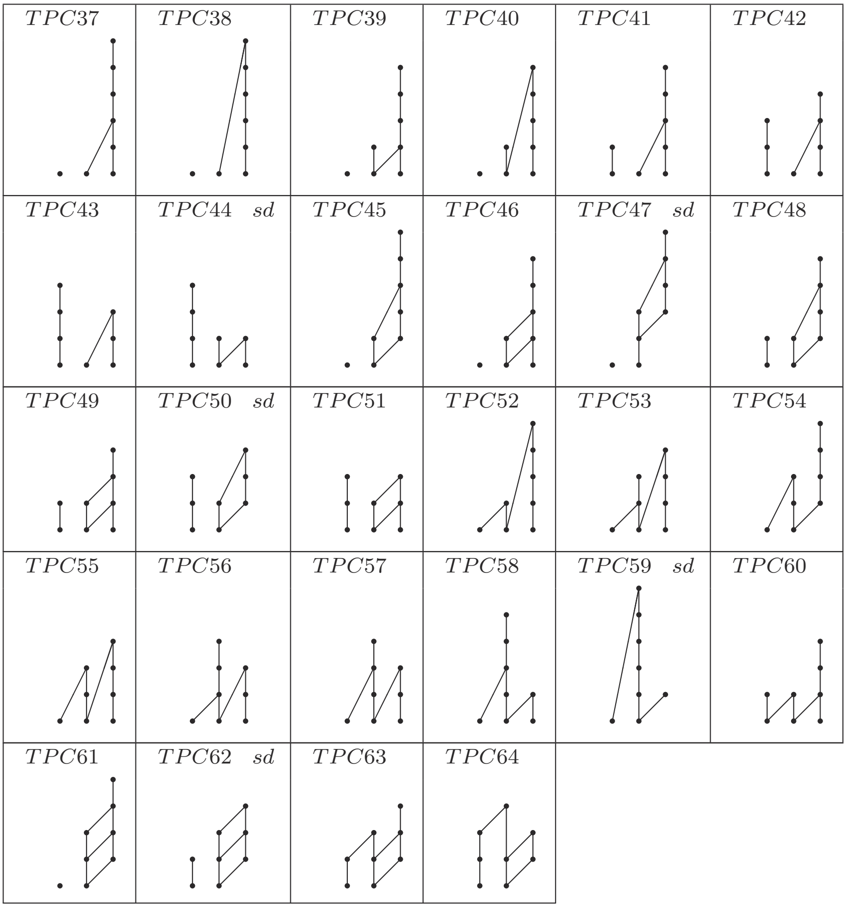
In this section we write out a table (Table 2) of all Tits P-critical posets up to isomorphism and duality, i.e. Table 1 [7] (see also [16]) of the P-critical posets without the posets from Table 1 of Section 1.

Table 2 contains 64 posets; self-dual posets are marked (in the upper right corners) by *sd*. If we add all the posets dual to unmarked ones, we obtain the Tits P-critical posets up to isomorphism; their number is 115.

TABLE 2.



$TPC1$	$TPC2$ *sd*	$TPC3$	$TPC4$ *sd*	$TPC5$ *sd*	$TPC6$
$TPC7$	$TPC8$	$TPC9$	$TPC10$	$TPC11$	$TPC12$ *sd*
$TPC13$	$TPC14$	$TPC15$ *sd*	$TPC16$	$TPC17$	$TPC18$
$TPC19$	$TPC20$	$TPC21$	$TPC22$	$TPC23$	$TPC24$
$TPC25$	$TPC26$ *sd*	$TPC27$	$TPC28$	$TPC29$	$TPC30$
$TPC31$	$TPC32$	$TPC33$	$TPC34$	$TPC35$ *sd*	$TPC36$ *sd*



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