Optimalisatie van de energie-efficiënte werking
van grootschalige drinkwaternetten: modellen en algoritmes

Optimization of Large-Scale Water Supply Networks
for Energy Efficient Operations: Models and Algorithms

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Promotor: prof. dr. E.-H. Aghezzaf
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A certain type of perfection can only be realized through a limitless accumulation of the imperfect.

– Haruki Murakami, *Kafka on the Shore*

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Gent, september 2015
Derek Verleye

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List of Acronyms

B

B&B	Branch & Bound

C

CPU	Central Processing Unit

D

DE	Differential Evolution
DP	Dynamic Programming
D-W	Darcy-Weisbach

F

FP	Feasibility Pump

G

GA	Genetic Algorithm
GBD	Generalized Benders Decomposition
GIS	Geographic Information System
GRG	Generalized Reduced Gradient

H

H-W	Hazen-Williams

I

IP	Integer Program

L

LB	Lower bound
LP	Linear Program

M

MILP	Mixed Integer Linear Program
MINLP	Mixed Integer Nonlinear Program

N

NLP	Nonlinear Program
NP-hard	Non-deterministic Polynomial-time hard

O

OA	Outer Approximation

P

PWL	Piecewise Linear Approximation

R

RMP	Reduced Master Problem

S

SP	Subproblem

U

UB	Upper bound

V

VSP	Variable Speed Pump

W

WPC	Water Production Center

Nederlandse samenvatting

Als gevolg van de steeds toenemende kost van waterbehandeling en distributie waarmee drinkwaterbedrijven worden geconfronteerd, neemt het belang van kost-efficiënte waterproductie en -levering toe. Efficiëntie binnen een waterproductie en -distributienet legt zich niet enkel toe op kostenbesparing tijdens de productiefase maar ook op het doeltreffend transport naar elk leveringspunt. Na de initiële ontwerpfase van het toevoernet, kan een optimaal beheer van de daaropvolgende operationele activiteiten aanleiding geven tot significante besparingen van de betrokken middelen. Zo is bijvoorbeeld het hoofdnetwerk van Berlijn gekoppeld aan een elektronisch optimalisatiesysteem [4, 5] waarbij productie- en energiekosten worden geminimaliseerd, terwijl het pijpleidingnet in Pittsburgh en Adelaide is uitgerust met een softwarepakket dat suboptimale pompconfiguraties berekent [6, 7]. In [8] stelt de auteur een geïntegreerde optimalisatiemethode voor waarin productie en distributie gelijktijdig geanalyseerd worden, wat resulteert in een substantiële verbetering van de nettowinst.

Tegenwoordig wordt de planning van de dagelijkse productie en distributie in vele drinkwatermaatschappijen ondersteund door simulatiesoftware zoals EPANET [9]. Hiermee worden de waarden van de debieten en drukken in het netwerk iteratief geüpdatet door een zgn. ‘gradient based method’. Dit is ook het geval voor drinkwatermaatschappijen in Vlaanderen.

In tegenstelling tot simulatie kunnen optimalisatiemethoden voor waterproductie en -distributie drinkwatermaatschappijen helpen om hun middelen wijselijk te besteden, zowel op economisch als ecologisch vlak. Optimale operationele oplossingen voor watertoevoernetwerken resulteren ontegensprekelijk in lagere productiekosten en een efficiënter gebruik van netwerkpompen. Recent is het onderzoek naar zulke methodes toegenomen, maar vaak zijn de bestudeerde netwerken relatief klein en niet representatief voor realistische netwerken.

Aangezien de wereldbevolking aan een gestaag tempo toeneemt, wordt de schaarsheid van water een steeds belangrijker gegeven. Daarom is een generiek operationeel ondersteuningsmodel voor waterbeheer onontbeerlijk. Dit model dient niet alleen kostenefficiënt te zijn, maar moet ook overal in het netwerk de watervoorziening garanderen en kunnen reageren op veranderingen in de infrastructuur en de vraag naar water. Talrijke modellen zijn reeds voorgesteld in voorgaand onderzoek, maar meestal zijn deze gericht op kleinere netwerken. Voor het bestuderen van grotere netwerken zijn echter technieken nodig die op efficiënte wijze de structuur van het netwerk benutten om oplossingen van hoge kwaliteit te bekomen binnen redelijke rekentijd.

Eén van de doelstellingen van dit werk is om een model te bekomen dat voldoet aan deze eisen en voornamelijk gebaseerd is op het Vlaamse drinkwaternet maar potentieel gebruikt kan worden voor allerlei types drinkwaternetwerken. Aan de basis van dit model ligt een 'minimum cost flow' probleem met bijkomende restricties die eigen zijn aan een drinkwaternetwerk. Meerbepaald omvat het model restricties die de ingewikkelde instroomwerking aan buffers en de frequentiegestuurde pompcurves modelleren met behulp van binaire variabelen. Ladingsverliezen worden gemodelleerd met de formule van Prandtl-Kármán voor de wrijvingsfactor, wat leidt tot een nauwkeurig model over een groot gebied van mogelijke waarden voor het debiet. Daarnaast is deze formule rekenkundig gemakkelijker te evalueren in vergelijking met andere formules. Om een productieschema te evalueren op zijn juistheid wordt een doelfunctie opgesteld die bestaat uit de productiekost (per waterproductiecenter) en de energiekost van elke pomp. Daarenboven kunnen drinkwaterbedrijven onderling water aan- of verkopen op plaatsen waar hun netwerken elkaar ontmoeten. Deze kost vertegenwoordigt een derde term in de doelfunctie. Twee subnetwerken van het eigenlijke net dat in gebruik is door De Watergroep worden aangewend als testnetwerken voor dit wiskundige model. Als bestaande solvers worden losgelaten op dit model, blijkt het vinden van een (globaal) optimale oplossing binnen redelijke tijd een zeer zware taak. Daarom worden andere methodes voorgesteld in dit werk.

De niet-lineaire restricties die gebruikt worden om ladingsverliezen te modelleren, kunnen door een stuksgewijze lineaire benadering vervangen worden. Ook pompcurves kunnen op die manier benaderd worden. Om goede oplossingen te vinden is het belangrijk om de maximale benaderingsfout in elk interval te reduceren. Dit kan bereikt worden door het aantal intervallen te vergroten, wat onvermijdbaar leidt tot een zeer groot aantal binaire variabelen en restricties. Het aantal binaire variabelen kan logaritmisch worden verminderd middels een efficiëntere aanpak waarbij het aantal intervallen op een binaire vector wordt geprojecteerd. Aangezien het model frequentiegestuurde pompen bevat, wordt een apart model voorgesteld voor multivariate functies. In dit model is de convergentiesnelheid echter afhankelijk van de beoogde nauwkeurigheid, waardoor een lage benaderingsfout pas bereikt kan worden na een significant grote rekentijd.

Om de hoge rekentijd tegen te gaan, wordt het voorgestelde model gekoppeld aan een 'gradient method', een methode die courant gebruikt wordt in simulatiesoftware voor watertoevoernetten. In tegenstelling tot deze standaardsimulaties, wisselt de voorgestelde hybride methode informatie uit waardoor de productie- en distributiekosten geminimaliseerd worden. De (suboptimale) oplossing van het gelineariseerde model kan immers gebruikt worden als startpunt voor de gradient method, waarbij de waarden voor de debieten iteratief worden gewijzigd tot de approximatie is weggewerkt en de oplossing overeenkomt met de oorspronkelijke niet-lineaire functies. De resultaten op de testnetwerken vertonen een duidelijke verbetering ten opzichte van deze bekomen met het oorspronkelijke model, hoewel de grootte van het netwerk een snelle convergentie van de methode kan verhinderen.

In [10] werden de 'Generalized Benders Decomposition (GBD)' en 'Outer Approximation (OA)' methodes getest op modellen voor waterbeheer. De modellen in desbetreffende paper zijn nietconvexe MINLP's met binaire variabelen en bilineaire functies, maar kunnen eenvoudig worden gelineariseerd zodat het globaal optimum kan gevonden worden. Aangezien het masterprobleem van de OA methode veel groter is dan dat van de GBD methode, besluiten de auteurs dat de GBD methode efficiënter is wanneer grote modellen dienen te worden opgelost. Een interessante werkwijze wordt gebruikt in [11], waar de auteurs GBD gebruiken op twee niet-lineaire modellen voor waterbeheer. Aangezien de functies binnen deze modellen niet-separabel zijn is het vinden van een globaal optimum geen zekerheid. In onderhavig werk wordt deze werkwijze uitgebreid naar een model dat niet enkel niet-lineaire restricties bevat, maar ook binaire variabelen voor pompactivaties en bufferwerking. Daarenboven zijn sommige pompen frequentiegestuurd waardoor de selectie van bemoeilijkende variabelen een zware taak wordt. Slackvariabelen worden toegevoegd zodat het subprobleem altijd een toegelaten oplossing bevat en vanwege de niet-separabiliteit van de niet-lineaire functies worden approximatiecuts toegevoegd aan het masterprobleem. De methode convergeert maar is erg afhankelijk van de keuze van bemoeilijkende variabelen. Daarenboven hebben de keuze van bepaalde restricties en de grootte van de 'penalty factor' een significante invloed op de rekentijd. Resultaten van experimenten op de twee netwerken worden uitgebreid besproken.

English summary

As a result of an ever increasing cost of water treatment and distribution drinking water companies are confronted with, they need to produce water and deliver it to their customers as efficiently and cost-effectively as possible. Efficiency in a water production and distribution network is concerned with water related cost savings during its production phase as well as its efficient transportation to each final delivery point. After the initial design phase of the water supply network, an optimal management of the subsequent operational activities can lead to significant savings of the involved resources. As an example, the Berlin network is coupled with an electronic optimization system [4, 5] that minimizes production and energy costs, while the piping systems in the cities of Pittsburgh and Adelaide have been equipped with a software package for calculating a suboptimal pumping configuration [6, 7]. In [8] the author proposes an integrated optimization approach for which production and distribution are analyzed simultaneously, which results in a substantial improvement of the net profit.

Nowadays many water companies still plan daily production and distribution in their water supply network by using simulation software such as EPANET [9]. These tools seek to iteratively update the values of flows and pressures in the network by using an iterative gradient based method. This is also the case for water companies in Flanders, Belgium.

Unlike simulation, optimization approaches for water supply production and distribution can assist drinking water companies to wisely use their water resources, both economically and ecologically. Undeniably, optimal operating solutions for water supply networks yield lower production costs and more efficient usage of network pumps. Recently more research is being devoted to this type of optimization, but many of the common benchmark networks are relatively small and not representative of a real-world network.

Because of the never-decreasing population, the scarcity of water becomes increasingly important. Therefore it is useful to obtain a generic operational support model that is cost efficient, guarantees water supply in all parts of the network and can react on changes in water demand and infrastructure. Many models have been proposed in the literature, but they are usually aimed at small networks. When larger networks are envisaged however, techniques that efficiently exploit the structure of the network are required to obtain high-quality solutions in reasonable computational time.

One of the aims of this work is to propose such a model that is targeted at the Flemish water supply networks but potentially covers many networks worldwide. At the basis of this model lies a minimum cost flow problem. Specific constraints include complicated inflow mechanics at buffer entrances and variable speed pumps that require additional binary variables. Pressure loss equations are modeled using the formula of Prandtl-Kármán for the friction factor, which leads to an accurate model for most flow ranges while being computationally inexpensive. To evaluate a valid production plan, the objective function consists of the production cost (per water production center) and an energy cost for each pump. Furthermore water companies can exchange water at the borders of their networks, which represents a third cost term. The resulting model is a nonconvex Mixed Integer Nonlinear Program (MINLP). Two subnets of the water network of De Watergroep, a major drinking water company in Flanders, Belgium, serve as case studies for this realistic model. When they are modeled and tackled using established MINLP solvers, finding a (global) optimal solution within reasonable computational time is a very hard task. Therefore different methods are proposed.

Here, a piecewise linear (PWL) approach is used on the pressure loss equality constraints in pipes and pumps. The pump curves are approximated in a similar way. In order to find good solutions, it is important to reduce the maximum error on each interval. This can be achieved at the price of a large number of intervals. However, this inevitably leads to a very large number of additional binary variables and constraints. The number of binary variables in the PWL approximation is reduced logarithmically by using an efficient approach that projects the number of intervals on a binary vector. Since the model contains variable speed pumps, an additional model is proposed for multivariate functions. The convergence speed clearly depends on the desired accuracy. For a low (acceptable) maximum error the computation time is still significant.

As a way to remedy the long computational time, the above method is coupled with a gradient algorithm, a typical method used in water supply simulation software. Whereas in those standard methods no cost optimization is done, the hybrid approach proposed here will effectively exchange information in such a way that costs are minimized. The (suboptimal) solution of the PWL model can be used as a starting point for the gradient method, that will gradually alter the flows until the approximations are repaired and the solutions correspond with the original nonlinear equations. The results on several test networks show a clear improvement over the MINLP model, although the network size may still restrict fast convergence of the method.

In [10] both a Generalized Benders Decomposition (GBD) method and an outer approximation (OA) method to solve water resource models were tested. The proposed models are nonconvex MINLP's with binary variables and bilinear functions that are however easily linearized so that the global optimum can be found. Since the master problem of the OA method is much larger than that of the GBD method, the authors stated that the GBD method was generally more efficient than OA for solving large MINLP problems. An interesting Benders decomposition approach was also used on two different nonlinear water resource models where finding the global optimum is no longer certain since the functions are nonseparable [11]. In this dissertation the approach is extended to a model which not only contains nonlinear pressure loss equality constraints, but also binary variables to control pumps and water exchange at the buffers. Furthermore, some of these pumps are variable speed pumps, which makes the selection of complicating variables an even harder task. Slack variables are used to make the subproblem always feasible and because of the nonseparability of the nonlinear functions, approximation cuts are added to the master problem. The method converges, but is greatly dependent on the choice of the complicating variables. Additionally, the choice of restrictions to which the slack variables are added and the penalty factor significantly impact the computational time. Results of the experiments on the two networks are shown and discussed.

1
Introduction

1.1 Research motivation and challenges in drinking water production and distribution systems

Effectively and efficiently managing the production and distribution of drinking water is becoming a vital issue from economic, social, as well as environmental perspectives. Reliable production and distribution of drinkable water is a strategic problem that is growing to become more important over time. The EU water framework directive states that the correct usage and management of water on earth will be one of the biggest challenges of the 21st century [12]. For example, the yearly average ground water extraction is not allowed to exceed the import. As a result, alternative sources have to be used in an efficient way [13]. Water treatment and its transport are becoming more and more expensive as a result of chemical and microbial contamination of natural water bodies and the steadily increasing cost of energy. Moreover, the upward trend of the densely populated urban areas has changed the patterns of the drinking water demand, making the current water production and supply infrastructure expensive to operate. As a consequence, many water supply companies would benefit from decision support tools to efficiently manage operations of their water supply networks instead of relying on simulation software and the experience of their operators. Research in this direction has shown that 'optimally' (re)designing and efficiently managing these, usually very old, water supply networks enables the companies to achieve significant water and energy savings, which ultimately result in important financial savings [14].

Furthermore, this imperative requirement of efficiency and its resulting financial savings enable companies to provide their consumers with a high quality service at a minimum cost. The latter can then be used as a competitive advantage. A couple of successful implementations have been accomplished: the Berlin network is coupled with an electronic optimization system [4, 5] that minimizes production and energy costs. A software package for calculating a pumping configuration at a minimal cost has been coded for the city of Pittsburgh [6]. The piping system in Adelaide has been upgraded with a similar system [7].

The process of optimizing and monitoring water production and distribution in large water-supply networks involves several equally important phases, starting from the strategic design phase and ending with the operational planning phase (see among others [15], [16]). However, the network design phase and network operational planning phase are the two major and critical ones in this process, and most research papers focus on either one of these stages. Whereas the actual objective of the studied networks vary, they have many restrictions in common. The optimal design phase of a water network considers various aspects such as optimal layout of the network, optimal dimensions of the systems components (pipes, pumps, valves, reservoirs, etc). The optimization process here involves a very large number of combinations of pipe materials, diameters, pumping stations locations and capacities which makes the underlying optimization model for this phase rather complex. The optimal operational planning phase of water production and distribution in a designed water-supply network involves operational parameters such as hydraulic pressure zone boundaries, demand patterns, control valve settings, and pump operating schedules. The relation of the flow to the pressure is modeled using energy conservation or pressure loss equations, possibly joined by characteristic pump functions. This leads to complex formulations with nonlinear restrictions and binary variables and corresponding large computational times. Dropping or relaxing some of these constraints leads to simplistic models that are often not usable in practice. Furthermore the test instances that researchers focus their research on are very small and not representative for real-world networks. These models are not sufficiently generic for implementation in other drinking water networks and in particular the one of Flanders, Belgium. This network contains variable speed pumps and complex buffer mechanics over multiple periods, which are usually not considered. Given these limitations, the need for complete and realistic model formulations and mathematical techniques to solve these, rises.

For historical reasons, the Flemish drinking water network is being managed by a number of different operators (see figure 1.1). The policy objectives of the Flemish government mention the desire for mutual cooperation and the valorization of acquired expertise with the aim of efficiency gain [17]. It is beyond doubt that one single model for the full water supply network in Flanders will not only guarantee drinking water delivery but will also lower operational costs by more efficient usage of the available water sources.

These challenges bring with them the need for a generic operational support model that is cost efficient, guarantees water supply in all parts of the network and can react on changes in water demand and infrastructure. Such a model can potentially motivate cooperation between the different drinking water operators because mutual management of some parts of the network can lead to a more efficient flow configuration which results in lower operational costs. In 2010 this research started as a Master Thesis that had the goal to develop a first model for part of the drinking water network in West-Flanders operated by De Watergroep (which at that time was called VMW, Vlaamse Maatschappij voor Watervoorziening). A network configuration with a suboptimal cost was generated using state-of-the-art solvers [18]. When larger networks are envisaged however, techniques that efficiently exploit the structure of the network are required to obtain high-quality solutions in reasonable computational time.

1.2 The optimal water supply system operations network problem

This section focuses on the operational aspects of water supply systems and briefly introduces a way of modeling this. In what follows, we consider a fixed-topology network i.e. the network is already designed so that no expansions/alterations on the network will be done.

A general network for this purpose can be seen in figure 1.2.

In general water supply systems, we distinguish 2 major decisions that have to be made. In each water production center (WPC), a certain amount of water is to be produced either for storage or to be injected in the distribution net. The total amount of water produced over all WPCs has to be sufficient to satisfy the total demand in the network. The amount is limited by the production capacity and every WPC has a different production cost. A second decision concerns pump operations. During each period, the status (on/off) and speed setting of the pump has to be determined. Associated with this operation are limits imposed by the pump curve and energy consumption costs.

The general model for water supply operations is based on the minimal cost flow problem [19, 20]. Given is a directed graph $\mathcal{G} = [\mathcal{N}, \mathcal{A}]$, where $\mathcal{N}$ denotes the nodes and $\mathcal{A}$ represents the arcs. An arc (i, j) in a directed graph is an ordered pair in which i is the start node and j the end node. In this network it is assumed that two nodes are never directly connected by more than one arc.

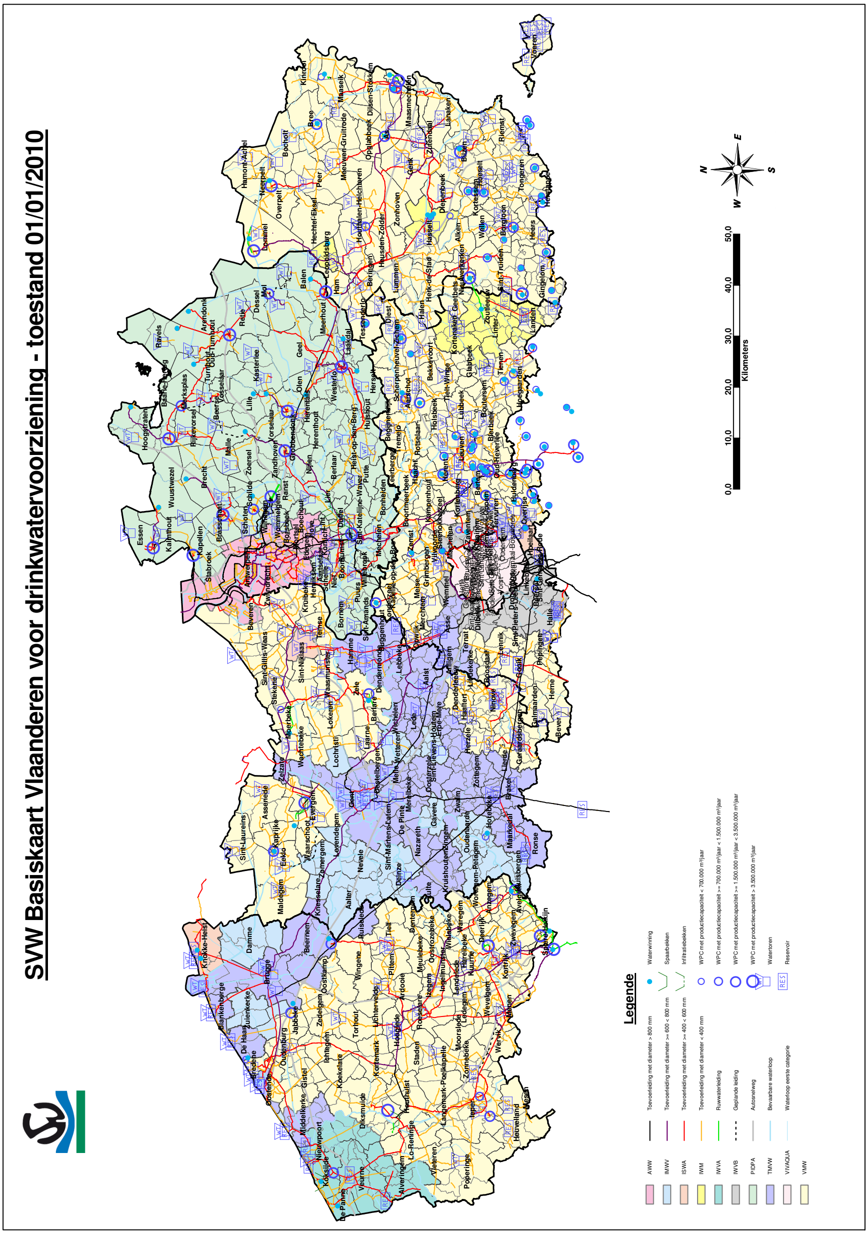


Figure 1.1: SVW map for water supply in Flanders. Each color depicts a different drinking water company

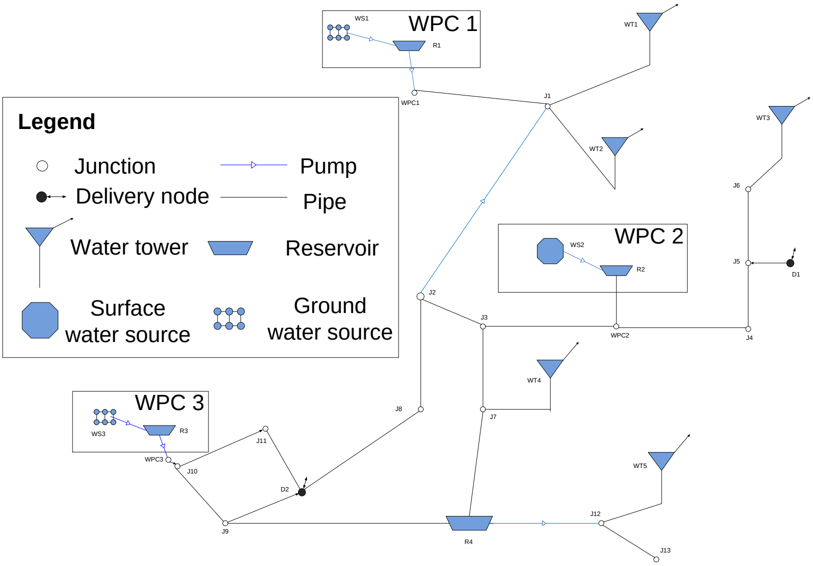


Figure 1.2: Typical components of a drinking water network

If we define:

- Q^{ij}: the (water) flow from i to j;
- c^{ij}: the cost for sending one unit from i to j;
- l^{ij} and u^{ij}: the lower bound and upper bound for the flow going from i to j;
- d^i: the demand in node i;

then the minimum cost flow problem is given by:

$$\text{Minimize} \sum_{(i,j)\in\mathcal{A}} c^{ij} Q^{ij}$$

subject to

$$\sum_{k:(k,i)\in\mathcal{A}} Q^{ki} - \sum_{j:(i,j)\in\mathcal{A}} Q^{ij} = d^i \qquad \forall i \in \mathcal{N}$$
$$l^{ij} \leq Q^{ij} \leq u^{ij} \qquad \forall (i,j) \in \mathcal{A}$$

This can be interpreted as 'Given the demand in each node and the capacities in each arc, send flow through the network at minimal cost so that all demand and capacity restrictions are respected'. Please note that Q is the variable (to be determined) in this problem whereas cost c, capacity l, u and demand d are the parameters of the problem.

The problem treated in this work is basically a minimum cost flow problem, in which arcs represent the pipes of the network. The bounds imposed will be linked to the maximum flow velocity while d_i and c_{ij} represent clustered demand in a node and production/pumping or delivery cost, respectively. Additional hydraulic constraints need to be formulated.

Next to the flow Q, pressure is another important variable in the water supply network. Work by moving the water through pressure differences is related to changes in gravitational potential and kinetic energy. Water in a pipe flows from places with high pressure to lower pressure areas, and steadily loses pressure due to friction of the pipe. When neglecting friction losses, these energy exchanges can be derived by using Bernoulli's law between two points in a pipe [21]:

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + h_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + h_2$$

where p_1 and p_2 are the manometric pressures, g is the gravitational acceleration, $\gamma = \rho g$ is the specific weight of water (with ρ the density), v_1 and v_2 are the velocities, and h_1, h_2 are the elevations with respect to the sea level.

Instead of pressure, the components in the equation are referred to as head. p/γ is the pressure head (associated with flow work), $v^2/2g$ the velocity head (associated with kinetic energy) and h the elevation head (associated with potential energy). The total sum of these components is the so-called 'piezometric head', denoted by the symbol H. Since velocity heads are negligible, the relation becomes $H = h + p/\gamma$. This variable will be used to model the pressure in this dissertation. Pressure losses caused by friction h_f are defined as the difference in total head from the beginning of the pipe to the end over a certain distance. When accounting for these losses and assuming water flows from point 1 to point 2, Bernoulli's law can be rewritten as:

$$H_1 = H_2 + h_f$$

When a pump is active in a pipeline, it discharges water to a higher head at its discharge side than the original head at the pump inlet. It thus adds an amount of pressure that we call 'pump head' (ΔH) and the pressure loss equation is rewritten as:

$$H_1 + \Delta H = H_2 + h_f$$

The pump head ΔH varies with the amount of water Q according to the pump characteristic curve.

For modeling purposes, we define the horizon $1, ..., T$ which usually spans one 24-h day divided over several discrete intervals. Each interval has a length τ_t that is usually composed of several hours. Subsequently, the hourly state of the network is considered to be constant: the pressure and flow rate do not change and are considered as an average of the real values within the interval. Furthermore, this distribution is based on the daily water consumption pattern and the energy cost. No more than one day will be modeled since the same optimized production plan is assumed to be repeatedly used on a daily basis. Since multiple periods are covered, buffers (water towers and reservoirs) play an important role in the network: they can be used to store water with the help of pumps when electricity tariff is low and add robustness to the network for demand variability. Pressure losses, pump curves, and buffers restrictions will be further detailed in chapter 3.

The modeling is restricted to the main supply network. The underlying distribution net which connects every single household with the supply network is thus not taken into account. This implies that the actual demand needs to be clustered in several dedicated demand nodes in the modeled network, which can be done using the GIS (Geographic Information System) database. In this dissertation we will not make a distinction between the terms ‘water supply network’ and ‘water distribution network’, both of which denote the generally studied networks.

Lastly, the cost function $c^{ij}\,Q^{ij}$ consists of three major parts:

- **Production cost at WPCs.** This cost consists of 4 major components: electricity (lighting + energy raw water pumps), taxes (ground water extraction), loans (personnel) and chemicals.
- **Energy cost for pure water pumps.** This is the cost for pumping water in the water supply net. Prices are higher during the day (‘normal hours’) than at night (‘silent hours’).
- **Delivery costs**. In some parts of the network, water can be sold or bought from other drinking water companies at a fixed cost.

1.3 Research objectives

This dissertation is concerned with the optimization of water production and distribution operations in real-world large-scale mesh structured water supply networks, based on the water supply infrastructure of De Watergroep. The main goal is to develop a decision support model that provides cost efficient configurations, guarantees water delivery and is able to respond to demand changes or network defects in reasonable computation time. The main characteristics of such a model are:

- Generality: many different hydraulic components are included in the mathematical model. Because of this, a large array of different water supply network configurations can be solved with the proposed model. At the least, every water supply network provider in Belgium should be able to implement and use the models and methods on their infrastructure. Many of the proposed networks found in the literature are composed of elements that are contained in the model that is proposed here. Because of this we believe the model to be suitable for obtaining good solutions on many different network configurations worldwide.

- Accuracy: a full hydraulic model that takes into account many of the important hydraulics such as pressure losses and pump characteristic curves, and that makes sure that pressure is within the limits everywhere in the network. Special attention goes to a detailed modeling of the buffer model, where additional binary variables have to be added to ensure an accurate representation of the actual situation. Delivery points that allow water exchange with other drinking water companies are also considered. Furthermore the optimal solution should be very close to and if possible equal to the global optimum.

- Reasonable computation time: methods will be proposed to overcome the computational difficulties inherent to the problem formulation. These methods should not only be robust but the computational time should be within reasonable limits. In contrast to many of the proposed (meta)heuristics found in literature, the goal is not to find a (sub)optimal solution in the shortest computation time possible. Getting a good estimate of the global optimal solution by exploiting the characteristics of the water supply system, and within a reasonable time frame, is the main goal.

1.4 Research outline

This dissertation starts with an overview of past research that has been conducted on water supply network optimization (chapter 2). A general way of structuring such networks is covered, and afterwards several different models and methods are discussed. Emphasis will be put on both operational aspects and optimal design during this review.

In order to generate an optimal configuration (production scheme and pump control), an accurate hydraulic model is needed. In chapter 3 such a model is proposed. Specific focus is put on the modeling of the buffer, in which water flows in or out depending on the relation of the net pressure with the water level in the tank. Further emphasis is put on the highly nonlinear pressure loss equations, for which different hydraulic laws have been proposed. A graphical representation allows comparison of these approximations. Most research papers, discussed in chapter 2, that are concerned with optimal pump configurations do not take into account pumps with variable speed settings. Here, a representation for these VSPs (variable speed pumps) is added, where the degree of nonlinearity is decreased as much as possible without giving in too much on accuracy. The total cost function is composed of three terms: production, energy and delivery cost. The nonconvex Mixed Integer NonLinear Programming (MINLP) model is tested on several instances using a state-of-the-art solver.

The proposed model is very hard to solve, mostly due to its nonlinear nature. Therefore, an alternative model is proposed in chapter 4. Using piecewise linear functions, nonlinearities can be avoided at the price of additional binary variables and constraints. Since the resulting Mixed Integer Linear Programming (MILP) model computationally outperforms the MINLP model but at the cost of loss of accuracy, the goal here is to correct the optimal solution to get it within a very tight feasibility tolerance. In order to understand how this 'fixing' works, an overview of steady-state hydraulic analysis is given. An overview is given of different methods that can obtain a solution for network states where certain information, such as tank levels in buffers and pump configurations, is already known beforehand. Afterwards the MILP model is adjusted to be used with the Newton's method (or gradient method) for pipe equations. Information from the optimal MILP solution is given as a starting point for this method. All bounds on variables are dropped and only general flow and pressure variables are updated. The hybrid algorithm is again tested on realistic networks and results are compared with those found with the MINLP model.

A different approach on solving the proposed model is outlined in chapter 5. Here, a decomposition method called Generalized Benders Decomposition is proposed to solve the MINLP model. This algorithm is originally made for convex MINLP models or for nonconvex formulations that can be decomposed in convex subproblems under certain conditions. One of these conditions is the separability property of the nonlinear functions. The general model proposed in chapter 3 does not satisfy this property, potentially making the algorithm computationally ineffective. By using approximation cuts however, this issue is circumvented at a certain price: the possibility of loss of a global optimal solution. Great care is taken in adequately selecting the variables for the decomposed master and subproblem, as well as the selection of slack variables and their corresponding penalty costs. With these carefully fine-tuned parameters, the algorithm converges to what is likely the global optimal solution and greatly outperforms the state-of-the-art solvers with respect to computational time. Results on all test instances are reported.

In chapter 6 the most important results of this dissertation are highlighted, followed by an extensive discussion. Although this work aims at giving a complete model with good and efficient methods to solve it, a lot can still be improved. Some guidelines and recommendations for future research are therefore given.

1.5 Contributions and publications

The operational management of water supply in Flanders is currently done using simulation software and experience of the operators. In first instance, a decision support model is proposed that minimizes operational costs while guaranteeing water supply and valid pressures everywhere in the network. The main benefits this network has in comparison with other models from literature is an accurate modeling of water towers (in general: buffers), which play an important role in energy efficiency by storing water during periods with low average demand and feeding the network gravitationally during the day when electricity tariffs are high. A second contribution is a detailed modeling of variable speed pumps, where nonlinearities in the formulation are limited. The power term in the cost function was linearized without reducing the hydraulic accuracy of the model.

The accuracy of the proposed model unfortunately implies a highly nonlinear and nonconvex structure. Solutions that are (near) global optimal are hard to acquire using existing state-of-the-art solvers. Therefore a piecewise-linear (PWL) approximation method is proposed that generates over- and underestimators for the nonlinear pressure loss constraints and pump characteristic curves. This decreases the accuracy of the system depending on the number of intervals that are used. For few intervals a good solution can be acquired in very short time, whereas very accurate approximations are computationally ineffective. The PWL method is therefore coupled with the gradient method, which is very popular in simulation software. The quickly generated solution from the MILP is hereby passed on to this method and repaired to a solution that is hydraulically feasible under certain conditions.

The proposed hybrid MILP/PWL method is able to obtain good results, although the computational time can still be improved for larger network instances. This is especially useful when uncertainty is added to the water supply system e.g. a sudden failure of a critical pump or a pipe that breaks, which may require a new operational plan on a relatively short term. In order to accomplish this, a Generalized Benders Decomposition (GBD) approach is formulated. The application is unique in the sense that a water supply model is solved that is not only nonlinear but also contains many binary variables. By carefully tuning the parameters of the model, good results can be generated in decent computational time that are very competitive in comparison with those obtained by state-of-the-art solvers. It is shown that GBD is a worthwhile alternative to common heuristics and provides an efficient operating solution to large-scale water supply networks.

1.5.1 List of publications

1.5.1.1 Publications in international journals

1. D. Verleye and E.-H. Aghezzaf. *Generalized Benders Decomposition to Re-Optimize Water Production and Distribution Operations in a Real Water Supply Network.* J. Water Resour. Plann. Manage., 2015 (accepted for publication).

2. D. Verleye and E.-H. Aghezzaf. *Optimising production and distribution operations in large water supply networks: A piecewise linear optimisation approach.* International Journal of Production Research, 51(23-24):7170-7189, 2013.

1.5.1.2 Publications in international conferences

1. D. Verleye and E.-H. Aghezzaf. *A hybrid optimization algorithm for water production and distribution operations in a large real-world water network.* In Advances in Intelligent Systems and Computing, volume 360, pages 93-104. Springer, 2015.

2. D. Verleye and E.-H. Aghezzaf. *A hybrid piecewise-linear/Newton approach to solve multi-period production and distribution planning problems in large water supply networks.* 11th International Conference on Hydroinformatics: Informatics and the Environment: Data and Model Integration in a Heterogeneous Hydro World (HIC 2014).

3. D. Verleye, E.-H. Aghezzaf and D. Defillet. *Optimizing operations of large water supply networks : a case study.* Proceedings of the 1st International Conference on Operations Research and Enterprise Systems (ICORES 2012).

4. D. Verleye and E.-H. Aghezzaf. *Modeling and optimization of production and distribution of drinking water at VMW.* In LNCS, volume 6701, pages 315-326. Springer-Verlag Berlin Heidelberg, 2010.

1.5.1.3 Publications in national conferences

1. D. Verleye and E.-H. Aghezzaf. *A hybrid piecewise-linear/Newton's method with corrected Jacobian matrix for multi-period water production and distribution in large networks.* 28th Annual conference of the Belgian Operations Research Society, 2014.

2. D. Verleye and E.-H. Aghezzaf. *Solving multi-period water production and distribution problems in large-scale networks.* 20th Conference of the International Federation of Operational Research Societies, 2014.

3. D. Verleye and E.-H. Aghezzaf. *A decomposition approach for large-scale drinking water supply networks.* 26th Annual Conference of the Belgian Operations Research Society, 2012.

4. D. Verleye and E.-H. Aghezzaf. *Modelling and optimizing production and distribution of drinking water at VMW.* 25th Annual Conference of the Belgian Operations Research Society, 2011.

2

State of the art on water supply network optimization

Water supply optimization has been widely researched, however different kinds of optimization on different kinds of networks exist. It is therefore useful to describe the different optimization stages that exist in the domain of water supply as well as distinct the different kinds of studied networks. This chapter consists of 4 sections. The first one defines the two main research topics of water supply optimization, operations and design, and gives a general way of structuring these networks. Next a small section is devoted to demand forecasting. The two final sections describe the main part of this chapter: an analysis of different models to describe the complex problem that optimization of drinking water networks is and a listing of the most relevant methods and programs for solving these optimization problems.

2.1 General classification of water supply network problems

The water supply network optimization problem can be divided in two general phases. The first phase is the network design problem that is mainly concerned with static parameters such as pipe diameters, whereas the second phase is the search for an optimal operational planning for fixed-topology networks, that is concerned with the dynamics of the network such as pump rates and buffer mechanics in response to varying demand rates (for a review see [15, 16]). The first two sections describe these two phases, next is a small section on the structure of drinking water networks, and the final section covers benchmark networks to estimate the quality of an optimization strategy.

2.1.1 Optimization of drinking water network design

The design phase of a water supply network considers various aspects such as optimal layout of the network and optimal dimensions of the systems components. In most cases reservoirs, pumps and valves are disregarded and the model reduces to the optimal choice of pipe diameters given a set of discrete values, while demand has to be satisfied. The main restrictions are the flow conservation in nodes and the pressure loss equations in pipes. In their paper, [22] show that even the very simplest type of branching problem is solved in non-deterministic polynomial time (otherwise stated it is NP-hard) and suggest that research should be aimed at proposing good approximation methods to find optimal solutions.

2.1.2 Optimization of drinking water network operations

As previously mentioned, the operational planning and monitoring phase of water production and distribution in a water supply network involves operational parameters such as hydraulic pressure zone boundaries, demand patterns, control valve settings, and pump operating schedules. While pipe diameters are fixed, pump and valve parameters have to be controlled and production rates determined while delivering good-quality water to customers at reasonable flow and pressure. Obviously a decision support model for such a system has to be paired with a good design and the expertise of the operators. As stated in [23], water system operations are extremely complicated. Thousands of customers, pumps and tanks may be involved as well as several water sources. Nowadays most of the operations at water supply companies are supported by simulation software such as EPANET [9]. This software is able to calculate the pressures and flows in a network on a detailed level given the configuration. In chapter 4 the gradient algorithm, which is used in EPANET, will be discussed in detail.

As is pointed out in [16], the main difference between the design and operation problems is the contrast between static and dynamic modeling. When looking for a solution to optimally operate a network, demand patterns have to be taken into account. Furthermore pumps may be switched on and off over time. Nevertheless similar approaches may be exploited to solve both types of modeling.

2.1.3 Structure of drinking water networks

Another way to categorize water supply networks is topology. The linear networks tend to be easier to solve, whereas meshed networks need sophisticated algorithms to provide a solution. This is mainly because actions in individual branches may not affect the rest of the network in the first case, whereas hydraulics in each branch of the second network globally influence this network [1]. The authors make a classification of four types, as shown in figures 2.1- 2.4. The star-structured network contains a single reservoir as central source, from which water flows to the demand nodes with a fixed sense. There is no further branching apart from the central source and each branch contains at most one reservoir. Because of this, the pressure loss equations can be dropped. This kind of network is called a mass-balance model in [23]. The difficulty stems from the fact that the pumping cost corresponding to a certain flow is nonlinear.

In contrast to star structures, a tree structure contains multiple intermediate locations from which multiple branches depart to the demand nodes. If there is only one branch the resulting network is called a cascade structured network. In general these models can be solved quite easily by linearization methods. They are however only suitable for regional supply systems in which flow is carried primarily by major pipelines rather than distribution networks that contain loops.

The mesh structured network, finally, is a highly interconnected network with multiple reservoirs in which flows have no fixed sense. Apart from flow conservation constraints, energy conservation has to be respected in these networks which makes them much harder to solve.

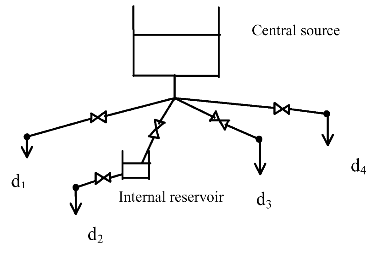


Figure 2.1: Network with star structure [1]

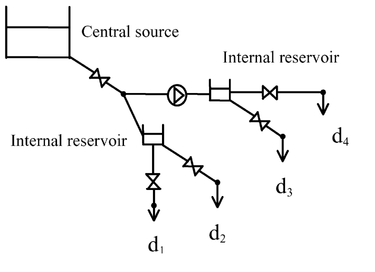


Figure 2.2: Network with tree structure [1]

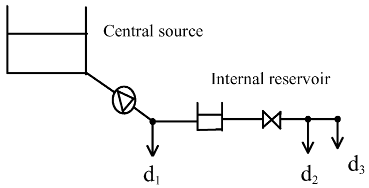


Figure 2.3: Network with cascade structure [1]

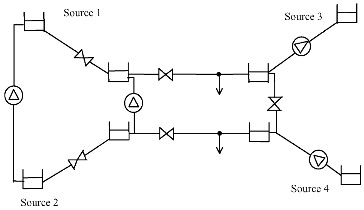


Figure 2.4: Network with mesh structure [1]

2.1.4 Benchmark water distribution networks

Many of the common test networks found in the literature are rather small and do not represent real-life water networks. Most of them are used for water supply network design.

2.1.4.1 Van Zyl Test Network

This network was proposed in [24] and consists of one main reservoir that feeds two tanks. The small network is composed of 2 pumping stations and several pipes.

2.1.4.2 Hanoi Network

The Hanoi network consists of one gravity-fed reservoir, 32 nodes and 34 pipes organized in 3 loops. No pumping facilities are considered since only a single fixed head source at elevation of 100 m is available. Only 6 possible pipe diameters are available for selection [25].

2.1.4.3 New York water supply system

This network was first mentioned by [26] and consists of a single reservoir and 21 pipes. The problem is to determine the most economically effective design for proposed additions to the primary water distribution system. This system is a gravity flow system drawing water from the single reservoir.

2.1.4.4 Anytown water distribution network

The objective of this network problem is to determine the most economically effective design to reinforce the existing system to meet demands. Pumping costs and capital expenditure are hereby taken into account [27]. The problem variables are 35 pipes considered for duplication or cleaning and lining and 6 additional new pipes. Furthermore 2 potential new tank locations are considered.

2.1.4.5 Richmond water distribution network

This network is one proposed for operational optimization. The primary objective is to determine the optimum trigger levels that minimize pumping costs. Two pairs of trigger levels are used for each level control pump, one for off-peak, and the other for peak electricity tariff periods. It contains about 40 pipes and junctions and additionally 7 pumps and 6 tanks fed by one main reservoir [24].

2.1.4.6 Balerma Network

This looped irrigation water distribution network is the largest network currently studied in literature [28]. It consists of 454 pipes connecting 447 junctions with 8 loops.

As is rightfully stated in [15], most of these benchmark networks are rather small and simplistic, leading to the fact that most of the proposed methods (discussed in the next sections) result in high-quality solutions. It is therefore not proven that fast metaheuristic methods succeed when applied on large and complex real-life water supply optimization models. More recently, the same authors have developed a tool called HydroGen that generates realistic water distribution configurations of varying size to allow researchers to test their methods on more, and importantly, larger realistic water supply networks [29]. Since most of these networks are optimized for design purposes, data is limited to a single time interval. Usually none or few pumps are considered, and variable speed pumps are usually not present in those networks. Complexity increases significantly when buffer volumes play an important role in multiperiod optimization and when the network size and number of pumps increases. The model proposed in this dissertation will be focused on real-life networks that contain all of these restrictions and as such will most likely be applicable on the benchmark networks presented in this section as well.

2.2 Demand forecast

Since the actual daily demand schedule is not known in advance, a forecast model has to be used. According to [23], three different approaches can be used to incorporate forecasted demands in decision support models for water supply systems. In a lumped approach, one single value is used that represents system demands. This kind of forecast is normally used with mass-balance hydraulic models i.e. models where pressure loss/energy constraints are not taken into account (usually star-structured or tree-structured networks). For models with pump curves and pressure losses (regression-based models) a second approach based on proportional demands is used. Regression relationships are derived from a demand pattern that may vary proportionally to total system demand. A distributed demand approach is a third type where demands are assigned both temporally and spatially to the network. Clearly this is the most accurate approach that will be used for detailed simulation and optimization models. Predictions are based on different factors such as weather conditions and seasons. The distribution of the demand among different junctions can be done using historical data or database information. Further disaggregation in hourly demands can be done based on the demand patterns, day of the week and seasonal variations [30, 31].

In [32] two different forecasting techniques are compared: one is based on time series and the other is a representation by neural networks. The author concludes that both methods are serviceable for demand forecasting purposes.

2.3 Models for solving water supply network optimization problems

Both the design and operations optimization problems have similarities since hydraulics play an important role in each. For the design problem, changing the diameter of one pipe causes the pressures to change throughout the network, whereas altering the pump speed in the operational problem does the same. In the following section, models for both optimization problems will be addressed. The general mathematical model is a MINLP (Mixed Integer Nonlinear Program), but many authors have proposed simplified models to overcome computational difficulties. An overview is given below.

2.3.1 Linear Programming (LP) Models

In water supply networks, nonlinearities stem from the pump curves and headloss equations, which are usually represented using regression curves. These can either be acquired by using a calibrated simulation model for several instances of tank levels and loading combinations or by using database information relating pump head, discharge, tank levels and demands [33]. The authors in [23] note that such regression curves may lead to errors if the forecasted demands are outside the range of the database.

During the early stages of water supply optimization, many researchers used simplified hydraulic models for this purpose. When spatial decomposition combined with dynamic programming is not possible, LP models could provide an advantageous alternative. The authors in [34] used a linear model based on 'duties' for pumps in a mesh structure: for each pump station a specific configuration is chosen that corresponds with a delivery in each reservoir during a certain time within each interval. The method is however network-dependent: the nodal heads need to be satisfied everywhere and the pump head is extremely large compared to network nodal changes elsewhere.

In [35] the authors use simulation techniques to find pump configurations that are feasible in terms of water demand and pressure constraints. Least-cost surfaces for each feasible configuration are then derived using numerical algorithms for convex polytopes. Each combination of pump configurations is then again tested and its associated cost is calculated.

The objective function of the linear problem formulated in [7] consists of costs for shortfalls below target storage levels, reservoir spillage costs and the negative of the value of water in the reservoirs at the end of the time period, apart from pumping electricity costs. The mass balance constraints are written for each reservoir as well as piecewise linearized pump cost curves. Pressure at nodes is not taken into account.

In [36] an LP model is linked with a simulation package to minimize the sum of penalties for deviations from target reservoir storages and performance levels. An extensive forecasting model is made for real-time reservoir operations, and the optimality of the solution is dependent on the quality of the forecast.

An LP model combined with a directed graph algorithm is used to tackle a simplified water supply system in [37]. The use of upstream and downstream nodes circumvents the implementation of the pressure loss constraints but the operating plan may not be feasible in practice. An equivalent formulation is used in [38].

2.3.2 Mixed Integer Linear Programming (MILP) Models

Mixed integer linear programming is used when binary variables are introduced to the model. Most of the time these variables represent the activity status of the pumps during a certain time period.

The authors in [39] use a simplified MILP model for a cascade structured network where the pressure in the central source was held constant. The binary variables represent the on/off status of the pumps.

An MILP model is proposed in [40] where discretized pump states are represented by binary variables and pressure losses are assumed to be negligible compared to the geographic height differences.

2.3.3 Nonlinear Programming (NLP) Models

When more accurate representations of the hydraulic laws in the network are required, nonlinear equations are introduced to the model. This will mostly be the case with highly interconnected mesh-structured networks, where the pressure losses/energy conservation between nodes can not be neglected.

In [41] a nonlinear objective term is introduced that represents the total power consumption of the pumps. Furthermore, energy conservation and the characteristic pump curves are taken into account. Instead of introducing binary variables that represent the activity status of the pumps, the authors multiply the pump pressure constraints with the continuous value of the flow, making them even more nonlinear. Furthermore penalty variables are added to these constraints and represented in the objective function with a big penalty value.

A large-scale multisource water supply system for cost minimization with additional penalties for demand, water quality, emergency sources and excess discharge is modeled as an NLP in [42]. The model contains nonlinear penalty functions and mixing constraints.

The general drawback of NLP methods is that they rely on the initial solution and do not guarantee the identification of a global optimum. In the case of network design where the pipe diameters are discrete market sizes the quality of the optimal solution is reduced.

2.3.4 Mixed Integer Nonlinear Programming (MINLP) Models

In these models most of the hydraulic components are accurately described. The pressure loss constraint and pump characteristic curves introduce nonlinearities, whereas binary variables are introduced for pump status or, in the case of design problems, to decide which market size of pipes is chosen.

The authors in [43] propose a MINLP model involving energy conservation along loops and pump characteristic curves that minimizes the sum of the pipe replacement, pipe rehabilitation, expected repair cost and energy cost. Binary variables are introduced that denote whether a pipe or pump is to be replaced or rehabilitated.

In [14] the authors transform their minimal pump cost MINLP model to an NLP by using continuous pump controls. The values of the optimal solution to this NLP are then used in the next stage, where the actual pump schedule is calculated. The big advantage of this method is that the model decomposes by time interval since the optimal values of the reservoir volumes in the continuous problem are used as estimates for the final solution.

A very complete model for the water supply network in Berlin is described in [4, 5]. The authors model smoothed versions of the pressure loss functions and nonlinear pump curves for fixed-speed and variable-speed pumps. Control variables for pumps and valves result in an MINLP with binary variables. It is noted that this formulation may be solvable by global optimization problems based on nonlinear branch and bound for small networks. The resulting model is however very large containing 1481 nodes and 192 arcs over 24 hourly intervals. Therefore the proposed model is modified to an NLP variant, in which negative flows for pumps are allowed but generate very high costs which the solver will avoid. As such the binary control variables can be omitted. Furthermore pump switching is prevented by introducing smoothed nonlinear complementarity functions, leading to good approximations.

In recent years, many heuristic approaches have been proposed to solve MINLP models (see section 2.4.5 for details).

2.4 Methods for solving water supply network optimization problems

Many optimization methods have been proposed, mostly focused on optimal design problems. Early methods cover simplified LP methods or dynamic programming, which are mostly limited to small tree-structured networks. NLP solvers and Branch and Bound have also been proposed, although these are less frequently encountered in literature. Heuristic approaches such as Genetic Algorithms on the other hand are an established value in water network design optimization.

Note that many of these optimization methods are coupled with simulation software. In [35] the network simulator GINAS5 was used to identify feasible pump schedules for fixed-speed as well as variable-speed pumps (tested at minimum and maximum speed) over a 24-h period. The selection procedure was used to identify the optimal pump selection for a part of London's water network.

2.4.1 LP methods

These LP methods are usually solved by the primal/dual simplex method or variations thereof (see [44]).

[7] used an LP solver to tackle a simplified model, which resulted in a 5-10 % reduction in operating costs for the Adelaide headworks system in South Australia.

Instead of simplifying the model, an approach such as the one proposed in [45] can be used. The authors solve a design problem in which the length of segments with a constant diameter in each link is to be determined. The authors use a decomposition approach in which a flow problem is solved and a separate problem in which all other variables are determined. The dual value of the energy conservation constraints is used to update the flows in the next iteration. A similar method is used in [46], where the LP is coupled with the Hardy Cross method (see section 4.2.1).

2.4.2 Dynamic Programming

Dynamic programming has been very popular with researchers in the early stages of research on water supply optimization. The dynamic programming recursion minimizes the total cost from the current stage (t) until the end of the problem, given that at stage t the state is known. The state is chosen from all possible states the system can be in at that stage. Note that both linear and nonlinear formulations can be tackled using this method.

The possible states are usually the tank water levels, defined over a number of discrete time steps (stages). The optimal solution is found by evaluating all state transitions between adjacent stages instead of evaluating every possible combination. The big advantage is that a complex problem involving different subproblems can be reduced to different problems with a single variable. The downside of this method is that the efficiency goes down substantially as the number of variables increases. Therefore most of the models solved by this method are single-tank systems (star or tree structured).

Extension of this method to networks with multiple tanks is possible when combined with spatial decomposition techniques. The system is broken down in various subnetworks, each of them with only one or two tanks and thus easily solvable. Policies for each of these networks are connected on a higher level in order to find a solution for the complete system. The paper [47] discusses a dynamic programming approach to solve a (simplified) tree structured network. The optimization function here is the total energy cost for operating the pumps (quadratic) and the constraints are the continuity equations for volume in reservoirs and bounds on flow and volume variables. The pressure constraints are not taken into account so all restrictions are linear. The model is decomposed in time steps and for each step the trajectories connecting two reservoirs are handled independently. More specifically the discharges are determined such that the cost is minimized for fixed volume levels (determined in an initial step).

In [6] a similar approach is used in which a tree structured network is solved using an LP model that is decomposed in space and time. Dynamic programming is used for the continuous relaxation of the problem, after which a simple heuristic is used to rearrange the discretized pump schedules in order to reduce the frequency of pump switching.

A variation on this method can be found in [48, 49]. Here the authors make a hierarchical decomposition in three distinct levels: an upper level in which reservoir storage is optimized using nonlinear dynamic programming, an intermediate level for source extraction and a lower level for the individual sources. The fixed tank trajectories are passed on to the lower levels where pump combinations are selected using binary variables.

In [50] the optimal pump operation plan is achieved in two phases: the first one is the development of an optimal tank trajectory, and secondly the development of an optimal pump operating policy. The optimal trajectory is determined using dynamic programming while the pump policy is determined using an explicit enumeration scheme. The authors in [51] also based their methodology on this idea. Simulations are used to determine the rate of change of tank water levels given an initial water level for every pump/demand combination. Avoiding excessive pump switching was also considered in the model. It was shown that acceptable solutions can be obtained on small networks with a single tank, however when extended towards multi-reservoirs models with many pumps the algorithm is not suitable. This shortcoming is also pointed out in [52].

2.4.3 NLP solvers

2.4.3.1 MINOS

This NLP solver is especially effective for problems with nonlinear objective function and sparse linear constraints. It is suitable for handling a large numbers of nonlinear constraints. The nonlinear functions should be smooth, but need not be convex [53]. When a nonlinear objective is present, the solver uses a reduced gradient algorithm in conjunction with a quasi-Newton method. A projected augmented Lagrangian algorithm is added to the package in case of nonlinear constraints.

In [41] a solution for an NLP model applied to a water supply network consisting of 34 pipes and 8 pumps connecting 16 junctions and 2 reservoirs. The optimization algorithm is able to find a solution, but the initial starting conditions need to be specified carefully.

The network of the Berliner Wasserbetreibe described in [5] was modeled as an NLP problem and solved using the commercial solver MINOS . The resulting model contained 25000 variables and was solved in very reasonable computational time.

MINOS was also successfully used on the NLP model in [42] for a network consisting of about 150 nodes and arcs.

2.4.3.2 CONOPT

CONOPT is a nonlinear programming solver that uses a generalized reduced gradient algorithm (GRG, [54]). The algorithm distinguishes two types of variables: basic variables that are calculated from the equality constraints, and super-basic variables which are allowed to change freely. Only for the latter type of variables is the gradient calculated. This solver is very efficient for models with few degrees of freedom.

The authors in [14] make use of this algorithm to solve the NLP subproblem of their MINLP formulation. They are able to solve a large reduced network that contains 252 nodes and 530 components, including 35 pumps, over 6 periods in reasonable computational time. A total saving of 14% on the operating cost is made.

2.4.4 Branch and bound (B&B)

In a B&B algorithm, a tree is formed from a root node which represents an initial solution (to the linearized version of the MILP). Next, an integer variable is chosen which has a non-integer value in the LP solution. This value is then rounded up to its nearest integer value, and a new subproblem (branch) is created with this value as lower bound for the branched variable. Likewise a second subproblem is created where the rounded down value is imposed as upper bound. Both subproblems are solved. If the LP solution is worse than the best-known IP (Integer Programming) solution, the branch is cut off (fathomed) and the subproblems discarded. If the solution to a subproblem is integer, the optimal value serves as an upper bound for the IP problem. Branching continues until there are no unfathomed sub-problems left.

In [43] a specific branch and bound structure is used to determine which pipes have to be replaced or rehabilitated. The values of the binary variables of the pipes that have not yet been assigned values are hereby relaxed. In every iteration the last optimal node is branched into three new nodes for the next pipes: one node represents the replacement, one the rehabilitation and the third neither of these. The three nonlinear subproblems are then again solved (with continuous variables for pipes that have not yet been assigned) and the node with the lowest objective value is then branched off again. Iterations continue until all pipes have fixed integer values. The nonlinear subproblems are solved using a gradient method coupled with simulation software through an augmented Lagrangian penalty method. Note that such a solution procedure is only effective when the number of pipes/pumps to be renewed is relatively small.

An extension of this method called nonlinear B&B can be used on MINLP formulations, where an NLP solver is used to tackle the subproblem at each node in the tree. Since the NLPs are generally nonconvex, pruning a node may be based on a local optimal solution and the global optimum is abandoned. The general optimal solution is not guaranteed to be a globally optimal one as a consequence. In [55] nonlinear B&B is applied on a water network design problem.

In [16] two other versions of this framework are discussed: LP/NLP based B&B and spatial B&B.

2.4.4.1 LP/NLP-based branch and bound

The feasible region of the MINLP is approximated by a polyhedron formed by first order taylor approximations of the nonlinear functions, a process called outer approximation (OA). The resulting MILP is solved to optimality and the optimal solution is used to refine the polyhedral relaxation. Instead of solving the entire B&B tree at each iteration, the corresponding NLP in every integer feasible node is solved to optimality and the OA cuts corresponding to that node are added. The LP relaxation with the newly added cuts is resolved and the search continues. This algorithm has been implemented in the open-source solver BONMIN [56]. Note that if the MINLP is nonconvex (usually the case with water supply optimization problems), the OA may cut off part of the feasible region, resulting in locally optimal solutions.

In [57] each network arc is copied for every possible diameter value in the design problem in order to get univariate pressure loss functions. Binary variables are introduced to indicate the direction of the flow on each of these arcs. As a consequence, the nonconvex constraints can be relaxed to two convex constraints, allowing the B&B algorithm to find a global optimum.

2.4.4.2 Spatial branch and bound

This general-purpose algorithm is able to solve nonconvex MINLP's to global optimality by making subproblems through branching on integer as well as continuous variables. When the relaxation of a nonlinear constraint becomes tighter when its domain is reduced, spatial branching gradually refines the relaxations until they are tight enough to provide ϵ-feasible solutions. This algorithm is implemented in the MINLP solver COUENNE [58].

2.4.5 (Meta)heuristic methods

Since solving a complete MINLP model for water supply networks is hard in practice, many researchers use heuristic approaches to find solutions for these networks. These approaches' main advantage is that they can come up with good solutions in short computational time, however there is no guarantee that a global optimal solution is found. In what follows, different techniques are briefly explained. Most of these methods are directed towards optimal design of the network. For an extensive review of different (meta)heuristic methods applied on water supply network design problems, the reader is referred to [15].

2.4.5.1 Simulated annealing (SA)

This method draws the analogy with the physical annealing of crystals to low energy states. The current solution is compared with other candidates in its neighbourhood. Better solutions will always be accepted and, to reduce getting stuck in local optima, worse solutions are accepted with a certain probability. This probability depends on the temperature parameter. The temperature decreases during the process and terminates the algorithm when it falls below the stopping value. In [59] SA is applied on the design of water distribution networks.

2.4.5.2 Genetic algorithms (GA)

This evolutionary algorithm mimics the process of natural selection. Starting from an initial population of solutions, usually chosen at random, selection procedures are devised that allow this population to evolve over a number of generations [60]. A chromosome stores values of decision variables for each individual. Fitness functions are used to measure how good each chromosome is with respect to an objective function. Based on this fitness value individuals are then selected and recombined with a certain probability, resulting in an offspring chromosome. This process is called crossover or recombination. Mutation then randomly alters certain genes (variable values) in the chromosome. Although crossover and mutation are the main genetic operators, variants exist e.g. regrouping or migration. Generally the mutation rate should be chosen small enough so that the loss of good solutions is avoided. Similarly, high recombination rates may lead to premature convergence. Because of the stochastic nature of GA there is no guarantee that the global optimum will be found but in general good solutions can be generated. An extensive overview of the current state of the art in GA's for water supply systems can be found in [61].

The authors in [62] code their decision variables for pipe diameters as binary strings made up of substrings that represent the pipe diameters. The fitness value is determined by running the gradient method (see section 4.2.4). Instead of ignoring infeasible solutions (solutions where pressure values are outside of their imposed limits), they are taken into the population with a penalty. A single gene is randomly chosen from each of the parents during recombination, while the mutation rate is set to a very low value. The method is tested on the simple Hanoi and New York water supply systems.

In [40] a GA approach is used to solve a simplified least-cost pump schedule. [63] combine the GA with a decomposition method that is limited to gravitational systems i.e. networks without pumping elements.

The GA can also be coupled with other local search algorithms to improve its convergence. Such a hybrid approach is suggested in [24], where the authors couple GA with a hillclimber method. It was shown that the hybrid algorithm performed better in terms of computational time and solution quality on the Richmond water distribution system.

2.4.5.3 Differential Evolution (DE)

Instead of relying on crossover methods, the focus with DE lies on mutation. The crossover in DE is uniform in the sense that child vector parameters from one parent are more often taken than from the other. The recombination shuffles information on successful combinations which lies the focus on the most promising area of the solution space. Compared to GA, DE is significantly faster and increases the chance of finding a global optimum. DE generates new vectors in each iteration by adding a weighted difference vector of two population vectors to a third population member vector. This is called the noisy random vector. Next crossover is applied on this vector and the target vector, resulting in the offspring called the trial vector. Lastly the objective value of this trial vector is matched with the one from the target vector. If the objective is better (lower) than the trial vector replaces the target vector. This is repeated for each member of the population to form the next generation. The process is continued until a termination criterion is met. In [64] DE is used to minimize the network design cost and maximize the resilience of the New York water supply system and the Hanoi water distribution system. The pipe diameter values found by the DE optimization algorithm are rounded to the nearest commercial pipe diameter before they are passed to EPANET. The conservation of flow and energy constraints are hereby checked with EPANET instead of in the DE framework which may result in infeasible solutions. The DE algorithm matched the feasible lowest cost solutions obtained from previous studies for both networks.

2.4.5.4 Ant Colony Optimization

This metaheuristic is inspired by the behavior of some species of ants. These ants are able to find an optimal path between nest and food through indirect communication by means of trails of pheromones laid by other ants along the way. This behavior is adapted in [65] where ants stochastically build a path in a graph. Nodes represent decision points and edges are possible choices. The ants decide at each node which edge they add to their path, based on the pheromone value of that edge. The higher this value, the greater the probability that this edge is chosen. The pheromone values decrease every iteration for every edge, while pheromone values associated with edges that are part of the best solution are increased. Furthermore the authors use a time-based trigger representation for pump switches. The method was tested on the Van Zyl and Richmond Test Networks. The advantage of their representation is that the number of potential solutions and thus the search space can be significantly reduced. Furthermore no penalty values need to be introduced since infeasible solutions are ranked in order of infeasibility.

2.4.5.5 Other local search algorithms

A greedy algorithm was coupled with an LP model and simulation software in [66]. The algorithm hereby started a search from the optimal pumping schedule from the linearized problem. The method was tested on the Anytown and Richmond Networks, providing good solutions in very short computation times when compared with genetic algorithms.

Even though heuristics and metaheuristics seem to be suitable methods to use for solving large water supply network problems, an important remark needs to be made at this stage. Most of the studies in the literature focus on water network design problems. In this process the heuristic methods used produce a set of discrete pipe diameters and then feed these solutions to the simulation software to check whether the corresponding configuration is feasible or not. This will often be the case for such ‘simple’ networks, especially when it is tree structured. When multiple time periods, buffer filling requirements and pumps are involved, the problem becomes much more complex. Consequently, producing feasible configurations is such setting is much harder and the chances that a heuristic solution would achieve feasibility is almost zero. Finding a feasible solution is an intricate task, and we therefore chose to start the solution process for the investigated operational water supply network with exact methods. With these exact methods, which can be hybridized with (meta)heuristics, we are able to produce solutions which are feasible and close to optimal.

3

Modeling Real-World Water Supply Systems

A drinking water network is a system of pipes, pumps, junctions and reservoirs designed to supply water to every demand node at adequate pressure and flow. This system of pipes has many components of which the hydraulic characteristics will be addressed in this chapter, along with a mathematical formulation for the constraints they induce. As already stated, the network that is discussed, modeled and analyzed in this dissertation is the supply network operated by De Watergroep. Specific features of this network include water towers with additional in- and outflow characteristics, variable speed pumps and multiple sources with distinct production costs. Only the supply network is modeled, this is the main pipeline structure containing large buffers, delivery points and clustered demand points connected by large diameter pipes. The lower level network consisting of pipes with small diameters through which water is distributed to the end user is not studied since it is assumed that a feasible solution for the main supply network will also be feasible for the underlying distribution network. This assumption relies on the fact that the pressures throughout the network are sufficiently high and that the clustered points are close to each other and at approximately the same elevation so that large pressure losses are prevented.

Before addressing these components, we introduce the discrete time setting that is required to be able to model the complex water supply system. Next, the different sets, variables and parameters that are used throughout the chapter are defined. The modeling is done for each set separately and additional attention is given to the modeling of pressure losses. Afterwards, different goal functions are formulated. The resulting model is a nonconvex MINLP (mixed integer nonlinear program).

3.1 Discrete time setting

Flow in a water network is a continuous process. From a practical point of view, modeling such a system is done in a discrete time setting. This means that a day is divided into discrete periods during each of which the state of the network is assumed to be constant. We will denote an interval by the subscript $t \in [1, T]$, where T represents the total number of periods. The length of each period is denoted by τ_t. Usually a division in 24 hours is proposed, as is the case in [5]. The authors note that discretization is motivated by the discrete nature of the forecasted demand and electricity tariffs.

A typical demand pattern (provided by the department of Water Technology of De Watergroep) is displayed in figure 3.1.

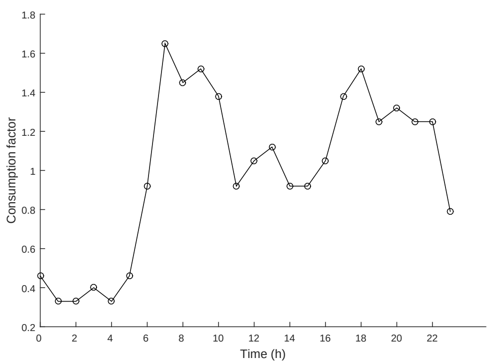


Figure 3.1: A typical daily water demand pattern

The consumption factor c_t represents the dimensionless relative usage.

Every time period that is added to the model introduces a very large amount of additional variables and constraints. Based on similar coefficients in subsequent periods and taking into account the different day and night tariffs for electricity, the number of periods can be reduced by clustering. The gain in computational time has to be weighed against the loss of accuracy; experiments with different divisions will be carried out. Figure 3.2 shows a division over five periods, in which periods 1 and 5 correspond to low energy tariff (10pm-7am) and periods 2-4 to high energy tariff (7am-10pm). The red lines indicate the start of each period. Note that during weekends the low energy tariff is used 24 hours per day.

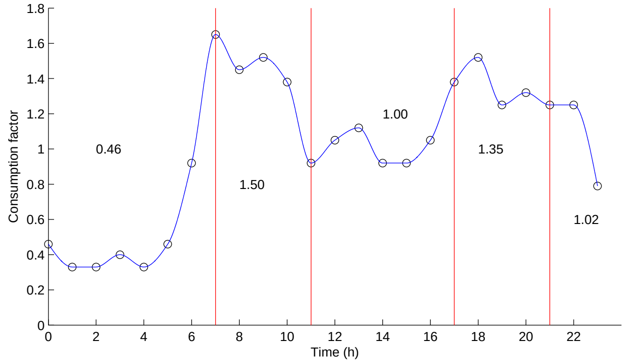


Figure 3.2: A division of the demand pattern in 5 periods with corresponding coefficients

Here, we find the following values for c_t and τ_t:

$$\tau_1 = 7,\ \tau_2 = 4,\ \tau_3 = 6,\ \tau_4 = 5,\ \tau_5 = 2$$
$$c_1 = 0.02,\ c_2 = 0.06,\ c_3 = 0.04,\ c_4 = 0.06,\ c_5 = 0.04$$

with $\sum_{t=1}^{5} \tau_t\ c_t = 1$.

Finally, seasonal variations in demand exist. For practical purposes we work with different demand coefficients for winter and summer.

3.2 Sets, variables and parameters

The general graph structure of the model was already introduced. First we define sets that represent components as nodes or arcs. Next, a reference list of variables and parameters is presented together with a short description. The network is modeled as a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ is the node set representing junctions, delivery points, buffers and raw water sources; $\mathcal{A}$ is the arc set representing simple pipes and pipes with pure and raw water pumps. In the remainder of this work, nodes will be indicated by the subscript i, whereas the subscript ij will be used to represent pipes connecting nodes i and j.

3.2.1 Sets

Nodes:

- $\mathcal{J}$: junctions;
- $\mathcal{D}$: delivery nodes;

- $\mathcal{B}$: buffers;
- $\mathcal{S}$: raw water source;
- $\mathcal{N} = \mathcal{J} \cup \mathcal{D} \cup \mathcal{B} \cup \mathcal{S}$.

Arcs:

- $\mathcal{P}i$: pipes;
- $\mathcal{P}u$: pure water pumps;
- $\mathcal{P}u^v \subseteq \mathcal{P}u$: variable speed pumps;
- $\mathcal{V}$: valves
- $\mathcal{P}r$: raw water pumps;
- $\mathcal{P}r^t \subseteq \mathcal{P}r$: raw water pumps type 2;
- $\mathcal{A} = \mathcal{P}i \cup \mathcal{P}u \cup \mathcal{V} \cup \mathcal{P}r$.

3.2.2 Model parameters with their corresponding units

- T: maximum number of periods (-);
- τ_t: length of period t (h);
- g: gravitational acceleration (m/s^2);
- $c_t(e)$: electricity cost in period t (€/kWh).

Nodes:

- h_i: geographic height in node i (m);
- d_{it}: (clustered) demand (m^3/h) at node i in period t ;
- l_{it}^{min}: minimum delivery at delivery node i in period t (m^3/day);
- l_{it}^{max}: maximum delivery at delivery node i in period t (m^3/day);
- h_{it}^{min}: minimal piezometric pressure at delivery node i in period t (m);
- h_{it}^{max}: maximal piezometric pressure at delivery node i in period t (m);
- p_i: price of water in delivery node i (€/m^3);
- A_i: cross-sectional area of tank i (m^2);

- l_i^{min}: minimum level in tank i (m);
- l_i^{max}: maximum level in tank i (m);
- h_i^{fl}: level of tank i's floor (m);
- h_i^{in}: level of inflow in tank i (m).

Arcs:

- k_{ij}: pipe/pump (i, j)'s roughness (m);
- d_{ij}: pipe/pump (i, j)'s diameter (m);
- l_{ij}: pipe/pump (i, j)'s length (m);
- κ_{ij}: loss coefficient of pipe/pump (i, j) (h^2/m^5);
- v_{ij}^{max}: maximum speed of flow in arc (i, j) (m/s);
- $h_{ij}^1, h_{ij}^2, h_{ij}^3$: head coefficients of pump (i, j)(varies);
- $e_{ij}^1, e_{ij}^2, e_{ij}^3$: efficiency coefficients of pump (i, j)(varies);
- p_{ij}^1, p_{ij}^2: power coefficients of pump (i, j)(varies);
- q_{ij}^{min}: pump (i, j)'s minimum working flow (m^3/h);
- q_{ij}^{max}: pump (i, j)'s maximum working flow (m^3/h);
- q_{ij}^{cap}: production capacity in WPC (i, j) (m^3/h);
- q_{ij}^{lim}: daily extraction limit in WPC (i, j) (m^3/h);
- f_{ij}: max fluctuation in water production from one period to the next in WPC (i, j) (m^3/h);
- $c_{ij}(p)$: production cost in a water production center (i, j) (€/m^3).

With the exception of the number of periods T and the tank's cross-sectional area (A), all parameters are denoted by lower-case letters.

3.2.3 Model variables with their corresponding units

Nodes:

- H_{it} : Piezometric head at node i in period t (m);
- I^+_{it} : Inflow at entrance buffer i in period t (m^3/h);
- I^-_{it} : Outflow at entrance buffer i in period t (m^3/h);
- O_{it} : Outflow at exit buffer i in period t (m^3/h);
- V_{it} : Volume in tank i at end of period t (m^3);
- L_{it} : Level of tank i at end of period t (m);
- H^M_{it} : Mean piezometric head of tank i in period t (m);
- ϑ^1_{it} : Binary variable inflow of tank i in period t (-);
- ϑ^2_{it}: Binary variable outflow of tank i in period t (-).

Arcs:

- Q_{ijt} : Flow on arc (i, j) during period t (m^3/h);
- ΔH_{ijt} : Pump (i, j)'s head increase during period t (m);
- P_{ijt} : Power pump (i, j) during period t (W);
- F_{ijt} : Frequency of variable speed pump (i, j) during period t (Hz);
- ϑ_{ijt} : Binary activity status pump/valve (i, j) during period t (-).

The central decision variables of the model are the amounts of water (Q_{ijt}) produced in the water production centers (WPC), the activity status (ϑ_{ijt}, on/off) and the working point ($Q_{ijt}, \Delta H_{ijt}$) of a pump. For variable speed pumps the frequency F_{ijt} at which the pump works needs to be determined. Knowledge about the values of these variables during each period will allow operators to optimally control the entire network. The other variables are thus dependent and can be determined based on the above sets of decision variables. Note that all variables are symbolized with a capital or greek letter.

The flow variables Q_{ijt} can take on negative values since we work in a directed graph. If Q is positive, water will flow from node i to j; if $Q < 0$ an amount of water $|Q|$ will be transferred from j to i. It is furthermore assumed that the flow in a pipe is constant within the same period, as opposed to research in other works [67].

The piezometric pressure (or head) H is conventionally measured as a height and is obtained as the sum of the elevation head (height) h and the pressure head $\frac{p}{\rho g}$, where p is the manometric pressure, ρ the density of water and g the gravitational acceleration.

3.3 MINLP model

In this section we will fully describe the working components of the drinking water network. For each of these components, the corresponding restrictions will be mathematically modeled. A typical drinking water network structure can be seen in figure 3.3.

The following restrictions are valid for each $t \in [1, T]$ unless otherwise stated.

3.3.1 Nodes ($i \in \mathcal{N}$)

The manometric pressure in every part of the network has to be kept below the design pressure. In the studied network a general upper bound of 10 bar is imposed. From a practical point of view, this pressure is calculated in the end points of each pipe. We implicitly assume that the geographic height at every position of the arc (i, j) lies between that of its endpoints. Otherwise stated, each pipe/pump's height is monotone increasing or decreasing between the two nodes that it connects.

Since the meter is used to quantify the piezometric head, the values need to be converted. The manometric pressure is given as $\frac{p}{\rho g}$ and

$$\rho g = 1000\,\frac{kg}{m^3} \times 9{,}81\,\frac{m}{s^2} \approx 10\,000\,\frac{Pa}{m} = 0.1\,\frac{bar}{m}$$

Subsequently the pressure restriction becomes (taking into account no negative pressure may occur):

$$0 \le H_{it} - h_i \le 100 \tag{3.1}$$

3.3.1.1 Junctions ($i \in \mathcal{J}$)

Junctions are the most common nodes in which either the pipe's characteristics change, the pipe splits up between two or more sections or a nonzero (clustered) demand d is assigned (see figure 3.4).

The flow conservation constraints at these nodes are given by:

$$\sum_{k:(k,i)\in\mathcal{A}} Q_{kit} - \sum_{j:(i,j)\in\mathcal{A}} Q_{ijt} = d_{it} \tag{3.2}$$

where d_{it} denotes the demand (+) or supply (-) in node i.

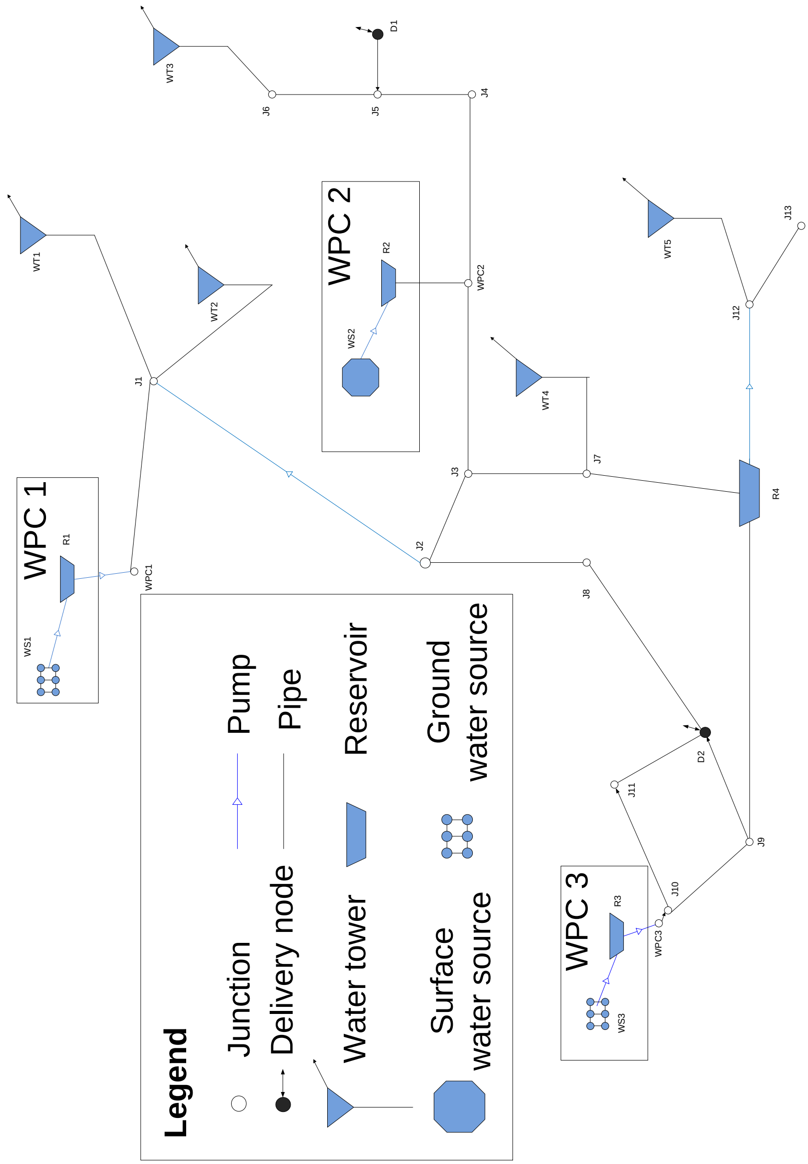


Figure 3.3: Example of the studied mesh structured drinking water network

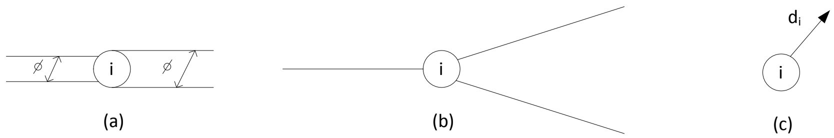


Figure 3.4: Junction (a) between two pipes with different diameter, (b) between 3 pipes, (c) with assigned demand

3.3.1.2 Delivery Nodes ($i \in \mathcal{D}$)

At borders of the network, interaction with other drinking water companies can occur. Since each organization has its own distribution network, exchange of drinking water can take place at these border nodes. Water can be bought as well as sold at a fixed price ap. Usually, the agreed upon amount has to be in between l^{min} and l^{max} levels:

$$l_{it}^{min} \leq \sum_{k:(k,i)\in\mathcal{A}} Q_{kit} - \sum_{j:(i,j)\in\mathcal{A}} Q_{ijt} \leq l_{it}^{max} \tag{3.3}$$

The pressure at these nodes is however dependent on the situation at the other side of the border. From historical data, the minimum and maximum pressure levels can be derived:

$$h_{it}^{min} \leq H_{it} \leq h_{it}^{max} \tag{3.4}$$

3.3.1.3 Buffers ($i \in \mathcal{B}$)

Buffers and their storage capacity play an essential role in drinking water supply networks. In addition to their deployment to hedge against demand variability, the possibility of buffering water in itself is of a major economic importance. At night, when the energy tariff is low, water can be stored in the reservoirs using pumps. The next day the stored amounts of water can flow gravitationally to the underlying distribution network or, albeit exceptionally, back into the supply network to economically meet part of the demand. Consequently, high energy costs due to excessive pumping during the day can be avoided. Both pure water reservoirs and water towers are categorized as buffers: the latter ones can be considered as elevated tanks on an underlying construction.

Figure 3.5 shows how a buffer typically operates. This figure is based on the detailed model in appendix A (property of De Watergroep). Despite the complexity of its operations and the multitude of connections, each buffer is modeled as a single conceptual node. Through the connection pipe (a) water flows in (I^+) or out (I^-) depending on the piezometric head H at this node. Actual inflow takes place only through connection (a_1) during periods in which the head at the conceptual

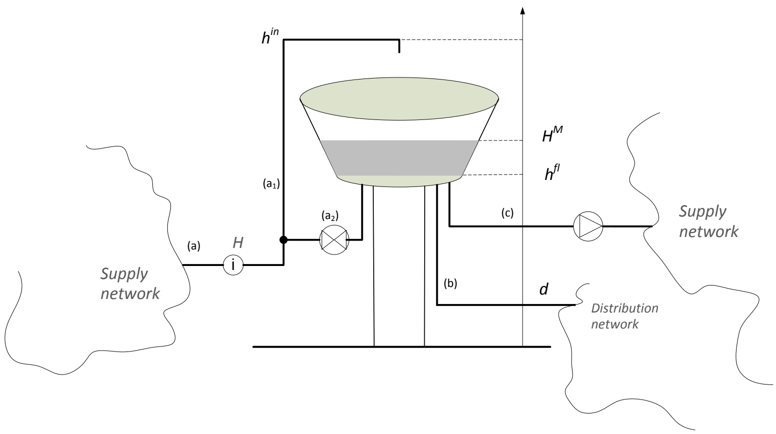


Figure 3.5: Model of a buffer: depending on the variable value of the head (H) at the entrance i, water can flow either in or out of the buffer.

node is larger than the height of the inflow point h^{in}, which is usually located $\approx$ 1 meter above the edge of the buffer's tank. Due to quality measures water never flows directly into the tank. This could cause the water in the highest part of the tank to leave less easily, thus reducing the water's quality. During periods in which the mean piezometric head of the water in the tank H^M is higher than the head at the conceptual node, water is re-injected into the network through connection pipe (a_2). This is expressed by the following relations:

$$I^+_{it} \leq A_i\, l_i^{max}\, \vartheta^1_{it} \tag{3.5}$$

$$I^-_{it} \leq A_i\, l_i^{max}\, \vartheta^2_{it} \tag{3.6}$$

$$H_{it} - h_i^{in} \geq (h_i - h_i^{in})\,(1 - \vartheta^1_{it}) \tag{3.7}$$

$$H^M_{it} - H_{it} \geq (h_i^{fl} - 100 - h_i)\,(1 - \vartheta^2_{it}) \tag{3.8}$$

$$\vartheta^1_{it} + \vartheta^2_{it} \leq 1 \tag{3.9}$$

In this set of equations, the binary variables ϑ^1_{it} and ϑ^2_{it} are used to model in- and outflow. Inflow $I^+_{it} > 0$ only takes place if the pressure H_{it} at the entrance is higher than the inflow level h_i^{in}. Similarly, water flows out of the buffer ($I^-_{it} > 0$) only if $H^M_{it} \geq H_{it}$. The big-M term 100 is derived from the fact that the manometric pressure cannot exceed 10 bar (100 m) in any part of the network - see also equations (3.1). Pipe (b) represents the connection with the underlying distribution network.

This network consists of pipes with a very small diameter and is too detailed to be modeled at this level. We cluster the demand of all the consumers in one single demand point d which can be fed directly by the buffer.

Some buffers can have an additional connection at the exit. Through this connection pipe (c), boosters or fresh water pumps are used to transport water out of the buffer. Denoting this outflow by O_{it}^-, the conservation of flow in the buffer is then given by:

$$\sum_{k:(k,i)\in\mathcal{A}} Q_{kit} - \sum_{j:(i,j)\in\mathcal{A}\setminus\mathcal{P}u} Q_{ijt} = I_{it}^+ - I_{it}^- + d_{it} \tag{3.10}$$

$$\sum_{j:(i,j)\in\mathcal{P}u} Q_{ijt} = O_{it}^- \tag{3.11}$$

An important variable is the buffer volume at the end of a period, V_{it}, because it links two subsequent periods through the filling rate.

$$V_{it} = V_{i,t-1} + (I_{it}^+ - I_{it}^- - O_{it})\tau_t \tag{3.12}$$

$$V_{i0} \leq V_{iT} \tag{3.13}$$

Observe that the second constraint prevents the optimal configuration from depleting all buffers in the last period, and essentially allows the optimal 24-hour plan to be extended to several days (if the demand parameters and electricity prices remain constant).

To obtain an approximation of the mean head of the water level in the buffer during a period, the mean fluid level is augmented by the geometric height of the tank floor:

$$H_{it}^M = h_i^{fl} + \frac{L_{it} + L_{i,t-1}}{2} \tag{3.14}$$

where $L_{it} = \dfrac{V_{it}}{A_i}$ is the water level in the buffer at the end of period t. The cross section of the water tank, A_i, is approximated by a circle with constant diameter over the entire height, despite the fact that the tank may be conical in shape. The fluid level is restricted by a lower bound due to water quality considerations and by an upper bound, the total height of the buffer.

$$l_i^{min} \leq L_{it} \leq l_i^{max} \tag{3.15}$$

Finally it has to be noted that the model presented here is a very general one. For some buffers the rerouting of water into the network is prohibited. In those cases, no additional binary variables are introduced.

3.3.2 Arcs ($(i, j) \in \mathcal{A}$).

We discern pipes, constant and variable speed pumps, valves and raw water pumps.

3.3.2.1 Pipes ($(i, j) \in \mathcal{P}i$).

As water flows through a pipe, pressure losses caused by friction occur. From a mathematical point of view, the general formula to describe this function is (see [16]):

$$H_{it} - H_{jt} = \Phi_{ij}(Q_{ijt})$$

where Φ is a strictly increasing uneven function that is concave on the negative axis of its domain and convex on the positive axis. Exact formulations depend on the viscosity of the water and the pipe characteristics. The Reynolds number is used to characterize the nature of the flow ([68]):

$$Re = \frac{d\,v\,\rho}{\mu}$$

where d is the diameter of the pipe, v the speed of the flow and ρ the density of water. The speed v (in m/s) is given by

$$v = \frac{4Q}{3600\pi d^2}$$

The dynamic viscosity μ is dependent of the temperature, but in analysis of water supply networks a constant temperature of 20 °C will be assumed. The dimensionless Reynolds number states whether a flow is turbulent or laminar. General values are given in table 3.1. In the transitional region, other factors (e.g. pipe roughness) will determine whether the flow is turbulent or laminar.

Table 3.1: Reynolds Number for various flow regimes

Flow Regime	Reynolds Number
Laminar	< 2000
Transitional	$2000 - 4000$
Turbulent	> 4000

The most commonly used pressure loss formulas are the Hazen-Williams (H-W) equation and the Darcy-Weisbach (D-W) equation:

$$H_{it} - H_{jt} = \frac{10.7\, l_{ij}}{3600^{1.852}\, C_{ij}^{1.852}\, d_{ij}^{4.87}}\, \text{sign}(Q_{ijt})|Q_{ijt}|^{1.852} \tag{H-W}$$

$$H_{it} - H_{jt} = \frac{8\, l_{ij}}{3600^2\, g\, \pi^2\, (d_{ij})^5}\, \lambda_{ij}\, Q_{ijt}|Q_{ijt}| \tag{D-W}$$

C is the Hazen-Williams coefficient that represents the carrying capacity. λ denotes the (dimensionless) friction coefficient. Both formulas introduce nonconvex and nonlinear constraints in the network. Furthermore, the H-W formula is not second order differentiable while the second order derivative of the D-W equation with constant friction coefficient is discontinuous for $Q = 0$. Here, the second (D-W) formula will be used. Different formulations for the friction coefficient λ exist. The most commonly used one is Colebrook-White:

$$\frac{1}{\sqrt{\lambda_{CW}}} = -0.86 \ln\left(\frac{k}{3.7d} + \frac{2.51}{Re\,\sqrt{\lambda_{CW}}}\right)$$

The letter k represents the roughness of the pipe. This formulation introduces two problems: it is implicit for λ and the Reynolds number Re, as a function of the variable Q, is logarithmic. This strongly increases the complexity of the mathematical model. The formula of Swamee-Jain [69].

$$\lambda_{SJ} = \frac{1.325}{\left(\ln\left(\frac{k}{3.7d} + \frac{5.47}{Re^{0.9}}\right)\right)^2}$$

circumvents the implicitness of the formulation but is still nonlinear in terms of the Reynolds number (and hence the flow). According to [70] the formula is accurate within 1 % compared to the Colebrook-White formula over the interval $4.10^3 \leq Re \leq 1.10^8$ and $1.10^{-6} \leq k/d \leq 1.10^{-2}$.

The formula of Prandtl-Kármán, finally, offers an approximation that is independent on the Reynolds number and is computationally very cost-efficient:

$$\lambda_{PK} = \left(2 \log \frac{k}{3.71\, d}\right)^{-2}$$

A comparison of these different formulations is displayed in figure 3.6 for a typical pipe with a diameter of $d = 500$mm and a roughness of $k = 0.5$.

The Swamee-Jain formula approximates the friction factor quite well, but it shows a jump discontinuity at the origin. Clearly, the PK formulation underestimates the friction losses. For small values of the flow, the relative difference is quite big. However, in a drinking water supply network such values for the flow are very uncommon, which makes this approximation sufficiently accurate for this model.

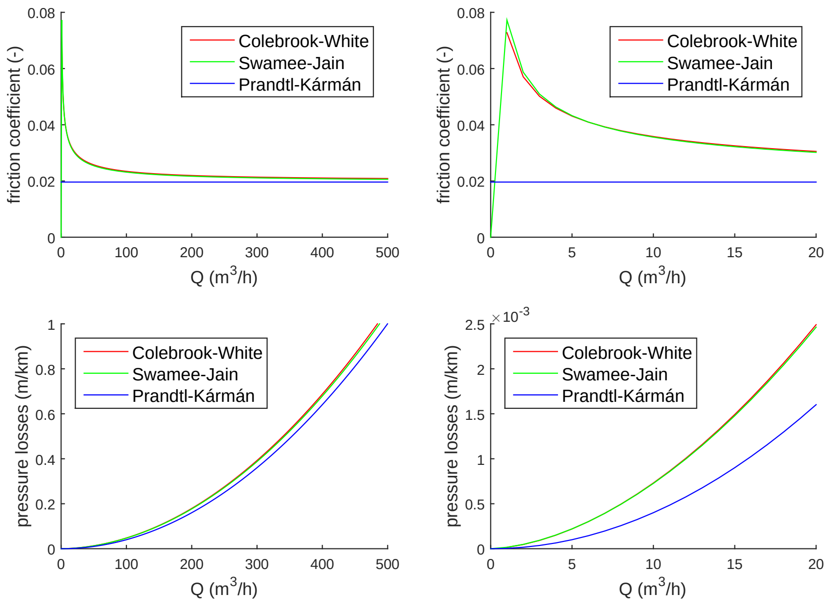


Figure 3.6: Friction coefficients (top) and pressure losses (bottom) for different formulations in function of large values of the flow (left) and small values (right).

Furthermore it is independent of Re making it the formula of choice here. For a more precise calculation of the friction losses, the reader is referred to [5]. Fig. 3.7 shows a plot of a nonconvex pressure loss constraint.

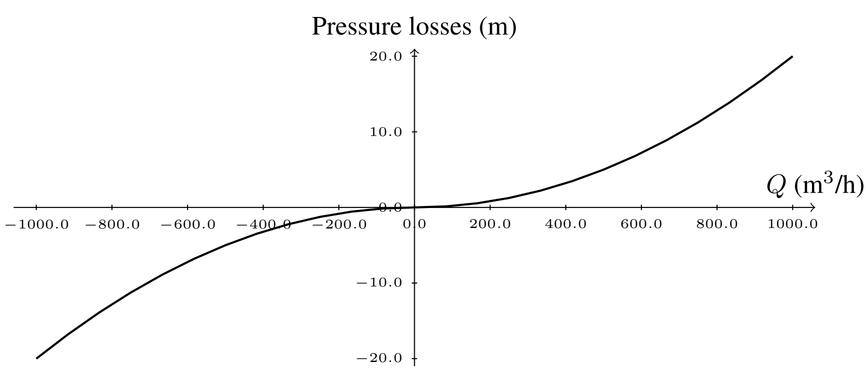


Figure 3.7: Pressure losses in a pipe with l = 5 km, d = 500 mm, k = 0.5

In general, the friction loss in pipes is given as:

$$H_{it} - H_{jt} = \kappa_{ij}\, Q_{ijt}|Q_{ijt}| \tag{3.16}$$

where the loss coefficient κ_{ij} is given by:

$$\kappa_{ij} = \frac{8\, l_{ij}}{3600^2\, g\, \pi^2\, (d_{ij})^5}\, \lambda_{ij}$$

The maximum value that water velocity (v_{max}) can reach in a pipe sets off bounds on the value of the flow of water through this pipe [71]:

$$-3600\frac{\pi}{4} v_{ij}^{max} (d_{ij})^2 \leq Q_{ijt} \leq 3600\frac{\pi}{4} v_{ij}^{max} (d_{ij})^2 \tag{3.17}$$

Note that these are general bounds and tighter bounds may be applied where needed.

3.3.2.2 Pure water pumps ($(i, j) \in \mathcal{P}u$)

Water flows from points of high pressure to points of low pressure. If water needs to be pushed to a place where a high pressure is present, pumps will be installed to increase the pressure. They also fulfill the role of providing water in tanks for intermediate storage. Pure water pumps are either delivery pumps or boosters. Delivery pumps add pressure to push water from the fresh water basements at the production centers into the network, whereas boosters are installed on well-chosen places, generally locations in the network where extra pressure is needed due to friction losses. Since a pump is connected to a pipe, they are modeled together as a single conceptual arc. The following bounds are imposed on the flow in these arcs:

$$\vartheta_{ijt}\, q_{ij}^{min} \leq Q_{ijt} \leq \vartheta_{ijt}\, q_{ij}^{max} \tag{3.18}$$

where ϑ_{ijt} is a binary variable which indicates whether the pump is active or shut down during period t and $[q_{ij}(min), q_{ij}(max)]$ is the feasible working range of the pump.

The head delivered by the pump, ΔH, is added to the right hand side of the pressure loss restriction related to the pipe in which it is active:

$$H_{it} - H_{jt} = \kappa_{ij}\, (Q_{ijt})^2 - \Delta H_{ijt} \quad \forall i \in \mathcal{N}\backslash\mathcal{B} \tag{3.19}$$

$$H_{it}^M - H_{jt} = \kappa_{ij}\, (Q_{ijt})^2 - \Delta H_{ijt} \quad \forall i \in \mathcal{B} \tag{3.20}$$

Note that always $Q > 0$ here, so the quadratic function correctly models the pressure loss. The second constraints apply to pumps transporting water out of a buffer (connection pipe (c) in figure 3.5). Here, the starting pressure is equal to the pressure in the tank H_{it}^M.

Fixed speed pumps $(i, j) \in \mathcal{P}u \backslash \mathcal{P}u^v$ The pump head depends on the flow that is pushed through the pump. Data provided by the pump supplier is used to approximate the relation between the head, ΔH, and the flow, Q. This is the pump characteristic curve. Head produced by a regular speed pump, when operating, is given by:

$$\Delta H_{ijt} = h^1_{ij}\,(Q_{ijt})^2 + h^2_{ij}\,Q_{ijt} + h^3_{ij}\,\vartheta_{ijt}$$

An inactive pump needs to have a flow rate of zero, but the pump head can take on any value. This is modeled as

$$-\,(1 - \vartheta_{ijt})\,200 \leq \Delta H_{ijt} - h^1_{ij}\,(Q_{ijt})^2 - h^2_{ij}\,Q_{ijt} - h^3_{ij}\,\vartheta_{ijt} \leq (1 - \vartheta_{ijt})\,200 \quad (3.21)$$

Here 200 is a strict bound on the maximum allowed pressure difference between two adjacent nodes. Clearly, if the pump is switched on during a certain period ($\vartheta_{ijt} = 1$) the original restriction is restored. If $\vartheta_{ijt} = 0$ then from (3.18) it follows that $Q_{ijt} = 0$ and ΔH_{ijt} can take on any value in the interval $[-200, 200]$.

The total pump efficiency E for a fixed speed pump is given as the product of mechanic efficiency E^{mech} and electric motor efficiency E^{elec}:

$$E = E^{mech} \times E^{elec}$$

The mathematical equation for pump efficiency can be derived from the pump data and is given as:

$$E_{ijt} = e^1_{ij}(Q_{ijt})^2 + e^2_{ij}\,Q_{ijt} + e^3_{ij}$$

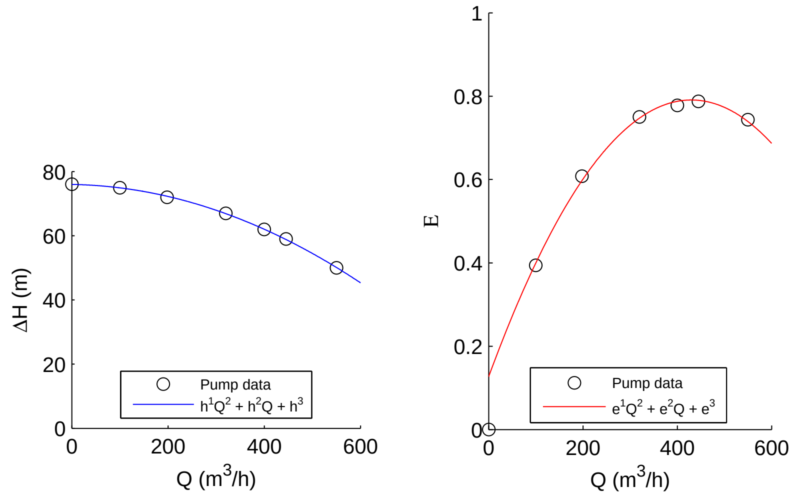
Figures 3.8(a) and 3.8(b) show approximating curves of the head and efficiency of a typical pump that is currently in use in the water supply network of De Watergroep.

The power term P is given by:

$$P = \frac{\gamma\,\Delta H\,Q}{E}$$

Recall that γ is the specific weight of water and is the product of g and ρ and is equal to $\approx 10\,000 kg/(m^2\,s^2)$. Since Q is expressed in m^3/h, this number needs to be divided by 3600 to get the correct unit for P (Watt). The expression then comes down to:

$$P_{ijt} = 2.73 \frac{\Delta H_{ijt}\,Q_{ijt}}{E_{ijt}} \quad \text{(P-1)}$$



(a) Fitted pump characteristic using pump data (b) Fitted efficiency curve using pump data

Figure 3.8: Graphical display of plotted pump data

To avoid numerical difficulties we force the coefficient e^3_{ij} in the efficiency inequality to be strictly positive. This prevents the efficiency, and thus the denominator of the power term P, from taking the value 0 when the pump is not active ($Q_{ijt} = 0$).

Since the previous expression for the power term is computationally very expensive, we choose to model it differently. The equations for efficiency and head are substituted in (P-1) to produce the curve equation for the power based on pump data (see Fig. 3.9). Then we fit a first-order curve given by the following expression:

$$P_{ijt} = p^1_{ij}\, Q_{ijt} + p^2_{ij}\, \vartheta_{ijt} \tag{3.22}$$

where p^1 and p^2 are the power coefficients. Here again, the binary variable ϑ_{ijt} is added to ensure that $P_{ijt} = 0$ when $Q_{ijt} = 0$.

Although this formula is physically not correct, its use in our model will not lead to any practical infeasibilities in the generated operating plan. The reason is that P is solely used to calculate the energy cost term and does not appear in any of the important hydraulic restrictions. Therefore the only effect of the approximation is a small error on the total energy cost.

Variable speed pumps $(i, j) \in \mathcal{P}u^v$. A cost efficient way to change the pump's characteristic curve is to change its speed by altering the frequency of the source voltage to the electric motor. By default the frequency is 50 Hz and by altering this

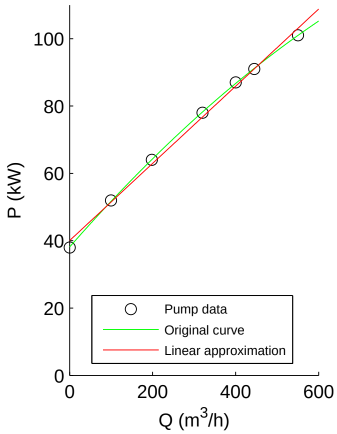


Figure 3.9: Fitted power function using pump data

frequency the turning speed of the pump changes, causing the pump characteristic to shift according to figure 3.10(a). A new working point is established which causes an increase in flow, pressure and power.
The relationship of these variables with the turning speed is given by:

$$\frac{\Delta H_1}{\Delta H_2} = \frac{n_1}{n_2}$$
$$\frac{Q_1}{Q_2} = \left(\frac{n_1}{n_2}\right)^2$$
$$\frac{P_1}{P_2} = \left(\frac{n_1}{n_2}\right)^3$$

where n_i $(i = 1, 2)$ is the turning speed. The efficiency in a variable speed pump can be expressed as the product of mechanic efficiency E^{mech}, electric motor efficiency E^{elec} and the efficiency of the frequency control E^{freq}:

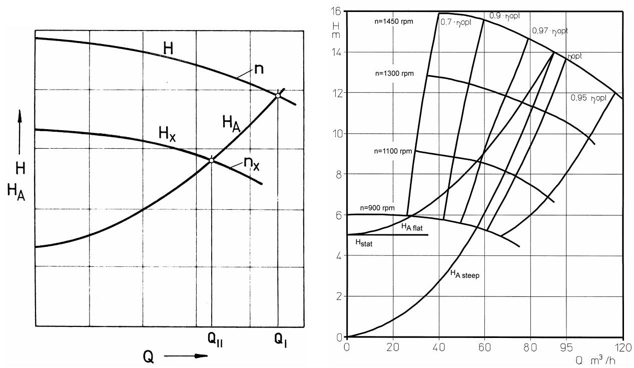
$$E = E^{mech} \times E^{elec} \times E^{freq}$$

For small changes of the turning speed, the efficiency remains approximately constant, whereas for larger changes it can be calculated by:

$$E_1 \approx 1 - (1 - E_2)(n_2/n_1)^{0.1}$$

Since this difference is only notable for small values of the efficiency (which are avoided in the optimal solution of this model), it is neglected here and efficiency is assumed to be independent of turning speed (and thus frequency).

Figure 3.10(b) displays the complete working area of a typical variable speed pump. This area is limited by the minimum and maximum speeds of the electro motor. In this model we impose a lower bound of 30 Hz and an upper bound of 55 Hz.



(a) Working point (b) Working area

Figure 3.10: Characteristics of a variable speed pump [2]

Using the previous relations, the head delivered by an operating variable speed pump can be modeled as:

$$\Delta H_{ijt} = h_{ij}^{1}\,(Q_{ijt})^2 + h_{ij}^{2}\,\frac{F_{ijt}}{f_{ij}^{ref}}\,Q_{ijt} + h_{ij}^{3}\left(\frac{F_{ijt}}{f_{ij}^{ref}}\right)^2$$

f_{ij}^{ref} is the reference frequency of 50 Hz. Subsequently the restriction is given by

$$-(1-\vartheta_{ijt})\,200 \leq \Delta H_{ijt} - h_{ij}^{1}\,(Q_{ijt})^2 - h_{ij}^{2}\,\frac{F_{ijt}}{f_{ij}^{ref}}\,Q_{ijt} - h_{ij}^{3}\left(\frac{F_{ijt}}{f_{ij}^{ref}}\right)^2 \leq (1-\vartheta_{ijt})\,200 \tag{3.23}$$

If $\vartheta_{ijt} = 0$, ΔH is not necessarily restricted to the interval $[-200, 200]$. In order to obtain this, we introduce the following limits on the frequency:

$$\vartheta_{ijt}\,30 \leq F_{ijt} \leq \vartheta_{ijt}\,55 \tag{3.24}$$

forcing F to be 0 when $\vartheta_{ijt} = 0$ and thus effectively validating constraint (3.23).

The approximated power term of the variable speed pump is given by the following expression:

$$P_{ijt} = p^1_{ij}\, Q_{ijt} \left(\frac{F_{ijt}}{f^{ref}_{ij}}\right)^2 + p^2_{ij} \left(\frac{F_{ijt}}{f^{ref}_{ij}}\right)^3 \tag{3.25}$$

Unfortunately this constraint again contains nonlinear functions.

Although not included in this model, it is worth noting that other transient conditions are often included by other authors. These restrictions involve the prevention of excessive pump switching. Additional integer variables and constraints have to be added. Due to the division of the optimization horizon in hourly intervals, motivated by demand forecasts and electricity prices, these minimum run- and downtimes are automatically satisfied.

3.3.2.3 Valves ($(i, j) \in \mathcal{V}$)

For completeness we also mention valves. These can also be modeled as pipes. The simplest kind of valves are those that prevent flow from going in a specific direction, so called check valves (CV). If a backward flow from node j to node i is to be avoided, one can simple add the restriction $Q_{ijt} \geq 0$ for all time steps.

Valves can also be used to separate two pipes when they are closed in the sense that they prevent flow from passing through. The variable ϑ_{ijt} denotes the activity status of the valve (on:1, off: 0). Given the bounds on flow and head values, the constraints can be written as

$$-3600\frac{\pi}{4}v^{max}_{ij}(d_{ij})^2\vartheta_{ijt} \leq Q_{ijt} \leq 3600\frac{\pi}{4}v^{max}_{ij}(d_{ij})^2\vartheta_{ijt} \tag{3.26}$$

$$-200\,(1-\vartheta_{ijt}) \leq H_{it} - H_{jt} - \kappa_{ij}\, Q_{ijt}|Q_{ijt}| \leq 200\,(1-\vartheta_{ijt}) \tag{3.27}$$

A closed valve forces the flow to be 0, while the pressure variables are decoupled (can take on any value in $[-200, 200]$); when the valve is open the regular pressure loss formula applies.

In [67] several other types of valves (gate valves, flow control valves and pressure breaker valve) are described and modeled in detail. In the current work only check valves appear in the network instances.

3.3.2.4 Raw water pumps ($(i, j) \in \mathcal{P}r$)

Raw water pumps are used to draw water from ground water wells or surface water basins. In this model, they conceptually represent the whole of a water production center (WPC). These arcs cover every production step, from pumping raw water out of surface water basins or ground water wells, through the treatment phases, to the storage of fresh water in the fresh water basins (modeled as a buffer). As such, the treatment capacities and daily extraction limits are linked to the amounts of water extracted by raw water pumps:

$$0 \leq Q_{ijt} \leq q_{ij}^{cap} \tag{3.28}$$

$$\sum_{t=1}^{T} Q_{ijt} \leq q_{ij}^{lim} \tag{3.29}$$

Ground water wells are modeled as sources with infinite capacity and the volume of surface water basins after each period is registered but in general these do not restrict the production center's capacity. The second set of constraints applies to ground water sources, where excessive pumping can harm the environment. These yearly restrictions are simplified to daily capacities here. For production centers with surface water basins this restriction is dropped.

To prevent water quality problems from occurring, the change in the flow levels between two subsequent periods has to be limited to some fixed percentage (f) of the treatment capacity:

$$-f_{ij}\, q_{ij}^{cap} \leq Q_{ijt} - Q_{ij,t-1} \leq f_{ij}\, q_{ij}^{cap} \tag{3.30}$$

$$-2\, f_{ij}\, q_{ij}^{cap} \leq Q_{ijT} - Q_{ij,0} \leq 2\, f_{ij}\, q_{ij}^{cap} \tag{3.31}$$

Again the second restriction is added to allow the optimal production plan to be repeated on a daily basis.

Raw ground water pumps type 2 ($(i, j) \in \mathcal{P}r^t$). In some ground water stations, each individual well is equipped with a 'type 2' pump. These pumps are set to a fixed flow to guarantee water quality. Furthermore, frequent on/off switching of these pumps is undesirable. The joint flow is therefore a discrete value of a set, $\mathcal{K}_{\mathcal{P}r^t}$ that contains all possible combinations ($q_k^{cap}, k \in \mathcal{K}_{\mathcal{P}r^t}$) of the configuration of type 2 pumps in set $\mathcal{P}r^t$. We define the variable $\alpha_k \in [0, 1]$ for each possible combination k. The total production flow in a type 2 station can then be defined as:

$$Q_{ijt} = \sum_{k \in \mathcal{K}_{\mathcal{P}r^t}} \alpha_k q_k^{cap} \tag{3.32}$$

Only allowing one possible combination throughout the planning horizon, we add the following restrictions:

$$\sum_{k \in \mathcal{K}_{\mathcal{P}r^t}} \alpha_k = 1 \tag{3.33}$$

3.3.3 Goal functions

We define three types of costs: water treatment and production costs at WPCs, energy costs at pumping stations and delivery costs at border nodes.

3.3.3.1 Production/electricity cost at the water production centers

This cost consists of many components: electricity, taxes, loans and chemicals. The variance of cost in each of these components results in big differences between the various water production centers.

Electricity The total yearly electricity cost contains the cost for lighting and electricity required for water treatment, as well as energy cost to power the raw water pumps. Since the pump head required to push water through the water production center is negligible, it is assumed to be dependent on the total production flow as opposed to the power. This gives a direct way to calculate the power, assuming a constant pump head. The average electricity cost is based on the electricity bill of 1 production week (168 hours) and as such is not dependent on day/night tariff.

Taxes This cost is notably different for surface water and ground water production centers. This is a direct consequence of the fact that drinking water companies pay taxes on ground water extraction. Excessive pumping can result in a lower growth of vegetation. The yearly cost on taxes is assumed to be proportional to the total extracted volume.

Loans Cost for personnel is assumed to be independent of produced amounts of water and therefore not included in the production cost. In reality however, transportation of work forces between WPCs takes place. Therefore, the indirect personnel cost is taken into account when the model is used to make strategic decisions.

Chemicals The usage of chemicals is very dependent on the type of WPC as well as the seasonal water quality in the case of surface water.

The total production cost is given as:

$$Z^{WP} = \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{P}r} c_{ij}(p)\,\tau_t\,Q_{ijt} \tag{3.34}$$

3.3.3.2 Energy cost of the pure water pumps

For fixed speed pumps the approximated cost function is:

$$Z^{FSP} = \sum_{t=1}^{T} \sum_{(i,j)\in \mathcal{P}u \setminus \mathcal{P}u^v} \frac{c_t(e)}{1000}\, \tau_t \, \left(p_{ij}^1 \, Q_{ijt} + p_{ij}^2 \vartheta_{ijt}\right) \tag{3.35}$$

For variable speed pumps, this function is given as:

$$Z^{VSP} = \sum_{t=1}^{T} \sum_{(i,j)\in \mathcal{P}u^v} \frac{c_t(e)}{1000}\, \tau_t \left(p_{ij}^1 \, Q_{ijt} \left(\frac{F_{ijt}}{f_{ij}^{ref}}\right)^2 + p_{ij}^2 \left(\frac{F_{ijt}}{f_{ij}^{ref}}\right)^3\right) \tag{3.36}$$

3.3.3.3 Cost at delivery nodes

At delivery points the company can buy/sell a certain quantity of water at some fixed unit price ap_i (dependent on the location of the exchange point i):

$$Z^{DP} = \sum_{t=1}^{T} \sum_{i\in \mathcal{D}} ap_i \, \tau_t \left(\sum_{j:(i,j)\in\mathcal{A}} Q_{ijt} - \sum_{k:(k,i)\in\mathcal{A}} Q_{kit}\right) \tag{3.37}$$

Finally, the total cost is to be minimized:

$$\text{Minimize} \;\; Z^{WP} + Z^{FSP} + Z^{VSP} + Z^{DP} \tag{3.38}$$

3.3.3.4 Complete MINLP model formulation

$$\text{Minimize } Z^{WP} + Z^{FSP} + Z^{VSP} + Z^{DP} \tag{3.38b}$$

s.t.

$$Z^{WP} = \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{P}r} c_{ij}(p)\,\tau_t\,Q_{ijt} \tag{3.34b}$$

$$Z^{FSP} = \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{P}u\setminus\mathcal{P}u^v} \frac{c_t(e)}{1000}\,\tau_t\,\left(p^1_{ij}\,Q_{ijt} + p^2_{ij}\vartheta_{ijt}\right) \tag{3.35b}$$

$$Z^{VSP} = \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{P}u^v} \frac{c_t(e)}{1000}\,\tau_t\,\left(p^1_{ij}\,Q_{ijt}\,\left(\frac{F_{ijt}}{f^{ref}_{ij}}\right)^2 p^2_{ij}\,\left(\frac{F_{ijt}}{f^{ref}_{ij}}\right)^3\right) \tag{3.36b}$$

$$Z^{DP} = \sum_{t=1}^{T}\sum_{i\in\mathcal{D}} ap_i\,\tau_t\,\left(\sum_{j:(i,j)\in\mathcal{A}} Q_{ijt} - \sum_{k:(k,i)\in\mathcal{A}} Q_{kit}\right) \tag{3.37b}$$

$\forall i \in \mathcal{N},\;\; t\in[1,T]:$

$$0 \le H_{it} - h_i \le 100 \tag{3.1b}$$

$\forall i \in \mathcal{J},\;\; t\in[1,T]:$

$$\sum_{k:(k,i)\in\mathcal{A}} Q_{kit} - \sum_{j:(i,j)\in\mathcal{A}} Q_{ijt} = d_{it} \tag{3.2b}$$

$\forall i \in \mathcal{D},\;\; t\in[1,T]:$

$$l^{min}_{it} \le \sum_{k:(k,i)\in\mathcal{A}} Q_{kit} - \sum_{j:(i,j)\in\mathcal{A}} Q_{ijt} \le l^{max}_{it} \tag{3.3b}$$

$$h^{min}_{it} \le H_{it} \le h^{max}_{it} \tag{3.4b}$$

$\forall i \in \mathcal{B}:$

$$V_{i0} \le V_{iT} \tag{3.13b}$$

$\forall i \in \mathcal{B},\;\; t\in[1,T]:$

$$I^+_{it} \le A_i\,l^{max}_i\,\vartheta^1_{it} \tag{3.5b}$$

$$I^-_{it} \le A_i\,l^{max}_i \tag{3.6b}$$

$$H_{it} - h_i^{in} \geq (h_i - h_i^{in})\,(1 - \vartheta_{it}^1) \tag{3.7b}$$

$$H_{it}^M - H_{it} \geq (h_i^{fl} - 100 - h_i)\,(1 - \vartheta_{it}^2) \tag{3.8b}$$

$$\vartheta_{it}^1 + \vartheta_{it}^2 \leq 1 \tag{3.9b}$$

$$\sum_{k:(k,i)\in\mathcal{A}} Q_{kit} - \sum_{j:(i,j)\in\mathcal{A}\setminus\mathcal{P}u} Q_{ijt} = I_{it}^+ - I_{it}^- + d_{it} \tag{3.10b}$$

$$\sum_{j:(i,j)\in\mathcal{P}u} Q_{ijt} = O_{it}^- \tag{3.11b}$$

$$V_{it} = V_{i,t-1} + (I_{it}^+ - I_{it}^- - O_{it})\tau_t \tag{3.12b}$$

$$H_{it}^M = h_i^{fl} + \frac{L_{it} + L_{i,t-1}}{2} \tag{3.14b}$$

$$l_i^{min} \leq L_{it} \leq l_i^{max} \tag{3.15b}$$

$\forall (i,j) \in \mathcal{P}i,\;\; t \in [1,T]:$

$$H_{it} - H_{jt} = \kappa_{ij}\, Q_{ijt}|Q_{ijt}| \tag{3.16b}$$

$$-3600\frac{\pi}{4}v_{ij}^{max}(d_{ij})^2 \leq Q_{ijt} \leq 3600\frac{\pi}{4}v_{ij}^{max}(d_{ij})^2 \tag{3.17b}$$

$\forall (i,j) \in \mathcal{P}u,\;\; t \in [1,T]:$

$$\vartheta_{ijt}\, q_{ij}^{min} \leq Q_{ijt} \leq \vartheta_{ijt}\, q_{ij}^{max} \tag{3.18b}$$

$$H_{it} - H_{jt} = \kappa_{ij}\,(Q_{ijt})^2 - \Delta H_{ijt}, \quad \forall i \in \mathcal{N}\setminus\mathcal{B} \tag{3.19b}$$

$$H_{it}^M - H_{jt} = \kappa_{ij}\,(Q_{ijt})^2 - \Delta H_{ijt}, \quad \forall i \in \mathcal{B} \tag{3.20b}$$

$$\begin{aligned} -(1-\vartheta_{ijt})\,200 &\leq \Delta H_{ijt} - h_{ij}^1\,(Q_{ijt})^2 - h_{ij}^2\,Q_{ijt} - h_{ij}^3\,\vartheta_{ijt} \\ &\leq (1-\vartheta_{ijt})\,200, \quad \forall (i,j) \in \mathcal{P}u\setminus\mathcal{P}u^v \end{aligned} \tag{3.21b}$$

$$P_{ijt} = p_{ij}^1\,Q_{ijt} + p_{ij}^2\,\vartheta_{ijt}, \quad \forall (i,j) \in \mathcal{P}u\setminus\mathcal{P}u^v \tag{3.22b}$$

$$\begin{aligned} -(1-\vartheta_{ijt})\,200 &\leq \Delta H_{ijt} - h_{ij}^1\,(Q_{ijt})^2 - h_{ij}^2\,\frac{F_{ijt}}{f_{ij}^{ref}}\,Q_{ijt} - h_{ij}^3\left(\frac{F_{ijt}}{f_{ij}^{ref}}\right)^2 \\ &\leq (1-\vartheta_{ijt})\,200, \quad \forall (i,j) \in \mathcal{P}u^v \end{aligned} \tag{3.23b}$$

$$\vartheta_{ijt}\,30 \leq F_{ijt} \leq \vartheta_{ijt}\,55, \quad \forall (i,j) \in \mathcal{P}u^v \tag{3.24b}$$

$$P_{ijt} = p_{ij}^1\,Q_{ijt}\left(\frac{F_{ijt}}{f_{ij}^{ref}}\right)^2 + p_{ij}^2\left(\frac{F_{ijt}}{f_{ij}^{ref}}\right)^3, \quad \forall (i,j) \in \mathcal{P}u^v \tag{3.25b}$$

$\forall (i,j) \in \mathcal{V},\;\; t \in [1,T]:$

$$-3600\frac{\pi}{4}v_{ij}^{max}(d_{ij})^2\vartheta_{ijt} \leq Q_{ijt} \leq 3600\frac{\pi}{4}v_{ij}^{max}(d_{ij})^2\vartheta_{ijt} \tag{3.26b}$$

$$-200\,(1-\vartheta_{ijt}) \leq H_{it} - H_{jt} - \kappa_{ij}\,Q_{ijt}|Q_{ijt}| \leq 200\,(1-\vartheta_{ijt}) \tag{3.27b}$$

$$\forall (i,j) \in \mathcal{P}r,\ \ t \in [1,T]:$$
$$0 \leq Q_{ijt} \leq q_{ij}^{cap} \tag{3.28b}$$

$$\forall (i,j) \in \mathcal{P}r,\ \ t \in [2,T]:$$
$$-f_{ij}\, q_{ij}^{cap} \leq Q_{ijt} - Q_{ij,t-1} \leq f_{ij}\, q_{ij}^{cap} \tag{3.30b}$$

$$\forall (i,j) \in \mathcal{P}r:$$
$$-2\, f_{ij}\, q_{ij}^{cap} \leq Q_{ijT} - Q_{ij,0} \leq 2\, f_{ij}\, q_{ij}^{cap} \tag{3.31b}$$
$$\sum_{t=1}^{T} Q_{ijt} \leq q_{ij}^{lim} \tag{3.29b}$$

$$\forall (i,j) \in \mathcal{P}r^t,\ \ t \in [1,T]:$$
$$Q_{ijt} = \sum_{k \in \mathcal{K}_{\mathcal{P}r^t}} \alpha_k q_k^{cap} \tag{3.32b}$$
$$\sum_{k \in \mathcal{K}_{\mathcal{P}r^t}} \alpha_k = 1 \tag{3.33b}$$

$$H_{it} \geq 0, \quad \forall i \in \mathcal{N}$$
$$I_{it}^{+},\, I_{it}^{-},\, O_{it},\, V_{it},\, L_{it},\, H_{it}^{M} \geq 0, \quad \forall i \in \mathcal{B}$$
$$\vartheta_{it}^{1},\, \vartheta_{it}^{2} \in \{0,1\}, \quad \forall i \in \mathcal{B}$$
$$Q_{ijt} \geq 0, \quad \forall i \in \mathcal{A}$$
$$\Delta H_{ijt},\, P_{ijt} \geq 0, \quad \forall i \in \mathcal{P}u$$
$$F_{ijt} \geq 0, \quad \forall i \in \mathcal{P}u^{v}$$
$$\vartheta_{ijt} \in \{0,1\}, \quad \forall i \in \mathcal{P}i \cup \mathcal{P}u \cup \mathcal{V}$$

3.4 Results on two realistic networks

In this section several networks will be described and tested with state-of-the-art solvers. Two different networks are considered. One is an existing part of the main supply network currently operated by De Watergroep. The second one is an artificial test network which is smaller in size and therefore ideal for preliminary testing purposes.

Coding is done in the mathematical programming language AMPL [72]. All tests were performed on an i7-2760QM CPU at 2.40 GHz, running Windows 7 Professional 64-bits with 8 GB of RAM. Since the nonconvex MINLP problem is NP-hard, obtaining a global optimal solution is very difficult. To find a solution for the MINLP model, the open-source solver BONMIN [56] is used. This solver generates local optimal solutions to nonconvex MINLP's. Bonmin's version is 1.5.1, Cbc 2.7.5 and Ipopt 3.10.1 are used as subsolvers. It has two interesting functionalities for our purpose. The first is a B-BB (branch & bound) algorithm, used to solve nonconvex problems to local optimality. The second is the Feasibility Pump (FP) heuristic, used to find feasible starting solutions for nonconvex MINLP's. The FP heuristic iteratively solves two subproblems: one with relaxed integer variables (NLP) and one with relaxed nonlinear constraints (MILP). The aim is to make both subproblems converge to a unique point that is feasible for the original problem. FP is called at the root node, when solving our problem with BONMIN, and after finding a feasible solution the branching phase starts [73].

3.4.1 Test network

This (smaller) network will be used to test both model formulations with readily available solvers and other algorithms described further in this work. It is based on an existing subnetwork of the water company De Watergroep and contains most of the components presented in the previous sections. However, fictional numbers are used for parameters. A graphical representation of this network is given in Figure 3.3. The network consists of the following nodes: 9 buffers (5 of which are water towers (WT) and the other 4 pure water reservoirs (R)), three raw water sources (WS), two delivery nodes (D) and 13 junctions (J). Artificial junctions were created for the WPCs to make a conceptual connection with the network (WPC). The arcs include 3 raw water pumps (one for each WPC), 5 regular speed pumps and 22 pipes. The corresponding data is given in appendix B, tables (B.1-B.4). Note that in these tables the capacity of the WPCs is always linked to the corresponding raw water pump, e.g. WS1-R1 has capacity 150 for WPC1. To divide a typical day into suitable periods, we refer to the daily consumption pattern presented in section 3.1, figure 3.1. The optimization is started at 7AM and ends at the same time the next day. This means that in the first three periods a day tariff (of 0,15 €/kWh) will be imposed, whereas the tariff during the last two periods is 0,1 €/kWh (night tariff).

The results for the MINLP model are presented in table 3.2.

Table 3.2: Results with BONMIN on the MINLP for the test network

method	CPU time (s)	obj.	prod. cost (€)	energy cost (€/day)	(€/night)
FP	16.89	3740.72	N/A	N/A	N/A
B-BB	448.35	3646.82	2058.63	1203.14	385.06

A solution with feasibility pump is already found after almost 17 s. This solution is improved by about 2.5 % using the B-BB method but this takes noticeably more time. Remark that if the relative optimality gap is reduced to 1 % the (sub)optimal solution of €3646.82 is already obtained by B-BB after 150.33 s. FP was called at the root node and apart from the objective value its results were not stored in the memory. For the sake of completeness, the solution obtained with B-BB can be found in appendix C.

3.4.2 High-level transport optimization model

As a result of the collaboration between different drinking water companies in Flanders, a significant amount of drinking water will be added to a part of the main supply network of De Watergroep. Hence arises the need of a high level decision model that investigates the impact of these changes on the operations of the drinking water supply network. More specifically, water will be traded with other drinking water companies in delivery points ($\mathcal{D}$) located at the border of their networks. Since a detailed modeling of the lower level (pipes with small diameters) of the water network is unnecessary to address this question, only the highest level transport network (the backbone subnetwork with large diameter pipes) is considered. The water supply network contains about 100 nodes and arcs including eight variable-speed pumps. Given the typical water usage over one day for this network, a division in 7 intervals was proposed in which periods with similar demand levels have been clustered. A part of this network is shown in figure 3.11.

The results for the MINLP model are presented in table 3.3.

Table 3.3: Results with BONMIN on the MINLP for the transport optimization network

method	CPU time (s)	obj.
FP	334.65	21596.30
B-BB	7200*	21399.331

* manually stopped, best possible solution 20795.016

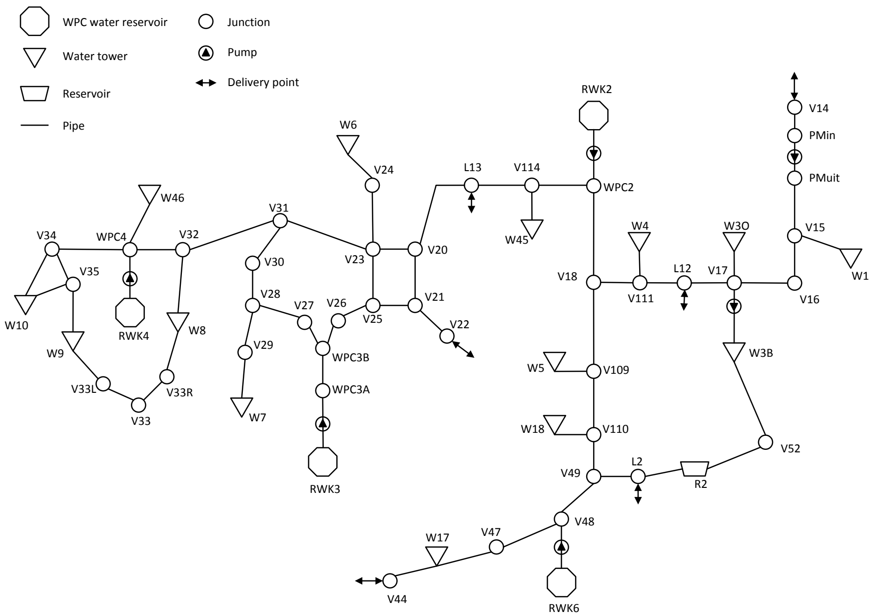


Figure 3.11: Part of the network for the high-level transport optimization model [3]

The Feasibility pump algorithm was able to find a solution after 334.65 seconds, which was slightly improved with the Branch and Bound method. Note that the B-BB method only found a lower bound and was stopped after 2 hours of CPU time.

3.5 Conclusion

A complete formulation for the Flemish water supply model leads to a large non-convex MINLP model. A careful analysis on network mechanics in buffers has been conducted, leading to additional constraints involving binary variables. After a discussion on different formulations of pressure losses, the Darcy-Weisbach formulation with the Prandtl-Kármán formula for the friction loss coefficient was chosen as the most beneficial model for our purpose. Pump characteristic curves were modeled as nonlinear functions based on manufacturer's data. In this study variable speed pumps are also included. Nonlinearity is avoided as much as possible by using an alternative formulation to model of these curves. To evaluate the quality of a solution, three different types of costs are taken into account. Since the network consists of multiple water production centers, each of them imposes a different production cost that is linear with the produced amount of water. Another important term is the energy cost of the pumps. The power term was linearized to decrease the complexity of the model. Lastly, the cost for delivering water to and receiving water from other water supply companies is added to the objective function. Using the Branch & Bound algorithm implemented in BONMIN, an locally optimal solution can be found for the test network within reasonable computational time, although the method does not converge within 1h of CPU time for the larger network.

4
A hybrid gradient algorithm for the piecewise linear MILP model

In an attempt to reduce the complexity, this chapter proposes a piecewise linear approximation of the nonlinear constraints which leads to an MILP model. To obtain an accurate solution to the original problem with this model, the number of segments in the piecewise linear functions has to be quite high. This leads to a higher number of binary variables and an increased computational time required to solve the problem. Furthermore, the obtained solution is not necessarily feasible for the more accurate MINLP problem formulation. Therefore, we use the solution of this model to serve as a starting point for the classical Newton's method. The second section resumes several well-known methods to find feasible solutions for (simple) pipe networks. The gradient algorithm is adapted in the third section to be used on the water supply model of the Flemish network. The section thereafter explains which modifications have to be made in order to couple the MILP model with the gradient algorithm and provide a final feasible solution of good quality. In the last section several results on different instances are presented.

4.1 MILP model

The above model is very complex in the sense that it contains many binary variables and nonlinear nonconvex pressure losses and pump constraints (3.16, 3.19-3.21, 3.23, 3.27). Moreover, the pressure loss related constraints are nonsmooth. This makes the resulting model very hard to solve, especially in large networks. Not many solvers exist that are able to tackle this kind of NP-had problems in reasonable computation time, let alone come up with a global optimum. Therefore, it might be beneficial to relax the nonlinearities and reduce the model to an (approximated) MILP (Mixed Integer Linear Programming) model. Solving this relaxed model by mixed integer linear techniques has the advantage that these methods are nowadays very robust and able to solve problems with many variables to global optimality. On the other hand, by eliminating the nonlinearities, the model does not accurately represent the original formulation anymore. As a consequence, solutions provided by these methods may be infeasible for the original problem depending on the tightness of the relaxation.

In this section we will first describe several methods to obtain piecewise linear over- and underestimates for general nonlinear functions. The method of choice here is a formulation that requires only a logarithmic amount of binary variables ([74, 75]). It is shown that the maximum approximation error can be determined a priori. Afterwards the extension to multivariate functions is made.

4.1.1 Piecewise linear formulations for univariate functions

The following is based on [76]. In general we consider a function of one variable $f(x)$ and a subdivision of the domain $[x_{min}, x_{max}]$ of f by introducing $K+1$ additional vertices $x_{min} = x_0 < x_1 < ... < x_K = x_{max}$. At each of these vertices we evaluate f and note this value as $y_k = f(x_k), k = 0..K$. Now the pairs (x_k, y_k) and (x_{k+1}, y_{k+1}) are connected by lines to obtain linear functions. The choice and number of these vertices will be discussed further. It should be clear that the more vertices are introduced, the more accurate the model becomes but the computational time will increase accordingly. Also note that the linear function obtained here is considered to be a strict lower or upper bound.

One of the most commonly used methods is the ‘convex combination method’. The variable x is expressed as a linear combination of x_{k-1}, x_k of one interval k. The variable Λ_k represents the weight that each vertex x_k contributes to x. After introduction of binary variables β_k, $k = 0..K-1$ the formulation is as follows (see [77, 78]):

$$
\begin{aligned}
x &= \sum_{k=0}^{K} \Lambda_k x_k \\
f(x) &= \sum_{k=0}^{K} \Lambda_k y_k \\
\sum_{k=0}^{K} \Lambda_k &= 1 \\
\Lambda_0 &\leq \beta_0 \\
\Lambda_k \leq \beta_{k-1} + \beta_k \quad \forall k &\in 1..K \\
\Lambda_K &\leq \beta_K \\
\sum_{k=0}^{K} \beta_k &= 1 \\
\Lambda_k \geq 0 \quad \forall k &\in 0..K \\
\beta_k \in \{0,1\} \quad \forall k &\in 0..K-1
\end{aligned}
$$

The first three restrictions express x and its function value as linear combinations of (x_k, y_k). Only one interval can be chosen ($\sum_{k=0}^{K} \beta_k = 1$) and the remaining restrictions make sure that only the Λ's corresponding to the vertices of this interval are positive.

The 'incremental method' (see also [77]) expresses x as lying in interval k with value $x_{k-1} + \delta$:

$$
\begin{aligned}
x &= x_0 + \sum_{k=1}^{K} \delta_k \\
f(x) &= y_0 + \sum_{k=1}^{K} \frac{y_k - y_{k-1}}{x_k - x_{k-1}} \delta_k \\
(x_k - x_{k-1})\beta_k \leq \delta_k \quad \forall k &\in 1..K-1 \\
\delta_{k+1} \leq (x_{k+1} - x_k)\beta_k \quad \forall k &\in 1..K-1 \\
0 \leq \delta_k \leq x_k - x_{k-1} \quad \forall k &\in 1..K \\
\beta_k \in \{0,1\} \quad \forall k &\in 1..K-1
\end{aligned}
$$

The variables β_k enforce the condition that if $\delta_k > 0$ then all δ_i with $i < k$ are at their upper bound of $x_i - x_{i-1}$, so that we can rewrite:

$$x = x_0 + \sum_{i=1}^{K} \delta_i = x_0 + \sum_{i=1}^{k-1} \delta_i + \delta_k = x_{k-1} + \delta_k$$

According to [76] both methods are generic and readily incorporated in MILPs. The main drawback of these methods is the fact that they introduce a large number of additional binary variables and restrictions to the model. Branching on these binary variables furthermore results in deep and unbalanced branch-and-bound trees. Vielma and Nemhauser [74] pointed out that the number of binary variables can be reduced drastically. Their 'logarithmic method' is described here, for more details we refer the reader to [74, 75]. First, we define the function $B : \{0..K-1\} \rightarrow \{0,1\}^{\log_2(K)}$ which projects every interval on a binary vector of length $\log_2(K)$. Since we work with SOS2 constraints, this function needs to be SOS2 compatible, meaning that the functions $B(k)$ and $B(k+1)$ should differ in at most 1 component. With $\sigma(B)$ the support vector of B, we then define:

$$\begin{aligned} K^+(l,B) &= \{k \in 1..K-1 : l \in \sigma(B(k)) \cap \sigma(B(k+1))\} \\ &\quad \cup \{0 \text{ if } l \in \sigma(B(1))\} \cup \{K \text{ if } l \in \sigma(B(K))\} \\ K^0(l,B) &= \{k \in 1..K-1 : l \notin \sigma(B(k)) \text{ and } l \notin \sigma(B(k+1))\} \\ &\quad \cup \{0 \text{ if } l \notin \sigma(B(1))\} \cup \{K \text{ if } l \notin \sigma(B(K))\} \end{aligned}$$

Essentially for each vertex x_k and each binary l it is determined whether k belongs to set $K^+(l,B)$, $K^0(l,B)$ or neither of those, based on the binary projections of intervals k and $k+1$. If both of these are equal to 1 in component l then $k \in K^+(l,B)$, if both are 0 then $k \in K^0(l,B)$. With this information, the PWL constraints can then be written as:

$$\begin{aligned} x &= \sum_{k=0}^{K} \Lambda_k x_k \\ f(x) &= \sum_{k=0}^{K} \Lambda_k y_k \\ \sum_{k=0}^{K} \Lambda_k &= 1 \end{aligned}$$

$$\sum_{k \in K^+(l,B)} \Lambda_k \leq \beta_l \quad \forall l \in 1..\log_2 K$$
$$\sum_{k \in K^0(l,B)} \Lambda_k \leq 1 - \beta_l \quad \forall l \in 1..\log_2 K$$
$$\Lambda_k \geq 0 \quad \forall k \in 0..K$$
$$\beta_l \in \{0,1\} \quad \forall l \in 1..\log_2 K$$

Here, β_l is a binary variable that is equal to 1 of component l of the binary vector is chosen. Each combination of β's thus determine a unique interval. With this formulation it is possible to describe each of the K intervals using only $\log_2(K)$ instead of K binary variables. This is of course even more interesting if we choose the number of intervals $K = 2^n$ with n an integer. This is the formulation that will now be applied on the model for the water supply network.

4.1.2 PWL formulation applied to the water supply model

We can write in general (for each arc/time combination individually, with K_{ijt} the total number of segments for that arc):

$$Q_{ijt} = \sum_{k=0}^{K_{ijt}} \Lambda_{ijtk} q_{ijtk} \tag{4.1}$$
$$f(H_{it}, H_{jt}, \Delta H_{ijt}) = \sum_{k=0}^{K_{ijt}} \Lambda_{ijtk}(g(q_{ijtk}) + c_{ijk}) \tag{4.2}$$
$$\sum_{k \in K^+(l,B)} \Lambda_{ijtk} \leq \beta_{ijtl} \quad \forall l \in 1..\log_2(K_{ijt}) \tag{4.3}$$
$$\sum_{k \in K^0(l,B)} \Lambda_{ijtk} \leq 1 - \beta_{ijtl} \quad \forall l \in 1..\log_2(K_{ijt}) \tag{4.4}$$
$$\sum_{k=0}^{K} \Lambda_{ijtk} = \begin{cases} 1 & \text{if } (i,j) \in \mathcal{P}i \\ \vartheta_{ijt} & \text{if } (i,j) \in \mathcal{P}u \backslash \mathcal{P}u^v \cup \mathcal{V} \end{cases} \tag{4.5}$$
$$\Lambda_{ijtk} \geq 0 \quad \forall k \in 0..K_{ijt} \tag{4.6}$$
$$\beta_{ijtl} \in \{0,1\} \quad \forall l \in 1..\log_2(K_{ijt}) \tag{4.7}$$

Here,

$$K^+(l,B) = \{k \in 1..K_{ijt} - 1 : l \in \sigma(B(k)) \cap \sigma(B(k+1))\}$$
$$\cup \{0 \text{ if } l \in \sigma(B(1))\} \cup \{K_{ijt} \text{ if } l \in \sigma(B(K_{ijt}))\}$$

$$K^0(l,B) = \{k \in 1..K_{ijt}-1 : l \notin \sigma(B(k)) \text{ and } l \notin \sigma(B(k+1))\} \\ \cup \{0 \text{ if } l \notin \sigma(B(1))\} \cup \{K_{ijt} \text{ if } l \notin \sigma(B(K_{ijt}))\}$$

For constraint (4.5) an additional case is made for pumps. When these are shut off ($\vartheta_{ijt} = 0$) it is desirable that $Q_{ijt} = 0$. In this case the constraint actually forces $\Lambda_{ijtk} = 0, \forall k \in 0..K_{ijt}$, giving the desired result through (4.1).

An example is shown in figure 4.1 with $K = 4$, $B(1) = \{0,0\}$, $B(2) = \{0,1\}$, $B(3) = \{1,1\}$, $B(4) = \{1,0\}$. For $l = 1$ (first component of the 2-dimensional binary vector) we find that $K^+(1,B) = \{3,4\}$ since their adjacent intervals both have a binary value of 1, whereas $K^0(1,B) = \{0,1\}$ because the projected values are equal to 0. For $l = 2$ these sets can be found in the same manner and our formulation then becomes:

$$\Lambda_{ijt3} + \Lambda_{ijt4} \leq \beta_{ijt1}, \quad \Lambda_{ijt0} + \Lambda_{ijt1} \leq 1 - \beta_{ijt1}$$
$$\Lambda_{ijt2} \leq \beta_{ijt2}, \quad \Lambda_{ijt0} + \Lambda_{ijt4} \leq 1 - \beta_{ijt2}$$
$$\Lambda_{ijtk} \geq 0 \quad \forall k \in 0..4$$
$$\beta_{ijt1}, \beta_{ijt2} \in \{0,1\}$$

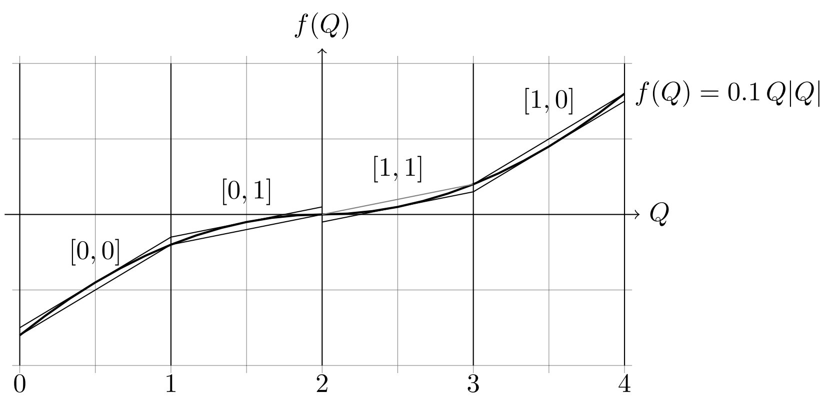


Figure 4.1: Example division in intervals with logarithmic number of binary variables

In order to get a comparison of the number of additional variables and restrictions introduced by each of these formulations, we remark the following. The constraints considered here are the univariate nonlinear constraints (3.16, 3.19-3.21, 3.27). Let us denote the subset of $\mathcal{A}$ that makes up this group as $\mathcal{A}^{uni}$. To reduce the complexity, we combine (3.19, 3.20) with (3.21) by substituting for ΔH. Furthermore we over- and underestimate the original constraints using linear segments as is shown on figure 4.1, effectively doubling the expressions for $f(Q)$ (more details on this will be given further on). This brings us to table 4.1.

Table 4.1: Comparison of the number of variables and constraints on the water supply model using different PWL formulations

Method	variables		constraints
	cont.	binary	
convex	$(K+1)\|\mathcal{A}^{uni}\|$	$K\|\mathcal{A}^{uni}\|$	$(K+6)\|\mathcal{A}^{uni}\|$
incremental	$K\|\mathcal{A}^{uni}\|$	$K\|\mathcal{A}^{uni}\|$	$(3K+1)\|\mathcal{A}^{uni}\|$
logarithmic	$(K+1)\|\mathcal{A}^{uni}\|$	$\log_2(K)\|\mathcal{A}^{uni}\|$	$(2\log_2(K)+3)\|\mathcal{A}^{uni}\|$

Clearly the logarithmic formulation has the biggest benefits, because for each value of K both the number of binary variables and constraints is strictly smaller than those in the other two formulations.

The proposed formulation approximates the pressure loss equations for pipes, pumps and valves. The feasible interval of the flow variable on each of these arcs is divided into an appropriate number of sub-intervals on which a linear approximation of the nonlinear function is satisfactory. Let us consider positive values for Q on figure 4.1. The pressure loss on each arc here is approximated from above by the defined linear functions over the chosen vertices. Rather than modeling these constraints as strict equalities, they are instead used as upper bounds for the original function. Furthermore the nonlinear functions can be bounded from below (underestimated) by subtracting from each linear function the maximal pressure loss error over the interval. Together they define a feasible region in which the pressure losses are approximated. The big advantage of approximating the pressure losses in this way is that a feasible (although approximate) solution will always be found, whereas a single linear approximation will often return infeasible solutions. The critical issue that arises is how many intervals are needed to guarantee an approximation error that is reasonably small. The following proposition states how large the maximum error is:

Proposition 4.1.1. *The maximum error resulting from a linear approximation to the quadratic function $f(q) = \gamma q^2$ in interval $[q_{k-1}, q_k]$ occurs at $\frac{q_{k-1}+q_k}{2}$ and its value is equal to $\frac{\gamma}{4}(q_{k-1}-q_k)^2$.*

This can easily be derived for a general quadratic function by writing down both the linear and nonlinear function for $Q \in [q_{k-1}, q_k]$:

$$f^1(Q) = q_{k-1}^2 + \frac{q_k^2 - q_{k-1}^2}{q_k - q_{k-1}}(Q - q_{k-1}) \tag{Q-1}$$

$$f^2(Q) = Q^2 \tag{Q-2}$$

Subtracting the right hand side of (Q-2) from (Q-1) and deriving to Q gives $\frac{q_{k-1}+q_k}{2}$ as value for Q. Substituting this value in (Q-1) gives the desired result. The function coefficient γ depends on the constraint and is equal to κ for pressure losses and h_1 for the pump curve. We observe that if we multiply the number of intervals by n, then the error will be reduced by a factor n^2.

For the sake of completeness the exact formulations for (4.2) are given for pipes, pumps and valves respectively. Pipes:

$$H_{it} - H_{jt} - \kappa_{ij} \sum_{k \in 0..K_{ijt}} \Lambda_{ijtk} q_{ijtk} |q_{ijtk}| \leq (1 - \beta_{ijt,\log_2(K_{ijt})})\epsilon_{ijt}^1 \tag{4.8}$$

$$H_{it} - H_{jt} - \kappa_{ij} \sum_{k \in 0..K_{ijt}} \Lambda_{ijtk} q_{ijtk} |q_{ijtk}| \geq -\beta_{ijt,\log_2(K_{ijt})}\epsilon_{ijt}^1 \tag{4.9}$$

Pumps:

$$H_{it}^M - H_{jt} - \kappa_{ij} \sum_{k \in 0..K_{ijt}} \Lambda_{ijtk} q_{ijtk}^2 + \Delta H_{ijt} \leq 0 \quad \forall i \in \mathcal{B} \tag{4.10}$$

$$H_{it}^M - H_{jt} - \kappa_{ij} \sum_{k \in 0..K_{ijt}} \Lambda_{ijtk} q_{ijtk}^2 + \Delta H_{ijt} \geq -\epsilon_{ijt}^2 \quad \forall i \in \mathcal{B} \tag{4.11}$$

$$H_{it} - H_{jt} - \kappa_{ij} \sum_{k \in 0..K_{ijt}} \Lambda_{ijtk} q_{ijtk}^2 + \Delta H_{ijt} \leq 0 \quad \forall i \in \mathcal{N} \backslash \mathcal{B} \tag{4.12}$$

$$H_{it} - H_{jt} - \kappa_{ij} \sum_{k \in 0..K_{ijt}} \Lambda_{ijtk} q_{ijtk}^2 + \Delta H_{ijt} \geq -\epsilon_{ijt}^2 \quad \forall i \in \mathcal{N} \backslash \mathcal{B} \tag{4.13}$$

$$\Delta H_{ijt} - \sum_{k \in 0..K_{ijt}} \Lambda_{ijtk}(h_{ij}^1 q_{ijtk}^2 + h_{ij}^2 q_{ijtk}) - h_{ij}^3 \vartheta_{ijt} + (1 - \vartheta_{ijt})100 \geq 0 \tag{4.14}$$

$$\Delta H_{ijt} - \sum_{k \in 0..K_{ijt}} \Lambda_{ijtk}(h_{ij}^1 q_{ijtk}^2 + h_{ij}^2 q_{ijtk}) - h_{ij}^3 \vartheta_{ijt} - (1 - \vartheta_{ijt})100 \leq \epsilon_{ijt}^3 \tag{4.15}$$

Valves:

$$H_{it} - H_{jt} - \kappa_{ij} \sum_{k \in 0..K_{ijt}} \Lambda_{ijtk} q_{ijtk}^2 - (1 - \vartheta_{ijt})100 \leq 0 \tag{4.16}$$

$$H_{it} - H_{jt} - \kappa_{ij} \sum_{k \in 0..K_{ijt}} \Lambda_{ijtk} q_{ijtk}^2 + (1 - \vartheta_{ijt})100 \geq -\epsilon_{ijt}^4 \tag{4.17}$$

Here the ϵ's denote the a-priori maximum error in the corresponding interval. For pipes this error is multiplied with an additional term to ensure the correct bounds for positive values of Q ($\beta_{\log_2(K)} = 1$) and negative values ($\beta_{\log_2(K)} = 0$).

Apart from increasing the number of intervals, the error can be further reduced by providing good upper- and lower bounds on the values of the components of the vector Q. Tighter bounds mean smaller intervals and, consequently, a smaller error. In addition to the readily available bounds (3.17),(3.18), we add another general bound for each time period:

$$|Q_t| \leq \max \left(\sum_{i \in \mathcal{N} \setminus \mathcal{D}} d_{it} + \sum_{i \in \mathcal{D}: l_{it}^{max} > 0} l_{it}^{max}, \sum_{(i,j) \in \mathcal{R}} q_{ij}^{cap} - \sum_{i \in \mathcal{D}: l_{it}^{max} < 0} l_{it}^{max} \right)$$

This bound is valid since the max flow through the network is either limited by the total capacity (in case the buffers are filled), or by the total demand (in case the buffers are used to satisfy demand), adjusted by delivered amounts. For pipes, we can deduce yet another bound. Combining (3.1) and (3.16), we find $\sqrt{\frac{100+|h_i-h_j|}{\kappa_{ij}}}$ to be a valid bound for Q_{ij} as well. For active regular speed pumps, we combine (3.1), (3.19) and (3.21) to find an upper bound of

$$\frac{h_{ij}^2 + \sqrt{(h_{ij}^2)^2 - 4(\kappa_{ij} - h_{ij}^1)\,(-h_i + h_j - 100 - h_{ij}^3)}}{2\,(\kappa_{ij} - h_{ij}^1)}$$

The smallest upper bound and largest lower bound for each arc and each timestep will then determine the length of interval ijt.

The MILP model can be strengthened further by adding some valid inequality cuts. For the example given in figure 4.1, possible inequality cuts are:

$$2\,\Lambda_{ijt0} + \Lambda_{ijt1} + \Lambda_{ijt4} \leq 2 - \beta_{ijt1} - \beta_{ijt2} \tag{4.18}$$

$$\Lambda_{ijt2} + \Lambda_{ijt3} + \Lambda_{ijt4} \leq \beta_{ijt1} + \beta_{ijt2} \tag{4.19}$$

For n intervals, a total of $2\sum_{i=2}^{\log_2(n)} C_{\log_2(n)}^i$ cuts can be added.

4.1.3 Multivariate functions

Until now the piecewise functions were defined for the nonlinear univariate functions that concern pipes, regular speed pumps, and valves. The constraints (3.23) and (3.25) were not yet addressed since they contain terms that exist of functions of the form xy and xy^2. As stated in [67] this kind of functions needs a two-dimensional triangulation in general. Two alternatives are provided here: one is based on decomposition and we also propose a reformulation.

4.1.3.1 Decomposition of the functions

A decomposition of nonlinear terms can be done by using the Binomial theorem to rewrite bivariate terms of the form xy in univariate ones:

$$xy = \frac{1}{2}(Z^2 - x^2 - y^2)$$
$$Z = x + y$$

Now each of the functions x^2, y^2, Z^2 can be reformulated using the technique described in the previous section. The functions xy^2 that appear in the power term (3.25) can then be decomposed as:

$$xy^2 = \frac{1}{2}(Z^2 - x^2 - y^4)$$
$$Z = x + y^2$$

Unfortunately, each of these requires additional linearizations for the variable Z which further enlarge the MILP. In [79] a more interesting approach is used. The author starts with the one-variable piecewise linear approximation for the variable x. For any value in the interval $[y_l, y_{l+1}]$ the approximated function value xy_{approx} is then given as:

$$xy_{approx} \leq \sum_{k \in 0..K} \Lambda_k \, x_k \, \tilde{y}_l + M(1 - \beta_l)$$
$$xy_{approx} \geq \sum_{k \in 0..K} \Lambda_k \, x_k \, \tilde{y}_l - M(1 - \beta_l)$$

where $\tilde{y}$ is a sampling coordinate in the associated interval (e.g. the middle of the interval). $\beta_l = 1$ if the corresponding interval l is chosen and M is a very large value.

Since we want to give more freedom to the model variables, we propose a decomposition based on lower- and upper bounds as is the case for univariate functions. Therefore, the above formulation is rewritten as

$$xy_{approx} \leq \sum_{k \in 0..K} \Lambda_k \, x_k \, y_l + M(1 - \beta_l)$$
$$xy_{approx} \geq \sum_{k \in 0..K} \Lambda_k \, x_k \, y_{l-1} - M(1 - \beta_l)$$

Applied on restriction (3.23), and substituting $\sum_{k\in 0..K_{ijt}} \Lambda_{ijtk} q_{ijtk} = Q_{ijt}$, this gives (for interval l):

$$\begin{aligned}
\Delta H_{ijt} \leq{} & h^1_{ij} \sum_{k\in 0..K_{ijt}} \Lambda_{ijtk} q^2_{ijtk} + h^2_{ij} \frac{f_{ijtl}}{f^{ref}_{ij}} Q_{ijt} \\
& + h^3_{ij} \sum_{m\in l-1..l} \Lambda^f_{ijtm} \left(\frac{f_{ijtm}}{f^{ref}_{ij}}\right)^2 + \epsilon^5_{ijt} + (1-\vartheta_{ijt})\,200 + (1-\beta_{ijtl})200 \\
\Delta H_{ijt} \geq{} & h^1_{ij} \sum_{k\in 0..K_{ijt}} \Lambda_{ijtk} q^2_{ijtk} + h^2_{ij} \frac{f_{ijt,l-1}}{f^{ref}_{ij}} Q_{ijt} \\
& + h^3_{ij} \sum_{m\in l-1..l} \Lambda^f_{ijtm} \left(\frac{f_{ijtm}}{f^{ref}_{ij}}\right)^2 - (1-\vartheta_{ijt})\,200 - (1-\beta_{ijtl})200
\end{aligned}$$

This effectively creates a larger feasible region and a predicted shorter solution time for the model.

4.1.3.2 Reformulation of F

As the method of decomposition will bring with it a considerable amount of additional variables and constraints, we look at a different approach. First, remark that the multivariate functions here are functions of Q and F. The variable F, denoting the frequency, lies within the interval $\{0\} \cup [30, 55]$. Instead of defining it as a continuous variable, it is plausible to model it as an integer variable:

$$F = 30\phi^1 + ... + 55\phi^p = \sum_{p=1}^{P} (p+29)\phi^p$$
$$\sum_{p=1}^{P} \phi^p \leq 1$$

ϕ is a vector of binary variables. Equivalently, each higher power univariate function of F can be rewritten as $F^i = \sum_{p=1}^{P} (p+29)^i \phi^p$. Based on the paper [80], the mixed integer product $Q\,\phi^p$ can be replaced by a continuous variable Z^p while adding the following constraints:

$$\begin{aligned}
Z^p &\geq Q - q^{max}(1-\phi^p) \\
Z^p &\geq q^{min}\phi^p \\
Z^p &\leq Q - q^{min}(1-\phi^p) \\
Z^p &\leq q^{max}\phi^p
\end{aligned}$$

This effectively introduces P additional binary and continuous variables and $4P$ constraints per variable speed pump and per period. It further limits the problem in the sense that F can not take on all continuous values in the interval $[30, 55]$. In comparison, the decomposition method introduces less binary variables ($2\log_2(K)$ in total for Z and F) and ($4\log_2(K) + 6$) additional constraints. Both methods will be modeled and experimented with. A visual comparison of the different approximations for the multivariate VSP curve can be seen in figures 4.2 - 4.4.

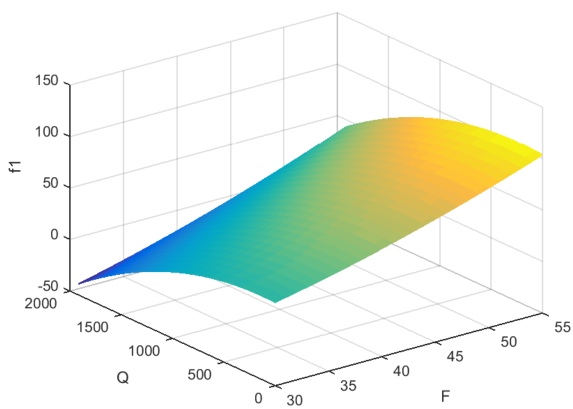


Figure 4.2: Original nonlinear formulation of the VSP curve

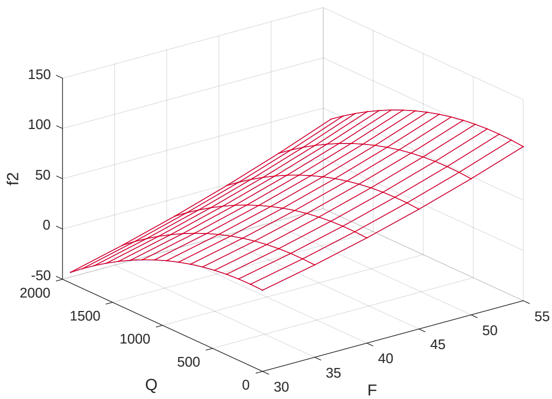


Figure 4.3: Approximation with decomposition

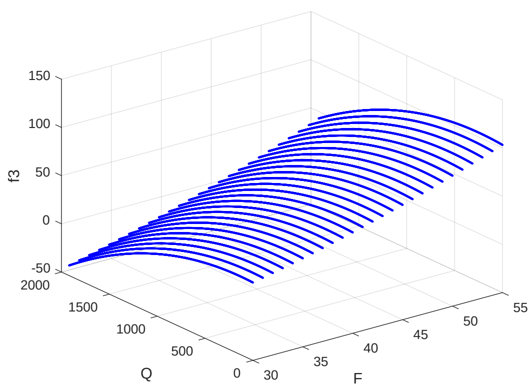


Figure 4.4: Approximation with reformulation

Note that the solution can only be found on the linear grid in figure 4.3 or the lines on 4.4.

4.2 Steady-state hydraulic analysis of pipe networks

In this section a very basic water supply network is considered in which the nodes only consist of junctions and nodes with fixed pressure, and the arcs represent pipes and regular speed pumps. For such a network we are now interested in a feasible flow and pressure distribution, e.g. one that satisfies the laws of conservation of mass and energy (= pressure losses in loops). All of the presented methods are iterative but are distinctive in the way the energy conservation laws are written; these can be formulated as node, loop or pipe equations [81].

4.2.1 Hardy Cross method

The oldest technique covered in this section was developed for solution by hand and is not computationally efficient for large systems [82]. The main equations used here are the conservation of mass (3.2) and the conservation of energy that can be written for closed loops (assume positive flow in the sense of the loop for simplicity):

$$\sum_{(i,j)\in\mathcal{P}i} \kappa_{ij} Q_{ijt}^2 + \sum_{(i,j)\in\mathcal{P}u} \left(\left(\kappa_{ij} - h_{ij}^1\right) Q_{ijt}^2 - h_{ij}^2\, Q_{ijt} - h_{ij}^3 \right) = 0$$

This equation is simply found by adding the RHS of equations (3.16) and (3.19) along the loop and setting the expression equal to zero. Note that all pumps are assumed to be working. Only the smallest loops (called primary loops) are identified so that each pipe appears at most twice in the set of loop equations. Between two nodes with fixed pressure (fixed-grade nodes, such as reservoirs with fixed volume tanks) the energy must also be conserved. We can then write down the independent equations in the form

$$\sum_{(i,j)\in\mathcal{P}i} \kappa_{ij} Q_{ijt}^2 + \sum_{(i,j)\in\mathcal{P}u} \left(\left(\kappa_{ij} - h_{ij}^1\right) Q_{ijt}^2 - h_{ij}^2\, Q_{ijt} - h_{ij}^3 \right) = \Delta E_{FGN}$$

ΔE_{FGN} is the piezometric pressure difference between the two fixed-grade nodes. Assuming $|\mathcal{L}|$ loops and $|\mathcal{F}|$ fixed-grade nodes, the $|\mathcal{N} + \mathcal{L} + \mathcal{F} - 1|$ equations can be solved using the following solution procedure. Starting with a solution for Q that satisfies the conservation of flow constraints, a correction ΔQ_L is determined for each loop during each iteration. These are determined so that conservation of flow is preserved at all times. With the initial values of Q the conservation of energy is likely not satisfied. For each iteration and each loop/FGN pair ΔQ_L is added to the equation:

$$\sum_{(i,j)\in\mathcal{P}i} \kappa_{ij}(Q_{ijt}+\Delta Q_L)^2 + \sum_{(i,j)\in\mathcal{P}u} \left((\kappa_{ij}-h^1_{ij})\,(Q_{ijt}+\Delta Q_L)^2 - h^2_{ij}\,(Q_{ijt}+\Delta Q_L) - h^3_{ij}\right) = \Delta E$$

ΔE is 0 for closed loops. This expression can be approximated using a Taylor expansion (assume ΔQ small):

$$\sum_{(i,j)\in\mathcal{P}i} \kappa_{ij}Q^2_{ijt} + \sum_{(i,j)\in\mathcal{P}i} 2\kappa_{ij}Q_{ijt}\Delta Q_L + \sum_{(i,j)\in\mathcal{P}u} \left((\kappa_{ij}-h^1_{ij})\,Q^2_{ijt} - h^2_{ij}\,Q_{ijt} - h^3_{ij}\right) + \sum_{(i,j)\in\mathcal{P}u} \left(2(\kappa_{ij}-h^1_{ij})\,(Q_{ijt}\Delta Q_L) - h^2_{ij}\Delta Q_L\right) = \Delta E$$

And subsequently ΔQ can be determined:

$$\Delta Q_L = -\frac{\sum_{(i,j)\in\mathcal{P}i} \kappa_{ij}Q^2_{ijt} + \sum_{(i,j)\in\mathcal{P}u} \left((\kappa_{ij}-h^1_{ij})Q^2_{ijt} - h^2_{ij}Q_{ijt} - h^3_{ij}\right) - \Delta E}{\sum_{(i,j)\in\mathcal{P}i} 2\kappa_{ij}Q_{ijt} + \sum_{(i,j)\in\mathcal{P}u} 2\left((\kappa_{ij}-h^1_{ij})Q_{ijt} - h^2_{ij}\right)}$$

The correction can either be computed for each loop seperately and applied before going to the next one or computed and applied simultaneously. For each iteration k, the next vector of flow $Q(k)$ is computed by $Q^{(k)} = Q^{(k-1)} + \Delta Q_L^{(k-1)}$. Repeat until ΔQ_L is smaller than some predefined value.

4.2.2 Linear theory method

Linear theory linearizes the energy equations and thus solves the following set of constraints:

$$\sum_{(i,j)\in\mathcal{P}i} \kappa_{ij}Q^{(k-1)}_{ijt}Q^{(k)}_{ijt} + \sum_{(i,j)\in\mathcal{P}u} \left((\kappa_{ij}-h^1_{ij})\,Q^{(k-1)}_{ijt}Q^{(k)}_{ijt} - h^2_{ij}\right) = 0, \quad \forall\,|\mathcal{L}| \text{ closed loops}$$

$$\sum_{(i,j)\in\mathcal{P}i} \kappa_{ij}Q^{(k-1)}_{ijt}Q^{(k)}_{ijt} + \sum_{(i,j)\in\mathcal{P}u} \left((\kappa_{ij}-h^1_{ij})\,Q^{(k-1)}_{ijt}Q^{(k)}_{ijt} - h^2_{ij}\right) = \Delta E_{FGN}, \quad \forall\,|\mathcal{F}|-1 \text{ independent pseudo-loops}$$

which, together with the conservation of flow constraints, form a set of linear equations that can be solved for $Q^{(k)}$. The absolute differences between successive flow estimates are compared to a convergence criterion. If this is not satisfied, the average of the flows from the previous two iterations should be used as an estimate for the next iterations.

4.2.3 Newton's method for node equations

The node equations are the flow conservation constraints written in terms of piezometric heads. This can be done by substituting the pressure loss and pump curve constraints in terms of Q. For pipes, this would give:

$$Q_{ijt} = sign(H_{it} - H_{jt})\sqrt{\frac{|H_{it} - H_{jt}|}{\kappa_{ijt}}}$$

To find a solution, start with an initial configuration $H^{(0)}$. At iteration k the set of node equations $f(H)$ is solved for $H^{(k)}$ using Newton's method:

$$f(H^{(k)}) = -f'(H^{(k)})\Delta H^{(k)} \tag{4.20}$$

after which the heads are updated: $\Delta H^{(k+1)} = H^{(k)} + \Delta H^{(k)}$. Continue until a stopping criterion is reached.

4.2.4 The gradient algorithm for pipe equations

This method, which is also implemented in EPANET [9], makes use of pipe equations. These are the constraints (3.16 - 3.21) as defined in section 3. The equations are solved to Q and H simultaneously [83]. The general structure is the one outlined in algorithm 1.

Initialization:
Initial configuration q^0(vectors of Q), Nonlinear equation $f(x) = 0$.

Iteration m:
Update q^m through Newton's iterative method
$f'(x^{(m)})(x^{(m+1)} - x^{(m)}) = -f(x^{(m)}), \quad x^{(0)} prescribed,\ m = 0, 1, 2, ...$

Until:
$\phi(q^{(m+1)}) = \sum_{(i,j)\in\mathcal{A},t\in 1..T} |Q_{ijt}^{(m+1)} - Q_{ijt}^{(m)}| / \sum_{(i,j)\in\mathcal{A},t\in 1..T} |Q_{ijt}^{(m+1)}| \leq \delta_{\text{stop}}$

Algorithm 1: (Newton's method)

A matrix structure containing these constraints in addition to the flow conservation equations is built. The incidence matrices identify node connections. Todini and Pilati [83] devised an efficient recursive formula for $H^{(k+1)}$ and $Q^{(k+1)}$ based on matrix inversions. More recently, Simpson and Elhay [84] revised this algorithm for the Darby-Weisbach headloss formula to gain quadratic convergence.

According to [81], a comparison of the 4 previous methods points out that all methods converge in reasonable computational time, except for the Hardy Cross method which is left out of the discussion. In general, the gradient method combined with pipe equations seemed to work best for different conditions.

4.3 The gradient method applied on the general water supply model

The gradient method is implemented directly in the MINLP model defined in chapter 3. Instead of recursive schemes, the original nonlinear equations are replaced by the linear ones. The nonlinear functions in the original system are the pressure losses and pump constraints (3.16, 3.19-3.21, 3.23). For friction losses in pipes (3.16), we easily find the linearized constraint in iteration m:

$$-2\kappa_{ij}\,|Q_{ijt}^{(m)}|Q_{ijt} = -H_{it} + H_{jt} - \kappa_{ij}\,Q_{ijt}^{(m)}|Q_{ijt}^{(m)}| \tag{4.21}$$

For regular speed pumps, the constraints are (substituting for ΔH and assuming the pumps are active):

$$\begin{aligned}(-2\kappa_{ij}\,Q_{ijt}^{(m)} + 2h_{ij}^{1}\,Q_{ijt}^{(m)} + h_{ij}^{2})Q_{ijt} = \\ - H_{it} + H_{jt} - (\kappa_{ij} - h_{ij}^{1})(Q_{ijt}^{(m)})^2 - h_{ij}^{3}, \\ \forall (i,j) \in \mathcal{P}u \backslash \mathcal{P}u^{v} : i \in \mathcal{N}\backslash\mathcal{B}\end{aligned} \tag{4.22}$$

$$\begin{aligned}(-2\kappa_{ij}\,Q_{ijt}^{(m)} + 2h_{ij}^{1}\,Q_{ijt}^{(m)} + h_{ij}^{2})Q_{ijt} = \\ - H_{it}^{M} + H_{jt} - (\kappa_{ij} - h_{ij}^{1})(Q_{ijt}^{(m)})^2 - h_{ij}^{3}, \\ \forall (i,j) \in \mathcal{P}u \backslash \mathcal{P}u^{v} : i \in \mathcal{B}\end{aligned} \tag{4.23}$$

For the variable speed pumps finally, the constraints are replaced by

$$\begin{aligned}(-2\kappa_{ij}\,Q_{ijt}^{(m)} + 2h_{ij}^{1}\,Q_{ijt}^{(m)} + h_{ij}^{2}\frac{F_{ijt}^{(m)}}{f_{ij}^{ref}})Q_{ijt} + (h_{ij}^{2}\,Q_{ijt}^{(m)} - 2h_{ij}^{3}\frac{F_{ijt}^{(m)}}{f_{ij}^{ref}})\frac{F_{ijt}}{f_{ij}^{ref}} = \\ - H_{it} + H_{jt} - (\kappa_{ij} - h_{ij}^{1})(Q_{ijt}^{(m)})^2 + h_{ij}^{2}\frac{F_{ijt}^{(m)}}{f_{ij}^{ref}}\,Q_{ijt}^{(m)} + h_{ij}^{3}\,\frac{(F_{ijt}^{(m)})^2}{(f_{ij}^{ref})^2}, \\ \forall (i,j) \in \mathcal{P}u^{v} : i \in \mathcal{N}\backslash\mathcal{B}\end{aligned} \tag{4.24}$$

$$(-2\kappa_{ij}\,Q_{ijt}^{(m)} + 2h_{ij}^{1}\,Q_{ijt}^{(m)} + h_{ij}^{2}\frac{F_{ijt}^{(m)}}{f_{ij}^{ref}})Q_{ijt} + (h_{ij}^{2}\,Q_{ijt}^{(m)} - 2h_{ij}^{3}\frac{F_{ijt}^{(m)}}{f_{ij}^{ref}})\frac{F_{ijt}}{f_{ij}^{ref}} =$$
$$- H_{it}^{M} + H_{jt} - (\kappa_{ij} - h_{ij}^{1})(Q_{ijt}^{(m)})^2 + h_{ij}^{2}\frac{F_{ijt}^{(m)}}{f_{ij}^{ref}}\,Q_{ijt}^{(m)} + h_{ij}^{3}\,\frac{(F_{ijt}^{(m)})^2}{(f_{ij}^{ref})^2},$$
$$\forall (i,j) \in \mathcal{P}u^{v} : i \in \mathcal{B} \quad (4.25)$$

4.4 The hybrid algorithm

Since solving the hydraulic model as an MINLP is computationally expensive, the MILP formulation provides a good alternative. As will be shown in the results section however, the solution time increases exponentially with the number of PWL intervals. For a lower number of K (and an approximated power term in the goal function), a solution can be found reasonably fast. However this solution is not necessarily feasible for the original MINLP formulation depending on the accuracy of the approximation. Therefore, the solution obtained by solving the MILP will now serve as a starting point for the presented gradient algorithm. The idea is to adjust the values for (Q, F, H) without altering any other variables so that the nonlinear pressure losses and pump constraints are satisfied. Afterwards the previously satisfied constraints have to be checked to make sure the configuration is globally feasible.

Algorithm 2 displays the pseudocode for the proposed algorithm. Start with $K = 2$ intervals and solve the MILP model described in this chapter. Set the convergence parameters to their initial values. Then start iterating the gradient algorithm described in the previous section. Update $Q^{(m)}, F^{(m)}$ - during the first iteration this means setting them equal to the values of the optimal solution of the MILP. Next, solve an iteration of Newton's method on the limited model described as follows:

$$\text{Maximize}(3.34 - 3.37)$$
$$\text{subject to}(3.2, 3.3, 3.10 - 3.12, 4.21 - 4.25)$$
$$Q, F, H, \Delta H, I^{+}, I^{-}, O^{-}\text{u.r.s.}$$

In order for this algorithm to work with our model formulation, it is necessary that we drop all bounds on variables and maintain only the equality constraints: flow conservation, storage volume in buffer and the newly defined linear functions. Furthermore all binary variables are fixed. Volume in buffers is also fixed and regarded as additional demand/supply in between periods. The production rate is fixed in order to ensure that the constraints (3.28-3.29) are satisfied.

$K \leftarrow 2, check \leftarrow 0$ ▷ Start with two intervals
repeat
  Solve MILP ▷ Solve complete problem with linearized constraints
  $numer \leftarrow 100, denom \leftarrow 1$
  $\delta \leftarrow 1e - 12$ ▷ convergence tolerance
  $iter \leftarrow 0$
  for $(i,j) \in \mathcal{P}u, t \in [1,T]$ **do**
    $q_{ijt}^{off} \leftarrow 0$
  repeat
    for $(i,j) \in \mathcal{P}i \cup \mathcal{P}u, t \in [1,T]$ **do**
      $Q_{ijt}^{(m)} \leftarrow Q_{ijt}$
    for $(i,j) \in \mathcal{P}u, t \in [1,T]$ **do**
      $F_{ijt}^{(m)} \leftarrow F_{ijt}$
    if $iter = 2,4,6,8$ **then**
      if $h_{ij}^3 < H_{jt} - H_{it}$ **then** ▷ If shutoff head pump smaller than pressure gain then switch off pump
        fix $\vartheta_{ijt} \leftarrow 0$, $Q_{ijt} \leftarrow 0$
        $Q_{ijt}^{(m)} \leftarrow 0$
        if $(i,j) \in \mathcal{P}u^v$ **then**
          fix $F_{ijt} \leftarrow 0$
          $F_{ijt}^{(m)} \leftarrow 0$
        $q_{ijt}^{off} \leftarrow 1$
      if $q_{ijt}^{off} = 1 \;\&\; h_{ij}^3 \geq H_{jt} - H_{it}$ **then** ▷ If pump is switched off check if it can be reactived
        fix $\vartheta_{ijt} \leftarrow 1$
        $Q_{ijt}^{(m)} \leftarrow \frac{h_{ij}^2 + \sqrt{(h_{ij}^2)^2 - 4(\kappa_{ij} - h_{ij}^1)\,(H_{jt} - H_{it} - h_{ij}^3)}}{2(\kappa_{ij} - h_{ij}^1)}$
        unfix Q_{ijt}
        if $(i,j) \in \mathcal{P}u^v$ **then**
          $F_{ijt}^{(m)} \leftarrow 50$
          unfix F_{ijt}
        $q_{ijt}^{off} \leftarrow 0$
    Solve Newton ▷ Solve limited model
    $iter \leftarrow iter + 1$
    $numer \leftarrow \sum_{(i,j)\in\mathcal{P}i\cup\mathcal{P}u} |Q_{ijt} - Q_{ijt}^{(m)}|$,
    $denom \leftarrow \sum_{(i,j)\in\mathcal{P}i\cup\mathcal{P}u} |Q_{ijt}|$
  until $numer/denom < \delta$
  if All constraints satisfied **then**
    $check \leftarrow 1$
  else $K \leftarrow K \times 2$
until $check = 1$

Algorithm 2: The hybrid algorithm

With the updated information, solve this limited model repeatedly until convergence is reached. Every other iteration (up until the 10th) a check is made on the pumps. If the shutoff head of the pump (maximum possible head delivered by pump) is smaller than the head gain (negative of head loss) then the flow is fixed to $Q = 0$ and the binary pump variable ϑ becomes 0 as well. If these conditions are not present, the pump becomes active again and the flow is calculated from the pump curve and the current value for H (This method is also used in [9]). Because this is not mentioned in the EPANET manual, an extension for variable speed pumps is made here. If the pump is to be switched off, the variable F is set to 0 as well. If the pump is reactivated the frequency becomes equal to the reference value of 50. Note that the respective constraint (4.22-4.25) is dropped if the pump gets switched off. Once the convergence criterion is satisfied, a final check on the solution is made. Since the gradient method can not handle bounds on variables, their value compared to the minimum and maximum values needs to be verified. Furthermore, in (3.5-3.9) water can essentially flow into the buffer although there is not enough pressure. The same may be true for water flowing out of the buffer. If one or more of these constraints are not satisfied within a certain tolerance, the number of intervals is doubled and the next iteration starts.

4.5 Computational results

For the MILP model a much wider array of solvers is available. Here, Gurobi [85] version 6.0.4 is used.

4.5.1 Test network

The MILP model can be solved for a varying number of intervals. PWL-1 is the formulation without additional inequalities, while PWL-2 contains those additional constraints. Table 4.2 summarizes the results for several values of K. Note that the same number of intervals is considered here for each pump and pipe, independent of the period: $K_{ijt} = K, \ \forall (i,j) \in \mathcal{P}i \cup \mathcal{P}u, t \in [1,T]$.

Table 4.2: Results with Gurobi on the MILP for the test network

# K	PWL-1			PWL-2				
	nvars	ncons	time (s)	nvars	ncons	time (s)	obj.	ϵ (m)
2	1268	2122	0.43	1268	2122	0.41	3646.50	28.75
4	1673	2392	3.32	1673	2662	4.98	3646.76	7.19
8	2348	2662	7.38	2348	3742	9.88	3646.82	1.80
16	3563	2932	24.43	3563	5902	65.72	3646.81	0.45
32	5858	3202	170.06	5858	10222	298.76	3646.82	0.11

nvars = # variables; ncons = # constraints

The number of variables and constraints increases substantially each time K doubles. The goal function is always a lower bound to the solution of the MINLP method. The displayed error ϵ is the maximum error among all segments in all time periods: $\max\{\epsilon^1, \epsilon^2, \epsilon^3, \epsilon^4\}$. Note that with 8 intervals the error is already relatively small (1.8 m, which corresponds to 0.18 bar). With 16 intervals the values of all variables are almost equal to those found by Bonmin's B-BB algorithm although the computational time is significantly smaller. From 32 intervals onwards the solution time goes up rapidly and the MINLP model outclasses the MILP model. The PWL-2 formulation is comparable with PWL-1 when the number of intervals is small, but it is obvious from the table that the number of additional constraints significantly slows down the solver starting from $K = 16$. Therefore the PWL-1 model will be used for the following real-world instances.

Table 4.3 shows the solution when the PWL formulation is coupled with the gradient algorithm. It displays per interval the final solution with Newton's method for every interval and the result after the final feasibility check. The final goal value is the objective value achieved with Newton's method. Note that the error is only present after the PWL optimization and disappears after the correction with Newton's method. The computation time for the Newton method is negligible compared with that of Gurobi.

Table 4.3: Results of the hybrid method on test network

# Intervals	goal	Feasibility check
K=2	INF	(INF)
4	3646.76	(INF)
8	3646.82	(INF)
16	3646.81	(FEAS)
32	3646.82	(FEAS)

nvars = # variables; ncons = # constraints

The algorithm converges for every instance of K except the first one. As mentioned previously, we note that the final solution is infeasible (INF) for small values of K. From values of $K = 16$ onwards the final solution satisfies all the restrictions from the original problem (FEAS, within a tolerance of 1%). In the first loop ($K = 2$) the Newton method did not converge because no solution was possible with the fixed values of the variables. For $K = 4, 8$ the method did converge but the solution was rejected because of infeasibilities in the dropped constraints. Most of the time infeasibility was due to the fact that water was flowing out of the buffer while the pressure in the network was actually too high. At $K = 16$ a solution is found that satisfies all the constraints and the algorithm is terminated. Note that the Newton's method did not alter the objective value found with the PWL model, indicating that the pump schedule was not modified. Most probably the global optimum has been found - this is also the same optimal value as found by BONMIN.

4.5.2 High-level transport optimization model

Two different formulations are proposed for the multivariate functions (3.23,3.25) in the MILP model: decomposition (MILP-d) and reformulation (MILP-r). For computational purposes Gurobi was set to solve to a relative optimality of 1%. Again K was given the same value over all arcs and time intervals. The term (3.25) in the objective function made it hard for the solver to come up with a solution in an acceptable computational time. Only for 2 and 4 intervals a solution was found within reasonable time for the MILP-d model; see Table 4.4. The MILP-r model performed considerably worse and no solutions was found within 3600 s.

Table 4.4: Results with Gurobi on the MILP-d for the transport optimization network

# K	nvars	ncons	time (s)	obj.	error (m)
2	5194	8917	0.87	20916.88	30.23
4	7210	10555	3504.53	21112.62	7.56

nvars = # variables; ncons = # constraints

Clearly the difference in computational time between 2 and 4 interval is extremely big. For a more accurate division in intervals the objective function was made cheaper to evaluate and (3.36) changed into:

$$\text{Minimize } Z^{FSP} = \sum_{t=1}^{T} \sum_{(i,j)\in \mathcal{P}u^v} \frac{c_t(e)}{1000}\, \tau_t \left(p_{ij}^1\, Q_{ijt} + p_{ij}^2\, \frac{F_{ijt}}{f_{ij}} \right) \tag{4.26}$$

This makes the objective function too steep for low values of F while the offset for $Q = 0$ is too high. Consequently, it is expected that the goal function will generally be overestimated. The results are found in table 4.5.

Table 4.5: Results with Gurobi on the MILP-d for the transport optimization network, altered goalfunction

# K	nvars	ncons	time (s)	obj.	obj. (corr)	error (m)
2	5138	8805	4.28	21337.92	21339.1	30.23
4	7154	10331	90.70	21421.49	21373.3	7.56

nvars = # variables; ncons = # constraints

In this table obj. (corr) displays the corrected cost according to the original goal function. For some reason the computation time for 2 intervals increases, whereas it decreases for larger numbers of intervals. From $K = 8$ intervals onwards, the computational time increased dramatically. Therefore these results are no longer displayed. Note that the solution times and objective values are otherwise very competitive to the best solution obtained by BONMIN on the MINLP model (21399.33).

In table 4.6 the result of the hybrid method is shown. For 4 intervals the approximation already seems tight enough so that the solution can be corrected using the gradient method. Another explanation for this ‘earlier’ convergence may be due to the presence of variable speed pumps, which give the model more flexibility.

Table 4.6: Results of the hybrid method on the transport optimization network

# Intervals	goal	Feasibility check
K=2	INF	(INF)
4	21373.3	(FEAS)

nvars = # variables; ncons = # constraints

4.6 Conclusion

The MILP formulation successfully reduces the complexity of the model by using a piecewise linear approximation of the nonlinear formulations. The results show a big gain in computational effort for the test network in comparison with the MINLP solver. For a small number of intervals this method is even faster than the FP heuristic. For the more complex network with variable speed pumps, the multivariate functions need to be approximated which requires more additional variables and constraints. This results in the fact that the method is not suitable for this network when the number of intervals is too high. For a small number of intervals, on the other hand, the approximation error may be too high for the solution to be realistic. Experiments show that a decomposition method is more beneficial than a reformulation of the multivariate functions. Using the gradient method, the solution can be repaired to satisfy the original restrictions. The clear advantage here is instead of doing a multiperiod simulation, the optimal values for the buffers and raw water pumps from the MILP model are fixed ensuring the transient conditions remain valid. Since the flow and pressure values are allowed to vary during this iterative process however, the final solution may not satisfy variable bounds and inflow constraints at buffer entrances. Consequently the number of intervals needs to be increased and the iterative process is restarted. Even for larger networks a solution may be found using this method, although the large number of binary variables may still prove to be a computational bottleneck.

5

Generalized Benders Decomposition to re-optimize water production and distribution operations

As stated before, finding an optimal solution for either the MINLP or the MILP with many intervals proposed in chapter 3 is not a trivial task. While the binary variables for pump activation and inflow constraints at the buffers complicate the model, the nonconvex nonlinear constraints (3.16-3.23) are the real computational bottlenecks. The resulting model is thus very challenging and hard to solve. Section 3.4 shows that the search for a global optimal solution for the MINLP leads to unacceptable computation times, even for small networks. In this chapter a method based on Generalized Benders Decomposition (GBD) is proposed to obtain good solutions for the MINLP model in a reasonable computational time. In general, decomposition is done to break the structure of a complicated model into one or more subproblems which are coupled by a master problem. This optimization problem gathers information from the optimal solution of the subproblems and, after it is solved, redirects information back to the subproblems for the next iteration. From a computational perspective the subproblems are easier to solve than the original problems. This can be either because they have a special structure (e.g. max flow problem) for which efficient algorithms exist, or simply because they are much smaller in size. The decomposition method proposed by Benders [86] facilitates the subproblems by finding the optimal value of so-called ‘complicating variables’ in the master problem and then fixing their values in the subproblem. The next section provides a detailed description of the general algorithm. Geoffrion [87] extended this approach for the nonlinear mixed integer case. In [11], the authors implement this 'Generalized Benders Decomposition' algorithm on two nonconvex water resource management systems, in which the Master problem contains most of the restrictions. Approximation cuts [88] are used to decrease computational time and lead to good solutions. In section 5.2 this approach is extended so that in can be used for the nonconvex mixed integer structure inherent to the water supply models in this dissertation. In the final section test results are presented and thoroughly discussed.

5.1 General background on Benders Decomposition algorithms

In this section the main ideas behind the classical Benders decomposition are given. Furthermore the extension of this method to nonlinear problems, known as Generalized Benders Decomposition, is outlined.

5.1.1 Benders Decomposition

The general MIP problem discussed in this section has the form

$$\text{Minimize}_{x,y} \{cx + fy \text{ subject to } Ax + By \geq b, \;\; x \geq 0, \;\; y \in Y\} \tag{5.1}$$

x and y represent vectors of continuous and integer variables respectively. For fixed values of y the remaining subproblem is stated as [86]:

$$\text{Minimize}_{x\geq 0} \{cx \text{ subject to } Ax \geq b - B\bar{y}\}$$

Transforming this inner minimization to its dual, the complete minimization problem can be rewritten as

$$\text{Minimize}_{\bar{y}\in Y} [f\bar{y} + \text{Maximize}_{u\geq 0} \{u(b - B\bar{y}) \text{ subject to } uA \leq c\}] \tag{5.2}$$

The vector u is the vector of dual variables. Note that for fixed values of y the subproblem is an LP problem. Its dual solution can either be bounded or unbounded. If the problem is unbounded (corresponding to an infeasible primal problem), the values of y resulting in an unbounded dual subproblem must be avoided. After obtaining the unbounded ray $\bar{u}$ the following cut is valid ($\forall y$):

$$\bar{u}(b - By) \leq 0$$

As a result, the problem 5.2 can be rewritten as Benders reformulation:

$$\text{Minimize}_{y \in Y} \, z + fy \tag{5.3}$$

$$\text{subject to } z \geq \bar{u}_k(b - By), \quad k = 1..K \tag{5.4}$$

$$\bar{u}_l(b - By) \leq 0, \quad l = 1..L \tag{5.5}$$

Since this problem can be extremely large, only a limited set of constraints is considered. The above formulation is therefore known as the reduced master problem (RMP). In order to find a solution to the complete problem, the procedure of algorithm 3 is followed.

$l = 0, k = 0$
$y \leftarrow y_0$ ▷ Start with a feasible solution by solving (5.3) without cuts
$LB \leftarrow -\infty$
$UB \leftarrow \infty$
while $UB - LB > \delta$ **do** ▷ $\delta =$ convergence tolerance
    solve SP: Maximize$_{u \geq 0}$ $\{u(b - B\bar{y})$ s.t. $uA \leq c\}$
    if Unbounded **then**
        $l \leftarrow l + 1$
        Get ray $\bar{u}_l$ and add cut $\bar{u}_l(b - By) \leq 0$ to (5.3)
    else
        $k \leftarrow k + 1$
        Get extreme point $\bar{u}_k$ and add $z \geq \bar{u}_k(b - By)$ to (5.3)
        $UB \leftarrow \min\{UB, fy + \bar{u}_k(b - By)\}$
    Solve (5.3)
    $LB \leftarrow \bar{z}$

Algorithm 3: Benders Decomposition algorithm

The master and subproblem are iteratively solved until convergence is reached. This method is very interesting when applied on difficult problems that decompose in easy-to-solve master and subproblems.

5.1.2 Generalized Benders Decomposition

In his seminal paper, Geoffrion [87] proposed an extension of Benders' famous algorithm for integer linear optimization problems to nonlinear optimization problems of the form:

$$\text{Minimize}_{x,y} \, \{f(x, y) \text{ subject to } G(x, y) \leq 0, \;\; x \in X, \;\; y \in Y\} \tag{5.6}$$

The variables are separated in two subsets x and y, where y is the vector of so-called complicating variables. In general, it is assumed that the original problem becomes easier to solve in x when y is held fixed. In his paper, Geoffrion states three different situations in which GBD may be beneficial:

1. For fixed y, (5.6) decomposes into a number of independent optimization problems, each with a different subvector x_i;

2. For fixed y, (5.6) the remaining problem has a specific structure for which efficient algorithms exist (e.g. transportation problems);

3. For fixed y, the nonconvex problem (5.6) becomes a convex problem in x.

The third situation is the one we wish to exploit here. After initialization of y, the following problems are solved to optimality at each iteration:

$$\text{Minimize}_{x \in X} \{f(x, \hat{y}) \text{ subject to } G(x, \hat{y}) \leq 0\}$$

which is the subproblem (SP) providing an upper bound (UB) and

$$\begin{aligned} &\min_{y \in Y, y_0} \quad y_0 \\ &\text{subject to} \qquad y_0 \geq L^*(y, u^j), \quad j = 1..p, \\ &\qquad L_*(y, \lambda^j) \leq 0, \quad j = 1..q, \end{aligned}$$

is now the reduced master problem (RMP). u_j is the vector of optimal dual variables for the j'th subproblem (if it is feasible) and λ_j the vector of extreme rays (in case of infeasibility). At every iteration the optimal solution is a lower bound (LB) to the original problem. Here, L^* and L_* are given by:

$$L^*(y, u_j) = \text{infimum}_{x \in X} \{f(x, y) + u_j^T G(x, y)\}$$
$$L_*(y, \lambda_j) = \text{infimum}_{x \in X} \{\lambda_j^T G(x, y)\}$$

The solution $\hat{y}$ of each RMP is plugged in the next SP. Solving this generates new cuts that can be added to the RMP. At each iteration, set $LB = y_0$ (the optimal solution to the RMP) and $UB = v(\hat{y})$, the optimal solution to the subproblem; then continue iterating until $(UB - LB)/LB < \delta$. The algorithm converges if [87]:

- f and g are convex on x for each fixed $y \in Y$,

- X is a nonempty compact convex set,

- the RMP can be solved to global optimality,

Furthermore, the functions L^* and L_* should essentially be explicitly obtained independent of y to make the algorithm computationally efficient (this is *Property (P)* introduced by [87]).

The idea here is to apply GBD on the MINLP model formulation for the water supply network. This is motivated by the fact that the complicating variable can be chosen in a quadratic term through the replacement of x^2 by $x\,x'$ and $x = x'$. Furthermore, the authors in [10] already proposed both a GBD method and an outer approximation (OA) method to solve water resource models. Because the master problem of the OA method is much larger than that of the GBD method, it can become a big bottleneck in terms of computational time. As a result, the GBD method was generally more efficient than OA for solving large MINLP problems, thus motivating the choice of this algorithm here. The proposed models are nonconvex MINLP's with binary variables and bilinear functions. However, these functions are easily linearized so the methods will converge to the global optimum. The paper concludes by stating that future work should include the extension of these methods to problems with nonconvex subproblems. In [11] the authors apply the method on two bilinear water resource models. Approximation cuts are used so that convergence to global optimality is not guaranteed. Nevertheless their approach gives good results in a reasonable number of iterations. Here, the method will be extended to be used with our model which not only contains nonlinear pressure loss equality constraints, but also binary variables to control pumps and water exchange at the buffers. Furthermore, some of these pumps are actually variable speed pumps, which adds to the complexity of the system.

5.2 GBD for the investigated water supply network

In this section a general selection of coupling restrictions is made. All restrictions of the MINLP model described in chapter 3 are taken into account. Next, two formulations (selection of complicating variables) are proposed for applying GBD: one with an integer RMP (GBD-a) and another with an integer SP (GBD-b). Along the same line as [11], we add slack variables to make the subproblem always feasible. Afterwards, the approximation cuts are explicitly derived. The final part outlines how one could theoretically solve the RMP in an exact fashion.

5.2.1 Selection of coupling constraints

The implementation of this approach for the model is based on [89]. First, the variables β, Q^p, Q^m, ϕ are introduced subject to the following constraints:

$$\beta_{ijt} = Q^p_{ijt} - Q^m_{ijt}, \quad \forall (i,j) \in \mathcal{P}i \tag{5.7}$$

$$\beta_{ijt} = Q_{ijt}, \quad \forall (i,j) \in \mathcal{P}u \tag{5.8}$$

$$\phi_{ijt} = F_{ijt}, \quad \forall (i,j) \in \mathcal{P}u^v. \tag{5.9}$$

$Q^p \geq 0, Q^m \leq 0$ are related by the following equalities:

$$Q_{ijt} = Q^p_{ijt} + Q^m_{ijt}, \quad \forall (i,j) \in \mathcal{P}i \tag{5.10}$$

$$Q^p_{ijt} \leq 3000\,\vartheta_{ijt}, \quad \forall (i,j) \in \mathcal{P}i \tag{5.11}$$

$$Q^m_{ijt} \geq -3000\,(1 - \vartheta_{ijt}), \quad \forall (i,j) \in \mathcal{P}i \tag{5.12}$$

where ϑ is a binary variable.

(3.16, 3.19 - 3.23) are now rewritten as follows:

$$H_{it} - H_{jt} - \kappa_{ij}\, Q_{ijt}\beta_{ijt} = 0, \;\; \forall (i,j) \in \mathcal{P}i \tag{5.13}$$

$$H_{it} - H_{jt} - \kappa_{ij}\, Q_{ijt}\beta_{ijt} + \Delta H_{ijt} = 0, \;\; \forall (i,j) \in \mathcal{P}u : i \in \mathcal{N}\backslash\mathcal{B} \tag{5.14}$$

$$H^M_{it} - H_{jt} - \kappa_{ij}\, Q_{ijt}\beta_{ijt} + \Delta H_{ijt} = 0, \;\; \forall (i,j) \in \mathcal{P}u : i \in \mathcal{B} \tag{5.15}$$

$$\Delta H_{ijt} - h^1_{ij}\, Q_{ijt}\beta_{ijt} - h^2_{ij}\,\beta_{ijt} - h^3_{ij}\,\vartheta_{ijt} = 0, \;\; \forall (i,j) \in \mathcal{P}u\backslash\mathcal{P}u^v \tag{5.16}$$

$$\Delta H_{ijt} - h^1_{ij}\,(Q_{ijt})\beta_{ijt} - h^2_{ij}\,\frac{Q_{ijt}\phi_{ijt}}{f^{ref}_{ij}} - h^3_{ij}\,\frac{F_{ijt}\phi_{ijt}}{(f^{ref}_{ij})^2} = 0, \;\; \forall (i,j) \in \mathcal{P}u^v \tag{5.17}$$

Furthermore, the part of the goal function accounting for the power of variable speed pumps (3.36) changes to $p^1_{ij}\, Q_{ijt}\,(\phi_{ijt}/f^{ref}_{ij})^2 + p^2_{ij}\, F_{ijt}\phi^2_{ijt}/(f^{ref}_{ij})^3$.

Two separate formulations of the GBD method are proposed, which we call GBD-a and GBD-b:

5.2.2 GBD-a: Integer RMP and Linear SP.

If one member of $((Q, \vartheta, F), (\beta, \phi))$ is fixed, the pressure loss constraints become linear. It seems logical to choose only (β, ϕ) as the set of complicating y variables, since their number is not very large. However, the authors in [11] report to have good results when making the SP as small as possible. When applied on our model, that would mean choosing (β, ϕ) as the vector x and all other variables as the vector y. Instead a more 'natural' decomposition is proposed here. By also taking the pressure variables H in the subproblem, the RMP becomes a flow problem with capacity restrictions and storage over multiple periods, whereas the subproblem is still small enough to be quickly solvable. This will also give the subproblem some more flexibility, thus improving the chance of finding feasible solutions. Consequently, we define $x = \{\beta, \phi, H\}$ and y the vector of all other variables. The set X consists of bounds on pressure variables and Y is defined by (3.2-3.3, 3.10-3.6, 3.9-3.15, 3.17-3.18, 3.24, 3.28-3.33, 5.10-5.12). Furthermore, the coupling constraints $G(x, y)$ are (5.7-5.9, 5.13-5.17), and the constraints in buffers linking binary variables and pressure in the tank (3.7-3.8).

5.2.3 GBD-b: Linear RMP and Integer SP.

With the previous decomposition, experiments showed that the RMP was the bottleneck since cuts containing binary variables are added in each iteration, thus increasing the computational time. Therefore, we place the binary variables in the subproblem, while maintaining the same structure as the GBD-a model. The resulting set x thus becomes $x = \{\beta, \phi, H, \vartheta\}$. The gain in computational time is traded off against the inaccuracy of the generated cuts (from the continuous version of the MIP subproblem).

In what follows, we analyze the structure of the GBD problem in the same fashion as in [11] to find an explicit formulation of the functions L^* and L_*.

5.2.3.1 Derivation and analysis of the cut functions: approximation cuts

For purposes of generality, the coupling constraints are rewritten as follows:

$$G(x,y) = f(y)x - b = 0 \tag{5.18}$$

where $f(y)$ is a vector of (possibly constant) coefficients of x in the subproblem and b is a vector of constants. Note that the equality sign is not necessarily strict.

Since we want to ensure that the subproblem is feasible for any value of $f(y)$, we add slack variables to the coupling constraints. Hereto, we partition $G(x,y)$ in two sets: $G^2(x,y)$, the set of restrictions with artificial slack variables s, e and $G^1(x,y)$, which contains all other restrictions. We now change (5.18) to:

$$G^1(x,y) = f(y)x - b = 0 \tag{5.19}$$
$$G^2(x,y) = f(y)x - b = s - e \tag{5.20}$$

The restrictions making up the set G^1 have to be chosen carefully so that convergence is improved while feasibility is ensured. The chosen restrictions will be discussed in the results section. The objective then becomes (we consider a linear function $c_x x$ for simplicity) :

$$\text{Minimize}\,[c_x x + c_y y + M(s+e)] \tag{5.21}$$

where the value of M has to be determined experimentally. We now define the subproblem (SP) as the minimization of (5.21) over x, s and e for fixed y subject to (5.19, 5.20) and $x \in X$.

The subproblem is always feasible and it is optimal if all slack variables (the penalty term $s+e$) are equal to zero. The Lagrangian function corresponding to this subproblem, where u, u_2 are the dual variables derived from the optimal solution, is given by:

$$L(x,y,s,e,u) = c_x x + c_y y + M(s+e) - u^T\,(f(y)x - b) - u_2^T\,(e-s)$$

and the optimality cut function is:

$$L^*(y,u) = \min_{x \in X, s \geq 0, e \geq 0} L(x,y,s,e,u)$$

As demonstrated by [11] the function L is linear and by complementary slackness can be rewritten as:

$$L^*(y,u) = u^T b + c_y y + \min_{x \in X}(c_x - u^T f(y))x \tag{5.22}$$

It can easily be seen that this function is non-separable and should be computed separately for each instance of y, and thus *Property (P)* [87] does not hold. This means that the functions L^* can not be obtained without knowing the value of y. Depending on the choice of variables in the set X, this can be computationally expensive. As a consequence, we will use approximate cuts and add them to our RMP without any guarantee that our algorithm will converge, as suggested in [88]. This means plugging the optimal values x^*, u^* for subproblem j in the j'th Lagrangian function, which the authors in [90] refer to as v-GBD. In that same paper it is stated that the Lagrangian cut function may cut off part of the feasible region and local optimal points or non-optimal points may be found. The authors state that finding an optimal solution is very dependent on the starting solution and the convergence speed on the tightness of the MINLP formulation. In [89] also, the authors warn for the convergence to local minima.

5.2.4 Towards exact cuts

Here we theoretically derive the exact formulation of the optimality cuts. For simplicity we only consider the GBD-a model, although the cut functions for GBD-b can be derived in the same fashion. As already mentioned, solving the RMP to global optimality will not be possible. In the case where the constraints $x \in X$ are reducible to $x_{min} \leq x \leq x_{max}$ it can be shown that L^* is a concave piecewise separable function, see [11]. Since $y_0 - L^*$ is a concave nonlinear function, the constraints $y_0 \geq L^*(y, u^j)$ define a reverse concave problem. We explicitly define these functions here. In a given iteration, the coefficient $c_x - u^T f(y)$ of β in (5.22), for $(i,j) \in \mathcal{P}i$, is given by:

$$-u^1_{ijt}\,\kappa_{ij} Q_{ijt} - u^2_{ijt}$$

where u^1 and u^2 are the dual prices of (5.13) and (5.7), respectively.

For pumps, these coefficients are also linear functions of Q and may be derived in the same way. The second set of variables composing the set X are the pressure variables H. Their coefficients in the function L are merely composed of dual prices of SP, denoted as u^3. The last term of (5.22) then comes down to:

$$\begin{aligned} &\text{Minimize } \left(-u^1 f(Q) - u^2\right)\beta - u^3\, H \\ &\text{subject to: } h_{min} \leq H \leq h_{max} \end{aligned}$$

Clearly, depending on the sign of u^3, H is either set to its lower or upper bound. The coefficient of β depends on Q. There are no explicit bounds put on β but these can actually be related to those imposed on Q. If $(-u^1 f(Q) - u^2) < 0$ then $\beta = |Q_{max}|$, otherwise $\beta = 0$. This requires adding one binary variable and two constraints to the RMP for every $x \in X$ in each iteration. As a result, the size of the RMP increases exponentially. Experiments show that after a few iterations the computation time already increases drastically and thus this model has been abandoned.

5.3 Results

The proposed approach is tested on the three networks described in chapter 3. The model varies depending on the choice of the constraints set G^2 and the size of the penalty factor M. Furthermore, some components of the goal function can be optimized at the level of the subproblem instead of having them all optimized in the RMP.

Experiments showed that the algorithm performs better if the set G^2 includes constraints (5.13 - 5.17), making the equality constraints (5.7 - 5.9) strict. Some constraints that contain binary variables needed additional slack variables to make sure that the subproblem is always feasible. Also considering the part of the energy cost function that is linear in β in the subproblem instead of keeping it in the RMP as a function of Q, turns out to be effective as this produces much better results. Deviating from the suggestion of the authors in [11], we choose the initial values of y by solving the RMP without any cut constraints. However, the original goal function is considered in this optimization step. We experimented with the penalty factor M varying it from 0.1 to 100 to assess its impact. Again, Gurobi is used as the MIP solver of our choice.

5.3.1 Test network

The findings are summarized in Table 5.1 below. For each method, the value of the goal function, the total processing time (in seconds) as well as the number of iterations are shown.

Table 5.1: Results of the used algorithms on the subnetwork

method	goal function	CPU time (s)	nr. iterations
GBD-a penalty=1.0	3646.29	196.61	208
GBD-a no penalty	3645.89	648.629	275
GBD-b penalty=1.2	3679.80	3.8	65

5.3.1.1 GBD-a

Clearly, the FP heuristic produces a good feasible solution quickly but the resulting optimality gap remains rather large. We distinguish two versions of the GBD-a algorithm. The first version includes the cost term $c\,\beta$ in the goal function at the level of the subproblem and sets the penalty factor to $M = 1$. The second version keeps the term $c\,Q$ in the original goal function of the RMP. The goal function of the subproblem thus consists of the sum of the artificial variables only, and the penalty factor M is dropped. The first version is solved in a computational time that is competitive with BONMIN's B-BB algorithm. Compared to this first version, the second one requires a lot more time before it converges. However, it still performs well compared to BONMIN. Other model variations were tested as well. However, these were not performing better than the two versions above. This confirms the fact that the algorithm is very sensitive to the choice of the complicating constraints, the artificial variables and the penalty size.

Figure 5.1 depicts the convergence behavior of the GBD-a algorithm (with the penalty term). The curve of the lower bound shows a rapid increase towards the final value during the first 50 iterations. The upper bound curve strongly fluctuates until it converges to the final value of 3646.29 after 208 iterations. The penalty curve $s + e$ from (5.21) is also displayed and converges to 0.

Fig. 5.2 depicts the impact of the penalty factor M on the behavior of the GBD-a algorithm. For $M = 0.1$ the resulting penalty is still unacceptably big, around 500, and the final value is 3494.4. The experiments show that choosing $M < 1$ does not ensure convergence, whereas for higher values ($M = 10$) the algorithm converges but is very slow.

5.3.1.2 GBD-b

When the binary variables are transferred to the primal subproblem, computational time decreases considerably. This is clearly a consequence of the fact that an 'easier' RMP is solved at each iteration. The disadvantage is that the cuts are not strong anymore, and thus the final objective value is slightly higher ($\approx 1\%$, see table 5.1). Still, if one prefers fast solutions, this model is a good choice as it still provides near-optimal solutions. Notice that the penalty factor is always present in the SP because of the binary variable ϑ in the goal function.

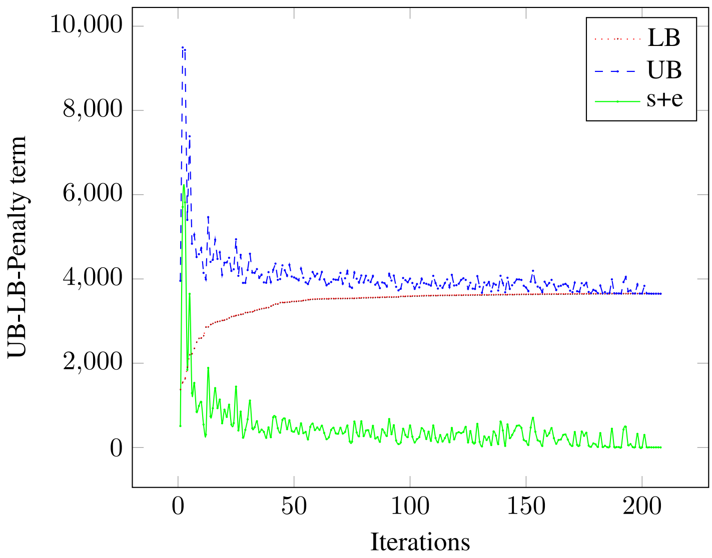


Figure 5.1: Upper bound, lower bound and penalty term in function of iterations (GBD-a, $M = 1.0$)

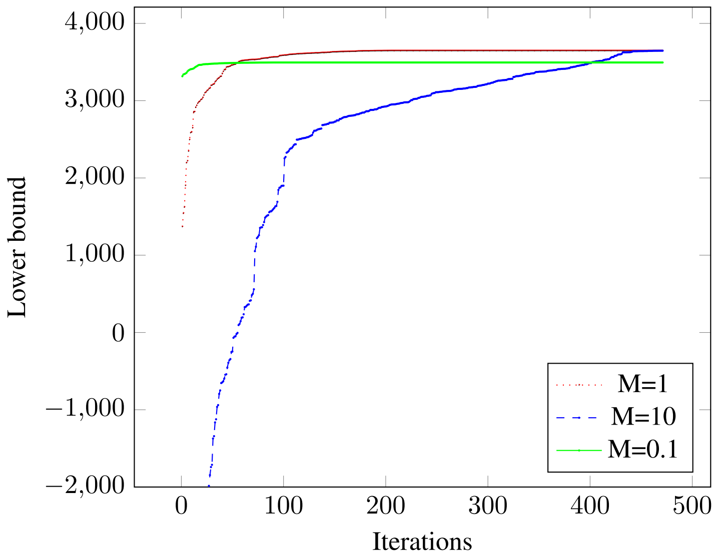


Figure 5.2: Lower bound in function of iterations for different instances of M (GBD-a)

In this case, varying the penalty coefficient M has a very unpredictable effect on the final value as well as on the remaining penalty (sum of slack variables). Table 5.2 summarizes these results for different values of M. Although it is difficult to draw general conclusions, the algorithm behaves badly for $M \leq 0.5$ (big penalty slack remaining at the end) and $M \geq 1.8$ (penalty + worse objective value). Some unexpected results are however obtained: when $M = 1.6$, the goal function and slack at the end are both large while for $M = 1.7$ a very low value for the slack and a good objective value are obtained. If the slack is lower than 10 for this specific model, the resulting solution is usually considered as acceptable. Choosing M smaller than 1 is therefore not recommended, while choosing M larger than 2 usually leads to a large objective value with slack values that are less evenly distributed. In general, one should carefully experiment with the penalty factor until a solution that satisfies the targeted trade-off (high objective vs. low slack) is found. The recommended range of values for M proved to be effective for the two cases we investigated in this paper.

Table 5.2: Effect of M on the goal function and remaining slack in GBD-b

M	goal function	remaining penalty	CPU time (s)	nr. iterations
0.1	3686.75	34.8	1.515	32
0.2	3699.82	61.28	1.567	29
0.4	3698.74	32.19	2.1	42
0.6	3692.35	10.94	2.422	40
0.8	3721.68	47.24	2.05	44
1	3693.85	5.55	3.385	62
1.2	**3679.80**	3.97	3.8	65
1.4	3711.30	4.19	3.57	68
1.6	3770.35	20.1322	3.77	70
1.7	3688.18	**1.48**	3.55	63
1.8	3804.82	47.24	5.45	87
2	3800.78	34.82	4.372	67
5	3925.15	7.85	5.34	79
10	4461.41	55.28	6.57	103
25	4387.41	9.27	5.59	91
50	4509.03	6.33	9.13	136
100	5247.90	10.05	6.99	115

Furthermore, to generate different final solutions, we also attempted to start the process from different initial solutions. This is carried out by using variants of the goal function. Figure 5.3 shows the convergence for these different initial solutions with $M = 1.2$.

The model in which only the pump costs are used in the goal function converges faster than the other ones, but leads to a worse objective value (3719.67). Further-

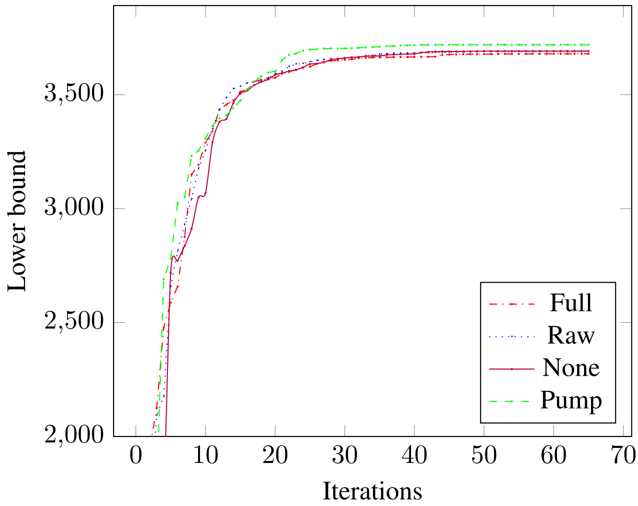


Figure 5.3: Lower bound in function of iterations for different initial solutions with GBD-b for $M = 1.2$ with goal functions: production + pumping cost (Full), production cost only (Raw), no objective (None) and pumping costs (Pump)

more, the final value of the penalty is still too large for the solution to be acceptable. Otherwise, the four models converge within approximately the same times, with only small ($< 0.3\%$) differences in the objective value.

To resume for the test network: GBD-b produces solutions faster than GBD-a; however the quality of the solutions obtained by GBD-a is better. GDB-b might nevertheless be useful to tackle very large networks, as is shown in the following test.

5.3.2 High-level transport optimization model

Findings for this case study are summarized in Table 5.3.

Table 5.3: Results of the algorithms on the transport optimization model

Solver/method	goal function	CPU time (s)	nr. iterations	
BONMIN/FP	21596.30	334.65	root node	
BONMIN/B-BB	21399.331	7200*	339104	*
Gurobi/hybrid method	21373.3	94.98	-	
Gurobi/GBD-b penalty=1	21327.9	3050.31	1000	

manually stopped, best possible solution 20795.016

Since the optimization problem for this case study is quite large and contains a significant number of binary variables, GBD-b is the most suitable method. Also,

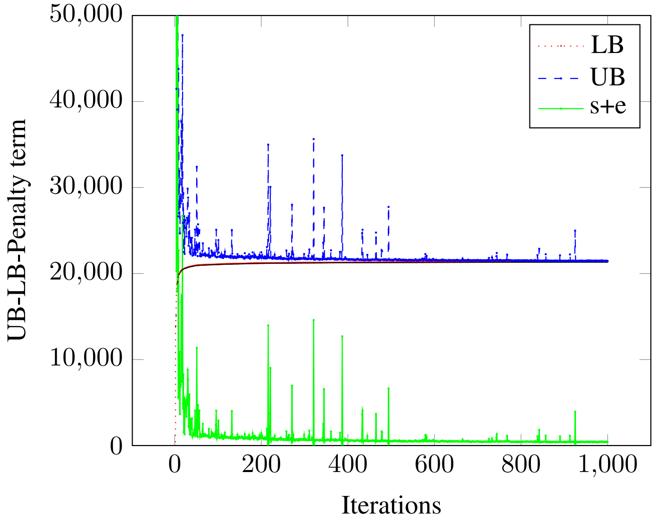


Figure 5.4: Upper bound, lower bound and penalty term in function of iterations (GBD-b, $M = 1.0$)

the objective function in the SP contains a quadratic term due to the presence of variable speed pumps. Even with the RMP solved as a linear program, it takes a lot of computational time to solve the problem (especially during the last couple of iterations). Figure 5.4 shows how the algorithm (LB + UB) converges after about 1000 iterations. Since the penalty term is large, a significant amount of iterations is needed. The results of GBD-a on this case are not presented as it did not converge to a good solution even after a very large amount of computational time. Again, GBD-b seems to be the best alternative for very large networks with variable speed pumps. Note that the result is also better than the one found with the hybrid method in chapter 4.

5.3.3 General observations

To conclude, the implementation and use of GBD with approximation cuts may have an edge, in terms of solution quality and computational time, over other nonconvex MINLP solvers. However, the underlying RMP and SP models and their related parameters must be chosen very carefully. Our findings for the investigated networks show that:

- A 'natural' decomposition where the RMP represents a flow problem and the SP contains the pressure variables is usually preferred;
- An objective term cx in the subproblem accelerates convergence of the

method;

- When the penalty factor is set to $M < 1$ the algorithm terminates while the penalty is still too high. On the other hand, increasing M too much seems to considerably slow down the convergence;
- If the variables and parameters are carefully selected convergence to a global optimum can be assured;
- GBD-a provides solutions with a better quality, but GBD-b seems to be the version to use for large networks with complicating features such as variable speed pumps.

5.4 Conclusion

The major contribution of this chapter is the extension of the method proposed in [10] and [11] to solve the nonconvex mixed integer models of water supply systems that contain binary pump variables and variable speed pumps with higher degree constraints. It has been shown that GBD is a worthwhile alternative to common heuristics and provides an efficient operating solution to large-scale water supply networks. Careful tuning of some parameters of the approach is however required to get good final solutions with an acceptable remaining penalty. A natural decomposition of the RMP in a pure flow model with several side constraints gives good results for the case studies investigated. When compared with results acquired with the Feasibility Pump algorithm and branch and bound (both implemented in BONMIN) the overall quality of our solutions is better but more importantly they are generated quickly, making them better suited to practical application in a water utility distribution system.

6 Conclusions and perspectives

A mathematical model can serve as a valuable tool for decision making in operational management of large-scale water supply systems. This has already been proven by several research papers, where the authors implemented their models on real-life networks. The Berliner Wasserbetreibe (Berlin's main water supply network) has been coupled with an electronic optimization system [4, 5] that minimizes production and energy costs. The authors hereby approximated the full MINLP formulation with a nonlinear one. For the cities of Pittsburgh [6] and Adelaide [7] similar approaches have been implemented, where the software is able to produce suboptimal control configurations. These systems have proven to save up to 20% on operational costs.

Recently, other approaches have been proposed for solving similar networks. In the last decades (meta)heuristics have begun to dominate the field of water supply network optimization, on operational as well as design aspects. However, it needs to be pointed out that these networks are often either too simplified or too small when they are compared with real-life instances.

The drinking water network in Flanders can also benefit from a decision support model for operational management. Such a model will not only guarantee drinking water delivery but also lower operational costs by more efficient usage of the available water sources. The water supply network of Flanders has a complex structure that includes variable speed pumps, multiple network loops and buffers with additional constraints. The proposed models found in literature and their accompanying methods can not be implemented on this network for various reasons.

Either these models discard nonlinearities, which lead to poor approximations. Dynamic programming is a method that is very beneficial for smaller networks but fails when applied on large-scale instances. Proposed NLP approximations may be worth considering but these methods are not suited for exact pump scheduling or models that require binary variables for flow dynamics at buffers. Finally, heuristics are definitely worth considering. We distinguish two types of heuristics: those for the design case and those constructed for operational optimization. Whereas the first type can be used to determine optimal pump schedules and let simulation software such as EPANET calculate the exact network hydraulics, the second type of heuristics has to cope with transient conditions such as buffer storage. For this, a multiperiod optimization is required. One can start with the first period, optimize the configuration and plug the final buffer volume values in the starting values for the next period. Clearly, this may lead to a solution where not enough water is available to satisfy either demand or buffer volumes in the last period, but the solution is definitely suboptimal.

Here, a full MINLP model is proposed that takes into account all the relevant active components in the Flemish drinking water network. The final model is nonconvex and contains many nonlinear constraints (pressure losses and pump characteristic curves) as well as binary variables for pump activation and buffer flows. The model is accurate up to a certain limit: pressure losses are modeled using the approximate law of Prandtl-Kármán, and pump curve equations are based on discrete data points delivered by the pump manufacturer. Demand uncertainty was not taken into account, which can of course influence the optimal solution more than the small inaccuracies inherent to the model. Furthermore a discretization of the time horizon was proposed, where the state of the network is assumed to be constant over the length of each time period.

Using the MINLP solver BONMIN to test the formulation led to high computational times. Instead of using an NLP approximation, the use of piecewise linear functions was proposed to reduce the complexity of the model. These functions serve as over- and underestimators for the nonlinear pressure loss and pump characteristic functions, reducing the model to an MILP variant. Given that the number of intervals is sufficiently large, the solution of this model closely approximates the one found by BONMIN in decreased computational effort. Special attention is dedicated to variable speed pumps, which are modeled as multivariate functions in the MINLP. Different formulations to overcome this difficulty are proposed.

Since the MILP model seems to have a significant computational benefit, work is continued to derive an accurate solution. Here the link with simulation software methods is made. Using a modified version of the gradient algorithm, the solution from the MILP is fixed to satisfy the original MINLP formulation, at a marginal computational cost. Since pressure and flow bounds may be invalid, the necessary number of intervals to ensure feasibility has to be determined experimentally. As a result the method may be suitable for medium-sized networks but the computational time for large-scale networks may be high due to the required number of intervals and, accordingly, binary variables. Future efforts to reduce the complexity may include even tighter bounds on a local level. For example, a pipe that is connected with a buffer at a border of the network will never transport more than the sum of the total volume of this buffer divided by the length of the period and the hourly demand in this buffer. To take this aspect into account, a more thorough analysis of the network is needed.

In an effort to find good solutions in a more efficient way, a Generalized Benders Decomposition algorithm was proposed. This method efficiently splits the model in a master and subproblem, both of which can be solved in short computational time. Careful selection of the complicating variables and fine-tuning of the parameters allows this method to converge rapidly using approximation cuts. For larger problems the computation time is still high but a good solution can be found that approximates what is probably the global optimal solution. In this regard the method may be preferable in comparison to the hybrid method, where the number of intervals may prevent the algorithm from converging. The RMP is clearly the bottleneck since its complexity increases with every iteration. It is a worthwhile direction to investigate the use of efficient network algorithms to solve a part of the problem. In addition, the knowledge of the properties and structure of the problem can be used to integrate the GDB algorithm with appropriate heuristics and take advantage of their speed. In this regard the master problem may be solved heuristically and convergence will still be achieved. Furthermore this may result in achieving a smaller gap to the optimal value compared with a purely heuristic approach.

Other perspectives include the application of the proposed model and its methods on network design optimization problems. This will involve adding discrete sets of pump diameters and an altered objective value. These discrete diameters may be reformulated with the use of binary variables. Additional pressure loss constraints need to be imposed for each possible diameter value. This will increase the complexity, but on the other hand optimization is done over only one period so computational difficulties are minimal unless a big number of pipes have to be renewed. This model could be coupled with the proposed hybrid method in this dissertation. A nonlinear way to go about this is described in [71], where the authors use area (A) instead of diameter to decrease the nonlinearity of the model. Heuristic methods such as genetic algorithms (GA), differential evolution (DE) or ant colonization may also prove to be beneficial here however.

Since maximum limits on extraction in ground water wells are determined on a yearly basis(3.29), a more accurate model should be optimized over 365 days.

Furthermore contractual obligations exist in the Flemish water supply network where the water supplier has to extract a minimal yearly volume from a certain source. These contracts impose additional costs to the water supply network problem. Since solving the model over a horizon of 365 days is computationally impossible, another approach can be proposed. By adding seasonal weights for example, the model can still be solved on a daily basis while taking into account the actual extraction during the previous periods.

Demand variability can also be taken into account. Using historical data, forecasting techniques can be adopted to generate expected values of the demand. Uncertainty could be added by making use of stochastic model structures. These will however increase the complexity even more, which will require new optimization methods to find good solutions in reasonable computational time.

Electrical power flow equations and pressure losses in gas pipes are quadratic equations that resemble the friction loss equations in water pipes. The similarities between water supply networks and electric grids/gas networks may allow the proposed methods to be used for designing and operating those networks as well.

The proposed methods are proven to be capable of solving large-scale water networks to near-optimality. A practical implementation can be carried out for weekly planning purposes. This would of course require a user-friendly interface and a close cooperation between hydraulic and industrial engineers.

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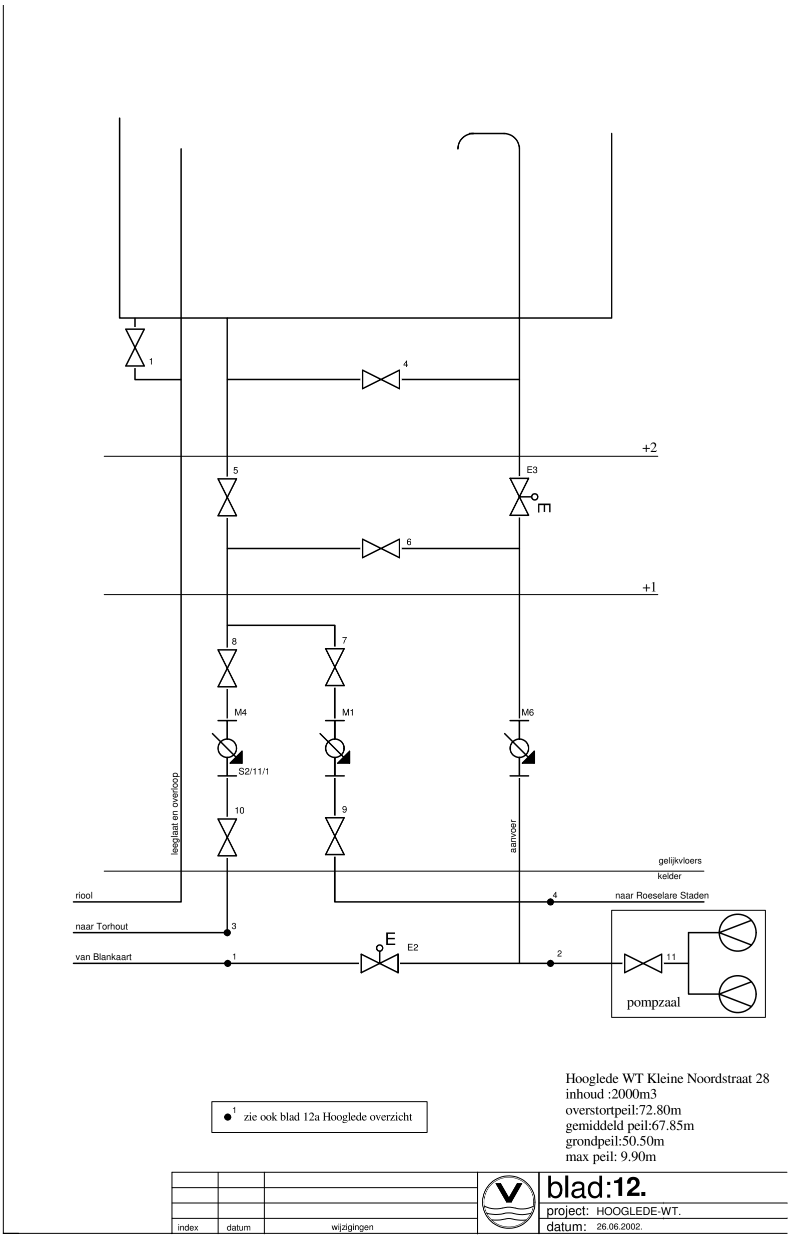
A
Detail of water tower in Hooglede



1
4
+2
5
E3
E
6
+1
8
7
M4
M1
M6
S2/11/1
10
9
leeglaat en overloop
aanvoer
gelijkvloers
kelder
riool
4
naar Roeselare Staden
naar Torhout
3
E
E2
van Blankaart
1
2
11
pompzaal
Hooglede WT Kleine Noordstraat 28
inhoud :2000m3
overstortpeil:72.80m
gemiddeld peil:67.85m
grondpeil:50.50m
max peil: 9.90m
1
zie ook blad 12a Hooglede overzicht
index
datum
wijzigingen
blad:12.
project: HOOGLEDE-WT.
datum: 26.06.2002.

B
Parameters of the test network

Table B.1: Node parameters for the example network

Node	A (m^2)	l^{min} (m)	l^{max} (m)	h^{fl} (m)	h^{in} (m)	d (m^3/day)	h (m)
WT1	150	1	6	40	47	2500	4
WT2	125	1	6	50	57	7000	15
WT3	200	1	7	40	48	1000	2
WT4	150	1	7	40	48	2500	5
WT5	200	1	5	70	76	2500	25
R1	150	2	4	14	19	0	10
R2	4000	2	5	2	8	0	0
R3	750	2	4	20	25	0	10
R4	2000	1	4	5	10	4000	0
WS1	-	-	-	-	-	0	10
WS2	-	-	-	-	-	0	10
WS3	-	-	-	-	-	0	10
D1	-	-	-	-	-	4000	10
D2	-	-	-	-	-	7500	10
J1	-	-	-	-	-	0	10
J2	-	-	-	-	-	0	20
J3	-	-	-	-	-	150	10
J4	-	-	-	-	-	3500	5
J5	-	-	-	-	-	1000	10
J6	-	-	-	-	-	0	5
J7	-	-	-	-	-	200	5
J8	-	-	-	-	-	0	20
J9	-	-	-	-	-	0	15
J10	-	-	-	-	-	0	25
J11	-	-	-	-	-	0	20
J12	-	-	-	-	-	0	20
J13	-	-	-	-	-	800	20
WPC1	-	-	-	-	-	0	10
WPC2	-	-	-	-	-	0	10
WPC3	-	-	-	-	-	0	10

Table B.2: Raw water pump parameters for the example network

Node	Node	q^{cap} (m^3/h)	q^{lim} (m)	$c(p)$ (€/m^3)
WS1	R1	150	3600	0.20
WS2	R2	1500	-	0.05
WS3	R3	175	4200	0.15

Table B.3: Regular speed pump parameters for the example network

Node	Node	l (m)	d (mm)	λ (-)	h^1 (m^{-5})	h^2 (m^{-2})	h^3 (m)	p^1 ($\frac{W}{m^3}$)	p^2 (W)	q^{min} ($\frac{m^3}{h}$)
R1	WPC1	0	400	0.021	-0.002	0.115	90.212	-58.143	36624	25
R3	WPC3	0	400	0.021	-0.002	0.115	90.212	-58.143	36624	25
J2	J1	20000	400	0.021	-0.0001	0.020	70.318	134.260	29606	100
R4	J12	5000	500	0.020	-0.0001	0.020	70.318	134.260	29606	100
R2	WPC2	0	1000	0.017	-0.00002	0.015	101.960	115.800	209032	300

Table B.4: Pipe parameters for the example network

Node	Node	l (m)	d (mm)	λ (-)
WPC1	J1	8000	500	0.01962
J1	WT1	10000	400	0.02073
J1	WT2	500	500	0.01962
J2	J3	5000	500	0.01962
WPC2	J3	1000	500	0.01962
WPC2	J4	1000	1000	0.01669
J4	J5	5000	600	0.01878
J5	J6	5000	300	0.02231
J6	WT3	3000	300	0.02231
J5	D1	5000	500	0.01962
J2	J8	8000	900	0.01709
J3	J7	2000	400	0.02073
J7	WT4	4000	400	0.02073
J7	R4	10000	400	0.02073
J8	D2	10000	500	0.01962
J9	R4	5000	400	0.02073
J9	J10	2000	400	0,02073
WPC3	J10	500	500	0.01962
J10	J11	1000	500	0.01962
J11	D2	2000	400	0.02073
J12	J13	1000	600	0.01878
J12	WT5	500	500	0.01962

C
Optimal solution for the test network

Table C.1: Optimal values for head in junctions (part I)

Node	Period	H(m)
J1	1	62.24
J1	2	61.40
J1	3	62.69
J1	4	97.22
J1	5	82.30
J2	1	54.55
J2	2	67.99
J2	3	54.18
J2	4	23.31
J2	5	45.89
J3	1	65.41
J3	2	79.54
J3	3	63.83
J3	4	23.40
J3	5	59.34
J4	1	70.68
J4	2	84.16
J4	3	69.05
J4	4	23.26
J4	5	67.63
J5	1	69.92
J5	2	83.83
J5	3	68.03
J5	4	23.25
J5	5	67.22
J6	1	69.92
J6	2	83.83
J6	3	65.94
J6	4	32.56
J6	5	62.00
J7	1	61.19
J7	2	77.05
J7	3	58.76
J7	4	25.11
J7	5	49.49
J8	1	54.42
J8	2	67.89
J8	3	54.09
J8	4	23.30
J8	5	45.54
J9	1	52.82
J9	2	65.13
J9	3	53.18
J9	4	25.01
J9	5	29.79

Table C.2: Optimal values for head in junctions (part II)

Node	Period	H(m)
J10	1	52.00
J10	2	65.07
J10	3	52.44
J10	4	25.00
J10	5	32.76
J11	1	51.87
J11	2	65.07
J11	3	52.32
J11	4	24.75
J11	5	33.22
J12	1	77.89
J12	2	77.84
J12	3	77.25
J12	4	55.70
J12	5	77.57
J13	1	77.89
J13	2	77.84
J13	3	77.24
J13	4	55.69
J13	5	77.57
WPC1	1	62.24
WPC1	2	61.40
WPC1	3	62.69
WPC1	4	97.51
WPC1	5	82.30
WPC2	1	70.71
WPC2	2	84.18
WPC2	3	69.08
WPC2	4	23.26
WPC2	5	67.64
WPC3	1	52.00
WPC3	2	65.07
WPC3	3	52.44
WPC3	4	25.10
WPC3	5	32.76
D1	1	68.67
D1	2	83.27
D1	3	67.02
D1	4	22.67
D1	5	67.10
D2	1	51.05
D2	2	65.01
D2	3	51.58
D2	4	23.11
D2	5	36.19

Table C.3: Optimal values for head, volume and mean level in buffers

Node	Period	H(m)	V (m^3)	H^M (m)
R1	1	60.81	500.00	17.33
R1	2	60.81	500.00	17.33
R1	3	60.81	500.00	17.33
R1	4	60.65	307.20	16.69
R1	5	63.11	500.00	16.69
R2	1	53.10	15274.27	5.91
R2	2	53.09	15691.32	5.87
R2	3	53.07	14658.65	5.79
R2	4	53.13	17658.65	6.04
R2	5	53.17	16000.00	6.21
R3	1	65.16	2638.48	23.43
R3	2	65.22	2843.58	23.65
R3	3	65.27	2957.20	23.87
R3	4	65.21	2500.00	23.64
R3	5	65.14	2500.00	23.33
R4	1	54.87	4507.10	10.25
R4	2	65.25	4193.92	7.18
R4	3	55.04	2340.66	6.63
R4	4	25.05	2000.00	6.09
R4	5	22.37	6000.00	7.00
WT1	1	61.41	445.35	43.98
WT1	2	57.30	889.34	44.45
WT1	3	62.69	188.79	43.59
WT1	4	96.24	150.00	41.13
WT1	5	79.99	750.00	43.00
WT2	1	61.97	315.22	53.66
WT2	2	61.20	445.62	53.04
WT2	3	62.30	702.47	54.59
WT2	4	97.22	125.00	53.31
WT2	5	82.22	600.00	52.90
WT3	1	69.92	750.00	44.38
WT3	2	83.83	500.00	43.13
WT3	3	64.69	641.83	42.85
WT3	4	38.14	200.00	42.10
WT3	5	58.87	1000.00	43.00
WT4	1	59.69	956.42	46.19
WT4	2	77.05	331.42	44.29
WT4	3	55.21	941.49	44.24
WT4	4	29.44	150.00	43.64
WT4	5	48.25	900.00	43.50
WT5	1	77.86	639.14	73.60
WT5	2	77.81	673.39	73.28
WT5	3	77.23	481.04	72.89
WT5	4	55.70	200.00	71.70
WT5	5	77.54	800.00	72.50

Table C.4: Optimal values for flow in pipes (part I)

Node	Node	Period	Q (m^3/h)
WPC1	J1	1	0.00
WPC1	J1	2	0.00
WPC1	J1	3	0.00
WPC1	J1	4	96.40
WPC1	J1	5	0.00
J1	WT1	1	80.09
J1	WT1	2	178.16
J1	WT1	3	0.00
J1	WT1	4	87.06
J1	WT1	5	133.79
J1	WT2	1	366.31
J1	WT2	2	313.40
J1	WT2	3	443.68
J1	WT2	4	9.34
J1	WT2	5	202.47
J2	J3	1	-736.70
J2	J3	2	-759.47
J2	J3	3	-694.24
J2	J3	4	-67.89
J2	J3	5	-819.63
WPC2	J3	1	1150.18
WPC2	J3	2	1076.33
WPC2	J3	3	1145.75
WPC2	J3	4	-183.62
WPC2	J3	5	1439.97
WPC2	J4	1	531.25
WPC2	J4	2	354.17
WPC2	J4	3	560.78
WPC2	J4	4	183.62
WPC2	J4	5	296.98
J4	J5	1	312.50
J4	J5	2	208.33
J4	J5	3	364.63
J4	J5	4	34.58
J4	J5	5	229.67
J5	J6	1	0.00
J5	J6	2	0.00
J5	J6	3	84.41
J5	J6	4	-178.33
J5	J6	5	133.52

Table C.5: Optimal values for flow in pipes (part II)

Node	Node	Period	Q (m^3/h)
J6	WT3	1	0.00
J6	WT3	2	0.00
J6	WT3	3	84.41
J6	WT3	4	-178.33
J6	WT3	5	133.52
J5	D1	1	250.00
J5	D1	2	166.67
J5	D1	3	224.18
J5	D1	4	170.33
J5	D1	5	76.92
J2	J8	1	290.31
J2	J8	2	267.90
J2	J8	3	250.56
J2	J8	4	67.89
J2	J8	5	483.36
J3	J7	1	404.11
J3	J7	2	310.61
J3	J7	3	443.10
J3	J7	4	-257.90
J3	J7	5	617.46
J7	WT4	1	170.35
J7	WT4	2	0.00
J7	WT4	3	262.12
J7	WT4	4	-289.29
J7	WT4	5	155.22
J7	R4	1	221.25
J7	R4	2	302.28
J7	R4	3	169.77
J7	R4	4	22.88
J7	R4	5	458.39
J8	D2	1	290.31
J8	D2	2	267.90
J8	D2	3	250.56
J8	D2	4	67.89
J8	D2	5	483.36
J9	R4	1	-178.44
J9	R4	2	-44.60
J9	R4	3	-169.77
J9	R4	4	-22.88
J9	R4	5	339.13

Table C.6: Optimal values for flow in pipes (part III)

Node	Node	Period	Q (m^3/h)
J9	J10	1	178.44
J9	J10	2	44.60
J9	J10	3	169.77
J9	J10	4	22.88
J9	J10	5	-339.13
WPC3	J10	1	0.00
WPC3	J10	2	0.00
WPC3	J10	3	0.00
WPC3	J10	4	228.60
WPC3	J10	5	0.00
J10	J11	1	178.44
J10	J11	2	44.60
J10	J11	3	169.77
J10	J11	4	251.48
J10	J11	5	-339.13
J11	D2	1	178.44
J11	D2	2	44.60
J11	D2	3	169.77
J11	D2	4	251.48
J11	D2	5	-339.13
J12	J13	1	50.00
J12	J13	2	33.33
J12	J13	3	44.84
J12	J13	4	34.07
J12	J13	5	15.38
J12	WT5	1	116.04
J12	WT5	2	109.87
J12	WT5	3	101.64
J12	WT5	4	-34.07
J12	WT5	5	133.79

Table C.7: Optimal values for flow in raw water pumps variables

Node	Node	Period	Q (m^3/h)
WS1	R1	1	0.00
WS1	R1	2	0.00
WS1	R1	3	0.00
WS1	R1	4	0.00
WS1	R1	5	27.54
WS3	R3	1	34.62
WS3	R3	2	34.18
WS3	R3	3	22.72
WS3	R3	4	0.00
WS3	R3	5	0.00
WS2	R2	1	1500.00
WS2	R2	2	1500.00
WS2	R2	3	1500.00
WS2	R2	4	1500.00
WS2	R2	5	1500.00

Table C.8: Optimal values for flow and head increase in regular speed pumps variables

Node	Node	Period	Q (m^3/h)	ΔH (m)
R1	WPC1	1	0.00	44.91
R1	WPC1	2	0.00	44.07
R1	WPC1	3	0.00	45.36
R1	WPC1	4	96.40	80.82
R1	WPC1	5	0.00	65.61
R3	WPC3	1	0.00	28.57
R3	WPC3	2	0.00	41.42
R3	WPC3	3	0.00	28.57
R3	WPC3	4	228.60	1.47
R3	WPC3	5	0.00	9.43
J2	J1	1	446.39	59.14
J2	J1	2	491.56	55.79
J2	J1	3	443.68	59.33
J2	J1	4	0.00	73.91
J2	J1	5	336.26	65.60
R4	J12	1	166.04	70.82
R4	J12	2	143.21	71.07
R4	J12	3	146.48	71.04
R4	J12	4	0.00	49.61
R4	J12	5	149.18	71.02
R2	WPC2	1	1681.43	64.80
R2	WPC2	2	1430.49	78.31
R2	WPC2	3	1706.53	63.29
R2	WPC2	4	0.00	17.22
R2	WPC2	5	1736.95	61.43